PRE CALC – §10-3 Notes

PRECALCULUS NOTES

10.3 Ellipses

Objectives: Use and determine the standard and general forms of the equation of an ellipse. Graph ellipses.

Warm-Up

1. Write the standard form of a circle.

$(x-h)^2 + (y-k)^2 = r^2$ center (h,k) radius r

2. Write the general form of a circle.

$x^2 + y^2 + Dx + Ey + F = 0$

3. Write $\frac{2x^2 + 2y^2 - 20x + 8y + 34}{2} = \frac{0}{2}$ in standard form.

$x^2 + y^2 - 10x + 4y + 17 = 0$

$(x^2 - 10x + \underline{25}) + (y^2 + 4y + \underline{4}) = -17 + \underline{25} + \underline{4}$

$\frac{1}{2}(-10) \to (-5)^2$ $\frac{1}{2}(4) \to (2)^2$

$(x-5)^2 + (y+2)^2 = 12$

General Form of the Equation of a ~~Circle~~ Ellipse: (standard form expanded):

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

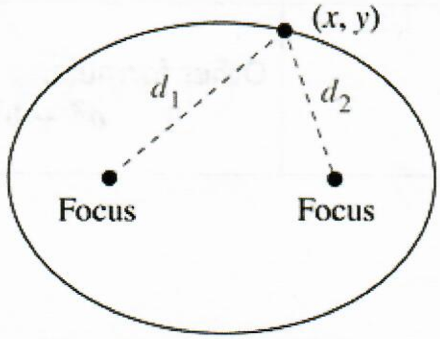
How is this different from the general form of a circle?

Bxy term

x^2 & y^2 have diff. coefficients

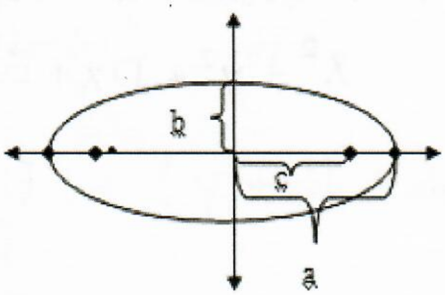
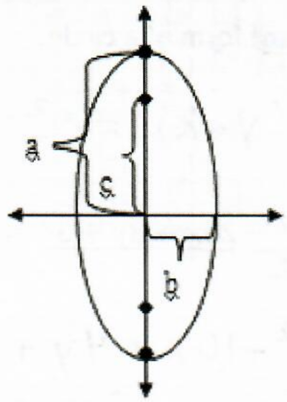
DEFINITION – ELLIPSE

An ellipse is the set of all points in a plane, the sum of whose distances from two distinct points (foci) is constant.



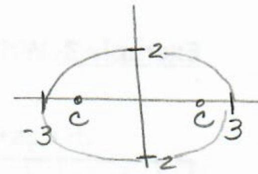
$d_1 + d_2$ is constant.

Standard Equations of Ellipses

Horizontal MAJOR axis	Vertical MAJOR axis
$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$$	$$\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$$
	
center: (h, k) foci: (h ± c, k) major axis: y = k ~~minor~~ major axis vertices: (h ± a, k) minor axis: x = h minor axis vertices: (h, k ± b)	center: (h, k) foci: (h, k ± c) major axis: x = h ~~minor~~ major axis vertices: (h, k ± a) minor axis: y = k minor axis vertices: (h ± b, k)
Other formula: $a^2-b^2=c^2$	Other formula: $a^2-b^2=c^2$

Example 1

Write an equation of an ellipse whose vertices are (-3, 0) and (3, 0) and whose minor axis length is 4. Find the foci.



