

Quadratic Functions and Inequalities

What You'll Learn

- **Lesson 6-1** Graph quadratic functions.
- **Lessons 6-2 through 6-5** Solve quadratic equations.
- **Lesson 6-3** Write quadratic equations and functions.
- **Lesson 6-6** Analyze graphs of quadratic functions.
- **Lesson 6-7** Graph and solve quadratic inequalities.

Key Vocabulary

- root (p. 294)
- zero (p. 294)
- completing the square (p. 307)
- Quadratic Formula (p. 313)
- discriminant (p. 316)

Why It's Important

Quadratic functions can be used to model real-world phenomena like the motion of a falling object. They can also be used to model the shape of architectural structures such as the supporting cables of a suspension bridge. *You will learn to calculate the value of the discriminant of a quadratic equation in order to describe the position of the supporting cables of the Golden Gate Bridge in Lesson 6-5.*

Getting Started

Prerequisite Skills To be successful in this chapter, you'll need to master these skills and be able to apply them in problem-solving situations. Review these skills before beginning Chapter 6.

For Lessons 6-1 and 6-2

Graph Functions

Graph each equation by making a table of values. (For review, see Lesson 2-1.)

1. $y = 2x + 3$ 2. $y = -x - 5$ 3. $y = x^2 + 4$ 4. $y = -x^2 - 2x + 1$

For Lessons 6-1, 6-2, and 6-5

Multiply Polynomials

Find each product. (For review, see Lesson 5-2.)

5. $(x - 4)(7x + 12)$ 6. $(x + 5)^2$ 7. $(3x - 1)^2$ 8. $(3x - 4)(2x - 9)$

For Lessons 6-3 and 6-4

Factor Polynomials

Factor completely. If the polynomial is not factorable, write *prime*. (For review, see Lesson 5-4.)

9. $x^2 + 11x + 30$ 10. $x^2 - 13x + 36$ 11. $x^2 - x - 56$ 12. $x^2 - 5x - 14$
13. $x^2 + x + 2$ 14. $x^2 + 10x + 25$ 15. $x^2 - 22x + 121$ 16. $x^2 - 9$

For Lessons 6-4 and 6-5

Simplify Radical Expressions

Simplify. (For review, see Lessons 5-6 and 5-9.)

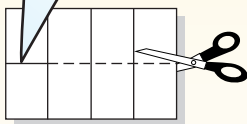
17. $\sqrt{225}$ 18. $\sqrt{48}$ 19. $\sqrt{180}$ 20. $\sqrt{68}$
21. $\sqrt{-25}$ 22. $\sqrt{-32}$ 23. $\sqrt{-270}$ 24. $\sqrt{-15}$

FOLDABLES™ Study Organizer

Make this Foldable to record information about quadratic functions and inequalities. Begin with one sheet of 11" × 17" paper.

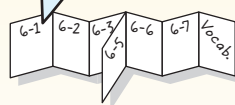
Step 1 Fold and Cut

Fold in half lengthwise. Then fold in fourths crosswise. Cut along the middle fold from the edge to the last crease as shown.



Step 2 Refold and Label

Refold along lengthwise fold and staple uncut section at top. Label the section with a lesson number and close to form a booklet.



Reading and Writing As you read and study the chapter, fill the journal with notes, diagrams, and examples for each lesson.

Vocabulary

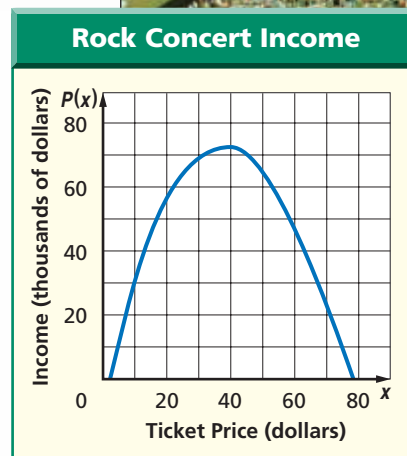
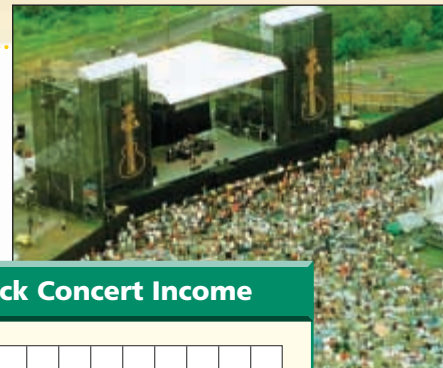
- quadratic function
- quadratic term
- linear term
- constant term
- parabola
- axis of symmetry
- vertex
- maximum value
- minimum value

What You'll Learn

- Graph quadratic functions.
- Find and interpret the maximum and minimum values of a quadratic function.

How can income from a rock concert be maximized?

Rock music managers handle publicity and other business issues for the artists they manage. One group's manager has found that based on past concerts, the predicted income for a performance is $P(x) = -50x^2 + 4000x - 7500$, where x is the price per ticket in dollars. The graph of this quadratic function is shown at the right. Notice that at first the income increases as the price per ticket increases, but as the price continues to increase, the income declines.



GRAPH QUADRATIC FUNCTIONS A **quadratic function** is described by an equation of the following form.

$$f(x) = ax^2 + bx + c, \text{ where } a \neq 0$$

linear term
quadratic term constant term

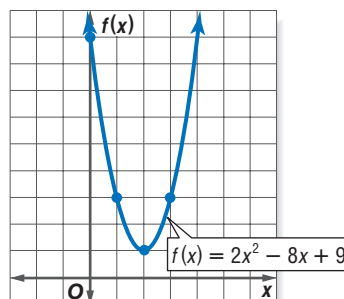
The graph of any quadratic function is called a **parabola**. One way to graph a quadratic function is to graph ordered pairs that satisfy the function.

Example 1 Graph a Quadratic Function

Graph $f(x) = 2x^2 - 8x + 9$ by making a table of values.

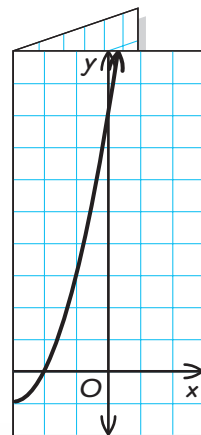
First, choose integer values for x . Then, evaluate the function for each x value. Graph the resulting coordinate pairs and connect the points with a smooth curve.

x	$2x^2 - 8x + 9$	$f(x)$	$(x, f(x))$
0	$2(0)^2 - 8(0) + 9$	9	(0, 9)
1	$2(1)^2 - 8(1) + 9$	3	(1, 3)
2	$2(2)^2 - 8(2) + 9$	1	(2, 1)
3	$2(3)^2 - 8(3) + 9$	3	(3, 3)
4	$2(4)^2 - 8(4) + 9$	9	(4, 9)



All parabolas have an **axis of symmetry**. If you were to fold a parabola along its axis of symmetry, the portions of the parabola on either side of this line would match.

The point at which the axis of symmetry intersects a parabola is called the **vertex**. The y -intercept of a quadratic function, the equation of the axis of symmetry, and the x -coordinate of the vertex are related to the equation of the function as shown below.

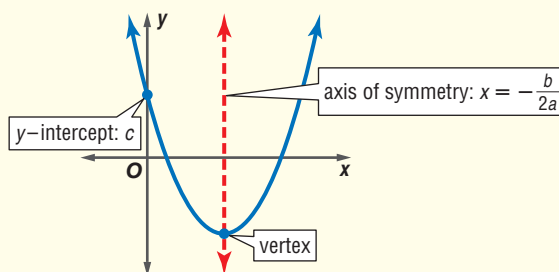


Key Concept

Graph of a Quadratic Function

- **Words** Consider the graph of $y = ax^2 + bx + c$, where $a \neq 0$.
 - The y -intercept is $a(0)^2 + b(0) + c$ or c .
 - The equation of the axis of symmetry is $x = -\frac{b}{2a}$.
 - The x -coordinate of the vertex is $-\frac{b}{2a}$.

- **Model**



Knowing the location of the axis of symmetry, y -intercept, and vertex can help you graph a quadratic function.

Example 2 Axis of Symmetry, y -Intercept, and Vertex

Consider the quadratic function $f(x) = x^2 + 9 + 8x$.

- a. Find the y -intercept, the equation of the axis of symmetry, and the x -coordinate of the vertex.

Begin by rearranging the terms of the function so that the quadratic term is first, the linear term is second, and the constant term is last. Then identify a , b , and c .

$$f(x) = ax^2 + bx + c$$

$$f(x) = x^2 + 9 + 8x \rightarrow f(x) = 1x^2 + 8x + 9$$

So, $a = 1$, $b = 8$, and $c = 9$.

The y -intercept is 9. You can find the equation of the axis of symmetry using a and b .

$$x = -\frac{b}{2a} \quad \text{Equation of the axis of symmetry}$$

$$x = -\frac{8}{2(1)} \quad a = 1, b = 8$$

$$x = -4 \quad \text{Simplify.}$$

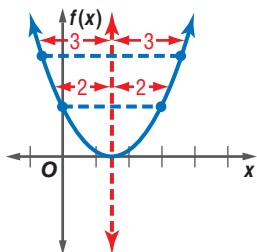
The equation of the axis of symmetry is $x = -4$. Therefore, the x -coordinate of the vertex is -4 .



Study Tip

Symmetry

Sometimes it is convenient to use symmetry to help find other points on the graph of a parabola. Each point on a parabola has a mirror image located the same distance from the axis of symmetry on the other side of the parabola.



b. Make a table of values that includes the vertex.

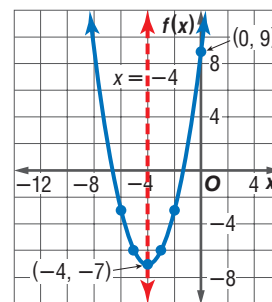
Choose some values for x that are less than -4 and some that are greater than -4 . This ensures that points on each side of the axis of symmetry are graphed.

x	$x^2 + 8x + 9$	$f(x)$	$(x, f(x))$
-6	$(-6)^2 + 8(-6) + 9$	-3	$(-6, -3)$
-5	$(-5)^2 + 8(-5) + 9$	-6	$(-5, -6)$
-4	$(-4)^2 + 8(-4) + 9$	-7	$(-4, -7)$
-3	$(-3)^2 + 8(-3) + 9$	-6	$(-3, -6)$
-2	$(-2)^2 + 8(-2) + 9$	-3	$(-2, -3)$

← Vertex

c. Use this information to graph the function.

Graph the vertex and y -intercept. Then graph the points from your table connecting them and the y -intercept with a smooth curve. As a check, draw the axis of symmetry, $x = -4$, as a dashed line. The graph of the function should be symmetrical about this line.



MAXIMUM AND MINIMUM VALUES The y -coordinate of the vertex of a quadratic function is the **maximum value** or **minimum value** obtained by the function.

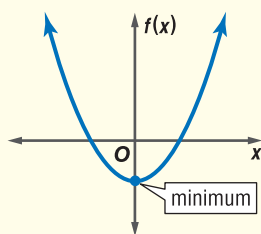
Key Concept

Maximum and Minimum Value

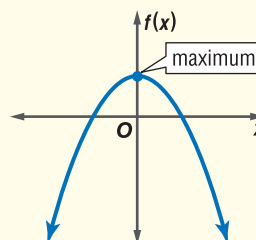
- **Words** The graph of $f(x) = ax^2 + bx + c$, where $a \neq 0$,
 - opens up and has a minimum value when $a > 0$, and
 - opens down and has a maximum value when $a < 0$.

- **Models**

a is positive.



a is negative.



Example 3 Maximum or Minimum Value

Consider the function $f(x) = x^2 - 4x + 9$.

a. Determine whether the function has a maximum or a minimum value.

For this function, $a = 1$, $b = -4$, and $c = 9$. Since $a > 0$, the graph opens up and the function has a minimum value.

Study Tip

Common Misconception

The terms *minimum point* and *minimum value* are not interchangeable. The minimum point on the graph of a quadratic function is the set of coordinates that describe the location of the vertex. The minimum value of a function is the y -coordinate of the minimum point. It is the smallest value obtained when $f(x)$ is evaluated for all values of x .

- b. State the maximum or minimum value of the function.

The minimum value of the function is the y -coordinate of the vertex.

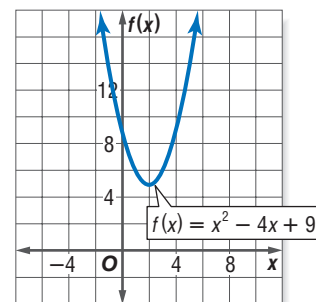
The x -coordinate of the vertex is $-\frac{-4}{2(1)}$ or 2.

Find the y -coordinate of the vertex by evaluating the function for $x = 2$.

$$f(x) = x^2 - 4x + 9 \quad \text{Original function}$$

$$f(2) = (2)^2 - 4(2) + 9 \text{ or } 5 \quad x = 2$$

Therefore, the minimum value of the function is 5.



When quadratic functions are used to model real-world situations, their maximum or minimum values can have real-world meaning.

Example 4 Find a Maximum Value

FUND-RAISING Four hundred people came to last year's winter play at Sunnybrook High School. The ticket price was \$5. This year, the Drama Club is hoping to earn enough money to take a trip to a Broadway play. They estimate that for each \$0.50 increase in the price, 10 fewer people will attend their play.

- a. How much should the tickets cost in order to maximize the income from this year's play?

Words The income is the number of tickets multiplied by the price per ticket.

Variables Let x = the number of \$0.50 price increases.
Then $5 + 0.50x$ = the price per ticket and
 $400 - 10x$ = the number of tickets sold.
Let $I(x)$ = income as a function of x .

	The income	is	the number of tickets	multiplied by	the price per ticket.
Equation	$I(x)$	=	$(400 - 10x)$	·	$(5 + 0.50x)$
					$= 400(5) + 400(0.50x) - 10x(5) - 10x(0.50x)$
					$= 2000 + 200x - 50x - 5x^2$ Multiply.
					$= 2000 + 150x - 5x^2$ Simplify.
					$= -5x^2 + 150x + 2000$ Rewrite in $ax^2 + bx + c$ form.

$I(x)$ is a quadratic function with $a = -5$, $b = 150$, and $c = 2000$. Since $a < 0$, the function has a maximum value at the vertex of the graph. Use the formula to find the x -coordinate of the vertex.

x -coordinate of the vertex =	$-\frac{b}{2a}$	Formula for the x-coordinate of the vertex
	$= -\frac{150}{2(-5)}$	$a = -5, b = 150$
	$= 15$	Simplify.

This means the Drama Club should make 15 price increases of \$0.50 to maximize their income. Thus, the ticket price should be $5 + 0.50(15)$ or \$12.50.

(continued on the next page)

More About...



Fund-Raising

The London Marathon, which has been run through the streets of London, England, annually since 1981, has historically raised more money than any other charity sports event. In 2000, this event raised an estimated £20 million (\$31.6 million U.S. dollars).

Source: Guinness World Records

b. What is the maximum income the Drama Club can expect to make?

To determine maximum income, find the maximum value of the function by evaluating $I(x)$ for $x = 15$.

$$I(x) = -5x^2 + 150x + 2000 \quad \text{Income function}$$

$$I(15) = -5(15)^2 + 150(15) + 2000 \quad x = 15$$

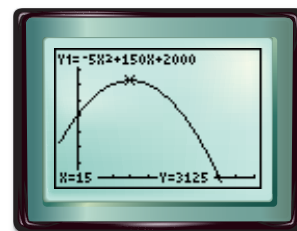
$$= 3125 \quad \text{Use a calculator.}$$

Thus, the maximum income the Drama Club can expect is \$3125.

CHECK Graph this function on a graphing calculator, and use the CALC menu to confirm this solution.

KEYSTROKES: $\boxed{2\text{nd}}$ $\boxed{[CALC]}$ $\boxed{4}$
 $\boxed{0}$ \boxed{ENTER} $\boxed{25}$ \boxed{ENTER} \boxed{ENTER}

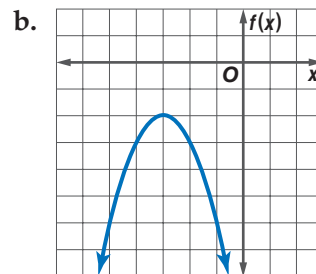
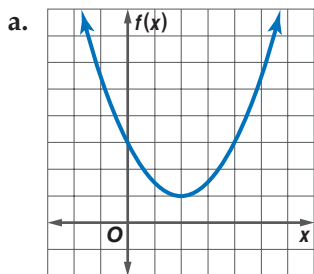
At the bottom of the display are the coordinates of the maximum point on the graph of $y = -5x^2 + 150x + 2000$. The y value of these coordinates is the maximum value of the function, or 3125. ✓



$[-5, 50]$ scl: 5 by $[-100, 4000]$ scl: 500

Check for Understanding

- Concept Check**
- OPEN ENDED** Give an example of a quadratic function. Identify its quadratic term, linear term, and constant term.
 - Identify** the vertex and the equation of the axis of symmetry for each function graphed below.



- State** whether the graph of each quadratic function opens *up* or *down*. Then state whether the function has a *maximum* or *minimum* value.
 - $f(x) = 3x^2 + 4x - 5$
 - $f(x) = -2x^2 + 9$
 - $f(x) = -5x^2 - 8x + 2$
 - $f(x) = 6x^2 - 5x$

Guided Practice Complete parts a–c for each quadratic function.

- Find the y -intercept, the equation of the axis of symmetry, and the x -coordinate of the vertex.
- Make a table of values that includes the vertex.
- Use this information to graph the function.
 - $f(x) = -4x^2$
 - $f(x) = -x^2 + 4x - 1$
 - $f(x) = 2x^2 - 4x + 1$
 - $f(x) = x^2 + 2x$
 - $f(x) = x^2 + 8x + 3$
 - $f(x) = 3x^2 + 10x$



Determine whether each function has a maximum or a minimum value. Then find the maximum or minimum value of each function.

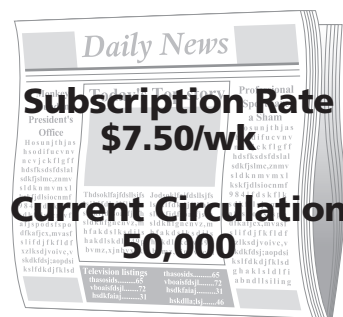
10. $f(x) = -x^2 + 7$

11. $f(x) = x^2 - x - 6$

12. $f(x) = 4x^2 + 12x + 9$

Application

13. **NEWSPAPERS** Due to increased production costs, the Daily News must increase its subscription rate. According to a recent survey, the number of subscriptions will decrease by about 1250 for each 25¢ increase in the subscription rate. What weekly subscription rate will maximize the newspaper's income from subscriptions?



Practice and Apply

Homework Help

For Exercises	See Examples
14–19	1
20–31	2
32–43, 54	3
44–53	4

Extra Practice

See page 839.

Complete parts a–c for each quadratic function.

- a. Find the y -intercept, the equation of the axis of symmetry, and the x -coordinate of the vertex.
- b. Make a table of values that includes the vertex.
- c. Use this information to graph the function.

14. $f(x) = 2x^2$

15. $f(x) = -5x^2$

16. $f(x) = x^2 + 4$

17. $f(x) = x^2 - 9$

18. $f(x) = 2x^2 - 4$

19. $f(x) = 3x^2 + 1$

20. $f(x) = x^2 - 4x + 4$

21. $f(x) = x^2 - 9x + 9$

22. $f(x) = x^2 - 4x - 5$

23. $f(x) = x^2 + 12x + 36$

24. $f(x) = 3x^2 + 6x - 1$

25. $f(x) = -2x^2 + 8x - 3$

26. $f(x) = -3x^2 - 4x$

27. $f(x) = 2x^2 + 5x$

28. $f(x) = 0.5x^2 - 1$

29. $f(x) = -0.25x^2 - 3x$

30. $f(x) = \frac{1}{2}x^2 + 3x + \frac{9}{2}$

31. $f(x) = x^2 - \frac{2}{3}x - \frac{8}{9}$

Determine whether each function has a maximum or a minimum value. Then find the maximum or minimum value of each function.

32. $f(x) = 3x^2$

33. $f(x) = -x^2 - 9$

34. $f(x) = x^2 - 8x + 2$

35. $f(x) = x^2 + 6x - 2$

36. $f(x) = 4x - x^2 + 1$

37. $f(x) = 3 - x^2 - 6x$

38. $f(x) = 2x + 2x^2 + 5$

39. $f(x) = x - 2x^2 - 1$

40. $f(x) = -7 - 3x^2 + 12x$

41. $f(x) = -20x + 5x^2 + 9$

42. $f(x) = -\frac{1}{2}x^2 - 2x + 3$

43. $f(x) = \frac{3}{4}x^2 - 5x - 2$

• **ARCHITECTURE** For Exercises 44 and 45, use the following information.

The shape of each arch supporting the Exchange House can be modeled by $h(x) = -0.025x^2 + 2x$, where $h(x)$ represents the height of the arch and x represents the horizontal distance from one end of the base in meters.

- 44. Write the equation of the axis of symmetry, and find the coordinates of the vertex of the graph of $h(x)$.
- 45. According to this model, what is the maximum height of the arch?



More About . . .

Architecture

The Exchange House in London, England, is supported by two interior and two exterior steel arches. V-shaped braces add stability to the structure.

Source: Council on Tall Buildings and Urban Habitat



PHYSICS For Exercises 46 and 47, use the following information.

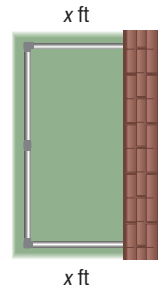
An object is fired straight up from the top of a 200-foot tower at a velocity of 80 feet per second. The height $h(t)$ of the object t seconds after firing is given by $h(t) = -16t^2 + 80t + 200$.

46. Find the maximum height reached by the object and the time that the height is reached.
47. Interpret the meaning of the y -intercept in the context of this problem.

