GLENCOE MATHEMATICS

Algebra 2

Chapter 8
Resource Masters

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Consumable Workbooks

Many of the worksheets contained in the Chapter Resource Masters booklets are available as consumable workbooks.

Study Guide and Intervention Workbook	0-07-828029-X
Skills Practice Workbook	0-07-828023-0
Practice Workbook	0-07-828024-9

ANSWERS FOR WORKBOOKS The answers for Chapter 8 of these workbooks can be found in the back of this Chapter Resource Masters booklet.

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Algebra 2
Chapter 8 Resource Masters

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Contents

Teacher's Guide to Using the Chapter 8 Resource Masters

The ***Fast File*** Chapter Resource system allows you to conveniently file the resources you use most often. The *Chapter 8 Resource Masters* includes the core materials needed for Chapter 8. These materials include worksheets, extensions, and assessment options. The answers for these pages appear at the back of this booklet.

All of the materials found in this booklet are included for viewing and printing in the *Algebra 2 TeacherWorks* CD-ROM.

Vocabulary Builder Pages vii–viii include a student study tool that presents up to twenty of the key vocabulary terms from the chapter. Students are to record definitions and/or examples for each term. You may suggest that students highlight or star the terms with which they are not familiar.

WHEN TO USE Give these pages to students before beginning Lesson 8-1. Encourage them to add these pages to their Algebra 2 Study Notebook. Remind them to add definitions and examples as they complete each lesson.

Study Guide and Intervention

Each lesson in *Algebra 2* addresses two objectives. There is one Study Guide and Intervention master for each objective.

WHEN TO USE Use these masters as reteaching activities for students who need additional reinforcement. These pages can also be used in conjunction with the Student Edition as an instructional tool for students who have been absent.

Skills Practice There is one master for each lesson. These provide computational practice at a basic level.

WHEN TO USE These masters can be used with students who have weaker mathematics backgrounds or need additional reinforcement.

Practice There is one master for each lesson. These problems more closely follow the structure of the Practice and Apply section of the Student Edition exercises. These exercises are of average difficulty.

WHEN TO USE These provide additional practice options or may be used as homework for second day teaching of the lesson.

Reading to Learn Mathematics

One master is included for each lesson. The first section of each master asks questions about the opening paragraph of the lesson in the Student Edition. Additional questions ask students to interpret the context of and relationships among terms in the lesson. Finally, students are asked to summarize what they have learned using various representation techniques.

WHEN TO USE This master can be used as a study tool when presenting the lesson or as an informal reading assessment after presenting the lesson. It is also a helpful tool for ELL (English Language Learner) students.

Enrichment There is one extension master for each lesson. These activities may extend the concepts in the lesson, offer an historical or multicultural look at the concepts, or widen students' perspectives on the mathematics they are learning. These are not written exclusively for honors students, but are accessible for use with all levels of students.

WHEN TO USE These may be used as extra credit, short-term projects, or as activities for days when class periods are shortened.

Assessment Options

The assessment masters in the *Chapter 8 Resource Masters* offer a wide range of assessment tools for intermediate and final assessment. The following lists describe each assessment master and its intended use.

Chapter Assessment

CHAPTER TESTS

- *Form 1* contains multiple-choice questions and is intended for use with basic level students.
- *Forms 2A and 2B* contain multiple-choice questions aimed at the average level student. These tests are similar in format to offer comparable testing situations.
- *Forms 2C and 2D* are composed of free-response questions aimed at the average level student. These tests are similar in format to offer comparable testing situations. Grids with axes are provided for questions assessing graphing skills.
- *Form 3* is an advanced level test with free-response questions. Grids without axes are provided for questions assessing graphing skills.

  All of the above tests include a free-response Bonus question.
- The **Open-Ended Assessment** includes performance assessment tasks that are suitable for all students. A scoring rubric is included for evaluation guidelines. Sample answers are provided for assessment.
- A **Vocabulary Test**, suitable for all students, includes a list of the vocabulary words in the chapter and ten questions assessing students' knowledge of those terms. This can also be used in conjunction with one of the chapter tests or as a review worksheet.

Intermediate Assessment

- Four free-response **quizzes** are included to offer assessment at appropriate intervals in the chapter.
- A **Mid-Chapter Test** provides an option to assess the first half of the chapter. It is composed of both multiple-choice and free-response questions.

Continuing Assessment

- The **Cumulative Review** provides students an opportunity to reinforce and retain skills as they proceed through their study of Algebra 2. It can also be used as a test. This master includes free-response questions.
- The **Standardized Test Practice** offers continuing review of algebra concepts in various formats, which may appear on the standardized tests that they may encounter. This practice includes multiple-choice, grid-in, and quantitative-comparison questions. Bubble-in and grid-in answer sections are provided on the master.

Answers

- Page A1 is an answer sheet for the Standardized Test Practice questions that appear in the Student Edition on pages 468–469. This improves students' familiarity with the answer formats they may encounter in test taking.
- The answers for the lesson-by-lesson masters are provided as reduced pages with answers appearing in red.
- Full-size answer keys are provided for the assessment masters in this booklet.

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8

Reading to Learn Mathematics

Vocabulary Builder

Vocabulary Builder

This is an alphabetical list of the key vocabulary terms you will learn in Chapter 8. As you study the chapter, complete each term's definition or description. Remember to add the page number where you found the term. Add these pages to your Algebra Study Notebook to review vocabulary at the end of the chapter.

Vocabulary Term	Found on Page	Definition/Description/Example
asymptote A·suhm(p)·TOHT		
center of a circle		
center of an ellipse		
circle		
conic section		
conjugate axis KAHN·jih·guht		
directrix duh·REHK·trihks		
distance formula		
ellipse ih·LIHPS		

(continued on the next page)

8 Reading to Learn Mathematics

Vocabulary Builder *(continued)*

Vocabulary Term	Found on Page	Definition/Description/Example
foci of an ellipse		
focus of a parabola FOH·kuhs		
hyperbola hy·PUHR·buh·luh		
latus rectum LA·tuhs REHK·tuhm		
major axis		
midpoint formula		
minor axis		
parabola puh·RA·buh·luh		
tangent TAN·juhnt		
transverse axis		

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8-1 Study Guide and Intervention

Midpoint and Distance Formulas

The Midpoint Formula

Midpoint Formula	The midpoint M of a segment with endpoints (x_1, y_1) and (x_2, y_2) is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

Example 1 **Find the midpoint of the line segment with endpoints at (4, −7) and (−2, 3).**

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{4 + (-2)}{2}, \frac{-7 + 3}{2}\right)$$

$$= \left(\frac{2}{2}, \frac{-4}{2}\right) \text{ or } (1, -2)$$

The midpoint of the segment is (1, −2).

Example 2 **A diameter $\overline{AB}$ of a circle has endpoints $A(5, -11)$ and $B(-7, 6)$. What are the coordinates of the center of the circle?**

The center of the circle is the midpoint of all of its diameters.

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{5 + (-7)}{2}, \frac{-11 + 6}{2}\right)$$

$$= \left(\frac{-2}{2}, \frac{-5}{2}\right) \text{ or } \left(-1, -2\frac{1}{2}\right)$$

The circle has center $\left(-1, -2\frac{1}{2}\right)$.

Exercises

Find the midpoint of each line segment with endpoints at the given coordinates.

1. (12, 7) and (−2, 11)

2. (−8, −3) and (10, 9)

3. (4, 15) and (10, 1)

4. (−3, −3) and (3, 3)

5. (15, 6) and (12, 14)

6. (22, −8) and (−10, 6)

7. (3, 5) and (−6, 11)

8. (8, −15) and (−7, 13)

9. (2.5, −6.1) and (7.9, 13.7)

10. (−7, −6) and (−1, 24)

11. (3, −10) and (30, −20)

12. (−9, 1.7) and (−11, 1.3)

13. Segment $\overline{MN}$ has midpoint P. If M has coordinates (14, −3) and P has coordinates (−8, 6), what are the coordinates of N?

14. Circle R has a diameter $\overline{ST}$. If R has coordinates (−4, −8) and S has coordinates (1, 4), what are the coordinates of T?

15. Segment $\overline{AD}$ has midpoint B, and $\overline{BD}$ has midpoint C. If A has coordinates (−5, 4) and C has coordinates (10, 11), what are the coordinates of B and D?

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8-1 Study Guide and Intervention *(continued)*

Midpoint and Distance Formulas

The Distance Formula

Distance Formula	The distance between two points (x_1, y_1) and (x_2, y_2) is given by $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

Example 1 **What is the distance between (8, −2) and (−6, −8)?**

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Distance Formula

$= \sqrt{(-6 - 8)^2 + [-8 - (-2)]^2}$ Let $(x_1, y_1) = (8, -2)$ and $(x_2, y_2) = (-6, -8)$.

$= \sqrt{(-14)^2 + (-6)^2}$ Subtract.

$= \sqrt{196 + 36}$ or $\sqrt{232}$ Simplify.

The distance between the points is $\sqrt{232}$ or about 15.2 units.

Example 2 **Find the perimeter and area of square *PQRS* with vertices *P*(−4, 1), *Q*(−2, 7), *R*(4, 5), and *S*(2, −1).**

Find the length of one side to find the perimeter and the area. Choose $\overline{PQ}$.

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Distance Formula

$= \sqrt{[-4 - (-2)]^2 + (1 - 7)^2}$ Let $(x_1, y_1) = (-4, 1)$ and $(x_2, y_2) = (-2, 7)$.

$= \sqrt{(-2)^2 + (-6)^2}$ Subtract.

$= \sqrt{40}$ or $2\sqrt{10}$ Simplify.

Since one side of the square is $2\sqrt{10}$, the perimeter is $8\sqrt{10}$ units. The area is $(2\sqrt{10})^2$, or 40 units^2.

Exercises

Find the distance between each pair of points with the given coordinates.

1. (3, 7) and (−1, 4)

2. (−2, −10) and (10, −5)

3. (6, −6) and (−2, 0)

4. (7, 2) and (4, −1)

5. (−5, −2) and (3, 4)

6. (11, 5) and (16, 9)

7. (−3, 4) and (6, −11)

8. (13, 9) and (11, 15)

9. (−15, −7) and (2, 12)

10. Rectangle *ABCD* has vertices *A*(1, 4), *B*(3, 1), *C*(−3, −2), and *D*(−5, 1). Find the perimeter and area of *ABCD*.

11. Circle *R* has diameter $\overline{ST}$ with endpoints *S*(4, 5) and *T*(−2, −3). What are the circumference and area of the circle? (Express your answer in terms of π.)

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8-1 Skills Practice

Midpoint and Distance Formulas

Lesson 8-1

Find the midpoint of each line segment with endpoints at the given coordinates.

1. $(4, -1), (-4, 1)$

2. $(-1, 4), (5, 2)$

3. $(3, 4), (5, 4)$

4. $(6, 2), (2, -1)$

5. $(3, 9), (-2, -3)$

6. $(-3, 5), (-3, -8)$

7. $(3, 2), (-5, 0)$

8. $(3, -4), (5, 2)$

9. $(-5, -9), (5, 4)$

10. $(-11, 14), (0, 4)$

11. $(3, -6), (-8, -3)$

12. $(0, 10), (-2, -5)$

Find the distance between each pair of points with the given coordinates.

13. $(4, 12), (-1, 0)$

14. $(7, 7), (-5, -2)$

15. $(-1, 4), (1, 4)$

16. $(11, 11), (8, 15)$

17. $(1, -6), (7, 2)$

18. $(3, -5), (3, 4)$

19. $(2, 3), (3, 5)$

20. $(-4, 3), (-1, 7)$

21. $(-5, -5), (3, 10)$

22. $(3, 9), (-2, -3)$

23. $(6, -2), (-1, 3)$

24. $(-4, 1), (2, -4)$

25. $(0, -3), (4, 1)$

26. $(-5, -6), (2, 0)$

8-1 Practice

Midpoint and Distance Formulas

Find the midpoint of each line segment with endpoints at the given coordinates.

1. $(8, -3), (-6, -11)$

2. $(-14, 5), (10, 6)$

3. $(-7, -6), (1, -2)$

4. $(8, -2), (8, -8)$

5. $(9, -4), (1, -1)$

6. $(3, 3), (4, 9)$

7. $(4, -2), (3, -7)$

8. $(6, 7), (4, 4)$

9. $(-4, -2), (-8, 2)$

10. $(5, -2), (3, 7)$

11. $(-6, 3), (-5, -7)$

12. $(-9, -8), (8, 3)$

13. $(2.6, -4.7), (8.4, 2.5)$

14. $\left(-\frac{1}{3}, 6\right), \left(\frac{2}{3}, 4\right)$

15. $(-2.5, -4.2), (8.1, 4.2)$

16. $\left(\frac{1}{8}, \frac{1}{2}\right), \left(-\frac{5}{8}, -\frac{1}{2}\right)$

Find the distance between each pair of points with the given coordinates.

17. $(5, 2), (2, -2)$

18. $(-2, -4), (4, 4)$

19. $(-3, 8), (-1, -5)$

20. $(0, 1), (9, -6)$

21. $(-5, 6), (-6, 6)$

22. $(-3, 5), (12, -3)$

23. $(-2, -3), (9, 3)$

24. $(-9, -8), (-7, 8)$

25. $(9, 3), (9, -2)$

26. $(-1, -7), (0, 6)$

27. $(10, -3), (-2, -8)$

28. $(-0.5, -6), (1.5, 0)$

29. $\left(\frac{2}{5}, \frac{3}{5}\right), \left(1, \frac{7}{5}\right)$

30. $(-4\sqrt{2}, -\sqrt{5}), (-5\sqrt{2}, 4\sqrt{5})$

31. GEOMETRY Circle O has a diameter $\overline{AB}$. If A is at $(-6, -2)$ and B is at $(-3, 4)$, find the center of the circle and the length of its diameter.

32. GEOMETRY Find the perimeter of a triangle with vertices at $(1, -3)$, $(-4, 9)$, and $(-2, 1)$.

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8-1 Reading to Learn Mathematics

Midpoint and Distance Formulas

Pre-Activity **How are the Midpoint and Distance Formulas used in emergency medicine?**

Read the introduction to Lesson 8-1 at the top of page 412 in your textbook.

How do you find distances on a road map?

Lesson 8-1

Reading the Lesson

1. a. Write the coordinates of the midpoint of a segment with endpoints (x_1, y_1) and (x_2, y_2).

b. Explain how to find the midpoint of a segment if you know the coordinates of the endpoints. Do not use subscripts in your explanation.

2. a. Write an expression for the distance between two points with coordinates (x_1, y_1) and (x_2, y_2).

b. Explain how to find the distance between two points. Do not use subscripts in your explanation.

3. Consider the segment connecting the points $(-3, 5)$ and $(9, 11)$.

a. Find the midpoint of this segment.

b. Find the length of the segment. Write your answer in simplified radical form.

Helping You Remember

4. How can the "mid" in *midpoint* help you remember the midpoint formula?

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8-1 Enrichment

Quadratic Form

Consider two methods for solving the following equation.

$$(y - 2)^2 - 5(y - 2) + 6 = 0$$

One way to solve the equation is to simplify first, then use factoring.

$$\begin{aligned} y^2 - 4y + 4 - 5y + 10 + 6 &= 0 \\ y^2 - 9y + 20 &= 0 \\ (y - 4)(y - 5) &= 0 \end{aligned}$$

Thus, the solution set is {4, 5}.

Another way to solve the equation is first to replace $y - 2$ by a single variable. This will produce an equation that is easier to solve than the original equation. Let $t = y - 2$ and then solve the new equation.

$$\begin{aligned} (y - 2)^2 - 5(y - 2) + 6 &= 0 \\ t^2 - 5t + 6 &= 0 \\ (t - 2)(t - 3) &= 0 \end{aligned}$$

Thus, t is 2 or 3. Since $t = y - 2$, the solution set of the original equation is {4, 5}.

Solve each equation using two different methods.

1. $(z + 2)^2 + 8(z + 2) + 7 = 0$

2. $(3x - 1)^2 - (3x - 1) - 20 = 0$

3. $(2t + 1)^2 - 4(2t + 1) + 3 = 0$

4. $(y^2 - 1)^2 - (y^2 - 1) - 2 = 0$

5. $(a^2 - 2)^2 - 2(a^2 - 2) - 3 = 0$

6. $\left(1 + \sqrt{c}\right)^2 + \left(1 + \sqrt{c}\right) - 6 = 0$

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8-2 Study Guide and Intervention

Parabolas

Equations of Parabolas A parabola is a curve consisting of all points in the coordinate plane that are the same distance from a given point (the **focus**) and a given line (the **directrix**). The following chart summarizes important information about parabolas.

Standard Form of Equation	$y = a(x - h)^2 + k$	$x = a(y - k)^2 + h$
Axis of Symmetry	$x = h$	$y = k$
Vertex	(h, k)	(h, k)
Focus	$\left(h, k + \frac{1}{4a}\right)$	$\left(h + \frac{1}{4a}, k\right)$
Directrix	$y = k - \frac{1}{4a}$	$x = h - \frac{1}{4a}$
Direction of Opening	upward if $a > 0$, downward if $a < 0$	right if $a > 0$, left if $a < 0$
Length of Latus Rectum	$\left\vert\frac{1}{a}\right\vert$ units	$\left\vert\frac{1}{a}\right\vert$ units

Example **Identify the coordinates of the vertex and focus, the equations of the axis of symmetry and directrix, and the direction of opening of the parabola with equation $y = 2x^2 - 12x - 25$.**

$y = 2x^2 - 12x - 25$ Original equation
$y = 2(x^2 - 6x) - 25$ Factor 2 from the x-terms.
$y = 2(x^2 - 6x + ■) - 25 - 2(■)$ Complete the square on the right side.
$y = 2(x^2 - 6x + 9) - 25 - 2(9)$ The 9 added to complete the square is multiplied by 2.
$y = 2(x - 3)^2 - 43$ Write in standard form.

The vertex of this parabola is located at $(3, -43)$, the focus is located at $\left(3, -42\frac{7}{8}\right)$, the equation of the axis of symmetry is $x = 3$, and the equation of the directrix is $y = -43\frac{1}{8}$. The parabola opens upward.

Exercises

Identify the coordinates of the vertex and focus, the equations of the axis of symmetry and directrix, and the direction of opening of the parabola with the given equation.

1. $y = x^2 + 6x - 4$

2. $y = 8x - 2x^2 + 10$

3. $x = y^2 - 8y + 6$

Write an equation of each parabola described below.

4. focus $(-2, 3)$, directrix $x = -2\frac{1}{12}$

5. vertex $(5, 1)$, focus $\left(4\frac{11}{12}, 1\right)$

Lesson 8-2

NAME ______________________________ DATE ____________ PERIOD _____

8-2 Study Guide and Intervention *(continued)*

Parabolas

Graph Parabolas To graph an equation for a parabola, first put the given equation in standard form.

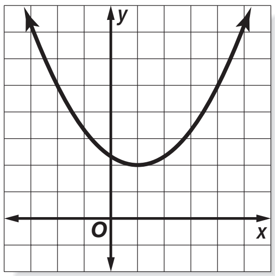
$y = a(x - h)^2 + k$ for a parabola opening up or down, or
$x = a(y - k)^2 + h$ for a parabola opening to the left or right

Use the values of a, h, and k to determine the vertex, focus, axis of symmetry, and length of the latus rectum. The vertex and the endpoints of the latus rectum give three points on the parabola. If you need more points to plot an accurate graph, substitute values for points near the vertex.

Example **Graph $y = \frac{1}{3}(x - 1)^2 + 2$.**

In the equation, $a = \frac{1}{3}$, $h = 1$, $k = 2$.

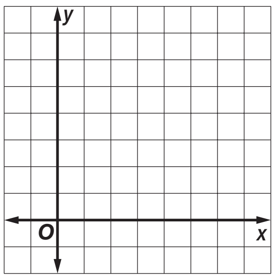
The parabola opens up, since $a > 0$.
vertex: (1, 2)
axis of symmetry: $x = 1$
focus: $\left(1, 2 + \frac{1}{4\left(\frac{1}{3}\right)}\right)$ or $\left(1, 2\frac{3}{4}\right)$
length of latus rectum: $\left|\frac{1}{\frac{1}{3}}\right|$ or 3 units
endpoints of latus rectum: $\left(2\frac{1}{2}, 2\frac{3}{4}\right), \left(-\frac{1}{2}, 2\frac{3}{4}\right)$



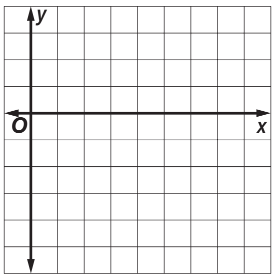
Exercises

The coordinates of the focus and the equation of the directrix of a parabola are given. Write an equation for each parabola and draw its graph.

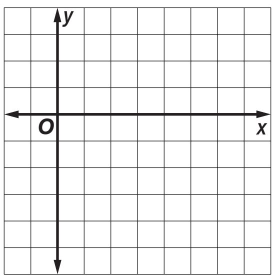
1. (3, 5), $y = 1$



2. (4, −4), $y = -6$



3. (5, −1), $x = 3$



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8-2 Skills Practice

Parabolas

Write each equation in standard form.

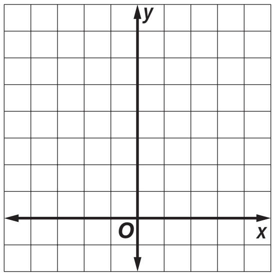
1. $y = x^2 + 2x + 2$

2. $y = x^2 - 2x + 4$

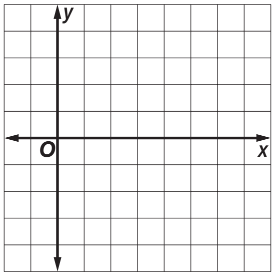
3. $y = x^2 + 4x + 1$

Identify the coordinates of the vertex and focus, the equations of the axis of symmetry and directrix, and the direction of opening of the parabola with the given equation. Then find the length of the latus rectum and graph the parabola.

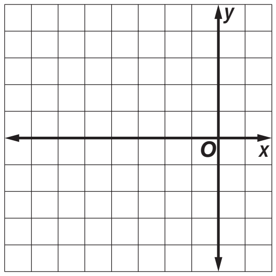
4. $y = (x - 2)^2$



5. $x = (y - 2)^2 + 3$

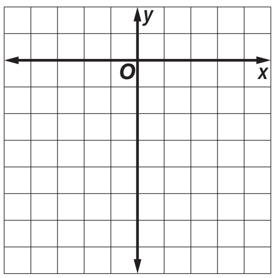


6. $y = -(x + 3)^2 + 4$

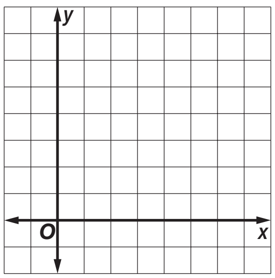


Write an equation for each parabola described below. Then draw the graph.

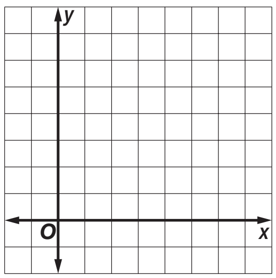
7. vertex (0, 0), focus $\left(0, -\frac{1}{12}\right)$



8. vertex (5, 1), focus $\left(5, \frac{5}{4}\right)$



9. vertex (1, 3), directrix $x = \frac{7}{8}$



8-2 Practice

Parabolas

Write each equation in standard form.

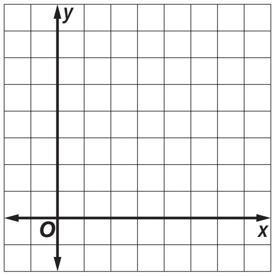
1. $y = 2x^2 - 12x + 19$

2. $y = \frac{1}{2}x^2 + 3x + \frac{1}{2}$

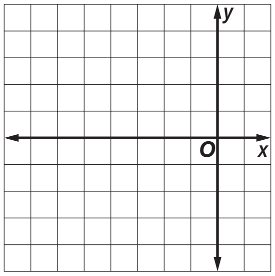
3. $y = -3x^2 - 12x - 7$

Identify the coordinates of the vertex and focus, the equations of the axis of symmetry and directrix, and the direction of opening of the parabola with the given equation. Then find the length of the latus rectum and graph the parabola.

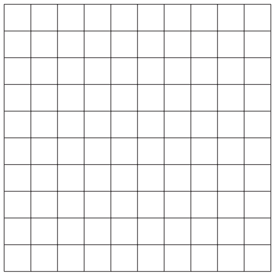
4. $y = (x - 4)^2 + 3$



5. $x = -\frac{1}{3}y^2 + 1$

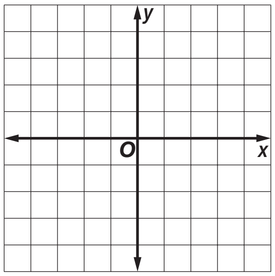


6. $x = 3(y + 1)^2 - 3$

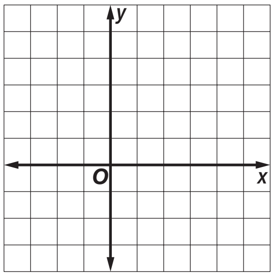


Write an equation for each parabola described below. Then draw the graph.

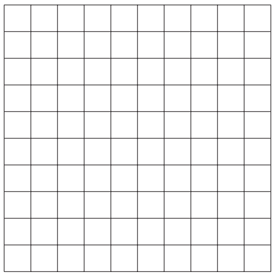
7. vertex $(0, -4)$, focus $\left(0, -3\frac{7}{8}\right)$



8. vertex $(-2, 1)$, directrix $x = -3$



9. vertex $(1, 3)$, axis of symmetry $x = 1$, latus rectum: 2 units, $a < 0$



10. TELEVISION Write the equation in the form $y = ax^2$ for a satellite dish. Assume that the bottom of the upward-facing dish passes through (0, 0) and that the distance from the bottom to the focus point is 8 inches.

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8-2 Reading to Learn Mathematics

Parabolas

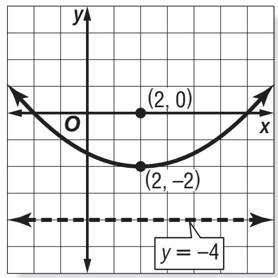
Pre-Activity **How are parabolas used in manufacturing?**

Read the introduction to Lesson 8-2 at the top of page 419 in your textbook.

Name at least two reflective objects that might have the shape of a parabola.

Reading the Lesson

1. In the parabola shown in the graph, the point (2, −2) is called the ________________________ and the point (2, 0) is called the ________________________. The line $y = -4$ is called the ________________________, and the line $x = 2$ is called the ________________________.



2. a. Write the standard form of the equation of a parabola that opens upward or downward.

b. The parabola opens downward if ____________ and opens upward if ____________. The equation of the axis of symmetry is ____________, and the coordinates of the vertex are ____________.

3. A parabola has equation $x = -\frac{1}{8}(y - 2)^2 + 4$. This parabola opens to the ____________. It has vertex ____________ and focus ____________. The directrix is ____________. The length of the latus rectum is ____________ units.

Helping You Remember

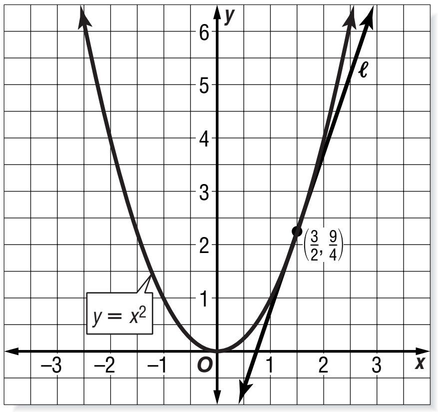
4. How can the way in which you plot points in a rectangular coordinate system help you to remember what the sign of a tells you about the direction in which a parabola opens?

Lesson 8-2

8-2 Enrichment

Tangents to Parabolas

A line that intersects a parabola in exactly one point without crossing the curve is a **tangent** to the parabola. The point where a tangent line touches a parabola is the **point of tangency.** The line perpendicular to a tangent to a parabola at the point of tangency is called the **normal** to the parabola at that point. In the diagram, line ℓ is tangent to the parabola that is the graph of $y = x^2$ at $\left(\frac{3}{2}, \frac{9}{4}\right)$. The x-axis is tangent to the parabola at O, and the y-axis is the normal to the parabola at O.



Solve each problem.

1. Find an equation for line ℓ in the diagram. *Hint:* A nonvertical line with an equation of the form $y = mx + b$ will be tangent to the graph of $y = x^2$ at $\left(\frac{3}{2}, \frac{9}{4}\right)$ if and only if $\left(\frac{3}{2}, \frac{9}{4}\right)$ is the only pair of numbers that satisfies both $y = x^2$ and $y = mx + b$.

2. If a is any real number, then (a, a^2) belongs to the graph of $y = x^2$. Express m and b in terms of a to find an equation of the form $y = mx + b$ for the line that is tangent to the graph of $y = x^2$ at (a, a^2).

3. Find an equation for the normal to the graph of $y = x^2$ at $\left(\frac{3}{2}, \frac{9}{4}\right)$.

4. If a is a nonzero real number, find an equation for the normal to the graph of $y = x^2$ at (a, a^2).

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8-3 Study Guide and Intervention

Circles

Equations of Circles The equation of a circle with center (h, k) and radius r units is $(x - h)^2 + (y - k)^2 = r^2$.

Example **Write an equation for a circle if the endpoints of a diameter are at (−4, 5) and (6, −3).**

Use the midpoint formula to find the center of the circle.

$(h, k) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ Midpoint formula

$= \left(\frac{-4 + 6}{2}, \frac{5 + (-3)}{2}\right)$ $(x_1, y_1) = (-4, 5), (x_2, y_2) = (6, -3)$

$= \left(\frac{2}{2}, \frac{2}{2}\right)$ or $(1, 1)$ Simplify.

Use the coordinates of the center and one endpoint of the diameter to find the radius.

$r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Distance formula

$r = \sqrt{(-4 - 1)^2 + (5 - 1)^2}$ $(x_1, y_1) = (1, 1), (x_2, y_2) = (-4, 5)$

$= \sqrt{(-5)^2 + 4^2} = \sqrt{41}$ Simplify.

The radius of the circle is $\sqrt{41}$, so $r^2 = 41$.

An equation of the circle is $(x - 1)^2 + (y - 1)^2 = 41$.

Exercises

Write an equation for the circle that satisfies each set of conditions.

1. center (8, −3), radius 6

2. center (5, −6), radius 4

3. center (−5, 2), passes through (−9, 6)

4. endpoints of a diameter at (6, 6) and (10, 12)

5. center (3, 6), tangent to the x-axis

6. center (−4, −7), tangent to $x = 2$

7. center at (−2, 8), tangent to $y = -4$

8. center (7, 7), passes through (12, 9)

9. endpoints of a diameter are (−4, −2) and (8, 4)

10. endpoints of a diameter are (−4, 3) and (6, −8)

Lesson 8-3

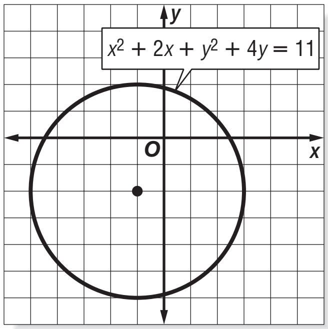
8-3 Study Guide and Intervention *(continued)*

Circles

Graph Circles To graph a circle, write the given equation in the standard form of the equation of a circle, $(x - h)^2 + (y - k)^2 = r^2$.

Plot the center (h, k) of the circle. Then use r to calculate and plot the four points $(h + r, k)$, $(h - r, k)$, $(h, k + r)$, and $(h, k - r)$, which are all points on the circle. Sketch the circle that goes through those four points.

Example **Find the center and radius of the circle whose equation is $x^2 + 2x + y^2 + 4y = 11$. Then graph the circle.**



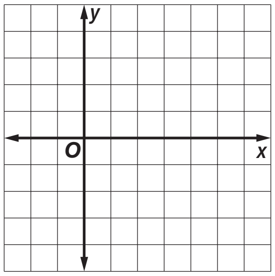
$$x^2 + 2x + y^2 + 4y = 11$$
$$x^2 + 2x + \blacksquare + y^2 + 4y + \blacksquare = 11 + \blacksquare$$
$$x^2 + 2x + 1 + y^2 + 4y + 4 = 11 + 1 + 4$$
$$(x + 1)^2 + (y + 2)^2 = 16$$

Therefore, the circle has its center at $(-1, -2)$ and a radius of $\sqrt{16} = 4$. Four points on the circle are $(3, -2)$, $(-5, -2)$, $(-1, 2)$, and $(-1, -6)$.

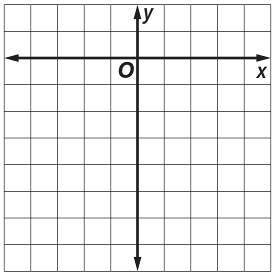
Exercises

Find the center and radius of the circle with the given equation. Then graph the circle.

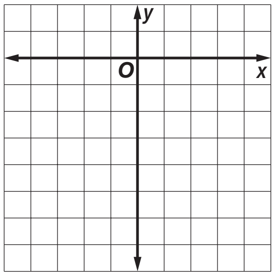
1. $(x - 3)^2 + y^2 = 9$



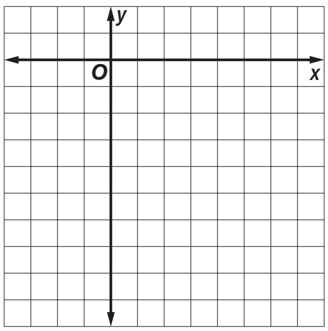
2. $x^2 + (y + 5)^2 = 4$



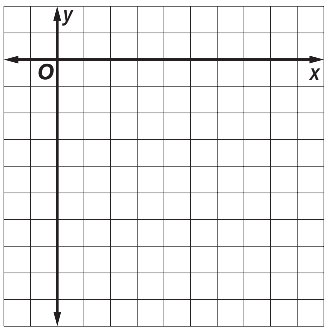
3. $(x - 1)^2 + (y + 3)^2 = 9$



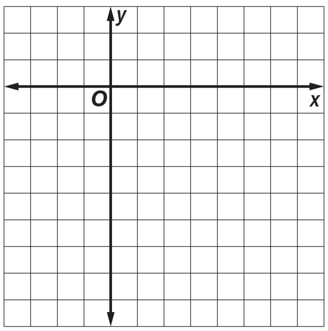
4. $(x - 2)^2 + (y + 4)^2 = 16$



5. $x^2 + y^2 - 10x + 8y + 16 = 0$



6. $x^2 + y^2 - 4x + 6y = 12$



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8-3 Skills Practice

Circles

Write an equation for the circle that satisfies each set of conditions.

