GLENCOE MATHEMATICS

Algebra 2

Chapter 3 Resource Masters

Glencoe McGraw-Hill

New York, New York
Columbus, Ohio
Chicago, Illinois
Peoria, Illinois
Woodland Hills, California

Consumable Workbooks

Many of the worksheets contained in the Chapter Resource Masters booklets are available as consumable workbooks.

Study Guide and Intervention Workbook	0-07-828029-X
Skills Practice Workbook	0-07-828023-0
Practice Workbook	0-07-828024-9

ANSWERS FOR WORKBOOKS The answers for Chapter 3 of these workbooks can be found in the back of this Chapter Resource Masters booklet.

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8787 Orion Place
Columbus, OH 43240-4027

ISBN: 0-07-828006-0

Algebra 2
Chapter 3 Resource Masters

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Contents

Teacher's Guide to Using the Chapter 3 Resource Masters

The ***Fast File*** Chapter Resource system allows you to conveniently file the resources you use most often. The *Chapter 3 Resource Masters* includes the core materials needed for Chapter 3. These materials include worksheets, extensions, and assessment options. The answers for these pages appear at the back of this booklet.

All of the materials found in this booklet are included for viewing and printing in the *Algebra 2 TeacherWorks* CD-ROM.

Vocabulary Builder Pages vii–viii include a student study tool that presents up to twenty of the key vocabulary terms from the chapter. Students are to record definitions and/or examples for each term. You may suggest that students highlight or star the terms with which they are not familiar.

WHEN TO USE Give these pages to students before beginning Lesson 3-1. Encourage them to add these pages to their Algebra 2 Study Notebook. Remind them to add definitions and examples as they complete each lesson.

Study Guide and Intervention

Each lesson in *Algebra 2* addresses two objectives. There is one Study Guide and Intervention master for each objective.

WHEN TO USE Use these masters as reteaching activities for students who need additional reinforcement. These pages can also be used in conjunction with the Student Edition as an instructional tool for students who have been absent.

Skills Practice There is one master for each lesson. These provide computational practice at a basic level.

WHEN TO USE These masters can be used with students who have weaker mathematics backgrounds or need additional reinforcement.

Practice There is one master for each lesson. These problems more closely follow the structure of the Practice and Apply section of the Student Edition exercises. These exercises are of average difficulty.

WHEN TO USE These provide additional practice options or may be used as homework for second day teaching of the lesson.

Reading to Learn Mathematics

One master is included for each lesson. The first section of each master asks questions about the opening paragraph of the lesson in the Student Edition. Additional questions ask students to interpret the context of and relationships among terms in the lesson. Finally, students are asked to summarize what they have learned using various representation techniques.

WHEN TO USE This master can be used as a study tool when presenting the lesson or as an informal reading assessment after presenting the lesson. It is also a helpful tool for ELL (English Language Learner) students.

Enrichment There is one extension master for each lesson. These activities may extend the concepts in the lesson, offer an historical or multicultural look at the concepts, or widen students' perspectives on the mathematics they are learning. These are not written exclusively for honors students, but are accessible for use with all levels of students.

WHEN TO USE These may be used as extra credit, short-term projects, or as activities for days when class periods are shortened.

Assessment Options

The assessment masters in the *Chapter 3 Resource Masters* offer a wide range of assessment tools for intermediate and final assessment. The following lists describe each assessment master and its intended use.

Chapter Assessment

CHAPTER TESTS

- *Form 1* contains multiple-choice questions and is intended for use with basic level students.
- *Forms 2A and 2B* contain multiple-choice questions aimed at the average level student. These tests are similar in format to offer comparable testing situations.
- *Forms 2C and 2D* are composed of free-response questions aimed at the average level student. These tests are similar in format to offer comparable testing situations. Grids with axes are provided for questions assessing graphing skills.
- *Form 3* is an advanced level test with free-response questions. Grids without axes are provided for questions assessing graphing skills.

  All of the above tests include a free-response Bonus question.
- The **Open-Ended Assessment** includes performance assessment tasks that are suitable for all students. A scoring rubric is included for evaluation guidelines. Sample answers are provided for assessment.
- A **Vocabulary Test**, suitable for all students, includes a list of the vocabulary words in the chapter and ten questions assessing students' knowledge of those terms. This can also be used in conjunction with one of the chapter tests or as a review worksheet.

Intermediate Assessment

- Four free-response **quizzes** are included to offer assessment at appropriate intervals in the chapter.
- A **Mid-Chapter Test** provides an option to assess the first half of the chapter. It is composed of both multiple-choice and free-response questions.

Continuing Assessment

- The **Cumulative Review** provides students an opportunity to reinforce and retain skills as they proceed through their study of Algebra 2. It can also be used as a test. This master includes free-response questions.
- The **Standardized Test Practice** offers continuing review of algebra concepts in various formats, which may appear on the standardized tests that they may encounter. This practice includes multiple-choice, grid-in, and quantitative-comparison questions. Bubble-in and grid-in answer sections are provided on the master.

Answers

- Page A1 is an answer sheet for the Standardized Test Practice questions that appear in the Student Edition on pages 150–151. This improves students' familiarity with the answer formats they may encounter in test taking.
- The answers for the lesson-by-lesson masters are provided as reduced pages with answers appearing in red.
- Full-size answer keys are provided for the assessment masters in this booklet.

NAME ____ DATE ____________ PERIOD _____

3 Reading to Learn Mathematics

Vocabulary Builder

This is an alphabetical list of the key vocabulary terms you will learn in Chapter 3. As you study the chapter, complete each term's definition or description. Remember to add the page number where you found the term. Add these pages to your Algebra Study Notebook to review vocabulary at the end of the chapter.

Vocabulary Term	Found on Page	Definition/Description/Example
bounded region		
consistent system		
constraints kuhn·STRAYNTS		
dependent system		
elimination method		
feasible region FEE·zuh·buhl		
inconsistent system ihn·kuhn·SIHS·tuhnt		
independent system		

(continued on the next page)

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3

Reading to Learn Mathematics

Vocabulary Builder *(continued)*

Vocabulary Term	Found on Page	Definition/Description/Example
linear programming		
ordered triple		
substitution method		
system of equations		
system of inequalities		
unbounded region		
vertices		

3-1 Study Guide and Intervention

Solving Systems of Equations by Graphing

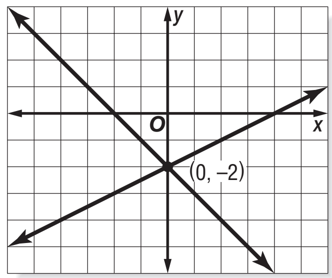
Graph Systems of Equations A system of equations is a set of two or more equations containing the same variables. You can solve a system of linear equations by graphing the equations on the same coordinate plane. If the lines intersect, the solution is that intersection point.

Example **Solve the system of equations by graphing.** $\boldsymbol{x - 2y = 4}$
$\boldsymbol{x + y = -2}$

Write each equation in slope-intercept form.

$x - 2y = 4 \quad \rightarrow \quad y = \frac{x}{2} - 2$

$x + y = -2 \quad \rightarrow \quad y = -x - 2$



The graphs appear to intersect at (0, −2).

CHECK Substitute the coordinates into each equation.

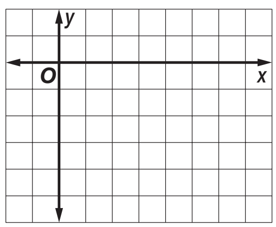
$x - 2y = 4$	$x + y = -2$
$0 - 2(-2) \stackrel{?}{=} 4$	$0 + (-2) \stackrel{?}{=} -2$
$4 = 4$ ✓	$-2 = -2$ ✓

The solution of the system is (0, −2).

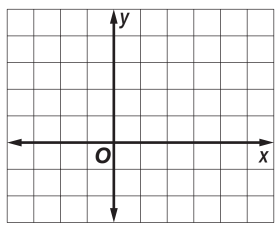
Exercises

Solve each system of equations by graphing.

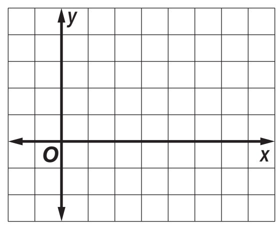
1. $y = -\frac{x}{3} + 1$
$y = \frac{x}{2} - 4$



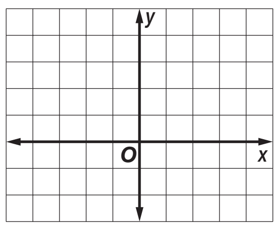
2. $y = 2x - 2$
$y = -x + 4$



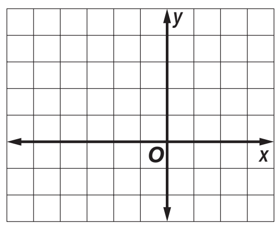
3. $y = -\frac{x}{2} + 3$
$y = \frac{x}{4}$



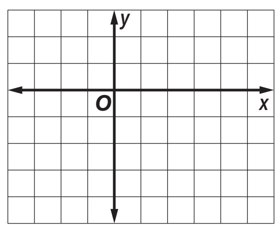
4. $3x - y = 0$
$x - y = -2$



5. $2x + \frac{y}{3} = -7$
$\frac{x}{2} + y = 1$



6. $\frac{x}{2} - y = 2$
$2x - y = -1$



3-1 Study Guide and Intervention *(continued)*

Solving Systems of Equations by Graphing

Classify Systems of Equations The following chart summarizes the possibilities for graphs of two linear equations in two variables.

Graphs of Equations	Slopes of Lines	Classification of System	Number of Solutions
Lines intersect	Different slopes	Consistent and independent	One
Lines coincide (same line)	Same slope, same *y*-intercept	Consistent and dependent	Infinitely many
Lines are parallel	Same slope, different *y*-intercepts	Inconsistent	None

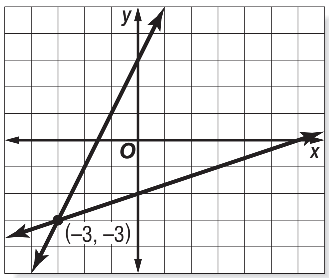
Example **Graph the system of equations and describe it as *consistent and independent*, *consistent and dependent*, or *inconsistent*.** $x - 3y = 6$ $2x - y = -3$

Write each equation in slope-intercept form.

$x - 3y = 6 \quad \rightarrow \quad y = \frac{1}{3}x - 2$

$2x - y = -3 \quad \rightarrow \quad y = 2x + 3$

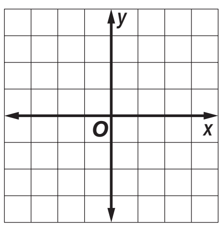
The graphs intersect at $(-3, -3)$. Since there is one solution, the system is consistent and independent.



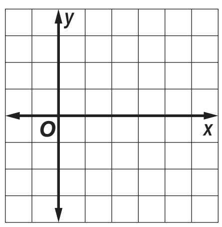
Exercises

Graph the system of equations and describe it as *consistent and independent*, *consistent and dependent*, or *inconsistent*.

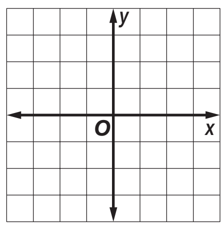
1. $3x + y = -2$
$6x + 2y = 10$



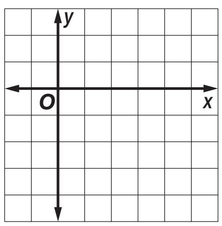
2. $x + 2y = 5$
$3x - 15 = -6y$



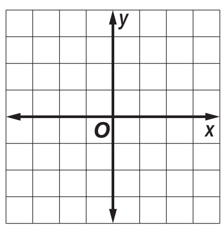
3. $2x - 3y = 0$
$4x - 6y = 3$



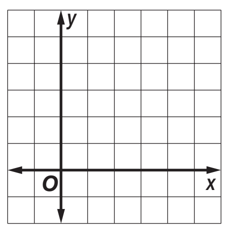
4. $2x - y = 3$
$x + 2y = 4$



5. $4x + y = -2$
$2x + \frac{y}{2} = -1$



6. $3x - y = 2$
$x + y = 6$



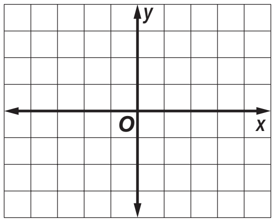
NAME ____ DATE ____________ PERIOD _____

3-1 Skills Practice

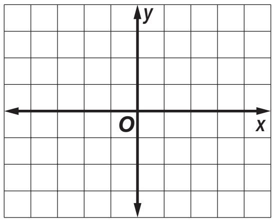
Solving Systems of Equations By Graphing

Solve each system of equations by graphing.

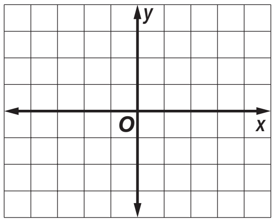
1. $x = 2$
$y = 0$



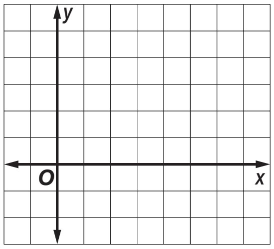
2. $y = -3x + 6$
$y = 2x - 4$



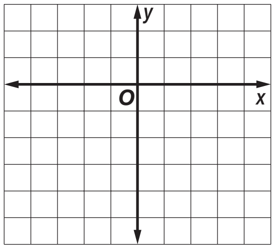
3. $y = 4 - 3x$
$y = -\frac{1}{2}x - 1$



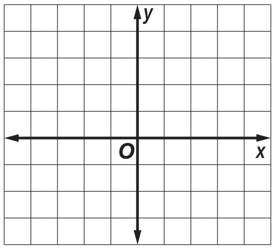
4. $y = 4 - x$
$y = x - 2$



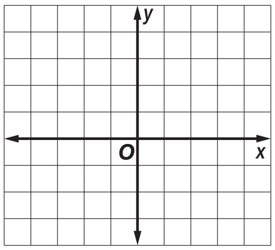
5. $y = -2x + 2$
$y = \frac{1}{3}x - 5$



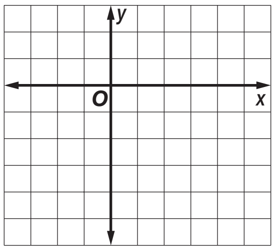
6. $y = x$
$y = -3x + 4$



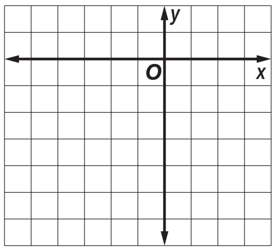
7. $x + y = 3$
$x - y = 1$



8. $x - y = 4$
$2x - 5y = 8$

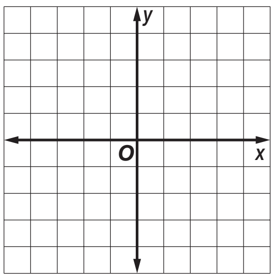


9. $3x - 2y = 4$
$2x - y = 1$

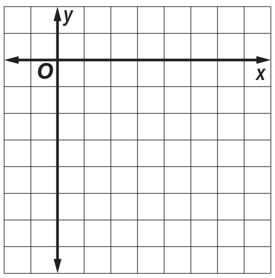


Graph each system of equations and describe it as *consistent and independent*, *consistent and dependent*, or *inconsistent*.

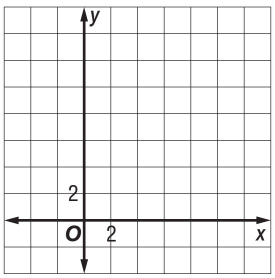
10. $y = -3x$
$y = -3x + 2$



11. $y = x - 5$
$-2x + 2y = -10$



12. $2x - 5y = 10$
$3x + y = 15$



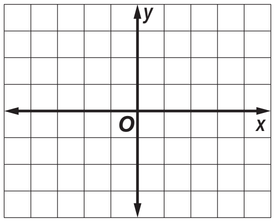
Lesson 3-1

3-1 Practice

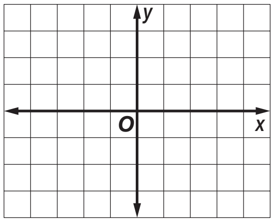
Solving Systems of Equations By Graphing

Solve each system of equations by graphing.

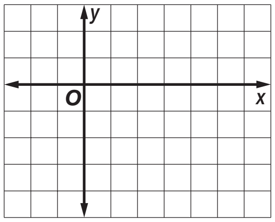
1. $x - 2y = 0$
$y = 2x - 3$



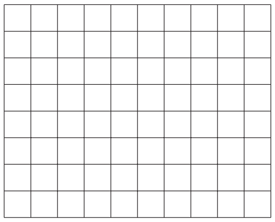
2. $x + 2y = 4$
$2x - 3y = 1$



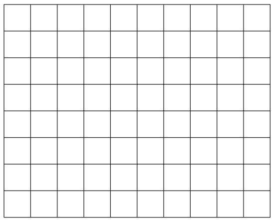
3. $2x + y = 3$
$y = \frac{1}{2}x - \frac{9}{2}$



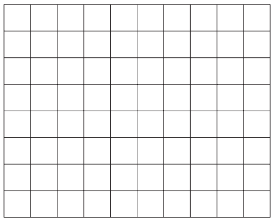
4. $y - x = 3$
$y = 1$



5. $2x - y = 6$
$x + 2y = -2$

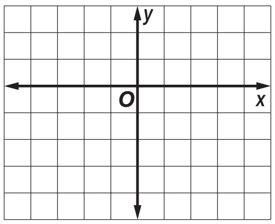


6. $5x - y = 4$
$-2x + 6y = 4$

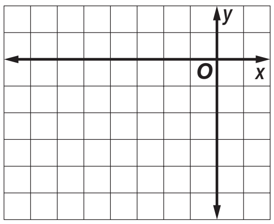


Graph each system of equations and describe it as *consistent and independent*, *consistent and dependent*, or *inconsistent*.

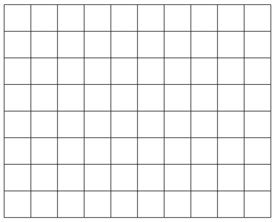
7. $2x - y = 4$
$x - y = 2$



8. $y = -x - 2$
$x + y = -4$



9. $2y - 8 = x$
$y = \frac{1}{2}x + 4$



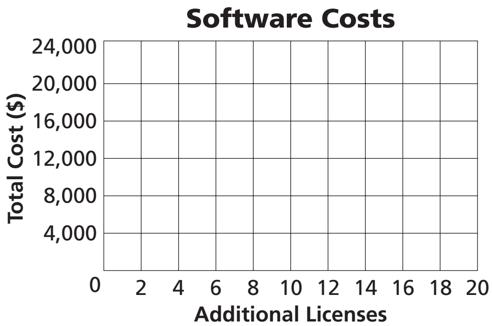
SOFTWARE For Exercises 10–12, use the following information.

Location Mapping needs new software. Software A costs \$13,000 plus \$500 per additional site license. Software B costs \$2500 plus \$1200 per additional site license.

10. Write two equations that represent the cost of each software.

11. Graph the equations. Estimate the break-even point of the software costs.

12. If Location Mapping plans to buy 10 additional site licenses, which software will cost less?



3-1 Reading to Learn Mathematics

Solving Systems of Equations by Graphing

Pre-Activity **How can a system of equations be used to predict sales?**

Read the introduction to Lesson 3-1 at the top of page 110 in your textbook.

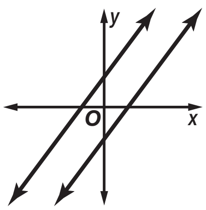
- Which are growing faster, in-store sales or online sales?

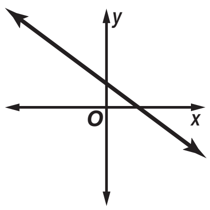
- In what year will the in-store and online sales be the same?

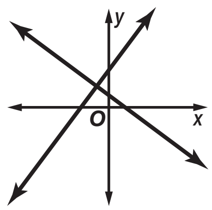
Reading the Lesson

1. The Study Tip on page 110 of your textbook says that when you solve a system of equations by graphing and find a point of intersection of the two lines, you must always check the ordered pair in *both* of the original equations. Why is it not good enough to check the ordered pair in just one of the equations?

2. Under each system graphed below, write all of the following words that apply: *consistent*, *inconsistent*, *dependent*, and *independent*.







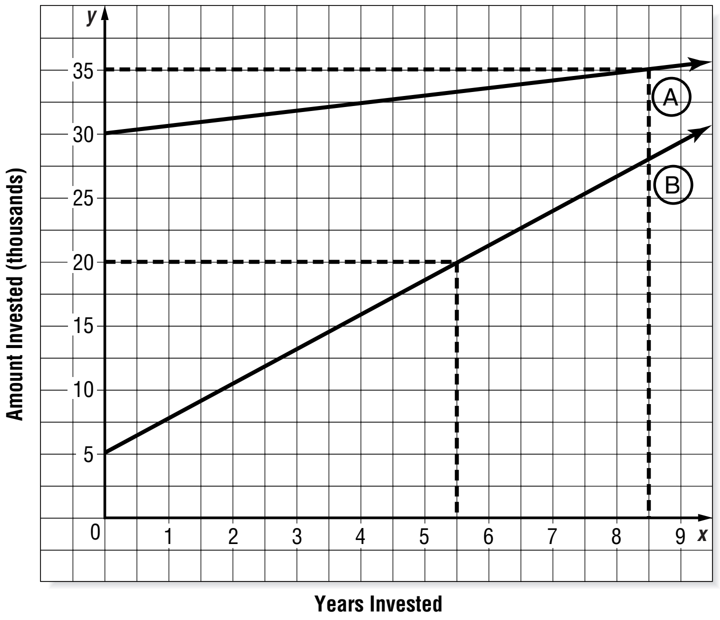
Helping You Remember

3. Look up the words *consistent* and *inconsistent* in a dictionary. How can the meaning of these words help you distinguish between consistent and inconsistent systems of equations?

3-1 Enrichment

Investments

The following graph shows the value of two different investments over time. Line A represents an initial investment of $30,000 with a bank paying passbook savings interest. Line B represents an initial investment of $5,000 in a profitable mutual fund with dividends reinvested and capital gains accepted in shares. By deriving the linear equation $y = mx + b$ for A and B, you can predict the value of these investments for years to come.



1. The y-intercept, b, is the initial investment. Find b for each of the following.
a. line A **b.** line B

2. The slope of the line, m, is the rate of return. Find m for each of the following.
a. line A **b.** line B

3. What are the equations of each of the following lines?
a. line A **b.** line B

4. What will be the value of the mutual fund after 11 years of investment?

5. What will be the value of the bank account after 11 years of investment?

6. When will the mutual fund and the bank account have equal value?

7. Which investment has the greatest payoff after 11 years of investment?

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3-2 Study Guide and Intervention

Solving Systems of Equations Algebraically

Substitution To solve a system of linear equations by **substitution**, first solve for one variable in terms of the other in one of the equations. Then substitute this expression into the other equation and simplify.

Example **Use substitution to solve the system of equations.** $\mathbf{2x - y = 9}$
$\mathbf{x + 3y = -6}$

Solve the first equation for y in terms of x.

$2x - y = 9$	First equation
$-y = -2x + 9$	Subtract $2x$ from both sides.
$y = 2x - 9$	Multiply both sides by -1.

Substitute the expression $2x - 9$ for y into the second equation and solve for x.

$x + 3y = -6$	Second equation
$x + 3(2x - 9) = -6$	Substitute $2x - 9$ for y.
$x + 6x - 27 = -6$	Distributive Property
$7x - 27 = -6$	Simplify.
$7x = 21$	Add 27 to each side.
$x = 3$	Divide each side by 7.

Now, substitute the value 3 for x in either original equation and solve for y.

$2x - y = 9$	First equation
$2(3) - y = 9$	Replace x with 3.
$6 - y = 9$	Simplify.
$-y = 3$	Subtract 6 from each side.
$y = -3$	Multiply each side by -1.

The solution of the system is $(3, -3)$.

Exercises

Solve each system of linear equations by using substitution.

1. $3x + y = 7$
$4x + 2y = 16$

2. $2x + y = 5$
$3x - 3y = 3$

3. $2x + 3y = -3$
$x + 2y = 2$

4. $2x - y = 7$
$6x - 3y = 14$

5. $4x - 3y = 4$
$2x + y = -8$

6. $5x + y = 6$
$3 - x = 0$

7. $x + 8y = -2$
$x - 3y = 20$

8. $2x - y = -4$
$4x + y = 1$

9. $x - y = -2$
$2x - 3y = 2$

10. $x - 4y = 4$
$2x + 12y = 13$

11. $x + 3y = 2$
$4x + 12y = 8$

12. $2x + 2y = 4$
$x - 2y = 0$

Lesson 3-2

3-2 Study Guide and Intervention *(continued)*

Solving Systems of Equations Algebraically

Elimination To solve a system of linear equations by **elimination**, add or subtract the equations to eliminate one of the variables. You may first need to multiply one or both of the equations by a constant so that one of the variables has the same (or opposite) coefficient in one equation as it has in the other.

Example 1 **Use the elimination method to solve the system of equations.**

$\mathbf{2x - 4y = -26}$
$\mathbf{3x - y = -24}$

Multiply the second equation by 4. Then subtract the equations to eliminate the y variable.

$2x - 4y = -26$ → $2x - 4y = -26$
$3x - y = -24$ → Multiply by 4. → $12x - 4y = -96$

$$-10x = 70$$
$$x = -7$$

Replace x with -7 and solve for y.

$$2x - 4y = -26$$
$$2(-7) - 4y = -26$$
$$-14 - 4y = -26$$
$$-4y = -12$$
$$y = 3$$

The solution is $(-7, 3)$.

Example 2 **Use the elimination method to solve the system of equations.**

$\mathbf{3x - 2y = 4}$
$\mathbf{5x + 3y = -25}$

Multiply the first equation by 3 and the second equation by 2. Then add the equations to eliminate the y variable.

$3x - 2y = 4$ → Multiply by 3. → $9x - 6y = 12$
$5x + 3y = -25$ → Multiply by 2. → $10x + 6y = -50$

$$19x = -38$$
$$x = -2$$

Replace x with -2 and solve for y.

$$3x - 2y = 4$$
$$3(-2) - 2y = 4$$
$$-6 - 2y = 4$$
$$-2y = 10$$
$$y = -5$$

The solution is $(-2, -5)$.

Exercises

Solve each system of equations by using elimination.

1. $2x - y = 7$
$3x + y = 8$

2. $x - 2y = 4$
$-x + 6y = 12$

3. $3x + 4y = -10$
$x - 4y = 2$

4. $3x - y = 12$
$5x + 2y = 20$

5. $4x - y = 6$
$2x - \frac{y}{2} = 4$

6. $5x + 2y = 12$
$-6x - 2y = -14$

7. $2x + y = 8$
$3x + \frac{3}{2}y = 12$

8. $7x + 2y = -1$
$4x - 3y = -13$

9. $3x + 8y = -6$
$x - y = 9$

10. $5x + 4y = 12$
$7x - 6y = 40$

11. $-4x + y = -12$
$4x + 2y = 6$

12. $5m + 2n = -8$
$4m + 3n = 2$

3-2 Skills Practice

Solving Systems of Equations Algebraically

Solve each system of equations by using substitution.