CONSTRUCTION For Exercises 48–50, use the following information.

Steve has 120 feet of fence to make a rectangular kennel for his dogs. He will use his house as one side.

48. Write an algebraic expression for the kennel's length.
49. What dimensions produce a kennel with the greatest area?
50. Find the maximum area of the kennel.

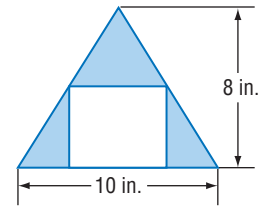


TOURISM For Exercises 51 and 52, use the following information.

A tour bus in the historic district of Savannah, Georgia, serves 300 customers a day. The charge is \$8 per person. The owner estimates that the company would lose 20 passengers a day for each \$1 fare increase.

51. What charge would give the most income for the company?
52. If the company raised their fare to this price, how much daily income should they expect to bring in?

53. **GEOMETRY** A rectangle is inscribed in an isosceles triangle as shown. Find the dimensions of the inscribed rectangle with maximum area. (*Hint*: Use similar triangles.)



54. **CRITICAL THINKING** Write an expression for the minimum value of a function of the form $y = ax^2 + c$, where $a > 0$. Explain your reasoning. Then use this function to find the minimum value of $y = 8.6x^2 - 12.5$.

55. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How can income from a rock concert be maximized?

Include the following in your answer:

- an explanation of why income increases and then declines as the ticket price increases, and
- an explanation of how to algebraically and graphically determine what ticket price should be charged to achieve maximum income.

56. The graph of which of the following equations is symmetrical about the y -axis?

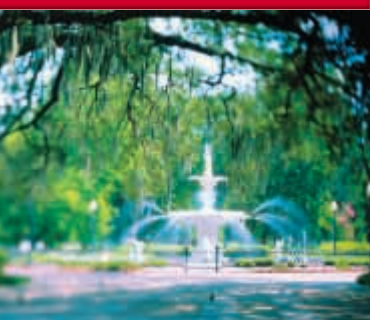
(A) $y = x^2 + 3x - 1$

(B) $y = -x^2 + x$

(C) $y = 6x^2 + 9$

(D) $y = 3x^2 - 3x + 1$

More About . . .



Tourism

Known as the Hostess City of the South, Savannah, Georgia, is a popular tourist destination. One of the first planned cities in the Americas, Savannah's Historic District is based on a grid-like pattern of streets and alleys surrounding open spaces called squares.

Source: savannah-online.com

Standardized Test Practice

(A) (B) (C) (D)

57. Which of the following tables represents a quadratic relationship between the two variables x and y ?

(A)

x	1	2	3	4	5
y	3	3	3	3	3

(B)

x	1	2	3	4	5
y	5	4	3	2	1

(C)

x	1	2	3	4	5
y	6	3	2	3	6

(D)

x	1	2	3	4	5
y	-4	-3	-4	-3	-4



Graphing Calculator

MAXIMA AND MINIMA You can use the **MINIMUM** or **MAXIMUM** feature on a graphing calculator to find the minimum or maximum value of a quadratic function. This involves defining an interval that includes the vertex of the parabola. A lower bound is an x value left of the vertex, and an upper bound is an x value right of the vertex.

Step 1 Graph the function so that the vertex of the parabola is visible.

Step 2 Select 3:minimum or 4:maximum from the **CALC** menu.

Step 3 Using the arrow keys, locate a left bound and press **ENTER**.

Step 4 Locate a right bound and press **ENTER** twice. The cursor appears on the maximum or minimum value of the function, and the coordinates are displayed.

Find the coordinates of the maximum or minimum value of each quadratic function to the nearest hundredth.

58. $f(x) = 3x^2 - 7x + 2$

59. $f(x) = -5x^2 + 8x$

60. $f(x) = 2x^2 - 3x + 2$

61. $f(x) = -6x^2 + 9x$

62. $f(x) = 7x^2 + 4x + 1$

63. $f(x) = -4x^2 + 5x$

Maintain Your Skills

Mixed Review Simplify. (Lesson 5-9)

64. i^{14}

65. $(4 - 3i) - (5 - 6i)$

66. $(7 + 2i)(1 - i)$

Solve each equation. (Lesson 5-8)

67. $5 - \sqrt{b + 2} = 0$

68. $\sqrt[3]{x + 5} + 6 = 4$

69. $\sqrt{n + 12} - \sqrt{n} = 2$

Perform the indicated operations. (Lesson 4-2)

70. $[4 \ 1 \ -3] + [6 \ -5 \ 8]$

71. $[2 \ -5 \ 7] - [-3 \ 8 \ -1]$

72. $4 \begin{bmatrix} -7 & 5 & -11 \\ 2 & -4 & 9 \end{bmatrix}$

73. $-2 \begin{bmatrix} -3 & 0 & 12 \\ -7 & \frac{1}{3} & 4 \end{bmatrix}$

74. Graph the system of equations $y = -3x$ and $y - x = 4$. State the solution. Is the system of equations *consistent* and *independent*, *consistent* and *dependent*, or *inconsistent*? (Lesson 3-1)

Getting Ready for the Next Lesson

PREREQUISITE SKILL Evaluate each function for the given value.

(To review *evaluating functions*, see Lesson 2-1.)

75. $f(x) = x^2 + 2x - 3$, $x = 2$

76. $f(x) = -x^2 - 4x + 5$, $x = -3$

77. $f(x) = 3x^2 + 7x$, $x = -2$

78. $f(x) = \frac{2}{3}x^2 + 2x - 1$, $x = -3$



Solving Quadratic Equations by Graphing

What You'll Learn

- Solve quadratic equations by graphing.
- Estimate solutions of quadratic equations by graphing.

Vocabulary

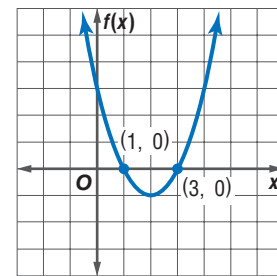
- quadratic equation
- root
- zero

How does a quadratic function model a free-fall ride?

As you speed to the top of a free-fall ride, you are pressed against your seat so that you feel like you're being pushed downward. Then as you free-fall, you fall at the same rate as your seat. Without the force of your seat pressing on you, you *feel* weightless. The height above the ground (in feet) of an object in free-fall can be determined by the quadratic function $h(t) = -16t^2 + h_0$, where t is the time in seconds and the initial height is h_0 feet.

SOLVE QUADRATIC EQUATIONS When a quadratic function is set equal to a value, the result is a quadratic equation. A **quadratic equation** can be written in the form $ax^2 + bx + c = 0$, where $a \neq 0$.

The solutions of a quadratic equation are called the **roots** of the equation. One method for finding the roots of a quadratic equation is to find the **zeros** of the related quadratic function. The zeros of the function are the x -intercepts of its graph. These are the solutions of the related equation because $f(x) = 0$ at those points. The zeros of the function graphed at the right are 1 and 3.



Study Tip

Reading Math

In general, equations have roots, functions have zeros, and graphs of functions have x -intercepts.

Example 1 Two Real Solutions

Solve $x^2 + 6x + 8 = 0$ by graphing.

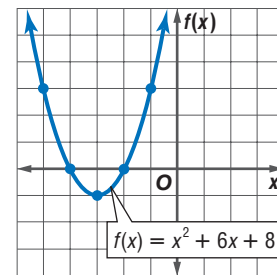
Graph the related quadratic function $f(x) = x^2 + 6x + 8$. The equation of the axis of symmetry is $x = -\frac{6}{2(1)}$ or -3 . Make a table using x values around -3 . Then, graph each point.

x	-5	-4	-3	-2	-1
$f(x)$	3	0	-1	0	3

From the table and the graph, we can see that the zeros of the function are -4 and -2 . Therefore, the solutions of the equation are -4 and -2 .

CHECK Check the solutions by substituting each solution into the equation to see if it is satisfied.

$$\begin{array}{l} x^2 + 6x + 8 = 0 \\ (-4)^2 + 6(-4) + 8 \stackrel{?}{=} 0 \\ 0 = 0 \checkmark \end{array} \qquad \begin{array}{l} x^2 + 6x + 8 = 0 \\ (-2)^2 + 6(-2) + 8 \stackrel{?}{=} 0 \\ 0 = 0 \checkmark \end{array}$$

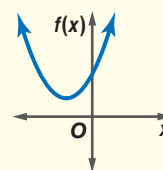
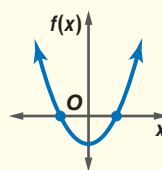
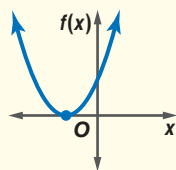


The graph of the related function in Example 1 had two zeros; therefore, the quadratic equation had two real solutions. This is one of the three possible outcomes when solving a quadratic equation.

Key Concept

Solutions of a Quadratic Equation

- **Words** A quadratic equation can have one real solution, two real solutions, or no real solution.
- **Models** One Real Solution Two Real Solutions No Real Solution



Study Tip

One Real Solution

When a quadratic equation has one real solution, it really has two solutions that are the same number.

Example 2 One Real Solution

Solve $8x - x^2 = 16$ by graphing.

Write the equation in $ax^2 + bx + c = 0$ form.

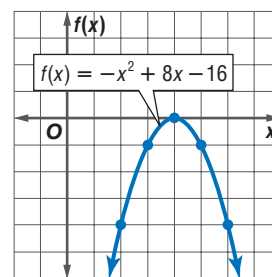
$$8x - x^2 = 16 \rightarrow -x^2 + 8x - 16 = 0 \quad \text{Subtract 16 from each side.}$$

Graph the related quadratic function

$$f(x) = -x^2 + 8x - 16.$$

x	2	3	4	5	6
$f(x)$	-4	-1	0	-1	-4

Notice that the graph has only one x -intercept, 4. Thus, the equation's only solution is 4.



Example 3 No Real Solution

NUMBER THEORY Find two real numbers whose sum is 6 and whose product is 10 or show that no such numbers exist.

Explore Let $x =$ one of the numbers. Then $6 - x =$ the other number.

Plan Since the product of the two numbers is 10, you know that $x(6 - x) = 10$.

$$x(6 - x) = 10 \quad \text{Original equation}$$

$$6x - x^2 = 10 \quad \text{Distributive Property}$$

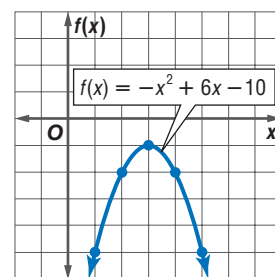
$$-x^2 + 6x - 10 = 0 \quad \text{Subtract 10 from each side.}$$

Solve You can solve $-x^2 + 6x - 10 = 0$ by graphing the related function $f(x) = -x^2 + 6x - 10$.

x	1	2	3	4	5
$f(x)$	-5	-2	-1	-2	-5

Notice that the graph has no x -intercepts. This means that the original equation has no real solution. Thus, it is *not* possible for two numbers to have a sum of 6 and a product of 10.

Examine Try finding the product of several pairs of numbers whose sum is 6. Is the product of each pair less than 10 as the graph suggests?



ESTIMATE SOLUTIONS Often exact roots cannot be found by graphing. In this case, you can estimate solutions by stating the consecutive integers between which the roots are located.

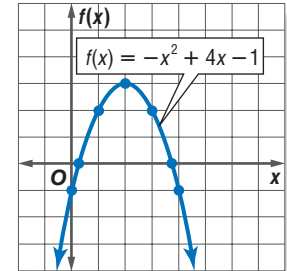
Example 4 Estimate Roots

Solve $-x^2 + 4x - 1 = 0$ by graphing. If exact roots cannot be found, state the consecutive integers between which the roots are located.

The equation of the axis of symmetry of the related function is $x = -\frac{4}{2(-1)}$ or 2.

x	0	1	2	3	4
f(x)	-1	2	3	2	-1

The x -intercepts of the graph are between 0 and 1 and between 3 and 4. So, one solution is between 0 and 1, and the other is between 3 and 4.



Study Tip

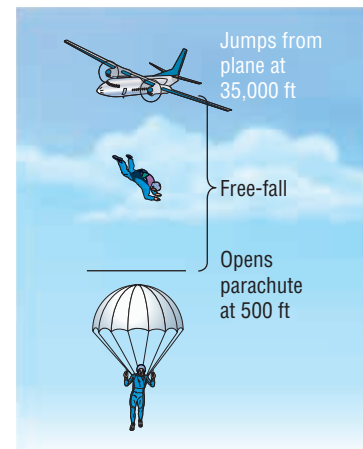
Location of Roots

Notice in the table of values that the value of the function changes from negative to positive between the x values of 0 and 1, and 3 and 4.

For many applications, an exact answer is not required, and approximate solutions are adequate. Another way to estimate the solutions of a quadratic equation is by using a graphing calculator.

Example 5 Write and Solve an Equation

EXTREME SPORTS On March 12, 1999, Adrian Nicholas broke the world record for the longest human flight. He flew 10 miles from his drop point in 4 minutes 55 seconds using a specially designed, aerodynamic suit. Using the information at the right and ignoring air resistance, how long would Mr. Nicholas have been in free-fall had he not used this special suit? Use the formula $h(t) = -16t^2 + h_0$, where the time t is in seconds and the initial height h_0 is in feet.



We need to find t when $h_0 = 35,000$ and $h(t) = 500$. Solve $500 = -16t^2 + 35,000$.

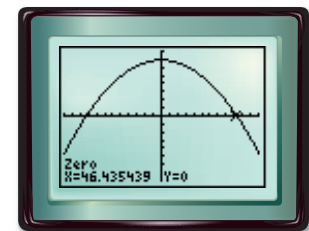
$$500 = -16t^2 + 35,000 \quad \text{Original equation}$$

$$0 = -16t^2 + 34,500 \quad \text{Subtract 500 from each side.}$$

Graph the related function $y = -16t^2 + 34,500$ using a graphing calculator. Adjust your window so that the x -intercepts of the graph are visible.

Use the ZERO feature, **2nd** [CALC], to find the positive zero of the function, since time cannot be negative. Use the arrow keys to locate a left bound for the zero and press **ENTER**.

Then, locate a right bound and press **ENTER** twice. The positive zero of the function is approximately 46.4. Mr. Nicholas would have been in free-fall for about 46 seconds.



[-60, 60] scl: 5 by
[-40000, 40000] scl: 5000

Check for Understanding

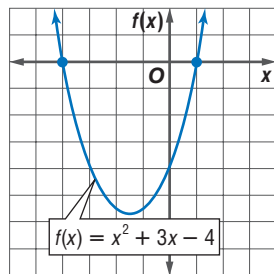
Concept Check

- Define each term and explain how they are related.
 - solution
 - root
 - zero of a function
 - x -intercept
- OPEN ENDED** Give an example of a quadratic function and state its related quadratic equation.
- Explain** how you can estimate the solutions of a quadratic equation by examining the graph of its related function.

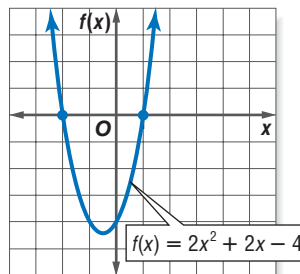
Guided Practice

Use the related graph of each equation to determine its solutions.

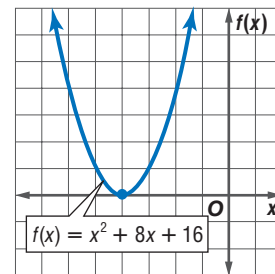
4. $x^2 + 3x - 4 = 0$



5. $2x^2 + 2x - 4 = 0$



6. $x^2 + 8x + 16 = 0$



Solve each equation by graphing. If exact roots cannot be found, state the consecutive integers between which the roots are located.

- $-x^2 - 7x = 0$
- $x^2 - 2x - 24 = 0$
- $x^2 + 3x = 28$
- $25 + x^2 + 10x = 0$
- $4x^2 - 7x - 15 = 0$
- $2x^2 - 2x - 3 = 0$

Application

- NUMBER THEORY** Use a quadratic equation to find two real numbers whose sum is 5 and whose product is -14 , or show that no such numbers exist.

Practice and Apply

Homework Help

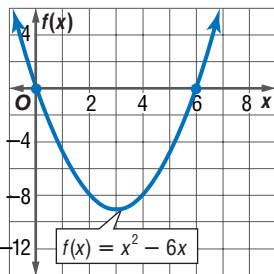
For Exercises	See Examples
14–19	1–3
20–37	1–4
38–41	3
42–46	5

Extra Practice

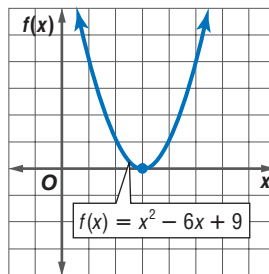
See page 840.

Use the related graph of each equation to determine its solutions.

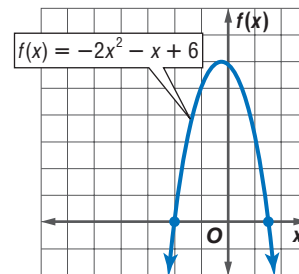
14. $x^2 - 6x = 0$



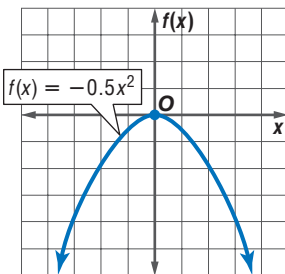
15. $x^2 - 6x + 9 = 0$



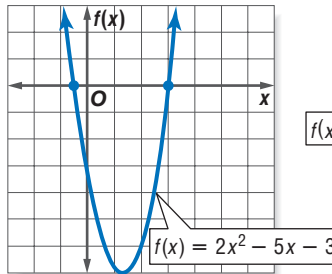
16. $-2x^2 - x + 6 = 0$



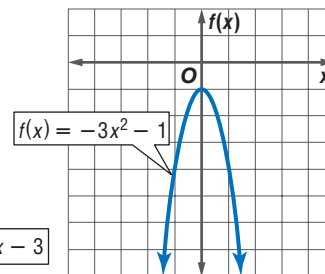
17. $-0.5x^2 = 0$



18. $2x^2 - 5x - 3 = 0$



19. $-3x^2 - 1 = 0$



Solve each equation by graphing. If exact roots cannot be found, state the consecutive integers between which the roots are located.

20. $x^2 - 3x = 0$

21. $-x^2 + 4x = 0$

22. $x^2 + 4x - 4 = 0$

23. $x^2 - 2x - 1 = 0$

24. $-x^2 + x = -20$

25. $x^2 - 9x = -18$

26. $14x + x^2 + 49 = 0$

27. $-12x + x^2 = -36$

28. $2x^2 - 3x = 9$

29. $4x^2 - 8x = 5$

30. $2x^2 = -5x + 12$

31. $2x^2 = x + 15$

32. $x^2 + 3x - 2 = 0$

33. $x^2 - 4x + 2 = 0$

34. $-2x^2 + 3x + 3 = 0$

35. $0.5x^2 - 3 = 0$

36. $x^2 + 2x + 5 = 0$

37. $-x^2 + 4x - 6 = 0$

NUMBER THEORY Use a quadratic equation to find two real numbers that satisfy each situation, or show that no such numbers exist.

38. Their sum is -17 , and their product is 72 .39. Their sum is 7 , and their product is 14 .40. Their sum is -9 , and their product is 24 .41. Their sum is 12 , and their product is -28 .

For Exercises 42–44, use the formula $h(t) = v_0t - 16t^2$ where $h(t)$ is the height of an object in feet, v_0 is the object's initial velocity in feet per second, and t is the time in seconds.

42. **ARCHERY** An arrow is shot upward with a velocity of 64 feet per second. Ignoring the height of the archer, how long after the arrow is released does it hit the ground?

43. **TENNIS** A tennis ball is hit upward with a velocity of 48 feet per second. Ignoring the height of the tennis player, how long does it take for the ball to fall to the ground?

44. **BOATING** A boat in distress launches a flare straight up with a velocity of 190 feet per second. Ignoring the height of the boat, how many seconds will it take for the flare to hit the water?

45. **LAW ENFORCEMENT** Police officers can use the length of skid marks to help determine the speed of a vehicle before the brakes were applied. If the skid marks are on dry concrete, the formula $\frac{s^2}{24} = d$ can be used. In the formula, s represents the speed in miles per hour, and d represents the length of the skid marks in feet. If the length of the skid marks on dry concrete are 50 feet, how fast was the car traveling?

46. **EMPIRE STATE BUILDING** Suppose you could conduct an experiment by dropping a small object from the Observatory of the Empire State Building. How long would it take for the object to reach the ground, assuming there is no air resistance? Use the information at the left and the formula $h(t) = -16t^2 + h_0$, where t is the time in seconds and the initial height h_0 is in feet.

47. **CRITICAL THINKING** A quadratic function has values $f(-4) = -11$, $f(-2) = 9$, and $f(0) = 5$. Between which two x values must $f(x)$ have a zero? Explain your reasoning.

More About . . .



Empire State Building

Located on the 86th floor, 1050 feet (320 meters) above the streets of New York City, the Observatory offers panoramic views from within a glass-enclosed pavilion and from the surrounding open-air promenade.

Source: www.esbnyc.com

48. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How does a quadratic function model a free-fall ride?

Include the following in your answer:

- a graph showing the height at any given time of a free-fall ride that lifts riders to a height of 185 feet, and
- an explanation of how to use this graph to estimate how long the riders would be in free-fall if the ride were allowed to hit the ground before stopping.



49. If one of the roots of the equation $x^2 + kx - 12 = 0$ is 4, what is the value of k ?
 (A) -1 (B) 0 (C) 1 (D) 3
50. For what value of x does $f(x) = x^2 + 5x + 6$ reach its minimum value?
 (A) -3 (B) $-\frac{5}{2}$ (C) -2 (D) -5

Extending the Lesson

SOLVE ABSOLUTE VALUE EQUATIONS BY GRAPHING Similar to quadratic equations, you can solve absolute value equations by graphing. Graph the related absolute value function for each equation using a graphing calculator. Then use the ZERO feature, **[2nd] [CALC]**, to find its real solutions, if any, rounded to the nearest hundredth.