1. center (0, 5), radius 1 unit

2. center (5, 12), radius 8 units

3. center (4, 0), radius 2 units

4. center (2, 2), radius 3 units

5. center (4, −4), radius 4 units

6. center (−6, 4), radius 5 units

7. endpoints of a diameter at (−12, 0) and (12, 0)

8. endpoints of a diameter at (−4, 0) and (−4, −6)

9. center at (7, −3), passes through the origin

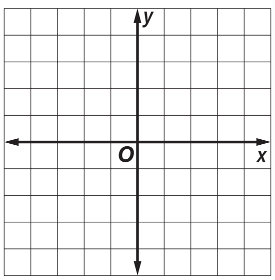
10. center at (−4, 4), passes through (−4, 1)

11. center at (−6, −5), tangent to y-axis

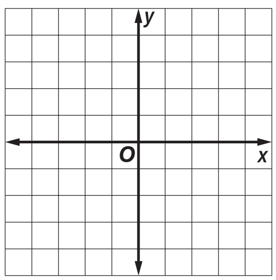
12. center at (5, 1), tangent to x-axis

Find the center and radius of the circle with the given equation. Then graph the circle.

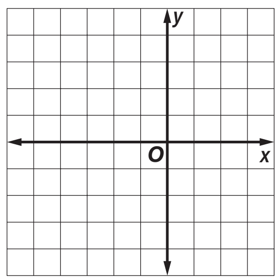
13. $x^2 + y^2 = 9$



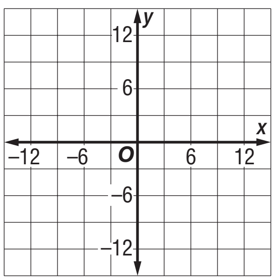
14. $(x - 1)^2 + (y - 2)^2 = 4$



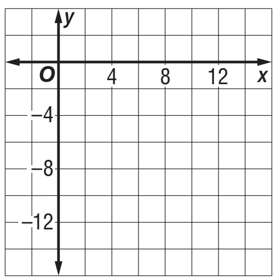
15. $(x + 1)^2 + y^2 = 16$



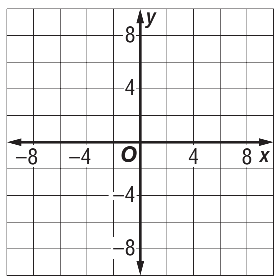
16. $x^2 + (y + 3)^2 = 81$



17. $(x - 5)^2 + (y + 8)^2 = 49$



18. $x^2 + y^2 - 4y - 32 = 0$



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8-3 Practice

Circles

Write an equation for the circle that satisfies each set of conditions.

1. center $(-4, 2)$, radius 8 units

2. center $(0, 0)$, radius 4 units

3. center $\left(-\frac{1}{4}, -\sqrt{3}\right)$, radius $5\sqrt{2}$ units

4. center $(2.5, 4.2)$, radius 0.9 unit

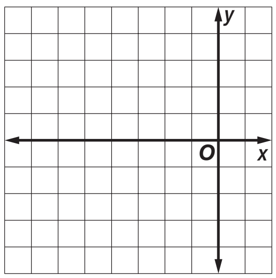
5. endpoints of a diameter at $(-2, -9)$ and $(0, -5)$

6. center at $(-9, -12)$, passes through $(-4, -5)$

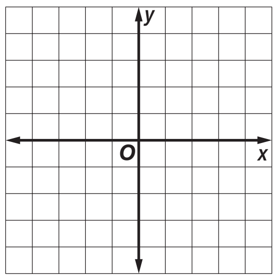
7. center at $(-6, 5)$, tangent to x-axis

Find the center and radius of the circle with the given equation. Then graph the circle.

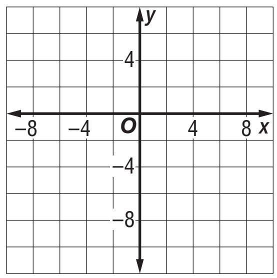
8. $(x + 3)^2 + y^2 = 16$



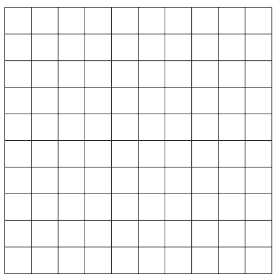
9. $3x^2 + 3y^2 = 12$



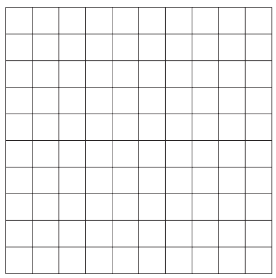
10. $x^2 + y^2 + 2x + 6y = 26$



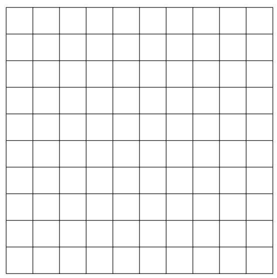
11. $(x - 1)^2 + y^2 + 4y = 12$



12. $x^2 - 6x + y^2 = 0$



13. $x^2 + y^2 + 2x + 6y = -1$



WEATHER For Exercises 14 and 15, use the following information.

On average, the circular eye of a hurricane is about 15 miles in diameter. Gale winds can affect an area up to 300 miles from the storm's center. In 1992, Hurricane Andrew devastated southern Florida. A satellite photo of Andrew's landfall showed the center of its eye on one coordinate system could be approximated by the point $(80, 26)$.

14. Write an equation to represent a possible boundary of Andrew's eye.

15. Write an equation to represent a possible boundary of the area affected by gale winds.

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8-3

Reading to Learn Mathematics

Circles

Pre-Activity **Why are circles important in air traffic control?**

Read the introduction to Lesson 8-3 at the top of page 426 in your textbook.

A large home improvement chain is planning to enter a new metropolitan area and needs to select locations for its stores. Market research has shown that potential customers are willing to travel up to 12 miles to shop at one of their stores. How can circles help the managers decide where to place their store?

Reading the Lesson

1. a. Write the equation of the circle with center (h, k) and radius r.

b. Write the equation of the circle with center $(4, -3)$ and radius 5.

c. The circle with equation $(x + 8)^2 + y^2 = 121$ has center ___________ and radius ___________.

d. The circle with equation $(x - 10)^2 + (y + 10)^2 = 1$ has center ___________ and radius ___________.

2. a. In order to find center and radius of the circle with equation $x^2 + y^2 + 4x - 6y - 3 = 0$, it is necessary to ________________________________. Fill in the missing parts of this process.

$$x^2 + y^2 + 4x - 6y - 3 = 0$$

$$x^2 + y^2 + 4x - 6y = \_\_\_\_\_\_$$

$$x^2 + 4x + \_\_\_\_\_\_ + y^2 - 6y + \_\_\_\_\_\_ = \_\_\_\_\_\_ + \_\_\_\_\_\_ + \_\_\_\_\_\_$$

$$(x + \_\_\_\_\_\_)^2 + (y - \_\_\_\_\_\_)^2 = \_\_\_\_\_\_$$

b. This circle has radius 4 and center at ___________.

Helping You Remember

3. How can the distance formula help you to remember the equation of a circle?

Lesson 8-3

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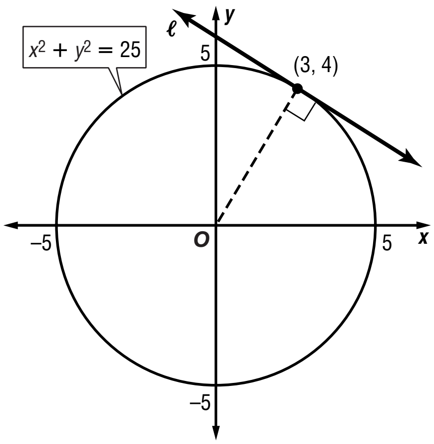
8-3 Enrichment

Tangents to Circles

A line that intersects a circle in exactly one point is a **tangent** to the circle. In the diagram, line ℓ is tangent to the circle with equation $x^2 + y^2 = 25$ at the point whose coordinates are (3, 4).

A line is tangent to a circle at a point P on the circle if and only if the line is perpendicular to the radius from the center of the circle to point P. This fact enables you to find an equation of the tangent to a circle at a point P if you know an equation for the circle and the coordinates of P.



Use the diagram above to solve each problem.

1. What is the slope of the radius to the point with coordinates (3, 4)? What is the slope of the tangent to that point?

2. Find an equation of the line ℓ that is tangent to the circle at (3, 4).

3. If k is a real number between -5 and 5, how many points on the circle have x-coordinate k? State the coordinates of these points in terms of k.

4. Describe how you can find equations for the tangents to the points you named for Exercise 3.

5. Find an equation for the tangent at $(-3, 4)$.

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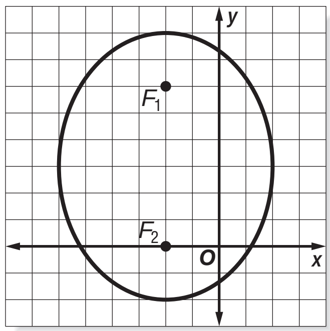
8-4 Study Guide and Intervention

Ellipses

Equations of Ellipses An **ellipse** is the set of all points in a plane such that the *sum* of the distances from two given points in the plane, called the foci, is constant. An ellipse has two axes of symmetry which contain the **major** and **minor axes**. In the table, the lengths a, b, and c are related by the formula $c^2 = a^2 - b^2$.

Standard Form of Equation	$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$	$\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1$
Center	(h, k)	(h, k)
Direction of Major Axis	Horizontal	Vertical
Foci	$(h + c, k)$, $(h - c, k)$	$(h, k - c)$, $(h, k + c)$
Length of Major Axis	$2a$ units	$2a$ units
Length of Minor Axis	$2b$ units	$2b$ units

Example **Write an equation for the ellipse shown.**



The length of the major axis is the distance between $(-2, -2)$ and $(-2, 8)$. This distance is 10 units.

$2a = 10$, so $a = 5$

The foci are located at $(-2, 6)$ and $(-2, 0)$, so $c = 3$.

$$b^2 = a^2 - c^2 = 25 - 9 = 16$$

The center of the ellipse is at $(-2, 3)$, so $h = -2$, $k = 3$, $a^2 = 25$, and $b^2 = 16$. The major axis is vertical.

An equation of the ellipse is $\frac{(y-3)^2}{25} + \frac{(x+2)^2}{16} = 1$.

Exercises

Write an equation for the ellipse that satisfies each set of conditions.

1. endpoints of major axis at $(-7, 2)$ and $(5, 2)$, endpoints of minor axis at $(-1, 0)$ and $(-1, 4)$

2. major axis 8 units long and parallel to the x-axis, minor axis 2 units long, center at $(-2, -5)$

3. endpoints of major axis at $(-8, 4)$ and $(4, 4)$, foci at $(-3, 4)$ and $(-1, 4)$

4. endpoints of major axis at $(3, 2)$ and $(3, -14)$, endpoints of minor axis at $(-1, -6)$ and $(7, -6)$

5. minor axis 6 units long and parallel to the x-axis, major axis 12 units long, center at $(6, 1)$

Lesson 8-4

8-4 Study Guide and Intervention *(continued)*

Ellipses

Graph Ellipses To graph an ellipse, if necessary, write the given equation in the standard form of an equation for an ellipse.

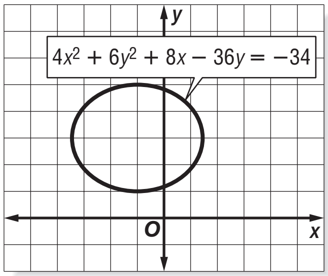
$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ (for ellipse with major axis horizontal) or

$\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1$ (for ellipse with major axis vertical)

Use the center (h, k) and the endpoints of the axes to plot four points of the ellipse. To make a more accurate graph, use a calculator to find some approximate values for x and y that satisfy the equation.

Example **Graph the ellipse $4x^2 + 6y^2 + 8x - 36y = -34$.**

$$4x^2 + 6y^2 + 8x - 36y = -34$$
$$4x^2 + 8x + 6y^2 - 36y = -34$$
$$4(x^2 + 2x + \blacksquare) + 6(y^2 - 6y + \blacksquare) = -34 + \blacksquare$$
$$4(x^2 + 2x + 1) + 6(y^2 - 6y + 9) = -34 + 58$$
$$4(x + 1)^2 + 6(y - 3)^2 = 24$$
$$\frac{(x+1)^2}{6} + \frac{(y-3)^2}{4} = 1$$

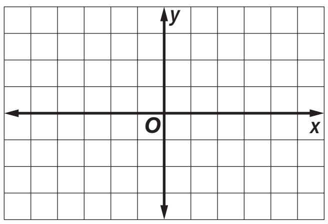


The center of the ellipse is $(-1, 3)$. Since $a^2 = 6$, $a = \sqrt{6}$.
Since $b^2 = 4$, $b = 2$.
The length of the major axis is $2\sqrt{6}$, and the length of the minor axis is 4. Since the x-term has the greater denominator, the major axis is horizontal. Plot the endpoints of the axes. Then graph the ellipse.

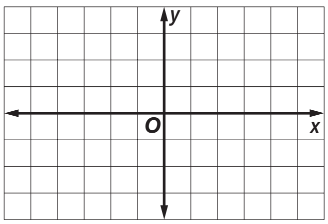
Exercises

Find the coordinates of the center and the lengths of the major and minor axes for the ellipse with the given equation. Then graph the ellipse.

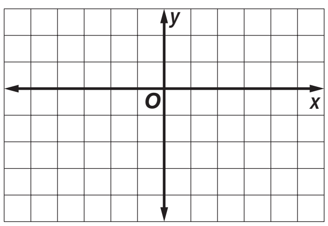
1. $\frac{y^2}{12} + \frac{x^2}{9} = 1$



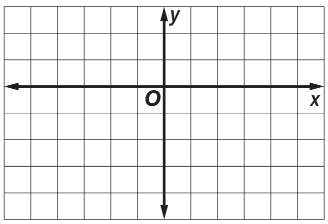
2. $\frac{x^2}{25} + \frac{y^2}{4} = 1$



3. $x^2 + 4y^2 + 24y = -32$



4. $9x^2 + 6y^2 - 36x + 12y = 12$



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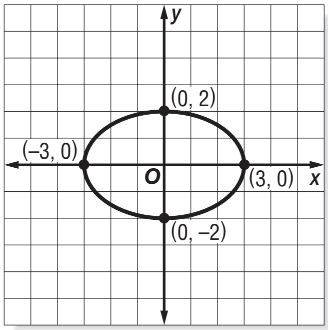
NAME ______________________ DATE ____________ PERIOD _____

8-4 Skills Practice

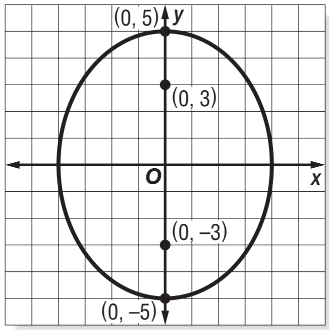
Ellipses

Write an equation for each ellipse.

1.



2.



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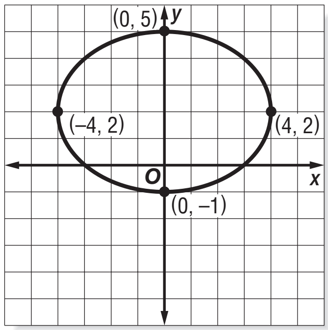


Write an equation for the ellipse that satisfies each set of conditions.

4. endpoints of major axis at $(0, 6)$ and $(0, -6)$, endpoints of minor axis at $(-3, 0)$ and $(3, 0)$

5. endpoints of major axis at $(2, 6)$ and $(8, 6)$, endpoints of minor axis at $(5, 4)$ and $(5, 8)$

6. endpoints of major axis at $(7, 3)$ and $(7, 9)$, endpoints of minor axis at $(5, 6)$ and $(9, 6)$

7. major axis 12 units long and parallel to x-axis, minor axis 4 units long, center at $(0, 0)$

8. endpoints of major axis at $(-6, 0)$ and $(6, 0)$, foci at $\left(-\sqrt{32}, 0\right)$ and $\left(\sqrt{32}, 0\right)$

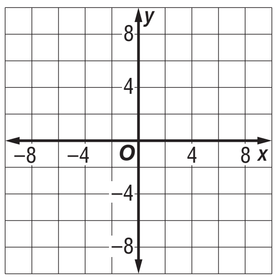
9. endpoints of major axis at $(0, 12)$ and $(0, -12)$, foci at $\left(0, \sqrt{23}\right)$ and $\left(0, -\sqrt{23}\right)$

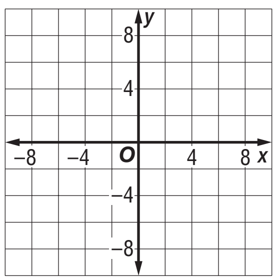
Find the coordinates of the center and foci and the lengths of the major and minor axes for the ellipse with the given equation. Then graph the ellipse.

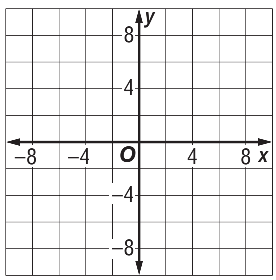
10. $\frac{y^2}{100} + \frac{x^2}{81} = 1$

11. $\frac{x^2}{81} + \frac{y^2}{9} = 1$

12. $\frac{y^2}{49} + \frac{x^2}{25} = 1$







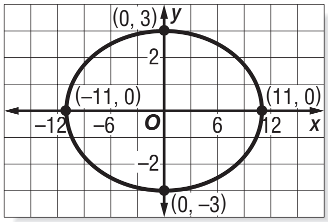
Lesson 8-4

8-4 Practice

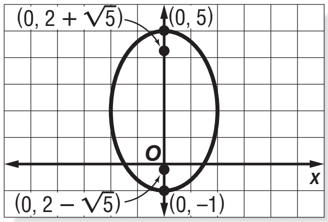
Ellipses

Write an equation for each ellipse.

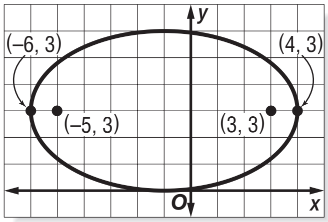
1.



2.



3.



Write an equation for the ellipse that satisfies each set of conditions.

4. endpoints of major axis at $(-9, 0)$ and $(9, 0)$, endpoints of minor axis at $(0, 3)$ and $(0, -3)$

5. endpoints of major axis at $(4, 2)$ and $(4, -8)$, endpoints of minor axis at $(1, -3)$ and $(7, -3)$

6. major axis 20 units long and parallel to x-axis, minor axis 10 units long, center at $(2, 1)$

7. major axis 10 units long, minor axis 6 units long and parallel to x-axis, center at $(2, -4)$

8. major axis 16 units long, center at $(0, 0)$, foci at $\left(0, 2\sqrt{15}\right)$ and $\left(0, -2\sqrt{15}\right)$

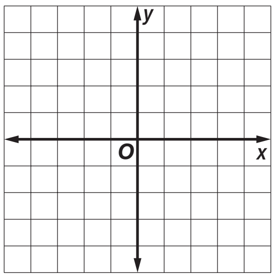
9. endpoints of minor axis at $(0, 2)$ and $(0, -2)$, foci at $(-4, 0)$ and $(4, 0)$

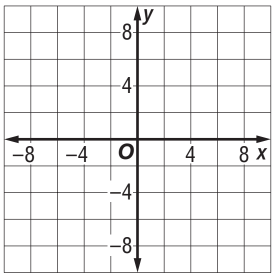
Find the coordinates of the center and foci and the lengths of the major and minor axes for the ellipse with the given equation. Then graph the ellipse.

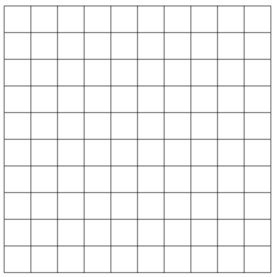
10. $\frac{y^2}{16} + \frac{x^2}{9} = 1$

11. $\frac{(y-1)^2}{36} + \frac{(x-3)^2}{1} = 1$

12. $\frac{(x+4)^2}{49} + \frac{(y+3)^2}{25} = 1$







13. SPORTS An ice skater traces two congruent ellipses to form a figure eight. Assume that the center of the first loop is at the origin, with the second loop to its right. Write an equation to model the first loop if its major axis (along the x-axis) is 12 feet long and its minor axis is 6 feet long. Write another equation to model the second loop.

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8-4 Reading to Learn Mathematics

Ellipses

Pre-Activity **Why are ellipses important in the study of the solar system?**

Read the introduction to Lesson 8-4 at the top of page 433 in your textbook.

Is the Earth always the same distance from the Sun? Explain your answer using the words *circle* and *ellipse*.

Reading the Lesson

1. An ellipse is the set of all points in a plane such that the __________________ of the distances from two fixed points is __________________. The two fixed points are called the __________________ of the ellipse.

2. Consider the ellipse with equation $\frac{x^2}{9} + \frac{y^2}{4} = 1$.

a. For this equation, $a =$ _________ and $b =$ _________.

b. Write an equation that relates the values of a, b, and c.

c. Find the value of c for this ellipse.

3. Consider the ellipses with equations $\frac{y^2}{25} + \frac{x^2}{16} = 1$ and $\frac{x^2}{9} + \frac{y^2}{4} = 1$. Complete the following table to describe characteristics of their graphs.

Standard Form of Equation	$\frac{y^2}{25} + \frac{x^2}{16} = 1$	$\frac{x^2}{9} + \frac{y^2}{4} = 1$
Direction of Major Axis		
Direction of Minor Axis		
Foci		
Length of Major Axis		
Length of Minor Axis		

Helping You Remember

4. Some students have trouble remembering the two standard forms for the equation of an ellipse. How can you remember which term comes first and where to place a and b in these equations?

Lesson 8-4

8-4 Enrichment

Eccentricity

In an ellipse, the ratio $\frac{c}{d}$ is called the **eccentricity** and is denoted by the letter e. Eccentricity measures the elongation of an ellipse. The closer e is to 0, the more an ellipse looks like a circle. The closer e is to 1, the more elongated it is. Recall that the equation of an ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ or $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$ where a is the length of the major axis, and that $c = \sqrt{a^2 - b^2}$.

Find the eccentricity of each ellipse rounded to the nearest hundredth.

1. $\frac{x^2}{9} + \frac{y^2}{36} = 1$

2. $\frac{x^2}{81} + \frac{y^2}{9} = 1$

3. $\frac{x^2}{4} + \frac{y^2}{9} = 1$

4. $\frac{x^2}{16} + \frac{y^2}{9} = 1$

5. $\frac{x^2}{36} + \frac{y^2}{16} = 1$

6. $\frac{x^2}{4} + \frac{y^2}{36} = 1$

7. Is a circle an ellipse? Explain your reasoning.

8. The center of the sun is one focus of Earth's orbit around the sun. The length of the major axis is 186,000,000 miles, and the foci are 3,200,000 miles apart. Find the eccentricity of Earth's orbit.

9. An artificial satellite orbiting the earth travels at an altitude that varies between 132 miles and 583 miles above the surface of the earth. If the center of the earth is one focus of its elliptical orbit and the radius of the earth is 3950 miles, what is the eccentricity of the orbit?

8-5 Study Guide and Intervention

Hyperbolas

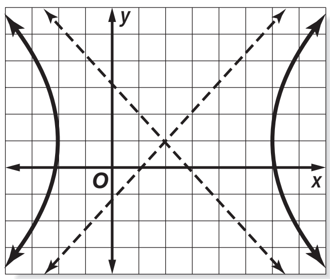
Equations of Hyperbolas A **hyperbola** is the set of all points in a plane such that the absolute value of the *difference* of the distances from any point on the hyperbola to any two given points in the plane, called the **foci**, is constant.

In the table, the lengths a, b, and c are related by the formula $c^2 = a^2 + b^2$.

Standard Form of Equation	$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$	$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$
Equations of the Asymptotes	$y - k = \pm\frac{b}{a}(x-h)$	$y - k = \pm\frac{a}{b}(x-h)$
Transverse Axis	Horizontal	Vertical
Foci	$(h-c, k), (h+c, k)$	$(h, k-c), (h, k+c)$
Vertices	$(h-a, k), (h+a, k)$	$(h, k-a), (h, k+a)$

Example **Write an equation for the hyperbola with vertices (−2, 1) and (6, 1) and foci (−4, 1) and (8, 1).**

Use a sketch to orient the hyperbola correctly. The center of the hyperbola is the midpoint of the segment joining the two vertices. The center is $\left(\frac{-2+6}{2}, 1\right)$, or (2, 1). The value of a is the distance from the center to a vertex, so $a = 4$. The value of c is the distance from the center to a focus, so $c = 6$.



$c^2 = a^2 + b^2$

$6^2 = 4^2 + b^2$

$b^2 = 36 - 16 = 20$

Use h, k, a^2, and b^2 to write an equation of the hyperbola.

$$\frac{(x-2)^2}{16} - \frac{(y-1)^2}{20} = 1$$

Exercises

Write an equation for the hyperbola that satisfies each set of conditions.

1. vertices (−7, 0) and (7, 0), conjugate axis of length 10

2. vertices (−2, −3) and (4, −3), foci (−5, −3) and (7, −3)

3. vertices (4, 3) and (4, −5), conjugate axis of length 4

4. vertices (−8, 0) and (8, 0), equation of asymptotes $y = \pm\frac{1}{6}x$

5. vertices (−4, 6) and (−4, −2), foci (−4, 10) and (−4, −6)

Lesson 8-5

NAME ______________________________ DATE ____________ PERIOD _____

8-5 Study Guide and Intervention *(continued)*

Hyperbolas

Graph Hyperbolas To graph a hyperbola, write the given equation in the standard form of an equation for a hyperbola

$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ if the branches of the hyperbola open left and right, or

$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$ if the branches of the hyperbola open up and down

Graph the point (h, k), which is the center of the hyperbola. Draw a rectangle with dimensions $2a$ and $2b$ and center (h, k). If the hyperbola opens left and right, the vertices are $(h - a, k)$ and $(h + a, k)$. If the hyperbola opens up and down, the vertices are $(h, k - a)$ and $(h, k + a)$.

Example **Draw the graph of $6y^2 - 4x^2 - 36y - 8x = -26$.**

Complete the squares to get the equation in standard form.

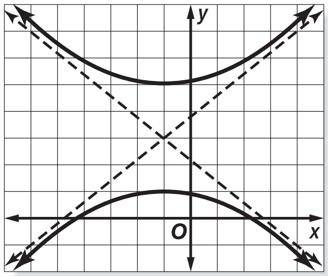
$6y^2 - 4x^2 - 36y - 8x = -26$

$6(y^2 - 6y + \blacksquare) - 4(x^2 + 2x + \blacksquare) = -26 + \blacksquare$

$6(y^2 - 6y + 9) - 4(x^2 + 2x + 1) = -26 + 50$

$6(y - 3)^2 - 4(x + 1)^2 = 24$

$\frac{(y-3)^2}{4} - \frac{(x+1)^2}{6} = 1$



The center of the hyperbola is $(-1, 3)$.

According to the equation, $a^2 = 4$ and $b^2 = 6$, so $a = 2$ and $b = \sqrt{6}$.

The transverse axis is vertical, so the vertices are $(-1, 5)$ and $(-1, 1)$. Draw a rectangle with vertical dimension 4 and horizontal dimension $2\sqrt{6} \approx 4.9$. The diagonals of this rectangle are the asymptotes. The branches of the hyperbola open up and down. Use the vertices and the asymptotes to sketch the hyperbola.

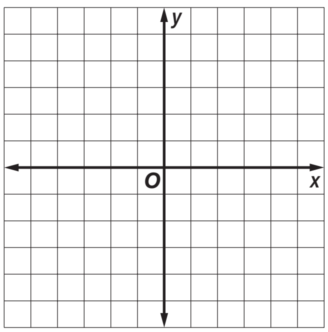
Exercises

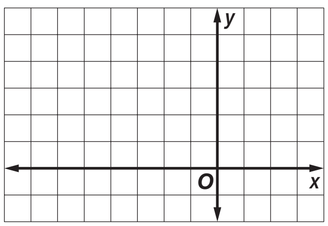
Find the coordinates of the vertices and foci and the equations of the asymptotes for the hyperbola with the given equation. Then graph the hyperbola.

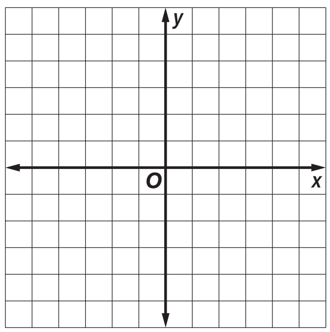
1. $\frac{x^2}{4} - \frac{y^2}{16} = 1$

2. $(y - 3)^2 - \frac{(x+2)^2}{9} = 1$

3. $\frac{y^2}{16} - \frac{x^2}{9} = 1$







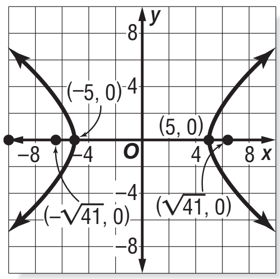
NAME ______________________________ DATE ____________ PERIOD _____

8-5 Skills Practice

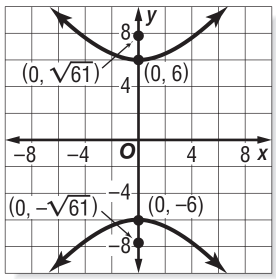
Hyperbolas

Write an equation for each hyperbola.

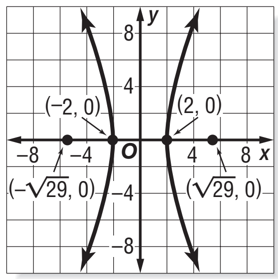
1.



2.



3.



Write an equation for the hyperbola that satisfies each set of conditions.

4. vertices $(-4, 0)$ and $(4, 0)$, conjugate axis of length 8

5. vertices $(0, 6)$ and $(0, -6)$, conjugate axis of length 14

6. vertices $(0, 3)$ and $(0, -3)$, conjugate axis of length 10

7. vertices $(-2, 0)$ and $(2, 0)$, conjugate axis of length 4

8. vertices $(-3, 0)$ and $(3, 0)$, foci $(\pm 5, 0)$

9. vertices $(0, 2)$ and $(0, -2)$, foci $(0, \pm 3)$

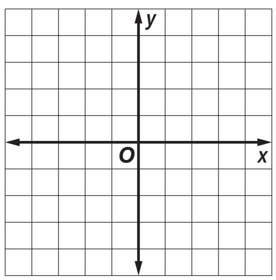
10. vertices $(0, -2)$ and $(6, -2)$, foci $\left(3 \pm \sqrt{13}, -2\right)$

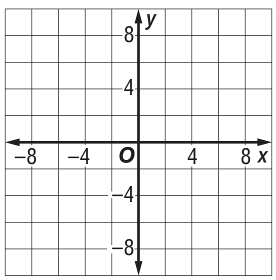
Find the coordinates of the vertices and foci and the equations of the asymptotes for the hyperbola with the given equation. Then graph the hyperbola.

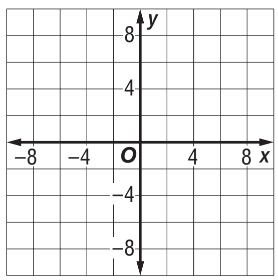
11. $\frac{x^2}{9} - \frac{y^2}{36} = 1$

12. $\frac{y^2}{49} - \frac{x^2}{9} = 1$

13. $\frac{x^2}{16} - \frac{y^2}{1} = 1$







Lesson 8-5

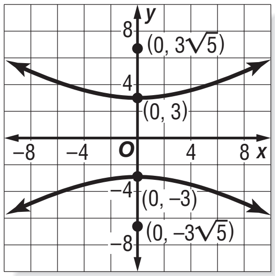
NAME ____ DATE ____________ PERIOD _____

8-5 Practice

Hyperbolas

Write an equation for each hyperbola.

1.



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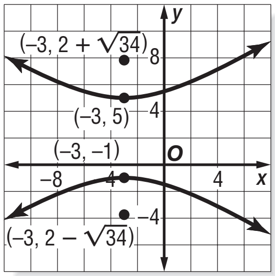
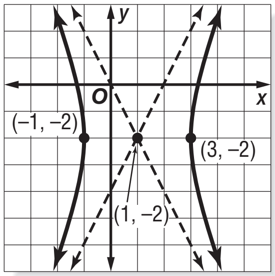


3.



Write an equation for the hyperbola that satisfies each set of conditions.

4. vertices (0, 7) and (0, −7), conjugate axis of length 18 units

5. vertices (1, −1) and (1, −9), conjugate axis of length 6 units

6. vertices (−5, 0) and (5, 0), foci $\left(\pm\sqrt{26}, 0\right)$

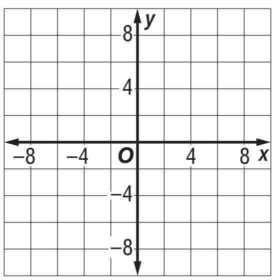
7. vertices (1, 1) and (1, −3), foci $\left(1, -1 \pm \sqrt{5}\right)$

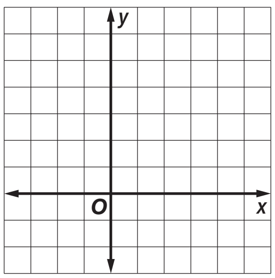
Find the coordinates of the vertices and foci and the equations of the asymptotes for the hyperbola with the given equation. Then graph the hyperbola.

