

**GLENCOE
MATHEMATICS**

Algebra 2

Chapter 7 Resource Masters



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Consumable Workbooks

Many of the worksheets contained in the Chapter Resource Masters booklets are available as consumable workbooks.

<i>Study Guide and Intervention Workbook</i>	0-07-828029-X
<i>Skills Practice Workbook</i>	0-07-828023-0
<i>Practice Workbook</i>	0-07-828024-9

ANSWERS FOR WORKBOOKS The answers for Chapter 7 of these workbooks can be found in the back of this Chapter Resource Masters booklet.

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Algebra 2
Chapter 7 Resource Masters

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Teacher's Guide to Using the Chapter 7 Resource Masters

The **Fast File** Chapter Resource system allows you to conveniently file the resources you use most often. The *Chapter 7 Resource Masters* includes the core materials needed for Chapter 7. These materials include worksheets, extensions, and assessment options. The answers for these pages appear at the back of this booklet.

All of the materials found in this booklet are included for viewing and printing in the *Algebra 2 TeacherWorks* CD-ROM.

Vocabulary Builder Pages vii–viii include a student study tool that presents up to twenty of the key vocabulary terms from the chapter. Students are to record definitions and/or examples for each term. You may suggest that students highlight or star the terms with which they are not familiar.

WHEN TO USE Give these pages to students before beginning Lesson 7-1. Encourage them to add these pages to their Algebra 2 Study Notebook. Remind them to add definitions and examples as they complete each lesson.

Study Guide and Intervention

Each lesson in *Algebra 2* addresses two objectives. There is one Study Guide and Intervention master for each objective.

WHEN TO USE Use these masters as reteaching activities for students who need additional reinforcement. These pages can also be used in conjunction with the Student Edition as an instructional tool for students who have been absent.

Skills Practice There is one master for each lesson. These provide computational practice at a basic level.

WHEN TO USE These masters can be used with students who have weaker mathematics backgrounds or need additional reinforcement.

Practice There is one master for each lesson. These problems more closely follow the structure of the Practice and Apply section of the Student Edition exercises. These exercises are of average difficulty.

WHEN TO USE These provide additional practice options or may be used as homework for second day teaching of the lesson.

Reading to Learn Mathematics

One master is included for each lesson. The first section of each master asks questions about the opening paragraph of the lesson in the Student Edition. Additional questions ask students to interpret the context of and relationships among terms in the lesson. Finally, students are asked to summarize what they have learned using various representation techniques.

WHEN TO USE This master can be used as a study tool when presenting the lesson or as an informal reading assessment after presenting the lesson. It is also a helpful tool for ELL (English Language Learner) students.

Enrichment There is one extension master for each lesson. These activities may extend the concepts in the lesson, offer an historical or multicultural look at the concepts, or widen students' perspectives on the mathematics they are learning. These are not written exclusively for honors students, but are accessible for use with all levels of students.

WHEN TO USE These may be used as extra credit, short-term projects, or as activities for days when class periods are shortened.

Assessment Options

The assessment masters in the *Chapter 7 Resource Masters* offer a wide range of assessment tools for intermediate and final assessment. The following lists describe each assessment master and its intended use.

Chapter Assessment

CHAPTER TESTS

- *Form 1* contains multiple-choice questions and is intended for use with basic level students.
- *Forms 2A and 2B* contain multiple-choice questions aimed at the average level student. These tests are similar in format to offer comparable testing situations.
- *Forms 2C and 2D* are composed of free-response questions aimed at the average level student. These tests are similar in format to offer comparable testing situations. Grids with axes are provided for questions assessing graphing skills.
- *Form 3* is an advanced level test with free-response questions. Grids without axes are provided for questions assessing graphing skills.

All of the above tests include a free-response Bonus question.

- The **Open-Ended Assessment** includes performance assessment tasks that are suitable for all students. A scoring rubric is included for evaluation guidelines. Sample answers are provided for assessment.
- A **Vocabulary Test**, suitable for all students, includes a list of the vocabulary words in the chapter and ten questions assessing students' knowledge of those terms. This can also be used in conjunction with one of the chapter tests or as a review worksheet.

Intermediate Assessment

- Four free-response **quizzes** are included to offer assessment at appropriate intervals in the chapter.
- A **Mid-Chapter Test** provides an option to assess the first half of the chapter. It is composed of both multiple-choice and free-response questions.

Continuing Assessment

- The **Cumulative Review** provides students an opportunity to reinforce and retain skills as they proceed through their study of Algebra 2. It can also be used as a test. This master includes free-response questions.
- The **Standardized Test Practice** offers continuing review of algebra concepts in various formats, which may appear on the standardized tests that they may encounter. This practice includes multiple-choice, grid-in, and quantitative-comparison questions. Bubble-in and grid-in answer sections are provided on the master.

Answers

- Page A1 is an answer sheet for the Standardized Test Practice questions that appear in the Student Edition on pages 406–407. This improves students' familiarity with the answer formats they may encounter in test taking.
- The answers for the lesson-by-lesson masters are provided as reduced pages with answers appearing in red.
- Full-size answer keys are provided for the assessment masters in this booklet.

7

Reading to Learn Mathematics***Vocabulary Builder***

This is an alphabetical list of the key vocabulary terms you will learn in Chapter 7. As you study the chapter, complete each term's definition or description. Remember to add the page number where you found the term. Add these pages to your Algebra Study Notebook to review vocabulary at the end of the chapter.

Vocabulary Term	Found on Page	Definition/Description/Example
composition of functions		
depressed polynomial		
end behavior		
Factor Theorem		
Fundamental Theorem of Algebra		
inverse function		
inverse relation		
leading coefficients		
location principle		
one-to-one		

(continued on the next page)

7

Reading to Learn Mathematics**Vocabulary Builder** *(continued)*

Vocabulary Term	Found on Page	Definition/Description/Example
polynomial function		
polynomial in one variable		
power function		
quadratic form		
Rational Zero Theorem		
relative maximum		
relative minimum		
remainder theorem		
square root function		
synthetic substitution sɪh·n·THEH·tɪhk		

7-1 Study Guide and Intervention

Polynomial Functions

Polynomial Functions

Polynomial in One Variable	A polynomial of degree n in one variable x is an expression of the form $a_0x^n + a_1x^{n-1} + \dots + a_{n-2}x^2 + a_{n-1}x + a_n$ where the coefficients $a_0, a_1, a_2, \dots, a_n$ represent real numbers, a_0 is not zero, and n represents a nonnegative integer.
-----------------------------------	---

The **degree of a polynomial** in one variable is the greatest exponent of its variable. The **leading coefficient** is the coefficient of the term with the highest degree.

Polynomial Function	A polynomial function of degree n can be described by an equation of the form $P(x) = a_0x^n + a_1x^{n-1} + \dots + a_{n-2}x^2 + a_{n-1}x + a_n$ where the coefficients $a_0, a_1, a_2, \dots, a_n$ represent real numbers, a_0 is not zero, and n represents a nonnegative integer.
----------------------------	--

Example 1

What are the degree and leading coefficient of $3x^2 - 2x^4 - 7 + x^3$?

Rewrite the expression so the powers of x are in decreasing order.

$$-2x^4 + x^3 + 3x^2 - 7$$

This is a polynomial in one variable. The degree is 4, and the leading coefficient is -2 .

Example 2

Find $f(-5)$ if $f(x) = x^3 + 2x^2 - 10x + 20$.

$f(x) = x^3 + 2x^2 - 10x + 20$	Original function
$f(-5) = (-5)^3 + 2(-5)^2 - 10(-5) + 20$	Replace x with -5 .
$= -125 + 50 + 50 + 20$	Evaluate.
$= -5$	Simplify.

Example 3

Find $g(a^2 - 1)$ if $g(x) = x^2 + 3x - 4$.

$g(x) = x^2 + 3x - 4$	Original function
$g(a^2 - 1) = (a^2 - 1)^2 + 3(a^2 - 1) - 4$	Replace x with $a^2 - 1$.
$= a^4 - 2a^2 + 1 + 3a^2 - 3 - 4$	Evaluate.
$= a^4 + a^2 - 6$	Simplify.

Exercises

State the degree and leading coefficient of each polynomial in one variable. If it is not a polynomial in one variable, explain why.

- | | | |
|-----------------------------|------------------------------|--|
| 1. $3x^4 + 6x^3 - x^2 + 12$ | 2. $100 - 5x^3 + 10x^7$ | 3. $4x^6 + 6x^4 + 8x^8 - 10x^2 + 20$ |
| 4. $4x^2 - 3xy + 16y^2$ | 5. $8x^3 - 9x^5 + 4x^2 - 36$ | 6. $\frac{x^2}{18} - \frac{x^6}{25} + \frac{x^3}{36} - \frac{1}{72}$ |

Find $f(2)$ and $f(-5)$ for each function.

- | | | |
|---------------------|----------------------------------|----------------------------------|
| 7. $f(x) = x^2 - 9$ | 8. $f(x) = 4x^3 - 3x^2 + 2x - 1$ | 9. $f(x) = 9x^3 - 4x^2 + 5x + 7$ |
|---------------------|----------------------------------|----------------------------------|

7-1 Study Guide and Intervention *(continued)*

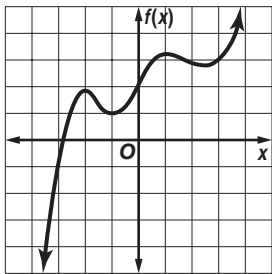
Polynomial Functions

Graphs of Polynomial Functions

End Behavior of Polynomial Functions	<p>If the degree is even and the leading coefficient is positive, then</p> $f(x) \rightarrow +\infty \text{ as } x \rightarrow -\infty$ $f(x) \rightarrow +\infty \text{ as } x \rightarrow +\infty$ <p>If the degree is even and the leading coefficient is negative, then</p> $f(x) \rightarrow -\infty \text{ as } x \rightarrow -\infty$ $f(x) \rightarrow -\infty \text{ as } x \rightarrow +\infty$ <p>If the degree is odd and the leading coefficient is positive, then</p> $f(x) \rightarrow -\infty \text{ as } x \rightarrow -\infty$ $f(x) \rightarrow +\infty \text{ as } x \rightarrow +\infty$ <p>If the degree is odd and the leading coefficient is negative, then</p> $f(x) \rightarrow +\infty \text{ as } x \rightarrow -\infty$ $f(x) \rightarrow -\infty \text{ as } x \rightarrow +\infty$
Real Zeros of a Polynomial Function	<p>The maximum number of zeros of a polynomial function is equal to the degree of the polynomial. A zero of a function is a point at which the graph intersects the x-axis. On a graph, count the number of real zeros of the function by counting the number of times the graph crosses or touches the x-axis.</p>

Example

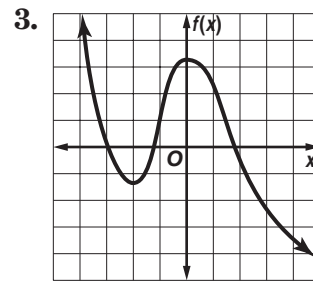
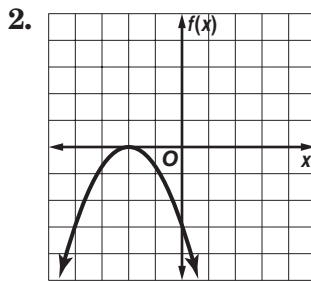
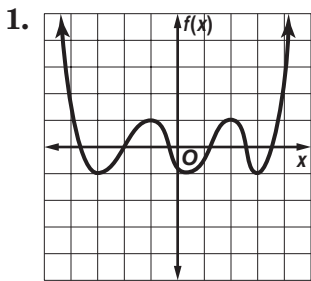
Determine whether the graph represents an odd-degree polynomial or an even-degree polynomial. Then state the number of real zeros.



As $x \rightarrow -\infty, f(x) \rightarrow -\infty$ and as $x \rightarrow +\infty, f(x) \rightarrow +\infty$, so it is an odd-degree polynomial function. The graph intersects the x -axis at 1 point, so the function has 1 real zero.

Exercises

Determine whether each graph represents an odd-degree polynomial or an even-degree polynomial. Then state the number of real zeros.



7-1 Skills Practice

Polynomial Functions

State the degree and leading coefficient of each polynomial in one variable. If it is not a polynomial in one variable, explain why.

1. $a + 8$

2. $(2x - 1)(4x^2 + 3)$

3. $-5x^5 + 3x^3 - 8$

4. $18 - 3y + 5y^2 - y^5 + 7y^6$

5. $u^3 + 4u^2v^2 + v^4$

6. $2r - r^2 + \frac{1}{r^2}$

Find $p(-1)$ and $p(2)$ for each function.

7. $p(x) = 4 - 3x$

8. $p(x) = 3x + x^2$

9. $p(x) = 2x^2 - 4x + 1$

10. $p(x) = -2x^3 + 5x + 3$

11. $p(x) = x^4 + 8x^2 - 10$

12. $p(x) = \frac{1}{3}x^2 - \frac{2}{3}x + 2$

If $p(x) = 4x^2 - 3$ and $r(x) = 1 + 3x$, find each value.

13. $p(a)$

14. $r(2a)$

15. $3r(a)$

16. $-4p(a)$

17. $p(a^2)$

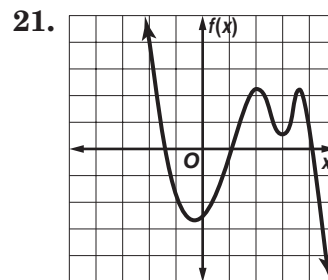
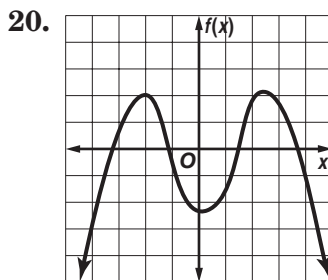
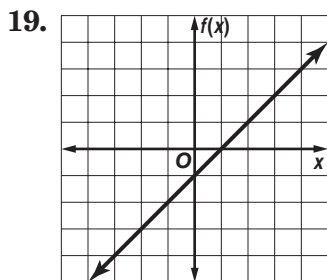
18. $r(x + 2)$

For each graph,

a. describe the end behavior,

b. determine whether it represents an odd-degree or an even-degree polynomial function, and

c. state the number of real zeroes.