1. $m + n = 20$
$m - n = -4$

2. $x + 3y = -3$
$4x + 3y = 6$

3. $w - z = 1$
$2w + 3z = 12$

4. $3r + s = 5$
$2r - s = 5$

5. $2b + 3c = -4$
$b + c = 3$

6. $x - y = 1$
$2x + 3y = 12$

Solve each system of equations by using elimination.

7. $2p - q = 5$
$3p + q = 5$

8. $2j - k = 3$
$3j + k = 2$

9. $3c - 2d = 2$
$3c + 4d = 50$

10. $2f + 3g = 9$
$f - g = 2$

11. $-2x + y = -1$
$x + 2y = 3$

12. $2x - y = 12$
$2x - y = 6$

Solve each system of equations by using either substitution or elimination.

13. $-r + t = 5$
$-2r + t = 4$

14. $2x - y = -5$
$4x + y = 2$

15. $x - 3y = -12$
$2x + y = 11$

16. $2p - 3q = 6$
$-2p + 3q = -6$

17. $6w - 8z = 16$
$3w - 4z = 8$

18. $c + d = 6$
$c - d = 0$

19. $2u + 4v = -6$
$u + 2v = 3$

20. $3a + b = -1$
$-3a + b = 5$

21. $2x + y = 6$
$3x - 2y = 16$

22. $3y - z = -6$
$-3y - z = 6$

23. $c + 2d = -2$
$-2c - 5d = 3$

24. $3r - 2s = 1$
$2r - 3s = 9$

25. The sum of two numbers is 12. The difference of the same two numbers is −4. Find the numbers.

26. Twice a number minus a second number is −1. Twice the second number added to three times the first number is 9. Find the two numbers.

3-2 Practice

Solving Systems of Equations Algebraically

Solve each system of equations by using substitution.

1. $2x + y = 4$
$3x + 2y = 1$

2. $x - 3y = 9$
$x + 2y = -1$

3. $g + 3h = 8$
$\frac{1}{3}g + h = 9$

4. $2a - 4b = 6$
$-a + 2b = -3$

5. $2m + n = 6$
$5m + 6n = 1$

6. $4x - 3y = -6$
$-x - 2y = 7$

7. $u - 2v = \frac{1}{2}$
$-u + 2v = 5$

8. $x - 3y = 16$
$4x - y = 9$

9. $w + 3z = 1$
$3w - 5z = -4$

Solve each system of equations by using elimination.

10. $2r + s = 5$
$3r - s = 20$

11. $2m - n = -1$
$3m + 2n = 30$

12. $6x + 3y = 6$
$8x + 5y = 12$

13. $3j - k = 10$
$4j - k = 16$

14. $2x - y = -4$
$-4x + 2y = 6$

15. $2g + h = 6$
$3g - 2h = 16$

16. $2t + 4v = 6$
$-t - 2v = -3$

17. $3x - 2y = 12$
$2x + \frac{2}{3}y = 14$

18. $\frac{1}{2}x + 3y = 11$
$8x - 5y = 17$

Solve each system of equations by using either substitution or elimination.

19. $8x + 3y = -5$
$10x + 6y = -13$

20. $8q - 15r = -40$
$4q + 2r = 56$

21. $3x - 4y = 12$
$\frac{1}{3}x - \frac{4}{9}y = \frac{4}{3}$

22. $4b - 2d = 5$
$-2b + d = 1$

23. $s + 3y = 4$
$s = 1$

24. $4m - 2p = 0$
$-3m + 9p = 5$

25. $5g + 4k = 10$
$-3g - 5k = 7$

26. $0.5x + 2y = 5$
$x - 2y = -8$

27. $h - z = 3$
$-3h + 3z = 6$

SPORTS For Exercises 28 and 29, use the following information.

Last year the volleyball team paid \$5 per pair for socks and \$17 per pair for shorts on a total purchase of \$315. This year they spent \$342 to buy the same number of pairs of socks and shorts because the socks now cost \$6 a pair and the shorts cost \$18.

28. Write a system of two equations that represents the number of pairs of socks and shorts bought each year.

29. How many pairs of socks and shorts did the team buy each year?

3-2 Reading to Learn Mathematics

Solving Systems of Equations Algebraically

Pre-Activity **How can systems of equations be used to make consumer decisions?**

Read the introduction to Lesson 3-2 at the top of page 116 in your textbook.

- How many more minutes of long distance time did Yolanda use in February than in January?
- How much more were the February charges than the January charges?
- Using your answers for the questions above, how can you find the rate per minute?

Reading the Lesson

1. Suppose that you are asked to solve the system of equations at the right by the substitution method.

$4x - 5y = 7$
$3x + y = -9$

The first step is to solve one of the equations for one variable in terms of the other. To make your work as easy as possible, which equation would you solve for which variable? Explain.

2. Suppose that you are asked to solve the system of equations at the right by the elimination method.

$2x + 3y = -2$
$7x - y = 39$

To make your work as easy as possible, which variable would you eliminate? Describe how you would do this.

Helping You Remember

3. The substitution method and elimination method for solving systems both have several steps, and it may be difficult to remember them. You may be able to remember them more easily if you notice what the methods have in common. What step is the same in both methods?

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3-2 Enrichment

Using Coordinates

From one observation point, the line of sight to a downed plane is given by $y = x - 1$. This equation describes the distance from the observation point to the plane in a straight line. From another observation point, the line of sight is given by $x + 3y = 21$. What are the coordinates of the point at which the crash occurred?

Solve the system of equations $\begin{cases} y = x - 1 \\ x + 3y = 21 \end{cases}$.

$$\begin{aligned} x + 3y &= 21 \\ x + 3(x - 1) &= 21 && \text{Substitute } x - 1 \text{ for } y. \\ x + 3x - 3 &= 21 \\ 4x &= 24 \\ x &= 6 \end{aligned}$$

$$\begin{aligned} x + 3y &= 21 \\ 6 + 3y &= 21 && \text{Substitute 6 for } x. \\ 3y &= 15 \\ y &= 5 \end{aligned}$$

The coordinates of the crash are (6, 5).

Solve the following.

1. The lines of sight to a forest fire are as follows.

From Ranger Station A: $3x + y = 9$
From Ranger Station B: $2x + 3y = 13$
Find the coordinates of the fire.

2. An airplane is traveling along the line $x - y = -1$ when it sees another airplane traveling along the line $5x + 3y = 19$. If they continue along the same lines, at what point will their flight paths cross?

3. Two mine shafts are dug along the paths of the following equations.

$x - y = 1400$
$2x + y = 1300$

If the shafts meet at a depth of 200 feet, what are the coordinates of the point at which they meet?

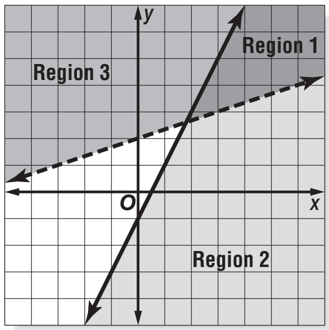
3-3 Study Guide and Intervention

Solving Systems of Inequalities by Graphing

Graph Systems of Inequalities To solve a system of inequalities, graph the inequalities in the same coordinate plane. The solution set is represented by the intersection of the graphs.

Example **Solve the system of inequalities by graphing.**

$y \le 2x - 1$ **and** $y > \frac{x}{3} + 2$



The solution of $y \le 2x - 1$ is Regions 1 and 2.

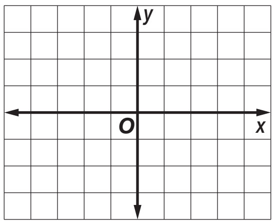
The solution of $y > \frac{x}{3} + 2$ is Regions 1 and 3.

The intersection of these regions is Region 1, which is the solution set of the system of inequalities.

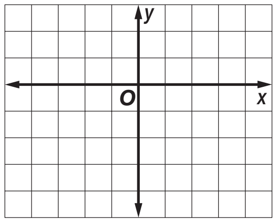
Exercises

Solve each system of inequalities by graphing.

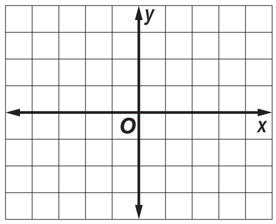
1. $x - y \le 2$
$x + 2y \ge 1$



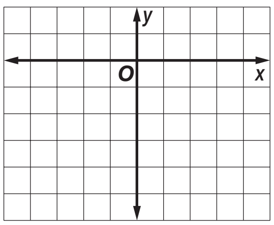
2. $3x - 2y \le -1$
$x + 4y \ge -12$



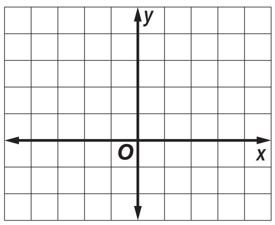
3. $|y| \le 1$
$x > 2$



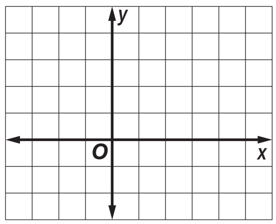
4. $y \ge \frac{x}{2} - 3$
$y < 2x$



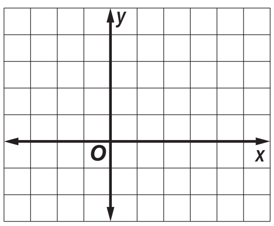
5. $y < \frac{x}{3} + 2$
$y < -2x + 1$



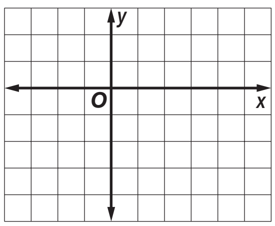
6. $y \ge -\frac{x}{4} + 1$
$y < 3x - 1$



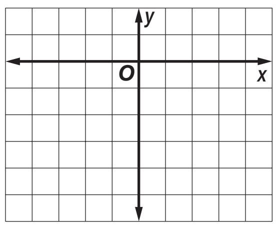
7. $x + y \ge 4$
$2x - y > 2$



8. $x + 3y < 3$
$x - 2y \ge 4$



9. $x - 2y > 6$
$x + 4y < -4$

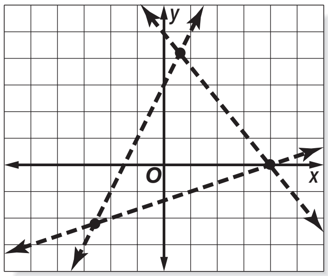


3-3 Study Guide and Intervention *(continued)*

Solving Systems of Inequalities by Graphing

Find Vertices of a Polygonal Region Sometimes the graph of a system of inequalities forms a bounded region. You can find the vertices of the region by a combination of the methods used earlier in this chapter: graphing, substitution, and/or elimination.

Example **Find the coordinates of the vertices of the figure formed by $5x + 4y < 20$, $y < 2x + 3$, and $x - 3y < 4$.**



Graph the boundary of each inequality. The intersections of the boundary lines are the vertices of a triangle.

The vertex (4, 0) can be determined from the graph. To find the coordinates of the second and third vertices, solve the two systems of equations

$$y = 2x + 3 \qquad\qquad y = 2x + 3$$
$$5x + 4y = 20 \quad \text{and} \quad x - 3y = 4$$

For the first system of equations, rewrite the first equation in standard form as $2x - y = -3$. Then multiply that equation by 4 and add to the second equation.

$2x - y = -3$ Multiply by 4. $8x - 4y = -12$

$5x + 4y = 20$ $\quad (+)\ 5x + 4y = 20$

$$13x = 8$$
$$x = \frac{8}{13}$$

Then substitute $x = \frac{8}{13}$ in one of the original equations and solve for y.

$$2\left(\frac{8}{13}\right) - y = -3$$
$$\frac{16}{13} - y = -3$$
$$y = \frac{55}{13}$$

The coordinates of the second vertex are $\left(\frac{8}{13}, 4\frac{3}{13}\right)$.

For the second system of equations, use substitution.

Substitute $2x + 3$ for y in the second equation to get

$$x - 3(2x + 3) = 4$$
$$x - 6x - 9 = 4$$
$$-5x = 13$$
$$x = -\frac{13}{5}$$

Then substitute $x = -\frac{13}{5}$ in the first equation to solve for y.

$$y = 2\left(-\frac{13}{5}\right) + 3$$
$$y = -\frac{26}{5} + 3$$
$$y = -\frac{11}{5}$$

The coordinates of the third vertex are $\left(-2\frac{3}{5}, -2\frac{1}{5}\right)$.

Thus, the coordinates of the three vertices are (4, 0), $\left(\frac{8}{13}, 4\frac{3}{13}\right)$, and $\left(-2\frac{3}{5}, -2\frac{1}{5}\right)$.

Exercises

Find the coordinates of the vertices of the figure formed by each system of inequalities.

1. $y \le -3x + 7$

$y < \frac{1}{2}x$

$y > -2$

2. $x > -3$

$y < -\frac{1}{3}x + 3$

$y > x - 1$

3. $y < -\frac{1}{2}x + 3$

$y > \frac{1}{2}x + 1$

$y < 3x + 10$

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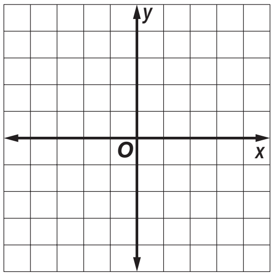
3-3 Skills Practice

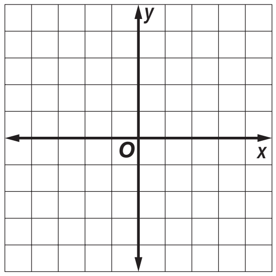
Solving Systems of Inequalities by Graphing

Solve each system of inequalities by graphing.

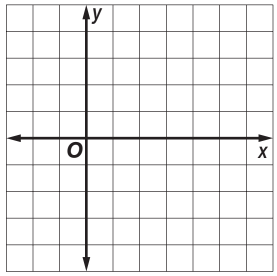
1. $x < 1$
$y \geq -1$



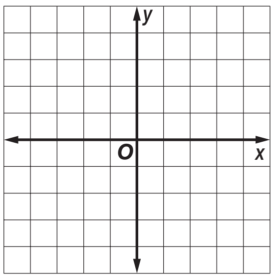
2. $x \geq -3$
$y \geq -3$



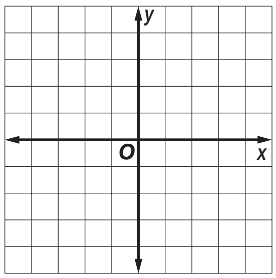
3. $x \leq 2$
$x > 4$



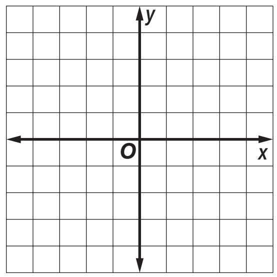
4. $y \geq x$
$y \geq -x$



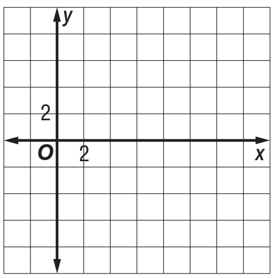
5. $y < -4x$
$y \geq 3x - 2$



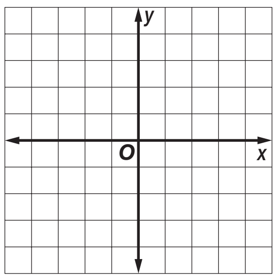
6. $x - y \geq -1$
$3x - y \leq 4$



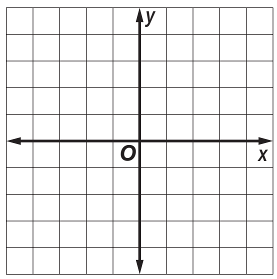
7. $y < 3$
$x + 2y < 12$



8. $y < -2x + 3$
$y \leq x - 2$



9. $x - y \leq 4$
$2x + y < 4$



Find the coordinates of the vertices of the figure formed by each system of inequalities.

10. $y < 0$
$x < 0$
$y \geq -x - 1$

11. $y < 3 - x$
$y \geq 3$
$x > -5$

12. $x \geq -2$
$y > x - 2$
$x + y \leq 2$

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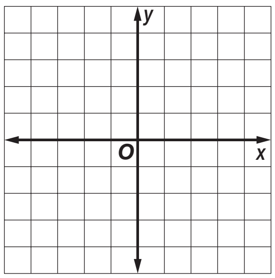
3-3 Practice

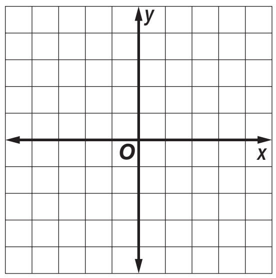
Solving Systems of Inequalities by Graphing

Solve each system of inequalities by graphing.

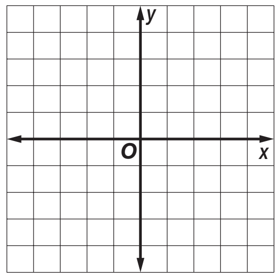
1. $y + 1 < -x$
$y \geq 1$



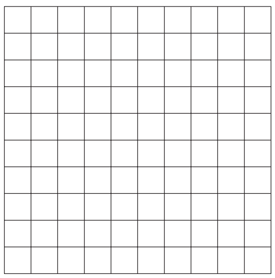
2. $x > -2$
$2y \geq 3x + 6$



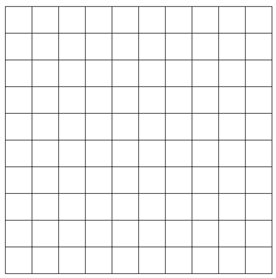
3. $y \leq 2x - 3$
$y \leq -\frac{1}{2}x + 2$



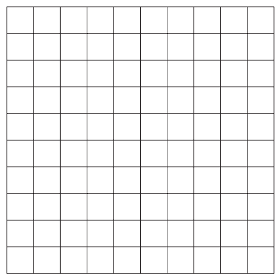
4. $x + y > -2$
$3x - y \geq -2$



5. $|y| \leq 1$
$y < x - 1$



6. $3y > 4x$
$2x - 3y > -6$



Find the coordinates of the vertices of the figure formed by each system of inequalities.

7. $y \geq 1 - x$
$y \leq x - 1$
$x \leq 3$

8. $x - y \leq 2$
$x + y \leq 2$
$x \geq -2$

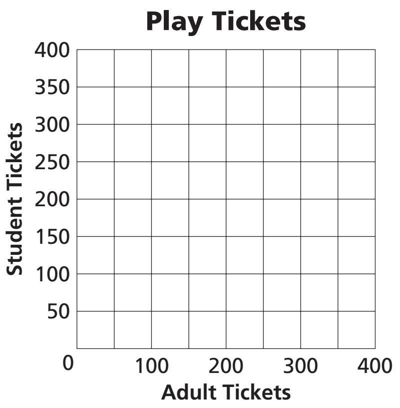
9. $y \geq 2x - 2$
$2x + 3y \geq 6$
$y < 4$

DRAMA For Exercises 10 and 11, use the following information.

The drama club is selling tickets to its play. An adult ticket costs $15 and a student ticket costs $11. The auditorium will seat 300 ticket-holders. The drama club wants to collect at least $3630 from ticket sales.

10. Write and graph a system of four inequalities that describe how many of each type of ticket the club must sell to meets its goal.

11. List three different combinations of tickets sold that satisfy the inequalities.



NAME ____ DATE ____________ PERIOD _____

3-3 Reading to Learn Mathematics

Solving Systems of Inequalities by Graphing

Pre-Activity **How can you determine whether your blood pressure is in a normal range?**

Read the introduction to Lesson 3-3 at the top of page 123 in your textbook.

Satish is 37 years old. He has a blood pressure reading of 135/99. Is his blood pressure within the normal range? Explain.

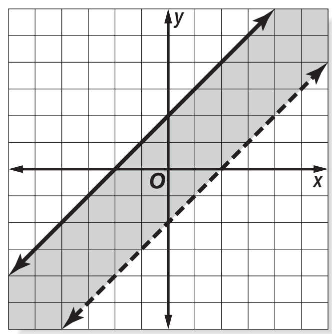
Reading the Lesson

1. Without actually drawing the graph, describe the boundary lines for the system of inequalities shown at the right.

$$|x| < 3$$
$$|y| \le 5$$

2. Think about how the graph would look for the system given above. What will be the shape of the shaded region? (It is not necessary to draw the graph. See if you can imagine it without drawing anything. If this is difficult to do, make a rough sketch to help you answer the question.)

3. Which system of inequalities matches the graph shown at the right?



A. $x - y \le -2$
$x - y > 2$

B. $x - y \ge -2$
$x - y < 2$

C. $x + y \le -2$
$x + y > 2$

D. $x - y > -2$
$x - y \le 2$

Helping You Remember

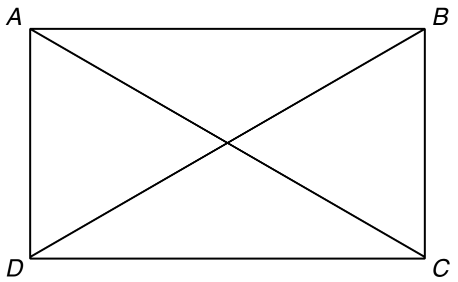
4. To graph a system of inequalities, you must graph two or more boundary lines. When you graph each of these lines, how can the inequality symbols help you remember whether to use a dashed or solid line?

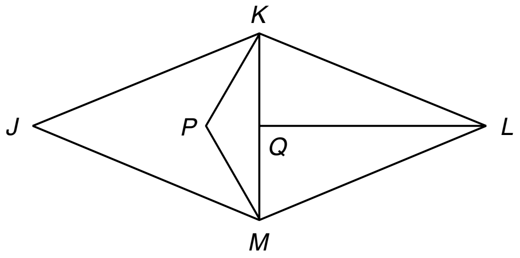
Lesson 3-3

3-3 Enrichment

Tracing Strategy

Try to trace over each of the figures below without tracing the same segment twice.

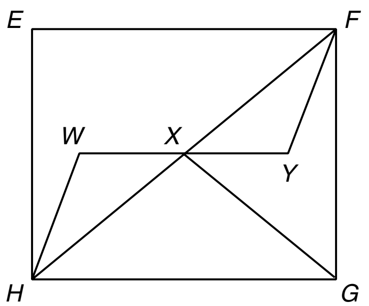




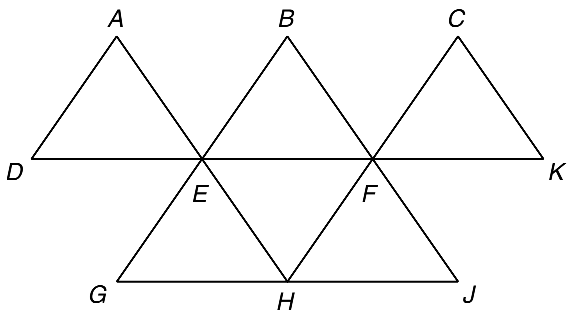
The figure at the left cannot be traced, but the one at the right can. The rule is that a figure is traceable if it has no more than two points where an odd number of segments meet. The figure at the left has three segments meeting at each of the four corners. However, the figure at the right has only two points, L and Q, where an odd number of segments meet.

Determine if each figure can be traced without tracing the same segment twice. If it can, then name the starting point and name the segments in the order they should be traced.

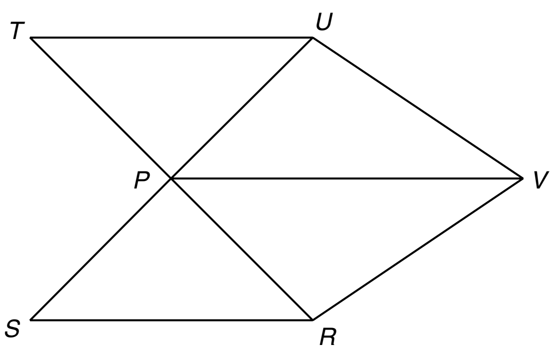
1.



2.



3.



3-4 Study Guide and Intervention

Linear Programming

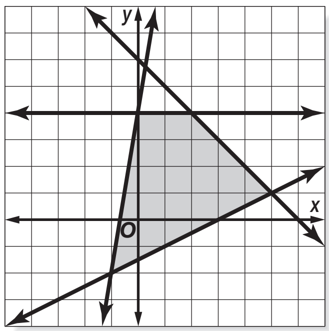
Maximum and Minimum Values When a system of linear inequalities produces a bounded polygonal region, the *maximum* or *minimum* value of a related function will occur at a vertex of the region.

Example **Graph the system of inequalities. Name the coordinates of the vertices of the feasible region. Find the maximum and minimum values of the function $f(x, y) = 3x + 2y$ for this polygonal region.**

$y \leq 4$
$y \leq -x + 6$
$y \geq \frac{1}{2}x - \frac{3}{2}$
$y \leq 6x + 4$

First find the vertices of the bounded region. Graph the inequalities.

The polygon formed is a quadrilateral with vertices at (0, 4), (2, 4), (5, 1), and (−1, −2). Use the table to find the maximum and minimum values of $f(x, y) = 3x + 2y$.



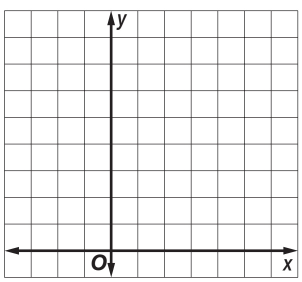
(x, y)	3x + 2y	f(x, y)
(0, 4)	3(0) + 2(4)	8
(2, 4)	3(2) + 2(4)	14
(5, 1)	3(5) + 2(1)	17
(−1, −2)	3(−1) + 2(−2)	−7

The maximum value is 17 at (5, 1). The minimum value is −7 at (−1, −2).

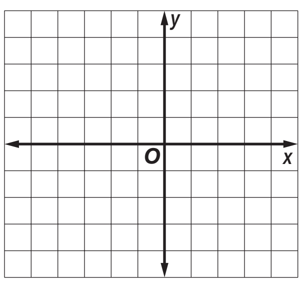
Exercises

Graph each system of inequalities. Name the coordinates of the vertices of the feasible region. Find the maximum and minimum values of the given function for this region.

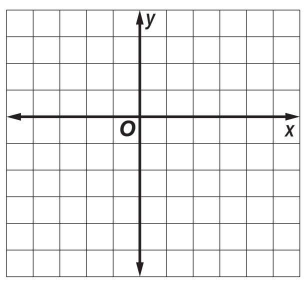
1. $y \geq 2$
$1 \leq x \leq 5$
$y \leq x + 3$
$f(x, y) = 3x - 2y$



2. $y \geq -2$
$y \geq 2x - 4$
$x - 2y \geq -1$
$f(x, y) = 4x - y$



3. $x + y \geq 2$
$4y \leq x + 8$
$y \geq 2x - 5$
$f(x, y) = 4x + 3y$



Lesson 3-4

3-4 Study Guide and Intervention *(continued)*

Linear Programming

Real-World Problems When solving **linear programming** problems, use the following procedure.