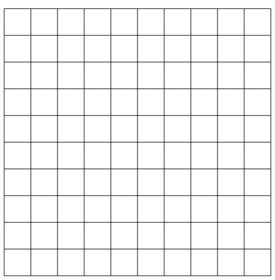
8. $\frac{y^2}{16} - \frac{x^2}{4} = 1$

9. $\frac{(y-2)^2}{1} - \frac{(x-1)^2}{4} = 1$

10. $\frac{(y+2)^2}{4} - \frac{(x-3)^2}{4} = 1$







11. ASTRONOMY Astronomers use special X-ray telescopes to observe the sources of celestial X rays. Some X-ray telescopes are fitted with a metal mirror in the shape of a hyperbola, which reflects the X rays to a focus. Suppose the vertices of such a mirror are located at (−3, 0) and (3, 0), and one focus is located at (5, 0). Write an equation that models the hyperbola formed by the mirror.

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8-5 Reading to Learn Mathematics

Hyperbolas

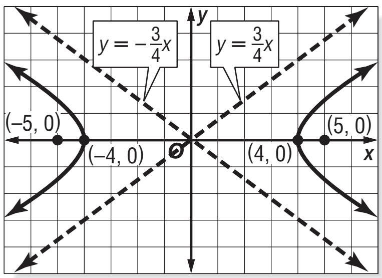
Pre-Activity **How are hyperbolas different from parabolas?**

Read the introduction to Lesson 8-5 at the top of page 441 in your textbook.

Look at the sketch of a hyperbola in the introduction to this lesson. List three ways in which hyperbolas are different from parabolas.

Reading the Lesson

1. The graph at the right shows the hyperbola whose equation in standard form is $\frac{x^2}{16} - \frac{y^2}{9} = 1$.



The point (0, 0) is the __________________ of the hyperbola.

The points (4, 0) and (−4, 0) are the __________________ of the hyperbola.

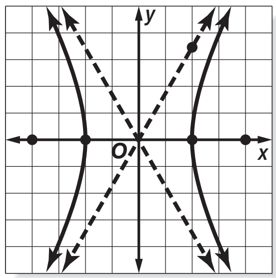
The points (5, 0) and (−5, 0) are the __________________ of the hyperbola.

The segment connecting (4, 0) and (−4, 0) is called the __________________ axis.

The segment connecting (0, 3) and (0, −3) is called the __________________ axis.

The lines $y = \frac{3}{4}x$ and $y = -\frac{3}{4}x$ are called the __________________.

2. Study the hyperbola graphed at the right.



The center is _________.

The value of a is _________.

The value of c is _________.

To find b^2, solve the equation _________ = _________ + _________.

The equation in standard form for this hyperbola is __________________.

Helping You Remember

3. What is an easy way to remember the equation relating the values of a, b, and c for a hyperbola?

Lesson 8-5

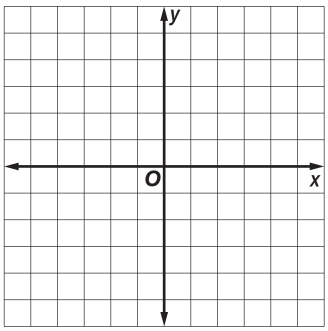
8-5 Enrichment

Rectangular Hyperbolas

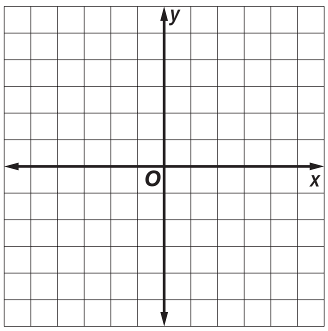
A **rectangular hyperbola** is a hyperbola with perpendicular asymptotes. For example, the graph of $x^2 - y^2 = 1$ is a rectangular hyperbola. A hyperbola with asymptotes that are not perpendicular is called a **nonrectangular hyperbola**. The graphs of equations of the form $xy = c$, where c is a constant, are rectangular hyperbolas.

Make a table of values and plot points to graph each rectangular hyperbola below. Be sure to consider negative values for the variables.

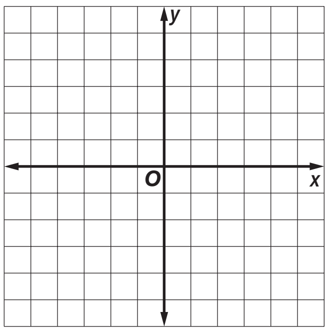
1. $xy = -4$



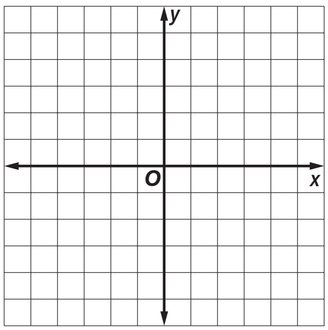
2. $xy = 3$



3. $xy = -1$



4. $xy = 8$



5. Make a conjecture about the asymptotes of rectangular hyperbolas.

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8-6 Study Guide and Intervention

Conic Sections

Lesson 8-6

Standard Form Any conic section in the coordinate plane can be described by an equation of the form

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0, \text{ where } A, B, \text{ and } C \text{ are not all zero.}$$

One way to tell what kind of conic section an equation represents is to rearrange terms and complete the square, if necessary, to get one of the standard forms from an earlier lesson. This method is especially useful if you are going to graph the equation.

Example **Write the equation $3x^2 - 4y^2 - 30x - 8y + 59 = 0$ in standard form. State whether the graph of the equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*.**

$3x^2 - 4y^2 - 30x - 8y + 59 = 0$	Original equation
$3x^2 - 30x - 4y^2 - 8y = -59$	Isolate terms.
$3(x^2 - 10x + \blacksquare) - 4(y^2 + 2y + \blacksquare) = -59 + \blacksquare + \blacksquare$	Factor out common multiples.
$3(x^2 - 10x + 25) - 4(y^2 + 2y + 1) = -59 + 3(25) + (-4)(1)$	Complete the squares.
$3(x - 5)^2 - 4(y + 1)^2 = 12$	Simplify.
$\frac{(x - 5)^2}{4} - \frac{(y + 1)^2}{3} = 1$	Divide each side by 12.

The graph of the equation is a hyperbola with its center at $(5, -1)$. The length of the transverse axis is 4 units and the length of the conjugate axis is $2\sqrt{3}$ units.

Exercises

Write each equation in standard form. State whether the graph of the equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*.

1. $x^2 + y^2 - 6x + 4y + 3 = 0$

2. $x^2 + 2y^2 + 6x - 20y + 53 = 0$

3. $6x^2 - 60x - y + 161 = 0$

4. $x^2 + y^2 - 4x - 14y + 29 = 0$

5. $6x^2 - 5y^2 + 24x + 20y - 56 = 0$

6. $3y^2 + x - 24y + 46 = 0$

7. $x^2 - 4y^2 - 16x + 24y - 36 = 0$

8. $x^2 + 2y^2 + 8x + 4y + 2 = 0$

9. $4x^2 + 48x + y + 158 = 0$

10. $3x^2 + y^2 - 48x - 4y + 184 = 0$

11. $-3x^2 + 2y^2 - 18x + 20y + 5 = 0$

12. $x^2 + y^2 + 8x + 2y + 8 = 0$

8-6 Study Guide and Intervention *(continued)*

Conic Sections

Identify Conic Sections If you are given an equation of the form

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0, \text{ with } B = 0,$$

you can determine the type of conic section just by considering the values of A and C. Refer to the following chart.

Relationship of *A* and *C*	Type of Conic Section
$A = 0$ or $C = 0$, but not both.	parabola
$A = C$	circle
A and C have the same sign, but $A \neq C$.	ellipse
A and C have opposite signs.	hyperbola

Example **Without writing the equation in standard form, state whether the graph of each equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*.**

a. $3x^2 - 3y^2 + 5x + 12 = 0$

$A = 3$ and $C = -3$ have opposite signs, so the graph of the equation is a hyperbola.

b. $y^2 = 7y - 2x + 13$

$A = 0$, so the graph of the equation is a parabola.

Exercises

Without writing the equation in standard form, state whether the graph of each equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*.

1. $x^2 = 17x - 5y + 8$

2. $2x^2 + 2y^2 - 3x + 4y = 5$

3. $4x^2 - 8x = 4y^2 - 6y + 10$

4. $8(x - x^2) = 4(2y^2 - y) - 100$

5. $6y^2 - 18 = 24 - 4x^2$

6. $y = 27x - y^2$

7. $x^2 = 4(y - y^2) + 2x - 1$

8. $10x - x^2 - 2y^2 = 5y$

9. $x = y^2 - 5y + x^2 - 5$

10. $11x^2 - 7y^2 = 77$

11. $3x^2 + 4y^2 = 50 + y^2$

12. $y^2 = 8x - 11$

13. $9y^2 - 99y = 3(3x - 3x^2)$

14. $6x^2 - 4 = 5y^2 - 3$

15. $111 = 11x^2 + 10y^2$

16. $120x^2 - 119y^2 + 118x - 117y = 0$

17. $3x^2 = 4y^2 + 12$

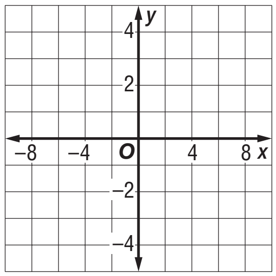
18. $150 - x^2 = 120 - y$

8-6 Skills Practice

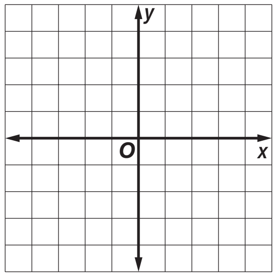
Conic Sections

Write each equation in standard form. State whether the graph of the equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*. Then graph the equation.

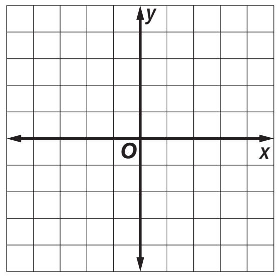
1. $x^2 - 25y^2 = 25$



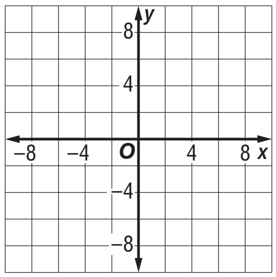
2. $9x^2 + 4y^2 = 36$



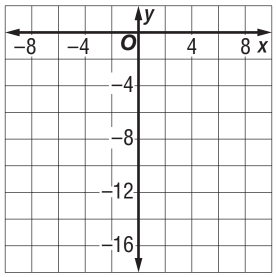
3. $x^2 + y^2 - 16 = 0$



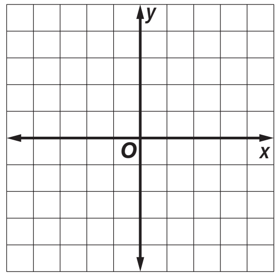
4. $x^2 + 8x + y^2 = 9$



5. $x^2 + 2x - 15 = y$



6. $100x^2 + 25y^2 = 400$



Without writing the equation in standard form, state whether the graph of each equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*.

7. $9x^2 + 4y^2 = 36$

8. $x^2 + y^2 = 25$

9. $y = x^2 + 2x$

10. $y = 2x^2 - 4x - 4$

11. $4y^2 - 25x^2 = 100$

12. $16x^2 + y^2 = 16$

13. $16x^2 - 4y^2 = 64$

14. $5x^2 + 5y^2 = 25$

15. $25y^2 + 9x^2 = 225$

16. $36y^2 - 4x^2 = 144$

17. $y = 4x^2 - 36x - 144$

18. $x^2 + y^2 - 144 = 0$

19. $(x + 3)^2 + (y - 1)^2 = 4$

20. $25y^2 - 50y + 4x^2 = 75$

21. $x^2 - 6y^2 + 9 = 0$

22. $x = y^2 + 5y - 6$

23. $(x + 5)^2 + y^2 = 10$

24. $25x^2 + 10y^2 - 250 = 0$

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8-6 Practice

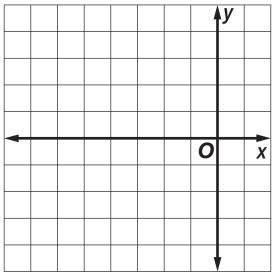
Conic Sections

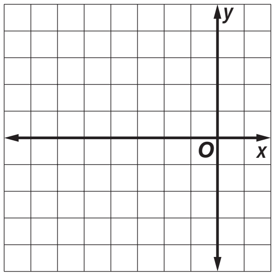
Write each equation in standard form. State whether the graph of the equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*. Then graph the equation.

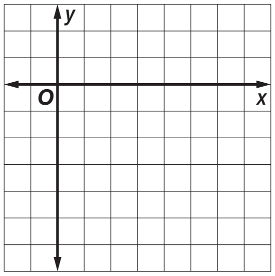
1. $y^2 = -3x$

2. $x^2 + y^2 + 6x = 7$

3. $5x^2 - 6y^2 - 30x - 12y = -9$



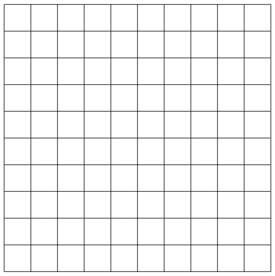


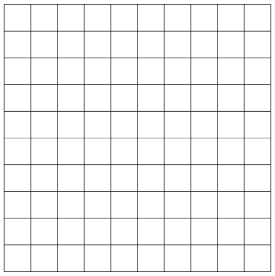


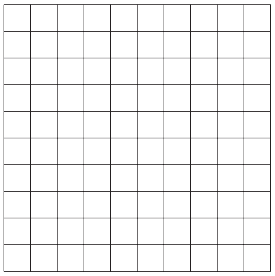
4. $196y^2 = 1225 - 100x^2$

5. $3x^2 = 9 - 3y^2 - 6y$

6. $9x^2 + y^2 + 54x - 6y = -81$







Without writing the equation in standard form, state whether the graph of each equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*.

7. $6x^2 + 6y^2 = 36$

8. $4x^2 - y^2 = 16$

9. $9x^2 + 16y^2 - 64y - 80 = 0$

10. $5x^2 + 5y^2 - 45 = 0$

11. $x^2 + 2x = y$

12. $4y^2 - 36x^2 + 4x - 144 = 0$

13. ASTRONOMY A satellite travels in an hyperbolic orbit. It reaches the vertex of its orbit at (5, 0) and then travels along a path that gets closer and closer to the line $y = \frac{2}{5}x$. Write an equation that describes the path of the satellite if the center of its hyperbolic orbit is at (0, 0).

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8-6 Reading to Learn Mathematics

Conic Sections

Lesson 8-6

Pre-Activity **How can you use a flashlight to make conic sections?**

Read the introduction to Lesson 8-6 at the top of page 449 in your textbook.

The figures in the introduction show how a plane can slice a double cone to form the conic sections. Name the conic section that is formed if the plane slices the double cone in each of the following ways:

- The plane is parallel to the base of the double cone and slices through one of the cones that form the double cone.
- The plane is perpendicular to the base of the double cone and slices through both of the cones that form the double cone.

Reading the Lesson

1. Name the conic section that is the graph of each of the following equations. Give the coordinates of the vertex if the conic section is a parabola and of the center if it is a circle, an ellipse, or a hyperbola.

a. $\frac{(x-3)^2}{36} + \frac{(y+5)^2}{15} = 1$

b. $x = -2(y+1)^2 + 7$

c. $(x-5)^2 - (y+5)^2 = 1$

d. $(x+6)^2 + (y-2)^2 = 1$

2. Each of the following is the equation of a conic section. For each equation, identify the values of A and C. Then, without writing the equation in standard form, state whether the graph of each equation is a *parabola*, *circle, ellipse*, or *hyperbola*.

a. $2x^2 + y^2 - 6x + 8y + 12 = 0$ A = _____; C = _____; type of graph: _____________

b. $2x^2 + 3x - 2y - 5 = 0$ A = _____; C = _____; type of graph: _____________

c. $5x^2 + 10x + 5y^2 - 20y + 1 = 0$ A = _____; C = _____; type of graph: _____________

d. $x^2 - y^2 + 4x + 2y - 5 = 0$ A = _____; C = _____; type of graph: _____________

Helping You Remember

3. What is an easy way to recognize that an equation represents a parabola rather than one of the other conic sections?

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8-6 Enrichment

Loci

A *locus* (plural, *loci*) is the set of all points, and only those points, that satisfy a given set of conditions. In geometry, figures often are defined as loci. For example, a circle is the locus of points of a plane that are a given distance from a given point. The definition leads naturally to an equation whose graph is the curve described.

Example **Write an equation of the locus of points that are the same distance from (3, 4) and $y = -4$.**

Recognizing that the locus is a parabola with focus (3, 4) and directrix $y = -4$, you can find that $h = 3$, $k = 0$, and $a = 4$ where (h, k) is the vertex and 4 units is the distance from the vertex to both the focus and directrix.

Thus, an equation for the parabola is $y = \frac{1}{16}(x - 3)^2$.

The problem also may be approached analytically as follows:

Let (x, y) be a point of the locus.

The distance from (3, 4) to (x, y) = the distance from $y = -4$ to (x, y).

$$\sqrt{(x-3)^2 + (y-4)^2} = \sqrt{(x-x)^2 + (y-(-4))^2}$$

$$(x-3)^2 + y^2 - 8y + 16 = y^2 + 8y + 16$$

$$(x-3)^2 = 16y$$

$$\frac{1}{16}(x-3)^2 = y$$

Describe each locus as a geometric figure. Then write an equation for the locus.

1. All points that are the same distance from (0, 5) and (4, 5).

2. All points that are 4 units from the origin.

3. All points that are the same distance from $(-2, -1)$ and $x = 2$.

4. The locus of points such that the sum of the distances from $(-2, 0)$ and $(2, 0)$ is 6.

5. The locus of points such that the absolute value of the difference of the distances from $(-3, 0)$ and $(3, 0)$ is 2.

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8-7 Study Guide and Intervention

Solving Quadratic Systems

Systems of Quadratic Equations Like systems of linear equations, systems of quadratic equations can be solved by substitution and elimination. If the graphs are a conic section and a line, the system will have 0, 1, or 2 solutions. If the graphs are two conic sections, the system will have 0, 1, 2, 3, or 4 solutions.

Example **Solve the system of equations.** $\mathbf{y = x^2 - 2x - 15}$
$\mathbf{x + y = -3}$

Rewrite the second equation as $y = -x - 3$ and substitute into the first equation.

$-x - 3 = x^2 - 2x - 15$

$0 = x^2 - x - 12$ Add $x + 3$ to each side.

$0 = (x - 4)(x + 3)$ Factor.

Use the Zero Product property to get
$x = 4$ or $x = -3$.

Substitute these values for x in $x + y = -3$:

$4 + y = -3$ or $-3 + y = -3$

$y = -7$ $\qquad$ $y = 0$

The solutions are $(4, -7)$ and $(-3, 0)$.

Exercises

Find the exact solution(s) of each system of equations.

1. $y = x^2 - 5$
$y = x - 3$

2. $x^2 + (y - 5)^2 = 25$
$y = -x^2$

3. $x^2 + (y - 5)^2 = 25$
$y = x^2$

4. $x^2 + y^2 = 9$
$x^2 + y = 3$

5. $x^2 - y^2 = 1$
$x^2 + y^2 = 16$

6. $y = x - 3$
$x = y^2 - 4$

Lesson 8-7

8-7 Study Guide and Intervention *(continued)*

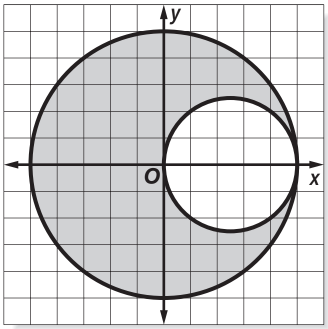
Solving Quadratic Systems

Systems of Quadratic Inequalities Systems of quadratic inequalities can be solved by graphing.

Example 1 **Solve the system of inequalities by graphing.**

$x^2 + y^2 \leq 25$

$\left(x - \frac{5}{2}\right)^2 + y^2 \geq \frac{25}{4}$

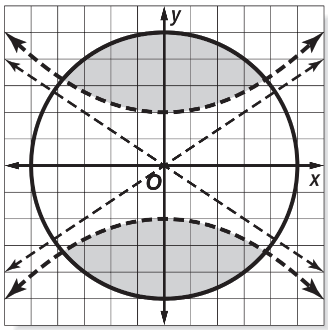


The graph of $x^2 + y^2 \leq 25$ consists of all points on or inside the circle with center (0, 0) and radius 5.The graph of $\left(x - \frac{5}{2}\right)^2 + y^2 \geq \frac{25}{4}$ consists of all points on or outside the circle with center $\left(\frac{5}{2}, 0\right)$ and radius $\frac{5}{2}$. The solution of the system is the set of points in both regions.

Example 2 **Solve the system of inequalities by graphing.**

$x^2 + y^2 \leq 25$

$\frac{y^2}{4} - \frac{x^2}{9} > 1$



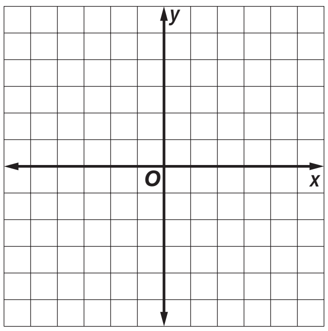
The graph of $x^2 + y^2 \leq 25$ consists of all points on or inside the circle with center (0, 0) and radius 5.The graph of $\frac{y^2}{4} - \frac{x^2}{9} > 1$ are the points "inside" but not on the branches of the hyperbola shown. The solution of the system is the set of points in both regions.

Exercises

Solve each system of inequalities below by graphing.

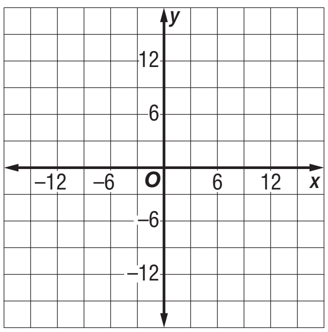
1. $\frac{x^2}{16} + \frac{y^2}{4} \leq 1$

$y > \frac{1}{2}x - 2$



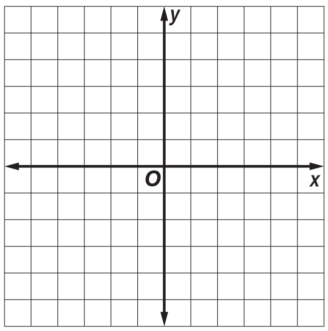
2. $x^2 + y^2 \leq 169$

$x^2 + 9y^2 \geq 225$



3. $y \geq (x - 2)^2$

$(x + 1)^2 + (y + 1)^2 \leq 16$



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8-7 Skills Practice

Solving Quadratic Systems

Find the exact solution(s) of each system of equations.

1. $y = x - 2$
$y = x^2 - 2$

2. $y = x + 3$
$y = 2x^2$

3. $y = 3x$
$x = y^2$

4. $y = x$
$x^2 + y^2 = 4$

5. $x = -5$
$x^2 + y^2 = 25$

6. $y = 7$
$x^2 + y^2 = 9$

7. $y = -2x + 2$
$y^2 = 2x$

8. $x - y + 1 = 0$
$y^2 = 4x$

9. $y = 2 - x$
$y = x^2 - 4x + 2$

10. $y = x - 1$
$y = x^2$

11. $y = 3x^2$
$y = -3x^2$

12. $y = x^2 + 1$
$y = -x^2 + 3$

13. $y = 4x$
$4x^2 + y^2 = 20$

14. $y = -1$
$4x^2 + y^2 = 1$

15. $4x^2 + 9y^2 = 36$
$x^2 - 9y^2 = 9$

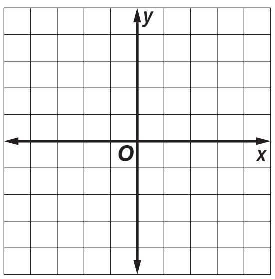
16. $3(y + 2)^2 - 4(x - 3)^2 = 12$
$y = -2x + 2$

17. $x^2 - 4y^2 = 4$
$x^2 + y^2 = 4$

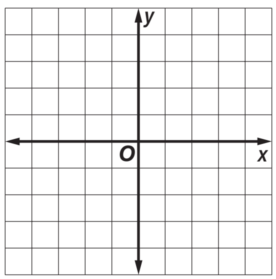
18. $y^2 - 4x^2 = 4$
$y = 2x$

Solve each system of inequalities by graphing.

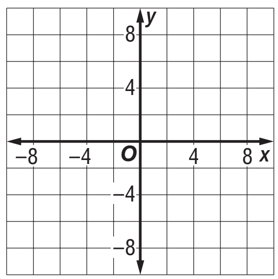
19. $y \leq 3x - 2$
$x^2 + y^2 < 16$



20. $y \leq x$
$y \geq -2x^2 + 4$



21. $4y^2 + 9x^2 < 144$
$x^2 + 8y^2 < 16$



Lesson 8-7

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8-7 Practice

Solving Quadratic Systems

Find the exact solution(s) of each system of equations.

1. $(x - 2)^2 + y^2 = 5$
$x - y = 1$

2. $x = 2(y + 1)^2 - 6$
$x + y = 3$

3. $y^2 - 3x^2 = 6$
$y = 2x - 1$

4. $x^2 + 2y^2 = 1$
$y = -x + 1$

5. $4y^2 - 9x^2 = 36$
$4x^2 - 9y^2 = 36$

6. $y = x^2 - 3$
$x^2 + y^2 = 9$

7. $x^2 + y^2 = 25$
$4y = 3x$

8. $y^2 = 10 - 6x^2$
$4y^2 = 40 - 2x^2$

9. $x^2 + y^2 = 25$
$x = 3y - 5$

10. $4x^2 + 9y^2 = 36$
$2x^2 - 9y^2 = 18$

11. $x = -(y - 3)^2 + 2$
$x = (y - 3)^2 + 3$

12. $\frac{x^2}{9} - \frac{y^2}{16} = 1$
$x^2 + y^2 = 9$

13. $25x^2 + 4y^2 = 100$
$x = -\frac{5}{2}$

14. $x^2 + y^2 = 4$
$\frac{x^2}{4} + \frac{y^2}{8} = 1$

15. $x^2 - y^2 = 3$
$y^2 - x^2 = 3$

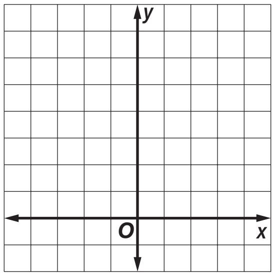
16. $\frac{x^2}{7} + \frac{y^2}{7} = 1$
$3x^2 - y^2 = 9$

17. $x + 2y = 3$
$x^2 + y^2 = 9$

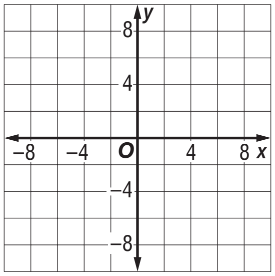
18. $x^2 + y^2 = 64$
$x^2 - y^2 = 8$

Solve each system of inequalities by graphing.

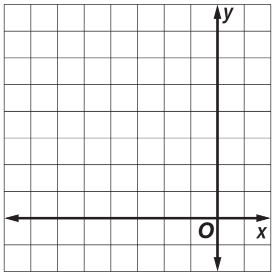
19. $y \geq x^2$
$y > -x + 2$



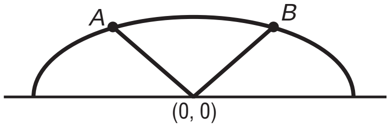
20. $x^2 + y^2 < 36$
$x^2 + y^2 \geq 16$



21. $\frac{(y - 3)^2}{16} + \frac{(x + 2)^2}{4} \leq 1$
$(x + 1)^2 + (y - 2)^2 \leq 4$



22. GEOMETRY The top of an iron gate is shaped like half an ellipse with two congruent segments from the center of the ellipse to the ellipse as shown. Assume that the center of the ellipse is at (0, 0). If the ellipse can be modeled by the equation $x^2 + 4y^2 = 4$ for $y \geq 0$ and the two congruent segments can be modeled by $y = \frac{\sqrt{3}}{2}x$ and $y = -\frac{\sqrt{3}}{2}x$, what are the coordinates of points A and B?



NAME ____ DATE ____________ PERIOD _____

8-7 Reading to Learn Mathematics

Solving Quadratic Systems

Pre-Activity **How do systems of equations apply to video games?**

Read the introduction to Lesson 8-7 at the top of page 455 in your textbook.

The figure in your textbook shows that the spaceship hits the circular force field in two points. Is it possible for the spaceship to hit the force field in either fewer or more than two points? State all possibilities and explain how these could happen.

Lesson 8-7

Reading the Lesson

1. Draw a sketch to illustrate each of the following possibilities.

a. a parabola and a line that intersect in 2 points

b. an ellipse and a circle that intersect in 4 points

c. a hyperbola and a line that intersect in 1 point

2. Consider the following system of equations.

$x^2 = 3 + y^2$

$2x^2 + 3y^2 = 11$

a. What kind of conic section is the graph of the first equation?

b. What kind of conic section is the graph of the second equation?

c. Based on your answers to parts a and b, what are the possible numbers of solutions that this system could have?

Helping You Remember

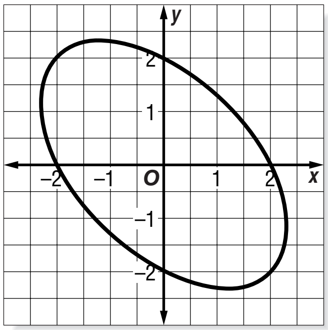
3. Suppose that the graph of a quadratic inequality is a region whose boundary is a circle. How can you remember whether to shade the interior or the exterior of the circle?

8-7 Enrichment

Graphing Quadratic Equations with xy-Terms

You can use a graphing calculator to examine graphs of quadratic equations that contain xy-terms.

Example **Use a graphing calculator to display the graph of $x^2 + xy + y^2 = 4$.**



Solve the equation for y in terms of x by using the quadratic formula.

$y^2 + xy + (x^2 - 4) = 0$

To use the formula, let $a = 1$, $b = x$, and $c = (x^2 - 4)$.

$$y = \frac{-x \pm \sqrt{x^2 - 4(1)(x^2 - 4)}}{2}$$

$$y = \frac{-x \pm \sqrt{16 - 3x^2}}{2}$$

To graph the equation on the graphing calculator, enter the two equations:

$y = \frac{-x + \sqrt{16 - 3x^2}}{2}$ and $y = \frac{-x - \sqrt{16 - 3x^2}}{2}$

Use a graphing calculator to graph each equation. State the type of curve each graph represents.

1. $y^2 + xy = 8$

2. $x^2 + y^2 - 2xy - x = 0$

3. $x^2 - xy + y^2 = 15$

4. $x^2 + xy + y^2 = -9$

5. $2x^2 - 2xy - y^2 + 4x = 20$

6. $x^2 - xy - 2y^2 + 2x + 5y - 3 = 0$

NAME ______________________ DATE ____________ PERIOD ________

8 Chapter 8 Test, Form 1

SCORE ________

Write the letter for the correct answer in the blank at the right of each question.

1. What is the midpoint of the line segment with endpoints at (12, 7) and (18, 19)?

A. (30, 26) **B.** (15, 13) **C.** (−6, −12) **D.** (3, 6) **1.** ______

2. Choose the midpoint of the line segment with endpoints at (5, 9) and (11, 15).

A. (8, 12) **B.** (16, 24) **C.** (6, 6) **D.** (−6, −6) **2.** ______

3. Find the distance between $A(12, 8)$ and $B(4, 2)$.

A. 14 units **B.** 100 units **C.** 10 units **D.** −10 units **3.** ______

4. What is the distance between $C(4, 3)$ and $D(7, 7)$?

A. −5 units **B.** 7 units **C.** 25 units **D.** 5 units **4.** ______

5. Write the equation of the parabola $y = x^2 + 10x + 16$ in standard form.

A. $y = (x + 5)^2 - 9$ **B.** $y = (x + 5)^2 + 41$

C. $y = (x + 5)^2 + 16$ **D.** $y = (x + 8)(x + 2)$ **5.** ______

6. Write an equation for the parabola with vertex (1, 0) if the length of the latus rectum is $\frac{1}{2}$ and the parabola opens down.

A. $y = -\frac{1}{2}(x - 1)^2$ **B.** $y = -2(x - 1)^2$ **C.** $x = -2(y - 1)^2$ **D.** $x = -\frac{1}{2}(y - 1)^2$ **6.** ______

7. Which is the equation of a parabola that opens downward and has axis of symmetry $x = -1$?

A. $y = (x - 1)^2 + 2$ **B.** $y = (x + 1)^2 + 2$

C. $y = -(x + 1)^2 + 2$ **D.** $y = -(x - 1)^2 + 2$ **7.** ______

8. Find the center and radius of the circle with equation $(x - 2)^2 + y^2 = 9$.

A. (−2, 0); 9 **B.** (0, 2); 9 **C.** (2, 0); 3 **D.** (0, −2); 3 **8.** ______

9. Write an equation for the circle with center (2, −3) that is tangent to the y-axis.

A. $(x + 2)^2 + (y - 3)^2 = 9$ **B.** $(x - 2)^2 + (y + 3)^2 = 9$

C. $(x + 2)^2 + (y - 3)^2 = 4$ **D.** $(x - 2)^2 + (y + 3)^2 = 4$ **9.** ______

10. Which is the equation of a circle with center (2, 1) that passes through (2, 4)?

A. $(x - 2)^2 + (y - 1)^2 = 9$ **B.** $(x - 2)^2 + (y - 1)^2 = 3$

C. $(x + 2)^2 + (y + 1)^2 = 9$ **D.** $(x + 2)^2 + (y + 1)^2 = 3$ **10.** ______

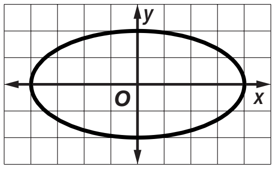
11. Which is the equation of an ellipse with foci at (0, 3) and (0, −3) that has the endpoints of its major axis at (0, 4) and (0, −4)?