7-1 Practice

Polynomial Functions

State the degree and leading coefficient of each polynomial in one variable. If it is not a polynomial in one variable, explain why.

1. $(3x^2 + 1)(2x^2 - 9)$

2. $\frac{1}{5}a^3 - \frac{3}{5}a^2 + \frac{4}{5}a$

3. $\frac{2}{m^2} + 3m - 12$

4. $27 + 3xy^3 - 12x^2y^2 - 10y$

Find $p(-2)$ and $p(3)$ for each function.

5. $p(x) = x^3 - x^5$

6. $p(x) = -7x^2 + 5x + 9$

7. $p(x) = -x^5 + 4x^3$

8. $p(x) = 3x^3 - x^2 + 2x - 5$

9. $p(x) = x^4 + \frac{1}{2}x^3 - \frac{1}{2}x$

10. $p(x) = \frac{1}{3}x^3 + \frac{2}{3}x^2 + 3x$

If $p(x) = 3x^2 - 4$ and $r(x) = 2x^2 - 5x + 1$, find each value.

11. $p(8a)$

12. $r(a^2)$

13. $-5r(2a)$

14. $r(x + 2)$

15. $p(x^2 - 1)$

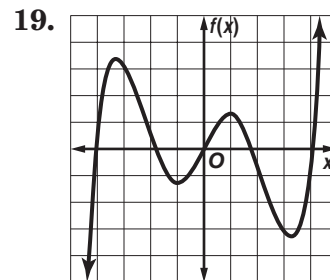
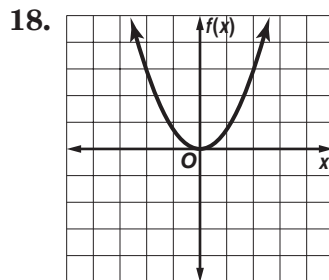
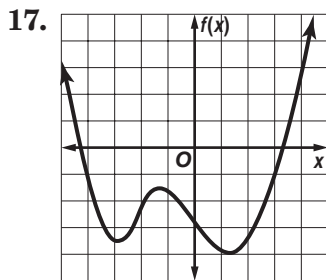
16. $5[p(x + 2)]$

For each graph,

a. describe the end behavior,

b. determine whether it represents an odd-degree or an even-degree polynomial function, and

c. state the number of real zeroes.



20. **WIND CHILL** The function $C(s) = 0.013s^2 - s - 7$ estimates the wind chill temperature $C(s)$ at 0°F for wind speeds s from 5 to 30 miles per hour. Estimate the wind chill temperature at 0°F if the wind speed is 20 miles per hour.

7-1

Reading to Learn Mathematics

Polynomial Functions

Pre-Activity Where are polynomial functions found in nature?

Read the introduction to Lesson 7-1 at the top of page 346 in your textbook.

- In the honeycomb cross section shown in your textbook, there is 1 hexagon in the center, 6 hexagons in the second ring, and 12 hexagons in the third ring. How many hexagons will there be in the fourth, fifth, and sixth rings?
- There is 1 hexagon in a honeycomb with 1 ring. There are 7 hexagons in a honeycomb with 2 rings. How many hexagons are there in honeycombs with 3 rings, 4 rings, 5 rings, and 6 rings?

Reading the Lesson

1. Give the degree and leading coefficient of each polynomial in one variable.

	degree	leading coefficient
a. $10x^3 + 3x^2 - x + 7$	_____	_____
b. $7y^2 - 2y^5 + y - 4y^3$	_____	_____
c. 100	_____	_____

2. Match each description of a polynomial function from the list on the left with the corresponding end behavior from the list on the right.

a. even degree, negative leading coefficient	i. $f(x) \rightarrow +\infty$ as $x \rightarrow +\infty$; $f(x) \rightarrow +\infty$ as $x \rightarrow -\infty$
b. odd degree, positive leading coefficient	ii. $f(x) \rightarrow -\infty$ as $x \rightarrow +\infty$; $f(x) \rightarrow +\infty$ as $x \rightarrow -\infty$
c. odd degree, negative leading coefficient	iii. $f(x) \rightarrow -\infty$ as $x \rightarrow +\infty$; $f(x) \rightarrow -\infty$ as $x \rightarrow -\infty$
d. even degree, positive leading coefficient	iv. $f(x) \rightarrow +\infty$ as $x \rightarrow +\infty$; $f(x) \rightarrow -\infty$ as $x \rightarrow -\infty$

Helping You Remember

3. What is an easy way to remember the difference between the end behavior of the graphs of even-degree and odd-degree polynomial functions?

7-1 Enrichment

Approximation by Means of Polynomials

Many scientific experiments produce pairs of numbers $[x, f(x)]$ that can be related by a formula. If the pairs form a function, you can fit a polynomial to the pairs in exactly one way. Consider the pairs given by the following table.

x	1	2	4	7
$f(x)$	6	11	39	-54

We will assume the polynomial is of degree three. Substitute the given values into this expression.

$$f(x) = A + B(x - x_0) + C(x - x_0)(x - x_1) + D(x - x_0)(x - x_1)(x - x_2)$$

You will get the system of equations shown below. You can solve this system and use the values for A , B , C , and D to find the desired polynomial.

$$\begin{aligned} 6 &= A \\ 11 &= A + B(2 - 1) = A + B \\ 39 &= A + B(4 - 1) + C(4 - 1)(4 - 2) = A + 3B + 6C \\ -54 &= A + B(7 - 1) + C(7 - 1)(7 - 2) + D(7 - 1)(7 - 2)(7 - 4) = A + 6B + 30C + 90D \end{aligned}$$

Solve.

1. Solve the system of equations for the values A , B , C , and D .
2. Find the polynomial that represents the four ordered pairs. Write your answer in the form $y = a + bx + cx^2 + dx^3$.
3. Find the polynomial that gives the following values.

x	8	12	15	20
$f(x)$	-207	169	976	3801

4. A scientist measured the volume $f(x)$ of carbon dioxide gas that can be absorbed by one cubic centimeter of charcoal at pressure x . Find the values for A , B , C , and D .

x	120	340	534	698
$f(x)$	3.1	5.5	7.1	8.3

7-2 Study Guide and Intervention

Graphing Polynomial Functions

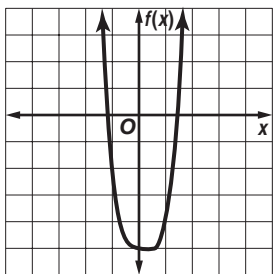
Graph Polynomial Functions

Location Principle	Suppose $y = f(x)$ represents a polynomial function and a and b are two numbers such that $f(a) < 0$ and $f(b) > 0$. Then the function has at least one real zero between a and b .
---------------------------	--

Example Determine the values of x between which each real zero of the function $f(x) = 2x^4 - x^3 - 5$ is located. Then draw the graph.

Make a table of values. Look at the values of $f(x)$ to locate the zeros. Then use the points to sketch a graph of the function.

x	$f(x)$
-2	35
-1	-2
0	-5
1	-4
2	19

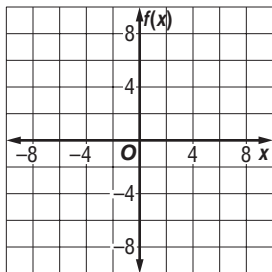


The changes in sign indicate that there are zeros between $x = -2$ and $x = -1$ and between $x = 1$ and $x = 2$.

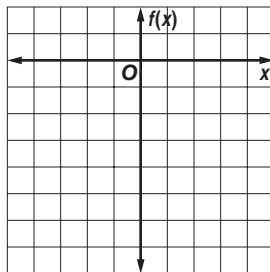
Exercises

Graph each function by making a table of values. Determine the values of x at which or between which each real zero is located.

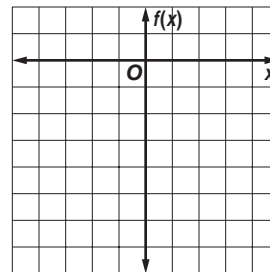
1. $f(x) = x^3 - 2x^2 + 1$



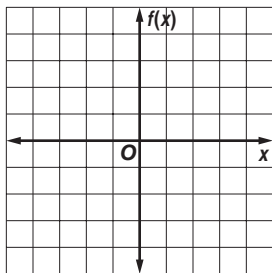
2. $f(x) = x^4 + 2x^3 - 5$



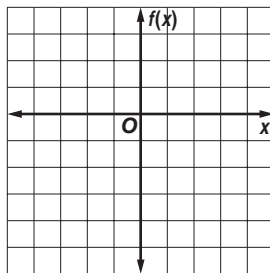
3. $f(x) = -x^4 + 2x^2 - 1$



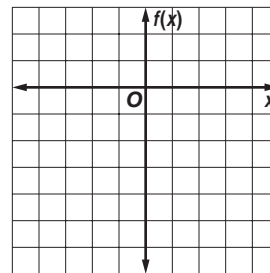
4. $f(x) = x^3 - 3x^2 + 4$



5. $f(x) = 3x^3 + 2x - 1$



6. $f(x) = x^4 - 3x^3 + 1$



7-2 Study Guide and Intervention *(continued)*

Graphing Polynomial Functions

Maximum and Minimum Points A quadratic function has either a maximum or a minimum point on its graph. For higher degree polynomial functions, you can find *turning points*, which represent **relative maximum** or **relative minimum** points.

Example

Graph $f(x) = x^3 + 6x^2 - 3$. Estimate the x -coordinates at which the relative maxima and minima occur.

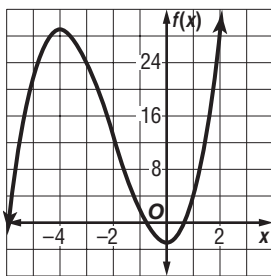
Make a table of values and graph the function.

x	$f(x)$
-5	22
-4	29
-3	24
-2	13
-1	2
0	-3
1	4
2	29

← indicates a relative maximum

← zero between $x = -1, x = 0$

← indicates a relative minimum

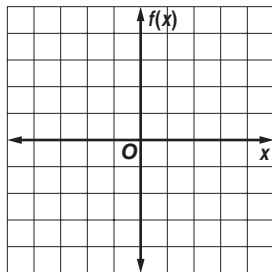


A relative maximum occurs at $x = -4$ and a relative minimum occurs at $x = 0$.

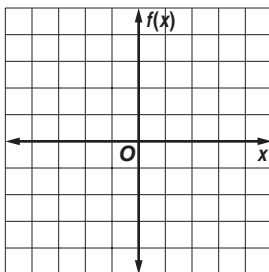
Exercises

Graph each function by making a table of values. Estimate the x -coordinates at which the relative maxima and minima occur.

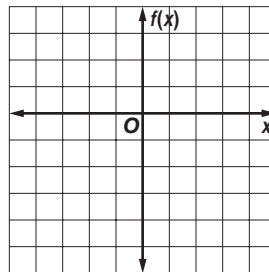
1. $f(x) = x^3 - 3x^2$



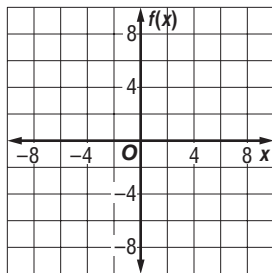
2. $f(x) = 2x^3 + x^2 - 3x$



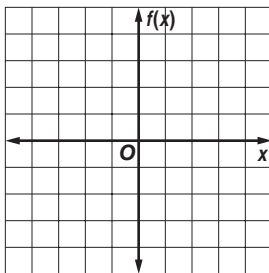
3. $f(x) = 2x^3 - 3x + 2$



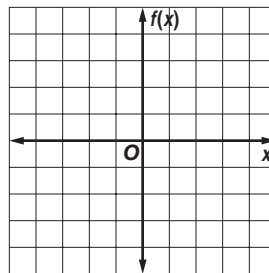
4. $f(x) = x^4 - 7x - 3$



5. $f(x) = x^5 - 2x^2 + 2$



6. $f(x) = x^3 + 2x^2 - 3$



7-2 Skills Practice

Graphing Polynomial Functions

Complete each of the following.

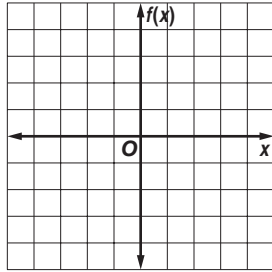
a. Graph each function by making a table of values.

b. Determine consecutive values of x between which each real zero is located.

c. Estimate the x -coordinates at which the relative maxima and minima occur.

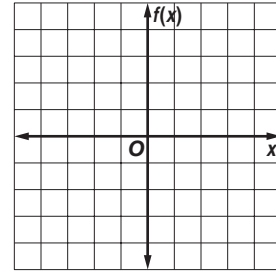
1. $f(x) = x^3 - 3x^2 + 1$

x	$f(x)$
-2	
-1	
0	
1	
2	
3	
4	



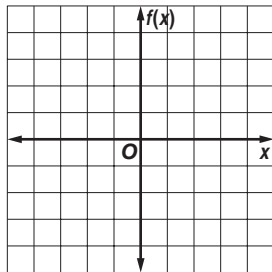
2. $f(x) = x^3 - 3x + 1$

x	$f(x)$
-3	
-2	
-1	
0	
1	
2	
3	



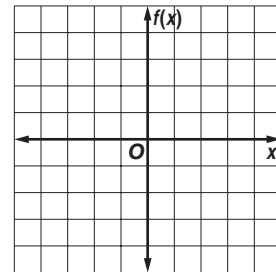
3. $f(x) = 2x^3 + 9x^2 + 12x + 2$

x	$f(x)$
-3	
-2	
-1	
0	
1	



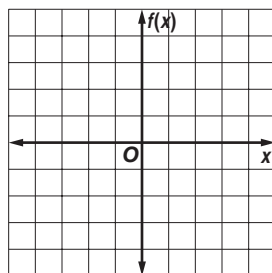
4. $f(x) = 2x^3 - 3x^2 + 2$

x	$f(x)$
-1	
0	
1	
2	
3	



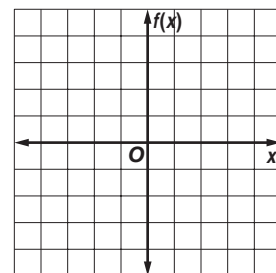
5. $f(x) = x^4 - 2x^2 - 2$

x	$f(x)$
-3	
-2	
-1	
0	
1	
2	
3	



6. $f(x) = 0.5x^4 - 4x^2 + 4$

x	$f(x)$
-3	
-2	
-1	
0	
1	
2	
3	



7-2 Practice

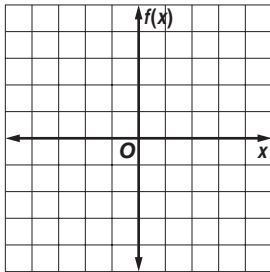
Graphing Polynomial Functions

Complete each of the following.