51. $|x + 1| = 0$ 52. $|x| - 3 = 0$
 53. $|x - 4| - 1 = 0$ 54. $-|x + 4| + 5 = 0$
 55. $2|3x| - 8 = 0$ 56. $|2x - 3| + 1 = 0$

Maintain Your Skills

Mixed Review

Find the y -intercept, the equation of the axis of symmetry, and the x -coordinate of the vertex for each quadratic function. Then graph the function by making a table of values. (Lesson 6-1)

57. $f(x) = x^2 - 6x + 4$ 58. $f(x) = -4x^2 + 8x - 1$ 59. $f(x) = \frac{1}{4}x^2 + 3x + 4$

Simplify. (Lesson 5-9)

60. $\frac{2i}{3+i}$ 61. $\frac{4}{5-i}$ 62. $\frac{1+i}{3-2i}$

Evaluate the determinant of each matrix. (Lesson 4-3)

63. $\begin{bmatrix} 6 & 4 \\ -3 & 2 \end{bmatrix}$ 64. $\begin{bmatrix} 2 & -1 & -6 \\ 5 & 0 & 3 \\ -3 & 2 & 11 \end{bmatrix}$ 65. $\begin{bmatrix} 6 & 5 & -2 \\ -3 & 0 & 6 \\ 1 & 4 & 2 \end{bmatrix}$

66. **COMMUNITY SERVICE** A drug awareness program is being presented at a theater that seats 300 people. Proceeds will be donated to a local drug information center. If every two adults must bring at least one student, what is the maximum amount of money that can be raised? (Lesson 3-4)



Getting Ready for the Next Lesson

PREREQUISITE SKILL Factor completely. (To review factoring trinomials, see Lesson 5-4.)

67. $x^2 + 5x$ 68. $x^2 - 100$ 69. $x^2 - 11x + 28$
 70. $x^2 - 18x + 81$ 71. $3x^2 + 8x + 4$ 72. $6x^2 - 14x - 12$





Graphing Calculator Investigation

A Follow-Up of Lesson 6-2

Modeling Real-World Data

You can use a TI-83 Plus to model data points whose curve of best fit is quadratic.

FALLING WATER Water is allowed to drain from a hole made in a 2-liter bottle. The table shows the level of the water y measured in centimeters from the bottom of the bottle after x seconds. Find and graph a linear regression equation and a quadratic regression equation. Determine which equation is a better fit for the data.

Time (s)	0	20	40	60	80	100	120	140	160	180	200	220
Water level (cm)	42.6	40.7	38.9	37.2	35.8	34.3	33.3	32.3	31.5	30.8	30.4	30.1

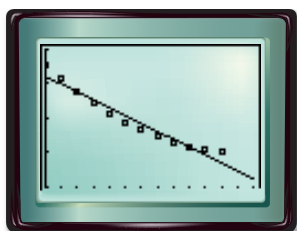
Step 1 Find a linear regression equation.

- Enter the times in L1 and the water levels in L2. Then find a linear regression equation.

KEYSTROKES: Review lists and finding a linear regression equation on page 87.

- Graph a scatter plot and the regression equation.

KEYSTROKES: Review graphing a regression equation on page 87.



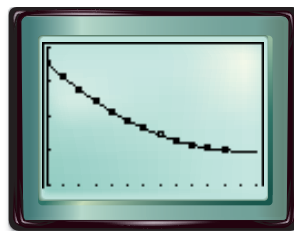
[0, 260] scl: 1 by [25, 45] scl: 5

Step 2 Find a quadratic regression equation.

- Find the quadratic regression equation. Then copy the equation to the Y= list and graph.

KEYSTROKES: STAT \blacktriangleright 5 ENTER Y=

VARS 5 \blacktriangleright \blacktriangleright ENTER GRAPH



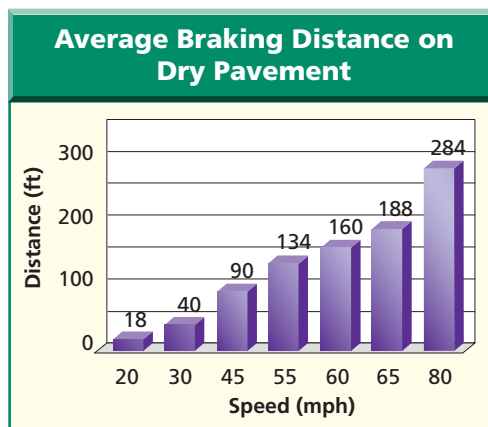
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The graph of the linear regression equation appears to pass through just two data points. However, the graph of the quadratic regression equation fits the data very well.

Exercises

For Exercises 1–4, use the graph of the braking distances for dry pavement.

- Find and graph a linear regression equation and a quadratic regression equation for the data. Determine which equation is a better fit for the data.
- Use the CALC menu with each regression equation to estimate the braking distance at speeds of 100 and 150 miles per hour.
- How do the estimates found in Exercise 2 compare?
- How might choosing a regression equation that does not fit the data well affect predictions made by using the equation?



Source: Missouri Department of Revenue

www.algebra2.com/other_calculator_keystrokes

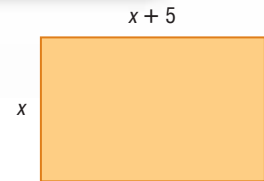
Solving Quadratic Equations by Factoring

What You'll Learn

- Solve quadratic equations by factoring.
- Write a quadratic equation with given roots.

How is the Zero Product Property used in geometry?

The length of a rectangle is 5 inches more than its width, and the area of the rectangle is 24 square inches. To find the dimensions of the rectangle you need to solve the equation $x(x + 5) = 24$ or $x^2 + 5x = 24$.



SOLVE EQUATIONS BY FACTORING In the last lesson, you learned to solve a quadratic equation like the one above by graphing. Another way to solve this equation is by factoring. Consider the following products.

$$\begin{aligned} 7(0) &= 0 \\ (6 - 6)(0) &= 0 \end{aligned}$$

$$\begin{aligned} 0(-2) &= 0 \\ -4(-5 + 5) &= 0 \end{aligned}$$

Notice that in each case, *at least one* of the factors is zero. These examples illustrate the **Zero Product Property**.

Key Concept

Zero Product Property

- **Words** For any real numbers a and b , if $ab = 0$, then either $a = 0$, $b = 0$, or both a and b equal zero.
- **Example** If $(x + 5)(x - 7) = 0$, then $x + 5 = 0$ and/or $x - 7 = 0$.

Example 1 Two Roots

Solve each equation by factoring.

a. $x^2 = 6x$

$$x^2 = 6x \quad \text{Original equation}$$

$$x^2 - 6x = 0 \quad \text{Subtract } 6x \text{ from each side.}$$

$$x(x - 6) = 0 \quad \text{Factor the binomial.}$$

$$x = 0 \quad \text{or} \quad x - 6 = 0 \quad \text{Zero Product Property}$$

$$x = 6 \quad \text{Solve the second equation.}$$

The solution set is $\{0, 6\}$.

CHECK Substitute 0 and 6 for x in the original equation.

$$\begin{array}{ll} x^2 = 6x & x^2 = 6x \\ (0)^2 \stackrel{?}{=} 6(0) & (6)^2 \stackrel{?}{=} 6(6) \\ 0 = 0 \checkmark & 36 = 36 \checkmark \end{array}$$

b. $2x^2 + 7x = 15$

$2x^2 + 7x = 15$ Original equation

$2x^2 + 7x - 15 = 0$ Subtract 15 from each side.

$(2x - 3)(x + 5) = 0$ Factor the trinomial.

$2x - 3 = 0$ or $x + 5 = 0$ Zero Product Property

$2x = 3$ $x = -5$ Solve each equation.

$x = \frac{3}{2}$

The solution set is $\left\{-5, \frac{3}{2}\right\}$. Check each solution.

Study Tip

Double Roots

The application of the Zero Product Property produced two identical equations, $x - 8 = 0$, both of which have a root of 8. For this reason, 8 is called the *double root* of the equation.

Example 2 Double Root

Solve $x^2 - 16x + 64 = 0$ by factoring.

$x^2 - 16x + 64 = 0$ Original equation

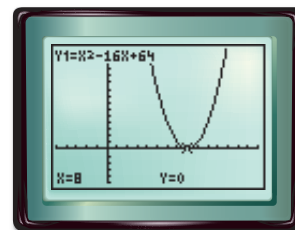
$(x - 8)(x - 8) = 0$ Factor.

$x - 8 = 0$ or $x - 8 = 0$ Zero Product Property

$x = 8$ $x = 8$ Solve each equation.

The solution set is $\{8\}$.

CHECK The graph of the related function, $f(x) = x^2 - 16x + 64$, intersects the x -axis only once. Since the zero of the function is 8, the solution of the related equation is 8.



Standardized Test Practice

A B C D

Example 3 Greatest Common Factor

Multiple-Choice Test Item

What is the positive solution of the equation $3x^2 - 3x - 60 = 0$?

(A) -4

(B) 2

(C) 5

(D) 10

Read the Test Item

You are asked to find the *positive* solution of the given quadratic equation. This implies that the equation also has a solution that is not positive. Since a quadratic equation can either have one, two, or no solutions, we should expect to find two solutions to this equation.

Solve the Test Item

Solve this equation by factoring. But before trying to factor $3x^2 - 3x - 60$ into two binomials, look for a greatest common factor. Notice that each term is divisible by 3.

$3x^2 - 3x - 60 = 0$ Original equation

$3(x^2 - x - 20) = 0$ Factor.

$x^2 - x - 20 = 0$ Divide each side by 3.

$(x + 4)(x - 5) = 0$ Factor.

$x + 4 = 0$ or $x - 5 = 0$ Zero Product Property

$x = -4$ $x = 5$ Solve each equation.

Both solutions, -4 and 5 , are listed among the answer choices. Since the question asked for the positive solution, the answer is C.

The Princeton Review

Test-Taking Tip

Because the problem asked for a *positive* solution, choice A could have been eliminated even before the expression was factored.

WRITE QUADRATIC EQUATIONS You have seen that a quadratic equation of the form $(x - p)(x - q) = 0$ has roots p and q . You can use this pattern to find a quadratic equation for a given pair of roots.

Study Tip

Writing an Equation

The pattern $(x - p)(x - q) = 0$ produces one equation with roots p and q . In fact, there are an infinite number of equations that have these same roots.

Example 4 Write an Equation Given Roots

Write a quadratic equation with $\frac{1}{2}$ and -5 as its roots. Write the equation in the form $ax^2 + bx + c = 0$, where a , b , and c are integers.

$$(x - p)(x - q) = 0 \quad \text{Write the pattern.}$$

$$\left(x - \frac{1}{2}\right)[x - (-5)] = 0 \quad \text{Replace } p \text{ with } \frac{1}{2} \text{ and } q \text{ with } -5.$$

$$\left(x - \frac{1}{2}\right)(x + 5) = 0 \quad \text{Simplify.}$$

$$x^2 + \frac{9}{2}x - \frac{5}{2} = 0 \quad \text{Use FOIL.}$$

$$2x^2 + 9x - 5 = 0 \quad \text{Multiply each side by 2 so that } b \text{ and } c \text{ are integers.}$$

A quadratic equation with roots $\frac{1}{2}$ and -5 and integral coefficients is $2x^2 + 9x - 5 = 0$. You can check this result by graphing the related function.

Check for Understanding

Concept Check

- Write the meaning of the Zero Product Property.
- OPEN ENDED** Choose two integers. Then, write an equation with those roots in the form $ax^2 + bx + c = 0$, where a , b , and c are integers.
- FIND THE ERROR** Lina and Kristin are solving $x^2 + 2x = 8$.

Lina

$$x^2 + 2x = 8$$

$$x(x + 2) = 8$$

$$x = 8 \text{ or } x + 2 = 8$$

$$x = 6$$

Kristin

$$x^2 + 2x = 8$$

$$x^2 + 2x - 8 = 0$$

$$(x + 4)(x - 2) = 0$$

$$x + 4 = 0 \text{ or } x - 2 = 0$$

$$x = -4 \quad x = 2$$

Who is correct? Explain your reasoning.

Guided Practice

Solve each equation by factoring.

4. $x^2 - 11x = 0$

5. $x^2 + 6x - 16 = 0$

6. $x^2 = 49$

7. $x^2 + 9 = 6x$

8. $4x^2 - 13x = 12$

9. $5x^2 - 5x - 60 = 0$

Write a quadratic equation with the given roots. Write the equation in the form $ax^2 + bx + c = 0$, where a , b , and c are integers.

10. $-4, 7$

11. $\frac{1}{2}, \frac{4}{3}$

12. $-\frac{3}{5}, -\frac{1}{3}$

13. Which of the following is the sum of the solutions of $x^2 - 2x - 8 = 0$?

(A) -6

(B) -4

(C) -2

(D) 2

Standardized Test Practice

(A) (B) (C) (D)

Practice and Apply

Homework Help

For Exercises	See Examples
14–33, 42–46	1, 2
34–41	4
51–52	3

Extra Practice

See page 840.

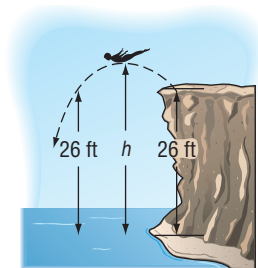
Solve each equation by factoring.

14. $x^2 + 5x - 24 = 0$
 15. $x^2 - 3x - 28 = 0$
 16. $x^2 = 25$
 17. $x^2 = 81$
 18. $x^2 + 3x = 18$
 19. $x^2 - 4x = 21$
 20. $3x^2 = 5x$
 21. $4x^2 = -3x$
 22. $x^2 + 36 = 12x$
 23. $x^2 + 64 = 16x$
 24. $4x^2 + 7x = 2$
 25. $4x^2 - 17x = -4$
 26. $4x^2 + 8x = -3$
 27. $6x^2 + 6 = -13x$
 28. $9x^2 + 30x = -16$
 29. $16x^2 - 48x = -27$
 30. $-2x^2 + 12x - 16 = 0$
 31. $-3x^2 - 6x + 9 = 0$
32. Find the roots of $x(x + 6)(x - 5) = 0$.
33. Solve $x^3 = 9x$ by factoring.

Write a quadratic equation with the given roots. Write the equation in the form $ax^2 + bx + c = 0$, where a , b , and c are integers.

- | | | | |
|----------------------|----------------------|---------------------------------|----------------------------------|
| 34. 4, 5 | 35. -2, 7 | 36. 4, -5 | 37. -6, -8 |
| 38. $\frac{1}{2}, 3$ | 39. $\frac{1}{3}, 5$ | 40. $-\frac{2}{3}, \frac{3}{4}$ | 41. $-\frac{3}{2}, -\frac{4}{5}$ |

42. **DIVING** To avoid hitting any rocks below, a cliff diver jumps up and out. The equation $h = -16t^2 + 4t + 26$ describes her height h in feet t seconds after jumping. Find the time at which she returns to a height of 26 feet.



43. **NUMBER THEORY** Find two consecutive even integers whose product is 224.

44. **PHOTOGRAPHY** A rectangular photograph is 8 centimeters wide and 12 centimeters long. The photograph is enlarged by increasing the length and width by an equal amount in order to double its area. What are the dimensions of the new photograph?

- **FORESTRY** For Exercises 45 and 46, use the following information.

Lumber companies need to be able to estimate the number of board feet that a given log will yield. One of the most commonly used formulas for estimating board feet is the *Doyle Log Rule*, $B = \frac{L}{16}(D^2 - 8D + 16)$, where B is the number of board feet, D is the diameter in inches, and L is the length of the log in feet.

45. Rewrite Doyle's formula for logs that are 16 feet long.
46. Find the root(s) of the quadratic equation you wrote in Exercise 45. What do the root(s) tell you about the kinds of logs for which Doyle's rule makes sense?
47. **CRITICAL THINKING** For a quadratic equation of the form $(x - p)(x - q) = 0$, show that the axis of symmetry of the related quadratic function is located halfway between the x -intercepts p and q .

CRITICAL THINKING Find a value of k that makes each statement true.

48. -3 is a root of $2x^2 + kx - 21 = 0$.
49. $\frac{1}{2}$ is a root of $2x^2 + 11x = -k$.

More About . . .



Forestry

A board foot is a measure of lumber volume. One piece of lumber 1 foot long by 1 foot wide by 1 inch thick measures one board foot.

Source: www.wood-worker.com

50. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How is the Zero Product Property used in geometry?

Include the following in your answer:

- an explanation of how to find the dimensions of the rectangle using the Zero Product Property, and
- why the equation $x(x + 5) = 24$ is not solved by using $x = 24$ and $x + 5 = 24$.

Standardized Test Practice

A B C D

51. Which quadratic equation has roots $\frac{1}{2}$ and $\frac{1}{3}$?
- (A) $5x^2 - 5x - 2 = 0$ (B) $5x^2 - 5x + 1 = 0$
 (C) $6x^2 + 5x - 1 = 0$ (D) $6x^2 - 5x + 1 = 0$
52. If the roots of a quadratic equation are 6 and -3 , what is the equation of the axis of symmetry?
- (A) $x = 1$ (B) $x = \frac{3}{2}$ (C) $x = \frac{1}{2}$ (D) $x = -2$

Maintain Your Skills

Mixed Review

Solve each equation by graphing. If exact roots cannot be found, state the consecutive integers between which the roots are located. (Lesson 6-2)

53. $f(x) = -x^2 - 4x + 5$ 54. $f(x) = 4x^2 + 4x + 1$ 55. $f(x) = 3x^2 - 10x - 4$

56. Determine whether $f(x) = 3x^2 - 12x - 7$ has a maximum or a minimum value. Then find the maximum or minimum value. (Lesson 6-1)

Simplify. (Lesson 5-6)

57. $\sqrt{3}(\sqrt{6} - 2)$ 58. $\sqrt{108} - \sqrt{48} + (\sqrt{3})^3$ 59. $(5 + \sqrt{8})^2$

Solve each system of equations. (Lesson 3-2)

60. $4a - 3b = -4$ 61. $2r + s = 1$ 62. $3x - 2y = -3$
 $3a - 2b = -4$ $r - s = 8$ $3x + y = 3$

Getting Ready for the Next Lesson

PREREQUISITE SKILL Simplify. (To review simplifying radicals, see Lesson 5-5.)

63. $\sqrt{8}$ 64. $\sqrt{20}$ 65. $\sqrt{27}$
 66. $\sqrt{-50}$ 67. $\sqrt{-12}$ 68. $\sqrt{-48}$

Practice Quiz 1

Lessons 6-1 through 6-3

- Find the y -intercept, the equation of the axis of symmetry, and the x -coordinate of the vertex for $f(x) = 3x^2 - 12x + 4$. Then graph the function by making a table of values. (Lesson 6-1)
- Determine whether $f(x) = 3 - x^2 + 5x$ has a maximum or minimum value. Then find this maximum or minimum value. (Lesson 6-1)
- Solve $2x^2 - 11x + 12 = 0$ by graphing. If exact roots cannot be found, state the consecutive integers between which the roots are located. (Lesson 6-2)
- Solve $2x^2 + 9x - 5 = 0$ by factoring. (Lesson 6-3)
- Write a quadratic equation with roots -4 and $\frac{1}{3}$. Write the equation in the form $ax^2 + bx + c = 0$, where a , b , and c are integers. (Lesson 6-3)

What You'll Learn

- Solve quadratic equations by using the Square Root Property.
- Solve quadratic equations by completing the square.

Vocabulary

- completing the square

How can you find the time it takes an accelerating race car to reach the finish line?

Under a yellow caution flag, race car drivers slow to a speed of 60 miles per hour. When the green flag is waved, the drivers can increase their speed.

Suppose the driver of one car is 500 feet from the finish line. If the driver accelerates at a constant rate of 8 feet per second squared, the equation $t^2 + 22t + 121 = 246$ represents the time t it takes the driver to reach this line. To solve this equation, you can use the Square Root Property.



SQUARE ROOT PROPERTY You have solved equations like $x^2 - 25 = 0$ by factoring. You can also use the **Square Root Property** to solve such an equation. This method is useful with equations like the one above that describes the race car's speed. In this case, the quadratic equation contains a perfect square trinomial set equal to a constant.

Study Tip**Reading Math**

$\pm\sqrt{n}$ is read *plus or minus the square root of n*.

Key Concept**Square Root Property**

For any real number n , if $x^2 = n$, then $x = \pm\sqrt{n}$.

Example 1 Equation with Rational Roots

Solve $x^2 + 10x + 25 = 49$ by using the Square Root Property.

$$x^2 + 10x + 25 = 49$$

Original equation

$$(x + 5)^2 = 49$$

Factor the perfect square trinomial.

$$x + 5 = \pm\sqrt{49}$$

Square Root Property

$$x + 5 = \pm 7$$

$$\sqrt{49} = 7$$

$$x = -5 \pm 7$$

Add -5 to each side.

$$x = -5 + 7 \quad \text{or} \quad x = -5 - 7$$

Write as two equations.

$$x = 2$$

$$x = -12$$

Solve each equation.

The solution set is $\{2, -12\}$. You can check this result by using factoring to solve the original equation.

Roots that are irrational numbers may be written as exact answers in radical form or as *approximate* answers in decimal form when a calculator is used.

Example 2 Equation with Irrational Roots

Solve $x^2 - 6x + 9 = 32$ by using the Square Root Property.

$$x^2 - 6x + 9 = 32$$

Original equation

$$(x - 3)^2 = 32$$

Factor the perfect square trinomial.

$$x - 3 = \pm\sqrt{32}$$

Square Root Property

$$x = 3 \pm 4\sqrt{2}$$

Add 3 to each side; $\sqrt{32} = 4\sqrt{2}$

$$x = 3 + 4\sqrt{2} \quad \text{or} \quad x = 3 - 4\sqrt{2}$$

Write as two equations.

$$x \approx 8.7$$

$$x \approx -2.7$$

Use a calculator.