A. $\frac{y^2}{16} + \frac{x^2}{9} = 1$ **B.** $x^2 + y^2 = 16$ **C.** $\frac{x^2}{16} + \frac{y^2}{7} = 1$ **D.** $\frac{y^2}{16} + \frac{x^2}{7} = 1$ **11.** ______

Assessment

8 Chapter 8 Test, Form 1 *(continued)*

12. Which equation is graphed at the right?



A. $\frac{x^2}{16} + \frac{y^2}{4} = 1$ **B.** $\frac{y^2}{16} + \frac{x^2}{4} = 1$

C. $\frac{x^2}{16} - \frac{y^2}{4} = 1$ **D.** $\frac{y^2}{16} - \frac{x^2}{4} = 1$

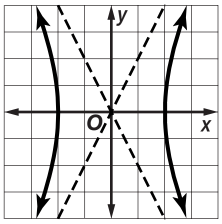
12. ______

13. Which is the equation of a hyperbola with vertices (0, 2) and (0, −2) and foci (0, 3) and (0, −3)?

A. $\frac{y^2}{5} - \frac{x^2}{4} = 1$ **B.** $\frac{y^2}{4} - \frac{x^2}{5} = 1$ **C.** $\frac{x^2}{4} - \frac{y^2}{5} = 1$ **D.** $\frac{x^2}{5} + \frac{y^2}{4} = 1$

13. ______

14. Which equation is graphed at the right?



A. $\frac{x^2}{16} - \frac{y^2}{4} = 1$ **B.** $\frac{x^2}{4} - \frac{y^2}{16} = 1$

C. $\frac{y^2}{16} - \frac{x^2}{4} = 1$ **D.** $\frac{y^2}{4} - \frac{x^2}{16} = 1$

14. ______

15. What is the standard form of the equation $5x^2 + 5y^2 - 20 = 0$?

A. $5x^2 + 5y^2 = 20$ **B.** $y^2 = -x^2 + 4$ **C.** $x^2 + y^2 - 4 = 0$ **D.** $x^2 + y^2 = 4$

15. ______

16. What is the graph of $x^2 + 4y^2 - 2y = 8$?

A. parabola **B.** circle **C.** ellipse **D.** hyperbola

16. ______

17. Which equation has a hyperbola as its graph?

A. $4x^2 - 4y^2 = 16$ **B.** $4x^2 - 4y = 16$ **C.** $4x^2 + 4y^2 = 16$ **D.** $x^2 + 4y^2 = 16$

17. ______

18. Find the exact solution(s) of the system of equations $x^2 + y^2 = 16$ and $x = y + 4$.

A. (−4, 0) and (0, 4) **B.** (4, 0) and (−4, 0)

C. (0, 4) and (0, −4) **D.** (4, 0) and (0, −4)

18. ______

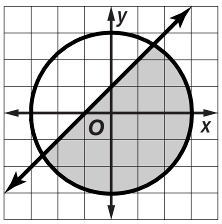
19. Solve the system of equations by graphing $y = x^2$ and $y = 2x$.

A. (0, 0) and (4, −2) **B.** (0, 0) and (−2, 4)

C. (0, 0) and (2, 4) **D.** (0, −1) and (2, 2)

19. ______

20. Which system of inequalities is graphed at the right?



A. $x^2 + y^2 \leq 9$, $y \leq x + 1$ **B.** $x^2 + y^2 \geq 9$, $y \leq x + 1$

C. $x^2 + y^2 \leq 9$, $y \geq x + 1$ **D.** $x^2 + y^2 \geq 9$, $y \geq x + 1$

20. ______

Bonus For the equation $4x^2 + ky^2 - 8x + 17y = 3$, find a value of k so that the graph of the equation is

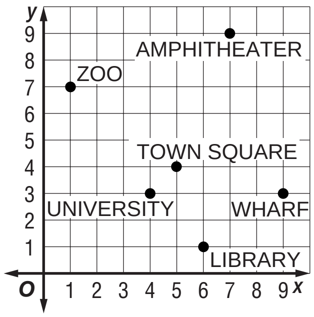
a. a circle **b.** an ellipse **c.** a hyperbola **d.** a parabola **B:** ______

8 Chapter 8 Test, Form 2A

SCORE ______

Write the letter for the correct answer in the blank at the right of each question.

For Questions 1 and 2, refer to the figure at the right showing six city locations. The origin is at the lower left corner of the grid.



1. What is the location of the point halfway between the wharf and library?

A. $(7, 2)$ **B.** $\left(\frac{15}{2}, \frac{5}{2}\right)$ **C.** $\left(7, \frac{5}{2}\right)$ **D.** $\left(\frac{15}{2}, 2\right)$ **1.** ______

2. What is the distance between the library and zoo?

A. 11 units **B.** $\sqrt{61}$ units **C.** 61 units **D.** $\frac{61}{2}$ units **2.** ______

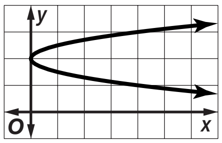
3. Write the equation of the parabola $y = 2x^2 - 8x + 1$ in standard form.

A. $y = 2(x - 2)^2 + 9$ **B.** $y = (x - 4)^2 - 15$
C. $y = 2(x - 2)^2 - 7$ **D.** $y = 2(x - 4)^2 - 15$ **3.** ______

4. Write an equation for the parabola with focus $(4, 0)$ and vertex $(2, 0)$.

A. $x = \frac{1}{8}y^2 + 2$ **B.** $x = -\frac{1}{8}y^2 - 2$ **C.** $y = \frac{1}{8}x^2 - 2$ **D.** $y = -\frac{1}{8}x^2 + 2$ **4.** ______

5. Which equation is graphed at the right?



A. $y = 4x^2 - 16x + 16$ **B.** $x = 4y^2 - 16y + 16$
C. $y = \frac{1}{4}x^2 - x + 1$ **D.** $x = \frac{1}{4}y^2 - y + 1$ **5.** ______

6. Write an equation for a circle if the endpoints of a diameter are at $(-7, 1)$ and $(5, 1)$.

A. $x^2 + (y - 1)^2 = 6$ **B.** $(x + 1)^2 + (y - 1)^2 = 36$
C. $(x - 1)^2 + y^2 = 6$ **D.** $(x - 1)^2 + (y + 1)^2 = 36$ **6.** ______

7. Which is the equation of a circle with center $(2, 0)$ and radius 2 units?

A. $x^2 + y^2 + 4x = 0$ **B.** $x^2 + y^2 - 4x = 0$
C. $x^2 + y^2 - 4y = 0$ **D.** $x^2 + y^2 + 4y = 0$ **7.** ______

8. Write an equation for an ellipse if the endpoints of the major axis are at $(-1, 5)$ and $(-1, -3)$ and the endpoints of the minor axis are at $(-4, 1)$ and $(2, 1)$.

A. $\frac{(y + 1)^2}{16} + \frac{(x - 1)^2}{9} = 1$ **B.** $\frac{(x + 1)^2}{16} + \frac{(y + 1)^2}{9} = 1$
C. $\frac{(x - 1)^2}{16} + \frac{(y + 1)^2}{9} = 1$ **D.** $\frac{(y - 1)^2}{16} + \frac{(x + 1)^2}{9} = 1$ **8.** ______

9. Which is the equation of an ellipse with center $(1, -2)$ and a vertical major axis?

A. $\frac{(y - 2)^2}{9} + \frac{(x + 1)^2}{4} = 1$ **B.** $\frac{(x + 1)^2}{9} + \frac{(y - 2)^2}{4} = 1$
C. $\frac{(y + 2)^2}{9} + \frac{(x - 1)^2}{4} = 1$ **D.** $\frac{(x - 1)^2}{9} + \frac{(y + 2)^2}{4} = 1$ **9.** ______

Assessment

8 Chapter 8 Test, Form 2A *(continued)*

10. Find the center and radius of the circle with equation $x^2 + (y - 4)^2 = 9$.

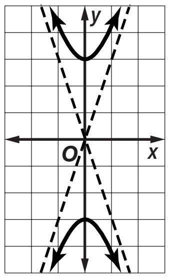
A. (0, 4); 9 **B.** (4, 0); 3 **C.** (−4, 0); 9 **D.** (0, 4); 3 **10.** ______

11. Write an equation for the hyperbola with vertices (−10, 1) and (6, 1) and foci (−12, 1) and (8, 1).

A. $\frac{(x+2)^2}{64} - \frac{(y-1)^2}{36} = 1$ **B.** $\frac{(x-2)^2}{36} - \frac{(y+1)^2}{64} = 1$

C. $\frac{(x-2)^2}{64} - \frac{(y+1)^2}{36} = 1$ **D.** $\frac{(x+2)^2}{36} - \frac{(y-1)^2}{64} = 1$ **11.** ______

12. Which equation is graphed at the right?



A. $x^2 - 9y^2 = 9$ **B.** $9y^2 - x^2 = 9$

C. $9x^2 - y^2 = 9$ **D.** $y^2 - 9x^2 = 9$ **12.** ______

13. Write the equation $x^2 - 2x + y^2 + 4y = 11$ in standard form.

A. $(x - 1)^2 + (y + 2)^2 = 16$ **B.** $(x + 1)^2 + (y - 2)^2 = 16$

C. $\frac{(x+1)^2}{1} + \frac{(y-2)^2}{4} = 1$ **D.** $\frac{(x-1)^2}{4} - \frac{(y-1)^2}{4} = 1$ **13.** ______

14. Write the equation $4x^2 + 24x - y + 34 = 0$ in standard form.

A. $y = 4(x - 3)^2 + 2$ **B.** $x = 4y^2 + 2$

C. $y = 4(x + 3)^2 - 2$ **D.** $x = 4(y + 3)^2 + 2$ **14.** ______

15. What is the graph of $4x^2 = y^2 + 8y + 32$?

A. parabola **B.** circle **C.** ellipse **D.** hyperbola **15.** ______

16. The graph of which equation is a circle?

A. $5x^2 + 10x = 9 + 5y^2$ **B.** $5x^2 - 10x = 9 - 5y^2$

C. $5x^2 + 5x + y^2 = 9$ **D.** $5x^2 + 10x + 5y = 9$ **16.** ______

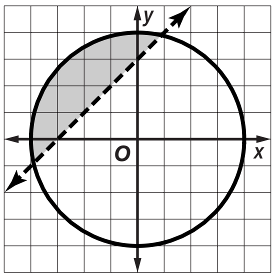
17. Solve the system of equations by graphing $x^2 + y^2 = 16$ and $y = -x + 4$.

A. (4, 0), (0, −4) **B.** (0, −4), (−4, 0) **C.** (−4, 0), (0, −4) **D.** (0, 4), (4, 0) **17.** ______

18. Which system of inequalities is graphed at the right?

A. $x^2 + y^2 \le 16$, $x - y > -3$ **B.** $x^2 + y^2 \le 16$, $x - y < -3$

C. $x^2 + y^2 \ge 16$, $x - y > -3$ **D.** $x^2 + y^2 \ge 16$, $x - y < -3$



18. ______

Find the exact solution(s) of each system of equations.

19. $x^2 + y^2 = 25$ and $9y = 4x^2$

A. (4, 3), (−4, 3) **B.** (3, 4), (3, −4) **C.** (4, 3), (4, −3) **D.** (3, 4), (−3, 4) **19.** ______

20. $y = x^2 + 1$ and $y = 2x$

A. (1, 2), (−1, 2) **B.** (−1, 2) **C.** (1, 2) **D.** (−1, 2), (0, 2) **20.** ______

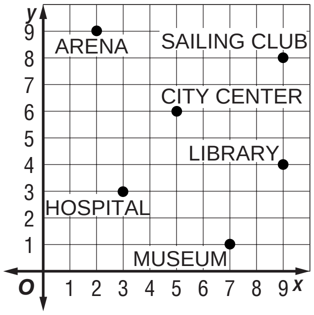
Bonus Solve the system of equations $(x - 2)^2 + y^2 = 1$ and $(x + 2)^2 + y^2 = 1$. **B:** ______________

8 Chapter 8 Test, Form 2B

SCORE ______

Write the letter for the correct answer in the blank at the right of each question.

For Questions 1 and 2, refer to the figure at the right showing six city locations. The origin is at the lower left corner of the grid.



1. What is the location of the point halfway between the hospital and arena?

A. $(2, 6)$ **B.** $\left(6, \frac{5}{2}\right)$ **C.** $\left(\frac{5}{2}, 6\right)$ **D.** $\left(\frac{1}{2}, 3\right)$ **1.** ______

2. What is the distance between the museum and sailing club?

A. $\sqrt{53}$ units **B.** 53 units **C.** $\frac{53}{2}$ units **D.** 9 units **2.** ______

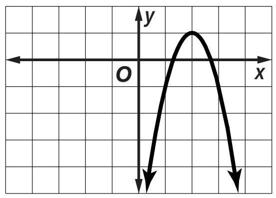
3. Write the equation of the parabola $y = 4x^2 - 8x + 1$ in standard form.

A. $y = (x - 4)^2 - 15$ **B.** $y = 4(x - 1)^2 + 5$
C. $y = 4(x - 1)^2 - 3$ **D.** $y = 4(x - 4)^2 - 15$ **3.** ______

4. Write an equation for the parabola with focus (1, 3) and vertex (0, 3).

A. $y = 4(x - 3)^2$ **B.** $x = -\frac{1}{4}(y + 3)^2$ **C.** $y = -4\left(x + \frac{3}{4}\right)^2$ **D.** $x = \frac{1}{4}(y - 3)^2$ **4.** ______

5. Which equation is graphed at the right?



A. $y = 2x^2 - 8x + 7$ **B.** $x = -2y^2 - 8y - 7$
C. $y = -2x^2 + 8x - 7$ **D.** $y = -2x^2 - 8x - 7$ **5.** ______

6. Write an equation for a circle if the endpoints of a diameter are at (1, 1) and (1, −9).

A. $(x - 1)^2 + (y + 4)^2 = 5$ **B.** $(x - 1)^2 + (y + 4)^2 = 25$
C. $(x + 1)^2 + (y - 4)^2 = 5$ **D.** $(x + 1)^2 + (y - 4)^2 = 25$ **6.** ______

7. Which is the equation of a circle with center (0, 1) and radius 2 units?

A. $x^2 + y^2 + 2y = 3$ **B.** $x^2 + y^2 + 2y = 1$
C. $x^2 + y^2 - 2y = 4$ **D.** $x^2 + y^2 - 2y = 3$ **7.** ______

8. Write an equation for an ellipse if the endpoints of the major axis are at (1, 6) and (1, −6) and the endpoints of the minor axis are at (5, 0) and (−3, 0).

A. $\frac{(x-1)^2}{36} + \frac{y^2}{16} = 1$ **B.** $\frac{(x+1)^2}{36} + \frac{y^2}{16} = 1$
C. $\frac{y^2}{36} + \frac{(x-1)^2}{16} = 1$ **D.** $\frac{y^2}{36} + \frac{x^2}{16} = 1$ **8.** ______

9. Which is the equation of an ellipse with center (−4, 2) and a horizontal major axis?

A. $\frac{(x+4)^2}{16} + \frac{(y-2)^2}{4} = 1$ **B.** $\frac{(x-4)^2}{16} + \frac{(y-2)^2}{4} = 1$
C. $\frac{(y+2)^2}{16} + \frac{(x+4)^2}{4} = 1$ **D.** $\frac{(y-2)^2}{16} + \frac{(x-4)^2}{4} = 1$ **9.** ______

Assessment

8 Chapter 8 Test, Form 2B *(continued)*

10. Find the center and radius of the circle with equation $(x - 1)^2 + y^2 = 16$.

A. (1, 0); 4 **B.** (−1, 0); 16 **C.** (0, 1); 4 **D.** (0, −1); 16 **10.** ______

11. Write an equation for the hyperbola with vertices (0, −1) and (0, 3) and foci (0, −3) and (0, 5).

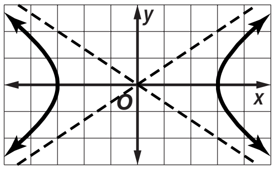
A. $\frac{(y+1)^2}{4} - \frac{x^2}{12} = 1$ **B.** $\frac{(x+1)^2}{12} - \frac{y^2}{4} = 1$

C. $\frac{(y-1)^2}{4} - \frac{x^2}{12} = 1$ **D.** $\frac{x^2}{4} - \frac{(y-1)^2}{12} = 1$ **11.** ______

12. Which equation is graphed at the right?

A. $9x^2 - 4y^2 = 36$ **B.** $4x^2 - 9y^2 = 36$

C. $9y^2 - 4x^2 = 36$ **D.** $4y^2 - 9x^2 = 36$ **12.** ______



13. Write the equation $4x^2 + 8x - y^2 - 4y - 4 = 0$ in standard form.

A. $\frac{(x+1)^2}{1} - \frac{(y+2)^2}{4} = 1$ **B.** $(x + 1)^2 + (y - 2)^2 = 4$

C. $\frac{(x-1)^2}{1} + \frac{(y-2)^2}{4} = 1$ **D.** $\frac{(x-1)^2}{4} + \frac{(y+2)^2}{1} = 1$ **13.** ______

14. Write the equation $2y^2 - 4y - x + 12 = 0$ in standard form.

A. $y = 2(x + 1)^2 + 6$ **B.** $x = 2(y - 1)^2 + 10$

C. $y = (x - 1)^2 + 10$ **D.** $x = 2(y + 1)^2 + 6$ **14.** ______

15. The graph of which equation is a circle?

A. $6x^2 - 12x = 6y^2 + 1$ **B.** $6x^2 - 12x + 6y^2 = 1$

C. $6x^2 - 6y^2 - 12x = 1$ **D.** $6x^2 + 6y + 12x = 1$ **15.** ______

16. What is the graph of $x^2 + 25y^2 = 50$?

A. parabola **B.** circle **C.** ellipse **D.** hyperbola **16.** ______

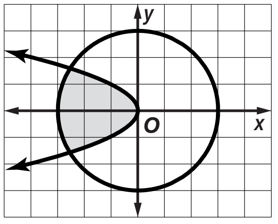
17. Solve the system of equations by graphing $y = x^2 - 2$ and $y = 2x - 2$.

A. (−2, 0), (2, 2) **B.** (2, 0), (0, −2) **C.** (−2, 0), (−2, 2) **D.** (0, −2), (2, 2) **17.** ______

18. Which system of inequalities is graphed at the right?

A. $x^2 + y^2 \leq 9$, $y^2 + x \geq 0$ **B.** $x^2 + y^2 \geq 9$, $y^2 + x \geq 0$

C. $x^2 + y^2 \geq 9$, $y^2 - x \leq 0$ **D.** $x^2 + y^2 \leq 9$, $y^2 + x \leq 0$ **18.** ______



Find the exact solution(s) of each system of equations.

19. $x^2 + 4y^2 = 16$ and $x + 2y = -4$

A. (0, −2), (0, 2) **B.** (0, −2), (4, 0) **C.** (0, −2), (−4, 0) **D.** (0, 2), (4, 0) **19.** ______

20. $x^2 + y^2 = 36$ and $y = x - 6$

A. (0, −6), (6, 0) **B.** (0, 6), (6, 0) **C.** (6, 0), (−6, 0) **D.** (−6, 0), (0, 6) **20.** ______

Bonus Solve the system of equations $x^2 + (y - 3)^2 = 4$ and $x^2 + (y + 3)^2 = 4$. **B:** ______________

NAME ______________________ DATE __________ PERIOD ______

8 Chapter 8 Test, Form 2C

SCORE ______

1. Find the midpoint of the line segment with endpoints at $(-2, 3)$ and $(14, 6)$. **1.** ____________

2. Find the distance between $A(4, -2)$ and $B(10, -7)$. **2.** ____________

3. Write an equation for the parabola with focus $(4, 4)$ and directrix $x = -2$. **3.** ____________

4. Write the $y = 3x^2 - 6x + 2$ in standard form. **4.** ____________

5. Identify the coordinates of the vertex and focus, the equations of the axis of symmetry and directrix, and the direction of opening of the parabola with equation $y^2 - 8y + 18 = x$. **5.** ____________

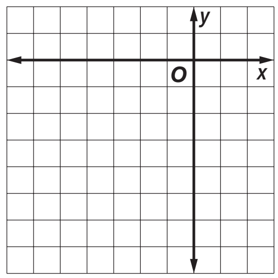
6. Write an equation for the circle with center $(-4, 2)$ that is tangent to the y-axis. **6.** ____________

Assessment

Graph each equation.

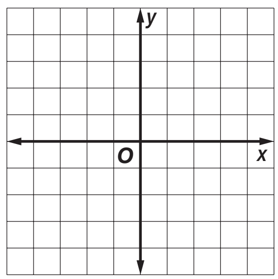
7. $x^2 + y^2 + 4x + 6y - 3 = 0$

7.



8. $9x^2 + 4y^2 = 36$

8.



For Questions 9 and 10, write an equation for the ellipse that satisfies each set of conditions.

9. endpoints of major axis at $(9, 3)$ and $(-11, 3)$, endpoints of minor axis at $(-1, 8)$ and $(-1, -2)$ **9.** ____________

10. major axis 12 units long and parallel to the y-axis, minor axis 8 units long, center at $(-2, 5)$ **10.** ____________

11. Find the exact solution(s) of the system of equations. **11.** ____________

$x^2 - y = 4$

$4x^2 + y^2 = 12$

Chapter 8 Test, Form 2C *(continued)*

For Questions 12 and 13, write an equation for the hyperbola that satisfies each set of conditions.

12. vertices (9, 0) and (−9, 0), conjugate axis of length 10 units **12.** ______________

13. vertices (−1, 4) and (−1, −8), foci $(-1, -2 \pm \sqrt{39})$ **13.** ______________

14. Find the coordinates of the vertices and foci and the equations of the asymptotes for the hyperbola $(x - 3)^2 - (y + 1)^2 = 4$. **14.** ______________

Write each equation in standard form. State whether the graph of the equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*.

15. $x^2 + y^2 + 2x + 2y = 23$ **15.** ______________

16. $4x^2 + 9y^2 + 24x - 18y + 9 = 0$ **16.** ______________

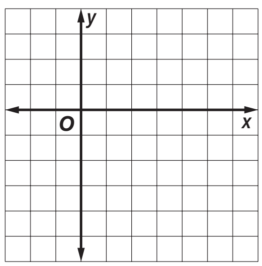
For Questions 17 and 18, state whether the graph of each equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*. State the values used to identify each conic section without writing each equation in standard form.

17. $3(x + 5)^2 + 3y - 15 = 0$ **17.** ______________

18. $4x^2 - 8x = 4(y^2 - 2y) + 7$ **18.** ______________

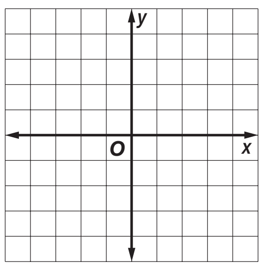
19. Graph the system of equations. Use the graph to solve the system.
$y = x^2 - 4x$
$y = x - 4$

19. ______________



20. Solve the system of inequalities by graphing.
$x^2 + y^2 < 16$
$y \leq -2x^2 + 1$

20.



Bonus Write an equation for the circle with the same center as the graph of $\frac{(x-3)^2}{4} - \frac{(y+1)^2}{16} = 1$ and the same radius as the graph of $x^2 + y^2 - 4x + 10y = 9$. **B:** ______________

NAME ______________________ DATE ____________ PERIOD ______

8 Chapter 8 Test, Form 2D

SCORE ______

1. Find the midpoint of the line segment with endpoints at (−4, 5) and (7, −3). **1.** ______________

2. Find the distance between $A(-7, 3)$ and $B(4, -6)$. **2.** ______________

3. Write an equation for the parabola with focus (−1, 1) and directrix $y = -7$. **3.** ______________

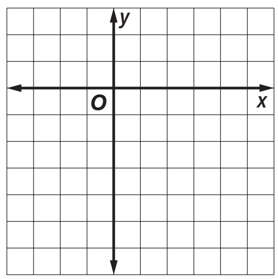
4. Write the equation of the parabola $x = 5y^2 - 10y + 2$ in standard form. **4.** ______________

5. Identify the coordinates of the vertex and focus, the equations of the axis of symmetry and directrix, and the direction of opening of the parabola with equation $y = 2x^2 - 4x + 5$. **5.** ______________

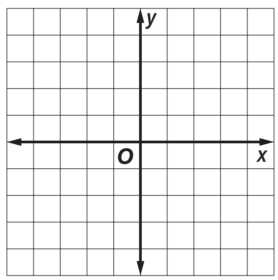
6. Write an equation for the circle with center $\left(\frac{1}{2}, -2\right)$ that is tangent to the x-axis. **6.** ______________

Graph each equation.

7. $x^2 + y^2 - 2x + 4y = 4$ **7.**



8. $9x^2 + 16y^2 = 144$ **8.**



For Questions 9 and 10, write an equation for the ellipse that satisfies each set of conditions.

9. endpoints of major axis at (2, −5) and (2, 9), endpoints of minor axis at (−4, 2) and (6, 2) **9.** ______________

10. major axis 16 units long and parallel to the x-axis, minor axis 6 units long, center at (1, −4) **10.** ______________

11. Find the exact solution(s) of the system of equations. **11.** ______________

$x^2 - 2y = 11$

$3x^2 + y^2 = 24$

NAME ______________________ DATE ____________ PERIOD ______

8 Chapter 8 Test, Form 2D *(continued)*

For Questions 12 and 13, write an equation for the hyperbola that satisfies each set of conditions.

12. vertices (0, 12) and (0, −12), conjugate axis of length 8 units **12.** ____________

13. vertices (−10, 1) and (4, 1), foci $(-3 \pm \sqrt{70}, 1)$ **13.** ____________

14. Find the coordinates of the vertices and foci and the equations of the asymptotes for the hyperbola $(x + 1)^2 - (y - 3)^2 = 4$. **14.** ____________

Write each equation in standard form. State whether the graph of the equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*.

15. $4x^2 - 16x - y + 21 = 0$ **15.** ____________

16. $y^2 - 6y - 4x^2 - 8x = 95$ **16.** ____________

For Questions 17 and 18, state whether the graph of each equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*. State the values used to identify each conic section without writing each equation in standard form.

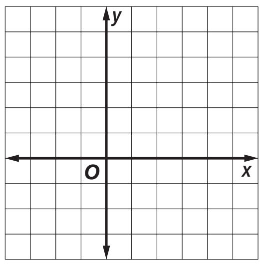
17. $2x^2 - 10x = 8y - 2y^2 + 5$ **17.** ____________

18. $3(y - 2)^2 + 8 = 9x - 10x^2$ **18.** ____________

19. Graph the system of equations. Use the graph to solve the system. **19.** ____________

$y^2 = 9 - x^2$

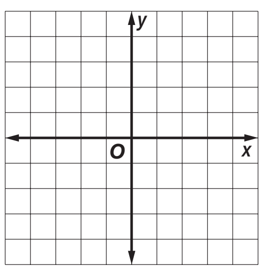
$y = -\frac{3}{4}x + 4$



20. Solve the system of inequalities by graphing. **20.**

$x^2 + 4y^2 < 1$

$x \geq 4(y + 2)^2$



Bonus Write an equation for the circle with the same center as the graph of $\frac{(x-5)^2}{16} - \frac{(y+2)^2}{9} = 1$ and the same radius as the graph of $x^2 + y^2 - 2y + 16x = 1$. **B:** ____________

8 Chapter 8 Test, Form 3

SCORE ______

1. Find the midpoint of the line segment with endpoints at $(-12, 3.5)$ and $(5.1, 4.8)$. **1.** ______

2. Find the distance between $A(4\sqrt{5}, -2)$ and $B(\sqrt{5}, 9)$. **2.** ______

3. Write the equation $x = -y^2 + 6y - 7$ in standard form. **3.** ______

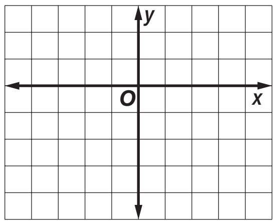
4. Write an equation for the parabola with vertex $(-5, 1)$ and directrix $x = -\frac{7}{2}$. **4.** ______

5. The path traveled by Pati's remote-controlled model airplane is shaped like a parabola. It took off from the ground and landed on the ground 160 feet away from where it took off. If the airplane reached a maximum height of 40 feet, write an equation for the parabola that models the path of the plane. Assume that the point of take-off is the origin. **5.** ______

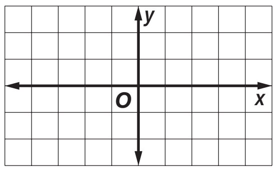
6. Identify the coordinates of the vertex and focus, the equations of the axis of symmetry and directrix, and the direction of opening of the parabola with equation $x = -y^2 - 2y + 9$. **6.**

7. Write an equation for a circle if its center is in the first quadrant, and it is tangent to $x = -2$, $x = 8$ and the x-axis. **7.** ______

8. Graph $x^2 + y^2 - 4x + 2y - 3 = 0$. **8.**



9. Graph $5x^2 - 2y^2 - 4y = 22$. **9.**



For Questions 10 and 11, write an equation for the ellipse that satisfies each set of conditions.

10. major axis 14 units long and parallel to the x-axis, minor axis 10 units long, center at $\left(5, -\frac{1}{2}\right)$ **10.** ______

11. endpoints of major axis at $(3, -8)$ and $(3, 4)$, foci at $(3, -2 + 2\sqrt{5})$ and $(3, -2 - 2\sqrt{5})$ **11.** ______

Assessment

NAME ______________________ DATE ____________ PERIOD ______

8 Chapter 8 Test, Form 3 *(continued)*

12. Find the coordinates of the center and foci and the lengths of the major and minor axes for the ellipse with equation $6x^2 + 5y^2 - 24x - 30y = -39$.

12. ____________

13. Write an equation for the hyperbola with vertices $(4, -5)$ and $(4, 1)$ and foci $(4, 3)$ and $(4, -7)$.

13. ____________

Write each equation in standard form. State whether the graph of the equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*.

14. $2x^2 + 3y^2 - 15 = 4(x - 2y)$

14. ____________

15. $\frac{1}{8}x + y^2 = -(y + 12)$

15. ____________

For Questions 16 and 17, state whether the graph of each equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*. State the values used to identify each conic section without writing each equation in standard form.

16. $3x^2 - 9x + y^2 = 2(24y - y^2 - 27)$

16. ____________

17. $34x^2 + 40y^2 - 18x - 25y = 17(2x^2 + 1)$

17. ____________

18. Find the exact solution(s) of the system of equations.

$\frac{x^2}{25} - \frac{y^2}{16} = 1$

$x = y$

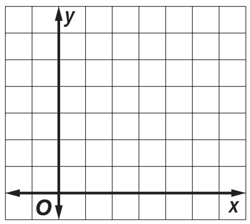
18. ____________

19. Solve the system of equations by graphing.

$x^2 + y^2 - 4x - 6y + 4 = 0$

$x^2 - 4x - 3y + 4 = 0$

19. ____________

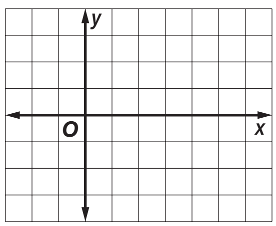


20. Solve the system of inequalities by graphing.

$x^2 + y^2 - 4x > -4 + 10$

$2 - y < (x - 1.75)^2$

20.



Bonus The parabolic curve of a certain camera lens can be represented by the equation $y = 10x^2 + 50x + 63.2$. What are the coordinates of the focus?

B: ____________

NAME ______________________ DATE ____________ PERIOD ______

8 Chapter 8 Open-Ended Assessment

SCORE ______

Demonstrate your knowledge by giving a clear, concise solution to each problem. Be sure to include all relevant drawings and justify your answers. You may show your solution in more than one way or investigate beyond the requirements of the problem.

1. Harry was asked to determine whether the graph of the equation $x^2 + y^2 + 8x - 6y + 30 = 0$ was a parabola, circle, ellipse, or hyperbola. At first glance, he identified the equation as that of a circle.

a. What made Harry think he was looking at the equation of a circle?

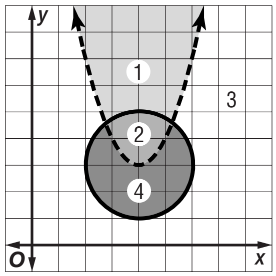
b. When Harry attempted to find the center and radius of the circle, he ran into a problem. What was the problem?

c. Change the equation so that Harry's problem no longer exists, then find the center and radius of the circle represented by your equation.

2. Do the graphs of any of the conic sections you have studied in this chapter represent relations that are functions? Explain your reasoning.

3. What do the graphs of the parabolas $y = (x - 2)^2 - 1$ and $x = (y + 1)^2 + 2$ have in common? How are the graphs different?

4. The graphs of the equations $(x - 4)^2 + (y - 3)^2 = 4$ and $y = (x - 4)^2 + 3$ are shown. For parts **a** and **b**, replace each of the ●s with one of the inequality symbols ($<, >, \leq, \geq$) so that the solution of the system is the region indicated. Explain your choices.

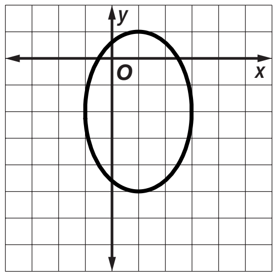


a. $(x - 4)^2 + (y - 3)^2$ ● 4
y ● $(x - 4)^2 + 3$
The solution of the system is region 2.

b. $(x - 4)^2 + (y - 3)^2$ ● 4
y ● $(x - 4)^2 + 3$
The solution of the system is region 3.

c. What region is represented by the system $(x - 4)^2 + (y - 3)^2 \geq 4$ and $y > (x - 4)^2 + 3$? Explain.

5. The graph of the equation $\frac{(x-1)^2}{4} + \frac{(y+2)^2}{9} = 1$ is shown. Find values of k for which the given system of equations has the given number of solutions. Explain the reasoning for your choices.



$$\frac{(x-1)^2}{4} + \frac{(y+2)^2}{9} = 1$$
$$y = k$$

a. For $k =$ ____ and $k =$ ____, the system has two solutions.

b. For $k =$ ____ and $k =$ ____, the system has one solution.

c. For $k =$ ____ and $k =$ ____, the system has no solutions.

Assessment

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NAME ______________________ DATE ____________ PERIOD ________

8 Chapter 8 Vocabulary Test/Review

SCORE ________

asymptote	conjugate axis	focus of a parabola	parabola
center of a circle	directrix	hyperbola	tangent
center of an ellipse	Distance Formula	latus rectum	transverse axis
center of a hyperbola	ellipse	major axis	vertex of a hyperbola
circle	foci of an ellipse	Midpoint Formula	
conic section	foci of a hyperbola	minor axis	

Choose from the terms above to complete each sentence.

1. A ______________________ is the set of all points in a plane that are the same distance from a given point and a given line. The given point is called the ______________________ and the given line is called the ______________________.

