

# PRECALC - §10-2 NOTES

## PRECALCULUS NOTES

### 10.2 Circles

**Objectives:** Use and determine the standard and general forms of the equation of a circle.  
Graph circles.

#### Standard form of the Equation of a Circle

Standard form of a circle with radius  $r$  and center at  $(h, k)$  :

$$(x - h)^2 + (y - k)^2 = r^2$$

#### General Form of the Equation of a Circle: (standard form expanded):

$$x^2 + y^2 + Dx + Ey + F = 0$$

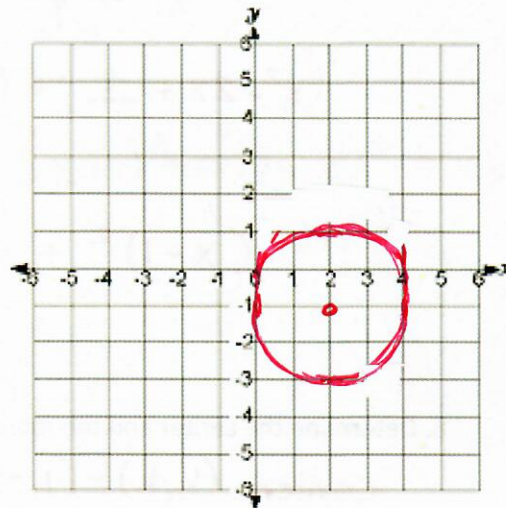
Note that  $x^2$  and  $y^2$  have a coefficient of 1. If the coefficient is not 1 then you will need to divide out the number so that the coefficients are 1.

**Example 1** Consider the equation  $(x - 2)^2 + (y + 1)^2 = 4$ .

a. Graph the equation.

center:  $(h, k) = (2, -1)$

radius  $r^2 = 4$   
 $r = 2$



b. Write the equation in general form of the equation.

$$(x - 2)^2 + (y + 1)^2 = 4$$

$$(x^2 - 4x + 4) + (y^2 + 2y + 1) = 4$$

$$x^2 + y^2 - 4x + 2y + 5 = 4$$

$$x^2 + y^2 - 4x + 2y + 1 = 0$$

**Example 2**

Write the equation  $x^2 + y^2 + 16y = 12$  in standard form. Determine the center and the radius.

$$x^2 + (y^2 + 16y + \underline{64}) = 12 + \underline{64}$$

$$\downarrow$$

$$\frac{1}{2}(16) \rightarrow (8)^2$$

$$(x-0)^2 + (y+8)^2 = 76$$

center  $(h, k) = (0, -8)$

radius  $r^2 = 76$

$$r = \sqrt{76}$$

$$r = 2\sqrt{19}$$

**Example 3**

Consider the equation  $\frac{2x^2 + 2y^2 - 4x + 12y - 18}{2} = \frac{0}{2}$ .

a. Write the equation in standard form.

$$x^2 + y^2 - 2x + 6y = 9$$

$$(x^2 - 2x + \underline{1}) + (y^2 + 6y + \underline{9}) = 9 + \underline{1} + \underline{9}$$

$$\downarrow$$

$$\frac{1}{2}(-2) \rightarrow (-1)^2 \quad \frac{1}{2}(6) \rightarrow (3)^2$$

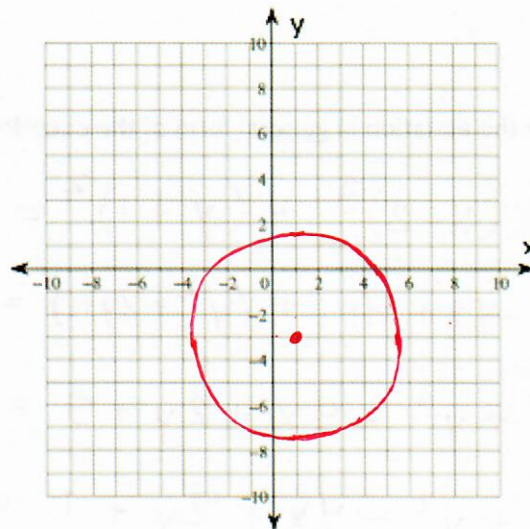
$$(x-1)^2 + (y+3)^2 = 19$$

b. Determine the center and the radius.

center  $(h, k) = (1, -3)$

radius  $r^2 = 19$   
 $r = \sqrt{19} \approx 4.4$

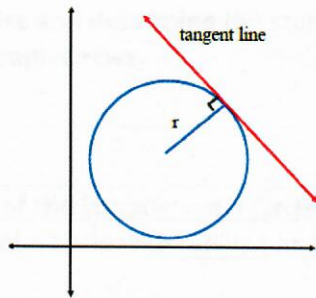
c. Graph the circle.



**Tangent Line-**

A line is tangent to a circle when it touches the circle at just one point.

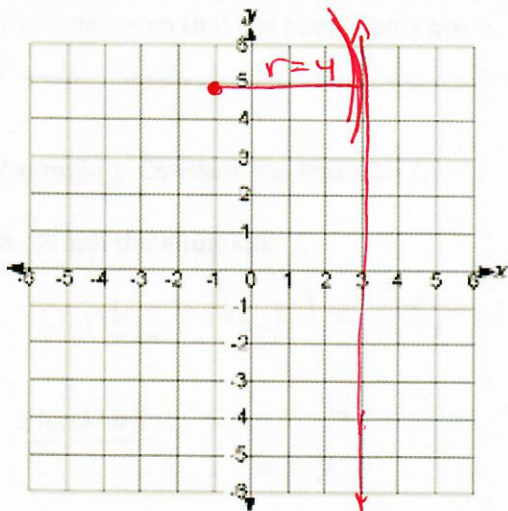
Tangent lines are perpendicular to a radius drawn to the same point of intersection.



**Example 4**

*h k*

Write an equation of a circle in standard form with center at  $(-1, 5)$  and tangent to the line  $x = 3$ .



$$(h, k) = (-1, 5)$$

$$\text{radius} = 4$$

Equation  $(x+1)^2 + (y-5)^2 = 16$