- Graph each function by making a table of values.
- Determine consecutive values of x between which each real zero is located.
- Estimate the x -coordinates at which the relative and relative minima occur.

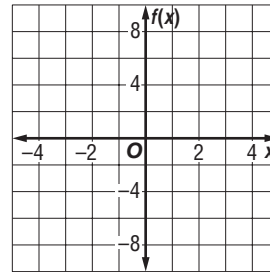
1. $f(x) = -x^3 + 3x^2 - 3$

x	$f(x)$
-2	
-1	
0	
1	
2	
3	
4	



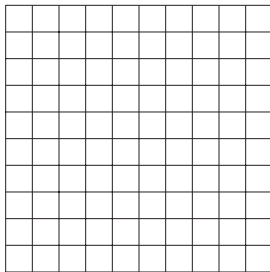
2. $f(x) = x^3 - 1.5x^2 - 6x + 1$

x	$f(x)$
-2	
-1	
0	
1	
2	
3	
4	



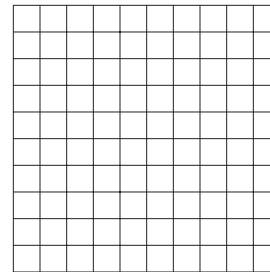
3. $f(x) = 0.75x^4 + x^3 - 3x^2 + 4$

x	$f(x)$



4. $f(x) = x^4 + 4x^3 + 6x^2 + 4x - 3$

x	$f(x)$

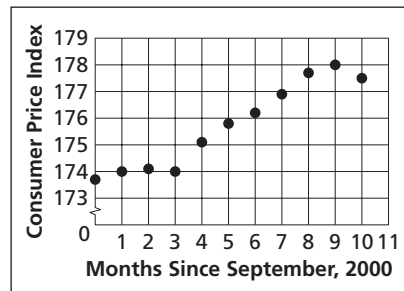


PRICES For Exercises 5 and 6, use the following information.

The Consumer Price Index (CPI) gives the relative price for a fixed set of goods and services. The CPI from September, 2000 to July, 2001 is shown in the graph.

Source: U. S. Bureau of Labor Statistics

- Describe the turning points of the graph.
- If the graph were modeled by a polynomial equation, what is the least degree the equation could have?
- LABOR** A town's jobless rate can be modeled by (1, 3.3), (2, 4.9), (3, 5.3), (4, 6.4), (5, 4.5), (6, 5.6), (7, 2.5), (8, 2.7). How many turning points would the graph of a polynomial function through these points have? Describe them.



7-2

Reading to Learn Mathematics

Graphing Polynomial Functions

Pre-Activity How can graphs of polynomial functions show trends in data?

Read the introduction to Lesson 7-2 at the top of page 353 in your textbook.

Three points on the graph shown in your textbook are $(0, 14)$, $(70, 3.78)$, and $(100, 9)$. Give the real-world meaning of the coordinates of these points.

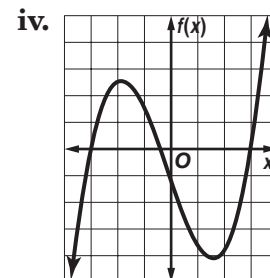
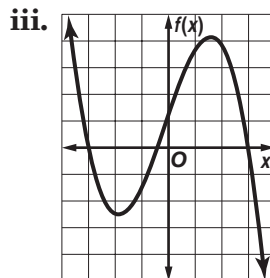
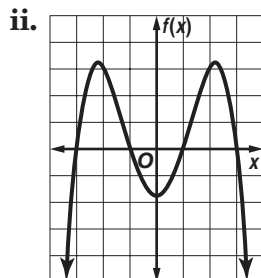
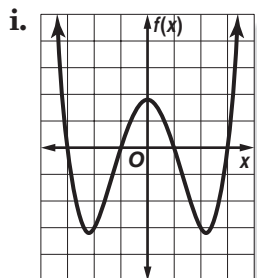
Reading the Lesson

1. Suppose that $f(x)$ is a third-degree polynomial function and that c and d are real numbers, with $d > c$. Indicate whether each statement is *true* or *false*. (Remember that *true* means *always* true.)

- a. If $f(c) > 0$ and $f(d) < 0$, there is exactly one real zero between c and d .
- b. If $f(c) = f(d) \neq 0$, there are no real zeros between c and d .
- c. If $f(c) < 0$ and $f(d) > 0$, there is at least one real zero between c and d .

2. Match each graph with its description.

- a. third-degree polynomial with one relative maximum and one relative minimum; leading coefficient negative
- b. fourth-degree polynomial with two relative minima and one relative maximum
- c. third-degree polynomial with one relative maximum and one relative minimum; leading coefficient positive
- d. fourth-degree polynomial with two relative maxima and one relative minimum



Helping You Remember

3. The origins of words can help you to remember their meaning and to distinguish between similar words. Look up *maximum* and *minimum* in a dictionary and describe their origins (original language and meaning).

7-2 Enrichment

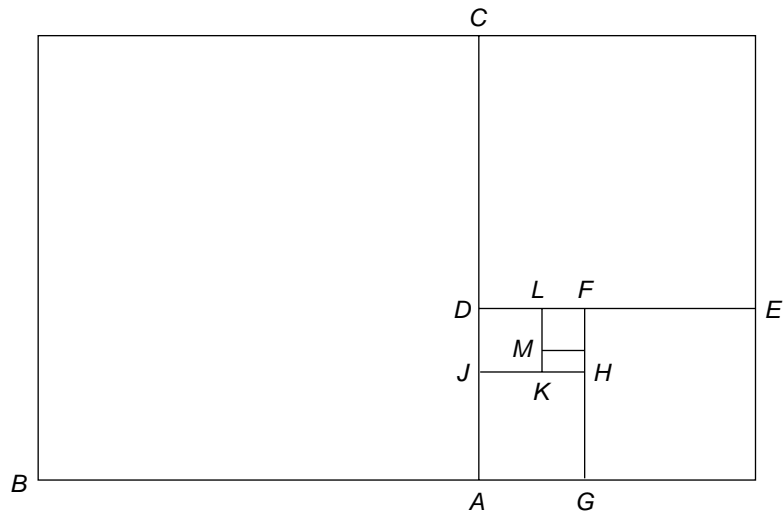
Golden Rectangles

Use a straightedge, a compass, and the instructions below to construct a golden rectangle.

1. Construct square $ABCD$ with sides of 2 centimeters.
2. Construct the midpoint of \overline{AB} . Call the midpoint M .
3. Using M as the center, set your compass opening at MC . Construct an arc with center M that intersects \overline{AB} . Call the point of intersection P .
4. Construct a line through P that is perpendicular to \overline{AB} .
5. Extend \overline{DC} so that it intersects the perpendicular. Call the intersection point Q . $APQD$ is a golden rectangle. Check this conclusion by finding the value of $\frac{QP}{AP}$.

A figure consisting of similar golden rectangles is shown below. Use a compass and the instructions below to draw quarter-circle arcs that form a spiral like that found in the shell of a chambered nautilus.

6. Using A as a center, draw an arc that passes through B and C .
7. Using D as a center, draw an arc that passes through C and E .
8. Using F as a center, draw an arc that passes through E and G .
9. Continue drawing arcs, using H , K , and M as the centers.



7-3

Study Guide and Intervention

Solving Equations Using Quadratic Techniques

Quadratic Form Certain polynomial expressions in x can be written in the quadratic form $au^2 + bu + c$ for any numbers a , b , and c , $a \neq 0$, where u is an expression in x .

Example

Write each polynomial in quadratic form, if possible.

a. $3a^6 - 9a^3 + 12$

Let $u = a^3$.

$$3a^6 - 9a^3 + 12 = 3(a^3)^2 - 9(a^3) + 12$$

b. $101b - 49\sqrt{b} + 42$

Let $u = \sqrt{b}$.

$$101b - 49\sqrt{b} + 42 = 101(\sqrt{b})^2 - 49(\sqrt{b}) + 42$$

c. $24a^5 + 12a^3 + 18$

This expression cannot be written in quadratic form, since $a^5 \neq (a^3)^2$.

Exercises

Write each polynomial in quadratic form, if possible.

1. $x^4 + 6x^2 - 8$

2. $4p^4 + 6p^2 + 8$

3. $x^8 + 2x^4 + 1$

4. $x^{\frac{1}{8}} + 2x^{\frac{1}{16}} + 1$

5. $6x^4 + 3x^3 + 18$

6. $12x^4 + 10x^2 - 4$

7. $24x^8 + x^4 + 4$

8. $18x^6 - 2x^3 + 12$

9. $100x^4 - 9x^2 - 15$

10. $25x^8 + 36x^6 - 49$

11. $48x^6 - 32x^3 + 20$

12. $63x^8 + 5x^4 - 29$

13. $32x^{10} + 14x^5 - 143$

14. $50x^3 - 15x\sqrt{x} - 18$

15. $60x^6 - 7x^3 + 3$

16. $10x^{10} - 7x^5 - 7$

7-3 Study Guide and Intervention *(continued)***Solving Equations Using Quadratic Techniques**

Solve Equations Using Quadratic Form If a polynomial expression can be written in quadratic form, then you can use what you know about solving quadratic equations to solve the related polynomial equation.

Example 1 Solve $x^4 - 40x^2 + 144 = 0$.

$$x^4 - 40x^2 + 144 = 0 \quad \text{Original equation}$$

$$(x^2)^2 - 40(x^2) + 144 = 0 \quad \text{Write the expression on the left in quadratic form.}$$

$$(x^2 - 4)(x^2 - 36) = 0 \quad \text{Factor.}$$

$$x^2 - 4 = 0 \quad \text{or} \quad x^2 - 36 = 0 \quad \text{Zero Product Property}$$

$$(x - 2)(x + 2) = 0 \quad \text{or} \quad (x - 6)(x + 6) = 0 \quad \text{Factor.}$$

$$x - 2 = 0 \text{ or } x + 2 = 0 \quad \text{or} \quad x - 6 = 0 \text{ or } x + 6 = 0 \quad \text{Zero Product Property}$$

$$x = 2 \text{ or } x = -2 \text{ or } x = 6 \text{ or } x = -6 \quad \text{Simplify.}$$

The solutions are ± 2 and ± 6 .

Example 2 Solve $2x + \sqrt{x} - 15 = 0$.

$$2x + \sqrt{x} - 15 = 0 \quad \text{Original equation}$$

$$2(\sqrt{x})^2 + \sqrt{x} - 15 = 0 \quad \text{Write the expression on the left in quadratic form.}$$

$$(2\sqrt{x} - 5)(\sqrt{x} + 3) = 0 \quad \text{Factor.}$$

$$2\sqrt{x} - 5 = 0 \quad \text{or} \quad \sqrt{x} + 3 = 0 \quad \text{Zero Product Property}$$

$$\sqrt{x} = \frac{5}{2} \quad \text{or} \quad \sqrt{x} = -3 \quad \text{Simplify.}$$

Since the principal square root of a number cannot be negative, $\sqrt{x} = -3$ has no solution.

The solution is $\frac{25}{4}$ or $6\frac{1}{4}$.

Exercises

Solve each equation.

1. $x^4 = 49$

2. $x^4 - 6x^2 = -8$

3. $x^4 - 3x^2 = 54$

4. $3t^6 - 48t^2 = 0$

5. $m^6 - 16m^3 + 64 = 0$

6. $y^4 - 5y^2 + 4 = 0$

7. $x^4 - 29x^2 + 100 = 0$

8. $4x^4 - 73x^2 + 144 = 0$

9. $\frac{1}{x^2} - \frac{7}{x} + 12 = 0$

10. $x - 5\sqrt{x} + 6 = 0$

11. $x - 10\sqrt{x} + 21 = 0$

12. $x^{\frac{2}{3}} - 5x^{\frac{1}{3}} + 6 = 0$

7-3

Skills Practice

Solving Equations Using Quadratic Techniques

Write each expression in quadratic form, if possible.

1. $5x^4 + 2x^2 - 8$

2. $3y^8 - 4y^2 + 3$

3. $100a^6 + a^3$

4. $x^8 + 4x^4 + 9$

5. $12x^4 - 7x^2$

6. $6b^5 + 3b^3 - 1$

7. $15v^6 - 8v^3 + 9$

8. $a^9 - 5a^5 + 7a$

Solve each equation.

9. $a^3 - 9a^2 + 14a = 0$

10. $x^3 = 3x^2$

11. $t^4 - 3t^3 - 40t^2 = 0$

12. $b^3 - 8b^2 + 16b = 0$

13. $m^4 = 4$

14. $w^3 - 6w = 0$

15. $m^4 - 18m^2 = -81$

16. $x^5 - 81x = 0$

17. $h^4 - 10h^2 = -9$

18. $a^4 - 9a^2 + 20 = 0$

19. $y^4 - 7y^2 + 12 = 0$

20. $v^4 - 12v^2 + 35 = 0$

21. $x^5 - 7x^3 + 6x = 0$

22. $c^{\frac{2}{3}} + 7c^{\frac{1}{3}} + 12 = 0$

23. $z - 5\sqrt{z} = -6$

24. $x - 30\sqrt{x} + 200 = 0$

7-3

Practice

Solving Equations Using Quadratic Techniques

Write each expression in quadratic form, if possible.