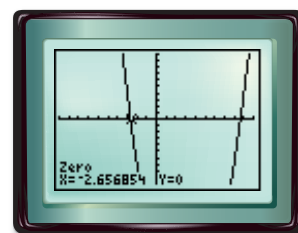
The exact solutions of this equation are $3 - 4\sqrt{2}$ and $3 + 4\sqrt{2}$. The approximate solutions are -2.7 and 8.7 . Check these results by finding and graphing the related quadratic function.

$$x^2 - 6x + 9 = 32 \quad \text{Original equation}$$

$$x^2 - 6x - 23 = 0 \quad \text{Subtract 32 from each side.}$$

$$y = x^2 - 6x - 23 \quad \text{Related quadratic function}$$

CHECK Use the ZERO function of a graphing calculator. The approximate zeros of the related function are -2.7 and 8.7 .



COMPLETE THE SQUARE The Square Root Property can only be used to solve quadratic equations when the side containing the quadratic expression is a perfect square. However, few quadratic expressions are perfect squares. To make a quadratic expression a perfect square, a method called **completing the square** may be used.

In a perfect square trinomial, there is a relationship between the coefficient of the linear term and the constant term. Consider the pattern for squaring a sum.

$$(x + 7)^2 = x^2 + 2(7)x + 7^2 \quad \text{Square of a sum pattern}$$

$$= x^2 + 14x + 49 \quad \text{Simplify.}$$

$$\begin{array}{c} \downarrow \quad \downarrow \\ \left(\frac{14}{2}\right)^2 \rightarrow 7^2 \end{array}$$

$$\left(\frac{14}{2}\right)^2 \rightarrow 7^2 \quad \text{Notice that 49 is } 7^2 \text{ and 7 is one-half of 14.}$$

You can use this pattern of coefficients to complete the square of a quadratic expression.

Key Concept

Completing the Square

- **Words** To complete the square for any quadratic expression of the form $x^2 + bx$, follow the steps below.

Step 1 Find one half of b , the coefficient of x .

Step 2 Square the result in Step 1.

Step 3 Add the result of Step 2 to $x^2 + bx$.

- **Symbols** $x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2$



Example 3 Complete the Square

Find the value of c that makes $x^2 + 12x + c$ a perfect square. Then write the trinomial as a perfect square.

- Step 1** Find one half of 12. $\frac{12}{2} = 6$
- Step 2** Square the result of Step 1. $6^2 = 36$
- Step 3** Add the result of Step 2 to $x^2 + 12x$. $x^2 + 12x + 36$

The trinomial $x^2 + 12x + 36$ can be written as $(x + 6)^2$.

You can solve any quadratic equation by completing the square. Because you are solving an equation, add the value you use to complete the square to each side.

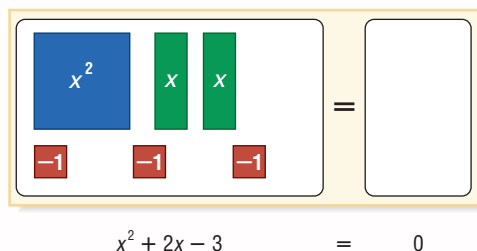


Algebra Activity

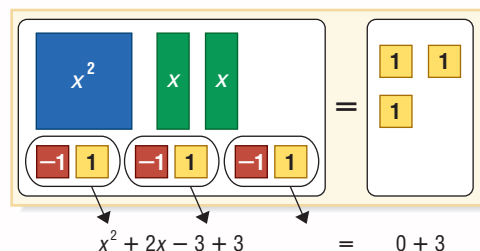
Completing the Square

Use algebra tiles to complete the square for the equation $x^2 + 2x - 3 = 0$.

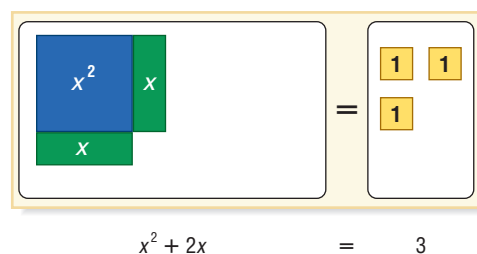
Step 1 Represent $x^2 + 2x - 3 = 0$ on an equation mat.



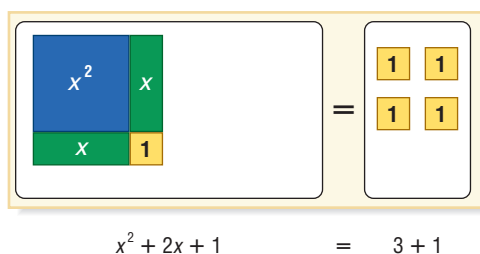
Step 2 Add 3 to each side of the mat. Remove the zero pairs.



Step 3 Begin to arrange the x^2 and x tiles into a square.



Step 4 To complete the square, add 1 yellow 1 tile to each side. The completed equation is $x^2 + 2x + 1 = 4$ or $(x + 1)^2 = 4$.



Model

Use algebra tiles to complete the square for each equation.

1. $x^2 + 2x - 4 = 0$
2. $x^2 + 4x + 1 = 0$
3. $x^2 - 6x = -5$
4. $x^2 - 2x = -1$

Study Tip

Common Misconception

When solving equations by completing the square, don't forget to add $(\frac{b}{2})^2$ to each side of the equation.

Example 4 Solve an Equation by Completing the Square

Solve $x^2 + 8x - 20 = 0$ by completing the square.

$$x^2 + 8x - 20 = 0$$

$$x^2 + 8x = 20$$

$$x^2 + 8x + 16 = 20 + 16$$

$$(x + 4)^2 = 36$$

Notice that $x^2 + 8x - 20$ is not a perfect square.

Rewrite so the left side is of the form $x^2 + bx$.

Since $(\frac{8}{2})^2 = 16$, add 16 to each side.

Write the left side as a perfect square by factoring.

$$x + 4 = \pm 6 \quad \text{Square Root Property}$$

$$x = -4 \pm 6 \quad \text{Add } -4 \text{ to each side.}$$

$$x = -4 + 6 \quad \text{or} \quad x = -4 - 6 \quad \text{Write as two equations.}$$

$$x = 2 \quad \quad \quad x = -10 \quad \text{The solution set is } \{-10, 2\}.$$

You can check this result by using factoring to solve the original equation.

When the coefficient of the quadratic term is not 1, you must first divide the equation by that coefficient before completing the square.

Example 5 Equation with $a \neq 1$

Solve $2x^2 - 5x + 3 = 0$ by completing the square.

$$2x^2 - 5x + 3 = 0 \quad \text{Notice that } 2x^2 - 5x + 3 \text{ is not a perfect square.}$$

$$x^2 - \frac{5}{2}x + \frac{3}{2} = 0 \quad \text{Divide by the coefficient of quadratic term, 2.}$$

$$x^2 - \frac{5}{2}x = -\frac{3}{2} \quad \text{Subtract } \frac{3}{2} \text{ from each side.}$$

$$x^2 - \frac{5}{2}x + \frac{25}{16} = -\frac{3}{2} + \frac{25}{16} \quad \text{Since } \left(-\frac{5}{2} \div 2\right)^2 = \frac{25}{16}, \text{ add } \frac{25}{16} \text{ to each side.}$$

$$\left(x - \frac{5}{4}\right)^2 = \frac{1}{16} \quad \text{Write the left side as a perfect square by factoring. Simplify the right side.}$$

$$x - \frac{5}{4} = \pm \frac{1}{4} \quad \text{Square Root Property}$$

$$x = \frac{5}{4} \pm \frac{1}{4} \quad \text{Add } \frac{5}{4} \text{ to each side.}$$

$$x = \frac{5}{4} + \frac{1}{4} \quad \text{or} \quad x = \frac{5}{4} - \frac{1}{4} \quad \text{Write as two equations.}$$

$$x = \frac{3}{2} \quad \quad \quad x = 1 \quad \text{The solution set is } \left\{1, \frac{3}{2}\right\}.$$

Not all solutions of quadratic equations are real numbers. In some cases, the solutions are complex numbers of the form $a + bi$, where $b \neq 0$.

Example 6 Equation with Complex Solutions

Solve $x^2 + 4x + 11 = 0$ by completing the square.

$$x^2 + 4x + 11 = 0 \quad \text{Notice that } x^2 + 4x + 11 \text{ is not a perfect square.}$$

$$x^2 + 4x = -11 \quad \text{Rewrite so the left side is of the form } x^2 + bx.$$

$$x^2 + 4x + 4 = -11 + 4 \quad \text{Since } \left(\frac{4}{2}\right)^2 = 4, \text{ add 4 to each side.}$$

$$(x + 2)^2 = -7 \quad \text{Write the left side as a perfect square by factoring.}$$

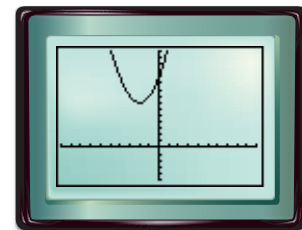
$$x + 2 = \pm \sqrt{-7} \quad \text{Square Root Property}$$

$$x + 2 = \pm i\sqrt{7} \quad \sqrt{-1} = i$$

$$x = -2 \pm i\sqrt{7} \quad \text{Subtract 2 from each side.}$$

The solution set is $\{-2 + i\sqrt{7}, -2 - i\sqrt{7}\}$. Notice that these are imaginary solutions.

CHECK A graph of the related function shows that the equation has no real solutions since the graph has no x -intercepts. Imaginary solutions must be checked algebraically by substituting them in the original equation.



Check for Understanding

Concept Check

1. Explain what it means to *complete the square*.
2. Determine whether the value of c that makes $ax^2 + bx + c$ a perfect square trinomial is *sometimes*, *always*, or *never* negative. Explain your reasoning.
3. **FIND THE ERROR** Rashid and Tia are solving $2x^2 - 8x + 10 = 0$ by completing the square.

Rashid

$$\begin{aligned}2x^2 - 8x + 10 &= 0 \\2x^2 - 8x &= -10 \\2x^2 - 8x + 16 &= -10 + 16 \\(x - 4)^2 &= 6 \\x - 4 &= \pm\sqrt{6} \\x &= 4 \pm \sqrt{6}\end{aligned}$$

Tia

$$\begin{aligned}2x^2 - 8x + 10 &= 0 \\x^2 - 4x &= 0 - 5 \\x^2 - 4x + 4 &= -5 + 4 \\(x - 2)^2 &= -1 \\x - 2 &= \pm i \\x &= 2 \pm i\end{aligned}$$

Who is correct? Explain your reasoning.

Guided Practice

Solve each equation by using the Square Root Property.

4. $x^2 + 14x + 49 = 9$
5. $9x^2 - 24x + 16 = 2$

Find the value of c that makes each trinomial a perfect square. Then write the trinomial as a perfect square.

6. $x^2 - 12x + c$
7. $x^2 - 3x + c$

Solve each equation by completing the square.

8. $x^2 + 3x - 18 = 0$
9. $x^2 - 8x + 11 = 0$
10. $x^2 + 2x + 6 = 0$
11. $2x^2 - 3x - 3 = 0$

Application

ASTRONOMY For Exercises 12 and 13, use the following information.

The height h of an object t seconds after it is dropped is given by $h = -\frac{1}{2}gt^2 + h_0$, where h_0 is the initial height and g is the acceleration due to gravity. The acceleration due to gravity near Earth's surface is 9.8 m/s^2 , while on Jupiter it is 23.1 m/s^2 . Suppose an object is dropped from an initial height of 100 meters from the surface of each planet.

12. On which planet should the object reach the ground first?
13. Find the time it takes for the object to reach the ground on each planet to the nearest tenth of a second.

Practice and Apply

Homework Help

For Exercises	See Examples
14–23, 48	1, 2
24–31	3
32–47,	4–6
49–50, 53	

Extra Practice

See page 840.

Solve each equation by using the Square Root Property.

14. $x^2 + 4x + 4 = 25$
15. $x^2 - 10x + 25 = 49$
16. $x^2 + 8x + 16 = 7$
17. $x^2 - 6x + 9 = 8$
18. $4x^2 - 28x + 49 = 5$
19. $9x^2 + 30x + 25 = 11$
20. $x^2 + x + \frac{1}{4} = \frac{9}{16}$
21. $x^2 + 1.4x + 0.49 = 0.81$

22. **MOVIE SCREENS** The area A in square feet of a projected picture on a movie screen is given by $A = 0.16d^2$, where d is the distance from the projector to the screen in feet. At what distance will the projected picture have an area of 100 square feet?

More About . . .



Engineering

Reverse ballistic testing—accelerating a target on a sled to impact a stationary test item at the end of the track—was pioneered at the Sandia National Laboratories' Rocket Sled Track Facility in Albuquerque, New Mexico. This facility provides a 10,000-foot track for testing items at very high speeds.

Source: www.sandia.gov

23. **ENGINEERING** In an engineering test, a rocket sled is propelled into a target. The sled's distance d in meters from the target is given by the formula $d = -1.5t^2 + 120$, where t is the number of seconds after rocket ignition. How many seconds have passed since rocket ignition when the sled is 10 meters from the target?

Find the value of c that makes each trinomial a perfect square. Then write the trinomial as a perfect square.

24. $x^2 + 16x + c$

26. $x^2 - 15x + c$

28. $x^2 + 0.6x + c$

30. $x^2 - \frac{8}{3}x + c$

25. $x^2 - 18x + c$

27. $x^2 + 7x + c$

29. $x^2 - 2.4x + c$

31. $x^2 + \frac{5}{2}x + c$

Solve each equation by completing the square.

32. $x^2 - 8x + 15 = 0$

34. $x^2 + 2x - 6 = 0$

36. $x^2 - 4x + 5 = 0$

38. $2x^2 + 3x - 5 = 0$

40. $3x^2 - 5x + 1 = 0$

42. $2x^2 - 7x + 12 = 0$

44. $x^2 + 1.4x = 1.2$

46. $x^2 - \frac{2}{3}x - \frac{26}{9} = 0$

33. $x^2 + 2x - 120 = 0$

35. $x^2 - 4x + 1 = 0$

37. $x^2 + 6x + 13 = 0$

39. $2x^2 - 3x + 1 = 0$

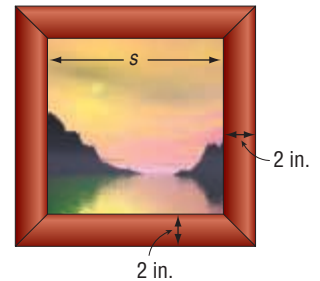
41. $3x^2 - 4x - 2 = 0$

43. $3x^2 + 5x + 4 = 0$

45. $x^2 - 4.7x = -2.8$

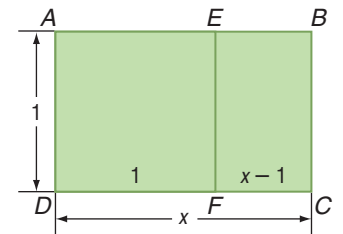
47. $x^2 - \frac{3}{2}x - \frac{23}{16} = 0$

48. **FRAMING** A picture has a square frame that is 2 inches wide. The area of the picture is one-third of the total area of the picture and frame. What are the dimensions of the picture to the nearest quarter of an inch?



GOLDEN RECTANGLE For Exercises 49–51, use the following information.

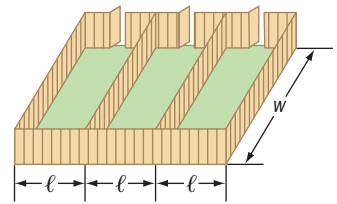
A *golden rectangle* is one that can be divided into a square and a second rectangle that is geometrically similar to the original rectangle. The ratio of the length of the longer side to the shorter side of a golden rectangle is called the *golden ratio*.



49. Find the ratio of the length of the longer side to the length of the shorter side for rectangle $ABCD$ and for rectangle $EBCF$.
50. Find the exact value of the golden ratio by setting the two ratios in Exercise 49 equal and solving for x . (*Hint*: The golden ratio is a positive value.)
51. **RESEARCH** Use the Internet or other reference to find examples of the golden rectangle in architecture. What applications does the reciprocal of the golden ratio have in music?
52. **CRITICAL THINKING** Find all values of n such that $x^2 + bx + \left(\frac{b}{2}\right)^2 = n$ has
- one real root.
 - two real roots.
 - two imaginary roots.



53. **KENNEL** A kennel owner has 164 feet of fencing with which to enclose a rectangular region. He wants to subdivide this region into three smaller rectangles of equal length, as shown. If the total area to be enclosed is 576 square feet, find the dimensions of the entire enclosed region. (*Hint*: Write an expression for ℓ in terms of w .)



54. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How can you find the time it takes an accelerating race car to reach the finish line?

Include the following in your answer:

- an explanation of why $t^2 + 22t + 121 = 246$ cannot be solved by factoring, and
- a description of the steps you would take to solve the equation $t^2 + 22t + 121 = 246$.

Standardized Test Practice

A B C D

55. What is the absolute value of the product of the two solutions for x in $x^2 - 2x - 2 = 0$?
- (A) -1 (B) 0 (C) 1 (D) 2
56. For which value of c will the roots of $x^2 + 4x + c = 0$ be real and equal?
- (A) 1 (B) 2 (C) 3 (D) 4 (E) 5

Maintain Your Skills

Mixed Review

Write a quadratic equation with the given root(s). Write the equation in the form $ax^2 + bx + c = 0$, where a , b , and c are integers. (*Lesson 6-3*)

57. 2, 1 58. -3, 9 59. $6, \frac{1}{3}$ 60. $-\frac{1}{3}, -\frac{3}{4}$

Solve each equation by graphing. If exact roots cannot be found, state the consecutive integers between which the roots are located. (*Lesson 6-2*)

61. $3x^2 = 4 - 8x$ 62. $x^2 + 48 = 14x$ 63. $2x^2 + 11x = -12$

64. Write the seventh root of 5 cubed using exponents. (*Lesson 5-7*)

Solve each system of equations by using inverse matrices. (*Lesson 4-8*)

65. $5x + 3y = -5$ 66. $6x + 5y = 8$
 $7x + 5y = -11$ $3x - y = 7$

CHEMISTRY For Exercises 67 and 68, use the following information.

For hydrogen to be a liquid, its temperature must be within 2°C of -257°C . (*Lesson 1-4*)

67. Write an equation to determine the greatest and least temperatures for this substance.
68. Solve the equation.

Getting Ready for the Next Lesson

PREREQUISITE SKILL Evaluate $b^2 - 4ac$ for the given values of a , b , and c . (*To review evaluating expressions, see Lesson 1-1.*)

69. $a = 1, b = 7, c = 3$ 70. $a = 1, b = 2, c = 5$
 71. $a = 2, b = -9, c = -5$ 72. $a = 4, b = -12, c = 9$

The Quadratic Formula and the Discriminant

Vocabulary

- Quadratic Formula
- discriminant

What You'll Learn

- Solve quadratic equations by using the Quadratic Formula.
- Use the discriminant to determine the number and type of roots of a quadratic equation.

How is blood pressure related to age?

As people age, their arteries lose their elasticity, which causes blood pressure to increase. For healthy women, average systolic blood pressure is estimated by $P = 0.01A^2 + 0.05A + 107$, where P is the average blood pressure in millimeters of mercury (mm Hg) and A is the person's age. For healthy men, average systolic blood pressure is estimated by $P = 0.006A^2 - 0.02A + 120$.



QUADRATIC FORMULA You have seen that exact solutions to some quadratic equations can be found by graphing, by factoring, or by using the Square Root Property. While completing the square can be used to solve any quadratic equation, the process can be tedious if the equation contains fractions or decimals. Fortunately, a formula exists that can be used to solve any quadratic equation of the form $ax^2 + bx + c = 0$. This formula can be derived by solving the general form of a quadratic equation.

$$ax^2 + bx + c = 0$$

General quadratic equation

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

Divide each side by a .

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

Subtract $\frac{c}{a}$ from each side.

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = -\frac{c}{a} + \frac{b^2}{4a^2}$$

Complete the square.

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

Factor the left side. Simplify the right side.

$$x + \frac{b}{2a} = \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

Square Root Property

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

Subtract $\frac{b}{2a}$ from each side.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Simplify.

This equation is known as the **Quadratic Formula**.

Study Tip

Reading Math

The Quadratic Formula is read *x equals the opposite of b, plus or minus the square root of b squared minus 4ac, all divided by 2a*.

Key Concept

Quadratic Formula

The solutions of a quadratic equation of the form $ax^2 + bx + c = 0$, where $a \neq 0$, are given by the following formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example 1 Two Rational Roots

Solve $x^2 - 12x = 28$ by using the Quadratic Formula.

First, write the equation in the form $ax^2 + bx + c = 0$ and identify a , b , and c .

$$\begin{array}{r}
 ax^2 + bx + c = 0 \\
 \downarrow \quad \downarrow \quad \downarrow \\
 x^2 - 12x = 28 \longrightarrow 1x^2 - 12x - 28 = 0
 \end{array}$$

Then, substitute these values into the Quadratic Formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{Quadratic Formula}$$

$$x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(1)(-28)}}{2(1)} \quad \text{Replace } a \text{ with } 1, b \text{ with } -12, \text{ and } c \text{ with } -28.$$

$$x = \frac{12 \pm \sqrt{144 + 112}}{2} \quad \text{Simplify.}$$

$$x = \frac{12 \pm \sqrt{256}}{2} \quad \text{Simplify.}$$

$$x = \frac{12 \pm 16}{2} \quad \sqrt{256} = 16$$

$$x = \frac{12 + 16}{2} \quad \text{or} \quad x = \frac{12 - 16}{2} \quad \text{Write as two equations.}$$

$$= 14 \quad \quad \quad = -2 \quad \quad \quad \text{Simplify.}$$

The solutions are -2 and 14 . Check by substituting each of these values into the original equation.