2. The set of all points in a plane the sum of whose distances from two fixed points is constant is a(n) ______________________. The two fixed points are called the ______________________.

3. The set of all points in a plane such that the absolute value of the difference of their distances from the two given points is constant is a(n) ______________________.

4. The points at which an ellipse intersects its axes of symmetry determine two segments on the ellipse. The shorter of these segments is called the ______________________ and the longer one is called the ______________________.

5. The segment that connects the two vertices of a hyperbola is called the ______________________.

6. A line that intersects a circle in exactly one point is ______________________ to the circle.

7. The line segment through the focus of a parabola and perpendicular to the line of symmetry is called the ______________________.

8. A line that the branches of a hyperbola approach but do not intersect is called a(n) ______________________.

9. The segment of length $2b$ units that is perpendicular to the transverse axis of a hyperbola at its center is called the ______________________.

10. The formula that can be used to find the length of a line segment if you know the coordinates of its endpoints is called the ______________________.

In your own words—
Define each term.

11. circle

12. vertex of a hyperbola

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NAME ______________ DATE ________ PERIOD ______

8

Chapter 8 Quiz

SCORE ______

(Lessons 8–1 and 8–2)

1. Find the midpoint of the line segment with endpoints at $(-7, -3)$ and $(5, 10)$. **1.** ______

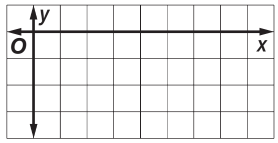
2. **Standardized Test Practice** Which point is farthest from $(2, -1)$?

A. $(3, 3)$ **B.** $(-2, -1)$ **C.** $(4, 0)$ **D.** $(-1, 0)$ **2.** ______

3. Write an equation for the parabola with focus $(1, 4)$ and directrix $y = -2$. **3.** ______

4. Identify the coordinates of the vertex and focus, the equations of the axis of symmetry and directrix, and the direction of opening of the parabola with equation $y = -2x^2 - 16x - 27$. **4.** ______

5. Graph the parabola $x = 6y^2 + 24y + 25$ and find the length of the latus rectum. **5.** ______



NAME ______________ DATE ________ PERIOD ______

8

Chapter 8 Quiz

SCORE ______

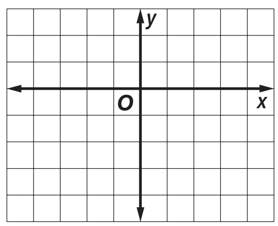
(Lessons 8–3 and 8–4)

For Questions 1 and 2, write an equation for the circle that satisfies each set of conditions.

1. center $(-7, 2)$, radius 9 units **1.** ______

2. endpoints of a diameter at $(-1, 1)$ and $(7, 1)$ **2.** ______

3. Find the center and radius of the circle with equation $x^2 + y^2 = 2x - 2y + 7$. Then graph the circle. **3.** ______



4. Write an equation for an ellipse if the endpoints of the major axis are at $(5, 1)$ and $(-5, 1)$ and the endpoints of the minor axis are at $(0, 5)$ and $(0, -3)$. **4.** ______

5. Find the coordinates of the center and foci and the lengths of the major and minor axes for the ellipse with equation $\frac{(x-3)^2}{16} + \frac{y^2}{4} = 1$. **5.** ______

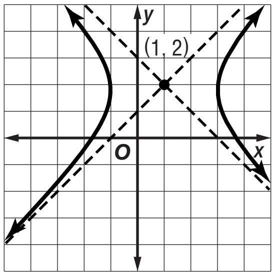
NAME ______________________ DATE ____________ PERIOD ______

8 Chapter 8 Quiz

SCORE ______

(Lessons 8–5 and 8–6)

1. Write an equation for the hyperbola whose graph is shown.



2. Write an equation for the hyperbola with vertices $(2, -5)$ and $(2, 3)$, foci $(2, -6)$ and $(2, 4)$

3. Find the coordinates of the vertices and foci and the equations of the asymptotes for the hyperbola $\frac{x^2}{4} - \frac{y^2}{36} = 1$. Then graph the hyperbola.

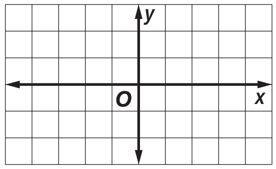
4. Write $2x^2 - 12x - y = 5$ in standard form. Then state whether the graph of the equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*.

5. State whether the graph of $x^2 - 2x + 4y^2 + 24y - 37 = 0$ is a *parabola*, *circle*, *ellipse*, or *hyperbola*. State the values used to identify the conic section without writing the equation in standard form.

1. ______________

2. ______________

3. ______________



4. ______________

5. ______________

NAME ______________________ DATE ____________ PERIOD ______

8 Chapter 8 Quiz

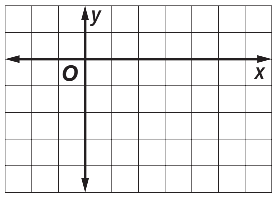
SCORE ______

(Lesson 8–7)

Graph each system of equations. Use the graph to solve the system.

1. $y = (x - 2)^2 - 3$
$\frac{(x-2)^2}{1} + \frac{(y+2)^2}{4} = 1$

1. ______________



For Questions 3 and 4, find the exact solution(s) of each system of equations.

2. $x^2 - y^2 = -8$
$y = 2x + 1$

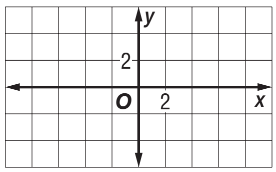
2. ______________

3. $2x^2 + 5y^2 = 22$
$y^2 - 3x^2 = 1$

3. ______________

4. Solve the system of inequalities by graphing.
$\frac{x^2}{9} - \frac{y^2}{4} \geq 1$
$\frac{x^2}{25} + \frac{y^2}{9} \leq 1$

4.



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NAME ______________________ DATE ____________ PERIOD ________

8 Chapter 8 Mid-Chapter Test

(Lessons 8–1 through 8–4)

SCORE ________

Part I ***Write the letter for the correct answer in the blank at the right of each question.***

1. What is the midpoint of the line segment with endpoints at $(-6, 3)$ and $(-10, 7)$?

A. $(-8, 5)$ **B.** $(-16, 10)$ **C.** $(2, -2)$ **D.** $(4, -4)$ **1.** ________

2. Find the distance between $A(-3, 1)$ and $B(5, -5)$.

A. 100 **B.** 32 **C.** 4 **D.** 10 **2.** ________

3. Write an equation for the parabola with vertex $(1, 2)$ and directrix $x = \frac{3}{4}$.

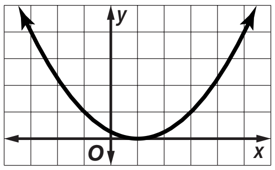
A. $y = (x - 2)^2 + 1$ **B.** $x = (y - 2)^2 + 1$

C. $y = (x + 2)^2 - 1$ **D.** $x = (y - 1)^2 - 2$ **3.** ________

4. Which equation is graphed?

A. $y = 4x^2 + 8x + 4$ **B.** $x = 4y^2 + 1$

C. $y = \frac{1}{4}x^2 - \frac{1}{2}x + \frac{1}{4}$ **D.** $x = \frac{1}{4}y^2 - \frac{1}{2}y + \frac{1}{4}$ **4.** ________



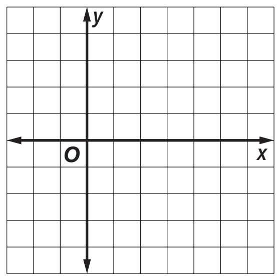
5. Write an equation of the circle with center $(-2, 7)$ that is tangent to the y-axis.

A. $(x + 2)^2 + (y - 7)^2 = 4$ **B.** $(x + 2)^2 + (y - 7)^2 = 49$

C. $(x - 2)^2 + (y + 7)^2 = 4$ **D.** $(x - 2)^2 + (y + 7)^2 = 49$ **5.** ________

Part II

6. Graph $x^2 + y^2 - 4x = 12$. **6.**



7. Write an equation of the ellipse centered at $(4, 1)$ if its minor axis is 8 units long and its major axis is 10 units long and parallel to the x-axis. **7.** ________________

8. Write the equation of the parabola $y = -3x^2 + 18x + 5$ in standard form. **8.** ________________

9. Write an equation for a circle if the endpoints of a diameter are at $(-2, -1)$ and $(8, 9)$. **9.** ________________

Assessment

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8 Chapter 8 Cumulative Review

(Chapters 1–8)

1. Evaluate $-3|5a + b|$ if $a = -3.5$ and $b = 10$. (Lesson 1-4) — **1.** ____________

2. Write an equation in slope-intercept form for the line that has a slope of 4 and passes through (2, 5). (Lesson 2-4) — **2.** ____________

3. Solve the system of equations by using substitution.
$y = 6x + 5$
$2x - 3y = 1$. (Lesson 3-2) — **3.** ____________

4. Perform the indicated matrix operation. If the matrix does not exist, write *impossible*. $\begin{bmatrix} 3 & 6 & 9 \\ -2 & -1 & 0 \end{bmatrix} + \begin{bmatrix} 4 & 3 & 7 \\ -2 & 9 & 5 \end{bmatrix}$ (Lesson 4-2) — **4.** ____________

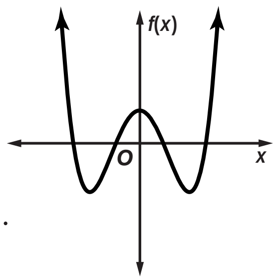
5. Simplify $\frac{2x^2 - 7x - 4}{2x^2 + 7x + 3}$. Assume that the denominator is not equal to 0. (Lesson 5-4) — **5.** ____________

6. Simplify $(3 + 4i) - (2 - 5i)$. (Lesson 5-9) — **6.** ____________

7. Find the exact solutions to $2x^2 - 7x + 5 = 0$ by using the Quadratic Formula. (Lesson 6-5) — **7.** ____________

8. Solve the inequality $-2x + 3 \leq x^2$ algebraically. (Lesson 6-7) — **8.** ____________

9. Determine whether the graph represents an odd degree or an even degree polynomial function. Then state the number of real zeros. (Lesson 7-1) — **9.** ____________



10. One factor of $2x^3 - 7x^2 + 2x + 3$ is $x - 3$. Find the remaining factors. (Lesson 7-4) — **10.** ____________

11. If $g(x) = 4x$ and $h(x) = 3x - 5$, find $[gh](x)$. (Lesson 7-7) — **11.** ____________

12. Find the midpoint of the line segment with end points at (−10, 8) and (2, −3). (Lesson 8-1) — **12.** ____________

13. Write an equation for the parabola with focus (−4, 0) and directrix $x = 6$. (Lesson 8-2) — **13.** ____________

14. Find the coordinates of the center and foci and the lengths of the major and minor axes for the ellipse with equation $9x^2 + y^2 = 9$. Then graph the ellipse. (Lesson 8-4) — **14.** ____________

15. Write the equation $4x^2 + 9y^2 + 24x - 18y + 9 = 0$ in standard form. Then state whether the graph of the equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*. (Lesson 8-6) — **15.** ____________

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8 Standardized Test Practice

(Chapters 1–8)

Part 1: Multiple Choice

Instructions: Fill in the appropriate oval for the best answer.

1. Which of the following is the sum of two consecutive prime numbers?

A. 9 **B.** 11 **C.** 17 **D.** 24

1. (A) (B) (C) (D)

2. If $(r + 2)(r + 1) > (r - 2)(r + 6)$, which of the following is true?

E. $r > 14$ **F.** $r > -14$ **G.** $r < 14$ **H.** $r < 19$

2. (E) (F) (G) (H)

3. What is the value of $10m - 3$ if $2m = 9$?

A. 2 **B.** 8 **C.** 42 **D.** 48

3. (A) (B) (C) (D)

4. If 10 pears cost c cents, how many pears will d dollars buy?

E. $\frac{1000d}{c}$ **F.** $\frac{d}{10c}$ **G.** $\frac{10d}{c}$ **H.** $\frac{10c}{d}$

4. (E) (F) (G) (H)

5. What is the value of $\frac{x}{y}$ is $2.5x = \frac{11}{3}y$ and $y \neq 0$?

A. $\frac{55}{6}$ **B.** $\frac{2}{3}$ **C.** $\frac{22}{15}$ **D.** $\frac{3}{2}$

5. (A) (B) (C) (D)

6. The sum of five integers is what percent of the average of the same five integers?

E. 5 **F.** 50 **G.** 500 **H.** 5000

6. (E) (F) (G) (H)

7. Which of the following are always true statements?

I. $x^2 > 0$ **II.** $x^2 > x$ **III.** $x + 1 > x$ **IV.** $x > -x$

A. I and II only **B.** I and IV only

C. III and IV only **D.** III only

7. (A) (B) (C) (D)

8. The table shows the distribution of quiz scores for a group of students. No student scored less than 50 or greater than 90. What is the mean of the scores?

E. 70 **F.** 72.5

G. 75 **H.** 74.5

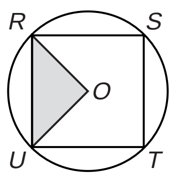
Score	Number of students
90	2
80	6
70	8
60	3
50	1

8. (E) (F) (G) (H)

9. Square *RSTU* is inscribed in circle *O*. If the circumference of circle *O* is 16π, find the area of triangle *ROU*.

A. 32 **B.** 32π

C. 64 **D.** 16π



9. (A) (B) (C) (D)

10. What is the value of t if $r = -1$ and $t = (r + 1)(r + 2)(r + 3)$?

E. 0 **F.** 2 **G.** 6 **H.** 24

10. (E) (F) (G) (H)

Assessment

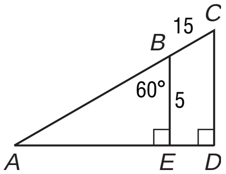
8 Standardized Test Practice *(continued)*

Part 2: Grid In

Instructions: Enter your answer by writing each digit of the answer in a column box and then shading in the appropriate oval that corresponds to that entry.

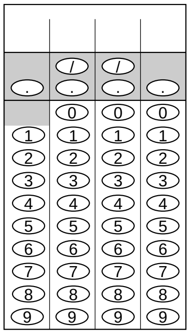
11. A jar contains 2 white marbles, 5 red marbles, and 13 blue marbles. How many white marbles must be added to the jar to make the probability of randomly selecting a white marble $\frac{1}{4}$?

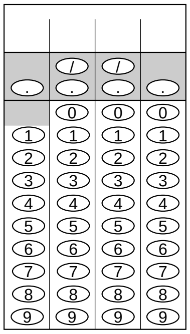
12. In the figure shown, what is the length of $\overline{CD}$?

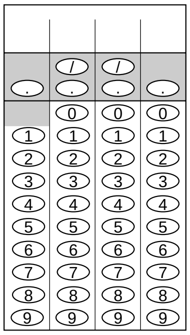


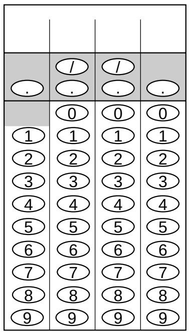
13. If the sales tax on a \$22.00 purchase is \$1.32, what is the total cost of an item priced at \$8.50?

14. Evaluate $7 + 3 \cdot 5 \div 2^2$.

11.


12.


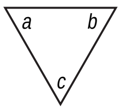
13.


14.


Part 3: Quantitative Comparison

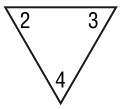
Instructions: Compare the quantities in columns A and B. Shade in
- (A) if the quantity in column A is greater;
- (B) if the quantity in column B is greater;
- (C) if the quantities are equal; or
- (D) if the relationship cannot be determined from the information given.

	Column A	Column B	
15. $2 + d = e + 2$	$e - d$	$d - e$	**15.** (A) (B) (C) (D)
16. The average of a, b, and c is x.	$3x$	$3a$	**16.** (A) (B) (C) (D)
17.	$\sqrt[3]{64}$	$\sqrt[4]{16^2}$	**17.** (A) (B) (C) (D)

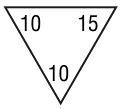
18. 

$= \frac{a+b}{c}$ for all real numbers a, b, and c **18.** (A) (B) (C) (D)

Column A:



Column B:



NAME ______________________________ DATE ____________ PERIOD _____

8 Standardized Test Practice

Student Record Sheet *(Use with pages 468–469 of the Student Edition.)*

Part 1 *Multiple Choice*

Select the best answer from the choices given and fill in the corresponding oval.

1 Ⓐ Ⓑ Ⓒ Ⓓ
2 Ⓐ Ⓑ Ⓒ Ⓓ
3 Ⓐ Ⓑ Ⓒ Ⓓ
4 Ⓐ Ⓑ Ⓒ Ⓓ
5 Ⓐ Ⓑ Ⓒ Ⓓ
6 Ⓐ Ⓑ Ⓒ Ⓓ
7 Ⓐ Ⓑ Ⓒ Ⓓ
8 Ⓐ Ⓑ Ⓒ Ⓓ
9 Ⓐ Ⓑ Ⓒ Ⓓ
10 Ⓐ Ⓑ Ⓒ Ⓓ

Part 2 *Short Response/Grid In*

Solve the problem and write your answer in the blank.

Also enter your answer by writing each number or symbol in a box. Then fill in the corresponding oval for that number or symbol.

11 ____________

12 ____________

13 ____________

14 ____________

15 ____________

16 ____________

17 ____________

Part 3 *Quantitative Comparison*

Select the best answer from the choices given and fill in the corresponding oval.

18 Ⓐ Ⓑ Ⓒ Ⓓ
19 Ⓐ Ⓑ Ⓒ Ⓓ
20 Ⓐ Ⓑ Ⓒ Ⓓ
21 Ⓐ Ⓑ Ⓒ Ⓓ

Answers

NAME ______________ DATE ________ PERIOD _____

8-1 Study Guide and Intervention

Midpoint and Distance Formulas

The Midpoint Formula

Midpoint Formula	The midpoint M of a segment with endpoints (x_1, y_1) and (x_2, y_2) is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

Example 1 **Find the midpoint of the line segment with endpoints at (4, −7) and (−2, 3).**

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{4 + (-2)}{2}, \frac{-7 + 3}{2}\right) = \left(\frac{2}{2}, \frac{-4}{2}\right) \text{ or } (1, -2)$$

The midpoint of the segment is (1, −2).

Example 2 **A diameter $\overline{AB}$ of a circle has endpoints $A(5, -11)$ and $B(-7, 6)$. What are the coordinates of the center of the circle?**

The center of the circle is the midpoint of all of its diameters.

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{5 + (-7)}{2}, \frac{-11 + 6}{2}\right) = \left(\frac{-2}{2}, \frac{-5}{2}\right) \text{ or } \left(-1, -2\frac{1}{2}\right)$$

The circle has center $\left(-1, -2\frac{1}{2}\right)$.

Exercises

Find the midpoint of each line segment with endpoints at the given coordinates.

1. (12, 7) and (−2, 11) **(5, 9)**
2. (−8, −3) and (10, 9) **(1, 3)**
3. (4, 15) and (10, 1) **(7, 8)**
4. (−3, −3) and (3, 3) **(0, 0)**
5. (15, 6) and (12, 14) **(13.5, 10)**
6. (22, −8) and (−10, 6) **(6, −1)**
7. (3, 5) and (−6, 11) $\left(-\frac{3}{2}, 8\right)$
8. (8, −15) and (−7, 13) $\left(\frac{1}{2}, -1\right)$
9. (2.5, −6.1) and (7.9, 13.7) **(5.2, 3.8)**
10. (−7, −6) and (−1, 24) **(−4, 9)**
11. (3, −10) and (30, −20) $\left(\frac{33}{2}, -15\right)$
12. (−9, 1.7) and (−11, 1.3) **(−10, 1.5)**

13. Segment $\overline{MN}$ has midpoint P. If M has coordinates (14, −3) and P has coordinates (−8, 6), what are the coordinates of N? **(−30, 15)**

14. Circle R has a diameter $\overline{ST}$. If R has coordinates (−4, −8) and S has coordinates (1, 4), what are the coordinates of T? **(−9, −20)**

15. Segment $\overline{AD}$ has midpoint B, and $\overline{BD}$ has midpoint C. If A has coordinates (−5, 4) and C has coordinates (10, 11), what are the coordinates of B and D? ***B* is $\left(5, 8\frac{2}{3}\right)$, *D* is $\left(15, 13\frac{1}{3}\right)$.**

Lesson 8-1

NAME ______________ DATE ________ PERIOD _____

8-1 Study Guide and Intervention *(continued)*

Midpoint and Distance Formulas

The Distance Formula

Distance Formula	The distance between two points (x_1, y_1) and (x_2, y_2) is given by $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

Example 1 **What is the distance between (8, −2) and (−6, −8)?**

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Distance Formula

$= \sqrt{(-6 - 8)^2 + [-8 - (-2)]^2}$ Let $(x_1, y_1) = (8, -2)$ and $(x_2, y_2) = (-6, -8)$.

$= \sqrt{(-14)^2 + (-6)^2}$ Subtract.

$= \sqrt{196 + 36}$ or $\sqrt{232}$ Simplify.

The distance between the points is $\sqrt{232}$ or about 15.2 units.

Example 2 **Find the perimeter and area of square *PQRS* with vertices $P(-4, 1)$, $Q(-2, 7)$, $R(4, 5)$, and $S(2, -1)$.**

Find the length of one side to find the perimeter and the area. Choose $\overline{PQ}$.

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Distance Formula

$= \sqrt{[-4 - (-2)]^2 + (1 - 7)^2}$ Let $(x_1, y_1) = (-4, 1)$ and $(x_2, y_2) = (-2, 7)$.

$= \sqrt{(-2)^2 + (-6)^2}$ Subtract.

$= \sqrt{40}$ or $2\sqrt{10}$ Simplify.

Since one side of the square is $2\sqrt{10}$, the perimeter is $8\sqrt{10}$ units. The area is $(2\sqrt{10})^2$, or 40 units^2.

Exercises

Find the distance between each pair of points with the given coordinates.

1. (3, 7) and (−1, 4) **5 units**
2. (−2, −10) and (10, −5) **13 units**
3. (6, −6) and (−2, 0) **10 units**
4. (7, 2) and (4, −1) **$3\sqrt{2}$ units**
5. (−5, −2) and (3, 4) **10 units**
6. (11, 5) and (16, 9) **$\sqrt{41}$ units**
7. (−3, 4) and (6, −11) **$3\sqrt{34}$ units**
8. (13, 9) and (11, 15) **$2\sqrt{10}$ units**
9. (−15, −7) and (2, 12) **$5\sqrt{26}$ units**

10. Rectangle $ABCD$ has vertices $A(1, 4)$, $B(3, 1)$, $C(-3, -2)$, and $D(-5, 1)$. Find the perimeter and area of $ABCD$. **$2\sqrt{13} + 6\sqrt{5}$ units; $3\sqrt{65}$ units^2**

11. Circle R has diameter $\overline{ST}$ with endpoints $S(4, 5)$ and $T(-2, -3)$. What are the circumference and area of the circle? (Express your answer in terms of π.) **10π units; 25π units^2**

NAME ______________ DATE ________ PERIOD _____

8-1 Skills Practice

Midpoint and Distance Formulas

Find the midpoint of each line segment with endpoints at the given coordinates.

1. $(4, -1), (-4, 1)$ **(0, 0)**

2. $(-1, 4), (5, 2)$ **(2, 3)**

3. $(3, 4), (5, 4)$ **(4, 4)**

4. $(6, 2), (2, -1)$ $\left(4, \frac{1}{2}\right)$

5. $(3, 9), (-2, -3)$ $\left(\frac{1}{2}, 3\right)$

6. $(-3, 5), (-3, -8)$ $\left(-3, -\frac{3}{2}\right)$

7. $(3, 2), (-5, 0)$ **(−1, 1)**

8. $(3, -4), (5, 2)$ **(4, −1)**

9. $(-5, -9), (5, 4)$ $\left(0, -\frac{5}{2}\right)$

10. $(-11, 14), (0, 4)$ $\left(-\frac{11}{2}, 9\right)$

11. $(3, -6), (-8, -3)$ $\left(-\frac{5}{2}, -\frac{9}{2}\right)$

12. $(0, 10), (-2, -5)$ $\left(-1, \frac{5}{2}\right)$

Find the distance between each pair of points with the given coordinates.

13. $(4, 12), (-1, 0)$ **13 units**

14. $(7, 7), (-5, -2)$ **15 units**

15. $(-1, 4), (1, 4)$ **2 units**

16. $(11, 11), (8, 15)$ **5 units**

17. $(1, -6), (7, 2)$ **10 units**

18. $(3, -5), (3, 4)$ **9 units**

19. $(2, 3), (3, 5)$ $\sqrt{5}$ **units**

20. $(-4, 3), (-1, 7)$ **5 units**

21. $(-5, -5), (3, 10)$ **17 units**

22. $(3, 9), (-2, -3)$ **13 units**

23. $(6, -2), (-1, 3)$ $\sqrt{74}$ **units**

24. $(-4, 1), (2, -4)$ $\sqrt{61}$ **units**

25. $(0, -3), (4, 1)$ $4\sqrt{2}$ **units**

26. $(-5, -6), (2, 0)$ $\sqrt{85}$ **units**

Lesson 8-1

NAME ______________ DATE ________ PERIOD _____

8-1 Practice (Average)

Midpoint and Distance Formulas

Find the midpoint of each line segment with endpoints at the given coordinates.

1. $(8, -3), (-6, -11)$ **(1, −7)**

2. $(-14, 5), (10, 6)$ $\left(-2, \frac{11}{2}\right)$

3. $(-7, -6), (1, -2)$ **(−3, −4)**

4. $(8, -2), (8, -8)$ **(8, −5)**

5. $(9, -4), (1, -1)$ $\left(5, -\frac{5}{2}\right)$

6. $(3, 3), (4, 9)$ $\left(\frac{7}{2}, 6\right)$

7. $(4, -2), (3, -7)$ $\left(\frac{7}{2}, -\frac{9}{2}\right)$

8. $(6, 7), (4, 4)$ $\left(5, \frac{11}{2}\right)$

9. $(-4, -2), (-8, 2)$ **(−6, 0)**

10. $(5, -2), (3, 7)$ $\left(4, \frac{5}{2}\right)$

11. $(-6, 3), (-5, -7)$ $\left(-\frac{11}{2}, -2\right)$

12. $(-9, -8), (8, 3)$ $\left(-\frac{1}{2}, -\frac{5}{2}\right)$

13. $(2.6, -4.7), (8.4, 2.5)$ **(5.5, −1.1)**

14. $\left(-\frac{1}{3}, 6\right), \left(\frac{2}{3}, 4\right)$ $\left(\frac{1}{6}, 5\right)$

15. $(-2.5, -4.2), (8.1, 4.2)$ **(2.8, 0)**

16. $\left(\frac{1}{8}, \frac{1}{2}\right), \left(-\frac{5}{8}, -\frac{1}{2}\right)$ $\left(-\frac{1}{4}, 0\right)$

Find the distance between each pair of points with the given coordinates.

17. $(5, 2), (2, -2)$ **5 units**

18. $(-2, -4), (4, 4)$ **10 units**

19. $(-3, 8), (-1, -5)$ $\sqrt{173}$ **units**

20. $(0, 1), (9, -6)$ $\sqrt{130}$ **units**

21. $(-5, 6), (-6, 6)$ **1 unit**

22. $(-3, 5), (12, -3)$ **17 units**

23. $(-2, -3), (9, 3)$ $\sqrt{157}$ **units**

24. $(-9, -8), (-7, 8)$ $2\sqrt{65}$ **units**

25. $(9, 3), (9, -2)$ **5 units**

26. $(-1, -7), (0, 6)$ $\sqrt{170}$ **units**

27. $(10, -3), (-2, -8)$ **13 units**

28. $(-0.5, -6), (1.5, 0)$ $2\sqrt{10}$ **units**

29. $\left(\frac{2}{5}, \frac{3}{5}\right), \left(1, \frac{7}{5}\right)$ **1 unit**

30. $(-4\sqrt{2}, -\sqrt{5}), (-5\sqrt{2}, 4\sqrt{5})$ $\sqrt{127}$ **units**

31. GEOMETRY Circle O has a diameter $\overline{AB}$. If A is at $(-6, -2)$ and B is at $(-3, 4)$, find the center of the circle and the length of its diameter. $\left(-\frac{9}{2}, 1\right)$; $3\sqrt{5}$ **units**

32. GEOMETRY Find the perimeter of a triangle with vertices at $(1, -3)$, $(-4, 9)$, and $(-2, 1)$. $18 + 2\sqrt{17}$ **units**

NAME ______________________ DATE ________ PERIOD _____

8-1 Reading to Learn Mathematics

Midpoint and Distance Formulas

Pre-Activity **How are the Midpoint and Distance Formulas used in emergency medicine?**

Read the introduction to Lesson 8-1 at the top of page 412 in your textbook.

How do you find distances on a road map?

Sample answer: Use the scale of miles on the map. You might also use a ruler.

Reading the Lesson

1. a. Write the coordinates of the midpoint of a segment with endpoints (x_1, y_1) and (x_2, y_2).
$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

b. Explain how to find the midpoint of a segment if you know the coordinates of the endpoints. Do not use subscripts in your explanation.
Sample answer: To find the *x*-coordinate of the midpoint, add the *x*-coordinates of the endpoints and divide by two. To find the *y*-coordinate of the midpoint, do the same with the *y*-coordinates of the endpoints.

2. a. Write an expression for the distance between two points with coordinates (x_1, y_1) and (x_2, y_2). $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

b. Explain how to find the distance between two points. Do not use subscripts in your explanation.
Sample answer: Find the difference between the *x*-coordinates and square it. Find the difference between the *y*-coordinates and square it. Add the squares. Then find the square root of the sum.

3. Consider the segment connecting the points $(-3, 5)$ and $(9, 11)$.

a. Find the midpoint of this segment. **(3, 8)**

b. Find the length of the segment. Write your answer in simplified radical form. $6\sqrt{5}$

Helping You Remember

4. How can the "mid" in *midpoint* help you remember the midpoint formula?
Sample answer: The *midpoint* is the point in the *middle* of a segment. It is halfway between the endpoints. The coordinates of the midpoint are found by finding the average of the two *x*-coordinates (add them and divide by 2) and the average of the two *y*-coordinates.

Lesson 8-1

NAME ______________________ DATE ________ PERIOD _____

8-1 Enrichment

Quadratic Form

Consider two methods for solving the following equation.

$(y - 2)^2 - 5(y - 2) + 6 = 0$

One way to solve the equation is to simplify first, then use factoring.

$$y^2 - 4y + 4 - 5y + 10 + 6 = 0$$
$$y^2 - 9y + 20 = 0$$
$$(y - 4)(y - 5) = 0$$

Thus, the solution set is {4, 5}.

Another way to solve the equation is first to replace $y - 2$ by a single variable. This will produce an equation that is easier to solve than the original equation. Let $t = y - 2$ and then solve the new equation.

$$(y - 2)^2 - 5(y - 2) + 6 = 0$$
$$t^2 - 5t + 6 = 0$$
$$(t - 2)(t - 3) = 0$$

Thus, t is 2 or 3. Since $t = y - 2$, the solution set of the original equation is {4, 5}.

Solve each equation using two different methods.

1. $(z + 2)^2 + 8(z + 2) + 7 = 0$
{−3, −9}

2. $(3x - 1)^2 - (3x - 1) - 20 = 0$
{2, −1}

3. $(2t + 1)^2 - 4(2t + 1) + 3 = 0$
{0, 1}

4. $(y^2 - 1)^2 - (y^2 - 1) - 2 = 0$
$\{0, \pm\sqrt{3}\}$

5. $(a^2 - 2)^2 - 2(a^2 - 2) - 3 = 0$
$\{\pm 1, \pm\sqrt{5}\}$

6. $(1 + \sqrt{c})^2 + (1 + \sqrt{c}) - 6 = 0$
{1}

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8-2 Study Guide and Intervention

Parabolas

Equations of Parabolas A parabola is a curve consisting of all points in the coordinate plane that are the same distance from a given point (the **focus**) and a given line (the **directrix**). The following chart summarizes important information about parabolas.

Standard Form of Equation	$y = a(x - h)^2 + k$	$x = a(y - k)^2 + h$
Axis of Symmetry	$x = h$	$y = k$
Vertex	(h, k)	(h, k)
Focus	$\left(h, k + \frac{1}{4a}\right)$	$\left(h + \frac{1}{4a}, k\right)$
Directrix	$y = k - \frac{1}{4a}$	$x = h - \frac{1}{4a}$
Direction of Opening	upward if $a > 0$, downward if $a < 0$	right if $a > 0$, left if $a < 0$
Length of Latus Rectum	$\left\lvert\frac{1}{a}\right\rvert$ units	$\left\lvert\frac{1}{a}\right\rvert$ units

Example **Identify the coordinates of the vertex and focus, the equations of the axis of symmetry and directrix, and the direction of opening of the parabola with equation $y = 2x^2 - 12x - 25$.**

$y = 2x^2 - 12x - 25$ — Original equation
$y = 2(x^2 - 6x) - 25$ — Factor 2 from the x-terms.
$y = 2(x^2 - 6x + ■) - 25 - 2(■)$ — Complete the square on the right side.
$y = 2(x^2 - 6x + 9) - 25 - 2(9)$ — The 9 added to complete the square is multiplied by 2.
$y = 2(x - 3)^2 - 43$ — Write in standard form.

The vertex of this parabola is located at $(3, -43)$, the focus is located at $\left(3, -42\frac{7}{8}\right)$, the equation of the axis of symmetry is $x = 3$, and the equation of the directrix is $y = -43\frac{1}{8}$. The parabola opens upward.

Exercises

Identify the coordinates of the vertex and focus, the equations of the axis of symmetry and directrix, and the direction of opening of the parabola with the given equation.

1. $y = x^2 + 6x - 4$
$(-3, -13)$, $\left(-3, -12\frac{3}{4}\right)$, $x = -3$, $y = -13\frac{1}{4}$, up

2. $y = 8x - 2x^2 + 10$
$(2, 18)$, $\left(2, 17\frac{1}{8}\right)$, $x = 2$, $y = 18\frac{1}{8}$, down

3. $x = y^2 - 8y + 6$
$(-10, 4)$, $\left(-9\frac{3}{4}, 4\right)$, $y = 4$, $x = -10\frac{1}{4}$, right

Write an equation of each parabola described below.