1. $10b^4 + 3b^2 - 11$

2. $-5x^8 + x^2 + 6$

3. $28d^6 + 25d^3$

4. $4s^8 + 4s^4 + 7$

5. $500x^4 - x^2$

6. $8b^5 - 8b^3 - 1$

7. $32w^5 - 56w^3 + 8w$

8. $e^{\frac{2}{3}} + 7e^{\frac{1}{3}} - 10$

9. $x^{\frac{1}{5}} + 29x^{\frac{1}{10}} + 2$

Solve each equation.

10. $y^4 - 7y^3 - 18y^2 = 0$

11. $s^5 + 4s^4 - 32s^3 = 0$

12. $m^4 - 625 = 0$

13. $n^4 - 49n^2 = 0$

14. $x^4 - 50x^2 + 49 = 0$

15. $t^4 - 21t^2 + 80 = 0$

16. $4r^6 - 9r^4 = 0$

17. $x^4 - 24 = -2x^2$

18. $d^4 = 16d^2 - 48$

19. $t^3 - 343 = 0$

20. $x^{\frac{1}{2}} - 5x^{\frac{1}{4}} + 6 = 0$

21. $x^{\frac{4}{3}} - 29x^{\frac{2}{3}} + 100 = 0$

22. $y^3 - 28y^{\frac{3}{2}} + 27 = 0$

23. $n - 10\sqrt{n} + 25 = 0$

24. $w - 12\sqrt{w} + 27 = 0$

25. $x - 2\sqrt{x} - 80 = 0$

26. PHYSICS A proton in a magnetic field follows a path on a coordinate grid modeled by the function $f(x) = x^4 - 2x^2 - 15$. What are the x -coordinates of the points on the grid where the proton crosses the x -axis?

27. SURVEYING Vista county is setting aside a large parcel of land to preserve it as open space. The county has hired Meghan's surveying firm to survey the parcel, which is in the shape of a right triangle. The longer leg of the triangle measures 5 miles less than the square of the shorter leg, and the hypotenuse of the triangle measures 13 miles less than twice the square of the shorter leg. The length of each boundary is a whole number. Find the length of each boundary.

7-3

Reading to Learn Mathematics

Solving Equations Using Quadratic Techniques

Pre-Activity How can solving polynomial equations help you to find dimensions?

Read the introduction to Lesson 7-3 at the top of page 360 in your textbook.

Explain how the formula given for the volume of the box can be obtained from the dimensions shown in the figure.

Reading the Lesson

1. Which of the following expressions can be written in quadratic form?

a. $x^3 + 6x^2 + 9$

b. $x^4 - 7x^2 + 6$

c. $m^6 + 4m^3 + 4$

d. $y - 2y^{\frac{1}{2}} - 15$

e. $x^5 + x^3 + 1$

f. $r^4 + 6 - r^8$

g. $p^{\frac{1}{4}} + 8p^{\frac{1}{2}} + 12$

h. $r^{\frac{1}{3}} + 2r^{\frac{1}{6}} - 3$

i. $5\sqrt{z} + 2z - 3$

2. Match each expression from the list on the left with its factorization from the list on the right.

a. $x^4 - 3x^2 - 40$

i. $(x^3 + 3)(x^3 - 3)$

b. $x^4 - 10x^2 + 25$

ii. $(\sqrt{x} + 3)(\sqrt{x} - 3)$

c. $x^6 - 9$

iii. $(\sqrt{x} + 3)^2$

d. $x - 9$

iv. $(x^2 + 1)(x^4 - x^2 + 1)$

e. $x^6 + 1$

v. $(x^2 - 5)^2$

f. $x + 6\sqrt{x} + 9$

vi. $(x^2 + 5)(x^2 - 8)$

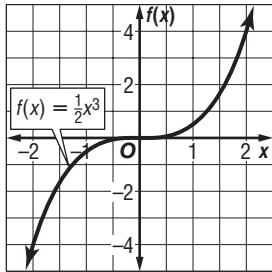
Helping You Remember

3. What is an easy way to tell whether a trinomial in one variable containing one constant term can be written in quadratic form?

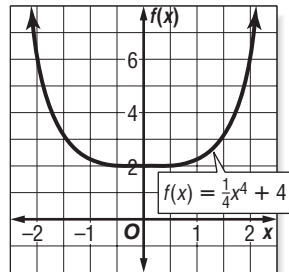
7-3 Enrichment

Odd and Even Polynomial Functions

Functions whose graphs are symmetric with respect to the origin are called *odd* functions. If $f(-x) = -f(x)$ for all x in the domain of $f(x)$, then $f(x)$ is odd.



Functions whose graphs are symmetric with respect to the y -axis are called *even* functions. If $f(-x) = f(x)$ for all x in the domain of $f(x)$, then $f(x)$ is even.



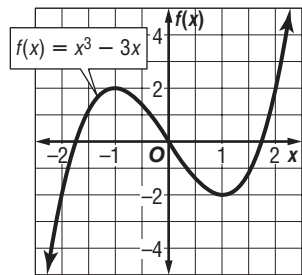
Example

Determine whether $f(x) = x^3 - 3x$ is odd, even, or neither.

$$\begin{aligned}
 f(x) &= x^3 - 3x \\
 f(-x) &= (-x)^3 - 3(-x) && \text{Replace } x \text{ with } -x. \\
 &= -x^3 + 3x && \text{Simplify.} \\
 &= -(x^3 - 3x) && \text{Factor out } -1. \\
 &= -f(x) && \text{Substitute.}
 \end{aligned}$$

Therefore, $f(x)$ is odd.

The graph at the right verifies that $f(x)$ is odd. The graph of the function is symmetric with respect to the origin.



Determine whether each function is *odd*, *even*, or *neither* by graphing or by applying the rules for odd and even functions.

- | | |
|-------------------------------|--|
| 1. $f(x) = 4x^2$ | 2. $f(x) = -7x^4$ |
| 3. $f(x) = x^7$ | 4. $f(x) = x^3 - x^2$ |
| 5. $f(x) = 3x^3 + 1$ | 6. $f(x) = x^8 - x^5 - 6$ |
| 7. $f(x) = -8x^5 - 2x^3 + 6x$ | 8. $f(x) = x^4 - 3x^3 + 2x^2 - 6x + 1$ |
| 9. $f(x) = x^4 + 3x^2 + 11$ | 10. $f(x) = x^7 - 6x^5 + 2x^3 + x$ |

11. Complete the following definitions: A polynomial function is odd if and only if all the terms are of _____ degrees. A polynomial function is even if and only if all the terms are of _____ degrees.

7-4 Study Guide and Intervention

The Remainder and Factor Theorems

Synthetic Substitution

Remainder Theorem	The remainder, when you divide the polynomial $f(x)$ by $(x - a)$, is the constant $f(a)$. $f(x) = q(x) \cdot (x - a) + f(a)$, where $q(x)$ is a polynomial with degree one less than the degree of $f(x)$.
--------------------------	--

Example 1 If $f(x) = 3x^4 + 2x^3 - 5x^2 + x - 2$, find $f(-2)$.

Method 1 Synthetic Substitution
By the Remainder Theorem, $f(-2)$ should be the remainder when you divide the polynomial by $x + 2$.

$$\begin{array}{r|rrrrr} -2 & 3 & 2 & -5 & 1 & -2 \\ & & -6 & 8 & -6 & 10 \\ \hline & 3 & -4 & 3 & -5 & 8 \end{array}$$

The remainder is 8, so $f(-2) = 8$.

Method 2 Direct Substitution

Replace x with -2 .

$$\begin{aligned} f(x) &= 3x^4 + 2x^3 - 5x^2 + x - 2 \\ f(-2) &= 3(-2)^4 + 2(-2)^3 - 5(-2)^2 + (-2) - 2 \\ &= 48 - 16 - 20 - 2 - 2 \text{ or } 8 \\ \text{So } f(-2) &= 8. \end{aligned}$$

Example 2 If $f(x) = 5x^3 + 2x - 1$, find $f(3)$.

Again, by the Remainder Theorem, $f(3)$ should be the remainder when you divide the polynomial by $x - 3$.

$$\begin{array}{r|rrrr} 3 & 5 & 0 & 2 & -1 \\ & & 15 & 45 & 141 \\ \hline & 5 & 15 & 47 & 140 \end{array}$$

The remainder is 140, so $f(3) = 140$.

Exercises

Use synthetic substitution to find $f(-5)$ and $f\left(\frac{1}{2}\right)$ for each function.

1. $f(x) = -3x^2 + 5x - 1$

2. $f(x) = 4x^2 + 6x - 7$

3. $f(x) = -x^3 + 3x^2 - 5$

4. $f(x) = x^4 + 11x^2 - 1$

Use synthetic substitution to find $f(4)$ and $f(-3)$ for each function.

5. $f(x) = 2x^3 + x^2 - 5x + 3$

6. $f(x) = 3x^3 - 4x + 2$

7. $f(x) = 5x^3 - 4x^2 + 2$

8. $f(x) = 2x^4 - 4x^3 + 3x^2 + x - 6$

9. $f(x) = 5x^4 + 3x^3 - 4x^2 - 2x + 4$

10. $f(x) = 3x^4 - 2x^3 - x^2 + 2x - 5$

11. $f(x) = 2x^4 - 4x^3 - x^2 - 6x + 3$

12. $f(x) = 4x^4 - 4x^3 + 3x^2 - 2x - 3$

7-4 Study Guide and Intervention *(continued)***The Remainder and Factor Theorems**

Factors of Polynomials The **Factor Theorem** can help you find all the factors of a polynomial.

Factor Theorem	The binomial $x - a$ is a factor of the polynomial $f(x)$ if and only if $f(a) = 0$.
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Example Show that $x + 5$ is a factor of $x^3 + 2x^2 - 13x + 10$. Then find the remaining factors of the polynomial.

By the Factor Theorem, the binomial $x + 5$ is a factor of the polynomial if -5 is a zero of the polynomial function. To check this, use synthetic substitution.

$$\begin{array}{r|rrrr} -5 & 1 & 2 & -13 & 10 \\ & & -5 & 15 & -10 \\ \hline & 1 & -3 & 2 & 0 \end{array}$$

Since the remainder is 0, $x + 5$ is a factor of the polynomial. The polynomial $x^3 + 2x^2 - 13x + 10$ can be factored as $(x + 5)(x^2 - 3x + 2)$. The depressed polynomial $x^2 - 3x + 2$ can be factored as $(x - 2)(x - 1)$.

$$\text{So } x^3 + 2x^2 - 13x + 10 = (x + 5)(x - 2)(x - 1).$$

Exercises

Given a polynomial and one of its factors, find the remaining factors of the polynomial. Some factors may not be binomials.

1. $x^3 + x^2 - 10x + 8; x - 2$

2. $x^3 - 4x^2 - 11x + 30; x + 3$

3. $x^3 + 15x^2 + 71x + 105; x + 7$

4. $x^3 - 7x^2 - 26x + 72; x + 4$

5. $2x^3 - x^2 - 7x + 6; x - 1$

6. $3x^3 - x^2 - 62x - 40; x + 4$

7. $12x^3 - 71x^2 + 57x - 10; x - 5$

8. $14x^3 + x^2 - 24x + 9; x - 1$

9. $x^3 + x + 10; x + 2$

10. $2x^3 - 11x^2 + 19x - 28; x - 4$

11. $3x^3 - 13x^2 - 34x + 24; x - 6$

12. $x^4 + x^3 - 11x^2 - 9x + 18; x - 1$

7-4 Skills Practice***The Remainder and Factor Theorems***

Use synthetic substitution to find $f(2)$ and $f(-1)$ for each function.

1. $f(x) = x^2 + 6x + 5$

2. $f(x) = x^2 - x + 1$

3. $f(x) = x^2 - 2x - 2$

4. $f(x) = x^3 + 2x^2 + 5$

5. $f(x) = x^3 - x^2 - 2x + 3$

6. $f(x) = x^3 + 6x^2 + x - 4$

7. $f(x) = x^3 - 3x^2 + x - 2$

8. $f(x) = x^3 - 5x^2 - x + 6$

9. $f(x) = x^4 + 2x^2 - 9$

10. $f(x) = x^4 - 3x^3 + 2x^2 - 2x + 6$

11. $f(x) = x^5 - 7x^3 - 4x + 10$

12. $f(x) = x^6 - 2x^5 + x^4 + x^3 - 9x^2 - 20$

Given a polynomial and one of its factors, find the remaining factors of the polynomial. Some factors may not be binomials.

13. $x^3 + 2x^2 - x - 2; x + 1$

14. $x^3 + x^2 - 5x + 3; x - 1$

15. $x^3 + 3x^2 - 4x - 12; x + 3$

16. $x^3 - 6x^2 + 11x - 6; x - 3$

17. $x^3 + 2x^2 - 33x - 90; x + 5$

18. $x^3 - 6x^2 + 32; x - 4$

19. $x^3 - x^2 - 10x - 8; x + 2$

20. $x^3 - 19x + 30; x - 2$

21. $2x^3 + x^2 - 2x - 1; x + 1$

22. $2x^3 + x^2 - 5x + 2; x + 2$

23. $3x^3 + 4x^2 - 5x - 2; 3x + 1$

24. $3x^3 + x^2 + x - 2; 3x - 2$

7-4 Practice***The Remainder and Factor Theorems***

Use synthetic substitution to find $f(-3)$ and $f(4)$ for each function.

1. $f(x) = x^2 + 2x + 3$

2. $f(x) = x^2 - 5x + 10$

3. $f(x) = x^2 - 5x - 4$

4. $f(x) = x^3 - x^2 - 2x + 3$

5. $f(x) = x^3 + 2x^2 + 5$

6. $f(x) = x^3 - 6x^2 + 2x$

7. $f(x) = x^3 - 2x^2 - 2x + 8$

8. $f(x) = x^3 - x^2 + 4x - 4$

9. $f(x) = x^3 + 3x^2 + 2x - 50$

10. $f(x) = x^4 + x^3 - 3x^2 - x + 12$

11. $f(x) = x^4 - 2x^2 - x + 7$

12. $f(x) = 2x^4 - 3x^3 + 4x^2 - 2x + 1$

13. $f(x) = 2x^4 - x^3 + 2x^2 - 26$

14. $f(x) = 3x^4 - 4x^3 + 3x^2 - 5x - 3$

15. $f(x) = x^5 + 7x^3 - 4x - 10$

16. $f(x) = x^6 + 2x^5 - x^4 + x^3 - 9x^2 + 20$

Given a polynomial and one of its factors, find the remaining factors of the polynomial. Some factors may not be binomials.