Study Tip

Quadratic Formula

Although factoring may be an easier method to solve the equations in Examples 1 and 2, the Quadratic Formula can be used to solve any quadratic equation.

When the value of the radicand in the Quadratic Formula is 0, the quadratic equation has exactly one rational root.

Example 2 One Rational Root

Solve $x^2 + 22x + 121 = 0$ by using the Quadratic Formula.

Identify a , b , and c . Then, substitute these values into the Quadratic Formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{Quadratic Formula}$$

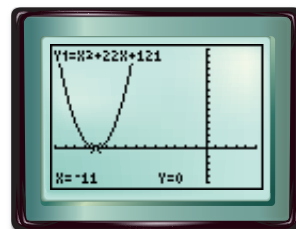
$$x = \frac{-(22) \pm \sqrt{(22)^2 - 4(1)(121)}}{2(1)} \quad \text{Replace } a \text{ with } 1, b \text{ with } 22, \text{ and } c \text{ with } 121.$$

$$x = \frac{-22 \pm \sqrt{0}}{2} \quad \text{Simplify.}$$

$$x = \frac{-22}{2} \quad \text{or} \quad -11 \quad \quad \quad \sqrt{0} = 0$$

The solution is -11 .

CHECK A graph of the related function shows that there is one solution at $x = -11$.



$[-15, 5]$ scl: 1 by $[-5, 15]$ scl: 1

You can express irrational roots exactly by writing them in radical form.

Example 3 Irrational Roots

Solve $2x^2 + 4x - 5 = 0$ by using the Quadratic Formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Quadratic Formula

$$x = \frac{-(-4) \pm \sqrt{(4)^2 - 4(2)(-5)}}{2(2)}$$

Replace a with 2, b with 4, and c with -5 .

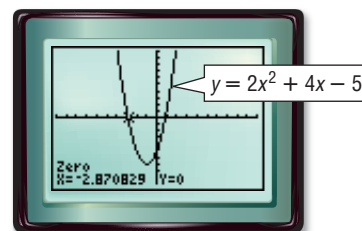
$$x = \frac{-4 \pm \sqrt{56}}{4}$$

Simplify.

$$x = \frac{-4 \pm 2\sqrt{14}}{4} \quad \text{or} \quad \frac{-2 \pm \sqrt{14}}{2} \quad \sqrt{56} = \sqrt{4 \cdot 14} \text{ or } 2\sqrt{14}$$

The exact solutions are $\frac{-2 - \sqrt{14}}{2}$ and $\frac{-2 + \sqrt{14}}{2}$. The approximate solutions are -2.9 and 0.9 .

CHECK Check these results by graphing the related quadratic function, $y = 2x^2 + 4x - 5$. Using the ZERO function of a graphing calculator, the approximate zeros of the related function are -2.9 and 0.9 .



$[-10, 10]$ scl: 1 by $[-10, 10]$ scl: 1

When using the Quadratic Formula, if the radical contains a negative value, the solutions will be complex. Complex solutions always appear in conjugate pairs.

Example 4 Complex Roots

Solve $x^2 - 4x = -13$ by using the Quadratic Formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Quadratic Formula

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(13)}}{2(1)}$$

Replace a with 1, b with -4 , and c with 13.

$$x = \frac{4 \pm \sqrt{-36}}{2}$$

Simplify.

$$x = \frac{4 \pm 6i}{2}$$

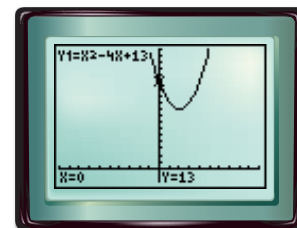
$$\sqrt{-36} = \sqrt{36(-1)} \text{ or } 6i$$

$$x = 2 \pm 3i$$

Simplify.

The solutions are the complex numbers $2 + 3i$ and $2 - 3i$.

A graph of the related function shows that the solutions are complex, but it cannot help you find them.



$[-15, 5]$ scl: 1 by $[-2, 18]$ scl: 1

Study Tip

Using the Quadratic Formula

Remember that to correctly identify a , b , and c for use in the Quadratic Formula, the equation must be written in the form $ax^2 + bx + c = 0$.



CHECK To check complex solutions, you must substitute them into the original equation. The check for $2 + 3i$ is shown below.

$$\begin{aligned} x^2 - 4x &= -13 && \text{Original equation} \\ (2 + 3i)^2 - 4(2 + 3i) &\stackrel{?}{=} -13 && x = 2 + 3i \\ 4 + 12i + 9i^2 - 8 - 12i &\stackrel{?}{=} -13 && \text{Sum of a square; Distributive Property} \\ -4 + 9i^2 &\stackrel{?}{=} -13 && \text{Simplify.} \\ -4 - 9 &= -13 \checkmark && i^2 = -1 \end{aligned}$$

Study Tip

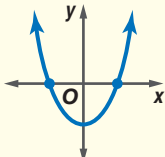
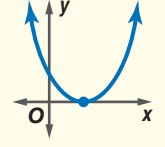
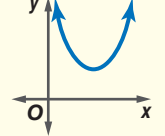
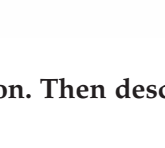
Reading Math

Remember that the solutions of an equation are called *roots*.

ROOTS AND THE DISCRIMINANT In Examples 1, 2, 3, and 4, observe the relationship between the value of the expression under the radical and the roots of the quadratic equation. The expression $b^2 - 4ac$ is called the **discriminant**.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \leftarrow \text{discriminant}$$

The value of the discriminant can be used to determine the number and type of roots of a quadratic equation.

Key Concept		Discriminant
Consider $ax^2 + bx + c = 0$.		
Value of Discriminant	Type and Number of Roots	Example of Graph of Related Function
$b^2 - 4ac > 0$; $b^2 - 4ac$ is a perfect square.	2 real, rational roots	
$b^2 - 4ac > 0$; $b^2 - 4ac$ is not a perfect square.	2 real, irrational roots	
$b^2 - 4ac = 0$	1 real, rational root	
$b^2 - 4ac < 0$	2 complex roots	

Study Tip

Using the Discriminant

The discriminant can help you check the solutions of a quadratic equation. Your solutions must match in number and in type to those determined by the discriminant.

Example 5 Describe Roots

Find the value of the discriminant for each quadratic equation. Then describe the number and type of roots for the equation.

a. $9x^2 - 12x + 4 = 0$

$$a = 9, b = -12, c = 4$$

$$\begin{aligned} b^2 - 4ac &= (-12)^2 - 4(9)(4) \\ &= 144 - 144 \\ &= 0 \end{aligned}$$

The discriminant is 0, so there is one rational root.

b. $2x^2 + 16x + 33 = 0$

$$a = 2, b = 16, c = 33$$

$$\begin{aligned} b^2 - 4ac &= (16)^2 - 4(2)(33) \\ &= 256 - 264 \\ &= -8 \end{aligned}$$

The discriminant is negative, so there are two complex roots.

c. $-5x^2 + 8x - 1 = 0$
 $a = -5, b = 8, c = -1$
 $b^2 - 4ac = (8)^2 - 4(-5)(-1)$
 $= 64 - 20$
 $= 44$

The discriminant is 44, which is not a perfect square. Therefore, there are two irrational roots.

d. $-7x + 15x^2 - 4 = 0$
 $a = 15, b = -7, c = -4$
 $b^2 - 4ac = (-7)^2 - 4(15)(-4)$
 $= 49 + 240$
 $= 289 \text{ or } 17^2$

The discriminant is 289, which is a perfect square. Therefore, there are two rational roots.

You have studied a variety of methods for solving quadratic equations. The table below summarizes these methods.

Concept Summary		Solving Quadratic Equations
Method	Can be Used	When to Use
Graphing	sometimes	Use only if an exact answer is not required. Best used to check the reasonableness of solutions found algebraically.
Factoring	sometimes	Use if the constant term is 0 or if the factors are easily determined. Example $x^2 - 3x = 0$
Square Root Property	sometimes	Use for equations in which a perfect square is equal to a constant. Example $(x + 13)^2 = 9$
Completing the Square	always	Useful for equations of the form $x^2 + bx + c = 0$, where b is even. Example $x^2 + 14x - 9 = 0$
Quadratic Formula	always	Useful when other methods fail or are too tedious. Example $3.4x^2 - 2.5x + 7.9 = 0$

Check for Understanding

- Concept Check**
- OPEN ENDED** Sketch the graph of a quadratic equation whose discriminant is
 - positive.
 - negative.
 - zero.
 - Explain** why the roots of a quadratic equation are complex if the value of the discriminant is less than 0.
 - Describe** the relationship that must exist between a , b , and c in the equation $ax^2 + bx + c = 0$ in order for the equation to have exactly one solution.

Guided Practice Complete parts a–c for each quadratic equation.

- Find the value of the discriminant.
 - Describe the number and type of roots.
 - Find the exact solutions by using the Quadratic Formula.
- $8x^2 + 18x - 5 = 0$
 - $2x^2 - 4x + 1 = 0$
 - $4x^2 + 4x + 1 = 0$
 - $x^2 + 3x + 8 = 5$



Solve each equation using the method of your choice. Find exact solutions.

8. $x^2 + 8x = 0$

9. $x^2 + 5x + 6 = 0$

10. $x^2 - 2x - 2 = 0$

11. $4x^2 + 20x + 25 = -2$

Application **PHYSICS** For Exercises 12 and 13, use the following information.

The height $h(t)$ in feet of an object t seconds after it is propelled straight up from the ground with an initial velocity of 85 feet per second is modeled by $h(t) = -16t^2 + 85t$.

12. When will the object be at a height of 50 feet?

13. Will the object ever reach a height of 120 feet? Explain your reasoning.

Practice and Apply

Homework Help

For Exercises	See Examples
14–27	1–5
28–39, 42–44	1–4
40–41	5

Extra Practice

See page 841.

Complete parts a–c for each quadratic equation.

a. Find the value of the discriminant.

b. Describe the number and type of roots.

c. Find the exact solutions by using the Quadratic Formula.

14. $x^2 + 3x - 3 = 0$

15. $x^2 - 16x + 4 = 0$

16. $x^2 - 2x + 5 = 0$

17. $x^2 - x + 6 = 0$

18. $-12x^2 + 5x + 2 = 0$

19. $-3x^2 - 5x + 2 = 0$

20. $x^2 + 4x + 3 = 4$

21. $2x - 5 = -x^2$

22. $9x^2 - 6x - 4 = -5$

23. $25 + 4x^2 = -20x$

24. $4x^2 + 7 = 9x$

25. $3x + 6 = -6x^2$

26. $\frac{3}{4}x^2 - \frac{1}{3}x - 1 = 0$

27. $0.4x^2 + x - 0.3 = 0$

Solve each equation by using the method of your choice. Find exact solutions.

28. $x^2 - 30x - 64 = 0$

29. $7x^2 + 3 = 0$

30. $x^2 - 4x + 7 = 0$

31. $2x^2 + 6x - 3 = 0$

32. $4x^2 - 8 = 0$

33. $4x^2 + 81 = 36x$

34. $-4(x + 3)^2 = 28$

35. $3x^2 - 10x = 7$

36. $x^2 + 9 = 8x$

37. $10x^2 + 3x = 0$

38. $2x^2 - 12x + 7 = 5$

39. $21 = (x - 2)^2 + 5$

BRIDGES For Exercises 40 and 41, use the following information.

The supporting cables of the Golden Gate Bridge approximate the shape of a parabola. The parabola can be modeled by the quadratic function $y = 0.00012x^2 + 6$, where x represents the distance from the axis of symmetry and y represents the height of the cables. The related quadratic equation is $0.00012x^2 + 6 = 0$.

40. Calculate the value of the discriminant.

41. What does the discriminant tell you about the supporting cables of the Golden Gate Bridge?

FOOTBALL For Exercises 42 and 43, use the following information.

The average NFL salary $A(t)$ (in thousands of dollars) from 1975 to 2000 can be estimated using the function $A(t) = 2.3t^2 - 12.4t + 73.7$, where t is the number of years since 1975.

42. Determine a domain and range for which this function makes sense.

43. According to this model, in what year did the average salary first exceed 1 million dollars?



Online Research Data Update What is the current average NFL salary? How does this average compare with the average given by the function used in Exercises 42 and 43? Visit www.algebra2.com/data_update to learn more.

More About...



Bridges

The Golden Gate, located in San Francisco, California, is the tallest bridge in the world, with its towers extending 746 feet above the water and the floor of the bridge extending 220 feet above water.

Source:

www.goldengatebridge.org

44. **HIGHWAY SAFETY** Highway safety engineers can use the formula $d = 0.05s^2 + 1.1s$ to estimate the minimum stopping distance d in feet for a vehicle traveling s miles per hour. If a car is able to stop after 125 feet, what is the fastest it could have been traveling when the driver first applied the brakes?
45. **CRITICAL THINKING** Find all values of k such that $x^2 - kx + 9 = 0$ has
 a. one real root. b. two real roots. c. no real roots.
46. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How is blood pressure related to age?

Include the following in your answer:

- an expression giving the average systolic blood pressure for a person of your age, and
- an example showing how you could determine A in either formula given a specific value of P .



47. If $2x^2 - 5x - 9 = 0$, then x could equal which of the following?
 (A) -1.12 (B) 1.54 (C) 2.63 (D) 3.71
48. Which best describes the nature of the roots of the equation $x^2 - 3x + 4 = 0$?
 (A) real and equal (B) real and unequal
 (C) complex (D) real and complex

Maintain Your Skills

Mixed Review

Solve each equation by using the Square Root Property. (Lesson 6-4)

49. $x^2 + 18x + 81 = 25$ 50. $x^2 - 8x + 16 = 7$ 51. $4x^2 - 4x + 1 = 8$

Solve each equation by factoring. (Lesson 6-3)

52. $4x^2 + 8x = 0$ 53. $x^2 - 5x = 14$ 54. $3x^2 + 10 = 17x$

Simplify. (Lesson 5-5)

55. $\sqrt{a^8b^{20}}$ 56. $\sqrt{100p^{12}q^2}$ 57. $\sqrt[3]{64b^6c^6}$

58. **ANIMALS** The fastest-recorded physical action of any living thing is the wing beat of the common midge. This tiny insect normally beats its wings at a rate of 133,000 times per minute. At this rate, how many times would the midge beat its wings in an hour? Write your answer in scientific notation. (Lesson 5-1)

Solve each system of inequalities. (Lesson 3-3)

59. $x + y \leq 9$
 $x - y \leq 3$
 $y - x \geq 4$

60. $x \geq 1$
 $y \leq -1$
 $y \leq x$

Getting Ready for the Next Lesson

PREREQUISITE SKILL State whether each trinomial is a perfect square. If it is, factor it. (To review perfect square trinomials, see Lesson 5-4.)

61. $x^2 - 5x - 10$ 62. $x^2 - 14x + 49$
 63. $4x^2 + 12x + 9$ 64. $25x^2 + 20x + 4$
 65. $9x^2 - 12x + 16$ 66. $36x^2 - 60x + 25$





Graphing Calculator

A Preview of Lesson 6-6

Families of Parabolas

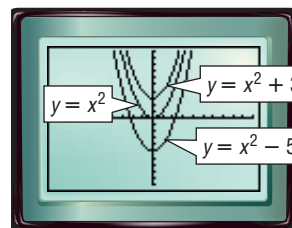
The general form of a quadratic equation is $y = a(x - h)^2 + k$. Changing the values of a , h , and k results in a different parabola in the family of quadratic functions. You can use a TI-83 Plus graphing calculator to analyze the effects that result from changing each of these parameters.

Example 1

Graph each set of equations on the same screen in the standard viewing window. Describe any similarities and differences among the graphs.

$$y = x^2, y = x^2 + 3, y = x^2 - 5$$

The graphs have the same shape, and all open up. The vertex of each graph is on the y -axis. However, the graphs have different vertical positions.



Example 1 shows how changing the value of k in the equation $y = a(x - h)^2 + k$ translates the parabola along the y -axis. If $k > 0$, the parabola is translated k units up, and if $k < 0$, it is translated k units down.

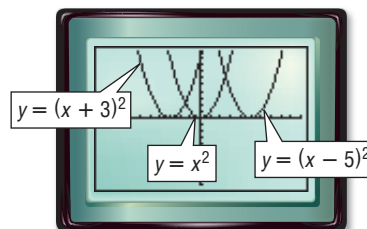
How do you think changing the value of h will affect the graph of $y = x^2$?

Example 2

Graph each set of equations on the same screen in the standard viewing window. Describe any similarities and differences among the graphs.

$$y = x^2, y = (x + 3)^2, y = (x - 5)^2$$

These three graphs all open up and have the same shape. The vertex of each graph is on the x -axis. However, the graphs have different horizontal positions.



Example 2 shows how changing the value of h in the equation $y = a(x - h)^2 + k$ translates the graph horizontally. If $h > 0$, the graph translates to the right h units. If $h < 0$, the graph translates to the left h units.



www.algebra2.com/other_calculator_keystrokes

Investigation

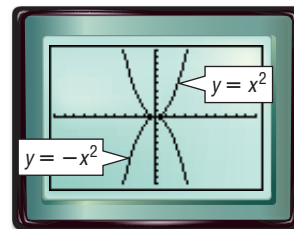
How does the value a affect the graph of $y = x^2$?

Example 3

Graph each set of equations on the same screen in the standard viewing window. Describe any similarities and differences among the graphs.

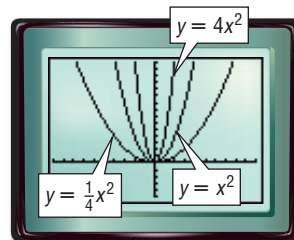
a. $y = x^2$, $y = -x^2$

The graphs have the same vertex and the same shape. However, the graph of $y = x^2$ opens up and the graph of $y = -x^2$ opens down.



b. $y = x^2$, $y = 4x^2$, $y = \frac{1}{4}x^2$

The graphs have the same vertex, $(0, 0)$, but each has a different shape. The graph of $y = 4x^2$ is narrower than the graph of $y = x^2$. The graph of $y = \frac{1}{4}x^2$ is wider than the graph of $y = x^2$.



$[-10, 10]$ scl: 1 by $[-5, 15]$ scl: 1

Changing the value of a in the equation $y = a(x - h)^2 + k$ can affect the direction of the opening and the shape of the graph. If $a > 0$, the graph opens up, and if $a < 0$, the graph opens down or is *reflected* over the x -axis. If $|a| > 1$, the graph is narrower than the graph of $y = x^2$. If $|a| < 1$, the graph is wider than the graph of $y = x^2$. Thus, a change in the absolute value of a results in a *dilation* of the graph of $y = x^2$.

Exercises

Consider $y = a(x - h)^2 - k$.

1. How does changing the value of h affect the graph? Give an example.
2. How does changing the value of k affect the graph? Give an example.
3. How does using $-a$ instead of a affect the graph? Give an example.

Examine each pair of equations and predict the similarities and differences in their graphs. Use a graphing calculator to confirm your predictions. Write a sentence or two comparing the two graphs.

- | | |
|--|--|
| 4. $y = x^2$, $y = x^2 + 2.5$ | 5. $y = -x^2$, $y = x^2 - 9$ |
| 6. $y = x^2$, $y = 3x^2$ | 7. $y = x^2$, $y = -6x^2$ |
| 8. $y = x^2$, $y = (x + 3)^2$ | 9. $y = -\frac{1}{3}x^2$, $y = -\frac{1}{3}x^2 + 2$ |
| 10. $y = x^2$, $y = (x - 7)^2$ | 11. $y = x^2$, $y = 3(x + 4)^2 - 7$ |
| 12. $y = x^2$, $y = -\frac{1}{4}x^2 + 1$ | 13. $y = (x + 3)^2 - 2$, $y = (x + 3)^2 + 5$ |
| 14. $y = 3(x + 2)^2 - 1$,
$y = 6(x + 2)^2 - 1$ | 15. $y = 4(x - 2)^2 - 3$,
$y = \frac{1}{4}(x - 2)^2 - 1$ |

Analyzing Graphs of Quadratic Functions

What You'll Learn

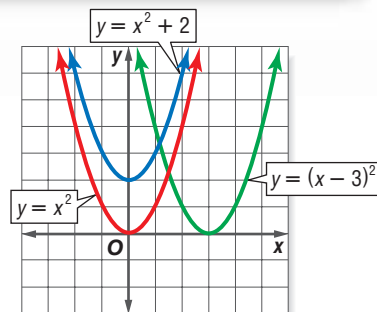
- Analyze quadratic functions of the form $y = a(x - h)^2 + k$.
- Write a quadratic function in the form $y = a(x - h)^2 + k$.

Vocabulary

- vertex form

How can the graph of $y = x^2$ be used to graph any quadratic function?

A *family of graphs* is a group of graphs that displays one or more similar characteristics. The graph of $y = x^2$ is called the *parent graph* of the family of quadratic functions. Study the graphs of $y = x^2$, $y = x^2 + 2$, and $y = (x - 3)^2$. Notice that adding a constant to x^2 moves the graph up. Subtracting a constant from x before squaring it moves the graph to the right.



ANALYZE QUADRATIC FUNCTIONS

Notice that each function above can be written in the form $y = (x - h)^2 + k$, where (h, k) is the vertex of the parabola and $x = h$ is its axis of symmetry. This is often referred to as the **vertex form** of a quadratic function.

In Chapter 4, you learned that a translation slides a figure on the coordinate plane without changing its shape or size. As the values of h and k change, the graph of $y = a(x - h)^2 + k$ is the graph of $y = x^2$ translated

- $|h|$ units *left* if h is negative or $|h|$ units *right* if h is positive, and
- $|k|$ units *up* if k is positive or $|k|$ units *down* if k is negative.