$a = 3$

$b = 2$

$a = 3$

$b = 2$

center $(0,0)$

$c^2 = a^2 - b^2$

$c^2 = 3^2 - 2^2$

$c^2 = 9 - 4$

$c^2 = 5$

$c = \pm\sqrt{5}$

foci $(\pm\sqrt{5}, 0)$

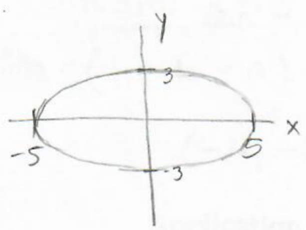
Equation (Horizontal axis)

$$\frac{(x-0)^2}{a^2} + \frac{(y-0)^2}{b^2} = 1$$

$$\frac{x^2}{9} + \frac{y^2}{4} = 1$$

Example 2

Write an equation of an ellipse whose vertices are (-5, 0) and (5, 0) and whose co-vertices are (0, -3) and (0, 3). Find the foci.



$a = 5$

$b = 3$

center $(0,0)$

$c^2 = a^2 - b^2$

$c^2 = 25 - 9$

$c^2 = 16$

$c = \pm 4$

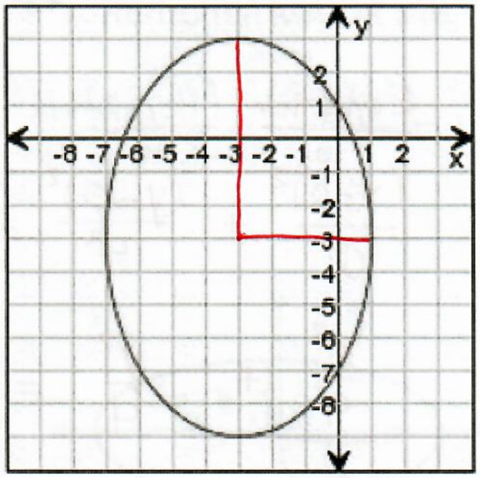
foci $(\pm 4, 0)$

Equation

$$\frac{(x-0)^2}{5^2} + \frac{(y-0)^2}{3^2} = 1$$

$$\frac{x^2}{25} + \frac{y^2}{9} = 1$$

Example 3 Write the equation of the graph.



major axis: vertical (y)

center: (-3, -3)

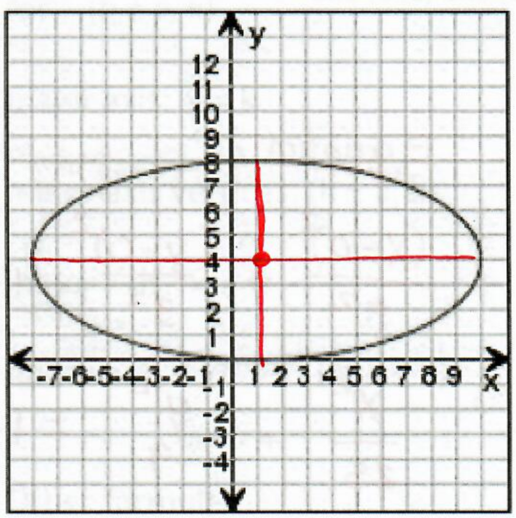
$a = 6$

$b = 4$

Equation: $\frac{(x-(-3))^2}{4^2} + \frac{(y-(-3))^2}{6^2} = 1$

$$\frac{(x+3)^2}{16} + \frac{(y+3)^2}{36} = 1$$

Example 4 Write the equation of the graph.



major axis: horizontal (x)

center: (1, 4)

$a = 9$

$b = 4$

Equation: $\frac{(x-1)^2}{9^2} + \frac{(y-4)^2}{4^2} = 1$

$$\frac{(x-1)^2}{81} + \frac{(y-4)^2}{16} = 1$$

Example 5

Determine the coordinates of the center, foci, and vertices of the ellipse, then graph it.