17. $x^3 + 3x^2 - 6x - 8; x - 2$

18. $x^3 + 7x^2 + 7x - 15; x - 1$

19. $x^3 - 9x^2 + 27x - 27; x - 3$

20. $x^3 - x^2 - 8x + 12; x + 3$

21. $x^3 + 5x^2 - 2x - 24; x - 2$

22. $x^3 - x^2 - 14x + 24; x + 4$

23. $3x^3 - 4x^2 - 17x + 6; x + 2$

24. $4x^3 - 12x^2 - x + 3; x - 3$

25. $18x^3 + 9x^2 - 2x - 1; 2x + 1$

26. $6x^3 + 5x^2 - 3x - 2; 3x - 2$

27. $x^5 + x^4 - 5x^3 - 5x^2 + 4x + 4; x + 1$

28. $x^5 - 2x^4 + 4x^3 - 8x^2 - 5x + 10; x - 2$

29. POPULATION The projected population in thousands for a city over the next several years can be estimated by the function $P(x) = x^3 + 2x^2 - 8x + 520$, where x is the number of years since 2000. Use synthetic substitution to estimate the population for 2005.

30. VOLUME The volume of water in a rectangular swimming pool can be modeled by the polynomial $2x^3 - 9x^2 + 7x + 6$. If the depth of the pool is given by the polynomial $2x + 1$, what polynomials express the length and width of the pool?

7-4 Reading to Learn Mathematics

The Remainder and Factor Theorems

Pre-Activity How can you use the Remainder Theorem to evaluate polynomials?

Read the introduction to Lesson 7-4 at the top of page 365 in your textbook.

Show how you would use the model in the introduction to estimate the number of international travelers (in millions) to the United States in the year 2000. (Show how you would substitute numbers, but do not actually calculate the result.)

Reading the Lesson

1. Consider the following synthetic division.

$$\begin{array}{r|rrrr} 1 & 3 & 2 & -6 & 4 \\ & & 3 & 5 & -1 \\ \hline & 3 & 5 & -1 & 3 \end{array}$$

a. Using the division symbol \div , write the division problem that is represented by this synthetic division. (Do not include the answer.)

b. Identify each of the following for this division.

dividend _____ divisor _____

quotient _____ remainder _____

c. If $f(x) = 3x^3 + 2x^2 - 6x + 4$, what is $f(1)$?

2. Consider the following synthetic division.

$$\begin{array}{r|rrrr} -3 & 1 & 0 & 0 & 27 \\ & & -3 & 9 & -27 \\ \hline & 1 & -3 & 9 & 0 \end{array}$$

a. This division shows that _____ is a factor of _____.

b. The division shows that _____ is a zero of the polynomial function $f(x) =$ _____.

c. The division shows that the point _____ is on the graph of the polynomial function $f(x) =$ _____.

Helping You Remember

3. Think of a mnemonic for remembering the sentence, "Dividend equals quotient times divisor plus remainder."

7-4 Enrichment

Using Maximum Values

Many times maximum solutions are needed for different situations. For instance, what is the area of the largest rectangular field that can be enclosed with 2000 feet of fencing?

Let x and y denote the length and width of the field, respectively.

$$\text{Perimeter: } 2x + 2y = 2000 \rightarrow y = 1000 - x$$

$$\text{Area: } A = xy = x(1000 - x) = -x^2 + 1000x$$

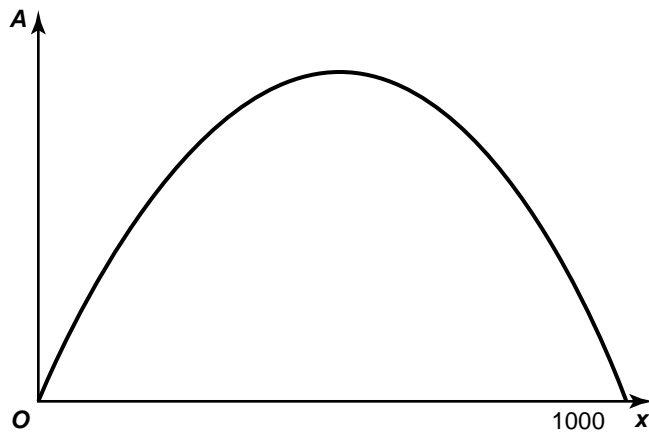


This problem is equivalent to finding the highest point on the graph of $A(x) = -x^2 + 1000x$ shown on the right.

Complete the square for $-x^2 + 1000x$.

$$\begin{aligned} A &= -(x^2 - 1000x + 500^2) + 500^2 \\ &= -(x - 500)^2 + 500^2 \end{aligned}$$

Because the term $-(x - 500)^2$ is either negative or 0, the greatest value of A is 500^2 . The maximum area enclosed is 500^2 or 250,000 square feet.



Solve each problem.

1. Find the area of the largest rectangular garden that can be enclosed by 300 feet of fence.
2. A farmer will make a rectangular pen with 100 feet of fence using part of his barn for one side of the pen. What is the largest area he can enclose?
3. An area along a straight stone wall is to be fenced. There are 600 meters of fencing available. What is the greatest rectangular area that can be enclosed?

7-5 Study Guide and Intervention

Roots and Zeros

Types of Roots The following statements are equivalent for any polynomial function $f(x)$.

- c is a zero of the polynomial function $f(x)$.
- $(x - c)$ is a factor of the polynomial $f(x)$.
- c is a root or solution of the polynomial equation $f(x) = 0$.

If c is real, then $(c, 0)$ is an intercept of the graph of $f(x)$.

Fundamental Theorem of Algebra	Every polynomial equation with degree greater than zero has at least one root in the set of complex numbers.
Corollary to the Fundamental Theorem of Algebras	A polynomial equation of the form $P(x) = 0$ of degree n with complex coefficients has exactly n roots in the set of complex numbers.
Descartes' Rule of Signs	<p>If $P(x)$ is a polynomial with real coefficients whose terms are arranged in descending powers of the variable,</p> <ul style="list-style-type: none"> • the number of positive real zeros of $y = P(x)$ is the same as the number of changes in sign of the coefficients of the terms, or is less than this by an even number, and • the number of negative real zeros of $y = P(x)$ is the same as the number of changes in sign of the coefficients of the terms of $P(-x)$, or is less than this number by an even number.

Example 1 Solve the equation $6x^3 + 3x = 0$ and state the number and type of roots.

$$6x^3 + 3x = 0$$

$$3x(2x^2 + 1) = 0$$

Use the Zero Product Property.

$$3x = 0 \text{ or } 2x^2 + 1 = 0$$

$$x = 0 \text{ or } 2x^2 = -1$$

$$x = \pm \frac{i\sqrt{2}}{2}$$

The equation has one real root, 0, and two imaginary roots, $\pm \frac{i\sqrt{2}}{2}$.

Example 2 State the number of positive real zeros, negative real zeros, and imaginary zeros for $p(x) = 4x^4 - 3x^3 + x^2 + 2x - 5$.

Since $p(x)$ has degree 4, it has 4 zeros. Use Descartes' Rule of Signs to determine the number and type of real zeros. Since there are three sign changes, there are 3 or 1 positive real zeros. Find $p(-x)$ and count the number of changes in sign for its coefficients.

$$p(-x) = 4(-x)^4 - 3(-x)^3 + (-x)^2 + 2(-x) - 5$$

$$= 4x^4 + 3x^3 + x^2 - 2x - 5$$

Since there is one sign change, there is exactly 1 negative real zero.

Exercises

Solve each equation and state the number and type of roots.

1. $x^2 + 4x - 21 = 0$

2. $2x^3 - 50x = 0$

3. $12x^3 + 100x = 0$

State the number of positive real zeros, negative real zeros, and imaginary zeros for each function.

4. $f(x) = 3x^3 + x^2 - 8x - 12$

5. $f(x) = 2x^4 - x^3 - 3x + 7$

7-5 Study Guide and Intervention *(continued)***Roots and Zeros****Find Zeros****Complex Conjugate Theorem**

Suppose a and b are real numbers with $b \neq 0$. If $a + bi$ is a zero of a polynomial function with real coefficients, then $a - bi$ is also a zero of the function.

Example

Find all of the zeros of $f(x) = x^4 - 15x^2 + 38x - 60$.

Since $f(x)$ has degree 4, the function has 4 zeros.

$$f(x) = x^4 - 15x^2 + 38x - 60 \quad f(-x) = x^4 - 15x^2 - 38x - 60$$

Since there are 3 sign changes for the coefficients of $f(x)$, the function has 3 or 1 positive real zeros. Since there is 1 sign change for the coefficients of $f(-x)$, the function has 1 negative real zero. Use synthetic substitution to test some possible zeros.

$$\begin{array}{r|rrrrr} 2 & 1 & 0 & -15 & 38 & -60 \\ & & 2 & 4 & -22 & 32 \\ \hline & 1 & 2 & -11 & 16 & -28 \end{array}$$

$$\begin{array}{r|rrrrr} 3 & 1 & 0 & -15 & 38 & -60 \\ & & 3 & 9 & -18 & 60 \\ \hline & 1 & 3 & -6 & 20 & 0 \end{array}$$

So 3 is a zero of the polynomial function. Now try synthetic substitution again to find a zero of the depressed polynomial.

$$\begin{array}{r|rrrr} -2 & 1 & 3 & -6 & 20 \\ & & -2 & -2 & 16 \\ \hline & 1 & 1 & -8 & 36 \end{array}$$

$$\begin{array}{r|rrrr} -4 & 1 & 3 & -6 & 20 \\ & & -4 & 4 & 8 \\ \hline & 1 & -1 & -2 & 28 \end{array}$$

$$\begin{array}{r|rrrr} -5 & 1 & 3 & -6 & 20 \\ & & -5 & 10 & -20 \\ \hline & 1 & -2 & 4 & 0 \end{array}$$

So -5 is another zero. Use the Quadratic Formula on the depressed polynomial $x^2 - 2x + 4$ to find the other 2 zeros, $1 \pm i\sqrt{3}$.

The function has two real zeros at 3 and -5 and two imaginary zeros at $1 \pm i\sqrt{3}$.

Exercises

Find all of the zeros of each function.

1. $f(x) = x^3 + x^2 + 9x + 9$

2. $f(x) = x^3 - 3x^2 + 4x - 12$

3. $p(a) = a^3 - 10a^2 + 34a - 40$

4. $p(x) = x^3 - 5x^2 + 11x - 15$

5. $f(x) = x^3 + 6x + 20$

6. $f(x) = x^4 - 3x^3 + 21x^2 - 75x - 100$

7-5 Skills Practice

Roots and Zeros

Solve each equation. State the number and type of roots.

1. $5x + 12 = 0$

2. $x^2 - 4x + 40 = 0$

3. $x^5 + 4x^3 = 0$

4. $x^4 + 625 = 0$

5. $4x^2 - 4x - 1 = 0$

6. $x^5 - 81x = 0$

State the possible number of positive real zeros, negative real zeros, and imaginary zeros of each function.

7. $g(x) = 3x^3 - 4x^2 - 17x + 6$

8. $h(x) = 4x^3 - 12x^2 - x + 3$

9. $f(x) = x^3 - 8x^2 + 2x - 4$

10. $p(x) = x^3 - x^2 + 4x - 6$

11. $q(x) = x^4 + 7x^2 + 3x - 9$

12. $f(x) = x^4 - x^3 - 5x^2 + 6x + 1$

Find all the zeros of each function.

13. $h(x) = x^3 - 5x^2 + 5x + 3$

14. $g(x) = x^3 - 6x^2 + 13x - 10$

15. $h(x) = x^3 + 4x^2 + x - 6$

16. $q(x) = x^3 + 3x^2 - 6x - 8$

17. $g(x) = x^4 - 3x^3 - 5x^2 + 3x + 4$

18. $f(x) = x^4 - 21x^2 + 80$

Write a polynomial function of least degree with integral coefficients that has the given zeros.

19. $-3, -5, 1$

20. $3i$

21. $-5 + i$

22. $-1, \sqrt{3}, -\sqrt{3}$

23. $i, 5i$

24. $-1, 1, i\sqrt{6}$

7-5 Practice**Roots and Zeros****Solve each equation. State the number and type of roots.**

1. $-9x - 15 = 0$

2. $x^4 - 5x^2 + 4 = 0$

3. $x^5 = 81x$

4. $x^3 + x^2 - 3x - 3 = 0$

5. $x^3 + 6x + 20 = 0$

6. $x^4 - x^3 - x^2 - x - 2 = 0$

State the possible number of positive real zeros, negative real zeros, and imaginary zeros of each function.

7. $f(x) = 4x^3 - 2x^2 + x + 3$

8. $p(x) = 2x^4 - 2x^3 + 2x^2 - x - 1$

9. $q(x) = 3x^4 + x^3 - 3x^2 + 7x + 5$

10. $h(x) = 7x^4 + 3x^3 - 2x^2 - x + 1$

Find all the zeros of each function.

11. $h(x) = 2x^3 + 3x^2 - 65x + 84$

12. $p(x) = x^3 - 3x^2 + 9x - 7$

13. $h(x) = x^3 - 7x^2 + 17x - 15$

14. $q(x) = x^4 + 50x^2 + 49$

15. $g(x) = x^4 + 4x^3 - 3x^2 - 14x - 8$

16. $f(x) = x^4 - 6x^3 + 6x^2 + 24x - 40$

Write a polynomial function of least degree with integral coefficients that has the given zeros.

17. $-5, 3i$

18. $-2, 3 + i$

19. $-1, 4, 3i$

20. $2, 5, 1 + i$

21. CRAFTS Stephan has a set of plans to build a wooden box. He wants to reduce the volume of the box to 105 cubic inches. He would like to reduce the length of each dimension in the plan by the same amount. The plans call for the box to be 10 inches by 8 inches by 6 inches. Write and solve a polynomial equation to find out how much Stephen should take from each dimension.