Equation	Vertex	Axis of Symmetry
$y = x^2$ or $y = (x - 0)^2 + 0$	$(0, 0)$	$x = 0$
$y = x^2 + 2$ or $y = (x - 0)^2 + 2$	$(0, 2)$	$x = 0$
$y = (x - 3)^2$ or $y = (x - 3)^2 + 0$	$(3, 0)$	$x = 3$

Example 1 Graph a Quadratic Function in Vertex Form

Analyze $y = (x + 2)^2 + 1$. Then draw its graph.

This function can be rewritten as $y = [x - (-2)]^2 + 1$. Then $h = -2$ and $k = 1$.

The vertex is at (h, k) or $(-2, 1)$, and the axis of symmetry is $x = -2$. The graph has the same shape as the graph of $y = x^2$, but is translated 2 units left and 1 unit up.

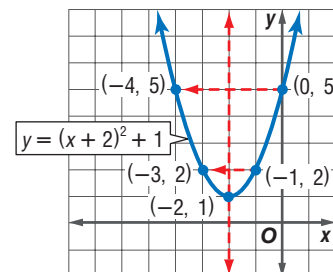
Now use this information to draw the graph.

Step 1 Plot the vertex, $(-2, 1)$.

Step 2 Draw the axis of symmetry, $x = -2$.

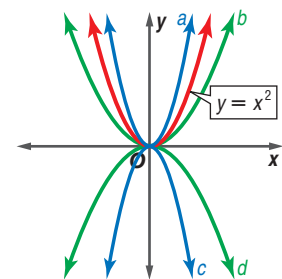
Step 3 Find and plot two points on one side of the axis of symmetry, such as $(-1, 2)$ and $(0, 5)$.

Step 4 Use symmetry to complete the graph.



How does the value of a in the general form $y = a(x - h)^2 + k$ affect a parabola? Compare the graphs of the following functions to the parent function, $y = x^2$.

- a. $y = 2x^2$ b. $y = \frac{1}{2}x^2$
 c. $y = -2x^2$ d. $y = -\frac{1}{2}x^2$



All of the graphs have the vertex $(0, 0)$ and axis of symmetry $x = 0$.

Notice that the graphs of $y = 2x^2$ and $y = \frac{1}{2}x^2$ are *dilations* of the graph of $y = x^2$. The graph of $y = 2x^2$ is narrower than the graph of $y = x^2$, while the graph of $y = \frac{1}{2}x^2$ is wider. The graphs of $y = -2x^2$ and $y = 2x^2$ are *reflections* of each other over the x -axis, as are the graphs of $y = -\frac{1}{2}x^2$ and $y = \frac{1}{2}x^2$.

Changing the value of a in the equation $y = a(x - h)^2 + k$ can affect the direction of the opening and the shape of the graph.

- If $a > 0$, the graph opens up.
- If $a < 0$, the graph opens down.
- If $|a| > 1$, the graph is narrower than the graph of $y = x^2$.
- If $|a| < 1$, the graph is wider than the graph of $y = x^2$.

Study Tip

Reading Math

$|a| < 1$ means that a is a rational number between 0 and 1, such as $\frac{2}{5}$, or a rational number between -1 and 0, such as -0.3 .

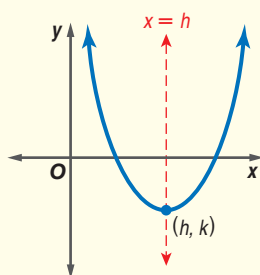
Concept Summary

Quadratic Functions in Vertex Form

The vertex form of a quadratic function is $y = a(x - h)^2 + k$.

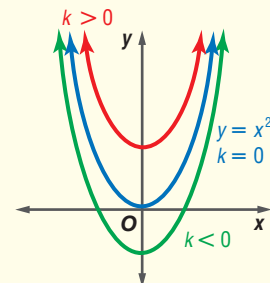
h and k

Vertex and Axis of Symmetry



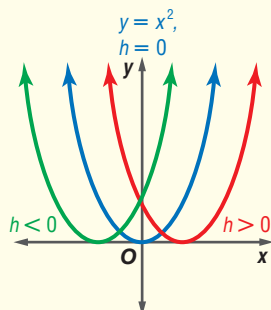
k

Vertical Translation



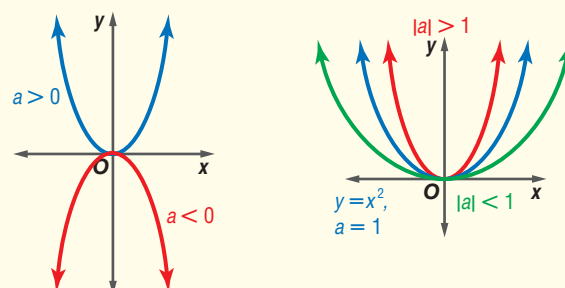
h

Horizontal Translation



a

Direction of Opening and Shape of Parabola



WRITE QUADRATIC FUNCTIONS IN VERTEX FORM Given a function of the form $y = ax^2 + bx + c$, you can complete the square to write the function in vertex form.

Example 2 Write $y = x^2 + bx + c$ in Vertex Form

Write $y = x^2 + 8x - 5$ in vertex form. Then analyze the function.

$y = x^2 + 8x - 5$ Notice that $x^2 + 8x - 5$ is not a perfect square.

$y = (x^2 + 8x + 16) - 5 - 16$ Complete the square by adding $(\frac{8}{2})^2$ or 16.
Balance this addition by subtracting 16.

$y = (x + 4)^2 - 21$ Write $x^2 + 8x + 16$ as a perfect square.

This function can be rewritten as $y = [x - (-4)]^2 + (-21)$. Written in this way, you can see that $h = -4$ and $k = -21$.

The vertex is at $(-4, -21)$, and the axis of symmetry is $x = -4$. Since $a = 1$, the graph opens up and has the same shape as the graph of $y = x^2$, but it is translated 4 units left and 21 units down.

CHECK You can check the vertex and axis of symmetry using the formula $x = -\frac{b}{2a}$. In the original equation, $a = 1$ and $b = 8$, so the axis of symmetry is $x = -\frac{8}{2(1)}$ or -4 . Thus, the x -coordinate of the vertex is -4 , and the y -coordinate of the vertex is $y = (-4)^2 + 8(-4) - 5$ or -21 .

Study Tip

Check

As an additional check, graph the function in Example 2 to verify the location of its vertex and axis of symmetry.

When writing a quadratic function in which the coefficient of the quadratic term is not 1 in vertex form, the first step is to factor out that coefficient from the quadratic and linear terms. Then you can complete the square and write in vertex form.

Example 3 Write $y = ax^2 + bx + c$ in Vertex Form, $a \neq 1$

Write $y = -3x^2 + 6x - 1$ in vertex form. Then analyze and graph the function.

$y = -3x^2 + 6x - 1$ Original equation

$y = -3(x^2 - 2x) - 1$ Group $ax^2 + bx$ and factor, dividing by a .

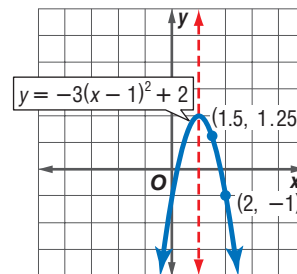
$y = -3(x - 2x + 1) - 1 - (-3)(1)$ Complete the square by adding 1 inside the parentheses.
Notice that this is an overall addition of $-3(1)$. Balance this addition by subtracting $-3(1)$.

$y = -3(x - 1)^2 + 2$ Write $x^2 - 2x + 1$ as a perfect square.

The vertex form of this function is $y = -3(x - 1)^2 + 2$. So, $h = 1$ and $k = 2$.

The vertex is at $(1, 2)$, and the axis of symmetry is $x = 1$. Since $a = -3$, the graph opens downward and is narrower than the graph of $y = x^2$. It is also translated 1 unit right and 2 units up.

Now graph the function. Two points on the graph to the right of $x = 1$ are $(1.5, 1.25)$ and $(2, -1)$. Use symmetry to complete the graph.



If the vertex and one other point on the graph of a parabola are known, you can write the equation of the parabola in vertex form.

Example 4 Write an Equation Given Points

Write an equation for the parabola whose vertex is at $(-1, 4)$ and passes through $(2, 1)$.

The vertex of the parabola is at $(-1, 4)$, so $h = -1$ and $k = 4$. Since $(2, 1)$ is a point on the graph of the parabola, let $x = 2$ and $y = 1$. Substitute these values into the vertex form of the equation and solve for a .

$$y = a(x - h)^2 + k \quad \text{Vertex form}$$

$$1 = a[2 - (-1)]^2 + 4 \quad \text{Substitute 1 for } y, 2 \text{ for } x, -1 \text{ for } h, \text{ and } 4 \text{ for } k.$$

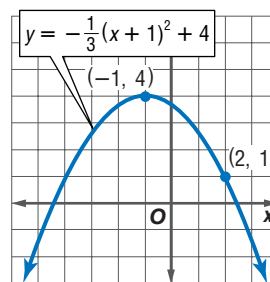
$$1 = a(9) + 4 \quad \text{Simplify.}$$

$$-3 = 9a \quad \text{Subtract 4 from each side.}$$

$$-\frac{1}{3} = a \quad \text{Divide each side by 9.}$$

The equation of the parabola in vertex form is $y = -\frac{1}{3}(x + 1)^2 + 4$.

CHECK A graph of $y = -\frac{1}{3}(x + 1)^2 + 4$ verifies that the parabola passes through the point at $(2, 1)$.



Check for Understanding

Concept Check

- Write a quadratic equation that transforms the graph of $y = 2(x + 1)^2 + 3$ so that it is:
 - 2 units up.
 - 3 units down.
 - 2 units to the left.
 - 3 units to the right.
 - narrower.
 - wider.
 - opening in the opposite direction.
- Explain how you can find an equation of a parabola using its vertex and one other point on its graph.
- OPEN ENDED** Write the equation of a parabola with a vertex of $(2, -1)$.
- FIND THE ERROR** Jenny and Ruben are writing $y = x^2 - 2x + 5$ in vertex form.

Jenny

$$\begin{aligned} y &= x^2 - 2x + 5 \\ y &= (x^2 - 2x + 1) + 5 - 1 \\ y &= (x - 1)^2 + 4 \end{aligned}$$

Ruben

$$\begin{aligned} y &= x^2 - 2x + 5 \\ y &= (x^2 - 2x + 1) + 5 + 1 \\ y &= (x - 1)^2 + 6 \end{aligned}$$

Who is correct? Explain your reasoning.

Guided Practice

Write each quadratic function in vertex form, if not already in that form. Then identify the vertex, axis of symmetry, and direction of opening.

5. $y = 5(x + 3)^2 - 1$

6. $y = x^2 + 8x - 3$

7. $y = -3x^2 - 18x + 11$

Graph each function.

8. $y = 3(x + 3)^2$

9. $y = \frac{1}{3}(x - 1)^2 + 3$

10. $y = -2x^2 + 16x - 31$

Write an equation for the parabola with the given vertex that passes through the given point.

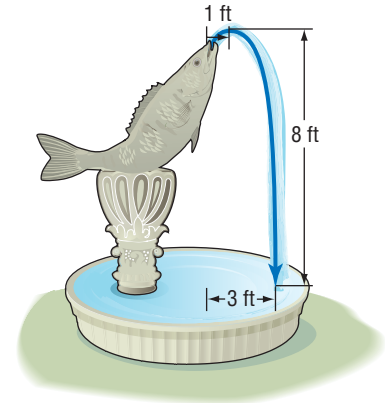
11. vertex: (2, 0)
point: (1, 4)

12. vertex: (-3, 6)
point: (-5, 2)

13. vertex: (-2, -3)
point: (-4, -5)

Application

14. **FOUNTAINS** The height of a fountain's water stream can be modeled by a quadratic function. Suppose the water from a jet reaches a maximum height of 8 feet at a distance 1 foot away from the jet. If the water lands 3 feet away from the jet, find a quadratic function that models the height $h(d)$ of the water at any given distance d feet from the jet.



Practice and Apply

Homework Help

For Exercises	See Examples
15–26	2
27–38, 47, 48, 50–52	1, 3
39–46, 49	4

Extra Practice

See page 841.

Write each quadratic function in vertex form, if not already in that form. Then identify the vertex, axis of symmetry, and direction of opening.

15. $y = -2(x + 3)^2$

16. $y = \frac{1}{3}(x - 1)^2 + 2$

17. $y = 5x^2 - 6$

18. $y = -8x^2 + 3$

19. $y = -x^2 - 4x + 8$

20. $y = x^2 - 6x + 1$

21. $y = -3x^2 + 12x$

22. $y = 4x^2 + 24x$

23. $y = 4x^2 + 8x - 3$

24. $y = -2x^2 + 20x - 35$

25. $y = 3x^2 + 3x - 1$

26. $y = 4x^2 - 12x - 11$

Graph each function.

27. $y = 4(x + 3)^2 + 1$

28. $y = -(x - 5)^2 - 3$

29. $y = \frac{1}{4}(x - 2)^2 + 4$

30. $y = \frac{1}{2}(x - 3)^2 - 5$

31. $y = x^2 + 6x + 2$

32. $y = x^2 - 8x + 18$

33. $y = -4x^2 + 16x - 11$

34. $y = -5x^2 - 40x - 80$

35. $y = -\frac{1}{2}x^2 + 5x - \frac{27}{2}$

36. $y = \frac{1}{3}x^2 - 4x + 15$

37. Write one sentence that compares the graphs of $y = 0.2(x + 3)^2 + 1$ and $y = 0.4(x + 3)^2 + 1$.

38. Compare the graphs of $y = 2(x - 5)^2 + 4$ and $y = 2(x - 4)^2 - 1$.

Write an equation for the parabola with the given vertex that passes through the given point.

39. vertex: (6, 1)
point: (5, 10)

40. vertex: (-4, 3)
point: (-3, 6)

41. vertex: (3, 0)
point: (6, -6)

42. vertex: (5, 4)
point: (3, -8)

43. vertex: (0, 5)
point: (3, 8)

44. vertex: (-3, -2)
point: (-1, 8)

WebQuest

You can use a quadratic function to model the world population. Visit www.algebra2.com/webquest to continue work on your WebQuest project.



More About . . .



Aerospace

The KC135A has the nickname "Vomit Comet." It starts its ascent at 24,000 feet. As it approaches maximum height, the engines are stopped, and the aircraft is allowed to free-fall at a determined angle. Zero gravity is achieved for 25 seconds as the plane reaches the top of its flight and begins its descent.

Source: NASA

45. Write an equation for a parabola whose vertex is at the origin and passes through $(2, -8)$.
46. Write an equation for a parabola with vertex at $(-3, -4)$ and y -intercept 8.
47. **AEROSPACE** NASA's KC135A aircraft flies in parabolic arcs to simulate the weightlessness experienced by astronauts in space. The height h of the aircraft (in feet) t seconds after it begins its parabolic flight can be modeled by the equation $h(t) = -9.09(t - 32.5)^2 + 34,000$. What is the maximum height of the aircraft during this maneuver and when does it occur?

DIVING

For Exercises 48–50, use the following information. The distance of a diver above the water $d(t)$ (in feet) t seconds after diving off a platform is modeled by the equation $d(t) = -16t^2 + 8t + 30$.

48. Find the time it will take for the diver to hit the water.
49. Write an equation that models the diver's distance above the water if the platform were 20 feet higher.
50. Find the time it would take for the diver to hit the water from this new height.

LAWN CARE

For Exercises 51 and 52, use the following information. The path of water from a sprinkler can be modeled by a quadratic function. The three functions below model paths for three different angles of the water.

Angle A: $y = -0.28(x - 3.09)^2 + 3.27$

Angle B: $y = -0.14(x - 3.57)^2 + 2.39$

Angle C: $y = -0.09(x - 3.22)^2 + 1.53$

51. Which sprinkler angle will send water the highest? Explain your reasoning.
52. Which sprinkler angle will send water the farthest? Explain your reasoning.
53. **CRITICAL THINKING** Given $y = ax^2 + bx + c$ with $a \neq 0$, derive the equation for the axis of symmetry by completing the square and rewriting the equation in the form $y = a(x - h)^2 + k$.

54. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How can the graph $y = x^2$ be used to graph any quadratic function?

Include the following in your answer:

- a description of the effects produced by changing a , h , and k in the equation $y = a(x - h)^2 + k$, and
- a comparison of the graph of $y = x^2$ and the graph of $y = a(x - h)^2 + k$ using values of your own choosing for a , h , and k .

55. If $f(x) = x^2 - 5x$ and $f(n) = -4$, then which of the following could be n ?

(A) -5 (B) -4 (C) -1 (D) 1

56. The vertex of the graph of $y = 2(x - 6)^2 + 3$ is located at which of the following points?

(A) $(2, 3)$ (B) $(6, 3)$ (C) $(6, -3)$ (D) $(-2, 3)$

Standardized Test Practice

(A) (B) (C) (D)



Maintain Your Skills

Mixed Review

Find the value of the discriminant for each quadratic equation. Then describe the number and type of roots for the equation. (Lesson 6-5)

57. $3x^2 - 6x + 2 = 0$

58. $4x^2 + 7x = 11$

59. $2x^2 - 5x + 6 = 0$

Solve each equation by completing the square. (Lesson 6-4)

60. $x^2 + 10x + 17 = 0$

61. $x^2 - 6x + 18 = 0$

62. $4x^2 + 8x = 9$

Find each quotient. (Lesson 5-3)

63. $(2t^3 - 2t - 3) \div (t - 1)$

64. $(t^3 - 3t + 2) \div (t + 2)$

65. $(n^4 - 8n^3 + 54n + 105) \div (n - 5)$

66. $(y^4 + 3y^3 + y - 1) \div (y + 3)$

67. **EDUCATION** The graph shows the number of U.S. students in study-abroad programs.

(Lesson 2-5)

- Write a prediction equation from the data given.
- Use your equation to predict the number of students in these programs in 2005.

Getting Ready for the Next Lesson

PREREQUISITE SKILL Determine whether the given value satisfies the inequality.

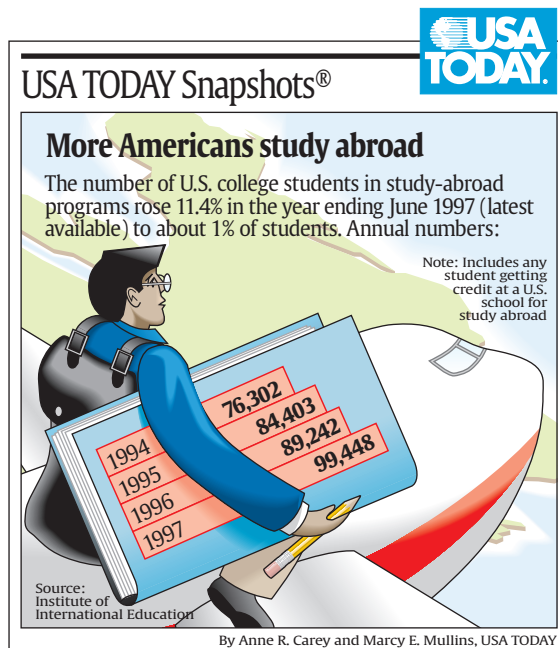
(To review **inequalities**, see Lesson 1-6.)

68. $-2x^2 + 3 < 0$; $x = 5$

69. $4x^2 + 2x - 3 \geq 0$; $x = -1$

70. $4x^2 - 4x + 1 \leq 10$; $x = 2$

71. $6x^2 + 3x > 8$; $x = 0$



Practice Quiz 2

Lessons 6-4 through 6-6

Solve each equation by completing the square. (Lesson 6-4)

1. $x^2 + 14x + 37 = 0$

2. $2x^2 - 2x + 5 = 0$

Find the value of the discriminant for each quadratic equation. Then describe the number and type of roots for the equation. (Lesson 6-5)

3. $5x^2 - 3x + 1 = 0$

4. $3x^2 + 4x - 7 = 0$

Solve each equation by using the Quadratic Formula. (Lesson 6-5)

5. $x^2 + 9x - 11 = 0$

6. $-3x^2 + 4x = 4$

7. Write an equation for a parabola with vertex at $(2, -5)$ that passes through $(-1, 1)$. (Lesson 6-6)

Write each equation in vertex form. Then identify the vertex, axis of symmetry, and direction of opening. (Lesson 6-6)

8. $y = x^2 + 8x + 18$

9. $y = -x^2 + 12x - 36$

10. $y = 2x^2 + 12x + 13$

Graphing and Solving Quadratic Inequalities

Vocabulary

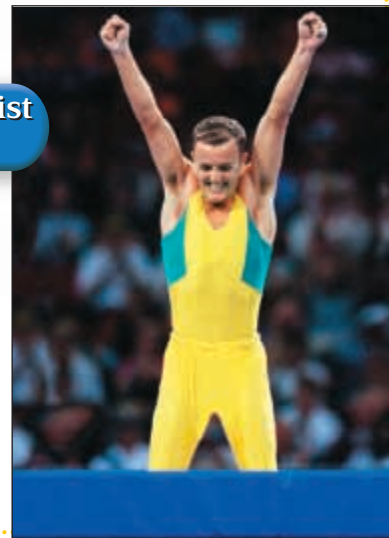
- quadratic inequality

What You'll Learn

- Graph quadratic inequalities in two variables.
- Solve quadratic inequalities in one variable.

How can you find the time a trampolinist spends above a certain height?

Trampolining was first featured as an Olympic sport at the 2000 Olympics in Sydney, Australia. The competitors performed two routines consisting of 10 different skills. Suppose the height $h(t)$ in feet of a trampolinist above the ground during one bounce is modeled by the quadratic function $h(t) = -16t^2 + 42t + 3.75$. We can solve a quadratic inequality to determine how long this performer is more than a certain distance above the ground.



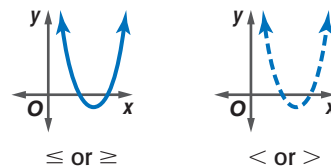
Study Tip

Look Back

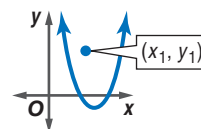
For review of graphing linear inequalities, see Lesson 2-7.