1. Define variables.
2. Write a system of inequalities.
3. Graph the system of inequalities.
4. Find the coordinates of the vertices of the feasible region.
5. Write an expression to be maximized or minimized.
6. Substitute the coordinates of the vertices in the expression.
7. Select the greatest or least result to answer the problem.

Example **A painter has exactly 32 units of yellow dye and 54 units of green dye. He plans to mix as many gallons as possible of color A and color B. Each gallon of color A requires 4 units of yellow dye and 1 unit of green dye. Each gallon of color B requires 1 unit of yellow dye and 6 units of green dye. Find the maximum number of gallons he can mix.**

Step 1 Define the variables.

x = the number of gallons of color A made

y = the number of gallons of color B made

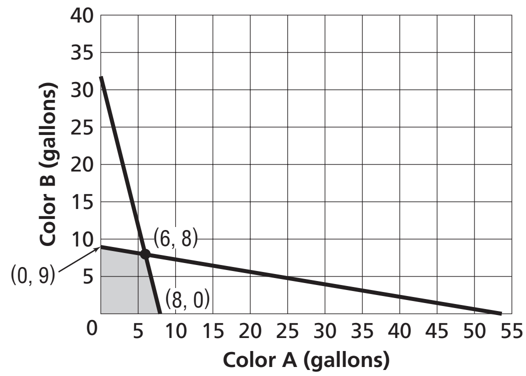
Step 2 Write a system of inequalities.

Since the number of gallons made cannot be negative, $x \geq 0$ and $y \geq 0$.

There are 32 units of yellow dye; each gallon of color A requires 4 units, and each gallon of color B requires 1 unit.

So $4x + y \leq 32$.

Similarly for the green dye, $x + 6y \leq 54$.



Steps 3 and 4 Graph the system of inequalities and find the coordinates of the vertices of the feasible region.

The vertices of the feasible region are (0, 0), (0, 9), (6, 8), and (8, 0).

Steps 5–7 Find the maximum number of gallons, $x + y$, that he can make.

The maximum number of gallons the painter can make is 14, 6 gallons of color A and 8 gallons of color B.

(x, y)	$x + y$	$f(x, y)$
(0, 0)	0 + 0	0
(0, 9)	0 + 9	9
(6, 8)	6 + 8	14
(8, 0)	8 + 0	8

Exercises

1. **FOOD** A delicatessen has 8 pounds of plain sausage and 10 pounds of garlic-flavored sausage. The deli wants to make as much bratwurst as possible. Each pound of bratwurst requires $\frac{3}{4}$ pound of plain sausage and $\frac{1}{4}$ pound of garlic-flavored sausage. Find the maximum number of pounds of bratwurst that can be made.

2. **MANUFACTURING** Machine A can produce 30 steering wheels per hour at a cost of $16 per hour. Machine B can produce 40 steering wheels per hour at a cost of $22 per hour. At least 360 steering wheels must be made in each 8-hour shift. What is the least cost involved in making 360 steering wheels in one shift?

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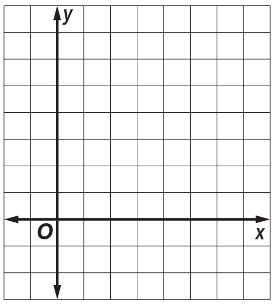
3-4 Skills Practice

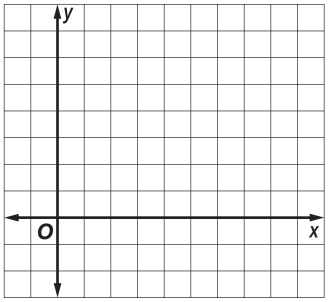
Linear Programming

Graph each system of inequalities. Name the coordinates of the vertices of the feasible region. Find the maximum and minimum values of the given function for this region.

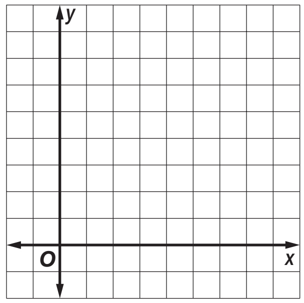
1. $x \geq 2$
$x \leq 5$
$y \geq 1$
$y \leq 4$
$f(x, y) = x + y$



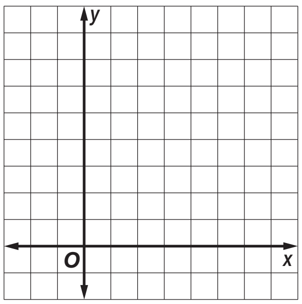
2. $x \geq 1$
$y \leq 6$
$y \geq x - 2$
$f(x, y) = x - y$



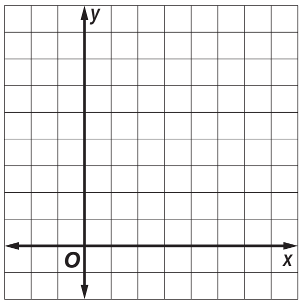
3. $x \geq 0$
$y \geq 0$
$y \leq 7 - x$
$f(x, y) = 3x + y$



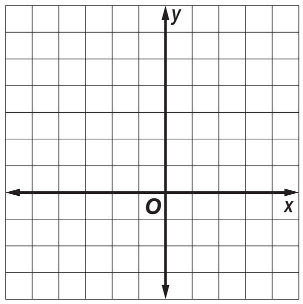
4. $x \geq -1$
$x + y \leq 6$
$f(x, y) = x + 2y$



5. $y \leq 2x$
$y \geq 6 - x$
$y \leq 6$
$f(x, y) = 4x + 3y$



6. $y \geq -x - 2$
$y \geq 3x + 2$
$y \leq x + 4$
$f(x, y) = -3x + 5y$



7. MANUFACTURING A backpack manufacturer produces an internal frame pack and an external frame pack. Let x represent the number of internal frame packs produced in one hour and let y represent the number of external frame packs produced in one hour. Then the inequalities $x + 3y \leq 18$, $2x + y \leq 16$, $x \geq 0$, and $y \geq 0$ describe the constraints for manufacturing both packs. Use the profit function $f(x) = 50x + 80y$ and the constraints given to determine the maximum profit for manufacturing both backpacks for the given constraints.

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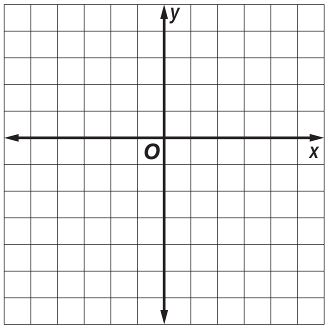
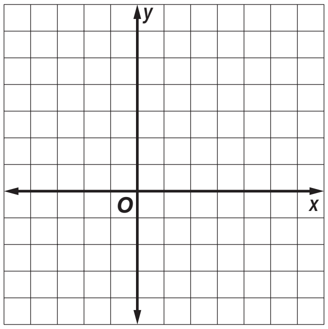
3-4 Practice
Linear Programming

Graph each system of inequalities. Name the coordinates of the vertices of the feasible region. Find the maximum and minimum values of the given function for this region.

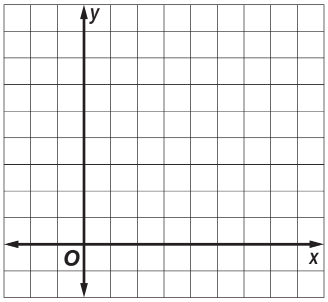
1. $2x - 4 \le y$
$-2x - 4 \le y$
$y \le 2$
$f(x, y) = -2x + y$



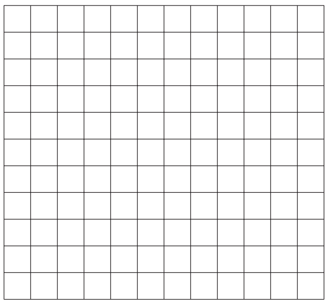
2. $3x - y \le 7$
$2x - y \ge 3$
$y \ge x - 3$
$f(x, y) = x - 4y$



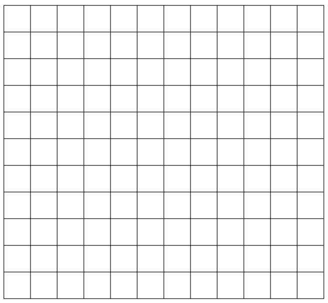
3. $x \ge 0$
$y \ge 0$
$y \le 6$
$y \le -3x + 15$
$f(x, y) = 3x + y$



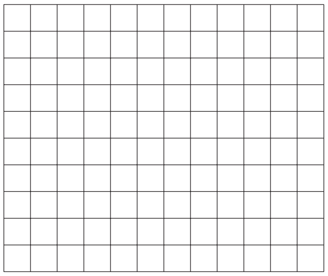
4. $x \le 0$
$y \le 0$
$4x + y \ge -7$
$f(x, y) = -x - 4y$



5. $y \le 3x + 6$
$4y + 3x \le 3$
$x \ge -2$
$f(x, y) = -x + 3y$



6. $2x + 3y \ge 6$
$2x - y \le 2$
$x \ge 0$
$y \ge 0$
$f(x, y) = x + 4y + 3$



PRODUCTION For Exercises 7–9, use the following information.

A glass blower can form 8 simple vases or 2 elaborate vases in an hour. In a work shift of no more than 8 hours, the worker must form at least 40 vases.

7. Let s represent the hours forming simple vases and e the hours forming elaborate vases. Write a system of inequalities involving the time spent on each type of vase.

8. If the glass blower makes a profit of \$30 per hour worked on the simple vases and \$35 per hour worked on the elaborate vases, write a function for the total profit on the vases.

9. Find the number of hours the worker should spend on each type of vase to maximize profit. What is that profit?

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3-4 Reading to Learn Mathematics

Linear Programming

Pre-Activity **How is linear programming used in scheduling work?**

Read the introduction to Lesson 3-4 at the top of page 129 in your textbook.

Name two or more facts that indicate that you will need to use inequalities to model this situation.

Reading the Lesson

1. Complete each sentence.

a. When you find the feasible region for a linear programming problem, you are solving a system of linear ____________________ called ____________________. The points in the feasible region are ____________________ of the system.

b. The corner points of a polygonal region are the ____________________ of the feasible region.

2. A polygonal region always takes up only a limited part of the coordinate plane. One way to think of this is to imagine a circle or rectangle that the region would fit inside. In the case of a polygonal region, you can always find a circle or rectangle that is large enough to contain all the points of the polygonal region. What word is used to describe a region that can be enclosed in this way? What word is used to describe a region that is too large to be enclosed in this way?

3. How do you find the corner points of the polygonal region in a linear programming problem?

4. What are some everyday meanings of the word *feasible* that remind you of the mathematical meaning of the term *feasible region*?

Lesson 3-4

Helping You Remember

5. Look up the word *constraint* in a dictionary. If more than one definition is given, choose the one that seems closest to the idea of a *constraint* in a linear programming problem. How can this definition help you to remember the meaning of *constraint* as it is used in this lesson?

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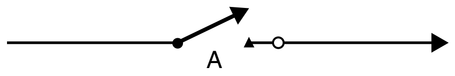
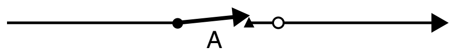
3-4 Enrichment

Computer Circuits and Logic

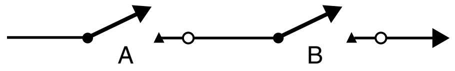
Computers operate according to the laws of logic. The circuits of a computer can be described using logic.

1. 

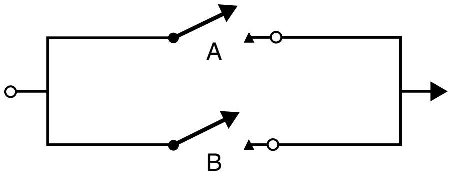
With switch A open, no current flows. The value 0 is assigned to an open switch.

2. 

With switch A closed, current flows. The value 1 is assigned to a closed switch.

3. 

With switches A and B open, no current flows. This circuit can be described by the conjunction, A · B.

4. 

In this circuit, current flows if either A or B is closed. This circuit can be described by the disjunction, A + B.

A	B	A + B
0	0	0
0	1	1
1	0	1
1	1	1

Truth tables are used to describe the flow of current in a circuit. The table at the left describes the circuit in diagram 4. According to the table, the only time current does not flow through the circuit is when both switches A and B are open.

Draw a circuit diagram for each of the following.

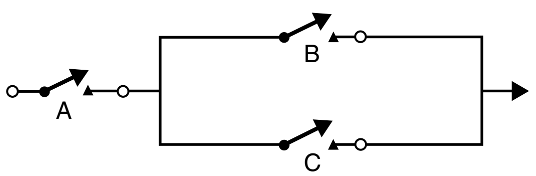
1. (A · B) + C

2. (A + B) · C

3. (A + B) · (C + D)

4. (A · B) + (C · D)

5. Construct a truth table for the following circuit.



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3-5 Study Guide and Intervention

Solving Systems of Equations in Three Variables

Systems in Three Variables Use the methods used for solving systems of linear equations in two variables to solve systems of equations in three variables. A system of three equations in three variables can have a unique solution, infinitely many solutions, or no solution. A solution is an **ordered triple**.

Example **Solve this system of equations.**

$$\begin{aligned} 3x + y - z &= -6 \\ 2x - y + 2z &= 8 \\ 4x + y - 3z &= -21 \end{aligned}$$

Step 1 Use elimination to make a system of two equations in two variables.

$3x + y - z = -6$	First equation	$2x - y + 2z = 8$	Second equation
$(+)\ 2x - y + 2z = 8$	Second equation	$(+)\ 4x + y - 3z = -21$	Third equation
$5x + z = 2$	Add to eliminate y.	$6x - z = -13$	Add to eliminate y.

Step 2 Solve the system of two equations.

$5x + z = 2$
$(+)\ 6x - z = -13$
$11x = -11$ Add to eliminate z.
$x = -1$ Divide both sides by 11.

Substitute -1 for x in one of the equations with two variables and solve for z.

$5x + z = 2$ Equation with two variables
$5(-1) + z = 2$ Replace x with -1.
$-5 + z = 2$ Multiply.
$z = 7$ Add 5 to both sides.

The result so far is $x = -1$ and $z = 7$.

Step 3 Substitute -1 for x and 7 for z in one of the original equations with three variables.

$3x + y - z = -6$ Original equation with three variables
$3(-1) + y - 7 = -6$ Replace x with -1 and z with 7.
$-3 + y - 7 = -6$ Multiply.
$y = 4$ Simplify.

The solution is $(-1, 4, 7)$.

Exercises

Solve each system of equations.

1. $2x + 3y - z = 0$
$x - 2y - 4z = 14$
$3x + y - 8z = 17$

2. $2x - y + 4z = 11$
$x + 2y - 6z = -11$
$3x - 2y - 10z = 11$

3. $x - 2y + z = 8$
$2x + y - z = 0$
$3x - 6y + 3z = 24$

4. $3x - y - z = 5$
$3x + 2y - z = 11$
$6x - 3y + 2z = -12$

5. $2x - 4y - z = 10$
$4x - 8y - 2z = 16$
$3x + y + z = 12$

6. $x - 6y + 4z = 2$
$2x + 4y - 8z = 16$
$x - 2y = 5$

Lesson 3-5

3-5 Study Guide and Intervention *(continued)*

Solving Systems of Equations in Three Variables

Real-World Problems

Example **The Laredo Sports Shop sold 10 balls, 3 bats, and 2 bases for \$99 on Monday. On Tuesday they sold 4 balls, 8 bats, and 2 bases for \$78. On Wednesday they sold 2 balls, 3 bats, and 1 base for \$33.60. What are the prices of 1 ball, 1 bat, and 1 base?**

First define the variables.

x = price of 1 ball
y = price of 1 bat
z = price of 1 base

Translate the information in the problem into three equations.

$10x + 3y + 2z = 99$
$4x + 8y + 2z = 78$
$2x + 3y + z = 33.60$

Subtract the second equation from the first equation to eliminate z.

$$\begin{array}{rl} & 10x + 3y + 2z = 99 \\ (-) & 4x + 8y + 2z = 78 \\ \hline & 6x - 5y \quad\quad = 21 \end{array}$$

Multiply the third equation by 2 and subtract from the second equation.

$$\begin{array}{rl} & 4x + 8y + 2z = 78 \\ (-) & 4x + 6y + 2z = 67.20 \\ \hline & 2y = 10.80 \\ & y = 5.40 \end{array}$$

Substitute 5.40 for y in the equation $6x - 5y = 21$.

$$\begin{aligned} 6x - 5(5.40) &= 21 \\ 6x &= 48 \\ x &= 8 \end{aligned}$$

Substitute 8 for x and 5.40 for y in one of the original equations to solve for z.

$$\begin{aligned} 10x + 3y + 2z &= 99 \\ 10(8) + 3(5.40) + 2z &= 99 \\ 80 + 16.20 + 2z &= 99 \\ 2z &= 2.80 \\ z &= 1.40 \end{aligned}$$

So a ball costs \$8, a bat \$5.40, and a base \$1.40.

Exercises

1. **FITNESS TRAINING** Carly is training for a triathlon. In her training routine each week, she runs 7 times as far as she swims, and she bikes 3 times as far as she runs. One week she trained a total of 232 miles. How far did she run that week?

2. **ENTERTAINMENT** At the arcade, Ryan, Sara, and Tim played video racing games, pinball, and air hockey. Ryan spent \$6 for 6 racing games, 2 pinball games, and 1 game of air hockey. Sara spent \$12 for 3 racing games, 4 pinball games, and 5 games of air hockey. Tim spent \$12.25 for 2 racing games, 7 pinball games, and 4 games of air hockey. How much did each of the games cost?

3. **FOOD** A natural food store makes its own brand of trail mix out of dried apples, raisins, and peanuts. One pound of the mixture costs \$3.18. It contains twice as much peanuts by weight as apples. One pound of dried apples costs \$4.48, a pound of raisins \$2.40, and a pound of peanuts \$3.44. How many ounces of each ingredient are contained in 1 pound of the trail mix?

NAME ______________________________ DATE ____________ PERIOD _____

3-5 Skills Practice

Solving Systems of Equations in Three Variables

Solve each system of equations.

1. $2a + c = -10$
$b - c = 15$
$a - 2b + c = -5$

2. $x + y + z = 3$
$13x + 2z = 2$
$-x - 5z = -5$

3. $2x + 5y + 2z = 6$
$5x - 7y = -29$
$z = 1$

4. $x + 4y - z = 1$
$3x - y + 8z = 0$
$x + 4y - z = 10$

5. $-2z = -6$
$2x + 3y - z = -2$
$x + 2y + 3z = 9$

6. $3x - 2y + 2z = -2$
$x + 6y - 2z = -2$
$x + 2y = 0$

7. $-x - 5z = -5$
$y - 3x = 0$
$13x + 2z = 2$

8. $-3r + 2t = 1$
$4r + s - 2t = -6$
$r + s + 4t = 3$

9. $x - y + 3z = 3$
$-2x + 2y - 6z = 6$
$y - 5z = -3$

10. $5m + 3n + p = 4$
$3m + 2n = 0$
$2m - n + 3p = 8$

11. $2x + 2y + 2z = -2$
$2x + 3y + 2z = 4$
$x + y + z = -1$

12. $x + 2y - z = 4$
$3x - y + 2z = 3$
$-x + 3y + z = 6$

13. $3x - 2y + z = 1$
$-x + y - z = 2$
$5x + 2y + 10z = 39$

14. $3x - 5y + 2z = -12$
$x + 4y - 2z = 8$
$-3x + 5y - 2z = 12$

15. $2x + y + 3z = -2$
$x - y - z = -3$
$3x - 2y + 3z = -12$

16. $2x - 4y + 3z = 0$
$x - 2y - 5z = 13$
$5x + 3y - 2z = 19$

17. $-2x + y + 2z = 2$
$3x + 3y + z = 0$
$x + y + z = 2$

18. $x - 2y + 2z = -1$
$x + 2y - z = 6$
$-3x + 6y - 6z = 3$

19. The sum of three numbers is 18. The sum of the first and second numbers is 15, and the first number is 3 times the third number. Find the numbers.

NAME ____ DATE ____________ PERIOD _____

3-5 Practice

Solving Systems of Equations in Three Variables

Solve each system of equations.

1. $2x - y + 2z = 15$
$-x + y + z = 3$
$3x - y + 2z = 18$

2. $x - 4y + 3z = -27$
$2x + 2y - 3z = 22$
$4z = -16$

3. $a + b = 3$
$-b + c = 3$
$a + 2c = 10$

4. $3m - 2n + 4p = 15$
$m - n + p = 3$
$m + 4n - 5p = 0$

5. $2g + 3h - 8j = 10$
$g - 4h = 1$
$-2g - 3h + 8j = 5$

6. $2x + y - z = -8$
$4x - y + 2z = -3$
$-3x + y + 2z = 5$

7. $2x - 5y + z = 5$
$3x + 2y - z = 17$
$4x - 3y + 2z = 17$

8. $2x + 3y + 4z = 2$
$5x - 2y + 3z = 0$
$x - 5y - 2z = -4$

9. $p + 4r = -7$
$p - 3q = -8$
$q + r = 1$

10. $4x + 4y - 2z = 8$
$3x - 5y + 3z = 0$
$2x + 2y - z = 4$

11. $d + 3e + f = 0$
$-d + 2e + f = -1$
$4d + e - f = 1$

12. $4x + y + 5z = -9$
$x - 4y - 2z = -2$
$2x + 3y - 2z = 21$

13. $5x + 9y + z = 20$
$2x - y - z = -21$
$5x + 2y + 2z = -21$

14. $2x + y - 3z = -3$
$3x + 2y + 4z = 5$
$-6x - 3y + 9z = 9$

15. $3x + 3y + z = 10$
$5x + 2y + 2z = 7$
$3x - 2y + 3z = -9$

16. $2u + v + w = 2$
$-3u + 2v + 3w = 7$
$-u - v + 2w = 7$

17. $x + 5y - 3z = -18$
$3x - 2y + 5z = 22$
$-2x - 3y + 8z = 28$

18. $x - 2y + z = -1$
$-x + 2y - z = 6$
$-4y + 2z = 1$

19. $2x - 2y - 4z = -2$
$3x - 3y - 6z = -3$
$-2x + 3y + z = 7$

20. $x - y + 9z = -27$
$2x - 4y - z = -1$
$3x + 6y - 3z = 27$

21. $2x - 5y - 3z = 7$
$-4x + 10y + 2z = 6$
$6x - 15y - z = -19$

22. The sum of three numbers is 6. The third number is the sum of the first and second numbers. The first number is one more than the third number. Find the numbers.

23. The sum of three numbers is −4. The second number decreased by the third is equal to the first. The sum of the first and second numbers is −5. Find the numbers.

24. SPORTS Alexandria High School scored 37 points in a football game. Six points are awarded for each touchdown. After each touchdown, the team can earn one point for the extra kick or two points for a 2-point conversion. The team scored one fewer 2-point conversions than extra kicks. The team scored 10 times during the game. How many touchdowns were made during the game?

3-5 Reading to Learn Mathematics

Solving Systems of Equations in Three Variables

Pre-Activity **How can you determine the number and type of medals U.S. Olympians won?**

Read the introduction to Lesson 3-5 at the top of page 138 in your textbook.

At the 1996 Summer Olympics in Atlanta, Georgia, the United States won 101 medals. The U.S. team won 12 more gold medals than silver and 7 fewer bronze medals than silver. Using the same variables as those in the introduction, write a system of equations that describes the medals won for the 1996 Summer Olympics.

Reading the Lesson

1. The planes for the equations in a system of three linear equations in three variables determine the number of solutions. Match each graph description below with the description of the number of solutions of the system. (Some of the items on the right may be used more than once, and not all possible types of graphs are listed.)

a. three parallel planes ________ **I.** one solution

b. three planes that intersect in a line ________ **II.** no solutions

c. three planes that intersect in one point ________ **III.** infinite solutions

d. one plane that represents all three equations ________

2. Suppose that three classmates, Monique, Josh, and Lilly, are studying for a quiz on this lesson. They work together on solving a system of equations in three variables, x, y, and z, following the algebraic method shown in your textbook. They first find that $z = 3$, then that $y = -2$, and finally that $x = -1$. The students agree on these values, but disagree on how to write the solution. Here are their answers:

Monique: $(3, -2, -1)$ Josh: $(-2, -1, 3)$ Lilly: $(-1, -2, 3)$

a. How do you think each student decided on the order of the numbers in the ordered triple?

b. Which student is correct?

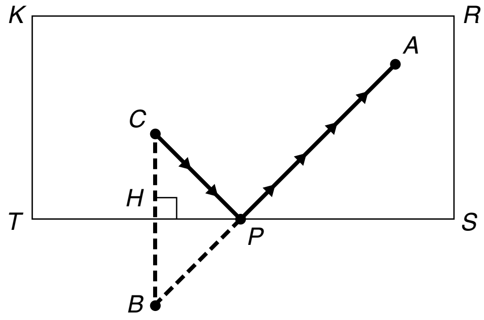
Helping You Remember

3. How can you remember that obtaining the equation $0 = 0$ indicates a system with infinitely many solutions, while obtaining an equation such as $0 = 8$ indicates a system with no solutions?

3-5 Enrichment

Billiards

The figure at the right shows a billiard table. The object is to use a cue stick to strike the ball at point C so that the ball will hit the sides (or cushions) of the table at least once before hitting the ball located at point A. In playing the game, you need to locate point P.



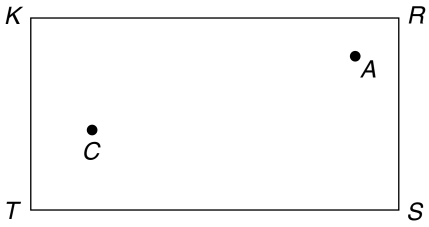
Step 1 Find point B so that $\overline{BC} \perp \overline{ST}$ and $\overline{BH} \cong \overline{CH}$. B is called the reflected image of C in $\overline{ST}$.

Step 2 Draw $\overline{AB}$.

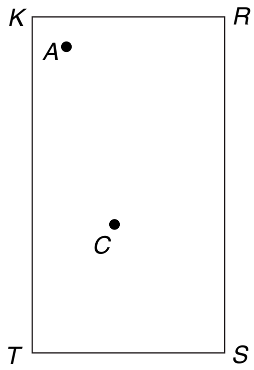
Step 3 $\overline{AB}$ intersects $\overline{ST}$ at the desired point P.

For each billiards problem, the cue ball at point *C* must strike the indicated cushion(s) and then strike the ball at point *A*. Draw and label the correct path for the cue ball using the process described above.