4. focus $(-2, 3)$, directrix $x = -2\frac{1}{12}$
$x = 6(y - 3)^2 - 2\frac{1}{24}$

5. vertex $(5, 1)$, focus $\left(4\frac{11}{12}, 1\right)$
$x = -3(y - 1)^2 + 5$

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8-2 Study Guide and Intervention *(continued)*

Parabolas

Graph Parabolas To graph an equation for a parabola, first put the given equation in standard form.

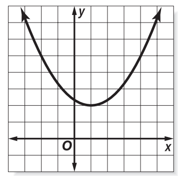
$y = a(x - h)^2 + k$ for a parabola opening up or down, or
$x = a(y - k)^2 + h$ for a parabola opening to the left or right

Use the values of a, h, and k to determine the vertex, focus, axis of symmetry, and length of the latus rectum. The vertex and the endpoints of the latus rectum give three points on the parabola. If you need more points to plot an accurate graph, substitute values for points near the vertex.

Example **Graph $y = \frac{1}{3}(x - 1)^2 + 2$.**

In the equation, $a = \frac{1}{3}$, $h = 1$, $k = 2$.

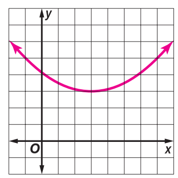
The parabola opens up, since $a > 0$.
vertex: $(1, 2)$
axis of symmetry: $x = 1$
focus: $\left(1, 2 + \frac{1}{4\left(\frac{1}{3}\right)}\right)$ or $\left(1, 2\frac{3}{4}\right)$
length of latus rectum: $\left\lvert\frac{1}{\frac{1}{3}}\right\rvert$ or 3 units
endpoints of latus rectum: $\left(2\frac{1}{2}, 2\frac{3}{4}\right)$, $\left(-\frac{1}{2}, 2\frac{3}{4}\right)$



Exercises

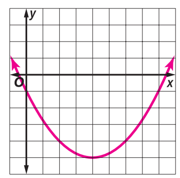
The coordinates of the focus and the equation of the directrix of a parabola are given. Write an equation for each parabola and draw its graph.

1. $(3, 5)$, $y = 1$



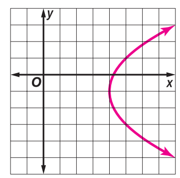
$y = \frac{1}{8}(x - 3)^2 + 3$

2. $(4, -4)$, $y = -6$



$y = \frac{1}{4}(x - 4)^2 - 5$

3. $(5, -1)$, $x = 3$



$x = \frac{1}{4}(y + 1)^2 + 4$

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8-2 Skills Practice

Parabolas

Write each equation in standard form.

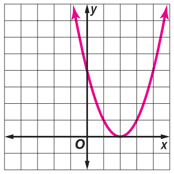
1. $y = x^2 + 2x + 2$
$y = [x - (-1)]^2 + 1$

2. $y = x^2 - 2x + 4$
$y = (x - 1)^2 + 3$

3. $y = x^2 + 4x + 1$
$y = [x - (-2)]^2 + (-3)$

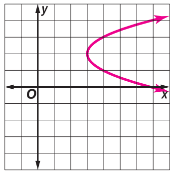
Identify the coordinates of the vertex and focus, the equations of the axis of symmetry and directrix, and the direction of opening of the parabola with the given equation. Then find the length of the latus rectum and graph the parabola.

4. $y = (x - 2)^2$



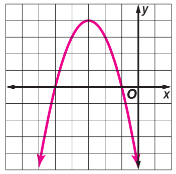
vertex: (2, 0);
focus: $\left(2, \frac{1}{4}\right)$;
axis of symmetry: $x = 2$;
directrix: $y = -\frac{1}{4}$;
opens up;
latus rectum: 1 unit

5. $x = (y - 2)^2 + 3$



vertex: (3, 2);
focus: $\left(3\frac{1}{4}, 2\right)$;
axis of symmetry: $y = 2$;
directrix: $x = 2\frac{3}{4}$;
opens right;
latus rectum: 1 unit

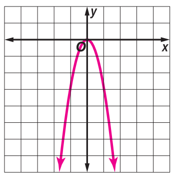
6. $y = -(x + 3)^2 + 4$



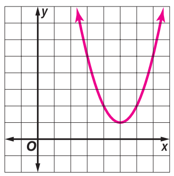
vertex: (−3, 4);
focus: $\left(-3, 3\frac{3}{4}\right)$;
axis of symmetry: $x = -3$;
directrix: $y = 4\frac{1}{4}$;
opens down;
latus rectum: 1 unit

Write an equation for each parabola described below. Then draw the graph.

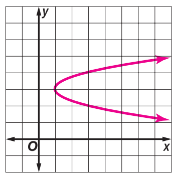
7. vertex (0, 0), focus $\left(0, -\frac{1}{12}\right)$
$y = -3x^2$



8. vertex (5, 1), focus $\left(5, \frac{5}{4}\right)$
$y = (x - 5)^2 + 1$



9. vertex (1, 3), directrix $x = \frac{7}{8}$
$x = 2(y - 3)^2 + 1$



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8-2 Practice (Average)

Parabolas

Write each equation in standard form.

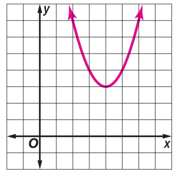
1. $y = 2x^2 - 12x + 19$
$y = 2(x - 3)^2 + 1$

2. $y = \frac{1}{2}x^2 + 3x + \frac{1}{2}$
$y = \frac{1}{2}[x - (-3)]^2 + (-4)$

3. $y = -3x^2 - 12x - 7$
$y = -3[x - (-2)]^2 + 5$

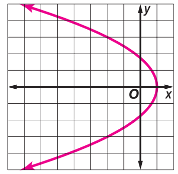
Identify the coordinates of the vertex and focus, the equations of the axis of symmetry and directrix, and the direction of opening of the parabola with the given equation. Then find the length of the latus rectum and graph the parabola.

4. $y = (x - 4)^2 + 3$



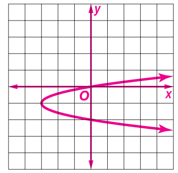
vertex: (4, 3);
focus: $\left(4, 3\frac{1}{4}\right)$;
axis: $x = 4$;
directrix: $y = 2\frac{3}{4}$;
opens up;
latus rectum: 1 unit

5. $x = -\frac{1}{3}y^2 + 1$



vertex: (1, 0);
focus: $\left(\frac{1}{4}, 0\right)$;
axis: $y = 0$;
directrix: $x = 1\frac{3}{4}$;
opens left;
latus rectum: 3 units

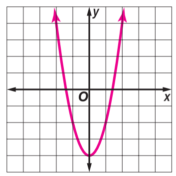
6. $x = 3(y + 1)^2 - 3$



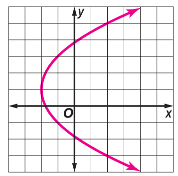
vertex: (−3, −1);
focus: $\left(-2\frac{11}{12}, -1\right)$;
axis: $y = -1$;
directrix: $x = -3\frac{1}{12}$;
opens right;
latus rectum: $\frac{1}{3}$ unit

Write an equation for each parabola described below. Then draw the graph.

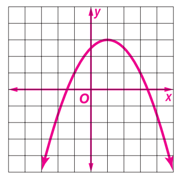
7. vertex (0, −4), focus $\left(0, -3\frac{7}{8}\right)$
$y = 2x^2 - 4$



8. vertex (−2, 1), directrix $x = -3$
$x = \frac{1}{4}(y - 1)^2 - 2$



9. vertex (1, 3), axis of symmetry $x = 1$, latus rectum: 2 units, $a < 0$
$y = -\frac{1}{2}(x - 1)^2 + 3$



10. TELEVISION Write the equation in the form $y = ax^2$ for a satellite dish. Assume that the bottom of the upward-facing dish passes through (0, 0) and that the distance from the bottom to the focus point is 8 inches. $y = \frac{1}{32}x^2$

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8-2 Reading to Learn Mathematics

Parabolas

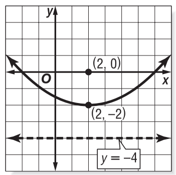
Pre-Activity **How are parabolas used in manufacturing?**

Read the introduction to Lesson 8-2 at the top of page 419 in your textbook.

Name at least two reflective objects that might have the shape of a parabola.

Sample answer: telescope mirror, satellite dish

Reading the Lesson



1. In the parabola shown in the graph, the point $(2, -2)$ is called the **vertex** and the point $(2, 0)$ is called the **focus**. The line $y = -4$ is called the **directrix**, and the line $x = 2$ is called the **axis of symmetry**.

2. a. Write the standard form of the equation of a parabola that opens upward or downward. $y = a(x - h)^2 + k$

b. The parabola opens downward if $a < 0$ and opens upward if $a > 0$. The equation of the axis of symmetry is $x = h$, and the coordinates of the vertex are (h, k).

3. A parabola has equation $x = -\frac{1}{8}(y - 2)^2 + 4$. This parabola opens to the **left**. It has vertex $(4, 2)$ and focus $(2, 2)$. The directrix is $x = 6$. The length of the latus rectum is **8** units.

Helping You Remember

4. How can the way in which you plot points in a rectangular coordinate system help you to remember what the sign of a tells you about the direction in which a parabola opens?

Sample answer: In plotting points, a positive *x*-coordinate tells you to move to the *right* and a negative *x*-coordinate tells you to move to the *left*. This is like a parabola whose equation is of the form "*x* = …"; it opens to the *right* if $a > 0$ and to the *left* if $a < 0$. Likewise, a positive *y*-coordinate tells you to move *up* and a negative *y*-coordinate tells you to move *down*. This is like a parabola whose equation is of the form "*y* = …"; it opens *upward* if $a > 0$ and *downward* if $a < 0$.

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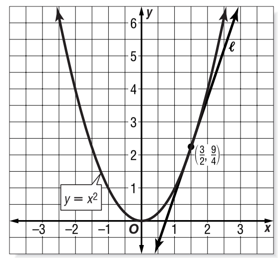
8-2 Enrichment

Tangents to Parabolas



A line that intersects a parabola in exactly one point without crossing the curve is a **tangent** to the parabola. The point where a tangent line touches a parabola is the **point of tangency.** The line perpendicular to a tangent to a parabola at the point of tangency is called the **normal** to the parabola at that point. In the diagram, line ℓ is tangent to the parabola that is the graph of $y = x^2$ at $\left(\frac{3}{2}, \frac{9}{4}\right)$. The x-axis is tangent to the parabola at O, and the y-axis is the normal to the parabola at O.

Solve each problem.

1. Find an equation for line ℓ in the diagram. *Hint:* A nonvertical line with an equation of the form $y = mx + b$ will be tangent to the graph of $y = x^2$ at $\left(\frac{3}{2}, \frac{9}{4}\right)$ if and only if $\left(\frac{3}{2}, \frac{9}{4}\right)$ is the only pair of numbers that satisfies both $y = x^2$ and $y = mx + b$.

$m = 3, b = -\frac{9}{4}, y = 3x - \frac{9}{4}$

2. If a is any real number, then (a, a^2) belongs to the graph of $y = x^2$. Express m and b in terms of a to find an equation of the form $y = mx + b$ for the line that is tangent to the graph of $y = x^2$ at (a, a^2).

$m = 2a, b = a^2, y = (2a)x + (-a^2)$ or $y = 2ax - a^2$

3. Find an equation for the normal to the graph of $y = x^2$ at $\left(\frac{3}{2}, \frac{9}{4}\right)$.

$y = -\frac{1}{3}x + \frac{11}{4}$

4. If a is a nonzero real number, find an equation for the normal to the graph of $y = x^2$ at (a, a^2).

$y = \left(-\frac{1}{2a}\right)x + \left(a^2 + \frac{1}{2}\right)$

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8-3 Study Guide and Intervention

Circles

Equations of Circles The equation of a circle with center (h, k) and radius r units is $(x - h)^2 + (y - k)^2 = r^2$.

Example **Write an equation for a circle if the endpoints of a diameter are at (−4, 5) and (6, −3).**

Use the midpoint formula to find the center of the circle.

$(h, k) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ Midpoint formula

$= \left(\frac{-4 + 6}{2}, \frac{5 + (-3)}{2}\right)$ $(x_1, y_1) = (-4, 5), (x_2, y_2) = (6, -3)$

$= \left(\frac{2}{2}, \frac{2}{2}\right)$ or $(1, 1)$ Simplify.

Use the coordinates of the center and one endpoint of the diameter to find the radius.

$r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Distance formula

$r = \sqrt{(-4 - 1)^2 + (5 - 1)^2}$ $(x_1, y_1) = (1, 1), (x_2, y_2) = (-4, 5)$

$= \sqrt{(-5)^2 + 4^2} = \sqrt{41}$ Simplify.

The radius of the circle is $\sqrt{41}$, so $r^2 = 41$.

An equation of the circle is $(x - 1)^2 + (y - 1)^2 = 41$.

Exercises

Write an equation for the circle that satisfies each set of conditions.

1. center $(8, -3)$, radius 6 **$(x - 8)^2 + (y + 3)^2 = 36$**

2. center $(5, -6)$, radius 4 **$(x - 5)^2 + (y + 6)^2 = 16$**

3. center $(-5, 2)$, passes through $(-9, 6)$ **$(x + 5)^2 + (y - 2)^2 = 32$**

4. endpoints of a diameter at $(6, 6)$ and $(10, 12)$ **$(x - 8)^2 + (y - 9)^2 = 13$**

5. center $(3, 6)$, tangent to the x-axis **$(x - 3)^2 + (y - 6)^2 = 36$**

6. center $(-4, -7)$, tangent to $x = 2$ **$(x + 4)^2 + (y + 7)^2 = 36$**

7. center at $(-2, 8)$, tangent to $y = -4$ **$(x + 2)^2 + (y - 8)^2 = 144$**

8. center $(7, 7)$, passes through $(12, 9)$ **$(x - 7)^2 + (y - 7)^2 = 29$**

9. endpoints of a diameter are $(-4, -2)$ and $(8, 4)$ **$(x - 2)^2 + (y - 1)^2 = 45$**

10. endpoints of a diameter are $(-4, 3)$ and $(6, -8)$ **$(x - 1)^2 + (y + 2.5)^2 = 55.25$**

Lesson 8-3

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8-3 Study Guide and Intervention *(continued)*

Circles

Graph Circles To graph a circle, write the given equation in the standard form of the equation of a circle, $(x - h)^2 + (y - k)^2 = r^2$.

Plot the center (h, k) of the circle. Then use r to calculate and plot the four points $(h + r, k)$, $(h - r, k)$, $(h, k + r)$, and $(h, k - r)$, which are all points on the circle. Sketch the circle that goes through those four points.

Example **Find the center and radius of the circle whose equation is $x^2 + 2x + y^2 + 4y = 11$. Then graph the circle.**

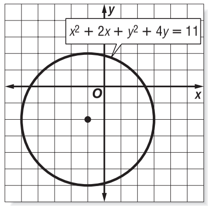
$x^2 + 2x + y^2 + 4y = 11$

$x^2 + 2x + ■ + y^2 + 4y + ■ = 11 + ■$

$x^2 + 2x + 1 + y^2 + 4y + 4 = 11 + 1 + 4$

$(x + 1)^2 + (y + 2)^2 = 16$

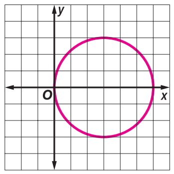
Therefore, the circle has its center at $(-1, -2)$ and a radius of $\sqrt{16} = 4$. Four points on the circle are $(3, -2)$, $(-5, -2)$, $(-1, 2)$, and $(-1, -6)$.



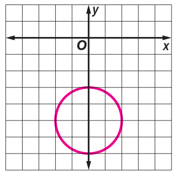
Exercises

Find the center and radius of the circle with the given equation. Then graph the circle.

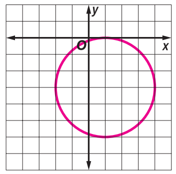
1. $(x - 3)^2 + y^2 = 9$
(3, 0), $r = 3$



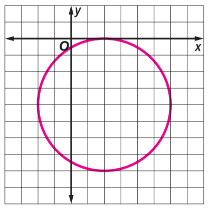
2. $x^2 + (y + 5)^2 = 4$
(0, −5), $r = 2$



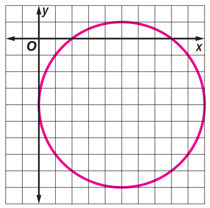
3. $(x - 1)^2 + (y + 3)^2 = 9$
(1, −3), $r = 3$



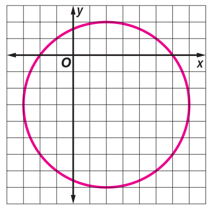
4. $(x - 2)^2 + (y + 4)^2 = 16$
(2, −4), $r = 4$



5. $x^2 + y^2 - 10x + 8y + 16 = 0$
(5, −4), $r = 5$



6. $x^2 + y^2 - 4x + 6y = 12$
(2, −3), $r = 5$



NAME ______________________ DATE __________ PERIOD _____

8-3 Skills Practice

Circles

Write an equation for the circle that satisfies each set of conditions.

1. center (0, 5), radius 1 unit
$x^2 + (y - 5)^2 = 1$

2. center (5, 12), radius 8 units
$(x - 5)^2 + (y - 12)^2 = 64$

3. center (4, 0), radius 2 units
$(x - 4)^2 + y^2 = 4$

4. center (2, 2), radius 3 units
$(x - 2)^2 + (y - 2)^2 = 9$

5. center (4, −4), radius 4 units
$(x - 4)^2 + (y + 4)^2 = 16$

6. center (−6, 4), radius 5 units
$(x + 6)^2 + (y - 4)^2 = 25$

7. endpoints of a diameter at (−12, 0) and (12, 0) $x^2 + y^2 = 144$

8. endpoints of a diameter at (−4, 0) and (−4, −6) $(x + 4)^2 + (y + 3)^2 = 9$

9. center at (7, −3), passes through the origin $(x - 7)^2 + (y + 3)^2 = 58$

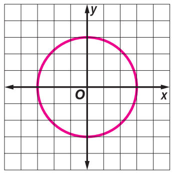
10. center at (−4, 4), passes through (−4, 1) $(x + 4)^2 + (y - 4)^2 = 9$

11. center at (−6, −5), tangent to y-axis $(x + 6)^2 + (y + 5)^2 = 36$

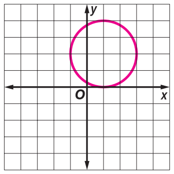
12. center at (5, 1), tangent to x-axis $(x - 5)^2 + (y - 1)^2 = 1$

Find the center and radius of the circle with the given equation. Then graph the circle.

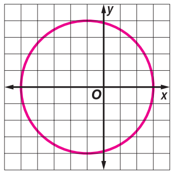
13. $x^2 + y^2 = 9$
(0, 0), 3 units



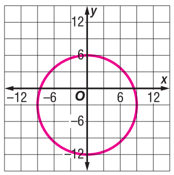
14. $(x - 1)^2 + (y - 2)^2 = 4$
(1, 2), 2 units



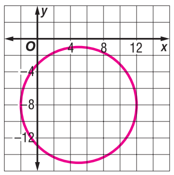
15. $(x + 1)^2 + y^2 = 16$
(−1, 0), 4 units



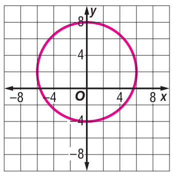
16. $x^2 + (y + 3)^2 = 81$
(0, −3), 9 units



17. $(x - 5)^2 + (y + 8)^2 = 49$
(5, −8), 7 units



18. $x^2 + y^2 - 4y - 32 = 0$
(0, 2), 6 units



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8-3 Practice (Average)

Circles

Write an equation for the circle that satisfies each set of conditions.

1. center (−4, 2), radius 8 units
$(x + 4)^2 + (y - 2)^2 = 64$

2. center (0, 0), radius 4 units
$x^2 + y^2 = 16$

3. center $\left(-\frac{1}{4}, -\sqrt{3}\right)$, radius $5\sqrt{2}$ units
$\left(x + \frac{1}{4}\right)^2 + (y + \sqrt{3})^2 = 50$

4. center (2.5, 4.2), radius 0.9 unit
$(x - 2.5)^2 + (y - 4.2)^2 = 0.81$

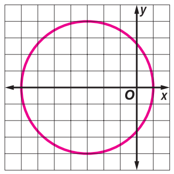
5. endpoints of a diameter at (−2, −9) and (0, −5) $(x + 1)^2 + (y + 7)^2 = 5$

6. center at (−9, −12), passes through (−4, −5) $(x + 9)^2 + (y + 12)^2 = 74$

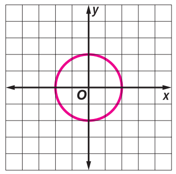
7. center at (−6, 5), tangent to x-axis $(x + 6)^2 + (y - 5)^2 = 25$

Find the center and radius of the circle with the given equation. Then graph the circle.

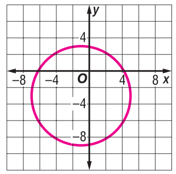
8. $(x + 3)^2 + y^2 = 16$
(−3, 0), 4 units



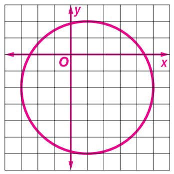
9. $3x^2 + 3y^2 = 12$
(0, 0), 2 units



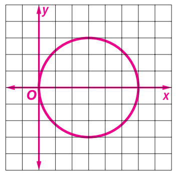
10. $x^2 + y^2 + 2x + 6y = 26$
(−1, −3), 6 units



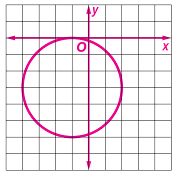
11. $(x - 1)^2 + y^2 + 4y = 12$
(1, −2), 4 units



12. $x^2 - 6x + y^2 = 0$
(3, 0), 3 units



13. $x^2 + y^2 + 2x + 6y = -1$
(−1, −3), 3 units



WEATHER For Exercises 14 and 15, use the following information.

On average, the circular eye of a hurricane is about 15 miles in diameter. Gale winds can affect an area up to 300 miles from the storm's center. In 1992, Hurricane Andrew devastated southern Florida. A satellite photo of Andrew's landfall showed the center of its eye on one coordinate system could be approximated by the point (80, 26).

14. Write an equation to represent a possible boundary of Andrew's eye.
$(x - 80)^2 + (y - 26)^2 = 56.25$

15. Write an equation to represent a possible boundary of the area affected by gale winds.
$(x - 80)^2 + (y - 26)^2 = 90{,}000$

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8-3 Reading to Learn Mathematics

Circles

Pre-Activity **Why are circles important in air traffic control?**

Read the introduction to Lesson 8-3 at the top of page 426 in your textbook.

A large home improvement chain is planning to enter a new metropolitan area and needs to select locations for its stores. Market research has shown that potential customers are willing to travel up to 12 miles to shop at one of their stores. How can circles help the managers decide where to place their store?

Sample answer: A store will draw customers who live inside a circle with center at the store and a radius of 12 miles. The management should select locations for which as many people as possible live within a circle of radius 12 miles around one of the stores.

Reading the Lesson

1. a. Write the equation of the circle with center (h, k) and radius r.
$(x - h)^2 + (y - k)^2 = r^2$

b. Write the equation of the circle with center $(4, -3)$ and radius 5.
$(x - 4)^2 + (y + 3)^2 = 25$

c. The circle with equation $(x + 8)^2 + y^2 = 121$ has center **(−8, 0)** and radius **11**.

d. The circle with equation $(x - 10)^2 + (y + 10)^2 = 1$ has center **(10, −10)** and radius **1**.

2. a. In order to find center and radius of the circle with equation $x^2 + y^2 + 4x - 6y - 3 = 0$, it is necessary to **complete the square**. Fill in the missing parts of this process.

$$x^2 + y^2 + 4x - 6y - 3 = 0$$
$$x^2 + y^2 + 4x - 6y = \underline{3}$$
$$x^2 + 4x + \underline{4} + y^2 - 6y + \underline{9} = \underline{3} + \underline{4} + \underline{9}$$
$$(x + \underline{2})^2 + (y - \underline{3})^2 = \underline{16}$$

b. This circle has radius 4 and center at **(−2, 3)**.

Helping You Remember

3. How can the distance formula help you to remember the equation of a circle?
Sample answer: Write the distance formula. Replace (x_1, y_1) with (h, k) and (x_2, y_2) with (x, y). Replace d with r. Square both sides. Now you have the equation of a circle.

Lesson 8-3

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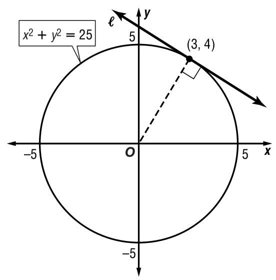
8-3 Enrichment

Tangents to Circles

A line that intersects a circle in exactly one point is a **tangent** to the circle. In the diagram, line ℓ is tangent to the circle with equation $x^2 + y^2 = 25$ at the point whose coordinates are (3, 4).

A line is tangent to a circle at a point P on the circle if and only if the line is perpendicular to the radius from the center of the circle to point P. This fact enables you to find an equation of the tangent to a circle at a point P if you know an equation for the circle and the coordinates of P.



Use the diagram above to solve each problem.

1. What is the slope of the radius to the point with coordinates (3, 4)? What is the slope of the tangent to that point?
$\frac{4}{3}, -\frac{3}{4}$

2. Find an equation of the line ℓ that is tangent to the circle at (3, 4).
$y = -\frac{3}{4}x + \frac{25}{4}$

3. If k is a real number between −5 and 5, how many points on the circle have x-coordinate k? State the coordinates of these points in terms of k.
two, $\left(k, \pm\sqrt{25 - k^2}\right)$

4. Describe how you can find equations for the tangents to the points you named for Exercise 3.
Use the coordinates of (0, 0) and of one of the given points. Find the slope of the radius to that point. Use the slope of the radius to find what the slope of the tangent must be. Use the slope of the tangent and the coordinates of the point on the circle to find an equation for the tangent.

5. Find an equation for the tangent at (−3, 4).
$y = \frac{3}{4}x + \frac{25}{x}$

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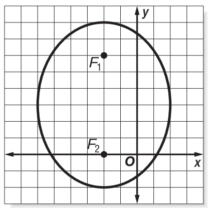
8-4 Study Guide and Intervention

Ellipses

Equations of Ellipses An **ellipse** is the set of all points in a plane such that the *sum* of the distances from two given points in the plane, called the foci, is constant. An ellipse has two axes of symmetry which contain the **major** and **minor axes**. In the table, the lengths a, b, and c are related by the formula $c^2 = a^2 - b^2$.

Standard Form of Equation	$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$	$\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1$
Center	(h, k)	(h, k)
Direction of Major Axis	Horizontal	Vertical
Foci	$(h + c, k), (h - c, k)$	$(h, k - c), (h, k + c)$
Length of Major Axis	$2a$ units	$2a$ units
Length of Minor Axis	$2b$ units	$2b$ units

Example **Write an equation for the ellipse shown.**



The length of the major axis is the distance between $(-2, -2)$ and $(-2, 8)$. This distance is 10 units.

$2a = 10$, so $a = 5$

The foci are located at $(-2, 6)$ and $(-2, 0)$, so $c = 3$.

$b^2 = a^2 - c^2$
$= 25 - 9$
$= 16$

The center of the ellipse is at $(-2, 3)$, so $h = -2$, $k = 3$, $a^2 = 25$, and $b^2 = 16$. The major axis is vertical.

An equation of the ellipse is $\frac{(y-3)^2}{25} + \frac{(x+2)^2}{16} = 1$.

Exercises

Write an equation for the ellipse that satisfies each set of conditions.

1. endpoints of major axis at $(-7, 2)$ and $(5, 2)$, endpoints of minor axis at $(-1, 0)$ and $(-1, 4)$
   $\frac{(x+1)^2}{36} + \frac{(y-2)^2}{4} = 1$

2. major axis 8 units long and parallel to the x-axis, minor axis 2 units long, center at $(-2, -5)$
   $\frac{(x+2)^2}{16} + (y+5)^2 = 1$

3. endpoints of major axis at $(-8, 4)$ and $(4, 4)$, foci at $(-3, 4)$ and $(-1, 4)$
   $\frac{(x+2)^2}{36} + \frac{(y-4)^2}{35} = 1$

4. endpoints of major axis at $(3, 2)$ and $(3, -14)$, endpoints of minor axis at $(-1, -6)$ and $(7, -6)$
   $\frac{(y+6)^2}{64} + \frac{(x-3)^2}{16} = 1$

5. minor axis 6 units long and parallel to the x-axis, major axis 12 units long, center at $(6, 1)$
   $\frac{(y-1)^2}{36} + \frac{(x-6)^2}{9} = 1$

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8-4 Study Guide and Intervention *(continued)*

Ellipses

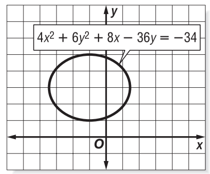
Graph Ellipses To graph an ellipse, if necessary, write the given equation in the standard form of an equation for an ellipse.

$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ (for ellipse with major axis horizontal) or

$\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1$ (for ellipse with major axis vertical)

Use the center (h, k) and the endpoints of the axes to plot four points of the ellipse. To make a more accurate graph, use a calculator to find some approximate values for x and y that satisfy the equation.

Example **Graph the ellipse $4x^2 + 6y^2 + 8x - 36y = -34$.**



$4x^2 + 6y^2 + 8x - 36y = -34$
$4x^2 + 8x + 6y^2 - 36y = -34$
$4(x^2 + 2x + ■) + 6(y^2 - 6y + ■) = -34 + ■$
$4(x^2 + 2x + 1) + 6(y^2 - 6y + 9) = -34 + 58$
$4(x + 1)^2 + 6(y - 3)^2 = 24$
$\frac{(x+1)^2}{6} + \frac{(y-3)^2}{4} = 1$

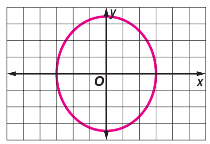
The center of the ellipse is $(-1, 3)$. Since $a^2 = 6$, $a = \sqrt{6}$.
Since $b^2 = 4$, $b = 2$.

The length of the major axis is $2\sqrt{6}$, and the length of the minor axis is 4. Since the x-term has the greater denominator, the major axis is horizontal. Plot the endpoints of the axes. Then graph the ellipse.

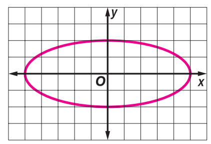
Exercises

Find the coordinates of the center and the lengths of the major and minor axes for the ellipse with the given equation. Then graph the ellipse.

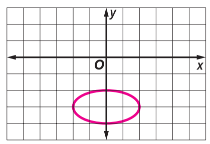
1. $\frac{y^2}{12} + \frac{x^2}{9} = 1$ **(0, 0), $4\sqrt{3}$, 6**



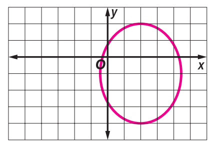
2. $\frac{x^2}{25} + \frac{y^2}{4} = 1$ **(0, 0), 10, 4**



3. $x^2 + 4y^2 + 24y = -32$ **(0, −3), 4, 2**



4. $9x^2 + 6y^2 - 36x + 12y = 12$ **(2, −1), 6, $2\sqrt{6}$**



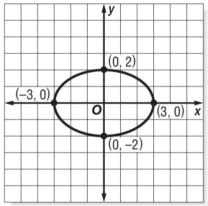
NAME ______________________ DATE __________ PERIOD _____

8-4 Skills Practice

Ellipses

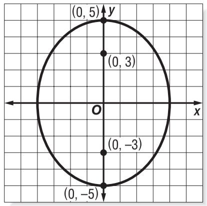
Write an equation for each ellipse.

1.



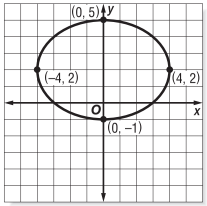
$\frac{x^2}{9} + \frac{y^2}{4} = 1$

2.



$\frac{y^2}{25} + \frac{x^2}{16} = 1$

3.



$\frac{x^2}{16} + \frac{(y-2)^2}{9} = 1$

Write an equation for the ellipse that satisfies each set of conditions.

4. endpoints of major axis at $(0, 6)$ and $(0, -6)$, endpoints of minor axis at $(-3, 0)$ and $(3, 0)$
$\frac{y^2}{36} + \frac{x^2}{9} = 1$

5. endpoints of major axis at $(2, 6)$ and $(8, 6)$, endpoints of minor axis at $(5, 4)$ and $(5, 8)$
$\frac{(x-5)^2}{9} + \frac{(y-6)^2}{4} = 1$

6. endpoints of major axis at $(7, 3)$ and $(7, 9)$, endpoints of minor axis at $(5, 6)$ and $(9, 6)$
$\frac{(y-6)^2}{9} + \frac{(x-7)^2}{4} = 1$

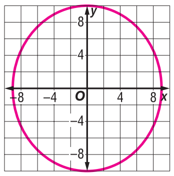
7. major axis 12 units long and parallel to x-axis, minor axis 4 units long, center at $(0, 0)$
$\frac{x^2}{36} + \frac{y^2}{4} = 1$

8. endpoints of major axis at $(-6, 0)$ and $(6, 0)$, foci at $(-\sqrt{32}, 0)$ and $(\sqrt{32}, 0)$
$\frac{x^2}{36} + \frac{y^2}{4} = 1$

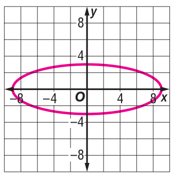
9. endpoints of major axis at $(0, 12)$ and $(0, -12)$, foci at $(0, \sqrt{23})$ and $(0, -\sqrt{23})$
$\frac{y^2}{144} + \frac{x^2}{121} = 1$

Find the coordinates of the center and foci and the lengths of the major and minor axes for the ellipse with the given equation. Then graph the ellipse.

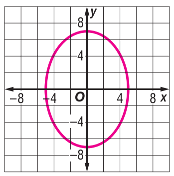
10. $\frac{y^2}{100} + \frac{x^2}{81} = 1$
$(0, 0)$; $(0, \pm\sqrt{19})$; 20; 18



11. $\frac{x^2}{81} + \frac{y^2}{9} = 1$
$(0, 0)$; $(\pm 6\sqrt{2}, 0)$; 18; 6



12. $\frac{y^2}{49} + \frac{x^2}{25} = 1$
$(0, 0)$, $(0, \pm 2\sqrt{6})$; 14; 10



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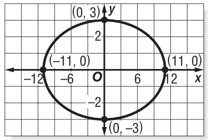
8-4 Practice (Average)

Ellipses

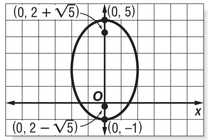
Write an equation for each ellipse.