GRAPH QUADRATIC INEQUALITIES You can graph **quadratic inequalities** in two variables using the same techniques you used to graph linear inequalities in two variables.

Step 1 Graph the related quadratic equation, $y = ax^2 + bx + c$. Decide if the parabola should be solid or dashed.

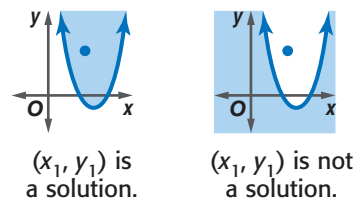


Step 2 Test a point (x_1, y_1) inside the parabola. Check to see if this point is a solution of the inequality.



$$y_1 \stackrel{?}{\geq} a(x_1)^2 + b(x_1) + c$$

Step 3 If (x_1, y_1) is a solution, shade the region *inside* the parabola. If (x_1, y_1) is *not* a solution, shade the region *outside* the parabola.

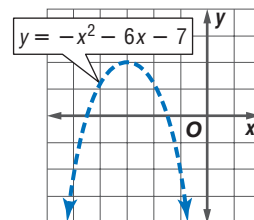


Example 1 Graph a Quadratic Inequality

Graph $y > -x^2 - 6x - 7$.

Step 1 Graph the related quadratic equation, $y = -x^2 - 6x - 7$.

Since the inequality symbol is $>$, the parabola should be dashed.



(continued on the next page)

Step 2 Test a point inside the parabola, such as $(-3, 0)$.

$$y > -x^2 - 6x - 7$$

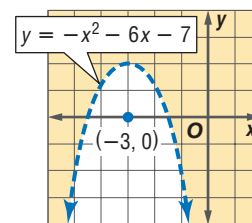
$$0 \stackrel{?}{>} -(-3)^2 - 6(-3) - 7$$

$$0 \stackrel{?}{>} -9 + 18 - 7$$

$$0 \stackrel{?}{>} 2 \quad \times$$

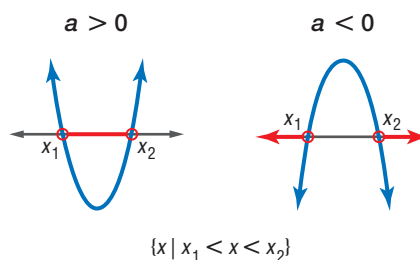
So, $(-3, 0)$ is *not* a solution of the inequality.

Step 3 Shade the region outside the parabola.



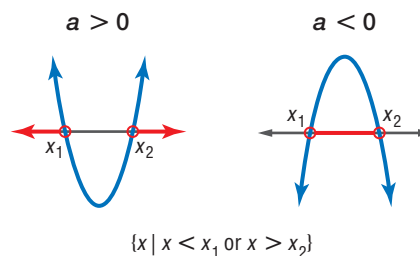
SOLVE QUADRATIC INEQUALITIES To solve a quadratic inequality in one variable, you can use the graph of the related quadratic function.

To solve $ax^2 + bx + c < 0$, graph $y = ax^2 + bx + c$. Identify the x values for which the graph lies *below* the x -axis.



For \leq , include the x -intercepts in the solution.

To solve $ax^2 + bx + c > 0$, graph $y = ax^2 + bx + c$. Identify the x values for which the graph lies *above* the x -axis.



For \geq , include the x -intercepts in the solution.

Example 2 Solve $ax^2 + bx + c > 0$

Solve $x^2 + 2x - 3 > 0$ by graphing.

The solution consists of the x values for which the graph of the related quadratic function lies *above* the x -axis. Begin by finding the roots of the related equation.

$$x^2 + 2x - 3 = 0 \quad \text{Related equation}$$

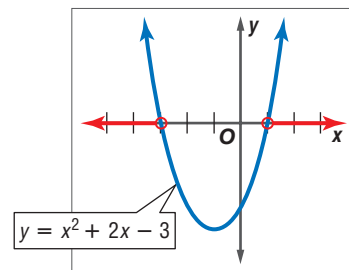
$$(x + 3)(x - 1) = 0 \quad \text{Factor.}$$

$$x + 3 = 0 \quad \text{or} \quad x - 1 = 0 \quad \text{Zero Product Property}$$

$$x = -3 \quad x = 1 \quad \text{Solve each equation.}$$

Sketch the graph of a parabola that has x -intercepts at -3 and 1 . The graph should open up since $a > 0$.

The graph lies above the x -axis to the left of $x = -3$ and to the right of $x = 1$. Therefore, the solution set is $\{x \mid x < -3 \text{ or } x > 1\}$.



Study Tip

Solving Quadratic Inequalities by Graphing

A precise graph of the related quadratic function is not necessary since the zeros of the function were found algebraically.



Example 3 Solve $ax^2 + bx + c \leq 0$

Solve $0 \geq 3x^2 - 7x - 1$ by graphing.

This inequality can be rewritten as $3x^2 - 7x - 1 \leq 0$. The solution consists of the x values for which the graph of the related quadratic function lies *on and below* the x -axis. Begin by finding the roots of the related equation.

$$3x^2 - 7x - 1 = 0$$

Related equation

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Use the Quadratic Formula.

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(3)(-1)}}{2(3)}$$

Replace a with 3, b with -7 , and c with -1 .

$$x = \frac{7 + \sqrt{61}}{6} \quad \text{or} \quad x = \frac{7 - \sqrt{61}}{6}$$

Simplify and write as two equations.

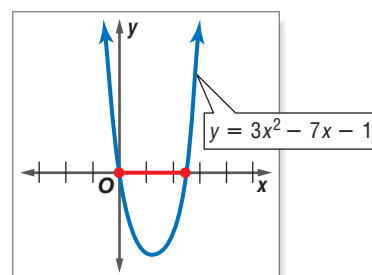
$$x \approx 2.47$$

$$x \approx -0.14$$

Simplify.

Sketch the graph of a parabola that has x -intercepts of 2.47 and -0.14 . The graph should open up since $a > 0$.

The graph lies on and below the x -axis at $x = -0.14$ and $x = 2.47$ and between these two values. Therefore, the solution set of the inequality is approximately $\{x \mid -0.14 \leq x \leq 2.47\}$.



CHECK Test one value of x less than -0.14 , one between -0.14 and 2.47 , and one greater than 2.47 in the original inequality.

Test $x = -1$.

$$0 \geq 3x^2 - 7x - 1$$

$$0 \stackrel{?}{\geq} 3(-1)^2 - 7(-1) - 1$$

$$0 \geq 9 \quad \times$$

Test $x = 0$.

$$0 \geq 3x^2 - 7x - 1$$

$$0 \stackrel{?}{\geq} 3(0)^2 - 7(0) - 1$$

$$0 \geq -1 \quad \checkmark$$

Test $x = 3$.

$$0 \geq 3x^2 - 7x - 1$$

$$0 \stackrel{?}{\geq} 3(3)^2 - 7(3) - 1$$

$$0 \geq 5 \quad \times$$

Real-world problems that involve vertical motion can often be solved by using a quadratic inequality.

Example 4 Write an Inequality

FOOTBALL The height of a punted football can be modeled by the function $H(x) = -4.9x^2 + 20x + 1$, where the height $H(x)$ is given in meters and the time x is in seconds. At what time in its flight is the ball within 5 meters of the ground?

The function $H(x)$ describes the height of the football. Therefore, you want to find the values of x for which $H(x) \leq 5$.

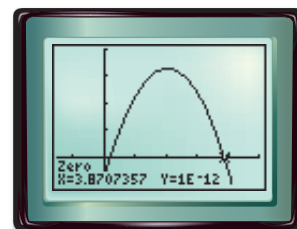
$$H(x) \leq 5 \quad \text{Original inequality}$$

$$-4.9x^2 + 20x + 1 \leq 5 \quad H(x) = -4.9x^2 + 20x + 1$$

$$-4.9x^2 + 20x - 4 \leq 0 \quad \text{Subtract 5 from each side.}$$

Graph the related function $y = -4.9x^2 + 20x - 4$ using a graphing calculator. The zeros of the function are about 0.21 and 3.87, and the graph lies below the x -axis when $x < 0.21$ or $x > 3.87$.

Thus, the ball is within 5 meters of the ground for the first 0.21 second of its flight and again after 3.87 seconds until the ball hits the ground at 4.13 seconds.



$[-1.5, 5]$ scl: 1 by $[-5, 20]$ scl: 5

More About . . .



Football

A long hang time allows the kicking team time to provide good coverage on a punt return. The suggested hang time for high school and college punters is 4.5–4.6 seconds.

Source: www.takeaknee.com



You can also solve quadratic inequalities algebraically.

Example 5 Solve a Quadratic Inequality

Solve $x^2 + x > 6$ algebraically.

First solve the related quadratic equation $x^2 + x = 6$.

$$x^2 + x = 6 \quad \text{Related quadratic equation}$$

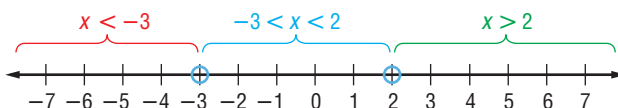
$$x^2 + x - 6 = 0 \quad \text{Subtract 6 from each side.}$$

$$(x + 3)(x - 2) = 0 \quad \text{Factor.}$$

$$x + 3 = 0 \quad \text{or} \quad x - 2 = 0 \quad \text{Zero Product Property}$$

$$x = -3 \quad \quad \quad x = 2 \quad \text{Solve each equation.}$$

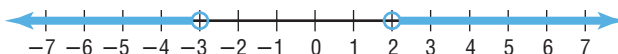
Plot -3 and 2 on a number line. Use circles since these values are not solutions of the original inequality. Notice that the number line is now separated into three intervals.



Test a value in each interval to see if it satisfies the original inequality.

$x < -3$	$-3 < x < 2$	$x > 2$
Test $x = -4$.	Test $x = 0$.	Test $x = 4$.
$x^2 + x > 6$	$x^2 + x > 6$	$x^2 + x > 6$
$(-4)^2 + (-4) \stackrel{?}{>} 6$	$0^2 + 0 \stackrel{?}{>} 6$	$4^2 + 4 \stackrel{?}{>} 6$
$12 > 6 \checkmark$	$0 > 6 \times$	$20 > 6 \checkmark$

The solution set is $\{x \mid x < -3 \text{ or } x > 2\}$. This is shown on the number line below.



Study Tip

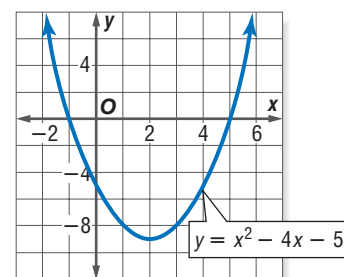
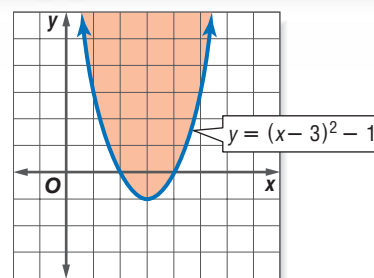
Solving Quadratic Inequalities Algebraically

As with linear inequalities, the solution set of a quadratic inequality can be all real numbers or the empty set, \emptyset . The solution is all real numbers when all three test points satisfy the inequality. It is the empty set when none of the test points satisfy the inequality.

Check for Understanding

Concept Check

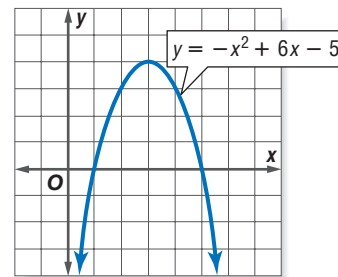
- Determine** which inequality, $y \geq (x - 3)^2 - 1$ or $y \leq (x - 3)^2 - 1$, describes the graph at the right.
- OPEN ENDED** List three points you might test to find the solution of $(x + 3)(x - 5) < 0$.
- Examine** the graph of $y = x^2 - 4x - 5$ at the right.
 - What are the solutions of $0 = x^2 - 4x - 5$?
 - What are the solutions of $x^2 - 4x - 5 \geq 0$?
 - What are the solutions of $x^2 - 4x - 5 \leq 0$?



Guided Practice Graph each inequality.

4. $y \geq x^2 - 10x + 25$ 5. $y < x^2 - 16$
 6. $y > -2x^2 - 4x + 3$ 7. $y \leq -x^2 + 5x + 6$

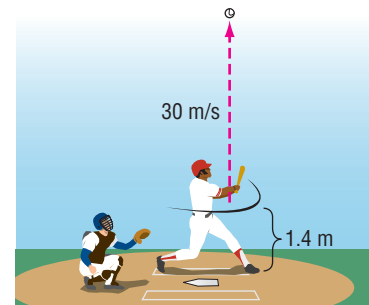
8. Use the graph of the related function of $-x^2 + 6x - 5 < 0$, which is shown at the right, to write the solutions of the inequality.



Solve each inequality algebraically.

9. $x^2 - 6x - 7 < 0$
 10. $x^2 - x - 12 > 0$
 11. $x^2 < 10x - 25$
 12. $x^2 \leq 3$

Application 13. **BASEBALL** A baseball player hits a high pop-up with an initial upward velocity of 30 meters per second, 1.4 meters above the ground. The height $h(t)$ of the ball in meters t seconds after being hit is modeled by $h(t) = -4.9t^2 + 30t + 1.4$. How long does a player on the opposing team have to catch the ball if he catches it 1.7 meters above the ground?



Practice and Apply

Homework Help

For Exercises	See Examples
14–25	1
26–29	2, 3
30–42	2, 3, 5
43–48	4

Graph each inequality.

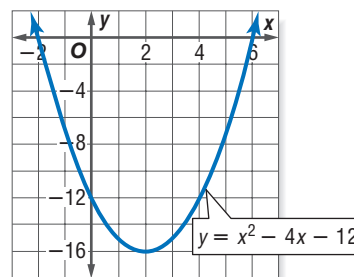
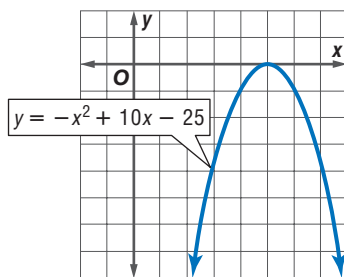
14. $y \geq x^2 + 3x - 18$ 15. $y < -x^2 + 7x + 8$ 16. $y \leq x^2 + 4x + 4$
 17. $y \leq x^2 + 4x$ 18. $y > x^2 - 36$ 19. $y > x^2 + 6x + 5$
 20. $y \leq -x^2 - 3x + 10$ 21. $y \geq -x^2 - 7x + 10$ 22. $y > -x^2 + 10x - 23$
 23. $y < -x^2 + 13x - 36$ 24. $y < 2x^2 + 3x - 5$ 25. $y \geq 2x^2 + x - 3$

Extra Practice

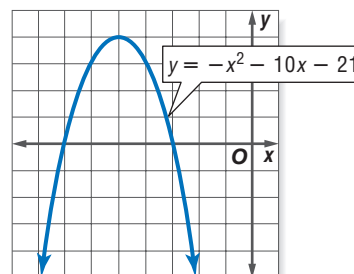
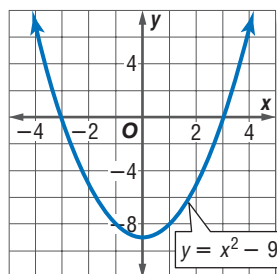
See page 841.

Use the graph of its related function to write the solutions of each inequality.

26. $-x^2 + 10x - 25 \geq 0$ 27. $x^2 - 4x - 12 \leq 0$



28. $x^2 - 9 > 0$ 29. $-x^2 - 10x - 21 < 0$



Solve each inequality algebraically.

30. $x^2 - 3x - 18 > 0$

32. $x^2 - 4x \leq 5$

34. $-x^2 - x + 12 \geq 0$

36. $9x^2 - 6x + 1 \leq 0$

38. $x^2 + 12x < -36$

40. $18x - x^2 \leq 81$

31. $x^2 + 3x - 28 < 0$

33. $x^2 + 2x \geq 24$

35. $-x^2 - 6x + 7 \leq 0$

37. $4x^2 + 20x + 25 \geq 0$

39. $-x^2 + 14x - 49 \geq 0$

41. $16x^2 + 9 < 24x$

42. Solve $(x - 1)(x + 4)(x - 3) > 0$.

43. **LANDSCAPING** Kinu wants to plant a garden and surround it with decorative stones. She has enough stones to enclose a rectangular garden with a perimeter of 68 feet, but she wants the garden to cover no more than 240 square feet. What could the width of her garden be?

44. **BUSINESS** A mall owner has determined that the relationship between monthly rent charged for store space r (in dollars per square foot) and monthly profit $P(r)$ (in thousands of dollars) can be approximated by the function $P(r) = -8.1r^2 + 46.9r - 38.2$. Solve each quadratic equation or inequality. Explain what each answer tells about the relationship between monthly rent and profit for this mall.

a. $-8.1r^2 + 46.9r - 38.2 = 0$

b. $-8.1r^2 + 46.9r - 38.2 > 0$

c. $-8.1r^2 + 46.9r - 38.2 > 10$

d. $-8.1r^2 + 46.9r - 38.2 < 10$

45. **GEOMETRY** A rectangle is 6 centimeters longer than it is wide. Find the possible dimensions if the area of the rectangle is more than 216 square centimeters.

FUND-RAISING For Exercises 46–48, use the following information.

The girls' softball team is sponsoring a fund-raising trip to see a professional baseball game. They charter a 60-passenger bus for \$525. In order to make a profit, they will charge \$15 per person if all seats on the bus are sold, but for each empty seat, they will increase the price by \$1.50 per person.

46. Write a quadratic function giving the softball team's profit $P(n)$ from this fund-raiser as a function of the number of passengers n .
47. What is the minimum number of passengers needed in order for the softball team not to lose money?
48. What is the maximum profit the team can make with this fund-raiser, and how many passengers will it take to achieve this maximum?

49. **CRITICAL THINKING** Graph the intersection of the graphs of $y \leq -x^2 + 4$ and $y \geq x^2 - 4$.

50. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How can you find the time a trampolinist spends above a certain height?

Include the following in your answer:

- a quadratic inequality that describes the time the performer spends more than 10 feet above the ground, and
- two approaches to solving this quadratic inequality.

Career Choices



Landscape Architect

Landscape architects design outdoor spaces so that they are not only functional, but beautiful and compatible with the natural environment.

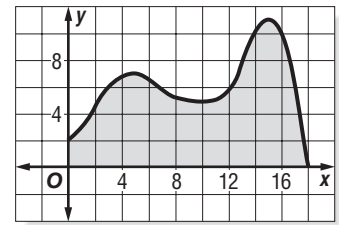
Online Research

For information about a career as a landscape architect, visit:

www.algebra2.com/careers

51. Which is a reasonable estimate of the area under the curve from $x = 0$ to $x = 18$?

- (A) 29 square units
- (B) 58 square units
- (C) 116 square units
- (D) 232 square units



52. If $(x + 1)(x - 2)$ is positive, then

- (A) $x < -1$ or $x > 2$.
- (B) $x > -1$ or $x < 2$.
- (C) $-1 < x < 2$.
- (D) $-2 < x < 1$.

**Extending
the Lesson**

SOLVE ABSOLUTE VALUE INEQUALITIES BY GRAPHING Similar to quadratic inequalities, you can solve absolute value inequalities by graphing.

Graph the related absolute value function for each inequality using a graphing calculator. For $>$ and \geq , identify the x values, if any, for which the graph lies below the x -axis. For $<$ and \leq , identify the x values, if any, for which the graph lies above the x -axis.

53. $|x - 2| > 0$

54. $|x| - 7 < 0$

55. $-|x + 3| + 6 < 0$

56. $2|x + 3| - 1 \geq 0$

57. $|5x + 4| - 2 \leq 0$

58. $|4x - 1| + 3 < 0$

Maintain Your Skills

Mixed Review

Write each equation in vertex form. Then identify the vertex, axis of symmetry, and direction of opening. (Lesson 6-6)

59. $y = x^2 - 2x + 9$

60. $y = -2x^2 + 16x - 32$

61. $y = \frac{1}{2}x^2 + 6x + 18$

Solve each equation using the method of your choice. Find exact solutions. (Lesson 6-5)

62. $x^2 + 12x + 32 = 0$

63. $x^2 + 7 = -5x$

64. $3x^2 + 6x - 2 = 3$

Simplify. (Lesson 5-2)

65. $(2a^2b - 3ab^2 + 5a - 6b) + (4a^2b^2 + 7ab^2 - b + 7a)$

66. $(x^3 - 3x^2y + 4xy^2 + y^3) - (7x^3 + x^2y - 9xy^2 + y^3)$

67. $x^{-3}y^2(x^4y + x^3y^{-1} + x^2y^{-2})$

68. $(5a - 3)(1 - 3a)$

Find each product, if possible. (Lesson 4-3)

69. $\begin{bmatrix} -6 & 3 \\ 4 & 7 \end{bmatrix} \cdot \begin{bmatrix} 2 & -5 \\ -3 & 6 \end{bmatrix}$

70. $[2 \quad -6 \quad 3] \cdot \begin{bmatrix} 3 & -3 \\ 9 & 0 \\ -2 & 4 \end{bmatrix}$

71. **LAW ENFORCEMENT** Thirty-four states classify drivers having at least a 0.1 blood alcohol content (BAC) as intoxicated. An infrared device measures a person's BAC through an analysis of his or her breath. A certain detector measures BAC to within 0.002. If a person's actual blood alcohol content is 0.08, write and solve an absolute value equation to describe the range of BACs that might register on this device. (Lesson 1-6)

Vocabulary and Concept Check

axis of symmetry (p. 287)

completing the square (p. 307)

constant term (p. 286)

discriminant (p. 316)

linear term (p. 286)

maximum value (p. 288)

minimum value (p. 288)

parabola (p. 286)

quadratic equation (p. 294)

Quadratic Formula (p. 313)

quadratic function (p. 286)

quadratic inequality (p. 329)

quadratic term (p. 286)

roots (p. 294)

Square Root Property (p. 306)

vertex (p. 287)

vertex form (p. 322)

Zero Product Property (p. 301)

zeros (p. 294)

Choose the letter of the term that best matches each phrase.