7-5 Reading to Learn Mathematics

Roots and Zeros

Pre-Activity How can the roots of an equation be used in pharmacology?

Read the introduction to Lesson 7-5 at the top of page 371 in your textbook.

Using the model given in the introduction, write a polynomial equation with 0 on one side that can be solved to find the time or times at which there is 100 milligrams of medication in a patient's bloodstream.

Reading the Lesson

1. Indicate whether each statement is *true* or *false*.
 - a. Every polynomial equation of degree greater than one has at least one root in the set of real numbers.
 - b. If c is a root of the polynomial equation $f(x) = 0$, then $(x - c)$ is a factor of the polynomial $f(x)$.
 - c. If $(x + c)$ is a factor of the polynomial $f(x)$, then c is a zero of the polynomial function f .
 - d. A polynomial function f of degree n has exactly $(n - 1)$ complex zeros.

2. Let $f(x) = x^6 - 2x^5 + 3x^4 - 4x^3 + 5x^2 + 6x - 7$.
 - a. What are the possible numbers of positive real zeros of f ?
 - b. Write $f(-x)$ in simplified form (with no parentheses).

What are the possible numbers of negative real zeros of f ?

- c. Complete the following chart to show the possible combinations of positive real zeros, negative real zeros, and imaginary zeros of the polynomial function f .

Number of Positive Real Zeros	Number of Negative Real Zeros	Number of Imaginary Zeros	Total Number of Zeros

Helping You Remember

3. It is easier to remember mathematical concepts and results if you relate them to each other. How can the Complex Conjugates Theorem help you remember the part of Descartes' Rule of Signs that says, "or is less than this number by an even number."

7-5 Enrichment

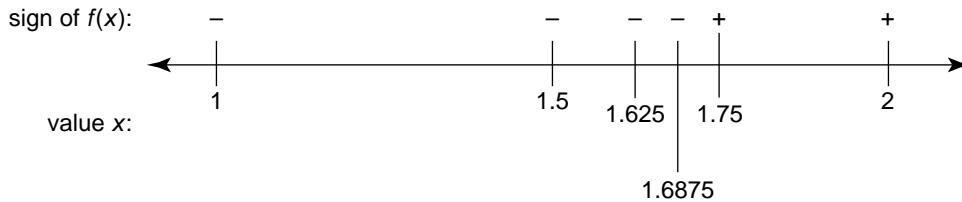
The Bisection Method for Approximating Real Zeros

The **bisection method** can be used to approximate zeros of polynomial functions like $f(x) = x^3 + x^2 - 3x - 3$.

Since $f(1) = -4$ and $f(2) = 3$, there is at least one real zero between 1 and 2.

The midpoint of this interval is $\frac{1+2}{2} = 1.5$. Since $f(1.5) = -1.875$, the zero is between 1.5 and 2. The midpoint of this interval is $\frac{1.5+2}{2} = 1.75$. Since $f(1.75)$ is about 0.172, the zero is between 1.5 and 1.75. The midpoint of this interval is $\frac{1.5+1.75}{2} = 1.625$ and $f(1.625)$ is about -0.94 . The zero is between 1.625 and 1.75. The midpoint of this interval is $\frac{1.625+1.75}{2} = 1.6875$. Since $f(1.6875)$ is about -0.41 , the zero is between 1.6875 and 1.75. Therefore, the zero is 1.7 to the nearest tenth.

The diagram below summarizes the results obtained by the bisection method.



Using the bisection method, approximate to the nearest tenth the zero between the two integral values of x for each function.

1. $f(x) = x^3 - 4x^2 - 11x + 2$, $f(0) = 2$, $f(1) = -12$

2. $f(x) = 2x^4 + x^2 - 15$, $f(1) = -12$, $f(2) = 21$

3. $f(x) = x^5 - 2x^3 - 12$, $f(1) = -13$, $f(2) = 4$

4. $f(x) = 4x^3 - 2x + 7$, $f(-2) = -21$, $f(-1) = 5$

5. $f(x) = 3x^3 - 14x^2 - 27x + 126$, $f(4) = -14$, $f(5) = 16$

7-6 Study Guide and Intervention**Rational Zero Theorem****Identify Rational Zeros**

Rational Zero Theorem	Let $f(x) = a_0x^n + a_1x^{n-1} + \dots + a_{n-2}x^2 + a_{n-1}x + a^n$ represent a polynomial function with integral coefficients. If $\frac{p}{q}$ is a rational number in simplest form and is a zero of $y = f(x)$, then p is a factor of a_n and q is a factor of a_0 .
Corollary (Integral Zero Theorem)	If the coefficients of a polynomial are integers such that $a_0 = 1$ and $a_n \neq 0$, any rational zeros of the function must be factors of a_n .

Example**List all of the possible rational zeros of each function.**

a. $f(x) = 3x^4 - 2x^2 + 6x - 10$

If $\frac{p}{q}$ is a rational root, then p is a factor of -10 and q is a factor of 3 . The possible values for p are $\pm 1, \pm 2, \pm 5, \text{ and } \pm 10$. The possible values for q are ± 1 and ± 3 . So all of the possible rational zeros are $\frac{p}{q} = \pm 1, \pm 2, \pm 5, \pm 10, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{5}{3}, \text{ and } \pm \frac{10}{3}$.

b. $g(x) = x^3 - 10x^2 + 14x - 36$

Since the coefficient of x^3 is 1 , the possible rational zeros must be the factors of the constant term -36 . So the possible rational zeros are $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 9, \pm 12, \pm 18, \text{ and } \pm 36$.

Exercises**List all of the possible rational zeros of each function.**

1. $f(x) = x^3 + 3x^2 - x + 8$

2. $g(x) = x^5 - 7x^4 + 3x^2 + x - 20$

3. $h(x) = x^4 - 7x^3 - 4x^2 + x - 49$

4. $p(x) = 2x^4 - 5x^3 + 8x^2 + 3x - 5$

5. $q(x) = 3x^4 - 5x^3 + 10x + 12$

6. $r(x) = 4x^5 - 2x + 18$

7. $f(x) = x^7 - 6x^5 - 3x^4 + x^3 + 4x^2 - 120$

8. $g(x) = 5x^6 - 3x^4 + 5x^3 + 2x^2 - 15$

9. $h(x) = 6x^5 - 3x^4 + 12x^3 + 18x^2 - 9x + 21$

10. $p(x) = 2x^7 - 3x^6 + 11x^5 - 20x^2 + 11$

7-6 Study Guide and Intervention *(continued)***Rational Zero Theorem****Find Rational Zeros****Example 1** Find all of the rational zeros of $f(x) = 5x^3 + 12x^2 - 29x + 12$.

From the corollary to the Fundamental Theorem of Algebra, we know that there are exactly 3 complex roots. According to Descartes' Rule of Signs there are 2 or 0 positive real roots and 1 negative real root. The possible rational zeros are $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12, \pm \frac{1}{5}, \pm \frac{2}{5}, \pm \frac{3}{5}, \pm \frac{4}{5}, \pm \frac{6}{5}, \pm \frac{12}{5}$. Make a table and test some possible rational zeros.

$\frac{p}{q}$	5	12	-29	12
1	5	17	-12	0

Since $f(1) = 0$, you know that $x = 1$ is a zero.

The depressed polynomial is $5x^2 + 17x - 12$, which can be factored as $(5x - 3)(x + 4)$.

By the Zero Product Property, this expression equals 0 when $x = \frac{3}{5}$ or $x = -4$.

The rational zeros of this function are $1, \frac{3}{5}$, and -4 .

Example 2 Find all of the zeros of $f(x) = 8x^4 + 2x^3 + 5x^2 + 2x - 3$.

There are 4 complex roots, with 1 positive real root and 3 or 1 negative real roots. The possible rational zeros are $\pm 1, \pm 3, \pm \frac{1}{2}, \pm \frac{1}{4}, \pm \frac{1}{8}, \pm \frac{3}{2}, \pm \frac{3}{4},$ and $\pm \frac{3}{8}$.

Make a table and test some possible values.

$\frac{p}{q}$	8	2	5	2	-3
1	8	10	15	17	14
2	8	18	41	84	165
$\frac{1}{2}$	8	6	8	6	0

Since $f\left(\frac{1}{2}\right) = 0$, we know that $x = \frac{1}{2}$ is a root.

The depressed polynomial is $8x^3 + 6x^2 + 8x + 6$.

Try synthetic substitution again. Any remaining rational roots must be negative.

$\frac{p}{q}$	8	6	8	6
$-\frac{1}{4}$	8	4	7	$4\frac{1}{4}$
$-\frac{3}{4}$	8	0	8	0

$x = -\frac{3}{4}$ is another rational root.

The depressed polynomial is $8x^2 + 8 = 0$, which has roots $\pm i$.

The zeros of this function are $\frac{1}{2}, -\frac{3}{4}$, and $\pm i$.

Exercises

Find all of the rational zeros of each function.

1. $f(x) = x^3 + 4x^2 - 25x - 28$

2. $f(x) = x^3 + 6x^2 + 4x + 24$

Find all of the zeros of each function.

3. $f(x) = x^4 + 2x^3 - 11x^2 + 8x - 60$

4. $f(x) = 4x^4 + 5x^3 + 30x^2 + 45x - 54$

7-6

Skills Practice

Rational Zero Theorem

List all of the possible rational zeros of each function.

1. $n(x) = x^2 + 5x + 3$

2. $h(x) = x^2 - 2x - 5$

3. $w(x) = x^2 - 5x + 12$

4. $f(x) = 2x^2 + 5x + 3$

5. $q(x) = 6x^3 + x^2 - x + 2$

6. $g(x) = 9x^4 + 3x^3 + 3x^2 - x + 27$

Find all of the rational zeros of each function.

7. $f(x) = x^3 - 2x^2 + 5x - 4 = 0$

8. $g(x) = x^3 - 3x^2 - 4x + 12$

9. $p(x) = x^3 - x^2 + x - 1$

10. $z(x) = x^3 - 4x^2 + 6x - 4$

11. $h(x) = x^3 - x^2 + 4x - 4$

12. $g(x) = 3x^3 - 9x^2 - 10x - 8$

13. $g(x) = 2x^3 + 7x^2 - 7x - 12$

14. $h(x) = 2x^3 - 5x^2 - 4x + 3$

15. $p(x) = 3x^3 - 5x^2 - 14x - 4 = 0$

16. $q(x) = 3x^3 + 2x^2 + 27x + 18$

17. $q(x) = 3x^3 - 7x^2 + 4$

18. $f(x) = x^4 - 2x^3 - 13x^2 + 14x + 24$

19. $p(x) = x^4 - 5x^3 - 9x^2 - 25x - 70$

20. $n(x) = 16x^4 - 32x^3 - 13x^2 + 29x - 6$

Find all of the zeros of each function.

21. $f(x) = x^3 + 5x^2 + 11x + 15$

22. $q(x) = x^3 - 10x^2 + 18x - 4$

23. $m(x) = 6x^4 - 17x^3 + 8x^2 + 8x - 3$

24. $g(x) = x^4 + 4x^3 + 5x^2 + 4x + 4$

7-6

Practice

Rational Zero Theorem

List all of the possible rational zeros of each function.

1. $h(x) = x^3 - 5x^2 + 2x + 12$

2. $s(x) = x^4 - 8x^3 + 7x - 14$

3. $f(x) = 3x^5 - 5x^2 + x + 6$

4. $p(x) = 3x^2 + x + 7$

5. $g(x) = 5x^3 + x^2 - x + 8$

6. $q(x) = 6x^5 + x^3 - 3$

Find all of the rational zeros of each function.

7. $q(x) = x^3 + 3x^2 - 6x - 8 = 0$

8. $v(x) = x^3 - 9x^2 + 27x - 27$

9. $c(x) = x^3 - x^2 - 8x + 12$

10. $f(x) = x^4 - 49x^2$

11. $h(x) = x^3 - 7x^2 + 17x - 15$

12. $b(x) = x^3 + 6x + 20$

13. $f(x) = x^3 - 6x^2 + 4x - 24$

14. $g(x) = 2x^3 + 3x^2 - 4x - 4$

15. $h(x) = 2x^3 - 7x^2 - 21x + 54 = 0$

16. $z(x) = x^4 - 3x^3 + 5x^2 - 27x - 36$

17. $d(x) = x^4 + x^3 + 16$

18. $n(x) = x^4 - 2x^3 - 3$

19. $p(x) = 2x^4 - 7x^3 + 4x^2 + 7x - 6$

20. $q(x) = 6x^4 + 29x^3 + 40x^2 + 7x - 12$

Find all of the zeros of each function.

21. $f(x) = 2x^4 + 7x^3 - 2x^2 - 19x - 12$

22. $q(x) = x^4 - 4x^3 + x^2 + 16x - 20$

23. $h(x) = x^6 - 8x^3$

24. $g(x) = x^6 - 1$

25. TRAVEL The height of a box that Joan is shipping is 3 inches less than the width of the box. The length is 2 inches more than twice the width. The volume of the box is 1540 in^3 . What are the dimensions of the box?

26. GEOMETRY The height of a square pyramid is 3 meters shorter than the side of its base. If the volume of the pyramid is 432 m^3 , how tall is it? Use the formula $V = \frac{1}{3}Bh$.

7-6

Reading to Learn Mathematics***Rational Zero Theorem***

Pre-Activity How can the Rational Zero Theorem solve problems involving large numbers?

Read the introduction to Lesson 7-6 at the top of page 378 in your textbook.

Rewrite the polynomial equation $w(w + 8)(w - 5) = 2772$ in the form $f(x) = 0$, where $f(x)$ is a polynomial written in descending powers of x .

Reading the Lesson

1. For each of the following polynomial functions, list all the possible values of p , all the possible values of q , and all the possible rational zeros $\frac{p}{q}$.

a. $f(x) = x^3 - 2x^2 - 11x + 12$

possible values of p :

possible values of q :

possible values of $\frac{p}{q}$:

b. $f(x) = 2x^4 + 9x^3 - 23x^2 - 81x + 45$

possible values of p :

possible values of q :

possible values of $\frac{p}{q}$:

2. Explain in your own words how Descartes' Rule of Signs, the Rational Zero Theorem, and synthetic division can be used together to find all of the rational zeros of a polynomial function with integer coefficients.