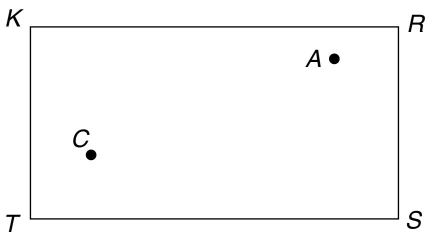
1. cushion $\overline{KR}$



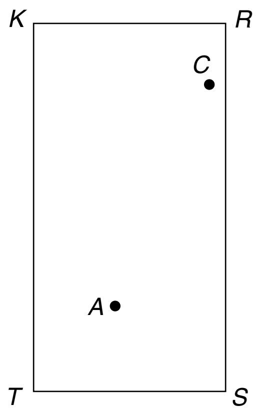
2. cushion $\overline{RS}$



3. cushion $\overline{TS}$, then cushion $\overline{RS}$



4. cushion $\overline{KT}$, then cushion $\overline{RS}$



NAME ____________________ DATE ____________ PERIOD ______

3 Chapter 3 Test, Form 1

SCORE ______

Assessment

Write the letter for the correct answer in the blank at the right of each question.

1. A system of equations may *not* have

A. exactly one solution. **B.** no solution.
C. infinitely many solutions. **D.** exactly two solutions. **1.** ______

Choose the correct description of each system of equations.

A. consistent and independent **B.** consistent and dependent
C. inconsistent **D.** inconsistent and dependent

2. $4x + 2y = -6$
$2x + y = 8$ **2.** ______

3. $3x + y = 3$
$x - 2y = 4$ **3.** ______

To solve each system of equations, which expression could be substituted for x into the first equation?

4. $5x - 2y = 8$
$x - y = 1$

A. $y + 1$ **B.** $y - 1$
C. $-\frac{2}{5}x + \frac{5}{8}$ **D.** $x - 1$ **4.** ______

5. $4x + 3y = 12$
$x + 3y = -5$

A. $3y - 5$ **B.** $y + 35$
C. $-3y - 5$ **D.** $-\frac{1}{3}x - \frac{5}{3}$ **5.** ______

The first equation of each system is multiplied by 4. By what number would you multiply the second equation in order to eliminate the x variable by adding?

6. $3x - 2y = 4$
$4x + 5y = 28$

A. 3 **B.** -3
C. 4 **D.** -4 **6.** ______

7. $-6x - 3y = 12$
$8x + 2y = 16$

A. 3 **B.** -3
C. 6 **D.** -6 **7.** ______

Solve each system of equations.

8. $3x - 2y = 5$
$x = y + 2$

A. (1, 1) **B.** (2, 0)
C. (0, -2) **D.** (1, -1) **8.** ______

9. $2x + 3y = 5$
$3x - 2y = 1$

A. (3, 4) **B.** (-2, 3)
C. (1, 1) **D.** (4, -1) **9.** ______

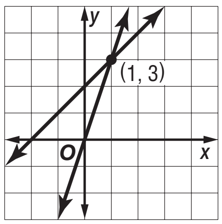
10. Which system of equations is graphed?

A. $y - \frac{1}{3}x = 0$
$x - y = -2$

B. $y - 3x = 0$
$x - y = -2$

C. $y - 3x = 0$
$x - y = 2$

D. $y - \frac{1}{3}x = 0$
$x - y = 2$



10. ______

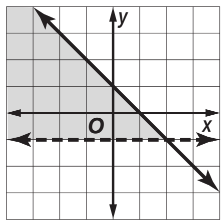
11. Which system of inequalities is graphed?

A. $y > -1$
$y \geq -2x+1$

B. $y > -1$
$y \leq -2x + 1$

C. $y \geq -1$
$y \geq -2x + 1$

D. $y > -1$
$y < -2x + 1$



11. ______

3 Chapter 3 Test, Form 1 *(continued)*

12. Find the coordinates of the vertices of the figure formed by the system $y \geq 0$, $x \geq 0$, $y \leq 2$, and $x \leq 3$.

A. (0, 0), (3, 0), (3, 2), (0, 2)
B. (0, 0), (2, 0), (2, 3), (0, 3)
C. (0, 0), (−3, 0), (−3, −2), (0, −2)
D. (0, 0), (−2, 0), (−2, −3), (0, −3)

12. ______

Use the system of inequalities $y \geq 0$, $x \geq 0$, and $y \leq -2x + 4$.

13. Find the coordinates of the vertices of the feasible region.

A. (0, 0), (−2, 0), (0, −4)
B. (0, 0), (2, 0), (0, 4)
C. (0, 0), (4, 0), (0, 2)
D. (0, 0), (−4, 0), (0, 2)

13. ______

14. Find the maximum value of $f(x, y) = 3x + y$ for the feasible region.

A. 2 **B.** 4 **C.** 6 **D.** 12

14. ______

15. Find the minimum value of $f(x, y) = 3x + y$ for the feasible region.

A. 6 **B.** 4 **C.** 2 **D.** 0

15. ______

For Questions 16–18, use the following information. A college arena sells tickets to students and to the public. Student tickets are $8 each and general public tickets are $32 each. The college reserves at least 5000 tickets for students. The arena seats 18,000.

16. Let s represent the number of student tickets and p represent the number of general public tickets. Which system of inequalities represents the number of tickets sold?

A. $s \geq 0, p \geq 0, s + p \leq 18{,}000$
B. $s \geq 5000, p \geq 0, s + p \leq 18{,}000$
C. $s \geq 8, p \geq 32, s + p \geq 40$
D. $s \geq 0, p \geq 0, s + p \geq 18{,}000$

16. ______

17. How many general public tickets should the college sell to maximize revenue (amount collected)?

A. 18,000 **B.** 0 **C.** 13,000 **D.** 5000

17. ______

18. What is the maximum revenue?

A. $456,000 **B.** $416,000 **C.** $40,000 **D.** $576,000

18. ______

19. What is the value of y in the solution of the system of equations?

$$2x + y + z = 13$$
$$2x - y - 3z = -3$$
$$x + 2y + 4z = 20$$

A. 1 **B.** 2 **C.** 3 **D.** 4

19. ______

20. The 300 students at Holmes School work a total of 5000 hours each month. Each student in group A works 10 hours, each in group B works 15 hours, and each in group C works 20 hours each month. There are twice as many students in group B as in group A. Which equation would *not* be included in the system used to solve this problem?

A. $A = 2B$
B. $10A + 15B + 20C = 5000$
C. $A + B + C = 300$
D. $B = 2A$

20. ______

Bonus Find the area of the region defined by the system of inequalities $x \geq 0$, $y \geq 0$, and $x + 2y \leq 4$.

B: ______________

NAME ______________________ DATE __________ PERIOD ______

3 Chapter 3 Test, Form 2A

SCORE ______

Assessment

Write the letter for the correct answer in the blank at the right of each question.

1. The system of equations $y = -3x + 5$ and $y = -3x - 7$ has

A. exactly one solution. **B.** no solution.
C. infinitely many solutions. **D.** exactly two solutions. **1.** ______

Choose the correct description of each system of equations.

A. consistent and independent **B.** consistent and dependent
C. inconsistent **D.** inconsistent and dependent **2.** ______

2. $2x - y = 4$
$4x - 2y = 6$

3. $9x - 3y = 15$
$6x = 2y + 10$ **3.** ______

To solve each system of equations, which expression could be substituted for *y* into the first equation?

4. $5x + 3y = 9$
$4x + y = 8$

A. $12x - 3y$ **B.** $-\frac{3}{5}x + 3$
C. $4x - 8$ **D.** $8 - 4x$ **4.** ______

5. $3x + 6y = 12$
$2x - y = 5$

A. $\frac{1}{2}y + \frac{5}{2}$ **B.** $2x - 5$
C. $2x + 5$ **D.** $12y - 5$ **5.** ______

6. The first equation of the system is multiplied by 3. By what number would you multiply the second equation to eliminate the *x* variable by adding?

$4x - 3y = 6$
$6x + 1y = 10$

A. -2 **B.** 2 **C.** 9 **D.** -9 **6.** ______

7. The first equation of the system is multiplied by 2. By what number would you multiply the second equation to eliminate the *y* variable by adding?

$4x - 3y = 6$
$6x + 1y = 10$

A. -2 **B.** -1 **C.** 6 **D.** -3 **7.** ______

For Questions 8 and 9, solve each system of equations.

8. $5x + 2y = 1$
$y = 1 - 3x$

A. $(1, -2)$ **B.** $(1, 2)$
C. $\left(0, \frac{1}{2}\right)$ **D.** $(-2, 1)$ **8.** ______

9. $3x + 4y = 12$
$2x - 3y = -9$

A. $(3, 0)$ **B.** $(4, 0)$
C. $(-1, 4)$ **D.** $(0, 3)$ **9.** ______

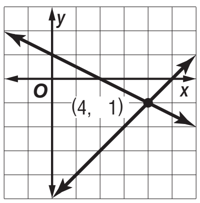
10. Which system of equations is graphed?

A. $x + y = 5$
$x - 2y = 2$

B. $x + y = 5$
$x + 2y = 2$

C. $x - y = 5$
$x - 2y = 2$

D. $x - y = 5$
$x + 2y = 2$



10. ______

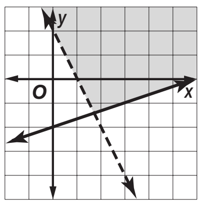
11. Which system of inequalities is graphed?

A. $2x - y \geq 2$
$x + 3y \leq 6$

B. $2x + y > 2$
$x - 3y \leq 6$

C. $2x + y \geq 2$
$x - 3y < 6$

D. $2x - y < 2$
$x + 3y > 6$



11. ______

3 Chapter 3 Test, Form 2A *(continued)*

12. Find the coordinates of the vertices of the figure formed by the system $x \geq -1$, $y \geq -2$, and $2x + y \leq 6$.

A. (0, 0), (3, 0), (0, 6)
B. (−1, 8), (−1, −2), (4, −2)
C. (0, 0), (0, 3), (6, 0)
D. (−1, −2), (−1, 6), (4, 0)

12. _____

For Questions 13–15, use the system of inequalities $x \geq 2$, $y - x \geq -3$, and $x + y \leq 5$.

13. Find the coordinates of the vertices of the feasible region.

A. (2, −1), (2, 3), (4, 1) **B.** (0, −3), (0, 5), (4, 1)
C. (2, 0), (3, 0), (4, 1), (2, 3) **D.** (0, 0), (0, 5), (3, 0), (4, 1)

13. _____

14. Find the maximum value of $f(x, y) = x - 4y$ for the feasible region.

A. 14 **B.** 0 **C.** 8 **D.** 6

14. _____

15. Find the minimum value of $f(x, y) = x - 4y$ for the feasible region.

A. −2 **B.** 0 **C.** −10 **D.** −4

15. _____

16. What is the value of z in the solution of the system of equations?

$$2x + 3y + z = 9$$
$$x - 2y - z = 4$$
$$x - 3y + 2z = -3$$

A. 4 **B.** 1 **C.** −2 **D.** $\frac{3}{4}$

16. _____

An office building containing 96,000 square feet of space is to be made into apartments. There will be at most 15 one-bedroom units, each with 800 square feet of space. The remaining units, each with 1200 square feet of space, will have two bedrooms. Rent for each one-bedroom unit will be $650 and for each two-bedroom unit will be $900.

17. Let x represent the number of one-bedroom apartments and y represent the number of two-bedroom apartments. Which system of inequalities represents the number of apartments to be built?

A. $x \geq 15$, $y \geq 0$, $650x + 900y \leq 96{,}000$
B. $x \leq 15$, $y \geq 0$, $800x + 1200y \geq 96{,}000$
C. $x \leq 650$, $y \leq 900$, $800x + 1200y \leq 96{,}000$
D. $x \leq 15$, $y \geq 0$, $800x + 1200y \leq 96{,}000$

17. _____

18. How many two-bedroom apartments should be built to maximize revenue?

A. 70 **B.** 15 **C.** 80 **D.** 120

18. _____

At a university, 1200 students are enrolled in engineering. There are twice as many in electrical engineering as in mechanical engineering, and three times as many in chemical engineering as in mechanical engineering.

19. Which system of equations represents the number of students in each program?

A. $c + m + e = 1200$, $2m = e$, $3m = c$ **B.** $c + m + e = 1200$, $3m = e$, $2m = c$
C. $c + m + e = 1200$, $2e = m$, $3c = m$ **D.** $c + m + e = 1200$, $2m = e$, $3m = 2e$

19. _____

20. How many students are enrolled in the mechanical engineering program?

A. 200 **B.** 400 **C.** 600 **D.** 1200

20. _____

Bonus Find the value of x in the solution of the system of equations $x + y = \frac{9}{8}$ and $x - 2y = \frac{9}{8}$.

B: ______________

NAME ______________________ DATE ____________ PERIOD ______

3 Chapter 3 Test, Form 2B

SCORE ______

Assessment

Write the letter for the correct answer in the blank at the right of each question.

1. The system of equations $y = 2x - 3$ and $y = 4x - 3$ has

A. exactly one solution. **B.** no solution.

C. infinitely many solutions. **D.** exactly two solutions. **1.** ______

Choose the correct description of each system of equations.

A. consistent and independent **B.** consistent and dependent

C. inconsistent **D.** inconsistent and dependent **2.** ______

2. $x + 2y = 7$
$3x - 2y = 5$

3. $2x + 3y = 10$
$4x + 6y = 20$ **3.** ______

To solve each system of equations, which expression could be substituted for x into the first equation?

4. $3x - 5y = 14$
$x + 4y = 10$

A. $10 - 4y$ **B.** $4y + 10$

C. $\frac{1}{4}x + \frac{5}{2}$ **D.** $-\frac{1}{4}x + \frac{5}{2}$ **4.** ______

5. $2x + 7y = 10$
$x - 2y = 15$

A. $\frac{1}{2}x + 15$ **B.** $\frac{1}{2}x - 15$

C. $2y + 15$ **D.** $2y - 15$ **5.** ______

6. The first equation of the system is multiplied by 2. By what number would you multiply the second equation to eliminate the x variable by adding?

$6x - 5y = 21$
$4x + 7y = 15$

A. 3 **B.** −3 **C.** 2 **D.** −2 **6.** ______

7. The first equation of the system is multiplied by 4. By what number would you multiply the second equation to eliminate the y variable by adding?

$2x + 5y = 16$
$8x - 4y = 10$

A. 5 **B.** −5 **C.** 2 **D.** −2 **7.** ______

For Questions 8 and 9, solve each system of equations.

8. $4x - 3y = 14$
$y = -3x + 4$

A. (1, 1) **B.** (5, 2)

C. (−4, −10) **D.** (2, −2) **8.** ______

9. $4x - 3y = 8$
$2x + 5y = -9$

A. (−2, 1) **B.** (2, 0)

C. (0, −83) **D.** $\left(\frac{1}{2}, -2\right)$ **9.** ______

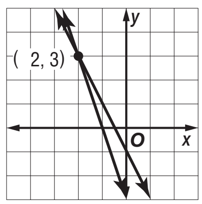
10. Which system of equations is graphed?

A. $2x + y = 1$
$-3x - y = 3$

B. $2x + y = -1$
$3x - y = 3$

C. $2x + y = 1$
$3x - y = 3$

D. $2x + y = -1$
$-3x - y = 3$



10. ______

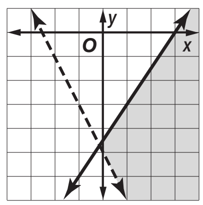
11. Which system of inequalities is graphed?

A. $2x + y \geq 5$
$3x + 2y \leq 9$

B. $2x - y \leq 5$
$3x + 2y < 9$

C. $2x + y > -5$
$3x - 2y \geq 9$

D. $-2x + y > 5$
$3x - 2y \leq 9$



11. ______

3 Chapter 3 Test, Form 2B *(continued)*

12. Find the coordinates of the vertices of the figure formed by the system $x \geq 0$, $y \geq -2$, and $2x + y \leq 4$.

A. (3, −2), (0, 4), (0, −2) **B.** (−2, 0), (4, 0), (−2, 3)
C. (0, 0), (0, 4), (2, 0) **D.** (−2, 3), (0, 4), (0, −2) **12.** ______

For Questions 13–15, use the system of inequalities $y \geq 1$, $y - x \leq 6$, and $x + 2y \leq 6$.

13. Find the coordinates of the vertices of the feasible region.

A. (−6, 0), (−2, 4), (6, 0) **B.** (−5, 1), (−2, 4), (4, 1)
C. (0, 1), (0, 3), (4, 1) **D.** (−5, 1), (−2, 4), (0, 3), (0, 1) **13.** ______

14. Find the maximum value of $f(x, y) = 2x + y$ for the feasible region.

A. 0 **B.** 11 **C.** 9 **D.** 8 **14.** ______

15. Find the minimum value of $f(x, y) = 2x + y$ for the feasible region.

A. −10 **B.** 0 **C.** −9 **D.** −4 **15.** ______

16. What is the value of z in the solution of the system of equations?

$$2x + 3y - z = 12$$
$$4x - y + z = -3$$
$$-2x + 2y + z = 3$$

A. −1 **B.** 12 **C.** 3 **D.** −2 **16.** ______

Tickets to a golf tournament are sold in advance for $40 each, and on the day of the event for $50 each. For the tournament to occur, at least 2000 of the 8000 tickets must be sold in advance.

17. Let a represent the number of advance tickets sold and d represent the number sold on the day of the tournament. Which system of inequalities represents the number of tickets sold?

A. $a \geq 2000, d \geq 0, a + d \leq 8000$ **B.** $a \geq 0, d \geq 0, a + d \leq 8000$
C. $a \geq 0, d \geq 0, a + d \leq 2000$ **D.** $a \leq 40, d \leq 50, a + d \leq 2000$ **17.** ______

18. How many advance tickets should be sold to maximize revenue?

A. 6000 **B.** 2000 **C.** 4000 **D.** 8000 **18.** ______

A local gas station sells low-grade (ℓ), mid-grade (m), and premium (p) gasoline. Mid-grade gasoline costs $0.10 per gallon more than low-grade, and premium gasoline costs $0.10 per gallon more than mid-grade gasoline. Five gallons of low-grade gas cost $9.

19. Which system of equations represents the cost of each type of gasoline?

A. $5\ell + m = 9, m = \ell + 0.10, p = m + 0.10$
B. $5\ell = 9, m = \ell - 0.10, p = m - 0.10$
C. $5\ell = 9, m = \ell + 0.10, p = m + 0.10$
D. $0.10\ell + 0.10m + 5p = 9, 0.10\ell + m = 0, 0.10m + p = 0$ **19.** ______

20. What is the cost of one gallon of premium gasoline?

A. $1.80 **B.** $1.90 **C.** $2.00 **D.** $2.10 **20.** ______

Bonus Solve the system of equations. **B:** ____________

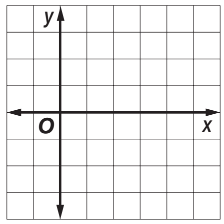
$a + b = 6$ $c + d = 4$ $f + a = 2$
$b + c = 5$ $d + f = 3$

Chapter 3 Test, Form 2C

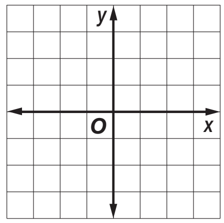
SCORE ______

Solve each system of equations by graphing.

1. $3x - 2y = 6$
$2x + y = 4$

1. 

2. $3x - y = 1$
$3y = 9x + 6$

2. 

Describe each system of equations as *consistent and independent*, *consistent and dependent*, or *inconsistent*.

3. $y = 2x + 5$
$y = -3x + 4$

4. $2x - y = 5$
$6x - 3y = 15$

Solve each system of equations by using substitution.

5. $3x + 7y = 19$
$x + y = 5$

6. $x + 3y = 12$
$5x + y = 4$

Solve each system of equations by using elimination.

7. $3x - 2y = 4$
$2x + 3y = 7$

8. $4x - y = 10$
$5x + 2y = 6$

3. ____________________

4. ____________________

5. ____________________

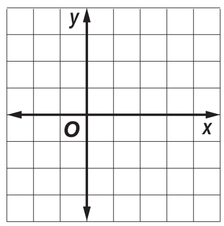
6. ____________________

7. ____________________

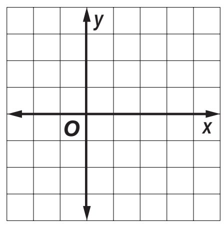
8. ____________________

Solve each system of inequalities by graphing.

9. $4x - 3y < 9$
$2x + y \geq 5$

9. 

10. $y \leq \frac{3}{2}x - 2$
$2y \geq x - 4$

10. 

Find the coordinates of the vertices of the figure formed by each system of inequalities.

11. $y \geq -3$
$y \leq 2x + 1$
$x \leq 2$

12. $x \leq 3$
$y \leq 2x + 4$
$x + y \geq -2$
$3y \leq -2x + 12$

11. ____________________

12. ____________________

3 Chapter 3 Test, Form 2C *(continued)*

Use the system of inequalities $x \geq -2$, $x + y \leq 7$, and $y \leq 2x + 1$.

13. Find the coordinates of the vertices of the feasible region. **13.** ____________

14. Find the maximum and minimum values of the function $f(x, y) = 3x - y$ for the feasible region. **14.** ____________

The area of a parking lot is 600 square meters. A car requires 6 square meters and a bus requires 30 square meters of space. The lot can handle a maximum of 60 vehicles.

15. Let c represent the number of cars and b represent the number of buses. Write a system of inequalities to represent the number of vehicles that can be parked. **15.** ____________

16. If a car costs \$3 and a bus costs \$8 to park in the lot, determine the number of each vehicle to maximize the amount collected. **16.** ____________

Solve each system of equations.

17. $x + 2y - 3z = 5$
$x - y + 2z = -3$
$x + y - z = 2$ **17.** ____________

18. $3x + y + 2z = 1$
$2x - y + z = -3$
$x + y - 4z = -3$ **18.** ____________

A printing company sells small packages of personalized stationery for \$7 each, medium packages for \$12 each, and large packages for \$15 each. Yesterday, the company sold 9 packages of stationery, collecting a total of \$86. Three times as many medium packages were sold as large packages.

19. Let s represent the number of small packages, m the number of medium packages, and ℓ the number of large packages. Write a system of three equations that represents the number of packages sold. **19.** ____________

20. Find the number of each size package sold. **20.** ____________

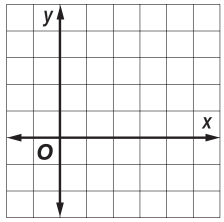
Bonus Find the perimeter of the region defined by the system of inequalities:
$-2 \leq x \leq 5$
$-4 \leq y \leq -1$ **B:** ____________

3 Chapter 3 Test, Form 2D

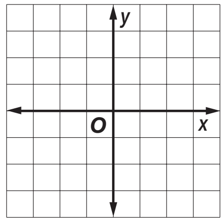
SCORE ______

Solve each system of equations by graphing.

1. $x + y = 5$
$2y = x - 2$

1. 

2. $y = \frac{2}{3}x - 1$
$2x + y = -1$

2. 

Describe each system of equations as *consistent and independent*, *consistent and dependent*, or *inconsistent*.

3. $3x - 4y = 5$
$6x - 8y = -5$

4. $2x - 7y = 14$
$x + 3y = 6$

3. ______________

4. ______________

Solve each system of equations by using substitution.

5. $4x - y = 10$
$y = 3x - 6$

6. $x - y = 6$
$3x + 2y = -22$

5. ______________

6. ______________

Solve each system of equations by using elimination.

7. $5x + 2y = 1$
$2x + 3y = 7$

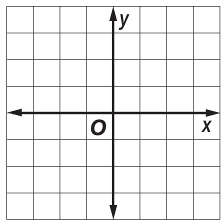
8. $5x - 3y = 16$
$2x + 7y = -10$

7. ______________

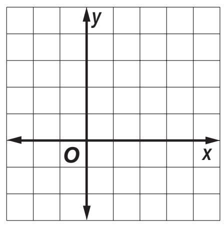
8. ______________

Solve each system of inequalities by graphing.

9. $2x - 3y \geq -3$
$3y > -2x - 6$

9. 

10. $x + 3y \geq 6$
$y < \frac{3}{2}x - 2$

10. 

Find the coordinates of the vertices of the figure formed by the solution of each system of inequalities.

11. $x \geq -3$
$y \geq -2$
$2x + y \leq -2$

12. $y \leq 3$
$4y \leq 3x + 12$
$x + y \geq -4$
$y \geq 2x - 1$

11. ______________

12. ______________

3 Chapter 3 Test, Form 2D *(continued)*

For Questions 13 and 14, use the system of inequalities $y \le 7, x + y \ge 2, y \ge 2x + 5$.

13. Find the coordinates of the vertices of the feasible region. **13.** ______________

14. Find the maximum and minimum values of the function $f(x, y) = 3x + y$ for the feasible region. **14.** ______________

Kristin earns \$7 per hour at a video store and \$10 per hour at a landscaping company. She must work at least 4 hours per week at the video store, but the total number of hours she works at both jobs cannot be greater than 15.

15. Let v represent the number of hours working at the video store and ℓ represent the number of hours working at the landscaping company. Write a system of inequalities to represent the number of hours worked in one week. **15.** ______________

16. Determine Kristin's maximum weekly earnings (before deductions). **16.** ______________

Solve each system of equations.

17. $2x + y + z = 4$
$3x - y + 4z = 11$
$x - y + 5z = 20$

18. $3x - y - z = 12$
$2x + 3y + z = 5$
$x + 2y - z = 9$

17. ______________

18. ______________

The price of a sweatshirt at a local shop is twice the price of a pair of shorts. The price of a T-shirt at the shop is \$4 less than the price of a pair of shorts. Brad purchased 3 sweatshirts, 2 pairs of shorts, and 5 T-shirts for a total cost of \$136.

19. Let w represent the price of one sweatshirt, t represent the price of one T-shirt, and h represent the price of one pair of shorts. Write a system of three equations that represents the prices of the clothing. **19.** ______________

20. Find the cost of one sweatshirt. **20.** ______________

Bonus Solve the system of equations. **B:** ______________

$$\frac{2}{3}x + \frac{1}{2}y = 4$$
$$-\frac{1}{6}x + \frac{1}{8}y = -3$$

NAME ________________________ DATE ____________ PERIOD ______

3 Chapter 3 Test, Form 3

SCORE ______

Solve each system of equations by graphing.

1. $\frac{1}{3}x - \frac{1}{2}y = 1$
$\frac{1}{2}x - \frac{1}{4}y = -\frac{1}{2}$

2. $x + 0.5y = 0.5$
$2.5 = 1.5x - y$

Describe each system of equations as *consistent and independent*, *consistent and dependent*, or *inconsistent*.

3. $2x + \frac{2}{3}y = \frac{10}{3}$
$9x + 3y = 15$

4. $\frac{3}{14}x - \frac{1}{14}y = \frac{1}{2}$
$6x = 2(y + 5)$

Solve each system of equations by using substitution.