- the graph of any quadratic function
- process used to create a perfect square trinomial
- the line passing through the vertex of a parabola and dividing the parabola into two mirror images
- a function described by an equation of the form $f(x) = ax^2 + bx + c$, where $a \neq 0$
- the solutions of an equation
- $y = a(x - h)^2 + k$
- in the Quadratic Formula, the expression under the radical sign, $b^2 - 4ac$
- $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

- axis of symmetry
- completing the square
- discriminant
- constant term
- linear term
- parabola
- Quadratic Formula
- quadratic function
- roots
- vertex form

Lesson-by-Lesson Review

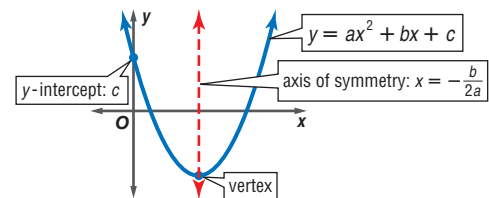
6-1 Graphing Quadratic Functions

See pages
286–293.

Concept Summary

The graph of $y = ax^2 + bx + c$, $a \neq 0$,

- opens up, and the function has a minimum value when $a > 0$, and
- opens down, and the function has a maximum value when $a < 0$.



Example

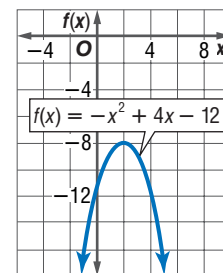
Find the maximum or minimum value of

$$f(x) = -x^2 + 4x - 12.$$

Since $a > 0$, the graph opens down and the function has a maximum value. The maximum value of the function is the y -coordinate of the vertex. The x -coordinate of the vertex is $x = -\frac{4}{2(-1)}$ or 2. Find the y -coordinate by evaluating the function for $x = 2$.

$$f(x) = -x^2 + 4x - 12 \quad \text{Original function}$$

$$f(2) = -(2)^2 + 4(2) - 12 \text{ or } -8 \quad \text{Replace } x \text{ with } 2.$$

Therefore, the maximum value of the function is -8 .

Exercises Complete parts a–c for each quadratic function.

- Find the y -intercept, the equation of the axis of symmetry, and the x -coordinate of the vertex.
 - Make a table of values that includes the vertex.
 - Use this information to graph the function. (See Example 2 on pages 287 and 288.)
- $f(x) = x^2 + 6x + 20$
 - $f(x) = x^2 - 2x - 15$
 - $f(x) = x^2 - 8x + 7$
 - $f(x) = -2x^2 + 12x - 9$
 - $f(x) = -x^2 - 4x - 3$
 - $f(x) = 3x^2 + 9x + 6$

Determine whether each function has a maximum or a minimum value. Then find the maximum or minimum value of each function.

(See Example 3 on pages 288 and 289.)

- $f(x) = 4x^2 - 3x - 5$
- $f(x) = -3x^2 + 2x - 2$
- $f(x) = -2x^2 + 7$

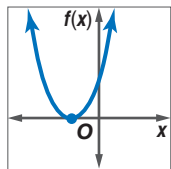
6-2 Solving Quadratic Equations by Graphing

See pages 294–299.

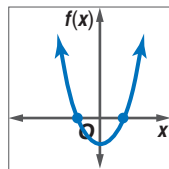
Concept Summary

- The solutions, or roots, of a quadratic equation are the zeros of the related quadratic function. You can find the zeros of a quadratic function by finding the x -intercepts of its graph.
- A quadratic equation can have one real solution, two real solutions, or no real solution.

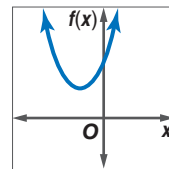
One Real Solution



Two Real Solutions



No Real Solution



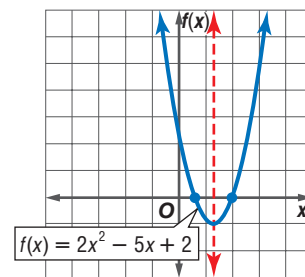
Example

Solve $2x^2 - 5x + 2 = 0$ by graphing.

The equation of the axis of symmetry is $x = -\frac{-5}{2(2)}$ or $x = \frac{5}{4}$.

x	0	$\frac{1}{2}$	$\frac{5}{4}$	2	$\frac{5}{2}$
$f(x)$	2	0	$-\frac{9}{8}$	0	2

The zeros of the related function are $\frac{1}{2}$ and 2. Therefore, the solutions of the equation are $\frac{1}{2}$ and 2.



Exercises Solve each equation by graphing. If exact roots cannot be found, state the consecutive integers between which the roots are located.

(See Examples 1–3 on pages 294 and 295.)

- $x^2 - 36 = 0$
- $-x^2 - 3x + 10 = 0$
- $2x^2 + x - 3 = 0$
- $-x^2 - 40x - 80 = 0$
- $-3x^2 - 6x - 2 = 0$
- $\frac{1}{5}(x + 3)^2 - 5 = 0$

6-3 Solving Quadratic Equations by Factoring

See pages
301–305.

Concept Summary

- **Zero Product Property:** For any real numbers a and b , if $ab = 0$, then either $a = 0$, $b = 0$, or both a and $b = 0$.

Example

Solve $x^2 + 9x + 20 = 0$ by factoring.

$$x^2 + 9x + 20 = 0$$

Original equation

$$(x + 4)(x + 5) = 0$$

Factor the trinomial.

$$x + 4 = 0 \quad \text{or} \quad x + 5 = 0$$

Zero Product Property

$$x = -4$$

$$x = -5$$

The solution set is $\{-5, -4\}$.

Exercises Solve each equation by factoring. (See Examples 1–3 on pages 301 and 302.)

24. $x^2 - 4x - 32 = 0$

25. $3x^2 + 6x + 3 = 0$

26. $5y^2 = 80$

27. $2c^2 + 18c - 44 = 0$

28. $25x^2 - 30x = -9$

29. $6x^2 + 7x = 3$

Write a quadratic equation with the given root(s). Write the equation in the form $ax^2 + bx + c$, where a , b , and c are integers. (See Example 4 on page 303.)

30. $-4, -25$

31. $10, -7$

32. $\frac{1}{3}, 2$

6-4 Completing the Square

See pages
306–312.

Concept Summary

- To complete the square for any quadratic expression $x^2 + bx$:

Step 1 Find one half of b , the coefficient of x .

Step 2 Square the result in Step 1.

Step 3 Add the result of Step 2 to $x^2 + bx$.

$$x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2$$

Example

Solve $x^2 + 10x - 39 = 0$ by completing the square.

$$x^2 + 10x - 39 = 0$$

Notice that $x^2 + 10x - 39 = 0$ is not a perfect square.

$$x^2 + 10x = 39$$

Rewrite so the left side is of the form $x^2 + bx$.

$$x^2 + 10x + 25 = 39 + 25$$

Since $\left(\frac{10}{2}\right)^2 = 25$, add 25 to each side.

$$(x + 5)^2 = 64$$

Write the left side as a perfect square by factoring.

$$x + 5 = \pm 8$$

Square Root Property

$$x + 5 = 8 \quad \text{or} \quad x + 5 = -8$$

Rewrite as two equations.

$$x = 3$$

$$x = -13$$

The solution set is $\{-13, 3\}$.

Exercises Find the value of c that makes each trinomial a perfect square. Then write the trinomial as a perfect square. (See Example 3 on page 307.)

33. $x^2 + 34x + c$

34. $x^2 - 11x + c$

35. $x^2 + \frac{7}{2}x + c$

Solve each equation by completing the square. (See Examples 4–6 on pages 308 and 309.)

36. $2x^2 - 7x - 15 = 0$

37. $2n^2 - 12n - 22 = 0$

38. $2x^2 - 5x + 7 = 3$

6-5 The Quadratic Formula and the Discriminant

See pages
313–319.

Concept Summary

- Quadratic Formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ where $a \neq 0$

Solve $x^2 - 5x - 66 = 0$ by using the Quadratic Formula.

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} && \text{Quadratic Formula} \\ &= \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(-66)}}{2(1)} && \text{Replace } a \text{ with } 1, b \text{ with } -5, \text{ and } c \text{ with } -66. \\ &= \frac{5 \pm 17}{2} && \text{Simplify.} \\ x &= \frac{5 + 17}{2} \quad \text{or} \quad x = \frac{5 - 17}{2} && \text{Write as two equations.} \\ &= 11 \qquad \qquad \qquad = -6 && \text{The solution set is } \{11, -6\}. \end{aligned}$$

Exercises Complete parts a–c for each quadratic equation.

- Find the value of the discriminant.
- Describe the number and type of roots.
- Find the exact solutions by using the Quadratic Formula.
(See Examples 1–4 on pages 314–316.)

39. $x^2 + 2x + 7 = 0$ 40. $-2x^2 + 12x - 5 = 0$ 41. $3x^2 + 7x - 2 = 0$

6-6 Analyzing Graphs of Quadratic Functions

See pages
322–328.

Concept Summary

- As the values of h and k change, the graph of $y = (x - h)^2 + k$ is the graph of $y = x^2$ translated
 - $|h|$ units left if h is negative or $|h|$ units right if h is positive.
 - $|k|$ units up if k is positive or $|k|$ units down if k is negative.
- Consider the equation $y = a(x - h)^2 + k$.
 - If $a > 0$, the graph opens up; if $a < 0$ the graph opens down.
 - If $|a| > 1$, the graph is narrower than the graph of $y = x^2$.
 - If $|a| < 1$, the graph is wider than the graph of $y = x^2$.

Example

Write the quadratic function $y = 3x^2 + 42x + 142$ in vertex form. Then identify the vertex, axis of symmetry, and direction of opening.

$$\begin{aligned} y &= 3x^2 + 42x + 142 && \text{Original equation} \\ y &= 3(x^2 + 14x) + 142 && \text{Group } ax^2 + bx \text{ and factor, dividing by } a. \\ y &= 3(x^2 + 14x + 49) + 142 - 3(49) && \text{Complete the square by adding } 3\left(\frac{14}{2}\right)^2. \\ &&& \text{Balance this with a subtraction of } 3(49). \\ y &= 3(x + 7)^2 - 5 && \text{Write } x^2 + 14x + 7 \text{ as a perfect square.} \end{aligned}$$

So, $a = 3$, $h = -7$, and $k = -5$. The vertex is at $(-7, -5)$, and the axis of symmetry is $x = -7$. Since a is positive, the graph opens up.



- Extra Practice, see pages 839–841.
- Mixed Problem Solving, see page 867.

Exercises Write each equation in vertex form, if not already in that form. Then identify the vertex, axis of symmetry, and direction of opening.

(See Examples 1 and 3 on pages 322 and 324.)

42. $y = -6(x + 2)^2 + 3$ 43. $y = 5x^2 + 35x + 58$ 44. $y = -\frac{1}{3}x^2 + 8x$

Graph each function. (See Examples 1–3 on pages 322 and 324.)

45. $y = (x - 2)^2 - 2$ 46. $y = 2x^2 + 8x + 10$ 47. $y = -9x^2 - 18x - 6$

Write an equation for the parabola with the given vertex that passes through the given point. (See Example 4 on page 325.)

48. vertex: (4, 1)
point: (2, 13) 49. vertex: (-2, 3)
point: (-6, 11) 50. vertex: (-3, -5)
point: (0, -14)

6-7 Graphing and Solving Quadratic Inequalities

See pages
329–335.

Concept Summary

- Graph quadratic inequalities in two variables as follows.
 - Step 1** Graph the related quadratic equation, $y = ax^2 + bx + c$. Decide if the parabola should be solid or dashed.
 - Step 2** Test a point (x_1, y_1) inside the parabola. Check to see if this point is a solution of the inequality.
 - Step 3** If (x_1, y_1) is a solution, shade the region *inside* the parabola. If (x_1, y_1) is *not* a solution, shade the region *outside* the parabola.
- To solve a quadratic inequality in one variable, graph the related quadratic function. Identify the x values for which the graph lies *below* the x -axis for $<$ and \leq . Identify the x values for which the graph lies *above* the x -axis for $>$ and \geq .

Example

Solve $x^2 + 3x - 10 < 0$ by graphing.

Find the roots of the related equation.

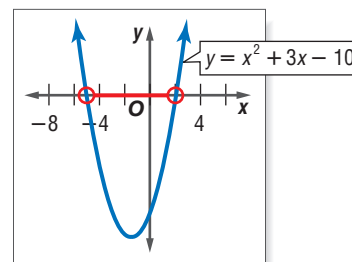
$0 = x^2 + 3x - 10$ Related equation

$0 = (x + 5)(x - 2)$ Factor.

$x + 5 = 0$ or $x - 2 = 0$ Zero Product Property

$x = -5$ $x = 2$ Solve each equation.

Sketch the graph of the parabola that has x -intercepts at -5 and 2 . The graph should open up since $a > 0$. The graph lies below the x -axis between $x = -5$ and $x = 2$. Therefore, the solution set is $\{x \mid -5 < x < 2\}$.



Exercises Graph each inequality. (See Example 1 on pages 329 and 330.)

51. $y > x^2 - 5x + 15$ 52. $y \leq 4x^2 - 36x + 17$ 53. $y \geq -x^2 + 7x - 11$

Solve each inequality. (See Examples 2, 3, and 5 on pages 330–332.)

54. $6x^2 + 5x > 4$ 55. $8x + x^2 \geq -16$ 56. $2x^2 + 5x < 12$

57. $2x^2 - 5x > 3$ 58. $4x^2 - 9 \leq -4x$ 59. $3x^2 - 5 > 6x$

Vocabulary and Concepts

Choose the word or term that best completes each statement.

- The y -coordinate of the vertex of the graph of $y = ax^2 + bx + c$ is the (*maximum*, *minimum*) value obtained by the function when a is positive.
- (*The Square Root Property*, *Completing the square*) can be used to solve any quadratic equation.

Skills and Applications

Complete parts a–c for each quadratic function.

- Find the y -intercept, the equation of the axis of symmetry, and the x -coordinate of the vertex.
- Make a table of values that includes the vertex.
- Use this information to graph the function.

3. $f(x) = x^2 - 2x + 5$

4. $f(x) = -3x^2 + 8x$

5. $f(x) = -2x^2 - 7x - 1$

Determine whether each function has a maximum or a minimum value.

Then find the maximum or minimum value of each function.

6. $f(x) = x^2 + 6x + 9$

7. $f(x) = 3x^2 - 12x - 24$

8. $f(x) = -x^2 + 4x$

- Write a quadratic equation with roots -4 and 5 . Write the equation in the form $ax^2 + bx + c = 0$, where a , b , and c are integers.

Solve each equation using the method of your choice. Find exact solutions.

10. $x^2 + x - 42 = 0$

11. $-1.6x^2 - 3.2x + 18 = 0$

12. $15x^2 + 16x - 7 = 0$

13. $x^2 + 8x - 48 = 0$

14. $x^2 + 12x + 11 = 0$

15. $x^2 - 9x - \frac{19}{4} = 0$

16. $3x^2 + 7x - 31 = 0$

17. $10x^2 + 3x = 1$

18. $-11x^2 - 174x + 221 = 0$

- BALLOONING** At a hot-air balloon festival, you throw a weighted marker straight down from an altitude of 250 feet toward a bull's eye below. The initial velocity of the marker when it leaves your hand is 28 feet per second. Find how long it will take the marker to hit the target by solving the equation $-16t^2 - 28t + 250 = 0$.

Write each equation in vertex form, if not already in that form. Then identify the vertex, axis of symmetry, and direction of opening.

20. $y = (x + 2)^2 - 3$

21. $y = x^2 + 10x + 27$

22. $y = -9x^2 + 54x - 8$

Graph each inequality.

23. $y \leq x^2 + 6x - 7$

24. $y > -2x^2 + 9$

25. $y \geq -\frac{1}{2}x^2 - 3x + 1$

Solve each inequality.

26. $(x - 5)(x + 7) < 0$

27. $3x^2 \geq 16$

28. $-5x^2 + x + 2 < 0$

- PETS** A rectangular turtle pen is 6 feet long by 4 feet wide. The pen is enlarged by increasing the length and width by an equal amount in order to double its area. What are the dimensions of the new pen?

- STANDARDIZED TEST PRACTICE** Which of the following is the sum of both solutions of the equation $x^2 + 8x - 48 = 0$?

Ⓐ -16

Ⓑ -8

Ⓒ -4

Ⓓ 12



Part 1 Multiple Choice

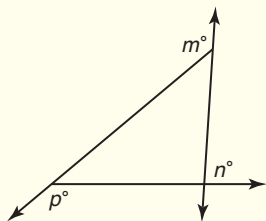
Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

1. In a class of 30 students, half are girls and 24 ride the bus to school. If 4 of the girls do not ride the bus to school, how many boys in this class ride the bus to school?

(A) 2 (B) 11
(C) 13 (D) 15

2. In the figure below, the measures of $\angle m + \angle n + \angle p =$?

(A) 90 (B) 180
(C) 270 (D) 360



3. Of the points $(-4, -2)$, $(1, -3)$, $(-1, 3)$, $(3, 1)$, and $(-2, 1)$, which three lie on the same side of the line $y - x = 0$?

(A) $(-4, -2)$, $(1, -3)$, $(-2, 1)$
(B) $(-4, -2)$, $(1, -3)$, $(3, 1)$
(C) $(-4, -2)$, $(-1, 3)$, $(-2, 1)$
(D) $(1, -3)$, $(-1, 3)$, $(3, 1)$

4. If k is an integer, then which of the following must also be integers?

I. $\frac{5k+5}{5k}$ II. $\frac{5k+5}{k+1}$ III. $\frac{5k^2+k}{5k}$

(A) I only (B) II only
(C) I and II (D) II and III

5. Which of the following is a factor of $x^2 - 7x - 8$?

(A) $x + 2$ (B) $x - 1$
(C) $x - 4$ (D) $x - 8$

6. If $x > 0$, then $\frac{\sqrt{16x^2 + 64x + 64}}{x + 2} =$?

(A) 2 (B) 4
(C) 8 (D) 16

7. If x and p are both greater than zero and $4x^2p^2 + xp - 33 = 0$, then what is the value of p in terms of x ?

(A) $-\frac{3}{x}$ (B) $-\frac{11}{4x}$
(C) $\frac{3}{4x}$ (D) $\frac{11}{4x}$

8. For all positive integers n , $\langle n \rangle = 3\sqrt{n}$. Which of the following equals 12?

(A) $\langle 4 \rangle$ (B) $\langle 8 \rangle$
(C) $\langle 16 \rangle$ (D) $\langle 32 \rangle$

9. Which number is the sum of both solutions of the equation $x^2 - 3x - 18 = 0$?

(A) -6 (B) -3
(C) 3 (D) 6

10. One of the roots of the polynomial

$6x^2 + kx + 20 = 0$ is $-\frac{5}{2}$. What is the value of k ?

(A) -23 (B) $-\frac{4}{3}$
(C) 23 (D) 7

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Test-Taking Tip

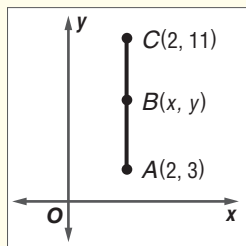
Questions 8, 11, 13, 16, 21, and 27 Be sure to use the information that describes the variables in any standardized test item. For example, if an item says that $x > 0$, check to be sure that your solution for x is not a negative number.

Part 2 Short Response/Grid In

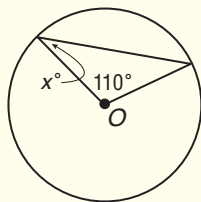
Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

- If n is a three-digit number that can be expressed as the product of three consecutive *even* integers, what is one possible value of n ?
- If x and y are *different* positive integers and $x + y = 6$, what is one possible value of $3x + 5y$?
- If a circle of radius 12 inches has its radius decreased by 6 inches, by what percent is its area decreased?
- What is the least positive integer k for which $12k$ is the cube of an integer?

- If $AB = BC$ in the figure, what is the y -coordinate of point B ?



- In the figure, if O is the center of the circle, what is the value of x ?



- Let $a \blacklozenge b$ be defined as the sum of all integers greater than a and less than b . For example, $6 \blacklozenge 10 = 7 + 8 + 9$ or 25. What is the value of $(75 \blacklozenge 90) - (76 \blacklozenge 89)$?
- If $x^2 - y^2 = 42$ and $x + y = 6$, what is the value of $x - y$?
- By what amount does the sum of the roots exceed the product of the roots of the equation $(x - 7)(x + 3) = 0$?
- If $x^2 = 36$ and $y^2 = 9$, what is the greatest possible value of $(x - y)^2$?

Part 3 Quantitative Comparison

Compare the quantity in Column A and the quantity in Column B. Then determine whether:

- the quantity in Column A is greater,
- the quantity in Column B is greater,
- the two quantities are equal, or
- the relationship cannot be determined from the information given.

Column A	Column B
$s > 0$	

s increased by 300% of s	$4s$
------------------------------	------

- In $\triangle ABC$, side \overline{AB} has length 8, and side \overline{BC} has length 4.

the length of side \overline{AC}	10
------------------------------------	----

the perimeter of a rectangle with area 8 units	the perimeter of a rectangle with area 10 units
--	---

$2^{350} - 2^{349}$	2^{349}
---------------------	-----------

$t + 5 > 9$	
$t + 3$	7

$x^2 + 12x + 36 = 0$	
x	-5

$p > q$	
$ p $	$ q $

the measure of side x	the measure of side y