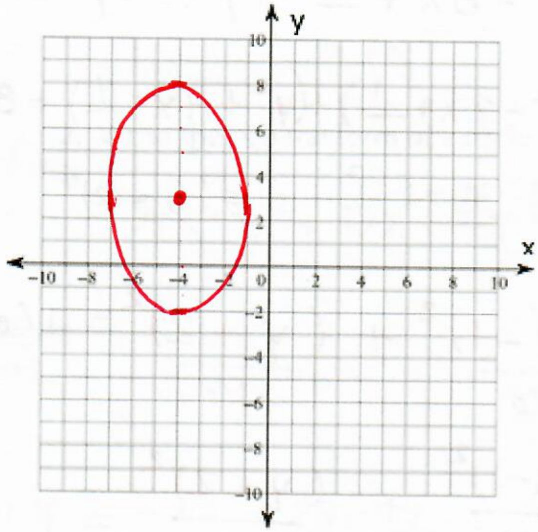
$$\underbrace{\frac{(y-3)^2}{25}}_{a^2}+\underbrace{\frac{(x+4)^2}{9}}_{b^2}=1$$

$a^2 = 25$ $\quad b^2 = 9$ $\quad c^2 = a^2 - b^2$

$a = \pm 5$ $\quad b = \pm 3$ $\quad c^2 = 25 - 9$

$c^2 = 16$

$c = \pm 4$



major axis : vertical (y)

center : $(h, k) = (-4, 3)$

foci : $(h, k \pm c) = (-4, 3 \pm 4)$

$(-4, -1)$ and $(-4, 7)$

major axis vertices

$(h, k \pm a) = (-4, 3 \pm 5)$

$(-4, -2)$ and $(-4, 8)$

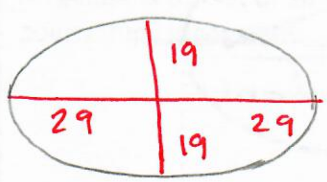
minor axis vertices

$(h \pm b, k) = (-4 \pm 3, 3)$

$(-7, 3)$ and $(-1, 3)$

Application

A skating park has a track shaped like an ellipse. If the length of the track is 58 meters and the Width of the track is 38 meters, find the equation of the ellipse.



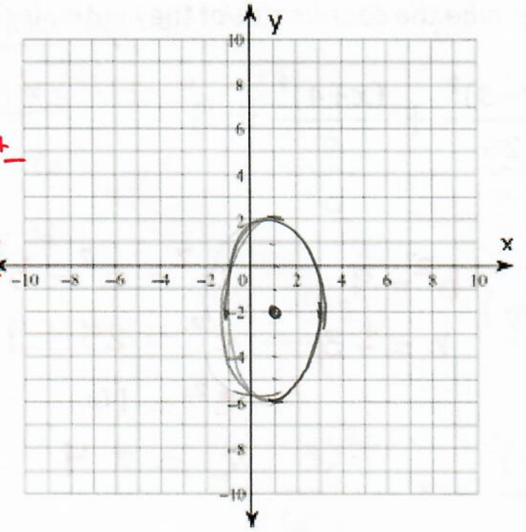
$$\frac{x^2}{29^2} + \frac{y^2}{19^2} = 1$$

$$\frac{x^2}{841} + \frac{y^2}{361} = 1$$

Example 6

Write the equation in standard form and graph the ellipse.

$$4x^2 + y^2 - 8x + 4y - 8 = 0$$



$4x^2 - 8x + \_ + y^2 + 4y + \_ = 8 + \_ + \_$

$4(x^2 - 2x + \underline{1}) + (y^2 + 4x + \underline{4}) = 8 + \underline{4} + \underline{4}$

$\frac{1}{2}(-2) \to (-1)^2$ $\qquad \frac{1}{2}(4) \to (2)^2$

$$\frac{4(x-1)^2}{16} + \frac{(y+2)^2}{16} = \frac{16}{16}$$

$$\frac{(x-1)^2}{4} + \frac{(y+2)^2}{16} = 1$$

major axis: vertical (y)

center $(1, -2)$

$a^2 = 16$ $\qquad b^2 = 4$

$a = \pm 4$ $\qquad b = \pm 2$

DEFINITION – ECCENTRICITY

To measure the ovalness of an ellipse, use the eccentricity formula $e = \frac{c}{a}$