5. $\frac{1}{2}x + \frac{1}{3}y = 1$
$\frac{2}{3}y - x = 6$

6. $\frac{4}{5}a + b + \frac{3}{4} = 0$
$2a = \frac{5}{4} - \frac{5}{2}b$

Solve each system of equations by using elimination.

7. $\frac{2}{3}x + 4y = -1$
$\frac{1}{2}x - 3y = -\frac{9}{4}$

8. $0.2c + 1.5d = -2.7$
$1.2c - 0.5d = 2.8$

Solve each system of inequalities by graphing.

9. $x + y \leq 5$
$x + y > -3$
$x - 2y \leq 6$
$x - 2y \geq -2$

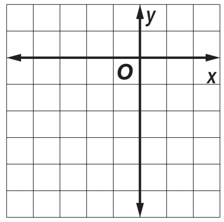
10. $|x| \leq 1$
$|y + 1| \leq 3$
$x + 3y > -6$

Find the coordinates of the vertices of the figure formed by the solution of each system of inequalities.

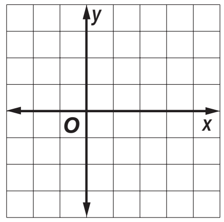
11. $x \leq 2$
$-4 \leq y \leq 3$
$x + y \geq -3$

12. $-2 \leq x \leq 5$
$\frac{1}{2}x + \frac{1}{2}y \leq 1$
$2x + y \geq -2$
$y \geq -4$

1.



2.



3. ____________________

4. ____________________

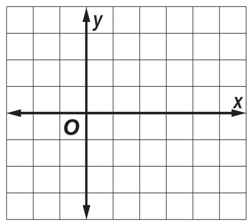
5. ____________________

6. ____________________

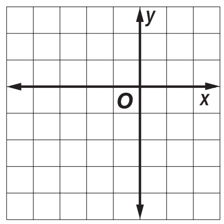
7. ____________________

8. ____________________

9.



10.



11. ____________________

12. ____________________

3 Chapter 3 Test, Form 3 *(continued)*

For Questions 13 and 14, use the system of inequalities $x \geq -3$, $x - y \geq -4$, $x \leq y + 1$, and $\frac{1}{3}x + \frac{1}{2}y \leq 2$.

13. Find the coordinates of the vertices of the feasible region. **13.** ______________

14. Find the maximum and minimum values of the function $f(x, y) = 3x - \frac{1}{2}y$ for the feasible region. **14.** ______________

A dog food manufacturer wants to advertise its products. A magazine charges \$60 per ad and requires the purchase of at least three ads. A radio station charges \$150 per commercial minute and requires the purchase of at least four minutes. Each magazine ad reaches 12,000 people while each commercial minute reaches 16,000 people. At most \$900 can be spent on advertising.

15. Let a represent the number of magazine ads and m represent the number of commercial minutes. Write a system of inequalities that represents the advertising plan for the company. **15.** ______________

16. How many ads and commercial minutes should be purchased to reach the most people? How many people would this be? **16.** ______________

Solve each system of equations.

17.
$$4x - 6y + z = 1$$
$$3x + \frac{1}{2}y - \frac{2}{3}z = \frac{9}{2}$$
$$5x - 3y + 2z = -\frac{11}{2}$$

18.
$$10a + b - \frac{1}{3}c = -6$$
$$2a - 3b + \frac{1}{7}c = \frac{32}{5}$$
$$-\frac{5}{2}a + \frac{1}{4}c = \frac{19}{4}$$

17. ______________

18. ______________

An electronic repair shop offers three types of service: on-site, at-store, and by-mail. On-site service costs 3 times as much as at-store service. By-mail service costs \$10 less than at-store service. Last week, the shop completed 15 services on-site, 40 services at-store, and 5 services by mail for total sales of \$2650.

19. Let s represent the cost of one on-site repair, a represent the cost of one at-store repair, and m represent the cost of one by-mail repair service. Write a system of three equations that represents the cost of each repair service. **19.** ______________

20. Determine the cost of one on-site repair service. **20.** ______________

Bonus Solve the system of equations. **B:** ______________

$$\frac{3}{x} + \frac{4}{y} = \frac{5}{2}$$
$$\frac{6}{x} - \frac{1}{y} = \frac{1}{2}$$

3 Chapter 3 Open-Ended Assessment

SCORE _____

Demonstrate your knowledge by giving a clear, concise solution to each problem. Be sure to include all relevant drawings and justify your answers. You may show your solution in more than one way or investigate beyond the requirements of the problem.

1. The square of Janet's age is 400 more than the square of the sum of Kim's and Sue's ages. Kim's and Sue's ages total 10 less than Janet's age. Find the square of the sum of the ages of Janet, Kim, and Sue. Explain your reasoning.

2. A feasible region has vertices at (0, 8), (6, 0), and (0, 0).

a. Write a system of inequalities whose graph forms this feasible region.

b. Explain how to find the maximum and minimum values of $f(x, y) = x - y$ for the region.

3. Explain what the algebraic solution of a system of two linear equations is and what that means in terms of the graphs of the equations of the system.

4. When describing a system of two linear equations, a student indicated that the system was inconsistent and dependent. Discuss the meaning of the student's description. Then assess the student's understanding of these terms.

5. A business owner asks the company finance manager to develop a formula, or function, for the cost incurred in the production of their products, and another function for the revenue (money collected) that the company earns when the products are sold. In preparing the report for the owner, the finance manager prepares a graph which shows both the cost function and the revenue function. The business owner, on seeing the graph of two parallel lines, makes a major business decision. What might that decision be and why would it be made?

6. Explain what it means for a system of two linear inequalities to have no solution. Sketch a graph of such a system and write the system of inequalities represented by your graph.

NAME ______________________ DATE ____________ PERIOD ______

3 Chapter 3 Vocabulary Test/Review

SCORE ______

bounded region	elimination method	linear programming	system of inequalities
consistent system	feasible region	ordered triple	unbounded region
constraints	inconsistent system	substitution method	vertices
dependent system	independent system	system of equations	

Choose from the terms above to complete each sentence.

1. A system of equations with no solutions is called a(n) ______________________.

2. A(n) ______________________ is a system of equations with an infinite number of solutions.

3. (3, −2, 7) is an example of a(n) ______________________.

4. If your first step in solving a system of equations is to solve one of the equations for one variable in terms of the others, you are using the ______________________.

5. In a linear programming problem, the inequalities are called ______________________.

6. A set of two or more inequalities that are considered together is called a(n) ______________________.

7. If you are solving a system of equations and one of your steps is to add the equations, you are using the ______________________.

8. If you are solving a system of equations by graphing and your graph shows two intersecting lines, the system can be described both as a(n) ______________________ and as a(n) ______________________.

9. Graphing, the substitution method, and the elimination method are all methods for solving a(n) ______________________.

10. The process of finding the maximum or minimum values of a function defined by a feasible region is called ______________________.

In your own words—
Define each term.

11. feasible region

12. unbounded region

NAME ______________________ DATE __________ PERIOD ______

3 Chapter 3 Quiz

SCORE ______

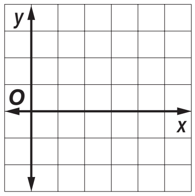
(Lessons 3–1 and 3–2)

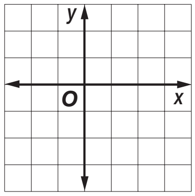
Assessment

1. Solve the system $x + y = 4$ and $x - 2y = 1$ by graphing.
2. Graph the system $y = x + 2$ and $2y = 2x - 4$. Describe it as *consistent and independent*, *consistent and dependent*, or *inconsistent*.
3. Solve the system $3y = 2x$, $y = \frac{2}{3}x - 2$, by using substitution.
4. Solve the system of equations by $4x - 5y = -2$, and $3x + 2y = -13$ using elimination.
5. **Standardized Test Practice** Compare the quantity in Column A and the quantity in Column B. Then determine whether:
   A. the quantity in Column A is greater,
   B. the quantity in Column B is greater,
   C. the two quantities are equal, or
   D. the relationship cannot be determined from the information given.

$$2x - 3y = 2$$
$$5x + y = 22$$

Column A	**Column B**
x	y

1. 

2. 

3. ______________

4. ______________

5. ______________

NAME ______________________ DATE __________ PERIOD ______

3 Chapter 3 Quiz

SCORE ______

(Lesson 3–3)

Solve each system of inequalities by graphing.

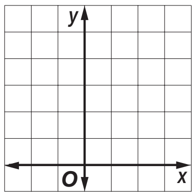
1. $x - y > -3$
   $2x + y < 6$

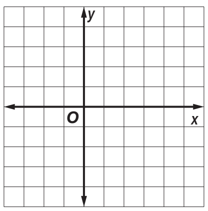
2. $y \leq \frac{1}{3}x + 1$
   $y > \frac{1}{3}x - 2$

Find the coordinates of the vertices of the figure formed by each system of inequalities.

3. $x \geq 0$
   $y \geq 0$
   $2x + y \leq 4$

4. $y \geq -2$
   $x \leq 3$
   $x + y \leq 2$
   $y \leq 2x - 4$

1. 

2. 

3. ______________

4. ______________

NAME ______________________ DATE ____________ PERIOD ______

Chapter 3 Quiz

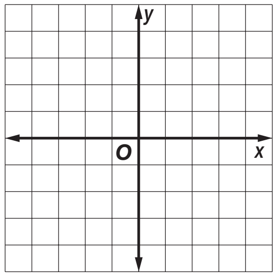
SCORE ______

(Lesson 3–4)

1. A feasible region has vertices at $(-2, 3)$, $(1, 6)$, $(1, -1)$, and $(-3, -2)$. Find the maximum and minimum of the function $f(x, y) = -x + 2y$ over this region.

1. ______________

2. Graph the system of inequalities. Name the coordinates of the vertices of the feasible region. Find the maximum and minimum values of the given function for this region.
$4y \leq x + 12$
$-4y \leq 3x + 4$
$5x - 4y \leq 4$
$f(x, y) = x - 5y$

2. 

A clothing company makes jackets and pants. Each jacket requires 1 hour of cutting and 4 hours of sewing. Each pair of pants requires 2 hours of cutting and 2 hours of sewing. The total time per day available for cutting is 20 hours and for sewing is 32 hours.

3. Let j represent the number of jackets and let p represent the number of pairs of pants. Write a system of inequalities to represent the number of items that can be produced.

3. ______________

4. If the profit on a jacket is \$14 and the profit on a pair of pants is \$8, determine the number of each that should be made each day to maximize profit. What is the maximum profit?

4. ______________

NAME ______________________ DATE ____________ PERIOD ______

Chapter 3 Quiz

SCORE ______

(Lesson 3–5)

Solve each system of equations.

1. $4x + 6y - 3z = 20$
$x - 5y + z = -15$
$-7x + y + 2z = 1$

2. $3x - y + 2z = 1$
$-2x = -4$
$x + 3y = 11$

3. $x + 2y - z = -7$
$3x - 3y + z = 13$
$2x + 5y + 2z = 0$

1. ______________

2. ______________

3. ______________

During one month, a rental car agency rented a total of 155 cars, vans, and trucks. Nine times as many cars were rented as vans, and three times as many vans were rented as trucks.

4. Let x represent the number of cars, let y represent the number of vans, and let z represent the number of trucks. Write a system of three equations that represents the number of vehicles rented.

5. Find the number of each type of vehicle rented.

4. ______________

5. ______________

NAME ______________________ DATE ____________ PERIOD _____

3

Chapter 3 Mid-Chapter Test

SCORE _____

(Lessons 3–1 through 3–3)

Assessment

Part I ***Write the letter for the correct answer in the blank at the right of each question.***

1. Choose the correct description of the system of equations.

$3x - y = 5$
$6x = 2y + 5$

A. consistent and independent
B. consistent and dependent
C. inconsistent
D. inconsistent and dependent

1. _____

2. Solve $3x + 2y = 7$ and $x - 4y = -21$ by using substitution.

A. $(3, -1)$ **B.** $\left(\frac{7}{3}, \frac{7}{2}\right)$ **C.** $(-1, 5)$ **D.** $(1, 5)$

2. _____

Solve each system of equations by using elimination.

3. $2x + 5y = 18$
$3x - 2y = -11$

A. $(-1, 4)$ **B.** $\left(9, \frac{18}{5}\right)$
C. $(1, 4)$ **D.** $(-3, 1)$

3. _____

4. $3x - 5y = 14$
$2x + 3y = 3$

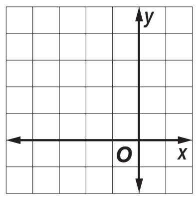
A. $(3, -1)$ **B.** $(8, 2)$
C. $(0, 1)$ **D.** $(6, -3)$

4. _____

Part II

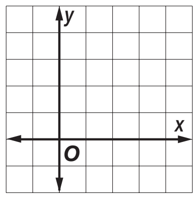
5. Solve $3x + y = -4$ and $x - 2y = -6$ by graphing.

5.



6. Solve $2x - y = -1$ and $x + y = 4$ by graphing.

6.



7. Classify the system as *consistent and independent*, *consistent and dependent*, or *inconsistent*.

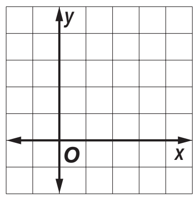
$3x - y = 5$
$6x = 2(y + 5)$

7. ______________

8. Solve the system of inequalities by graphing.

$4x - y \geq 4$
$3y < -x + 6$

8.



9. Find the coordinates of the vertices of the figure formed by the system of inequalities.

$x \geq -2$
$y \leq 6$
$y \leq -\frac{3}{2}x + 6$
$x + 2y \geq 4$

9. ______________

NAME ____________ DATE ________ PERIOD ______

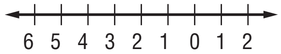
Chapter 3 Cumulative Review

(Chapters 1–3)

1. Simplify $\frac{1}{3}(6x - 21) - 4(x + 5)$. (Lesson 1-2) **1.** ________

2. Solve $7 - 2(m + 3) = 4 - m$. (Lesson 1-3) **2.** ________

3. Solve the inequality $|3 + 2x| > 7$. Then graph the solution set. (Lesson 1-6) **3.** ________



4. Find $f(2a)$ if $f(x) = -x^3 + 2x - 5$. (Lesson 2-1) **4.** ________

For Questions 5 and 6, state whether each equation or function is linear. If not, explain. (Lesson 2-2)

5. $f(x) = \frac{1}{x} + 5$ **5.** ________

6. $y + x^2 = 2$ **6.** ________

7. Write an equation for the line that passes through $(2, -3)$ and is parallel to the line whose equation is $y = -4x + 3$. (Lesson 2-4) **7.** ________

8. Evaluate $f\left(\frac{1}{8}\right)$ if $f(x) = 5 - 3x$. (Lesson 2-6) **8.** ________

9. Describe the system of equations as *consistent and independent*, *consistent and dependent*, or *inconsistent*. **9.** ________

$2x - 3y = 11$

$4x + 6y = 22$ (Lesson 3-1)

10. Solve the system of equations by using substitution. (Lesson 3-2) **10.** ________

$y = 2x + 5$

$4x - 5y = -1$

11. Solve the system of equations by using elimination. (Lesson 3-2) **11.** ________

$y - 3x = 5$

$4x - 9y = -22$

For Questions 12 and 13, use the system of inequalities $x \geq 1$, $y \geq -2$, and $x + y \leq 4$.

12. Find the coordinates of the vertices of the figure formed by the system of inequalities. (Lesson 3-3) **12.** ________

13. Find the maximum and minimum values of the function $f(x, y) = y - 3x$ for the feasible region. (Lesson 3-4) **13.** ________

14. Solve the system of equations. (Lesson 3-5) **14.** ________

$2x + y - 3z = 9$

$x - 2y + z = -8$

$x + 3y - 2z = 11$

NAME ____________________ DATE ____________ PERIOD ______

3 Standardized Test Practice

(Chapters 1–3)

Part 1: Multiple Choice

Instructions: Fill in the appropriate oval for the best answer.

1. A number is 8 more than the ratio of q and 2. What is three-fourths of the number in terms of q?

A. 6 **B.** $5 + \frac{3q}{8}$ **C.** $\frac{3q}{8}$ **D.** $6 + \frac{3q}{8}$ **1.** (A) (B) (C) (D)

2. Carlisle's salary was raised from \$19,000 to \$23,750. Find the percent of increase.

E. 20% **F.** 25% **G.** 30% **H.** 47.5% **2.** (E) (F) (G) (H)

3. Which of the following fractions is more than one-fourth but less than three-eights?

A. $\frac{2}{7}$ **B.** $\frac{2}{8}$ **C.** $\frac{3}{4}$ **D.** $\frac{1}{2}$ **3.** (A) (B) (C) (D)

4. If the area of a square is 49, the difference between the length of a diagonal and the length of a side is _____.

E. $7\sqrt{2} - 7$ **F.** 0 **G.** $\sqrt{2}$ **H.** $1 - \sqrt{2}$ **4.** (E) (F) (G) (H)

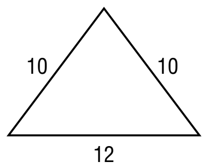
5. Which is not equal to $\sqrt{2}$?

A. $2^{\frac{1}{2}}$ **B.** $\sqrt[4]{4}$ **C.** $(\sqrt{2})^2$ **D.** $\sqrt[6]{8}$ **5.** (A) (B) (C) (D)

6. Find the area of the triangle at the right.

E. 32 **F.** 48

G. 60 **H.** 120



6. (E) (F) (G) (H)

7. If m is an integer greater than 5, then which of the following must represent an odd integer?

A. m^2 **B.** $m - 1$ **C.** $2m - 3$ **D.** $m + 2$ **7.** (A) (B) (C) (D)

8. Desiree and Mario together have \$30. Mario and Scott together have \$26. Desiree and Scott together have \$34. What is the least amount of money any of them has?

E. \$11 **F.** \$15 **G.** \$19 **H.** \$26 **8.** (E) (F) (G) (H)

9. If x, y, and z are negative integers, which of the following must be true?

A. $x(y + z) < 0$ **B.** $xyz < 0$

C. $x - (y - z) < 0$ **D.** $x < y - z$ **9.** (A) (B) (C) (D)

10. If $\frac{r + 2}{r} = \frac{4}{3}$, then $\frac{r}{2} =$ _____.

E. $\frac{1}{4}$ **F.** $\frac{2}{3}$ **G.** 1 **H.** 3 **10.** (E) (F) (G) (H)

3 Standardized Test Practice *(continued)*

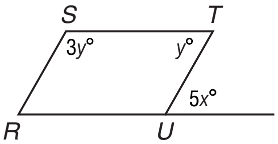
Part 2: Grid In

Instructions: Enter your answer by writing each digit of the answer in a column box and then shading in the appropriate oval that corresponds to that entry.

11. If the operation ★ is defined by the equation $a \star b = ab - b^2$, what is the value of n in the equation $4 \star n = 4$?

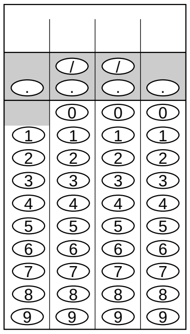
12. In a survey of high school sophomores, 60 students named Science as their favorite subject. If 70% of the students surveyed had a favorite subject other than Science, how many sophomores were surveyed?

13. If figure $RSTV$ is a parallelogram, what is the value of x?

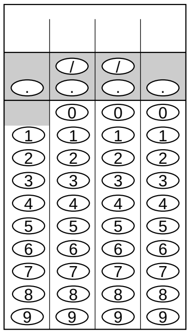


14. What number is in the hundreds place in the product of 540 and 9?

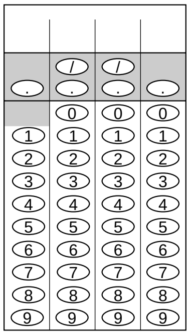
11.



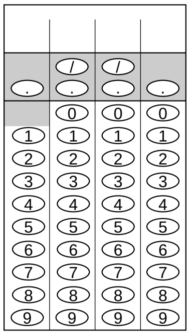
12.



13.



14.



Part 3: Quantitative Comparison

Instructions: Compare the quantities in columns A and B. Shade in
(A) if the quantity in column A is greater;
(B) if the quantity in column B is greater;
(C) if the quantities are equal; or
(D) if the relationship cannot be determined from the information given.

	Column A		Column B	
15.		$p = r^3 - 4,\ q = r^3 + 4$		**15.** (A) (B) (C) (D)
	p		q	
16.	$\frac{2}{5}$ of $6c$		$0.4 \times 2c$	**16.** (A) (B) (C) (D)
17.	$\sqrt{0.09}$		0.09	**17.** (A) (B) (C) (D)
18.		$x < 0$		**18.** (A) (B) (C) (D)
	$\frac{x^6}{x^2}$		$\frac{x^{10}}{x^6}$	
19.		$2x + 1 = 13$ and $5y = 14$		**19.** (A) (B) (C) (D)
	$2x$		$5y$	

NAME ______________________ DATE ____________ PERIOD ______

3 Standardized Test Practice

Student Record Sheet *(Use with pages 150–151 of the Student Edition.)*

Part 1 *Multiple Choice*

Select the best answer from the choices given and fill in the corresponding oval.

1 (A) (B) (C) (D)

2 (A) (B) (C) (D)

3 (A) (B) (C) (D)

4 (A) (B) (C) (D)

5 (A) (B) (C) (D)

6 (A) (B) (C) (D)

7 (A) (B) (C) (D)

8 (A) (B) (C) (D)

9 (A) (B) (C) (D)

10 (A) (B) (C) (D)

Part 2 *Short Response/Grid In*

Solve the problem and write your answer in the blank.

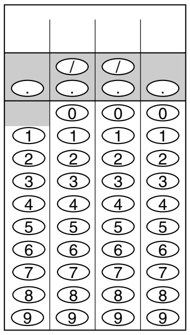
For Questions 14–18, also enter your answer by writing each number or symbol in a box. Then fill in the corresponding oval for that number or symbol.

11 ____________

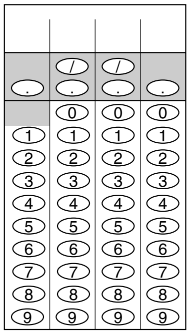
12 ____________

13 ____________

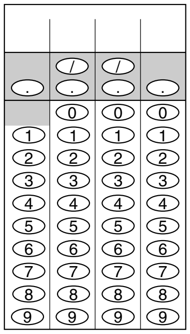
14 ____________



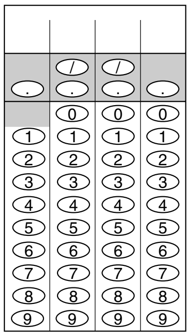
15 ____________



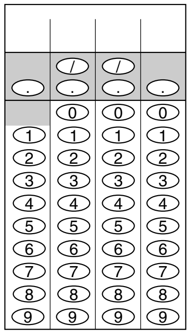
16 ____________



17 ____________



18 ____________



Part 3 *Quantitative Comparison*

Select the best answer from the choices given and fill in the corresponding oval.

19 (A) (B) (C) (D)

20 (A) (B) (C) (D)

21 (A) (B) (C) (D)

22 (A) (B) (C) (D)

Answers

NAME ______________________ DATE ____________ PERIOD _____

3-1 Study Guide and Intervention

Solving Systems of Equations by Graphing

Graph Systems of Equations A system of equations is a set of two or more equations containing the same variables. You can solve a system of linear equations by graphing the equations on the same coordinate plane. If the lines intersect, the solution is that intersection point.

Example **Solve the system of equations by graphing.** $x - 2y = 4$, $x + y = -2$

Write each equation in slope-intercept form.

$x - 2y = 4 \quad \rightarrow \quad y = \frac{x}{2} - 2$

$x + y = -2 \quad \rightarrow \quad y = -x - 2$

The graphs appear to intersect at (0, −2).

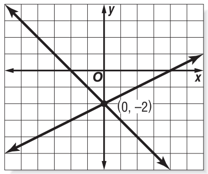
CHECK Substitute the coordinates into each equation.

$x - 2y = 4$ $x + y = -2$

$0 - 2(-2) \stackrel{?}{=} 4$ $0 + (-2) \stackrel{?}{=} -2$

$4 = 4$ ✓ $-2 = -2$ ✓

The solution of the system is (0, −2).

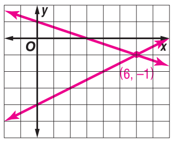


Lesson 3-1

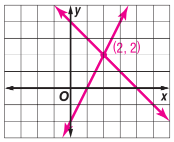
Exercises

Solve each system of equations by graphing.

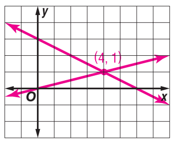
1. $y = -\frac{x}{3} + 1$, $y = \frac{x}{2} - 4$ **(6, −1)**



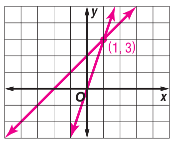
2. $y = 2x - 2$, $y = -x + 4$ **(2, 2)**



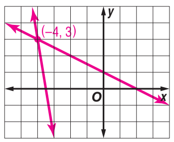
3. $y = -\frac{x}{2} + 3$, $y = \frac{x}{4}$ **(4, 1)**



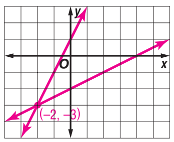
4. $3x - y = 0$, $x - y = -2$ **(1, 3)**



5. $2x + \frac{y}{3} = -7$, $\frac{x}{2} + y = 1$ **(−4, 3)**



6. $\frac{x}{2} - y = 2$, $2x - y = -1$ **(−2, −3)**



NAME ______________________ DATE ____________ PERIOD _____

3-1 Study Guide and Intervention *(continued)*

Solving Systems of Equations by Graphing

Classify Systems of Equations The following chart summarizes the possibilities for graphs of two linear equations in two variables.

Graphs of Equations	Slopes of Lines	Classification of System	Number of Solutions
Lines intersect	Different slopes	Consistent and independent	One
Lines coincide (same line)	Same slope, same *y*-intercept	Consistent and dependent	Infinitely many
Lines are parallel	Same slope, different *y*-intercepts	Inconsistent	None

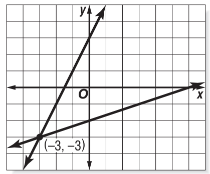
Example **Graph the system of equations and describe it as *consistent and independent*, *consistent and dependent*, or *inconsistent*.** $x - 3y = 6$, $2x - y = -3$

Write each equation in slope-intercept form.

$x - 3y = 6 \quad \rightarrow \quad y = \frac{1}{3}x - 2$

$2x - y = -3 \quad \rightarrow \quad y = 2x + 3$

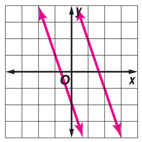
The graphs intersect at (−3, −3). Since there is one solution, the system is consistent and independent.