Helping You Remember

3. Some students have trouble remembering which numbers go in the numerators and which go in the denominators when forming a list of possible rational zeros of a polynomial function. How can you use the linear polynomial equation $ax + b = 0$, where a and b are nonzero integers, to remember this?

7-6 Enrichment

Infinite Continued Fractions

Some infinite expressions are actually equal to real numbers! The infinite continued fraction at the right is one example.

If you use x to stand for the infinite fraction, then the entire denominator of the first fraction on the right is also equal to x . This observation leads to the following equation:

$$x = 1 + \frac{1}{x}$$

$$x = 1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \dots}}}}$$

Write a decimal for each continued fraction.

1. $1 + \frac{1}{1}$

2. $1 + \frac{1}{1 + \frac{1}{1}}$

3. $1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1}}}$

4. $1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1}}}}$

5. $1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1}}}}}$

6. The more terms you add to the fractions above, the closer their value approaches the value of the infinite continued fraction. What value do the fractions seem to be approaching?

7. Rewrite $x = 1 + \frac{1}{x}$ as a quadratic equation and solve for x .

8. Find the value of the following infinite continued fraction.

$$3 + \frac{1}{3 + \frac{1}{3 + \frac{1}{3 + \frac{1}{3 + \frac{1}{3 + \dots}}}}}$$

7-7

Study Guide and Intervention

Operations on Functions

Arithmetic Operations

Operations with Functions	Sum	$(f + g)(x) = f(x) + g(x)$
	Difference	$(f - g)(x) = f(x) - g(x)$
	Product	$(f \cdot g)(x) = f(x) \cdot g(x)$
	Quotient	$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, g(x) \neq 0$

Example Find $(f + g)(x)$, $(f - g)(x)$, $(f \cdot g)(x)$, and $\left(\frac{f}{g}\right)(x)$ for $f(x) = x^2 + 3x - 4$ and $g(x) = 3x - 2$.

$$(f + g)(x) = f(x) + g(x)$$

$$= (x^2 + 3x - 4) + (3x - 2)$$

$$= x^2 + 6x - 6$$

Addition of functions

$$f(x) = x^2 + 3x - 4, g(x) = 3x - 2$$

Simplify.

$$(f - g)(x) = f(x) - g(x)$$

$$= (x^2 + 3x - 4) - (3x - 2)$$

$$= x^2 - 2$$

Subtraction of functions

$$f(x) = x^2 + 3x - 4, g(x) = 3x - 2$$

Simplify.

$$(f \cdot g)(x) = f(x) \cdot g(x)$$

$$= (x^2 + 3x - 4)(3x - 2)$$

$$= x^2(3x - 2) + 3x(3x - 2) - 4(3x - 2)$$

$$= 3x^3 - 2x^2 + 9x^2 - 6x - 12x + 8$$

$$= 3x^3 + 7x^2 - 18x + 8$$

Multiplication of functions

$$f(x) = x^2 + 3x - 4, g(x) = 3x - 2$$

Distributive Property

Distributive Property

Simplify.

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$$

Division of functions

$$= \frac{x^2 + 3x - 4}{3x - 2}, x \neq \frac{2}{3}$$

$$f(x) = x^2 + 3x - 4 \text{ and } g(x) = 3x - 2$$

Exercises

Find $(f + g)(x)$, $(f - g)(x)$, $(f \cdot g)(x)$, and $\left(\frac{f}{g}\right)(x)$ for each $f(x)$ and $g(x)$.

1. $f(x) = 8x - 3$; $g(x) = 4x + 5$

2. $f(x) = x^2 + x - 6$; $g(x) = x - 2$

3. $f(x) = 3x^2 - x + 5$; $g(x) = 2x - 3$

4. $f(x) = 2x - 1$; $g(x) = 3x^2 + 11x - 4$

5. $f(x) = x^2 - 1$; $g(x) = \frac{1}{x + 1}$

7-7

Study Guide and Intervention *(continued)*

Operations on Functions

Composition of Functions

Composition of Functions

Suppose f and g are functions such that the range of g is a subset of the domain of f . Then the composite function $f \circ g$ can be described by the equation $[f \circ g](x) = f[g(x)]$.

Example 1

For $f = \{(1, 2), (3, 3), (2, 4), (4, 1)\}$ and $g = \{(1, 3), (3, 4), (2, 2), (4, 1)\}$, find $f \circ g$ and $g \circ f$ if they exist.

$$f[g(1)] = f(3) = 3 \quad f[g(2)] = f(2) = 4 \quad f[g(3)] = f(4) = 1 \quad f[g(4)] = f(1) = 2$$

$$f \circ g = \{(1, 3), (2, 4), (3, 1), (4, 2)\}$$

$$g[f(1)] = g(2) = 2 \quad g[f(2)] = g(4) = 1 \quad g[f(3)] = g(3) = 4 \quad g[f(4)] = g(1) = 3$$

$$g \circ f = \{(1, 2), (2, 1), (3, 4), (4, 3)\}$$

Example 2

Find $[g \circ h](x)$ and $[h \circ g](x)$ for $g(x) = 3x - 4$ and $h(x) = x^2 - 1$.

$$[g \circ h](x) = g[h(x)]$$

$$= g(x^2 - 1)$$

$$= 3(x^2 - 1) - 4$$

$$= 3x^2 - 7$$

$$[h \circ g](x) = h[g(x)]$$

$$= h(3x - 4)$$

$$= (3x - 4)^2 - 1$$

$$= 9x^2 - 24x + 16 - 1$$

$$= 9x^2 - 24x + 15$$

Exercises

For each set of ordered pairs, find $f \circ g$ and $g \circ f$ if they exist.

1. $f = \{(-1, 2), (5, 6), (0, 9)\}$,
 $g = \{(6, 0), (2, -1), (9, 5)\}$

2. $f = \{(5, -2), (9, 8), (-4, 3), (0, 4)\}$,
 $g = \{(3, 7), (-2, 6), (4, -2), (8, 10)\}$

Find $[f \circ g](x)$ and $[g \circ f](x)$.

3. $f(x) = 2x + 7$; $g(x) = -5x - 1$

4. $f(x) = x^2 - 1$; $g(x) = -4x^2$

5. $f(x) = x^2 + 2x$; $g(x) = x - 9$

6. $f(x) = 5x + 4$; $g(x) = 3 - x$

7-7

Skills Practice

Operations on Functions

Find $(f + g)(x)$, $(f - g)(x)$, $(f \cdot g)(x)$, and $\left(\frac{f}{g}\right)(x)$ for each $f(x)$ and $g(x)$.

1. $f(x) = x + 5$

2. $f(x) = 3x + 1$

$g(x) = x - 4$

$g(x) = 2x - 3$

3. $f(x) = x^2$

4. $f(x) = 3x^2$

$g(x) = 4 - x$

$g(x) = \frac{5}{x}$

For each set of ordered pairs, find $f \circ g$ and $g \circ f$ if they exist.

5. $f = \{(0, 0), (4, -2)\}$
 $g = \{(0, 4), (-2, 0), (5, 0)\}$

6. $f = \{(0, -3), (1, 2), (2, 2)\}$
 $g = \{(-3, 1), (2, 0)\}$

7. $f = \{(-4, 3), (-1, 1), (2, 2)\}$
 $g = \{(1, -4), (2, -1), (3, -1)\}$

8. $f = \{(6, 6), (-3, -3), (1, 3)\}$
 $g = \{(-3, 6), (3, 6), (6, -3)\}$

Find $[g \circ h](x)$ and $[h \circ g](x)$.

9. $g(x) = 2x$
 $h(x) = x + 2$

10. $g(x) = -3x$
 $h(x) = 4x - 1$

11. $g(x) = x - 6$
 $h(x) = x + 6$

12. $g(x) = x - 3$
 $h(x) = x^2$

13. $g(x) = 5x$
 $h(x) = x^2 + x - 1$

14. $g(x) = x + 2$
 $h(x) = 2x^2 - 3$

If $f(x) = 3x$, $g(x) = x + 4$, and $h(x) = x^2 - 1$, find each value.

15. $f[g(1)]$

16. $g[h(0)]$

17. $g[f(-1)]$

18. $h[f(5)]$

19. $g[h(-3)]$

20. $h[f(10)]$

7-7

Practice

Operations on Functions

Find $(f + g)(x)$, $(f - g)(x)$, $(f \cdot g)(x)$, and $\left(\frac{f}{g}\right)(x)$ for each $f(x)$ and $g(x)$.

$$1. \begin{aligned} f(x) &= 2x + 1 \\ g(x) &= x - 3 \end{aligned}$$

$$2. \begin{aligned} f(x) &= 8x^2 \\ g(x) &= \frac{1}{x^2} \end{aligned}$$

$$3. \begin{aligned} f(x) &= x^2 + 7x + 12 \\ g(x) &= x^2 - 9 \end{aligned}$$

For each set of ordered pairs, find $f \circ g$ and $g \circ f$ if they exist.

$$4. \begin{aligned} f &= \{(-9, -1), (-1, 0), (3, 4)\} \\ g &= \{(0, -9), (-1, 3), (4, -1)\} \end{aligned}$$

$$5. \begin{aligned} f &= \{(-4, 3), (0, -2), (1, -2)\} \\ g &= \{(-2, 0), (3, 1)\} \end{aligned}$$

$$6. \begin{aligned} f &= \{(-4, -5), (0, 3), (1, 6)\} \\ g &= \{(6, 1), (-5, 0), (3, -4)\} \end{aligned}$$

$$7. \begin{aligned} f &= \{(0, -3), (1, -3), (6, 8)\} \\ g &= \{(8, 2), (-3, 0), (-3, 1)\} \end{aligned}$$

Find $[g \circ h](x)$ and $[h \circ g](x)$.

$$8. \begin{aligned} g(x) &= 3x \\ h(x) &= x - 4 \end{aligned}$$

$$9. \begin{aligned} g(x) &= -8x \\ h(x) &= 2x + 3 \end{aligned}$$

$$10. \begin{aligned} g(x) &= x + 6 \\ h(x) &= 3x^2 \end{aligned}$$

$$11. \begin{aligned} g(x) &= x + 3 \\ h(x) &= 2x^2 \end{aligned}$$

$$12. \begin{aligned} g(x) &= -2x \\ h(x) &= x^2 + 3x + 2 \end{aligned}$$

$$13. \begin{aligned} g(x) &= x - 2 \\ h(x) &= 3x^2 + 1 \end{aligned}$$

If $f(x) = x^2$, $g(x) = 5x$, and $h(x) = x + 4$, find each value.

$$14. f[g(1)]$$

$$15. g[h(-2)]$$

$$16. h[f(4)]$$

$$17. f[h(-9)]$$

$$18. h[g(-3)]$$

$$19. g[f(8)]$$

$$20. h[f(20)]$$

$$21. [f \circ (h \circ g)](-1)$$

$$22. [f \circ (g \circ h)](4)$$

23. BUSINESS The function $f(x) = 1000 - 0.01x^2$ models the manufacturing cost per item when x items are produced, and $g(x) = 150 - 0.001x^2$ models the service cost per item. Write a function $C(x)$ for the total manufacturing and service cost per item.

24. MEASUREMENT The formula $f = \frac{n}{12}$ converts inches n to feet f , and $m = \frac{f}{5280}$ converts feet to miles m . Write a composition of functions that converts inches to miles.

7-7

Reading to Learn Mathematics

Operations on Functions

Pre-Activity Why is it important to combine functions in business?

Read the introduction to Lesson 7-7 at the top of page 383 in your textbook.

Describe two ways to calculate Ms. Coffmon's profit from the sale of 50 birdhouses. (Do not actually calculate her profit.)

Reading the Lesson

- Determine whether each statement is *true* or *false*. (Remember that *true* means *always true*.)
 - If f and g are polynomial functions, then $f + g$ is a polynomial function.
 - If f and g are polynomial functions, then $\frac{f}{g}$ is a polynomial function.
 - If f and g are polynomial functions, the domain of the function $f \cdot g$ is the set of all real numbers.
 - If $f(x) = 3x + 2$ and $g(x) = x - 4$, the domain of the function $\frac{f}{g}$ is the set of all real numbers.
 - If f and g are polynomial functions, then $(f \circ g)(x) = (g \circ f)(x)$.
 - If f and g are polynomial functions, then $(f \cdot g)(x) = (g \cdot f)(x)$.
- Let $f(x) = 2x - 5$ and $g(x) = x^2 + 1$.
 - Explain in words how you would find $(f \circ g)(-3)$. (Do not actually do any calculations.)
 - Explain in words how you would find $(g \circ f)(-3)$. (Do not actually do any calculations.)

Helping You Remember

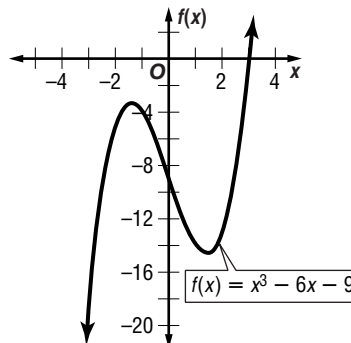
- Some students have trouble remembering the correct order in which to apply the two original functions when evaluating a composite function. Write three sentences, each of which explains how to do this in a slightly different way. (Hint: Use the word *closest* in the first sentence, the words *inside* and *outside* in the second, and the words *left* and *right* in the third.)

7-7 Enrichment

Relative Maximum Values

The graph of $f(x) = x^3 - 6x - 9$ shows a relative maximum value somewhere between $f(-2)$ and $f(-1)$. You can obtain a closer approximation by comparing values such as those shown in the table.

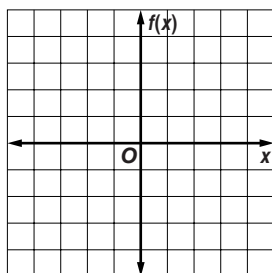
To the nearest tenth a relative maximum value for $f(x)$ is -3.3 .