1.



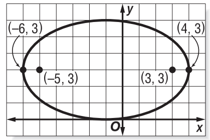
$\frac{x^2}{121} + \frac{y^2}{9} = 1$

2.



$\frac{(y-2)^2}{9} + \frac{x^2}{4} = 1$

3.



$\frac{(x+1)^2}{25} + \frac{(y-3)^2}{9} = 1$

Write an equation for the ellipse that satisfies each set of conditions.

4. endpoints of major axis at $(-9, 0)$ and $(9, 0)$, endpoints of minor axis at $(0, 3)$ and $(0, -3)$
$\frac{x^2}{81} + \frac{y^2}{9} = 1$

5. endpoints of major axis at $(4, 2)$ and $(4, -8)$, endpoints of minor axis at $(1, -3)$ and $(7, -3)$
$\frac{(y+3)^2}{25} + \frac{(x-4)^2}{9} = 1$

6. major axis 20 units long and parallel to x-axis, minor axis 10 units long, center at $(2, 1)$
$\frac{(x-2)^2}{100} + \frac{(y-1)^2}{25} = 1$

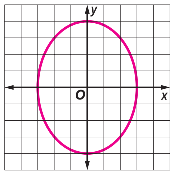
7. major axis 10 units long, minor axis 6 units long and parallel to x-axis, center at $(2, -4)$
$\frac{(y+4)^2}{25} + \frac{(x-2)^2}{9} = 1$

8. major axis 16 units long, center at $(0, 0)$, foci at $(0, 2\sqrt{15})$ and $(0, -2\sqrt{15})$
$\frac{y^2}{64} + \frac{x^2}{4} = 1$

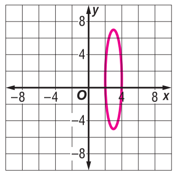
9. endpoints of minor axis at $(0, 2)$ and $(0, -2)$, foci at $(-4, 0)$ and $(4, 0)$
$\frac{x^2}{20} + \frac{y^2}{4} = 1$

Find the coordinates of the center and foci and the lengths of the major and minor axes for the ellipse with the given equation. Then graph the ellipse.

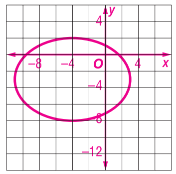
10. $\frac{y^2}{16} + \frac{x^2}{9} = 1$
$(0, 0)$; $(0, \pm\sqrt{7})$; 8; 6



11. $\frac{(y-1)^2}{36} + \frac{(x-3)^2}{1} = 1$
$(3, 1)$; $(3, 1 \pm \sqrt{35})$; 12; 2



12. $\frac{(x+4)^2}{49} + \frac{(y+3)^2}{25} = 1$
$(-4, -3)$; $(-4 \pm 2\sqrt{6}, -3)$; 14; 10



13. SPORTS An ice skater traces two congruent ellipses to form a figure eight. Assume that the center of the first loop is at the origin, with the second loop to its right. Write an equation to model the first loop if its major axis (along the x-axis) is 12 feet long and its minor axis is 6 feet long. Write another equation to model the second loop.
$\frac{x^2}{36} + \frac{y^2}{9} = 1$; $\frac{(x-12)^2}{36} + \frac{y^2}{9} = 1$

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8-4 Reading to Learn Mathematics

Ellipses

Pre-Activity **Why are ellipses important in the study of the solar system?**

Read the introduction to Lesson 8-4 at the top of page 433 in your textbook.

Is the Earth always the same distance from the Sun? Explain your answer using the words *circle* and *ellipse*. **No; if the Earth's orbit were a circle, it would always be the same distance from the Sun because every point on a circle is the same distance from the center. However, the Earth's orbit is an ellipse, and the points on an ellipse are not all the same distance from the center.**

Reading the Lesson

1. An ellipse is the set of all points in a plane such that the **sum** of the distances from two fixed points is **constant**. The two fixed points are called the **foci** of the ellipse.

2. Consider the ellipse with equation $\frac{x^2}{9} + \frac{y^2}{4} = 1$.

a. For this equation, $a =$ **3** and $b =$ **2**.

b. Write an equation that relates the values of a, b, and c. $c^2 = a^2 - b^2$

c. Find the value of c for this ellipse. $\sqrt{5}$

3. Consider the ellipses with equations $\frac{y^2}{25} + \frac{x^2}{16} = 1$ and $\frac{x^2}{9} + \frac{y^2}{4} = 1$. Complete the following table to describe characteristics of their graphs.

Standard Form of Equation	$\frac{y^2}{25} + \frac{x^2}{16} = 1$	$\frac{x^2}{9} + \frac{y^2}{4} = 1$
Direction of Major Axis	vertical	horizontal
Direction of Minor Axis	horizontal	vertical
Foci	(0, 3), (0, −3)	$(\sqrt{5}, 0), (-\sqrt{5}, 0)$
Length of Major Axis	10 units	6 units
Length of Minor Axis	8 units	4 units

Helping You Remember

4. Some students have trouble remembering the two standard forms for the equation of an ellipse. How can you remember which term comes first and where to place a and b in these equations? **The *x*-axis is horizontal. If the major axis is horizontal, the first term is $\frac{x^2}{a^2}$. The *y*-axis is vertical. If the major axis is vertical, the first term is $\frac{y^2}{a^2}$. *a* is always the larger of the numbers *a* and *b*.**

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8-4 Enrichment

Eccentricity

In an ellipse, the ratio $\frac{c}{d}$ is called the **eccentricity** and is denoted by the letter e. Eccentricity measures the elongation of an ellipse. The closer e is to 0, the more an ellipse looks like a circle. The closer e is to 1, the more elongated it is. Recall that the equation of an ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ or $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$ where a is the length of the major axis, and that $c = \sqrt{a^2 - b^2}$.

Find the eccentricity of each ellipse rounded to the nearest hundredth.

1. $\frac{x^2}{9} + \frac{y^2}{36} = 1$ **0.87**

2. $\frac{x^2}{81} + \frac{y^2}{9} = 1$ **0.94**

3. $\frac{x^2}{4} + \frac{y^2}{9} = 1$ **0.75**

4. $\frac{x^2}{16} + \frac{y^2}{9} = 1$ **0.66**

5. $\frac{x^2}{36} + \frac{y^2}{16} = 1$ **0.75**

6. $\frac{x^2}{4} + \frac{y^2}{36} = 1$ **0.94**

7. Is a circle an ellipse? Explain your reasoning.

Yes; it is an ellipse with eccentricity 0.

8. The center of the sun is one focus of Earth's orbit around the sun. The length of the major axis is 186,000,000 miles, and the foci are 3,200,000 miles apart. Find the eccentricity of Earth's orbit.

approximately 0.17

9. An artificial satellite orbiting the earth travels at an altitude that varies between 132 miles and 583 miles above the surface of the earth. If the center of the earth is one focus of its elliptical orbit and the radius of the earth is 3950 miles, what is the eccentricity of the orbit?

approximately 0.052

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8-5 Study Guide and Intervention

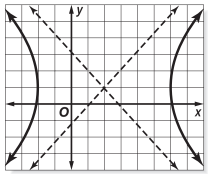
Hyperbolas

Equations of Hyperbolas A **hyperbola** is the set of all points in a plane such that the absolute value of the *difference* of the distances from any point on the hyperbola to any two given points in the plane, called the **foci**, is constant.

In the table, the lengths a, b, and c are related by the formula $c^2 = a^2 + b^2$.

Standard Form of Equation	$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$	$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$
Equations of the Asymptotes	$y - k = \pm\frac{b}{a}(x-h)$	$y - k = \pm\frac{a}{b}(x-h)$
Transverse Axis	Horizontal	Vertical
Foci	$(h-c, k), (h+c, k)$	$(h, k-c), (h, k+c)$
Vertices	$(h-a, k), (h+a, k)$	$(h, k-a), (h, k+a)$

Example **Write an equation for the hyperbola with vertices (−2, 1) and (6, 1) and foci (−4, 1) and (8, 1).**



Use a sketch to orient the hyperbola correctly. The center of the hyperbola is the midpoint of the segment joining the two vertices. The center is $\left(\frac{-2+6}{2}, 1\right)$, or (2, 1). The value of a is the distance from the center to a vertex, so $a = 4$. The value of c is the distance from the center to a focus, so $c = 6$.

$c^2 = a^2 + b^2$
$6^2 = 4^2 + b^2$
$b^2 = 36 - 16 = 20$

Use h, k, a^2, and b^2 to write an equation of the hyperbola.

$\frac{(x-2)^2}{16} - \frac{(y-1)^2}{20} = 1$

Exercises

Write an equation for the hyperbola that satisfies each set of conditions.

1. vertices (−7, 0) and (7, 0), conjugate axis of length 10 $\frac{x^2}{49} - \frac{y^2}{25} = 1$

2. vertices (−2, −3) and (4, −3), foci (−5, −3) and (7, −3) $\frac{(x-1)^2}{9} - \frac{(y+3)^2}{27} = 1$

3. vertices (4, 3) and (4, −5), conjugate axis of length 4 $\frac{(y+1)^2}{16} - \frac{(x-4)^2}{4} = 1$

4. vertices (−8, 0) and (8, 0), equation of asymptotes $y = \pm\frac{1}{6}x$ $\frac{x^2}{64} - \frac{9y^2}{16} = 1$

5. vertices (−4, 6) and (−4, −2), foci (−4, 10) and (−4, −6) $\frac{(y-2)^2}{16} - \frac{(x+4)^2}{48} = 1$

Lesson 8-5

NAME ______________________ DATE __________ PERIOD _____

8-5 Study Guide and Intervention *(continued)*

Hyperbolas

Graph Hyperbolas To graph a hyperbola, write the given equation in the standard form of an equation for a hyperbola

$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ if the branches of the hyperbola open left and right, or

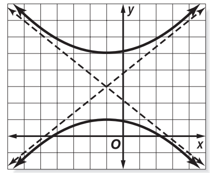
$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$ if the branches of the hyperbola open up and down

Graph the point (h, k), which is the center of the hyperbola. Draw a rectangle with dimensions $2a$ and $2b$ and center (h, k). If the hyperbola opens left and right, the vertices are $(h-a, k)$ and $(h+a, k)$. If the hyperbola opens up and down, the vertices are $(h, k-a)$ and $(h, k+a)$.

Example **Draw the graph of $6y^2 - 4x^2 - 36y - 8x = -26$.**

Complete the squares to get the equation in standard form.

$6y^2 - 4x^2 - 36y - 8x = -26$
$6(y^2 - 6y + ■) - 4(x^2 + 2x + ■) = -26 + ■$
$6(y^2 - 6y + 9) - 4(x^2 + 2x + 1) = -26 + 50$
$6(y-3)^2 - 4(x+1)^2 = 24$
$\frac{(y-3)^2}{4} - \frac{(x+1)^2}{6} = 1$



The center of the hyperbola is (−1, 3).

According to the equation, $a^2 = 4$ and $b^2 = 6$, so $a = 2$ and $b = \sqrt{6}$.

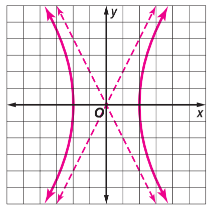
The transverse axis is vertical, so the vertices are (−1, 5) and (−1, 1). Draw a rectangle with vertical dimension 4 and horizontal dimension $2\sqrt{6} \approx 4.9$. The diagonals of this rectangle are the asymptotes. The branches of the hyperbola open up and down. Use the vertices and the asymptotes to sketch the hyperbola.

Exercises

Find the coordinates of the vertices and foci and the equations of the asymptotes for the hyperbola with the given equation. Then graph the hyperbola.

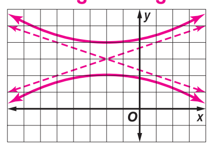
1. $\frac{x^2}{4} - \frac{y^2}{16} = 1$

(2, 0), (−2, 0); $(2\sqrt{5}, 0), (-2\sqrt{5}, 0)$; $y = \pm 2x$



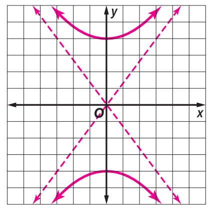
2. $(y-3)^2 - \frac{(x+2)^2}{9} = 1$

(−2, 4), (−2, 2); $(-2, 3 + \sqrt{10})$, $(-2, 3 - \sqrt{10})$; $y = \frac{1}{3}x + 3\frac{2}{3}$, $y = -\frac{1}{3}x + 2\frac{1}{3}$



3. $\frac{y^2}{16} - \frac{x^2}{9} = 1$

(0, 4), (0, −4); (0, 5), (0, −5); $y = \pm\frac{4}{3}x$



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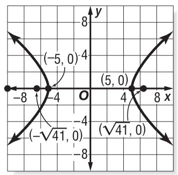
8-5 Skills Practice

Hyperbolas

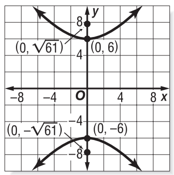
Write an equation for each hyperbola.

1.



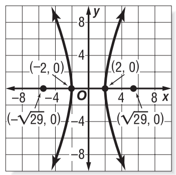
$\frac{x^2}{25} - \frac{y^2}{16} = 1$

2.



$\frac{y^2}{36} - \frac{x^2}{25} = 1$

3.



$\frac{x^2}{4} - \frac{y^2}{25} = 1$

Write an equation for the hyperbola that satisfies each set of conditions.

4. vertices $(-4, 0)$ and $(4, 0)$, conjugate axis of length 8 $\frac{x^2}{16} - \frac{y^2}{16} = 1$

5. vertices $(0, 6)$ and $(0, -6)$, conjugate axis of length 14 $\frac{y^2}{36} - \frac{x^2}{49} = 1$

6. vertices $(0, 3)$ and $(0, -3)$, conjugate axis of length 10 $\frac{y^2}{9} - \frac{x^2}{25} = 1$

7. vertices $(-2, 0)$ and $(2, 0)$, conjugate axis of length 4 $\frac{x^2}{4} - \frac{y^2}{4} = 1$

8. vertices $(-3, 0)$ and $(3, 0)$, foci $(\pm 5, 0)$ $\frac{x^2}{9} - \frac{y^2}{16} = 1$

9. vertices $(0, 2)$ and $(0, -2)$, foci $(0, \pm 3)$ $\frac{y^2}{4} - \frac{x^2}{5} = 1$

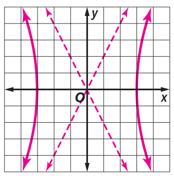
10. vertices $(0, -2)$ and $(6, -2)$, foci $(3 \pm \sqrt{13}, -2)$ $\frac{(x-3)^2}{9} - \frac{(y+2)^2}{4} = 1$

Find the coordinates of the vertices and foci and the equations of the asymptotes for the hyperbola with the given equation. Then graph the hyperbola.

11. $\frac{x^2}{9} - \frac{y^2}{36} = 1$

$(\pm 3, 0)$; $(\pm 3\sqrt{5}, 0)$;

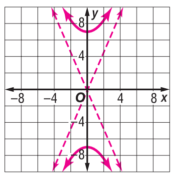
$y = \pm 2x$



12. $\frac{y^2}{49} - \frac{x^2}{9} = 1$

$(0, \pm 7)$; $(0, \pm\sqrt{58})$;

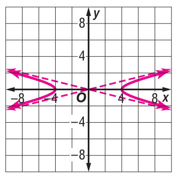
$y = \pm\frac{7}{3}x$



13. $\frac{x^2}{16} - \frac{y^2}{1} = 1$

$(\pm 4, 0)$; $(\pm\sqrt{17}, 0)$;

$y = \pm\frac{1}{4}x$



Lesson 8-5

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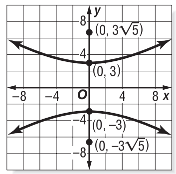
8-5 Practice (Average)

Hyperbolas

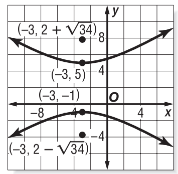
Write an equation for each hyperbola.

1.



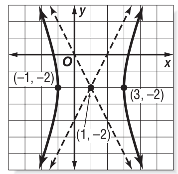
$\frac{y^2}{9} - \frac{x^2}{36} = 1$

2.



$\frac{(y-2)^2}{9} - \frac{(x+3)^2}{25} = 1$

3.



$\frac{(x-1)^2}{4} - \frac{(y+2)^2}{16} = 1$

Write an equation for the hyperbola that satisfies each set of conditions.

4. vertices $(0, 7)$ and $(0, -7)$, conjugate axis of length 18 units $\frac{y^2}{49} - \frac{x^2}{81} = 1$

5. vertices $(1, -1)$ and $(1, -9)$, conjugate axis of length 6 units $\frac{(y+5)^2}{16} - \frac{(x-1)^2}{9} = 1$

6. vertices $(-5, 0)$ and $(5, 0)$, foci $(\pm\sqrt{26}, 0)$ $\frac{x^2}{25} - \frac{y^2}{1} = 1$

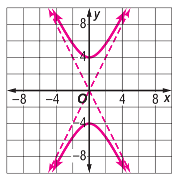
7. vertices $(1, 1)$ and $(1, -3)$, foci $(1, -1 \pm \sqrt{5})$ $\frac{(y+1)^2}{4} - \frac{(x-1)^2}{1} = 1$

Find the coordinates of the vertices and foci and the equations of the asymptotes for the hyperbola with the given equation. Then graph the hyperbola.

8. $\frac{y^2}{16} - \frac{x^2}{4} = 1$

$(0, \pm 4)$; $(0, \pm 2\sqrt{5})$;

$y = \pm 2x$

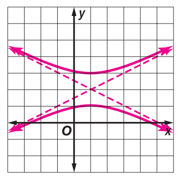


9. $\frac{(y-2)^2}{1} - \frac{(x-1)^2}{4} = 1$

$(1, 3), (1, 1)$;

$(1, 2 \pm \sqrt{5})$;

$y - 2 = \pm\frac{1}{2}(x - 1)$

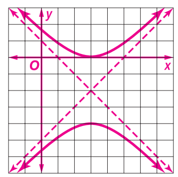


10. $\frac{(y+2)^2}{4} - \frac{(x-3)^2}{4} = 1$

$(3, 0), (3, -4)$;

$(3, -2 \pm 2\sqrt{2})$;

$y + 2 = \pm(x - 3)$



11. ASTRONOMY Astronomers use special X-ray telescopes to observe the sources of celestial X rays. Some X-ray telescopes are fitted with a metal mirror in the shape of a hyperbola, which reflects the X rays to a focus. Suppose the vertices of such a mirror are located at $(-3, 0)$ and $(3, 0)$, and one focus is located at $(5, 0)$. Write an equation that models the hyperbola formed by the mirror. $\frac{x^2}{9} - \frac{y^2}{16} = 1$

Answers

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8-5 Reading to Learn Mathematics

Hyperbolas

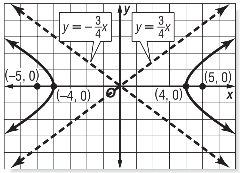
Pre-Activity **How are hyperbolas different from parabolas?**

Read the introduction to Lesson 8-5 at the top of page 441 in your textbook.

Look at the sketch of a hyperbola in the introduction to this lesson. List three ways in which hyperbolas are different from parabolas.
Sample answer: A hyperbola has two branches, while a parabola is one continuous curve. A hyperbola has two foci, while a parabola has one focus. A hyperbola has two vertices, while a parabola has one vertex.

Reading the Lesson

1. The graph at the right shows the hyperbola whose equation in standard form is $\frac{x^2}{16} - \frac{y^2}{9} = 1$.



The point (0, 0) is the **center** of the hyperbola.

The points (4, 0) and (−4, 0) are the **vertices** of the hyperbola.

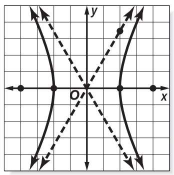
The points (5, 0) and (−5, 0) are the **foci** of the hyperbola.

The segment connecting (4, 0) and (−4, 0) is called the **transverse** axis.

The segment connecting (0, 3) and (0, −3) is called the **conjugate** axis.

The lines $y = \frac{3}{4}x$ and $y = -\frac{3}{4}x$ are called the **asymptotes**.

2. Study the hyperbola graphed at the right.



The center is **(0, 0)**.

The value of a is **2**.

The value of c is **4**.

To find b^2, solve the equation $c^2 = a^2 + b^2$.

The equation in standard form for this hyperbola is $\frac{x^2}{4} - \frac{y^2}{12} = 1$.

Helping You Remember

3. What is an easy way to remember the equation relating the values of a, b, and c for a hyperbola? **This equation looks just like the Pythagorean Theorem, although the variables represent different lengths in a hyperbola than in a right triangle.**

Lesson 8-5

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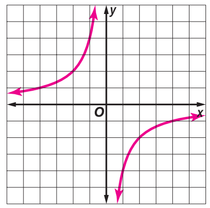
8-5 Enrichment

Rectangular Hyperbolas

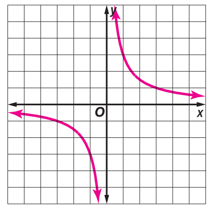
A **rectangular hyperbola** is a hyperbola with perpendicular asymptotes. For example, the graph of $x^2 - y^2 = 1$ is a rectangular hyperbola. A hyperbola with asymptotes that are not perpendicular is called a **nonrectangular hyperbola**. The graphs of equations of the form $xy = c$, where c is a constant, are rectangular hyperbolas.

Make a table of values and plot points to graph each rectangular hyperbola below. Be sure to consider negative values for the variables. **See students' tables.**

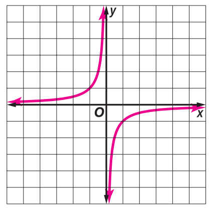
1. $xy = -4$



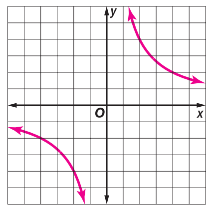
2. $xy = 3$



3. $xy = -1$



4. $xy = 8$



5. Make a conjecture about the asymptotes of rectangular hyperbolas.
The coordinate axes are the asymptotes.

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8-6 Study Guide and Intervention

Conic Sections

Standard Form Any conic section in the coordinate plane can be described by an equation of the form

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0, \text{ where } A, B, \text{ and } C \text{ are not all zero.}$$

One way to tell what kind of conic section an equation represents is to rearrange terms and complete the square, if necessary, to get one of the standard forms from an earlier lesson. This method is especially useful if you are going to graph the equation.

Example **Write the equation $3x^2 - 4y^2 - 30x - 8y + 59 = 0$ in standard form. State whether the graph of the equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*.**

$3x^2 - 4y^2 - 30x - 8y + 59 = 0$	Original equation
$3x^2 - 30x - 4y^2 - 8y = -59$	Isolate terms.
$3(x^2 - 10x + ■) - 4(y^2 + 2y + ■) = -59 + ■ + ■$	Factor out common multiples.
$3(x^2 - 10x + 25) - 4(y^2 + 2y + 1) = -59 + 3(25) + (-4)(1)$	Complete the squares.
$3(x - 5)^2 - 4(y + 1)^2 = 12$	Simplify.
$\frac{(x-5)^2}{4} - \frac{(y+1)^2}{3} = 1$	Divide each side by 12.

The graph of the equation is a hyperbola with its center at (5, −1). The length of the transverse axis is 4 units and the length of the conjugate axis is $2\sqrt{3}$ units.

Exercises

Write each equation in standard form. State whether the graph of the equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*.

1. $x^2 + y^2 - 6x + 4y + 3 = 0$
   $(x - 3)^2 + (y + 2)^2 = 10$; circle

2. $x^2 + 2y^2 + 6x - 20y + 53 = 0$
   $\frac{(x+3)^2}{6} + \frac{(y-5)^2}{3} = 1$; ellipse

3. $6x^2 - 60x - y + 161 = 0$
   $y = 6(x - 5)^2 + 11$; parabola

4. $x^2 + y^2 - 4x - 14y + 29 = 0$
   $(x - 2)^2 + (y - 7)^2 = 24$; circle

5. $6x^2 - 5y^2 + 24x + 20y - 56 = 0$
   $\frac{(x+2)^2}{10} - \frac{(y-2)^2}{12} = 1$; hyperbola

6. $3y^2 + x - 24y + 46 = 0$
   $x = -3(y - 4)^2 + 2$; parabola

7. $x^2 - 4y^2 - 16x + 24y - 36 = 0$
   $\frac{(x-8)^2}{64} - \frac{(y-3)^2}{16} = 1$; hyperbola

8. $x^2 + 2y^2 + 8x + 4y + 2 = 0$
   $\frac{(x+4)^2}{16} + \frac{(y+1)^2}{8} = 1$; ellipse

9. $4x^2 + 48x + y + 158 = 0$
   $y = -4(x + 6)^2 - 14$; parabola

10. $3x^2 + y^2 - 48x - 4y + 184 = 0$
    $\frac{(x-8)^2}{4} + \frac{(y-2)^2}{12} = 1$; ellipse

11. $-3x^2 + 2y^2 - 18x + 20y + 5 = 0$
    $\frac{(y+5)^2}{9} - \frac{(x+3)^2}{6} = 1$; hyperbola

12. $x^2 + y^2 + 8x + 2y + 8 = 0$
    $(x + 4)^2 + (y + 1)^2 = 9$; circle

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8-6 Study Guide and Intervention (continued)

Conic Sections

Identify Conic Sections If you are given an equation of the form

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0, \text{ with } B = 0,$$

you can determine the type of conic section just by considering the values of A and C. Refer to the following chart.

Relationship of A and C	Type of Conic Section
$A = 0$ or $C = 0$, but not both.	parabola
$A = C$	circle
A and C have the same sign, but $A \neq C$.	ellipse
A and C have opposite signs.	hyperbola

Example **Without writing the equation in standard form, state whether the graph of each equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*.**

a. $3x^2 - 3y^2 + 5x + 12 = 0$
$A = 3$ and $C = -3$ have opposite signs, so the graph of the equation is a hyperbola.

b. $y^2 = 7y - 2x + 13$
$A = 0$, so the graph of the equation is a parabola.

Exercises

Without writing the equation in standard form, state whether the graph of each equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*.

1. $x^2 = 17x - 5y + 8$
   parabola
2. $2x^2 + 2y^2 - 3x + 4y = 5$
   circle
3. $4x^2 - 8x = 4y^2 - 6y + 10$
   hyperbola
4. $8(x - x^2) = 4(2y^2 - y) - 100$
   circle
5. $6y^2 - 18 = 24 - 4x^2$
   ellipse
6. $y = 27x - y^2$
   parabola
7. $x^2 = 4(y - y^2) + 2x - 1$
   ellipse
8. $10x - x^2 - 2y^2 = 5y$
   ellipse
9. $x = y^2 - 5y + x^2 - 5$
   circle
10. $11x^2 - 7y^2 = 77$
    hyperbola
11. $3x^2 + 4y^2 = 50 + y^2$
    circle
12. $y^2 = 8x - 11$
    parabola
13. $9y^2 - 99y = 3(3x - 3x^2)$
    circle
14. $6x^2 - 4 = 5y^2 - 3$
    hyperbola
15. $111 = 11x^2 + 10y^2$
    ellipse
16. $120x^2 - 119y^2 + 118x - 117y = 0$
    hyperbola
17. $3x^2 = 4y^2 + 12$
    hyperbola
18. $150 - x^2 = 120 - y$
    parabola

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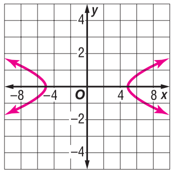
8-6 Skills Practice

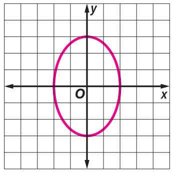
Conic Sections

Write each equation in standard form. State whether the graph of the equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*. Then graph the equation.

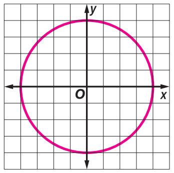
1. $x^2 - 25y^2 = 25$ **hyperbola**
$\frac{x^2}{25} - \frac{y^2}{1} = 1$



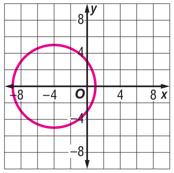
2. $9x^2 + 4y^2 = 36$ **ellipse**
$\frac{x^2}{4} + \frac{y^2}{9} = 1$



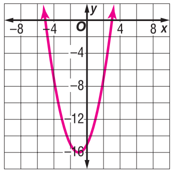
3. $x^2 + y^2 - 16 = 0$ **circle**
$x^2 + y^2 = 16$



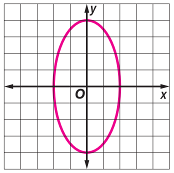
4. $x^2 + 8x + y^2 = 9$ **circle**
$(x + 4)^2 + y^2 = 25$



5. $x^2 + 2x - 15 = y$ **parabola**
$y = (x + 1)^2 - 16$



6. $100x^2 + 25y^2 = 400$ **ellipse**
$\frac{x^2}{4} + \frac{y^2}{16} = 1$



Without writing the equation in standard form, state whether the graph of each equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*.

7. $9x^2 + 4y^2 = 36$ **ellipse**

8. $x^2 + y^2 = 25$ **circle**

9. $y = x^2 + 2x$ **parabola**

10. $y = 2x^2 - 4x - 4$ **parabola**

11. $4y^2 - 25x^2 = 100$ **hyperbola**

12. $16x^2 + y^2 = 16$ **ellipse**

13. $16x^2 - 4y^2 = 64$ **hyperbola**

14. $5x^2 + 5y^2 = 25$ **circle**

15. $25y^2 + 9x^2 = 225$ **ellipse**

16. $36y^2 - 4x^2 = 144$ **hyperbola**

17. $y = 4x^2 - 36x - 144$ **parabola**

18. $x^2 + y^2 - 144 = 0$ **circle**

19. $(x + 3)^2 + (y - 1)^2 = 4$ **circle**

20. $25y^2 - 50y + 4x^2 = 75$ **ellipse**

21. $x^2 - 6y^2 + 9 = 0$ **hyperbola**

22. $x = y^2 + 5y - 6$ **parabola**

23. $(x + 5)^2 + y^2 = 10$ **circle**

24. $25x^2 + 10y^2 - 250 = 0$ **ellipse**

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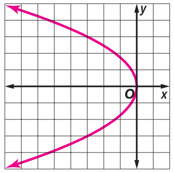
8-6 Practice (Average)

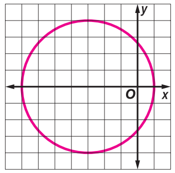
Conic Sections

Write each equation in standard form. State whether the graph of the equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*. Then graph the equation.

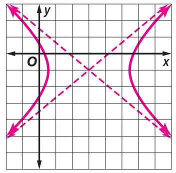
1. $y^2 = -3x$
parabola
$x = -\frac{1}{3}y^2$



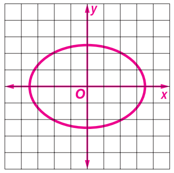
2. $x^2 + y^2 + 6x = 7$
circle
$(x + 3)^2 + y^2 = 16$



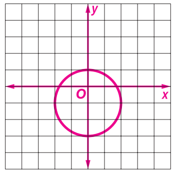
3. $5x^2 - 6y^2 - 30x - 12y = -9$
hyperbola
$\frac{(x - 3)^2}{6} - \frac{(y + 1)^2}{5} = 1$



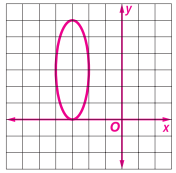
4. $196y^2 = 1225 - 100x^2$
ellipse
$\frac{x^2}{12.25} + \frac{y^2}{6.25} = 1$



5. $3x^2 = 9 - 3y^2 - 6y$
circle
$x^2 + (y + 1)^2 = 4$



6. $9x^2 + y^2 + 54x - 6y = -81$
ellipse
$\frac{(x + 3)^2}{1} + \frac{(y - 3)^2}{9} = 1$



Without writing the equation in standard form, state whether the graph of each equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*.

7. $6x^2 + 6y^2 = 36$
circle

8. $4x^2 - y^2 = 16$
hyperbola

9. $9x^2 + 16y^2 - 64y - 80 = 0$
ellipse

10. $5x^2 + 5y^2 - 45 = 0$
circle

11. $x^2 + 2x = y$
parabola

12. $4y^2 - 36x^2 + 4x - 144 = 0$
hyperbola

13. ASTRONOMY A satellite travels in an hyperbolic orbit. It reaches the vertex of its orbit at (5, 0) and then travels along a path that gets closer and closer to the line $y = \frac{2}{5}x$. Write an equation that describes the path of the satellite if the center of its hyperbolic orbit is at (0, 0).
$\frac{x^2}{25} - \frac{y^2}{4} = 1$

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8-6 Reading to Learn Mathematics

Conic Sections

Pre-Activity **How can you use a flashlight to make conic sections?**

Read the introduction to Lesson 8-6 at the top of page 449 in your textbook.

The figures in the introduction show how a plane can slice a double cone to form the conic sections. Name the conic section that is formed if the plane slices the double cone in each of the following ways:

- The plane is parallel to the base of the double cone and slices through one of the cones that form the double cone. **circle**
- The plane is perpendicular to the base of the double cone and slices through both of the cones that form the double cone. **hyperbola**

Reading the Lesson

1. Name the conic section that is the graph of each of the following equations. Give the coordinates of the vertex if the conic section is a parabola and of the center if it is a circle, an ellipse, or a hyperbola.

a. $\frac{(x-3)^2}{36} + \frac{(y+5)^2}{15} = 1$ **ellipse; (3, −5)**

b. $x = -2(y+1)^2 + 7$ **parabola; (7, −1)**

c. $(x-5)^2 - (y+5)^2 = 1$ **hyperbola; (5, −5)**

d. $(x+6)^2 + (y-2)^2 = 1$ **circle; (−6, 2)**

2. Each of the following is the equation of a conic section. For each equation, identify the values of A and C. Then, without writing the equation in standard form, state whether the graph of each equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*.

a. $2x^2 + y^2 - 6x + 8y + 12 = 0$ A = **2**; C = **1**; type of graph: **ellipse**

b. $2x^2 + 3x - 2y - 5 = 0$ A = **2**; C = **0**; type of graph: **parabola**

c. $5x^2 + 10x + 5y^2 - 20y + 1 = 0$ A = **5**; C = **5**; type of graph: **circle**

d. $x^2 - y^2 + 4x + 2y - 5 = 0$ A = **1**; C = **−1**; type of graph: **hyperbola**

Helping You Remember

3. What is an easy way to recognize that an equation represents a parabola rather than one of the other conic sections?

If the equation has an x^2 term and y term but no y^2 term, then the graph is a parabola. Likewise, if the equation has a y^2 term and x term but no x^2 term, then the graph is a parabola.