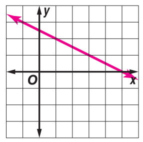
Exercises

Graph the system of equations and describe it as *consistent and independent*, *consistent and dependent*, or *inconsistent*.

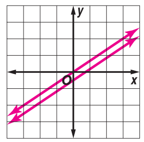
1. $3x + y = -2$, $6x + 2y = 10$ **inconsistent**



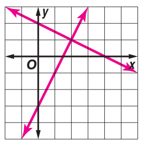
2. $x + 2y = 5$, $3x - 15 = -6y$ **consistent and dependent**



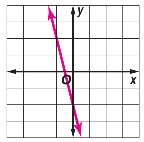
3. $2x - 3y = 0$, $4x - 6y = 3$ **inconsistent**



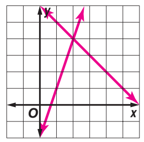
4. $2x - y = 3$, $x + 2y = 4$ **consistent and independent**



5. $4x + y = -2$, $2x + \frac{y}{2} = -1$ **consistent and dependent**



6. $3x - y = 2$, $x + y = 6$ **consistent and independent**



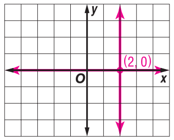
NAME ______________________ DATE ____________ PERIOD _____

3-1 Skills Practice

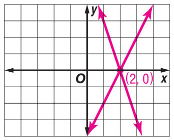
Solving Systems of Equations By Graphing

Solve each system of equations by graphing.

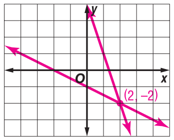
1. $x = 2$
$y = 0$ **(2, 0)**



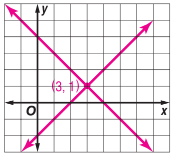
2. $y = -3x + 6$
$y = 2x - 4$ **(2, 0)**



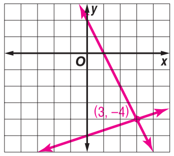
3. $y = 4 - 3x$
$y = -\frac{1}{2}x - 1$ **(2, −2)**



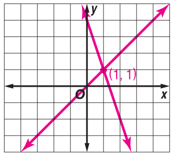
4. $y = 4 - x$
$y = x - 2$ **(3, 1)**



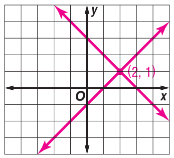
5. $y = -2x + 2$
$y = \frac{1}{3}x - 5$ **(3, −4)**



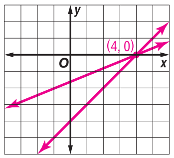
6. $y = x$
$y = -3x + 4$ **(1, 1)**



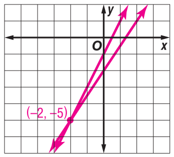
7. $x + y = 3$
$x - y = 1$ **(2, 1)**



8. $x - y = 4$
$2x - 5y = 8$ **(4, 0)**

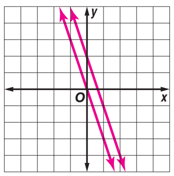


9. $3x - 2y = 4$
$2x - y = 1$ **(−2, −5)**



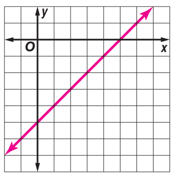
Graph each system of equations and describe it as *consistent and independent*, *consistent and dependent*, or *inconsistent*.

10. $y = -3x$
$y = -3x + 2$



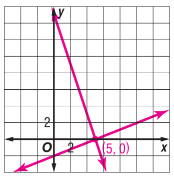
inconsistent

11. $y = x - 5$
$-2x + 2y = -10$



consistent and dependent

12. $2x - 5y = 10$
$3x + y = 15$



consistent and independent

Lesson 3-1

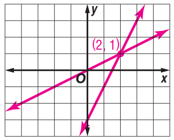
NAME ______________________ DATE ____________ PERIOD _____

3-1 Practice (Average)

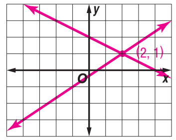
Solving Systems of Equations By Graphing

Solve each system of equations by graphing.

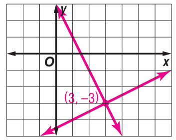
1. $x - 2y = 0$
$y = 2x - 3$ **(2, 1)**



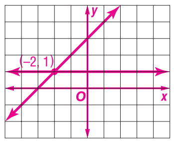
2. $x + 2y = 4$
$2x - 3y = 1$ **(2, 1)**



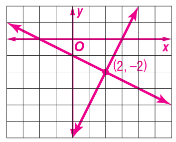
3. $2x + y = 3$
$y = \frac{1}{2}x - \frac{9}{2}$ **(3, −3)**



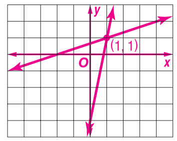
4. $y - x = 3$
$y = 1$ **(−2, 1)**



5. $2x - y = 6$
$x + 2y = -2$ **(2, −2)**

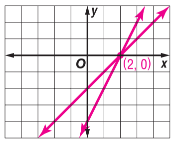


6. $5x - y = 4$
$-2x + 6y = 4$ **(1, 1)**



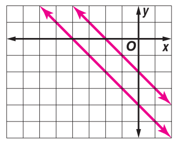
Graph each system of equations and describe it as *consistent and independent*, *consistent and dependent*, or *inconsistent*.

7. $2x - y = 4$
$x - y = 2$



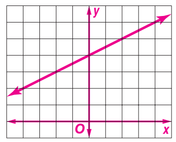
consistent and indep.

8. $y = -x - 2$
$x + y = -4$



inconsistent

9. $2y - 8 = x$
$y = \frac{1}{2}x + 4$



consistent and dep.

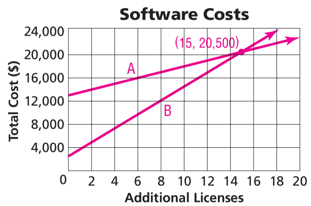
SOFTWARE For Exercises 10–12, use the following information.

Location Mapping needs new software. Software A costs $13,000 plus $500 per additional site license. Software B costs $2500 plus $1200 per additional site license.

10. Write two equations that represent the cost of each software. **A: $y = 13{,}000 + 500x$, B: $y = 2500 + 1200x$**

11. Graph the equations. Estimate the break-even point of the software costs. **15 additional licenses**

12. If Location Mapping plans to buy 10 additional site licenses, which software will cost less? **B**



Answers

NAME ______________________________ DATE __________ PERIOD _____

3-1 Reading to Learn Mathematics

Solving Systems of Equations by Graphing

Pre-Activity How can a system of equations be used to predict sales?

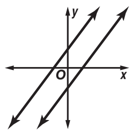
Read the introduction to Lesson 3-1 at the top of page 110 in your textbook.

- Which are growing faster, in-store sales or online sales? **online sales**
- In what year will the in-store and online sales be the same? **2005**

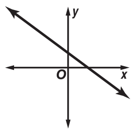
Reading the Lesson

1. The Study Tip on page 110 of your textbook says that when you solve a system of equations by graphing and find a point of intersection of the two lines, you must always check the ordered pair in *both* of the original equations. Why is it not good enough to check the ordered pair in just one of the equations?
Sample answer: To be a solution of the system, the ordered pair must make both of the equations true.

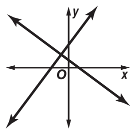
2. Under each system graphed below, write all of the following words that apply: *consistent*, *inconsistent*, *dependent*, and *independent*.



inconsistent



consistent; dependent



consistent; independent

Helping You Remember

3. Look up the words *consistent* and *inconsistent* in a dictionary. How can the meaning of these words help you distinguish between consistent and inconsistent systems of equations?
Sample answer: One meaning of *consistent* is "being in agreement," or "compatible," while one meaning of *inconsistent* is "not being in agreement" or "incompatible." When a system is consistent, the equations are compatible because both can be true at the same time (for the same values of x and y). When a system is inconsistent, the equations are incompatible because they can never be true at the same time.

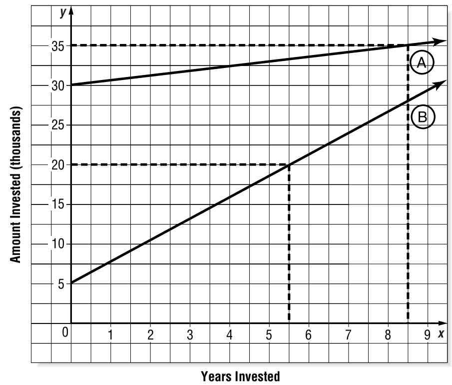
Lesson 3-1

NAME ______________________________ DATE __________ PERIOD _____

3-1 Enrichment

Investments

The following graph shows the value of two different investments over time. Line A represents an initial investment of $30,000 with a bank paying passbook savings interest. Line B represents an initial investment of $5,000 in a profitable mutual fund with dividends reinvested and capital gains accepted in shares. By deriving the linear equation $y = mx + b$ for A and B, you can predict the value of these investments for years to come.



1. The y-intercept, b, is the initial investment. Find b for each of the following.
a. line A **30,000** **b.** line B **5000**

2. The slope of the line, m, is the rate of return. Find m for each of the following.
a. line A **0.588** **b.** line B **2.73**

3. What are the equations of each of the following lines?
a. line A $y = 0.588x + 30$ **b.** line B $y = 2.73x + 5$

4. What will be the value of the mutual fund after 11 years of investment? **$35,030**

5. What will be the value of the bank account after 11 years of investment? **$36,468**

6. When will the mutual fund and the bank account have equal value?
after 11.67 years of investment

7. Which investment has the greatest payoff after 11 years of investment? **the mutual fund**

NAME ______________________ DATE __________ PERIOD _____

3-2 Study Guide and Intervention

Solving Systems of Equations Algebraically

Substitution To solve a system of linear equations by **substitution**, first solve for one variable in terms of the other in one of the equations. Then substitute this expression into the other equation and simplify.

Example **Use substitution to solve the system of equations.** $2x - y = 9$, $x + 3y = -6$

Solve the first equation for y in terms of x.

$2x - y = 9$ — First equation
$-y = -2x + 9$ — Subtract $2x$ from both sides.
$y = 2x - 9$ — Multiply both sides by −1.

Substitute the expression $2x - 9$ for y into the second equation and solve for x.

$x + 3y = -6$ — Second equation
$x + 3(2x - 9) = -6$ — Substitute $2x - 9$ for y.
$x + 6x - 27 = -6$ — Distributive Property
$7x - 27 = -6$ — Simplify.
$7x = 21$ — Add 27 to each side.
$x = 3$ — Divide each side by 7.

Now, substitute the value 3 for x in either original equation and solve for y.

$2x - y = 9$ — First equation
$2(3) - y = 9$ — Replace x with 3.
$6 - y = 9$ — Simplify.
$-y = 3$ — Subtract 6 from each side.
$y = -3$ — Multiply each side by −1.

The solution of the system is $(3, -3)$.

Exercises

Solve each system of linear equations by using substitution.

1. $3x + y = 7$, $4x + 2y = 16$ — **(−1, 10)**

2. $2x + y = 5$, $3x - 3y = 3$ — **(2, 1)**

3. $2x + 3y = -3$, $x + 2y = 2$ — **(−12, 7)**

4. $2x - y = 7$, $6x - 3y = 14$ — **no solution**

5. $4x - 3y = 4$, $2x + y = -8$ — **(−2, −4)**

6. $5x + y = 6$, $3 - x = 0$ — **(3, −9)**

7. $x + 8y = -2$, $x - 3y = 20$ — **(14, −2)**

8. $2x - y = -4$, $4x + y = 1$ — **$\left(-\frac{1}{2}, 3\right)$**

9. $x - y = -2$, $2x - 3y = 2$ — **(−8, −6)**

10. $x - 4y = 4$, $2x + 12y = 13$ — **$\left(5, \frac{1}{4}\right)$**

11. $x + 3y = 2$, $4x + 12y = 8$ — **infinitely many**

12. $2x + 2y = 4$, $x - 2y = 0$ — **$\left(\frac{4}{3}, \frac{2}{3}\right)$**

Lesson 3-2

NAME ______________________ DATE __________ PERIOD _____

3-2 Study Guide and Intervention *(continued)*

Solving Systems of Equations Algebraically

Elimination To solve a system of linear equations by **elimination**, add or subtract the equations to eliminate one of the variables. You may first need to multiply one or both of the equations by a constant so that one of the variables has the same (or opposite) coefficient in one equation as it has in the other.

Example 1 **Use the elimination method to solve the system of equations.**

$2x - 4y = -26$
$3x - y = -24$

Multiply the second equation by 4. Then subtract the equations to eliminate the y variable.

$2x - 4y = -26$ → $2x - 4y = -26$
$3x - y = -24$ (Multiply by 4.) → $12x - 4y = -96$
$-10x = 70$
$x = -7$

Replace x with −7 and solve for y.

$2x - 4y = -26$
$2(-7) - 4y = -26$
$-14 - 4y = -26$
$-4y = -12$
$y = 3$

The solution is $(-7, 3)$.

Example 2 **Use the elimination method to solve the system of equations.**

$3x - 2y = 4$
$5x + 3y = -25$

Multiply the first equation by 3 and the second equation by 2. Then add the equations to eliminate the y variable.

$3x - 2y = 4$ (Multiply by 3.) → $9x - 6y = 12$
$5x + 3y = -25$ (Multiply by 2.) → $10x + 6y = -50$
$19x = -38$
$x = -2$

Replace x with −2 and solve for y.

$3x - 2y = 4$
$3(-2) - 2y = 4$
$-6 - 2y = 4$
$-2y = 10$
$y = -5$

The solution is $(-2, -5)$.

Exercises

Solve each system of equations by using elimination.

1. $2x - y = 7$, $3x + y = 8$ — **(3, −1)**

2. $x - 2y = 4$, $-x + 6y = 12$ — **(12, 4)**

3. $3x + 4y = -10$, $x - 4y = 2$ — **(−2, −1)**

4. $3x - y = 12$, $5x + 2y = 20$ — **(4, 0)**

5. $4x - y = 6$, $2x - \frac{y}{2} = 4$ — **no solution**

6. $5x + 2y = 12$, $-6x - 2y = -14$ — **(2, 1)**

7. $2x + y = 8$, $3x + \frac{3}{2}y = 12$ — **infinitely many**

8. $7x + 2y = -1$, $4x - 3y = -13$ — **(−1, 3)**

9. $3x + 8y = -6$, $x - y = 9$ — **(6, −3)**

10. $5x + 4y = 12$, $7x - 6y = 40$ — **(4, −2)**

11. $-4x + y = -12$, $4x + 2y = 6$ — **$\left(\frac{5}{2}, -2\right)$**

12. $5m + 2n = -8$, $4m + 3n = 2$ — **(−4, 6)**

Answers

NAME ______________ DATE ________ PERIOD _____

3-2 Skills Practice

Solving Systems of Equations Algebraically

Solve each system of equations by using substitution.

1. $m + n = 20$, $m - n = -4$ **(8, 12)**

2. $x + 3y = -3$, $4x + 3y = 6$ **(3, −2)**

3. $w - z = 1$, $2w + 3z = 12$ **(3, 2)**

4. $3r + s = 5$, $2r - s = 5$ **(2, −1)**

5. $2b + 3c = -4$, $b + c = 3$ **(13, −10)**

6. $x - y = 1$, $2x + 3y = 12$ **(3, 2)**

Solve each system of equations by using elimination.

7. $2p - q = 5$, $3p + q = 5$ **(2, −1)**

8. $2j - k = 3$, $3j + k = 2$ **(1, −1)**

9. $3c - 2d = 2$, $3c + 4d = 50$ **(6, 8)**

10. $2f + 3g = 9$, $f - g = 2$ **(3, 1)**

11. $-2x + y = -1$, $x + 2y = 3$ **(1, 1)**

12. $2x - y = 12$, $2x - y = 6$ **no solution**

Solve each system of equations by using either substitution or elimination.

13. $-r + t = 5$, $-2r + t = 4$ **(1, 6)**

14. $2x - y = -5$, $4x + y = 2$ **$\left(-\frac{1}{2}, 4\right)$**

15. $x - 3y = -12$, $2x + y = 11$ **(3, 5)**

16. $2p - 3q = 6$, $-2p + 3q = -6$ **(3, 0)**

17. $6w - 8z = 16$, $3w - 4z = 8$ **infinitely many**

18. $c + d = 6$, $c - d = 0$ **(3, 3)**

19. $2u + 4v = -6$, $u + 2v = 3$ **no solution**

20. $3a + b = -1$, $-3a + b = 5$ **(−1, 2)**

21. $2x + y = 6$, $3x - 2y = 16$ **(4, −2)**

22. $3y - z = -6$, $-3y - z = 6$ **(−2, 0)**

23. $c + 2d = -2$, $-2c - 5d = 3$ **(−4, 1)**

24. $3r - 2s = 1$, $2r - 3s = 9$ **(−3, −5)**

25. The sum of two numbers is 12. The difference of the same two numbers is −4. Find the numbers. **4, 8**

26. Twice a number minus a second number is −1. Twice the second number added to three times the first number is 9. Find the two numbers. **1, 3**

Lesson 3-2

NAME ______________ DATE ________ PERIOD _____

3-2 Practice (Average)

Solving Systems of Equations Algebraically

Solve each system of equations by using substitution.

1. $2x + y = 4$, $3x + 2y = 1$ **(7, −10)**

2. $x - 3y = 9$, $x + 2y = -1$ **(3, −2)**

3. $g + 3h = 8$, $\frac{1}{3}g + h = 9$ **no solution**

4. $2a - 4b = 6$, $-a + 2b = -3$ **infinitely many**

5. $2m + n = 6$, $5m + 6n = 1$ **(5, −4)**

6. $4x - 3y = -6$, $-x - 2y = 7$ **(−3, −2)**

7. $u - 2v = \frac{1}{2}$, $-u + 2v = 5$ **no solution**

8. $x - 3y = 16$, $4x - y = 9$ **(1, −5)**

9. $w + 3z = 1$, $3w - 5z = -4$ **$\left(-\frac{1}{2}, \frac{1}{2}\right)$**

Solve each system of equations by using elimination.

10. $2r + s = 5$, $3r - s = 20$ **(5, −5)**

11. $2m - n = -1$, $3m + 2n = 30$ **(4, 9)**

12. $6x + 3y = 6$, $8x + 5y = 12$ **(−1, 4)**

13. $3j - k = 10$, $4j - k = 16$ **(6, 8)**

14. $2x - y = -4$, $-4x + 2y = 6$ **no solution**

15. $2g + h = 6$, $3g - 2h = 16$ **(4, −2)**

16. $2t + 4v = 6$, $-t - 2v = -3$ **infinitely many**

17. $3x - 2y = 12$, $2x + \frac{2}{3}y = 14$ **(6, 3)**

18. $\frac{1}{2}x + 3y = 11$, $8x - 5y = 17$ **(4, 3)**

Solve each system of equations by using either substitution or elimination.

19. $8x + 3y = -5$, $10x + 6y = -13$ **$\left(\frac{1}{2}, -3\right)$**

20. $8q - 15r = -40$, $4q + 2r = 56$ **(10, 8)**

21. $3x - 4y = 12$, $\frac{1}{3}x - \frac{4}{9}y = \frac{4}{3}$ **infinitely many**

22. $4b - 2d = 5$, $-2b + d = 1$ **no solution**

23. $s + 3y = 4$, $s = 1$ **(1, 1)**

24. $4m - 2p = 0$, $-3m + 9p = 5$ **$\left(\frac{1}{3}, \frac{2}{3}\right)$**

25. $5g + 4k = 10$, $-3g - 5k = 7$ **(6, −5)**

26. $0.5x + 2y = 5$, $x - 2y = -8$ **(−2, 3)**

27. $h - z = 3$, $-3h + 3z = 6$ **no solution**

SPORTS For Exercises 28 and 29, use the following information.

Last year the volleyball team paid \$5 per pair for socks and \$17 per pair for shorts on a total purchase of \$315. This year they spent \$342 to buy the same number of pairs of socks and shorts because the socks now cost \$6 a pair and the shorts cost \$18.

28. Write a system of two equations that represents the number of pairs of socks and shorts bought each year. **$5x + 17y = 315$, $6x + 18y = 342$**

29. How many pairs of socks and shorts did the team buy each year? **socks: 12, shorts: 15**

NAME ______________________ DATE __________ PERIOD _____

3-2 Reading to Learn Mathematics

Solving Systems of Equations Algebraically

Pre-Activity How can systems of equations be used to make consumer decisions?

Read the introduction to Lesson 3-2 at the top of page 116 in your textbook.

- How many more minutes of long distance time did Yolanda use in February than in January? **13 minutes**
- How much more were the February charges than the January charges? **$1.04**
- Using your answers for the questions above, how can you find the rate per minute? **Find $1.04 ÷ 13.**

Reading the Lesson

1. Suppose that you are asked to solve the system of equations at the right by the substitution method.

$$4x - 5y = 7$$
$$3x + y = -9$$

The first step is to solve one of the equations for one variable in terms of the other. To make your work as easy as possible, which equation would you solve for which variable? Explain.
Sample answer: Solve the second equation for *y* because in that equation the variable *y* has a coefficient of 1.

2. Suppose that you are asked to solve the system of equations at the right by the elimination method.

$$2x + 3y = -2$$
$$7x - y = 39$$

To make your work as easy as possible, which variable would you eliminate? Describe how you would do this.
Sample answer: Eliminate the variable *y*; multiply the second equation by 3 and then add the result to the first equation.

Helping You Remember

3. The substitution method and elimination method for solving systems both have several steps, and it may be difficult to remember them. You may be able to remember them more easily if you notice what the methods have in common. What step is the same in both methods?
Sample answer: After finding the value of one of the variables, you find the value of the other variable by substituting the value you have found in one of the original equations.

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3-2 Enrichment

Using Coordinates

From one observation point, the line of sight to a downed plane is given by $y = x - 1$. This equation describes the distance from the observation point to the plane in a straight line. From another observation point, the line of sight is given by $x + 3y = 21$. What are the coordinates of the point at which the crash occurred?

Solve the system of equations $\begin{cases} y = x - 1 \\ x + 3y = 21 \end{cases}$.

$$\begin{aligned} x + 3y &= 21 \\ x + 3(x - 1) &= 21 \quad \text{Substitute } x - 1 \text{ for } y. \\ x + 3x - 3 &= 21 \\ 4x &= 24 \\ x &= 6 \end{aligned}$$

$$\begin{aligned} x + 3y &= 21 \\ 6 + 3y &= 21 \quad \text{Substitute 6 for } x. \\ 3y &= 15 \\ y &= 5 \end{aligned}$$

The coordinates of the crash are (6, 5).

Solve the following.

1. The lines of sight to a forest fire are as follows.

From Ranger Station A: $3x + y = 9$
From Ranger Station B: $2x + 3y = 13$
Find the coordinates of the fire.
(2, 3)

2. An airplane is traveling along the line $x - y = -1$ when it sees another airplane traveling along the line $5x + 3y = 19$. If they continue along the same lines, at what point will their flight paths cross?
(2, 3)

3. Two mine shafts are dug along the paths of the following equations.

$x - y = 1400$
$2x + y = 1300$

If the shafts meet at a depth of 200 feet, what are the coordinates of the point at which they meet?
(900, −500)

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3-3 Study Guide and Intervention

Solving Systems of Inequalities by Graphing

Graph Systems of Inequalities To solve a system of inequalities, graph the inequalities in the same coordinate plane. The solution set is represented by the intersection of the graphs.

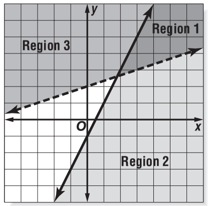
Example **Solve the system of inequalities by graphing.**

$y \le 2x - 1$ and $y > \frac{x}{3} + 2$

The solution of $y \le 2x - 1$ is Regions 1 and 2.

The solution of $y > \frac{x}{3} + 2$ is Regions 1 and 3.

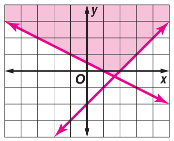
The intersection of these regions is Region 1, which is the solution set of the system of inequalities.



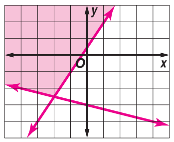
Exercises

Solve each system of inequalities by graphing.

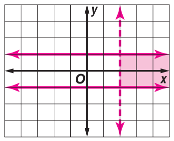
1. $x - y \le 2$
$x + 2y \ge 1$



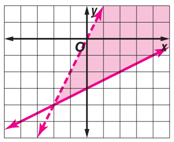
2. $3x - 2y \le -1$
$x + 4y \ge -12$



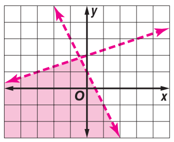
3. $|y| \le 1$
$x > 2$



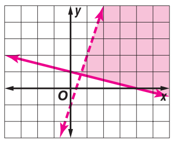
4. $y \ge \frac{x}{2} - 3$
$y < 2x$



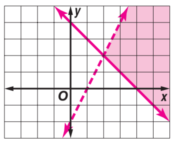
5. $y < \frac{x}{3} + 2$
$y < -2x + 1$



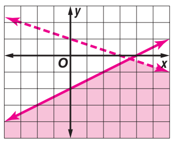
6. $y \ge -\frac{x}{4} + 1$
$y < 3x - 1$



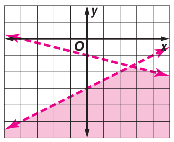
7. $x + y \ge 4$
$2x - y > 2$



8. $x + 3y < 3$
$x - 2y \ge 4$



9. $x - 2y > 6$
$x + 4y < -4$



Lesson 3-3

NAME ______________________ DATE __________ PERIOD _____

3-3 Study Guide and Intervention (continued)

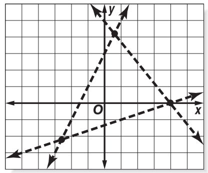
Solving Systems of Inequalities by Graphing

Find Vertices of a Polygonal Region Sometimes the graph of a system of inequalities forms a bounded region. You can find the vertices of the region by a combination of the methods used earlier in this chapter: graphing, substitution, and/or elimination.

Example **Find the coordinates of the vertices of the figure formed by $5x + 4y < 20$, $y < 2x + 3$, and $x - 3y < 4$.**

Graph the boundary of each inequality. The intersections of the boundary lines are the vertices of a triangle.

The vertex (4, 0) can be determined from the graph. To find the coordinates of the second and third vertices, solve the two systems of equations



$y = 2x + 3$, $5x + 4y = 20$ and $y = 2x + 3$, $x - 3y = 4$

For the first system of equations, rewrite the first equation in standard form as $2x - y = -3$. Then multiply that equation by 4 and add to the second equation.

$2x - y = -3$ (Multiply by 4.) $8x - 4y = -12$
$5x + 4y = 20$ $(+)\ 5x + 4y = 20$
$13x = 8$
$x = \frac{8}{13}$

Then substitute $x = \frac{8}{13}$ in one of the original equations and solve for y.

$2\left(\frac{8}{13}\right) - y = -3$

$\frac{16}{13} - y = -3$

$y = \frac{55}{13}$

The coordinates of the second vertex are $\left(\frac{8}{13}, 4\frac{3}{13}\right)$.

For the second system of equations, use substitution. Substitute $2x + 3$ for y in the second equation to get

$x - 3(2x + 3) = 4$
$x - 6x - 9 = 4$
$-5x = 13$
$x = -\frac{13}{5}$

Then substitute $x = -\frac{13}{5}$ in the first equation to solve for y.

$y = 2\left(-\frac{13}{5}\right) + 3$

$y = -\frac{26}{5} + 3$

$y = -\frac{11}{5}$

The coordinates of the third vertex are $\left(-2\frac{3}{5}, -2\frac{1}{5}\right)$.

Thus, the coordinates of the three vertices are (4, 0), $\left(\frac{8}{13}, 4\frac{3}{13}\right)$, and $\left(-2\frac{3}{5}, -2\frac{1}{5}\right)$.

Exercises

Find the coordinates of the vertices of the figure formed by each system of inequalities.

1. $y \le -3x + 7$
$y < \frac{1}{2}x$
$y > -2$
(2, 1), (−4, −2), (3, −2)

2. $x > -3$
$y < -\frac{1}{3}x + 3$
$y > x - 1$
(−3, 4), (−3, −4), (3, 2)

3. $y < -\frac{1}{2}x + 3$
$y > \frac{1}{2}x + 1$
$y < 3x + 10$
(−2, 4), (2, 2), $\left(-3\frac{3}{5}, -\frac{4}{5}\right)$

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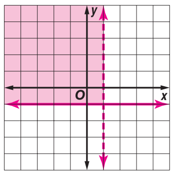
3-3 Skills Practice

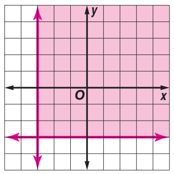
Solving Systems of Inequalities by Graphing

Solve each system of inequalities by graphing.

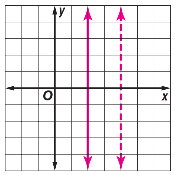
1. $x < 1$
$y \geq -1$



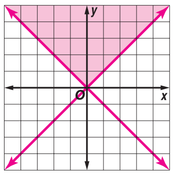
2. $x \geq -3$
$y \geq -3$



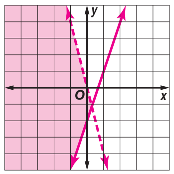
3. $x \leq 2$
$x > 4$ **no solution**



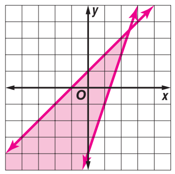
4. $y \geq x$
$y \geq -x$



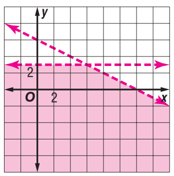
5. $y < -4x$
$y \geq 3x - 2$



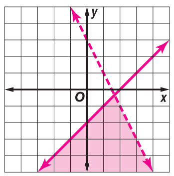
6. $x - y \geq -1$
$3x - y \leq 4$



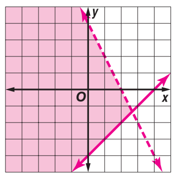
7. $y < 3$
$x + 2y < 12$



8. $y < -2x + 3$
$y \leq x - 2$



9. $x - y \leq 4$
$2x + y < 4$



Find the coordinates of the vertices of the figure formed by each system of inequalities.

10. $y < 0$
$x < 0$
$y \geq -x - 1$
(0, 0), (0, −1), (−1, 0)

11. $y < 3 - x$
$y \geq 3$
$x > -5$
(0, 3), (−5, 3), (−5, 8)

12. $x \geq -2$
$y > x - 2$
$x + y \leq 2$ **(−2, 4), (−2, −4), (2, 0)**

Lesson 3-3

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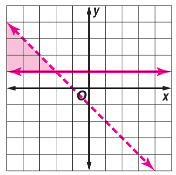
3-3 Practice (Average)

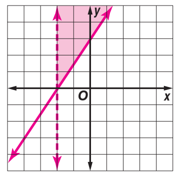
Solving Systems of Inequalities by Graphing

Solve each system of inequalities by graphing.

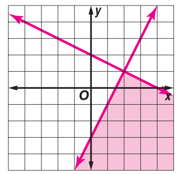
1. $y + 1 < -x$
$y \geq 1$



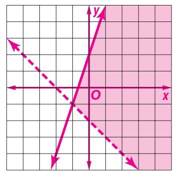
2. $x > -2$
$2y \geq 3x + 6$



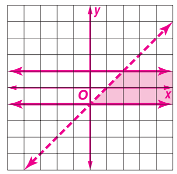
3. $y \leq 2x - 3$
$y \leq -\frac{1}{2}x + 2$



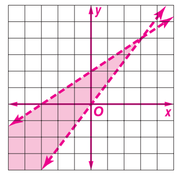
4. $x + y > -2$
$3x - y \geq -2$



5. $|y| \leq 1$
$y < x - 1$



6. $3y > 4x$
$2x - 3y > -6$



Find the coordinates of the vertices of the figure formed by each system of inequalities.

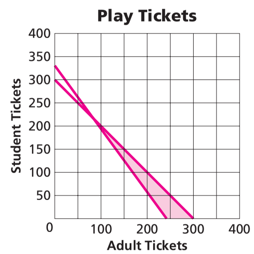
7. $y \geq 1 - x$
$y \leq x - 1$
$x \leq 3$
(1, 0), (3, 2), (3, −2)

8. $x - y \leq 2$
$x + y \leq 2$
$x \geq -2$
(−2, 4), (−2, −4), (2, 0)

9. $y \geq 2x - 2$
$2x + 3y \geq 6$
$y < 4$
(−3, 4), $\left(\frac{3}{2}, 1\right)$, (3, 4)

DRAMA For Exercises 10 and 11, use the following information.

The drama club is selling tickets to its play. An adult ticket costs $15 and a student ticket costs $11. The auditorium will seat 300 ticket-holders. The drama club wants to collect at least $3630 from ticket sales.



10. Write and graph a system of four inequalities that describe how many of each type of ticket the club must sell to meets its goal.
$x \geq 0, y \geq 0, x + y \leq 300, 15x + 11y \geq 3630$

11. List three different combinations of tickets sold that satisfy the inequalities. **Sample answer: 250 adult and 50 student, 200 adult and 100 student, 145 adult and 148 student**

Answers

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3-3 Reading to Learn Mathematics

Solving Systems of Inequalities by Graphing

Pre-Activity **How can you determine whether your blood pressure is in a normal range?**

Read the introduction to Lesson 3-3 at the top of page 123 in your textbook.

Satish is 37 years old. He has a blood pressure reading of 135/99. Is his blood pressure within the normal range? Explain.

Sample answer: No; his systolic pressure is normal, but his diastolic pressure is too high. It should be between 60 and 90.

Reading the Lesson

1. Without actually drawing the graph, describe the boundary lines for the system of inequalities shown at the right.

$|x| < 3$
$|y| \leq 5$

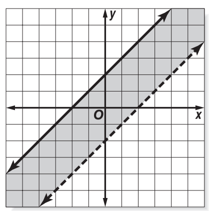
Two dashed vertical lines ($x = 3$ and $x = -3$) and two solid horizontal lines ($y = -5$ and $y = 5$)

2. Think about how the graph would look for the system given above. What will be the shape of the shaded region? (It is not necessary to draw the graph. See if you can imagine it without drawing anything. If this is difficult to do, make a rough sketch to help you answer the question.)

a rectangle

3. Which system of inequalities matches the graph shown at the right? **B**

A. $x - y \leq -2$, $x - y > 2$
B. $x - y \geq -2$, $x - y < 2$
C. $x + y \leq -2$, $x + y > 2$
D. $x - y > -2$, $x - y \leq 2$



Helping You Remember

4. To graph a system of inequalities, you must graph two or more boundary lines. When you graph each of these lines, how can the inequality symbols help you remember whether to use a dashed or solid line?

Use a dashed line if the inequality symbol is > or <, because these symbols do not include equality and the dashed line reminds you that the line itself is not included in the graph. Use a solid line if the symbol is ≥ or ≤, because these symbols include equality and tell you that the line itself is included in the graph.

Lesson 3-3

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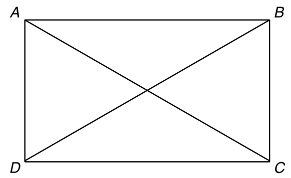
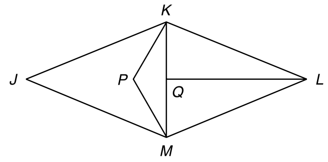
3-3 Enrichment

Tracing Strategy

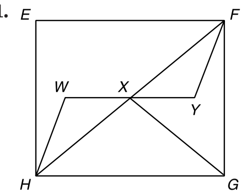
Try to trace over each of the figures below without tracing the same segment twice.



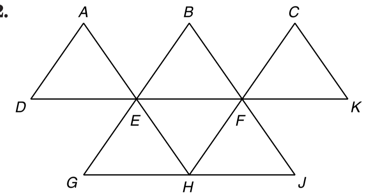


The figure at the left cannot be traced, but the one at the right can. The rule is that a figure is traceable if it has no more than two points where an odd number of segments meet. The figure at the left has three segments meeting at each of the four corners. However, the figure at the right has only two points, L and Q, where an odd number of segments meet.

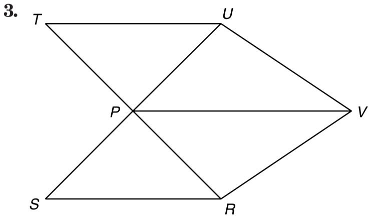
Determine if each figure can be traced without tracing the same segment twice. If it can, then name the starting point and name the segments in the order they should be traced.

1.


yes; X; $\overline{XY}$, $\overline{YF}$, $\overline{FX}$, $\overline{XG}$, $\overline{GF}$, $\overline{FE}$, $\overline{EH}$, $\overline{HX}$, $\overline{XW}$, $\overline{WH}$, $\overline{HG}$

2.


yes; E; $\overline{ED}$, $\overline{DA}$, $\overline{AE}$, $\overline{EB}$, $\overline{BF}$, $\overline{FC}$, $\overline{CK}$, $\overline{KF}$, $\overline{FJ}$, $\overline{JH}$, $\overline{HF}$, $\overline{FE}$, $\overline{EH}$, $\overline{HG}$, $\overline{GE}$

3.


not traceable

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3-4 Study Guide and Intervention

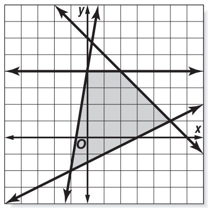
Linear Programming

Maximum and Minimum Values When a system of linear inequalities produces a bounded polygonal region, the *maximum* or *minimum* value of a related function will occur at a vertex of the region.

Example **Graph the system of inequalities. Name the coordinates of the vertices of the feasible region. Find the maximum and minimum values of the function $f(x, y) = 3x + 2y$ for this polygonal region.**

$y \le 4$
$y \le -x + 6$
$y \ge \frac{1}{2}x - \frac{3}{2}$
$y \le 6x + 4$

First find the vertices of the bounded region. Graph the inequalities.



The polygon formed is a quadrilateral with vertices at (0, 4), (2, 4), (5, 1), and (−1, −2). Use the table to find the maximum and minimum values of $f(x, y) = 3x + 2y$.

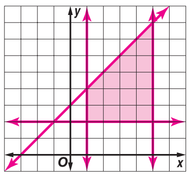
(x, y)	3x + 2y	f(x, y)
(0, 4)	3(0) + 2(4)	8
(2, 4)	3(2) + 2(4)	14
(5, 1)	3(5) + 2(1)	17
(−1, −2)	3(−1) + 2(−2)	−7

The maximum value is 17 at (5, 1). The minimum value is −7 at (−1, −2).

Exercises

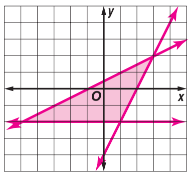
Graph each system of inequalities. Name the coordinates of the vertices of the feasible region. Find the maximum and minimum values of the given function for this region.

1. $y \ge 2$
$1 \le x \le 5$
$y \le x + 3$
$f(x, y) = 3x - 2y$



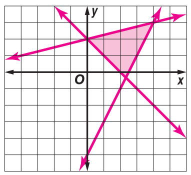
vertices: (1, 2), (1, 4), (5, 8), (5, 2); max: 11; min; −5

2. $y \ge -2$
$y \ge 2x - 4$
$x - 2y \ge -1$
$f(x, y) = 4x - y$



vertices: (−5, −2), (3, 2), (1, −2); max: 10; min: −18

3. $x + y \ge 2$
$4y \le x + 8$
$y \ge 2x - 5$
$f(x, y) = 4x + 3y$



vertices (0, 2), (4, 3), $\left(\frac{7}{3}, -\frac{1}{3}\right)$; max: 25; min: 6

Lesson 3-4

NAME ______________________ DATE __________ PERIOD _____

3-4 Study Guide and Intervention *(continued)*

Linear Programming

Real-World Problems When solving **linear programming** problems, use the following procedure.

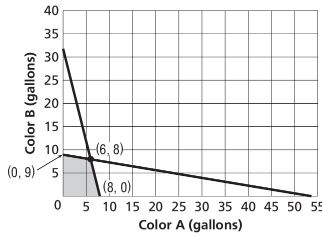
1. Define variables.
2. Write a system of inequalities.
3. Graph the system of inequalities.
4. Find the coordinates of the vertices of the feasible region.
5. Write an expression to be maximized or minimized.
6. Substitute the coordinates of the vertices in the expression.
7. Select the greatest or least result to answer the problem.

Example **A painter has exactly 32 units of yellow dye and 54 units of green dye. He plans to mix as many gallons as possible of color A and color B. Each gallon of color A requires 4 units of yellow dye and 1 unit of green dye. Each gallon of color B requires 1 unit of yellow dye and 6 units of green dye. Find the maximum number of gallons he can mix.**

Step 1 Define the variables.
x = the number of gallons of color A made
y = the number of gallons of color B made

Step 2 Write a system of inequalities.
Since the number of gallons made cannot be negative, $x \ge 0$ and $y \ge 0$.

There are 32 units of yellow dye; each gallon of color A requires 4 units, and each gallon of color B requires 1 unit.
So $4x + y \le 32$.
Similarly for the green dye, $x + 6y \le 54$.



Steps 3 and 4 Graph the system of inequalities and find the coordinates of the vertices of the feasible region.
The vertices of the feasible region are (0, 0), (0, 9), (6, 8), and (8, 0).

Steps 5–7 Find the maximum number of gallons, $x + y$, that he can make.
The maximum number of gallons the painter can make is 14, 6 gallons of color A and 8 gallons of color B.

(x, y)	x + y	f(x, y)
(0, 0)	0 + 0	0
(0, 9)	0 + 9	9
(6, 8)	6 + 8	14
(8, 0)	8 + 0	8

Exercises

1. FOOD A delicatessen has 8 pounds of plain sausage and 10 pounds of garlic-flavored sausage. The deli wants to make as much bratwurst as possible. Each pound of bratwurst requires $\frac{3}{4}$ pound of plain sausage and $\frac{1}{4}$ pound of garlic-flavored sausage. Find the maximum number of pounds of bratwurst that can be made. **$10\frac{2}{3}$ lb**

2. MANUFACTURING Machine A can produce 30 steering wheels per hour at a cost of $16 per hour. Machine B can produce 40 steering wheels per hour at a cost of $22 per hour. At least 360 steering wheels must be made in each 8-hour shift. What is the least cost involved in making 360 steering wheels in one shift? **$194**

Answers

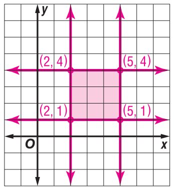
NAME ______________________ DATE ________ PERIOD _____

3-4 Skills Practice

Linear Programming

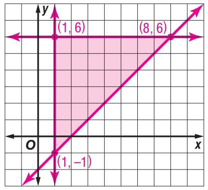
Graph each system of inequalities. Name the coordinates of the vertices of the feasible region. Find the maximum and minimum values of the given function for this region.

1. $x \geq 2$
$x \leq 5$
$y \geq 1$
$y \leq 4$
$f(x, y) = x + y$



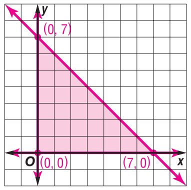
max.: 9, min.: 3

2. $x \geq 1$
$y \leq 6$
$y \geq x - 2$
$f(x, y) = x - y$



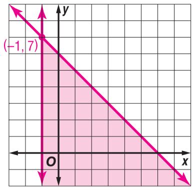
max.: 2, min.: –5

3. $x \geq 0$
$y \geq 0$
$y \leq 7 - x$
$f(x, y) = 3x + y$



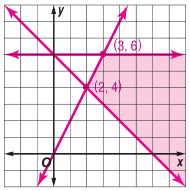
max.: 21, min.: 0

4. $x \geq -1$
$x + y \leq 6$
$f(x, y) = x + 2y$



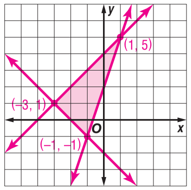
max.: 13, no min.

5. $y \leq 2x$
$y \geq 6 - x$
$y \leq 6$
$f(x, y) = 4x + 3y$



no max., min.: 20

6. $y \geq -x - 2$
$y \geq 3x + 2$
$y \leq x + 4$
$f(x, y) = -3x + 5y$



max.: 22, min.: –2

7. MANUFACTURING A backpack manufacturer produces an internal frame pack and an external frame pack. Let x represent the number of internal frame packs produced in one hour and let y represent the number of external frame packs produced in one hour. Then the inequalities $x + 3y \leq 18$, $2x + y \leq 16$, $x \geq 0$, and $y \geq 0$ describe the constraints for manufacturing both packs. Use the profit function $f(x) = 50x + 80y$ and the constraints given to determine the maximum profit for manufacturing both backpacks for the given constraints. **$620**

Lesson 3-4

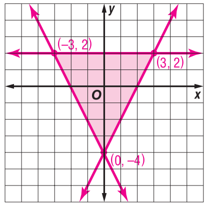
NAME ______________________ DATE ________ PERIOD _____

3-4 Practice (Average)

Linear Programming

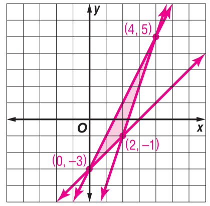
Graph each system of inequalities. Name the coordinates of the vertices of the feasible region. Find the maximum and minimum values of the given function for this region.

1. $2x - 4 \leq y$
$-2x - 4 \leq y$
$y \leq 2$
$f(x, y) = -2x + y$



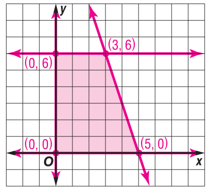
max.: 8, min.: –4

2. $3x - y \leq 7$
$2x - y \geq 3$
$y \geq x - 3$
$f(x, y) = x - 4y$



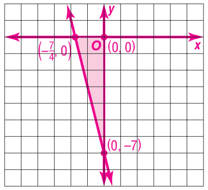
max.: 12, min.: –16

3. $x \geq 0$
$y \geq 0$
$y \leq 6$
$y \leq -3x + 15$
$f(x, y) = 3x + y$



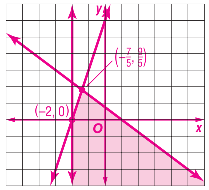
max.: 15, min.: 0

4. $x \leq 0$
$y \leq 0$
$4x + y \geq -7$
$f(x, y) = -x - 4y$



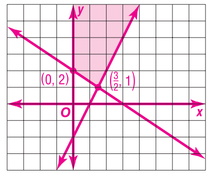
max.: 28, min.: 0

5. $y \leq 3x + 6$
$4y + 3x \leq 3$
$x \geq -2$
$f(x, y) = -x + 3y$



max.: $\frac{34}{5}$, no min.

6. $2x + 3y \geq 6$
$2x - y \leq 2$
$x \geq 0$
$y \geq 0$
$f(x, y) = x + 4y + 3$



no max., min.: $\frac{17}{2}$

PRODUCTION For Exercises 7–9, use the following information.

A glass blower can form 8 simple vases or 2 elaborate vases in an hour. In a work shift of no more than 8 hours, the worker must form at least 40 vases.

7. Let s represent the hours forming simple vases and e the hours forming elaborate vases. Write a system of inequalities involving the time spent on each type of vase.
$s \geq 0, e \geq 0, s + e \leq 8, 8s + 2e \geq 40$

8. If the glass blower makes a profit of $30 per hour worked on the simple vases and $35 per hour worked on the elaborate vases, write a function for the total profit on the vases.
$f(s, e) = 30s + 35e$

9. Find the number of hours the worker should spend on each type of vase to maximize profit. What is that profit? **4 h on each; $260**

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3-4 Reading to Learn Mathematics

Linear Programming

Pre-Activity How is linear programming used in scheduling work?

Read the introduction to Lesson 3-4 at the top of page 129 in your textbook.

Name two or more facts that indicate that you will need to use inequalities to model this situation.

Sample answer: The buoy tender can carry up to 8 new buoys. There seems to be a limit of 24 hours on the time the crew has at sea. The crew will want to repair or replace the maximum number of buoys possible.

Reading the Lesson

1. Complete each sentence.

a. When you find the feasible region for a linear programming problem, you are solving a system of linear **inequalities** called **constraints**. The points in the feasible region are **solutions** of the system.

b. The corner points of a polygonal region are the **vertices** of the feasible region.

2. A polygonal region always takes up only a limited part of the coordinate plane. One way to think of this is to imagine a circle or rectangle that the region would fit inside. In the case of a polygonal region, you can always find a circle or rectangle that is large enough to contain all the points of the polygonal region. What word is used to describe a region that can be enclosed in this way? What word is used to describe a region that is too large to be enclosed in this way? **bounded; unbounded**

3. How do you find the corner points of the polygonal region in a linear programming problem? **You solve a system of two linear equations.**

4. What are some everyday meanings of the word *feasible* that remind you of the mathematical meaning of the term *feasible region*?
Sample answer: possible or achievable

Helping You Remember

5. Look up the word *constraint* in a dictionary. If more than one definition is given, choose the one that seems closest to the idea of a *constraint* in a linear programming problem. How can this definition help you to remember the meaning of *constraint* as it is used in this lesson? **Sample answer: A *constraint* is a restriction or limitation. The constraints in a linear programming problem are restrictions on the variables that translate into inequality statements.**

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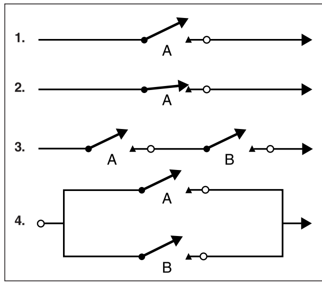
3-4 Enrichment

Computer Circuits and Logic

Computers operate according to the laws of logic. The circuits of a computer can be described using logic.



1. With switch A open, no current flows. The value 0 is assigned to an open switch.

2. With switch A closed, current flows. The value 1 is assigned to a closed switch.

3. With switches A and B open, no current flows. This circuit can be described by the conjunction, A · B.

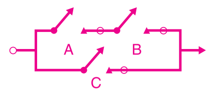
4. In this circuit, current flows if either A or B is closed. This circuit can be described by the disjunction, A + B.

A	B	A + B
0	0	0
0	1	1
1	0	1
1	1	1

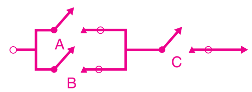
Truth tables are used to describe the flow of current in a circuit. The table at the left describes the circuit in diagram 4. According to the table, the only time current does not flow through the circuit is when both switches A and B are open.

Draw a circuit diagram for each of the following.

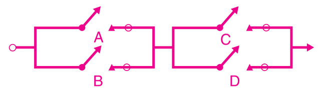
1. (A · B) + C



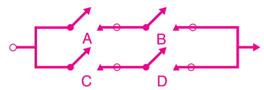
2. (A + B) · C



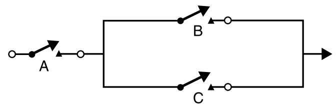
3. (A + B) · (C + D)



4. (A · B) + (C · D)



5. Construct a truth table for the following circuit.



A	B	C	(B + C)	A(B + C)
0	0	0	0	0
0	0	1	1	0
0	1	1	1	0
0	1	0	1	0
1	0	1	1	1
1	1	0	1	1
1	1	1	1	1
1	0	0	0	0

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3-5 Study Guide and Intervention

Solving Systems of Equations in Three Variables

Systems in Three Variables Use the methods used for solving systems of linear equations in two variables to solve systems of equations in three variables. A system of three equations in three variables can have a unique solution, infinitely many solutions, or no solution. A solution is an **ordered triple**.

Example **Solve this system of equations.**

$$3x + y - z = -6$$
$$2x - y + 2z = 8$$
$$4x + y - 3z = -21$$

Step 1 Use elimination to make a system of two equations in two variables.

$3x + y - z = -6$ First equation
$(+)\ 2x - y + 2z = 8$ Second equation
$5x + z = 2$ Add to eliminate y.

$2x - y + 2z = 8$ Second equation
$(+)\ 4x + y - 3z = -21$ Third equation
$6x - z = -13$ Add to eliminate y.

Step 2 Solve the system of two equations.

$5x + z = 2$
$(+)\ 6x - z = -13$
$11x = -11$ Add to eliminate z.
$x = -1$ Divide both sides by 11.

Substitute -1 for x in one of the equations with two variables and solve for z.

$5x + z = 2$ Equation with two variables
$5(-1) + z = 2$ Replace x with -1.
$-5 + z = 2$ Multiply.
$z = 7$ Add 5 to both sides.

The result so far is $x = -1$ and $z = 7$.

Step 3 Substitute -1 for x and 7 for z in one of the original equations with three variables.

$3x + y - z = -6$ Original equation with three variables
$3(-1) + y - 7 = -6$ Replace x with -1 and z with 7.
$-3 + y - 7 = -6$ Multiply.
$y = 4$ Simplify.

The solution is $(-1, 4, 7)$.

Exercises

Solve each system of equations.

1. $2x + 3y - z = 0$
$x - 2y - 4z = 14$
$3x + y - 8z = 17$
$(4, -3, -1)$

2. $2x - y + 4z = 11$
$x + 2y - 6z = -11$
$3x - 2y - 10z = 11$
$\left(2, -5, \frac{1}{2}\right)$

3. $x - 2y + z = 8$
$2x + y - z = 0$
$3x - 6y + 3z = 24$
infinitely many solutions

4. $3x - y - z = 5$
$3x + 2y - z = 11$
$6x - 3y + 2z = -12$
$\left(\frac{2}{3}, 2, -5\right)$

5. $2x - 4y - z = 10$
$4x - 8y - 2z = 16$
$3x + y + z = 12$
no solution

6. $x - 6y + 4z = 2$
$2x + 4y - 8z = 16$
$x - 2y = 5$
$\left(6, \frac{1}{2}, -\frac{1}{4}\right)$

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3-5 Study Guide and Intervention *(continued)*

Solving Systems of Equations in Three Variables

Real-World Problems

Example **The Laredo Sports Shop sold 10 balls, 3 bats, and 2 bases for $99 on Monday. On Tuesday they sold 4 balls, 8 bats, and 2 bases for $78. On Wednesday they sold 2 balls, 3 bats, and 1 base for $33.60. What are the prices of 1 ball, 1 bat, and 1 base?**

First define the variables.

x = price of 1 ball
y = price of 1 bat
z = price of 1 base

Translate the information in the problem into three equations.