Lesson 8-6

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8-6 Enrichment

Loci

A *locus* (plural, *loci*) is the set of all points, and only those points, that satisfy a given set of conditions. In geometry, figures often are defined as loci. For example, a circle is the locus of points of a plane that are a given distance from a given point. The definition leads naturally to an equation whose graph is the curve described.

Example **Write an equation of the locus of points that are the same distance from (3, 4) and $y = -4$.**

Recognizing that the locus is a parabola with focus (3, 4) and directrix $y = -4$, you can find that $h = 3$, $k = 0$, and $a = 4$ where (h, k) is the vertex and 4 units is the distance from the vertex to both the focus and directrix.

Thus, an equation for the parabola is $y = \frac{1}{16}(x-3)^2$.

The problem also may be approached analytically as follows:

Let (x, y) be a point of the locus.

The distance from (3, 4) to (x, y) = the distance from $y = -4$ to (x, y).

$$\sqrt{(x-3)^2 + (y-4)^2} = \sqrt{(x-x)^2 + (y-(-4))^2}$$
$$(x-3)^2 + y^2 - 8y + 16 = y^2 + 8y + 16$$
$$(x-3)^2 = 16y$$
$$\frac{1}{16}(x-3)^2 = y$$

Describe each locus as a geometric figure. Then write an equation for the locus.

1. All points that are the same distance from (0, 5) and (4, 5).

line, $x = 2$

2. All points that are 4 units from the origin.

circle, $x^2 + y^2 = 4$

3. All points that are the same distance from (−2, −1) and $x = 2$.

parabola, $x = \frac{-1}{8}(y^2 + 2y + 1)$

4. The locus of points such that the sum of the distances from (−2, 0) and (2, 0) is 6.

ellipse, $\frac{x^2}{9} + \frac{y^2}{5} = 1$

5. The locus of points such that the absolute value of the difference of the distances from (−3, 0) and (3, 0) is 2.

hyperbola, $\frac{x^2}{1} - \frac{y^2}{8} = 1$

Answers

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8-7 Study Guide and Intervention

Solving Quadratic Systems

Systems of Quadratic Equations Like systems of linear equations, systems of quadratic equations can be solved by substitution and elimination. If the graphs are a conic section and a line, the system will have 0, 1, or 2 solutions. If the graphs are two conic sections, the system will have 0, 1, 2, 3, or 4 solutions.

Example **Solve the system of equations.** $\mathbf{y = x^2 - 2x - 15}$, $\mathbf{x + y = -3}$

Rewrite the second equation as $y = -x - 3$ and substitute into the first equation.

$-x - 3 = x^2 - 2x - 15$

$0 = x^2 - x - 12$ Add $x + 3$ to each side.

$0 = (x - 4)(x + 3)$ Factor.

Use the Zero Product property to get

$x = 4$ or $x = -3$.

Substitute these values for x in $x + y = -3$:

$4 + y = -3$ or $-3 + y = -3$

$y = -7$ $\quad$ $y = 0$

The solutions are $(4, -7)$ and $(-3, 0)$.

Exercises

Find the exact solution(s) of each system of equations.

1. $y = x^2 - 5$
   $y = x - 3$
   (2, −1), (−1, −4)

2. $x^2 + (y - 5)^2 = 25$
   $y = -x^2$
   (0, 0)

3. $x^2 + (y - 5)^2 = 25$
   $y = x^2$
   (0, 0), (3, 9), (−3, 9)

4. $x^2 + y^2 = 9$
   $x^2 + y = 3$
   $(0, 3), (\sqrt{5}, -2), (-\sqrt{5}, -2)$

5. $x^2 - y^2 = 1$
   $x^2 + y^2 = 16$
   $\left(\frac{\sqrt{34}}{2}, \frac{\sqrt{30}}{2}\right), \left(\frac{\sqrt{34}}{2}, -\frac{\sqrt{30}}{2}\right), \left(-\frac{\sqrt{34}}{2}, \frac{\sqrt{30}}{2}\right), \left(-\frac{\sqrt{34}}{2}, -\frac{\sqrt{30}}{2}\right)$

6. $y = x - 3$
   $x = y^2 - 4$
   $\left(\frac{7 + \sqrt{29}}{2}, \frac{1 + \sqrt{29}}{2}\right), \left(\frac{7 - \sqrt{29}}{2}, \frac{1 - \sqrt{29}}{2}\right)$

NAME ______________________ DATE __________ PERIOD _____

8-7 Study Guide and Intervention *(continued)*

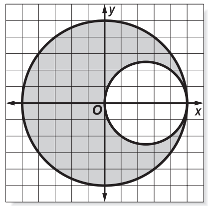
Solving Quadratic Systems

Systems of Quadratic Inequalities Systems of quadratic inequalities can be solved by graphing.

Example 1 **Solve the system of inequalities by graphing.**

$\mathbf{x^2 + y^2 \le 25}$

$\mathbf{\left(x - \frac{5}{2}\right)^2 + y^2 \ge \frac{25}{4}}$

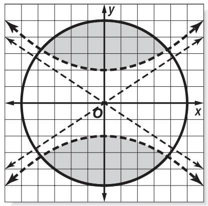


The graph of $x^2 + y^2 \le 25$ consists of all points on or inside the circle with center (0, 0) and radius 5.The graph of $\left(x - \frac{5}{2}\right)^2 + y^2 \ge \frac{25}{4}$ consists of all points on or outside the circle with center $\left(\frac{5}{2}, 0\right)$ and radius $\frac{5}{2}$. The solution of the system is the set of points in both regions.

Example 2 **Solve the system of inequalities by graphing.**

$\mathbf{x^2 + y^2 \le 25}$

$\mathbf{\frac{y^2}{4} - \frac{x^2}{9} > 1}$

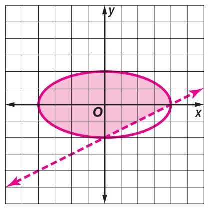


The graph of $x^2 + y^2 \le 25$ consists of all points on or inside the circle with center (0, 0) and radius 5.The graph of $\frac{y^2}{4} - \frac{x^2}{9} > 1$ are the points "inside" but not on the branches of the hyperbola shown. The solution of the system is the set of points in both regions.

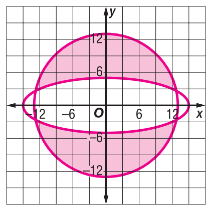
Exercises

Solve each system of inequalities below by graphing.

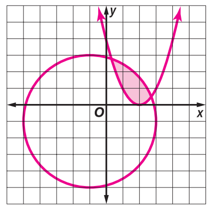
1. $\frac{x^2}{16} + \frac{y^2}{4} \le 1$
   $y > \frac{1}{2}x - 2$



2. $x^2 + y^2 \le 169$
   $x^2 + 9y^2 \ge 225$



3. $y \ge (x - 2)^2$
   $(x + 1)^2 + (y + 1)^2 \le 16$



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8-7 Skills Practice

Solving Quadratic Systems

Find the exact solution(s) of each system of equations.

1. $y = x - 2$, $y = x^2 - 2$ **(0, −2), (1, −1)**

2. $y = x + 3$, $y = 2x^2$ **(−1, 2), (1.5, 4.5)**

3. $y = 3x$, $x = y^2$ **(0, 0)**

4. $y = x$, $x^2 + y^2 = 4$ **$(\sqrt{2}, \sqrt{2})$, $(-\sqrt{2}, -\sqrt{2})$**

5. $x = -5$, $x^2 + y^2 = 25$ **(−5, 0)**

6. $y = 7$, $x^2 + y^2 = 9$ **no solution**

7. $y = -2x + 2$, $y^2 = 2x$ **(2, −2), $\left(\frac{1}{2}, 1\right)$**

8. $x - y + 1 = 0$, $y^2 = 4x$ **(1, 2)**

9. $y = 2 - x$, $y = x^2 - 4x + 2$ **(0, 2), (3, −1)**

10. $y = x - 1$, $y = x^2$ **no solution**

11. $y = 3x^2$, $y = -3x^2$ **(0, 0)**

12. $y = x^2 + 1$, $y = -x^2 + 3$ **(−1, 2), (1, 2)**

13. $y = 4x$, $4x^2 + y^2 = 20$ **(−1, −4), (1, 4)**

14. $y = -1$, $4x^2 + y^2 = 1$ **(0, −1)**

15. $4x^2 + 9y^2 = 36$, $x^2 - 9y^2 = 9$ **(−3, 0), (3, 0)**

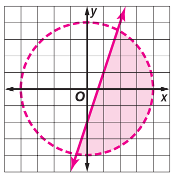
16. $3(y + 2)^2 - 4(x - 3)^2 = 12$, $y = -2x + 2$ **(0, 2), (3, −4)**

17. $x^2 - 4y^2 = 4$, $x^2 + y^2 = 4$ **(−2, 0), (2, 0)**

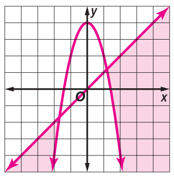
18. $y^2 - 4x^2 = 4$, $y = 2x$ **no solution**

Solve each system of inequalities by graphing.

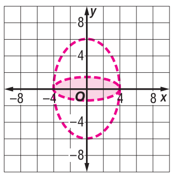
19. $y \le 3x - 2$, $x^2 + y^2 < 16$



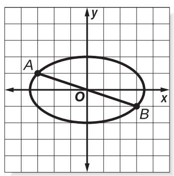
20. $y \le x$, $y \ge -2x^2 + 4$



21. $4y^2 + 9x^2 < 144$, $x^2 + 8y^2 < 16$



22. GARDENING An elliptical garden bed has a path from point A to point B. If the bed can be modeled by the equation $x^2 + 3y^2 = 12$ and the path can be modeled by the line $y = -\frac{1}{3}x$, what are the coordinates of points A and B? **(−3, 1) and (3, −1)**



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Lesson 8-7

NAME ______________ DATE ________ PERIOD ____

8-7 Practice (Average)

Solving Quadratic Systems

Find the exact solution(s) of each system of equations.

1. $(x - 2)^2 + y^2 = 5$, $x - y = 1$ **(0, −1), (3, 2)**

2. $x = 2(y + 1)^2 - 6$, $x + y = 3$ **(2, 1), (6.5, −3.5)**

3. $y^2 - 3x^2 = 6$, $y = 2x - 1$ **(−1, −3), (5, 9)**

4. $x^2 + 2y^2 = 1$, $y = -x + 1$ **(1, 0), $\left(\frac{1}{3}, \frac{2}{3}\right)$**

5. $4y^2 - 9x^2 = 36$, $4x^2 - 9y^2 = 36$ **no solution**

6. $y = x^2 - 3$, $x^2 + y^2 = 9$ **(0, −3), $(\pm\sqrt{5}, 2)$**

7. $x^2 + y^2 = 25$, $4y = 3x$ **(4, 3), (−4, −3)**

8. $y^2 = 10 - 6x^2$, $4y^2 = 40 - 2x^2$ **$(0, \pm\sqrt{10})$**

9. $x^2 + y^2 = 25$, $x = 3y - 5$ **(−5, 0), (4, 3)**

10. $4x^2 + 9y^2 = 36$, $2x^2 - 9y^2 = 18$ **(±3, 0)**

11. $x = -(y - 3)^2 + 2$, $x = (y - 3)^2 + 3$ **no solution**

12. $\frac{x^2}{9} - \frac{y^2}{16} = 1$, $x^2 + y^2 = 9$ **(±3, 0)**

13. $25x^2 + 4y^2 = 100$, $x = -\frac{5}{2}$ **no solution**

14. $x^2 + y^2 = 4$, $\frac{x^2}{4} + \frac{y^2}{8} = 1$ **(±2, 0)**

15. $x^2 - y^2 = 3$, $y^2 - x^2 = 3$ **no solution**

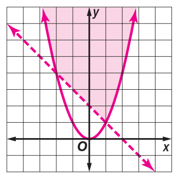
16. $\frac{x^2}{7} + \frac{y^2}{7} = 1$, $3x^2 - y^2 = 9$ **$(\pm 2, \pm\sqrt{3})$**

17. $x + 2y = 3$, $x^2 + y^2 = 9$ **(3, 0), $\left(-\frac{9}{5}, \frac{12}{5}\right)$**

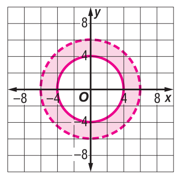
18. $x^2 + y^2 = 64$, $x^2 - y^2 = 8$ **$(\pm 6, \pm 2\sqrt{7})$**

Solve each system of inequalities by graphing.

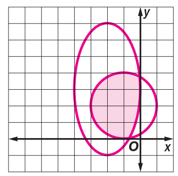
19. $y \ge x^2$, $y > -x + 2$



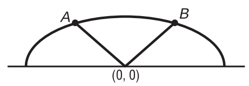
20. $x^2 + y^2 < 36$, $x^2 + y^2 \ge 16$



21. $\frac{(y - 3)^2}{16} + \frac{(x + 2)^2}{4} \le 1$, $(x + 1)^2 + (y - 2)^2 \le 4$



22. GEOMETRY The top of an iron gate is shaped like half an ellipse with two congruent segments from the center of the ellipse to the ellipse as shown. Assume that the center of the ellipse is at (0, 0). If the ellipse can be modeled by the equation $x^2 + 4y^2 = 4$ for $y \ge 0$ and the two congruent segments can be modeled by $y = \frac{\sqrt{3}}{2}x$ and $y = -\frac{\sqrt{3}}{2}x$, what are the coordinates of points A and B?



$\left(-1, \frac{\sqrt{3}}{2}\right)$ and $\left(1, \frac{\sqrt{3}}{2}\right)$

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Answers

NAME ______________________ DATE ____________ PERIOD _____

8-7 Reading to Learn Mathematics

Solving Quadratic Systems

Pre-Activity **How do systems of equations apply to video games?**

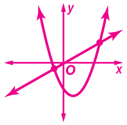
Read the introduction to Lesson 8-7 at the top of page 455 in your textbook.

The figure in your textbook shows that the spaceship hits the circular force field in two points. Is it possible for the spaceship to hit the force field in either fewer or more than two points? State all possibilities and explain how these could happen. **Sample answer: The spaceship could hit the force field in zero points if the spaceship missed the force field all together. The spaceship could also hit the force field in one point if the spaceship just touched the edge of the force field.**

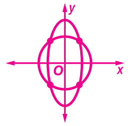
Reading the Lesson

1. Draw a sketch to illustrate each of the following possibilities.

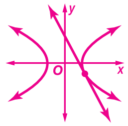
a. a parabola and a line that intersect in 2 points



b. an ellipse and a circle that intersect in 4 points



c. a hyperbola and a line that intersect in 1 point



2. Consider the following system of equations.

$x^2 = 3 + y^2$

$2x^2 + 3y^2 = 11$

a. What kind of conic section is the graph of the first equation? **hyperbola**

b. What kind of conic section is the graph of the second equation? **ellipse**

c. Based on your answers to parts a and b, what are the possible numbers of solutions that this system could have? **0, 1, 2, 3, or 4**

Helping You Remember

3. Suppose that the graph of a quadratic inequality is a region whose boundary is a circle. How can you remember whether to shade the interior or the exterior of the circle? **Sample answer: The solutions of an inequality of the form $x^2 + y^2 < r^2$ are all points that are *less than r* units from the origin, so the graph is the *interior* of the circle. The solutions of an inequality of the form $x^2 + y^2 > r^2$ are the points that are *more than r* units from the origin, so the graph is the *exterior* of the circle.**

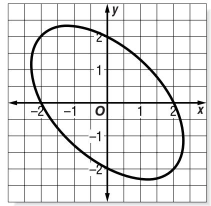
NAME ______________________ DATE ____________ PERIOD _____

8-7 Enrichment

Graphing Quadratic Equations with xy-Terms

You can use a graphing calculator to examine graphs of quadratic equations that contain xy-terms.

Example **Use a graphing calculator to display the graph of $x^2 + xy + y^2 = 4$.**



Solve the equation for y in terms of x by using the quadratic formula.

$y^2 + xy + (x^2 - 4) = 0$

To use the formula, let $a = 1$, $b = x$, and $c = (x^2 - 4)$.

$$y = \frac{-x \pm \sqrt{x^2 - 4(1)(x^2 - 4)}}{2}$$

$$y = \frac{-x \pm \sqrt{16 - 3x^2}}{2}$$

To graph the equation on the graphing calculator, enter the two equations:

$$y = \frac{-x + \sqrt{16 - 3x^2}}{2} \text{ and } y = \frac{-x - \sqrt{16 - 3x^2}}{2}$$

Use a graphing calculator to graph each equation. State the type of curve each graph represents.

1. $y^2 + xy = 8$
hyperbola

2. $x^2 + y^2 - 2xy - x = 0$
parabola

3. $x^2 - xy + y^2 = 15$
ellipse

4. $x^2 + xy + y^2 = -9$
graph is ∅

5. $2x^2 - 2xy - y^2 + 4x = 20$
hyperbola

6. $x^2 - xy - 2y^2 + 2x + 5y - 3 = 0$
two intersecting lines

Chapter 8 Assessment Answer Key

Form 1

Page 497

1. B
2. A
3. C
4. D
5. A
6. B
7. C
8. C
9. D
10. A
11. D

Page 498

12. A
13. B
14. B
15. D
16. C
17. A
18. D
19. C
20. A

B: a. 4; b. any positive number except 4; c. any negative number; d. 0

Form 2A

Page 499

1. D
2. B
3. C
4. A
5. B
6. B
7. B
8. D
9. C

Answers

(continued on the next page)

Chapter 8 Assessment Answer Key

Form 2A *(continued)*

Page 500

10. D

11. A

12. D

13. A

14. C

15. D

16. B

17. D

18. B

19. D

20. C

B: no solutions

Form 2B

Page 501

1. C

2. A

3. C

4. D

5. C

6. B

7. D

8. C

9. A

Page 502

10. A

11. C

12. B

13. A

14. B

15. B

16. C

17. D

18. D

19. C

20. A

B: no solutions

Chapter 8 Assessment Answer Key

Form 2C

Page 503

1. $\left(6, \frac{3}{2}\right)$

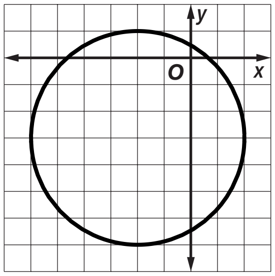
2. $\sqrt{61}$ units

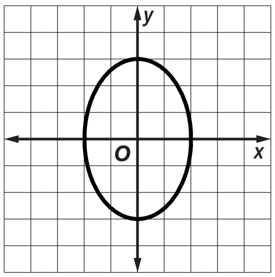
3. $x = \frac{1}{12}(y - 4)^2 + 1$

4. $y = 3(x - 1)^2 - 1$

5. $(2, 4)$; $\left(\frac{9}{4}, 4\right)$; $y = 4$; $x = \frac{7}{4}$; right

6. $(x + 4)^2 + (y - 2)^2 = 16$

7. 

8. 

9. $\frac{(x + 1)^2}{100} + \frac{(y - 3)^2}{25} = 1$

10. $\frac{(y - 5)^2}{36} + \frac{(x + 2)^2}{16} = 1$

11. $(\sqrt{2}, -2)$, $(-\sqrt{2}, -2)$

Page 504

12. $\frac{x^2}{81} - \frac{y^2}{25} = 1$

13. $\frac{(y + 2)^2}{36} - \frac{(x + 1)^2}{3} = 1$

14. $(5, -1)$, $(1, -1)$; $(3 \pm 2\sqrt{2}, -1)$; $y + 1 = \pm(x - 3)$

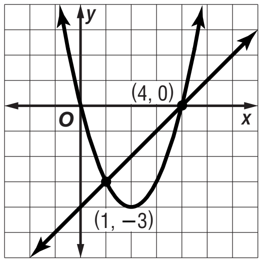
15. $(x + 1)^2 + (y + 1)^2 = 25$; circle

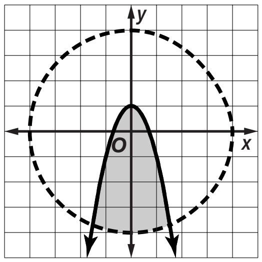
16. $\frac{(x + 3)^2}{9} + \frac{(y - 1)^2}{4} = 1$; ellipse

17. parabola; $C = 0$

18. hyperbola; $A = 4$, $C = -4$

19. $(4, 0)$, $(1, -3)$



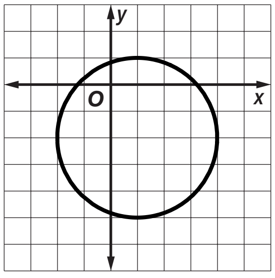
20. 

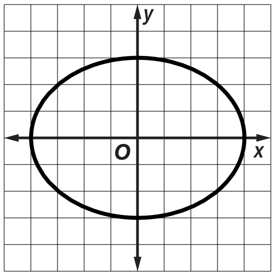
B: $(x - 3)^2 + (y + 1)^2 = 38$

Chapter 8 Assessment Answer Key

Form 2D

Page 505

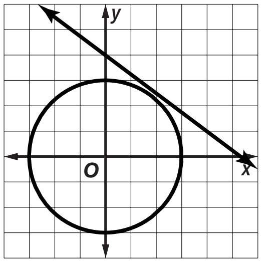
1. $\left(\frac{3}{2}, 1\right)$
2. $\sqrt{202}$ units
3. $y = \frac{1}{16}(x + 1)^2 - 3$
4.
5. $(1, 3); \left(1, \frac{25}{8}\right)$, $x = 1$; $y = \frac{23}{8}$; upward
6. $\left(x - \frac{1}{2}\right)^2 + (y + 2)^2 = 4$
7. 

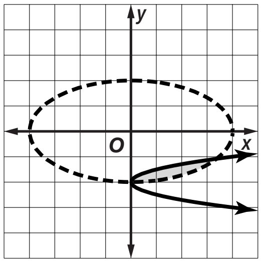
8. 

9. $\frac{(y - 2)^2}{49} + \frac{(x - 2)^2}{25} = 1$
10. $\frac{(x - 1)^2}{64} + \frac{(y + 4)^2}{9} = 1$
11. $(\sqrt{5}, -3), (-\sqrt{5}, -3)$

Page 506

12. $\frac{y^2}{144} - \frac{x^2}{16} = 1$
13. $\frac{(x + 3)^2}{49} - \frac{(y - 1)^2}{21} = 1$
14. $(1, 3), (-3, 3)$; $(-1 \pm 2\sqrt{2}, 3)$; $y - 3 = \pm(x + 1)$
15. $y = 4(x - 2)^2 + 5$; parabola
16. $\frac{(y - 3)^2}{100} - \frac{(x + 1)^2}{25} = 1$; hyperbola
17.
18. ellipse; $A = 10$; $C = 3$
19. no solution



20. 

B: $(x - 5)^2 + (y + 2)^2 = 66$

Chapter 8 Assessment Answer Key

Form 3

Page 507

1. $(-3.45, 4.15)$

2. $\sqrt{166}$ units

3. $x = -(y - 3)^2 + 2$

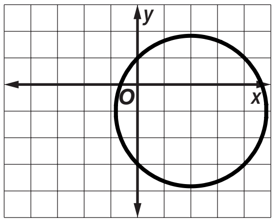
4. $x = -\frac{1}{6}(y - 1)^2 - 5$

5. $y = -\frac{1}{160}(x - 80)^2 + 40$

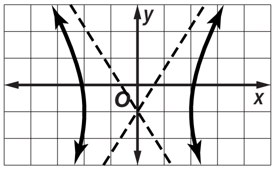
6. $(10, -1)$; $\left(\frac{39}{4}, -1\right)$, $y = -1$; $x = \frac{41}{4}$;

7. $(x - 3)^2 + (y - 5)^2 = 25$

8.



9.



10. $\frac{(x-5)^2}{49} + \frac{\left(y + \frac{1}{2}\right)^2}{25} = 1$

11. $\frac{(y+2)^2}{36} + \frac{(x-3)^2}{16} = 1$

Page 508

12. $(2, 3)$; $(2, 2)$, $(2, 4)$; $2\sqrt{6}$; $2\sqrt{5}$

13. $\frac{(y+2)^2}{9} - \frac{(x-4)^2}{16} = 1$

14. $2(x - 1)^2 + 3\left(y + \frac{4}{3}\right)^2 = \frac{67}{3}$; ellipse

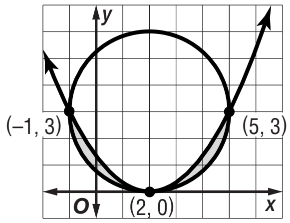
15. $x = -8\left(y + \frac{1}{2}\right)^2 - 94$; parabola

16. circle; $A = C = \frac{3}{2}$

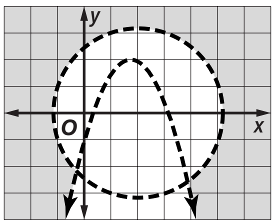
17. parabola; $A = 0$, $C = 4$

18. no solution

19. $(-1, 3)$, $(5, 3)$, $(2, 0)$



20.



B: $(-2.5, 0.725)$

Chapter 8 Assessment Answer Key

Page 509, Open-Ended Assessment
Scoring Rubric

Score	General Description	Specific Criteria
4	**Superior** A correct solution that is supported by well-developed, accurate explanations	• Shows thorough understanding of the concepts of *identifying, graphing, and writing equations of conic sections,* and *solving systems of quadratic equations and inequalities.* • Uses appropriate strategies to solve problems. • Computations are correct. • Written explanations are exemplary. • Goes beyond requirements of some or all problems.
3	**Satisfactory** A generally correct solution, but may contain minor flaws in reasoning or computation	• Shows an understanding of the concepts of *identifying, graphing, and writing equations of conic sections,* and *solving systems of quadratic equations and inequalities.* • Uses appropriate strategies to solve problems. • Computations are mostly correct. • Written explanations are effective. • Satisfies all requirements of problems.
2	**Nearly Satisfactory** A partially correct interpretation and/or solution to the problem	• Shows an understanding of most of the concepts of *identifying, graphing, and writing equations of conic sections,* and *solving systems of quadratic equations and inequalities.* • May not use appropriate strategies to solve problems. • Computations are mostly correct. • Written explanations are satisfactory. • Satisfies the requirements of most of the problems.
1	**Nearly Unsatisfactory** A correct solution with no supporting evidence or explanation	• Final computation is correct. • No written explanations or work is shown to substantiate the final computation. • Satisfies minimal requirements of some of the problems.
0	**Unsatisfactory** An incorrect solution indicating no mathematical understanding of the concept or task, or no solution is given	• Shows little or no understanding of most of the concepts of *identifying, graphing, and writing equations of conic sections,* and *solving systems of quadratic equations and inequalities.* • Does not use appropriate strategies to solve problems. • Computations are incorrect. • Written explanations are unsatisfactory. • Does not satisfy requirements of problems. • No answer may be given.

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Chapter 8 Assessment Answer Key

Page 509, Open-Ended Assessment
Sample Answers

In addition to the scoring rubric found on page A28, the following sample answers may be used as guidance in evaluating open-ended assessment items.

1a. The coefficients of the quadratic terms are the same number and have the same sign ($A = C = 1$).

1b. The radius would be the square root of a negative number (-5).

1c. Student responses should indicate that the constant in the original equation should be changed to a number less than 25, or that the constant obtained when the equation is written in standard form should be changed to a positive number, or that one or both of the coefficients of the linear terms, x and y, must be changed to a number sufficiently large to result in a positive number on the right side of the standard form of the equation. Sample answer: Change the constant in the original equation to 24. The center of the circle is $(-4, 3)$ and the radius is 1 unit.

2. The graphs of circles, ellipses, hyperbolas, and parabolas that open to the left and right never represent relations that are functions. Of all the conic sections studied in this chapter, only parabolas that open upward or downward have graphs which pass the vertical line test and are therefore functions.

3. The parabolas share the same vertex. Sample answer: The graph of $y = (x - 2)^2 - 1$ opens upward while the graph of $x = (y + 1)^2 + 2$ opens to the right.

4a. $(x - 4)^2 + (y - 3)^2 \le 4$
$y > (x - 4)^2 + 3$
Region 2 is the intersection of the region inside the circle, including its boundary ($\le$) and the region above the parabola, not including its boundary ($>$).

4b. $(x - 4)^2 + (y - 3)^2 \ge 4$
$y < (x - 4)^2 + 3$
Region 3 is the intersection of the region outside the circle, including its boundary ($\ge$) and the region below the parabola, not including its boundary ($<$).

4c. Region 1 is the intersection of the region outside the circle, including its boundary ($\ge$) and the region above the parabola, not including its boundary ($>$).

5a. Students must select both values such that $-5 < k < 1$ so that the graph of the horizontal line $y = k$ will intersect the graph of the ellipse twice.

5b. Students may select only $k = 1$ and $k = -5$, the equations of the only two horizontal lines that are tangent to the ellipse, each intersecting the ellipse in exactly one point.

5c. Students must select both values such that $k > 1$ or $k < -5$ so that the graph of the horizontal line $y = k$ will not intersect the graph of the ellipse.

Answers

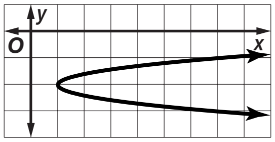
Chapter 8 Assessment Answer Key

Vocabulary Test/Review
Page 510

1. parabola; focus; directrix
2. ellipse; foci of the ellipse
3. hyperbola
4. minor axis; major axis
5. transverse axis
6. tangent
7. latus rectum
8. asymptote
9. conjugate axis
10. distance formula
11. Sample answer: A circle is the set of all points in a plane that are the same distance from a given point, which is the center.
12. Sample answer: A vertex of a hyperbola is the point on a branch of the hyperbola that is closest to the center of the hyperbola.

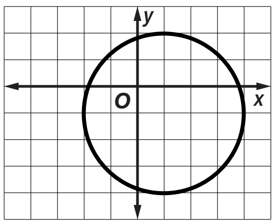
Quiz (Lessons 8–1 and 8–2)
Page 511

1. $\left(-1, \frac{7}{2}\right)$
2. A
3. $y = \frac{1}{12}(x - 1)^2 + 1$
4. $(-4, 5)$; $\left(-4, \frac{39}{8}\right)$; $x = -4$; $y = \frac{41}{8}$; downward
5. $\frac{1}{6}$ unit



Quiz (Lessons 8–3 and 8–4)
Page 511

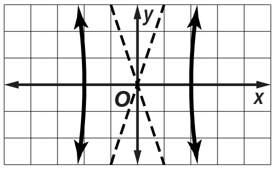
1. $(x + 7)^2 + (y - 2)^2 = 81$
2. $(x - 3)^2 + (y - 1)^2 = 16$
3. $(1, -1)$; 3 units



4. $\frac{x^2}{25} + \frac{(y - 1)^2}{16} = 1$
5. $(3, 0)$; $(3 \pm 2\sqrt{3}, 0)$; 8; 4

Quiz (Lessons 8–5 and 8–6)
Page 512

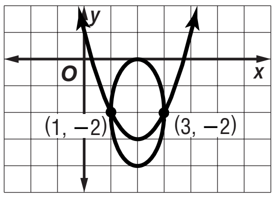
1. $\frac{(x - 1)^2}{4} - \frac{(y - 2)^2}{4} = 1$
2. $\frac{(y + 1)^2}{16} - \frac{(x - 2)^2}{9} = 1$
3. $(\pm 2, 0)$; $(\pm 2\sqrt{10}, 0)$; $y = \pm 3x$



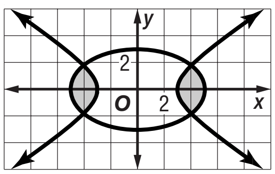
4. $y = 2(x - 3)^2 - 23$; parabola
5. ellipse; $A = 1$, $C = 4$

Quiz (Lesson 8–7)
Page 512

1. $(1, -2)$, $(3, -2)$



2. $\left(-\frac{7}{3}, -\frac{11}{3}\right)$, $(1, 3)$
3. $(1, 2)$, $(1, -2)$, $(-1, 2)$, $(-1, -2)$
4.



Chapter 8 Assessment Answer Key

Mid-Chapter Test
Page 513

1. A

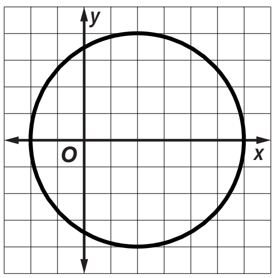
2. D

3. B

4. C

5. A

6.



7. $\frac{(x-4)^2}{25} + \frac{(y-1)^2}{16} = 1$

8. $y = -3(x-3)^2 + 32$

9. $(x-3)^2 + (y-4)^2 = 50$

Cumulative Review
Page 514

1. -22.5

2.

3. $(-1, -1)$

4. $\begin{bmatrix} 7 & 9 & 16 \\ -4 & 8 & 5 \end{bmatrix}$

5. $\frac{x-4}{x+3}$

6.

7. $1, \frac{5}{2}$

8. $\{x \mid x < -3 \text{ or } x > 1\}$

9. even; 4

10.

11.

12. $\left(-4, \frac{5}{2}\right)$

13. $x = -\frac{1}{20}y^2 + 1$

14. $(0, 0)$; $(0, \pm 2\sqrt{2})$; 6; 2

15. $\frac{(x+3)^2}{9} + \frac{(y-1)^2}{4} = 1$; ellipse

Answers

Chapter 8 Assessment Answer Key

Standardized Test Practice

Page 515

1. A B C **D**

2. E F **G** H

3. A B **C** D

4. **E** F G H

5. A B **C** D

6. E F **G** H

7. A B C **D**

8. E **F** G H

9. **A** B C D

10. **E** F G H

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11. 4

12. 12.5

13. 9.01

14. 43/4

15. A B **C** D

16. A B C **D**

17. A B **C** D

18. **A** B C D