$10x + 3y + 2z = 99$
$4x + 8y + 2z = 78$
$2x + 3y + z = 33.60$

Subtract the second equation from the first equation to eliminate z.

$10x + 3y + 2z = 99$
$(-)\ 4x + 8y + 2z = 78$
$6x - 5y = 21$

Multiply the third equation by 2 and subtract from the second equation.

$4x + 8y + 2z = 78$
$(-)\ 4x + 6y + 2z = 67.20$
$2y = 10.80$
$y = 5.40$

Substitute 5.40 for y in the equation $6x - 5y = 21$.

$6x - 5(5.40) = 21$
$6x = 48$
$x = 8$

Substitute 8 for x and 5.40 for y in one of the original equations to solve for z.

$10x + 3y + 2z = 99$
$10(8) + 3(5.40) + 2z = 99$
$80 + 16.20 + 2z = 99$
$2z = 2.80$
$z = 1.40$

So a ball costs $8, a bat $5.40, and a base $1.40.

Exercises

1. FITNESS TRAINING Carly is training for a triathlon. In her training routine each week, she runs 7 times as far as she swims, and she bikes 3 times as far as she runs. One week she trained a total of 232 miles. How far did she run that week? **56 miles**

2. ENTERTAINMENT At the arcade, Ryan, Sara, and Tim played video racing games, pinball, and air hockey. Ryan spent $6 for 6 racing games, 2 pinball games, and 1 game of air hockey. Sara spent $12 for 3 racing games, 4 pinball games, and 5 games of air hockey. Tim spent $12.25 for 2 racing games, 7 pinball games, and 4 games of air hockey. How much did each of the games cost? **Racing game: $0.50; pinball: $0.75; air hockey: $1.50**

3. FOOD A natural food store makes its own brand of trail mix out of dried apples, raisins, and peanuts. One pound of the mixture costs $3.18. It contains twice as much peanuts by weight as apples. One pound of dried apples costs $4.48, a pound of raisins $2.40, and a pound of peanuts $3.44. How many ounces of each ingredient are contained in 1 pound of the trail mix? **3 oz of apples, 7 oz of raisins, 6 oz of peanuts**

NAME ______________ DATE ________ PERIOD _____

3-5 Skills Practice

Solving Systems of Equations in Three Variables

Solve each system of equations.

1. $2a + c = -10$ **(5, −5, −20)**
$b - c = 15$
$a - 2b + c = -5$

2. $x + y + z = 3$ **(0, 2, 1)**
$13x + 2z = 2$
$-x - 5z = -5$

3. $2x + 5y + 2z = 6$ **(−3, 2, 1)**
$5x - 7y = -29$
$z = 1$

4. $x + 4y - z = 1$ **no solution**
$3x - y + 8z = 0$
$x + 4y - z = 10$

5. $-2z = -6$ **(2, −1, 3)**
$2x + 3y - z = -2$
$x + 2y + 3z = 9$

6. $3x - 2y + 2z = -2$ **(−2, 1, 3)**
$x + 6y - 2z = -2$
$x + 2y = 0$

7. $-x - 5z = -5$ **(0, 0, 1)**
$y - 3x = 0$
$13x + 2z = 2$

8. $-3r + 2t = 1$ **(1, −6, 2)**
$4r + s - 2t = -6$
$r + s + 4t = 3$

9. $x - y + 3z = 3$ **no solution**
$-2x + 2y - 6z = 6$
$y - 5z = -3$

10. $5m + 3n + p = 4$ **(−2, 3, 5)**
$3m + 2n = 0$
$2m - n + 3p = 8$

11. $2x + 2y + 2z = -2$ **infinitely many**
$2x + 3y + 2z = 4$
$x + y + z = -1$

12. $x + 2y - z = 4$ **(1, 2, 1)**
$3x - y + 2z = 3$
$-x + 3y + z = 6$

13. $3x - 2y + z = 1$ **(5, 7, 0)**
$-x + y - z = 2$
$5x + 2y + 10z = 39$

14. $3x - 5y + 2z = -12$ **infinitely many**
$x + 4y - 2z = 8$
$-3x + 5y - 2z = 12$

15. $2x + y + 3z = -2$ **(−1, 3, −1)**
$x - y - z = -3$
$3x - 2y + 3z = -12$

16. $2x - 4y + 3z = 0$ **(3, 0, −2)**
$x - 2y - 5z = 13$
$5x + 3y - 2z = 19$

17. $-2x + y + 2z = 2$ **(1, −2, 3)**
$3x + 3y + z = 0$
$x + y + z = 2$

18. $x - 2y + 2z = -1$ **infinitely many**
$x + 2y - z = 6$
$-3x + 6y - 6z = 3$

19. The sum of three numbers is 18. The sum of the first and second numbers is 15, and the first number is 3 times the third number. Find the numbers. **9, 6, 3**

Lesson 3-5

NAME ______________ DATE ________ PERIOD _____

3-5 Practice (Average)

Solving Systems of Equations in Three Variables

Solve each system of equations.

1. $2x - y + 2z = 15$
$-x + y + z = 3$
$3x - y + 2z = 18$
(3, 1, 5)

2. $x - 4y + 3z = -27$
$2x + 2y - 3z = 22$
$4z = -16$
(1, 4, −4)

3. $a + b = 3$
$-b + c = 3$
$a + 2c = 10$
(2, 1, 4)

4. $3m - 2n + 4p = 15$
$m - n + p = 3$
$m + 4n - 5p = 0$
(3, 3, 3)

5. $2g + 3h - 8j = 10$
$g - 4h = 1$
$-2g - 3h + 8j = 5$
no solution

6. $2x + y - z = -8$
$4x - y + 2z = -3$
$-3x + y + 2z = 5$
(−2, −3, 1)

7. $2x - 5y + z = 5$
$3x + 2y - z = 17$
$4x - 3y + 2z = 17$
(5, 1, 0)

8. $2x + 3y + 4z = 2$
$5x - 2y + 3z = 0$
$x - 5y - 2z = -4$
(2, 2, −2)

9. $p + 4r = -7$
$p - 3q = -8$
$q + r = 1$
(1, 3, −2)

10. $4x + 4y - 2z = 8$
$3x - 5y + 3z = 0$
$2x + 2y - z = 4$
infinitely many

11. $d + 3e + f = 0$
$-d + 2e + f = -1$
$4d + e - f = 1$
(1, −1, 2)

12. $4x + y + 5z = -9$
$x - 4y - 2z = -2$
$2x + 3y - 2z = 21$
(2, 3, −4)

13. $5x + 9y + z = 20$
$2x - y - z = -21$
$5x + 2y + 2z = -21$
(−7, 6, 1)

14. $2x + y - 3z = -3$
$3x + 2y + 4z = 5$
$-6x - 3y + 9z = 9$
infinitely many

15. $3x + 3y + z = 10$
$5x + 2y + 2z = 7$
$3x - 2y + 3z = -9$
(1, 3, −2)

16. $2u + v + w = 2$
$-3u + 2v + 3w = 7$
$-u - v + 2w = 7$
(0, −1, 3)

17. $x + 5y - 3z = -18$
$3x - 2y + 5z = 22$
$-2x - 3y + 8z = 28$
(1, −2, 3)

18. $x - 2y + z = -1$
$-x + 2y - z = 6$
$-4y + 2z = 1$
no solution

19. $2x - 2y - 4z = -2$
$3x - 3y - 6z = -3$
$-2x + 3y + z = 7$
(4, 5, 0)

20. $x - y + 9z = -27$
$2x - 4y - z = -1$
$3x + 6y - 3z = 27$
(2, 2, −3)

21. $2x - 5y - 3z = 7$
$-4x + 10y + 2z = 6$
$6x - 15y - z = -19$
(1, 2, −5)

22. The sum of three numbers is 6. The third number is the sum of the first and second numbers. The first number is one more than the third number. Find the numbers.
4, −1, 3

23. The sum of three numbers is −4. The second number decreased by the third is equal to the first. The sum of the first and second numbers is −5. Find the numbers. **−3, −2, 1**

24. SPORTS Alexandria High School scored 37 points in a football game. Six points are awarded for each touchdown. After each touchdown, the team can earn one point for the extra kick or two points for a 2-point conversion. The team scored one fewer 2-point conversions than extra kicks. The team scored 10 times during the game. How many touchdowns were made during the game? **5**

Answers

NAME ______________ DATE ________ PERIOD _____

3-5 Reading to Learn Mathematics

Solving Systems of Equations in Three Variables

Pre-Activity **How can you determine the number and type of medals U.S. Olympians won?**

Read the introduction to Lesson 3-5 at the top of page 138 in your textbook.

At the 1996 Summer Olympics in Atlanta, Georgia, the United States won 101 medals. The U.S. team won 12 more gold medals than silver and 7 fewer bronze medals than silver. Using the same variables as those in the introduction, write a system of equations that describes the medals won for the 1996 Summer Olympics.
$g + s + b = 101$; $g = s + 12$; $b = s - 7$

Reading the Lesson

1. The planes for the equations in a system of three linear equations in three variables determine the number of solutions. Match each graph description below with the description of the number of solutions of the system. (Some of the items on the right may be used more than once, and not all possible types of graphs are listed.)

a. three parallel planes **II**

b. three planes that intersect in a line **III**

c. three planes that intersect in one point **I**

d. one plane that represents all three equations **III**

I. one solution

II. no solutions

III. infinite solutions

2. Suppose that three classmates, Monique, Josh, and Lilly, are studying for a quiz on this lesson. They work together on solving a system of equations in three variables, x, y, and z, following the algebraic method shown in your textbook. They first find that $z = 3$, then that $y = -2$, and finally that $x = -1$. The students agree on these values, but disagree on how to write the solution. Here are their answers:

Monique: $(3, -2, -1)$ Josh: $(-2, -1, 3)$ Lilly: $(-1, -2, 3)$

a. How do you think each student decided on the order of the numbers in the ordered triple? **Sample answer: Monique arranged the values in the order in which she found them. Josh arranged them from smallest to largest. Lilly arranged them in alphabetical order of the variables.**

b. Which student is correct? **Lilly**

Helping You Remember

3. How can you remember that obtaining the equation $0 = 0$ indicates a system with infinitely many solutions, while obtaining an equation such as $0 = 8$ indicates a system with no solutions? **$0 = 0$ is always true, while $0 = 8$ is never true.**

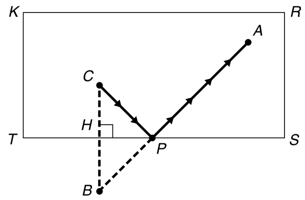
Lesson 3-5

NAME ______________ DATE ________ PERIOD _____

3-5 Enrichment

Billiards

The figure at the right shows a billiard table. The object is to use a cue stick to strike the ball at point C so that the ball will hit the sides (or cushions) of the table at least once before hitting the ball located at point A. In playing the game, you need to locate point P.



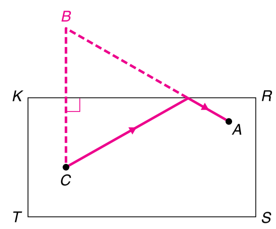
Step 1 Find point B so that $\overline{BC} \perp \overline{ST}$ and $\overline{BH} \cong \overline{CH}$. B is called the reflected image of C in $\overline{ST}$.

Step 2 Draw $\overline{AB}$.

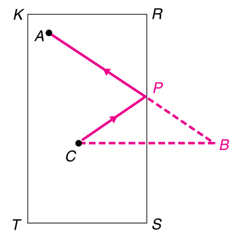
Step 3 $\overline{AB}$ intersects $\overline{ST}$ at the desired point P.

For each billiards problem, the cue ball at point C must strike the indicated cushion(s) and then strike the ball at point A. Draw and label the correct path for the cue ball using the process described above.

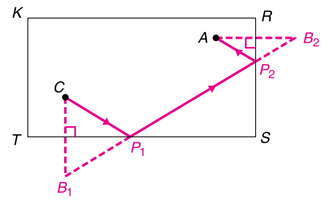
1. cushion $\overline{KR}$



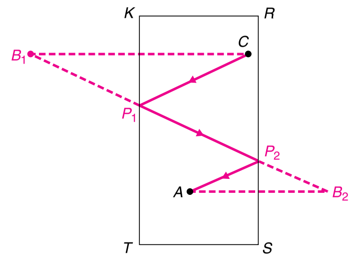
2. cushion $\overline{RS}$



3. cushion $\overline{TS}$, then cushion $\overline{RS}$



4. cushion $\overline{KT}$, then cushion $\overline{RS}$



Chapter 3 Assessment Answer Key

Form 1
Page 149

1. D
2. C
3. A
4. A
5. C
6. B
7. A
8. D
9. C
10. B
11. B

Page 150

12. A
13. B
14. C
15. D
16. B
17. C
18. A
19. B
20. A

B: 4 units^2

Form 2A
Page 151

1. B
2. C
3. B
4. D
5. B
6. A
7. C
8. A
9. D
10. D
11. B

(continued on the next page)

Answers

Chapter 3 Assessment Answer Key

Form 2A *(continued)*

Page 152

12. B
13. A
14. D
15. C
16. C
17. D
18. C
19. A
20. A

B: $\frac{9}{8}$

Form 2B

Page 153

1. A
2. A
3. B
4. A
5. C
6. B
7. A
8. D
9. D
10. D
11. C

Page 154

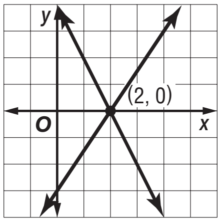
12. A
13. B
14. C
15. C
16. D
17. A
18. B
19. C
20. C

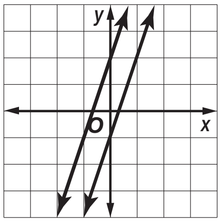
B: $a = 2$, $b = 4$, $c = 1$, $d = 3$, $f = 0$

Chapter 3 Assessment Answer Key

Form 2C

Page 155

1. 

2. 

no solution

3. consistent and independent

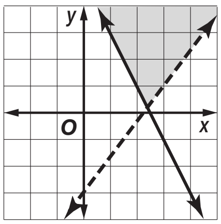
4. consistent and dependent

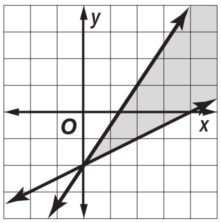
5. (4, 1)

6. (0, 4)

7. (2, 1)

8. (2, −2)

9. 

10. 

11. (−2, −3), (2, −3), (2, 5)

12. (−2, 0), (3, −5), (3, 2), (0, 4)

Page 156

13. (−2, −3), (−2, 9), (2, 5)

14. max: $f(2, 5) = 1$; min: $f(-2, 9) = -15$

15. $c \geq 0$, $b \geq 0$, $6c + 30b \leq 600$, $c + b \leq 60$

16. 50 cars, 10 busses

17. (0, 1, −1)

18. (−1, 2, 1)

19. $s + m + \ell = 9$
$7s + 12m + 15\ell = 86$
$m = 3\ell$

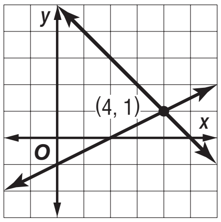
20. 5 small packages, 3 medium packages, 1 large package

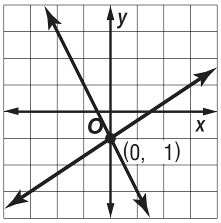
B: 20 units

Chapter 3 Assessment Answer Key

Form 2D

Page 157

1. 

2. 

3. inconsistent

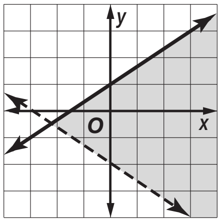
4. consistent and independent

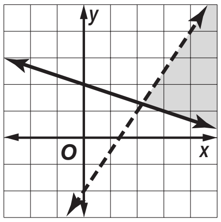
5. (4, 6)

6. (−2, −8)

7. (−1, 3)

8. (2, −2)

9. 

10. 

11. (−3, −2), (0, −2), (−3, 4)

12. (−1, −3), (−4, 0), (0, 3), (2, 3)

Page 158

13. (−5, 7), (1, 7), (−1, 3)

14. max: $f(1, 7) = 10$; min: $f(-5, 7) = -8$

15. $\ell \geq 0$, $v \geq 4$, $v + \ell \leq 15$

16. \$138

17. (−2, 3, 5)

18. (3, 1, −4)

19. $w = 2h$, $t = h - 4$, $3w + 2h + 5t = 136$

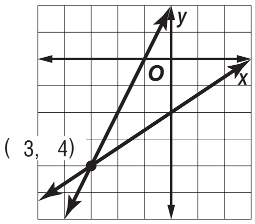
20. \$24

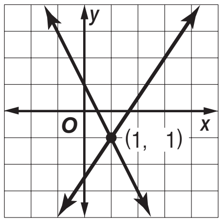
B: (12, −8)

Chapter 3 Assessment Answer Key

Form 3

Page 159

1. 

2. 

3. consistent and dependent

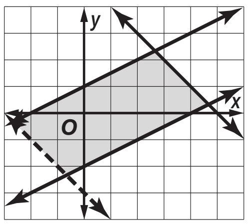
4. inconsistent

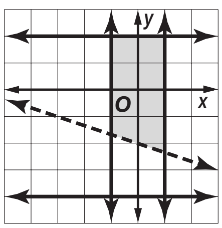
5. $(-2, 6)$

6. no solution

7. $\left(-3, \frac{1}{4}\right)$

8. $(1.5, -2)$

9. 

10. 

11. $(-6, 3), (2, 3), (2, -4), (1, -4)$

12. $(-2, 4), (5, -3), (5, -4), (1, -4), (-2, 2)$

Page 160

13. $(-3, -4), (-3, 1), (0, 4), (3, 2)$

14. max: $f(3, 2) = 8$; min: $f(-3, 1) = -\frac{19}{2}$

15. $60a + 150m \leq 900$, $a \geq 3$, $m \geq 4$

16. 5 ads and 4 commercial minutes; 124,000 people

17. $\left(\frac{1}{2}, -\frac{2}{3}, -5\right)$

18. $\left(\frac{1}{5}, -1, 21\right)$

19. $s = 3a$, $m = a - 10$, $15s + 40a + 5m = 2650$

20. $90

B: $(6, 2)$

Chapter 3 Assessment Answer Key

Page 161, Open-Ended Assessment
Scoring Rubric

Score	General Description	Specific Criteria
4	**Superior** A correct solution that is supported by well-developed, accurate explanations	• Shows thorough understanding of the concepts of *solving systems of linear equations by graphing, by substitution, and by elimination; solving systems of inequalities by graphing; solving problems using linear programming;* and *solving systems in three variables.* • Uses appropriate strategies to solve problems. • Computations are correct. • Written explanations are exemplary. • Graphs are accurate and appropriate. • Goes beyond requirements of some or all problems.
3	**Satisfactory** A generally correct solution, but may contain minor flaws in reasoning or computation	• Shows an understanding of the concepts of *solving systems of linear equations by graphing, by substitution, and by elimination; solving systems of inequalities by graphing; solving problems using linear programming;* and *solving systems in three variables.* • Uses appropriate strategies to solve problems. • Computations are mostly correct. • Written explanations are effective. • Graphs are mostly accurate and appropriate. • Satisfies all requirements of problems.
2	**Nearly Satisfactory** A partially correct interpretation and/or solution to the problem	• Shows an understanding of most of the concepts of *solving systems of linear equations by graphing, by substitution, and by elimination; solving systems of inequalities by graphing; solving problems using linear programming;* and *solving systems in three variables.* • May not use appropriate strategies to solve problems. • Computations are mostly correct. • Written explanations are satisfactory. • Graphs are mostly accurate. • Satisfies the requirements of most of the problems.
1	**Nearly Unsatisfactory** A correct solution with no supporting evidence or explanation	• Final computation is correct. • No written explanations or work is shown to substantiate the final computation. • Graphs may be accurate but lack detail or explanation. • Satisfies minimal requirements of some of the problems.
0	**Unsatisfactory** An incorrect solution indicating no mathematical understanding of the concept or task, or no solution is given	• Shows little or no understanding of most of *solving systems of linear equations by graphing, by substitution, and by elimination; solving systems of inequalities by graphing; solving problems using linear programming;* and *solving systems in three variables.* • Does not use appropriate strategies to solve problems. • Computations are incorrect. • Written explanations are unsatisfactory. • Graphs are inaccurate or inappropriate. • Does not satisfy requirements of problems. • No answer may be given.

Chapter 3 Assessment Answer Key

Page 161, Open-Ended Assessment
Sample Answers

In addition to the scoring rubric found on page A22, the following sample answers may be used as guidance in evaluating open-ended assessment items.

1. Let J = Janet's age, K = Kim's age, and S = Sue's age.

$J^2 = (K + S)^2 + 400$ and $K + S = J - 10$
$J^2 = (J - 10)^2 + 400$
$J^2 = J^2 - 20J + 100 + 400$
$20J = 500$
$J = 25$

Thus, $K + S = 25 - 10$ or 15 and $J + (K + S) = 25 + 15$ or 40. Therefore, $(J + K + S)^2 = 40^2$ or 1600.

2a. $x \geq 0$
$y \geq 0$
$y \leq -\frac{4}{3}x + 8$

2b. Sample answer: Evaluate $f(x, y)$ at each of the vertices of the region.

3. Student responses should indicate that the solution of a system of equations is an ordered pair representing the point at which the graphs of the equations intersect. Solutions should include the fact that a system may have no solution, indicating that the graphs of the lines are parallel and that a system may have an infinite number of solutions, indicating that the equations represent the same line.

4. Student responses should indicate that it is impossible for a system to be both inconsistent (a system of non-intersecting lines) and dependent (a system of two representations of the same line).

5. Students should demonstrate an understanding that, if the graphs of the cost and revenue functions are parallel, the cost and revenue have the same rate of change. This means that the company will never break even on the production and sale of these products and, therefore, never make a profit. The owner may decide to increase the price charged for the product, may look for ways to cut costs, may put the company resources into the production of other products, or may even decide to close up shop.

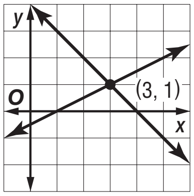
6. Students should indicate that a system of two linear inequalities has no solution when there are no ordered pairs which satisfy both inequalities. Graphically, this means that the shaded regions which represent the graphs of the inequalities do not intersect. Students' graphs should show two parallel lines with shading everywhere but between them.
Sample answer: $x \leq -3, x \geq 2$

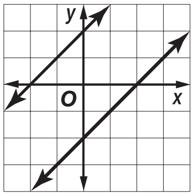
Chapter 3 Assessment Answer Key

Vocabulary Test/Review
Page 162

1. inconsistent system
2. dependent system
3. ordered triple
4. substitution method
5. constraints
6. system of inequalities
7. elimination method
8. consistent system independent system
9. system of equations
10. linear programming
11. Sample answer: In a linear programming problem, the region that is the intersection of the graphs of the constraints is called the feasible region.
12. Sample answer: When a system of linear inequalities is graphed, if the solutions form a region that is not a polygonal region, then we say the region is an unbounded region.

Quiz (Lessons 3–1 and 3–2)
Page 163

1. 

2. 

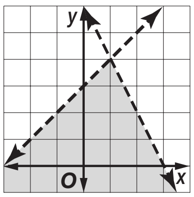
inconsistent

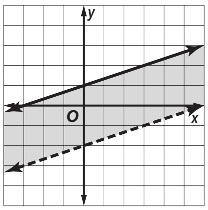
3. no solution; inconsistent system

4. (−3, −2)

5. A

Quiz (Lesson 3–3)
Page 163

1. 

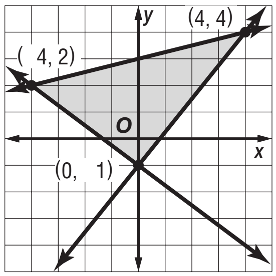
2. 

3. (0, 0), (0, 4), (2, 0)

4. (1, −2), (2, 0), (3, −1), (3, −2)

Quiz (Lesson 3–4)
Page 164

1. max: $f(1, 6) = 11$; min: $f(1, -1) = -3$

2. 

vertices: (−4, 2), (4, 4), (0, −1); max: $f(0, -1) = 5$; min: $f(4, 4) = -16$

3. $j \geq 0, p \geq 0,$ $j + 2p \leq 20,$ $4j + 2p \leq 32$

4. 4 jackets and 8 pairs of pants; $120

Quiz (Lesson 3–5)
Page 164

1. (−1, 2, −4)

2. (2, 3, −1)

3. (1, −2, 4)

4. $x + y + z = 155,$ $x = 9y, y = 3z$

5. 135 cars, 15 vans, 5 trucks

Chapter 3 Assessment Answer Key

Mid-Chapter Test
Page 165

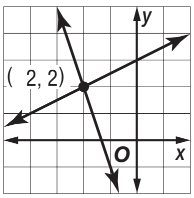
1. C

2. C

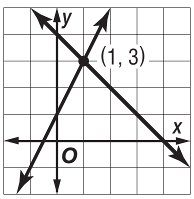
3. A

4. A

5.

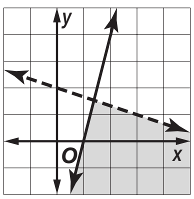


6.



7. consistent and dependent

8.



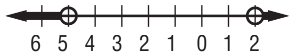
9. (−2, 3), (−2, 6), (0, 6), (4, 0)

Cumulative Review
Page 166

1.

2.

3. $\{x \mid x < -5 \text{ or } x > 2\}$ or $(-\infty, -5) \cup (2, +\infty)$



4. $-8a^3 + 4a - 5$

5. No, because the variable appears in a denominator.

6. No, because x has an exponent other than 1.

7.

8. $\frac{37}{8}$

9. consistent and independent

10.

11.

12. (1, 3), (6, −2), (1, −2)

13. max: $f(1, 3) = 0$; min: $f(6, -2) = -20$

14.

Answers

Chapter 3 Assessment Answer Key

Standardized Test Practice

Page 167

1. A B C **D**

2. E **F** G H

3. **A** B C D

4. **E** F G H

5. A B **C** D

6. E **F** G H

7. A B **C** D

8. **E** F G H

9. A **B** C D

10. E F G **H**

Page 168

11. 2

12. 200

13. 9

14. 8

15. A **B** C D

16. A B C **D**

17. **A** B C D

18. A B **C** D

19. A **B** C D