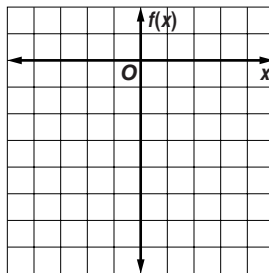
x	$f(x)$
-2	-5
-1.5	-3.375
-1.4	-3.344
-1.3	-3.397
-1	-4

Using a calculator to find points, graph each function. To the nearest tenth, find a relative maximum value of the function.

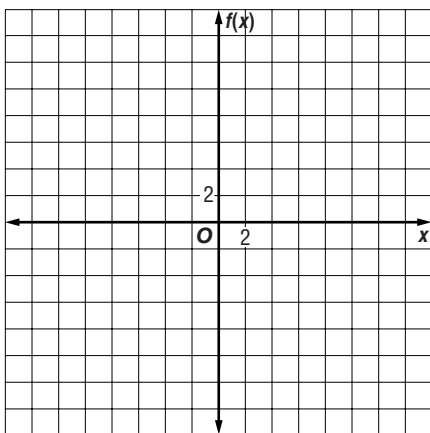
1. $f(x) = x(x^2 - 3)$



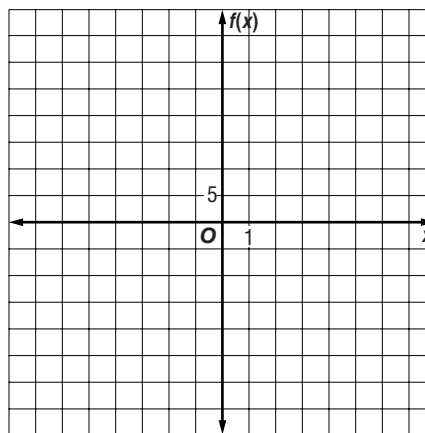
2. $f(x) = x^3 - 3x - 3$



3. $f(x) = x^3 - 9x - 2$



4. $f(x) = x^3 + 2x^2 - 12x - 24$



7-8 Study Guide and Intervention

Inverse Functions and Relations

Find Inverses

Inverse Relations	Two relations are inverse relations if and only if whenever one relation contains the element (a, b) , the other relation contains the element (b, a) .
Property of Inverse Functions	Suppose f and f^{-1} are inverse functions. Then $f(a) = b$ if and only if $f^{-1}(b) = a$.

Example Find the inverse of the function $f(x) = \frac{2}{5}x - \frac{1}{5}$. Then graph the function and its inverse.

Step 1 Replace $f(x)$ with y in the original equation.

$$f(x) = \frac{2}{5}x - \frac{1}{5} \rightarrow y = \frac{2}{5}x - \frac{1}{5}$$

Step 2 Interchange x and y .

$$x = \frac{2}{5}y - \frac{1}{5}$$

Step 3 Solve for y .

$$x = \frac{2}{5}y - \frac{1}{5}$$

$$5x = 2y - 1$$

$$5x + 1 = 2y$$

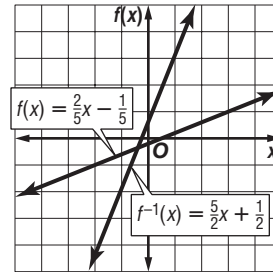
$$\frac{1}{2}(5x + 1) = y$$

Inverse

Multiply each side by 5.

Add 1 to each side.

Divide each side by 2.



The inverse of $f(x) = \frac{2}{5}x - \frac{1}{5}$ is $f^{-1}(x) = \frac{1}{2}(5x + 1)$.

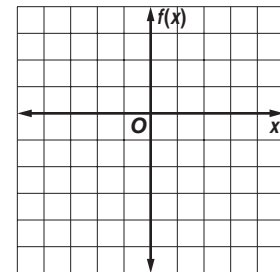
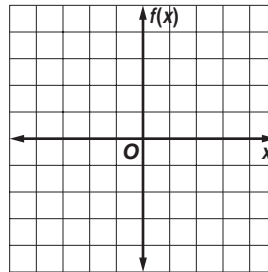
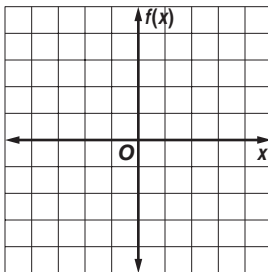
Exercises

Find the inverse of each function. Then graph the function and its inverse.

1. $f(x) = \frac{2}{3}x - 1$

2. $f(x) = 2x - 3$

3. $f(x) = \frac{1}{4}x - 2$



7-8 Study Guide and Intervention *(continued)***Inverse Functions and Relations****Inverses of Relations and Functions**

Inverse Functions	Two functions f and g are inverse functions if and only if $[f \circ g](x) = x$ and $[g \circ f](x) = x$.
--------------------------	--

Example 1 Determine whether $f(x) = 2x - 7$ and $g(x) = \frac{1}{2}(x + 7)$ are inverse functions.

$$\begin{aligned}
 [f \circ g](x) &= f[g(x)] & [g \circ f](x) &= g[f(x)] \\
 &= f\left[\frac{1}{2}(x + 7)\right] & &= g(2x - 7) \\
 &= 2\left[\frac{1}{2}(x + 7)\right] - 7 & &= \frac{1}{2}(2x - 7 + 7) \\
 &= x + 7 - 7 & &= x \\
 &= x & &
 \end{aligned}$$

The functions are inverses since both $[f \circ g](x) = x$ and $[g \circ f](x) = x$.

Example 2 Determine whether $f(x) = 4x + \frac{1}{3}$ and $g(x) = \frac{1}{4}x - 3$ are inverse functions.

$$\begin{aligned}
 [f \circ g](x) &= f[g(x)] \\
 &= f\left(\frac{1}{4}x - 3\right) \\
 &= 4\left(\frac{1}{4}x - 3\right) + \frac{1}{3} \\
 &= x - 12 + \frac{1}{3} \\
 &= x - 11\frac{2}{3}
 \end{aligned}$$

Since $[f \circ g](x) \neq x$, the functions are not inverses.

Exercises

Determine whether each pair of functions are inverse functions.

1. $f(x) = 3x - 1$
 $g(x) = \frac{1}{3}x + \frac{1}{3}$

2. $f(x) = \frac{1}{4}x + 5$
 $g(x) = 4x - 20$

3. $f(x) = \frac{1}{2}x - 10$
 $g(x) = 2x + \frac{1}{10}$

4. $f(x) = 2x + 5$
 $g(x) = 5x + 2$

5. $f(x) = 8x - 12$
 $g(x) = \frac{1}{8}x + 12$

6. $f(x) = -2x + 3$
 $g(x) = -\frac{1}{2}x + \frac{3}{2}$

7. $f(x) = 4x - \frac{1}{2}$
 $g(x) = \frac{1}{4}x + \frac{1}{8}$

8. $f(x) = 2x - \frac{3}{5}$
 $g(x) = \frac{1}{10}(5x + 3)$

9. $f(x) = 4x + \frac{1}{2}$
 $g(x) = \frac{1}{2}x - \frac{3}{2}$

10. $f(x) = 10 - \frac{x}{2}$
 $g(x) = 20 - 2x$

11. $f(x) = 4x - \frac{4}{5}$
 $g(x) = \frac{x}{4} + \frac{1}{5}$

12. $f(x) = 9 + \frac{3}{2}x$
 $g(x) = \frac{2}{3}x - 6$

7-8 Skills Practice

Inverse Functions and Relations

Find the inverse of each relation.

1. $\{(3, 1), (4, -3), (8, -3)\}$

2. $\{(-7, 1), (0, 5), (5, -1)\}$

3. $\{(-10, -2), (-7, 6), (-4, -2), (-4, 0)\}$

4. $\{(0, -9), (5, -3), (6, 6), (8, -3)\}$

5. $\{(-4, 12), (0, 7), (9, -1), (10, -5)\}$

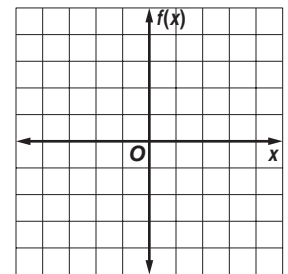
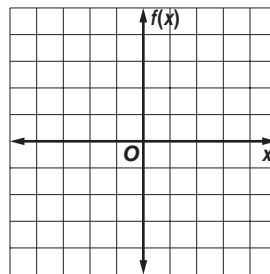
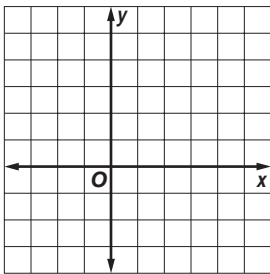
6. $\{(-4, 1), (-4, 3), (0, -8), (8, -9)\}$

Find the inverse of each function. Then graph the function and its inverse.

7. $y = 4$

8. $f(x) = 3x$

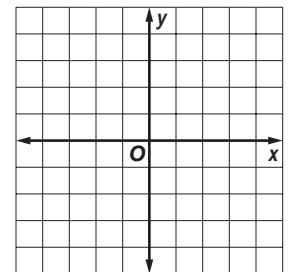
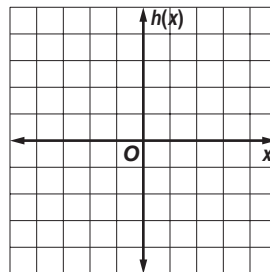
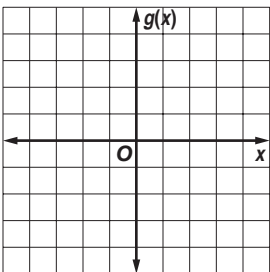
9. $f(x) = x + 2$



10. $g(x) = 2x - 1$

11. $h(x) = \frac{1}{4}x$

12. $y = \frac{2}{3}x + 2$



Determine whether each pair of functions are inverse functions.

13. $f(x) = x - 1$
 $g(x) = 1 - x$

14. $f(x) = 2x + 3$
 $g(x) = \frac{1}{2}(x - 3)$

15. $f(x) = 5x - 5$
 $g(x) = \frac{1}{5}x + 1$

16. $f(x) = 2x$
 $g(x) = \frac{1}{2}x$

17. $h(x) = 6x - 2$
 $g(x) = \frac{1}{6}x + 3$

18. $f(x) = 8x - 10$
 $g(x) = \frac{1}{8}x + \frac{5}{4}$

7-8 Practice

Inverse Functions and Relations

Find the inverse of each relation.

1. $\{(0, 3), (4, 2), (5, -6)\}$

2. $\{(-5, 1), (-5, -1), (-5, 8)\}$

3. $\{(-3, -7), (0, -1), (5, 9), (7, 13)\}$

4. $\{(8, -2), (10, 5), (12, 6), (14, 7)\}$

5. $\{(-5, -4), (1, 2), (3, 4), (7, 8)\}$

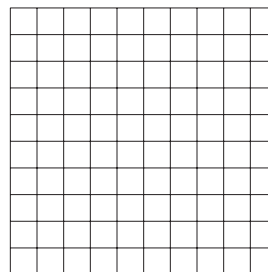
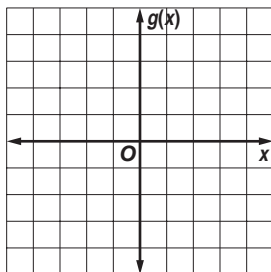
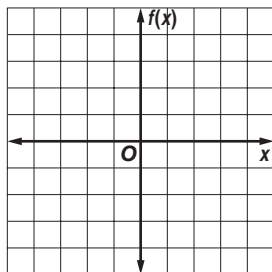
6. $\{(-3, 9), (-2, 4), (0, 0), (1, 1)\}$

Find the inverse of each function. Then graph the function and its inverse.

7. $f(x) = \frac{3}{4}x$

8. $g(x) = 3 + x$

9. $y = 3x - 2$



Determine whether each pair of functions are inverse functions.

10. $f(x) = x + 6$

$g(x) = x - 6$

11. $f(x) = -4x + 1$

$g(x) = \frac{1}{4}(1 - x)$

12. $g(x) = 13x - 13$

$h(x) = \frac{1}{13}x - 1$

13. $f(x) = 2x$

$g(x) = -2x$

14. $f(x) = \frac{6}{7}x$

$g(x) = \frac{7}{6}x$

15. $g(x) = 2x - 8$

$h(x) = \frac{1}{2}x + 4$

16. MEASUREMENT The points (63, 121), (71, 180), (67, 140), (65, 108), and (72, 165) give the weight in pounds as a function of height in inches for 5 students in a class. Give the points for these students that represent height as a function of weight.

REMODELING For Exercises 17 and 18, use the following information.

The Clearys are replacing the flooring in their 15 foot by 18 foot kitchen. The new flooring costs \$17.99 per square yard. The formula $f(x) = 9x$ converts square yards to square feet.

17. Find the inverse $f^{-1}(x)$. What is the significance of $f^{-1}(x)$ for the Clearys?

18. What will the new flooring cost the Cleary's?

7-8

Reading to Learn Mathematics***Inverse Functions and Relations*****Pre-Activity** How are inverse functions related to measurement conversions?

Read the introduction to Lesson 7-8 at the top of page 390 in your textbook.

A function multiplies a number by 3 and then adds 5 to the result. What does the inverse function do, and in what order?

Reading the Lesson**1.** Complete each statement.

- If two relations are inverses, the domain of one relation is the _____ of the other.
- Suppose that $g(x)$ is a relation and that the point $(4, -2)$ is on its graph. Then a point on the graph of $g^{-1}(x)$ is _____.
- The _____ test can be used on the graph of a function to determine whether the function has an inverse function.
- If you are given the graph of a function, you can find the graph of its inverse by reflecting the original graph over the line with equation _____.
- If f and g are inverse functions, then $(f \circ g)(x) = \underline{\hspace{2cm}}$ and $(g \circ f)(x) = \underline{\hspace{2cm}}$.
- A function has an inverse that is also a function only if the given function is _____.
- Suppose that $h(x)$ is a function whose inverse is also a function. If $h(5) = 12$, then $h^{-1}(12) = \underline{\hspace{2cm}}$.

2. Assume that $f(x)$ is a one-to-one function defined by an algebraic equation. Write the four steps you would follow in order to find the equation for $f^{-1}(x)$.

- _____
- _____
- _____
- _____

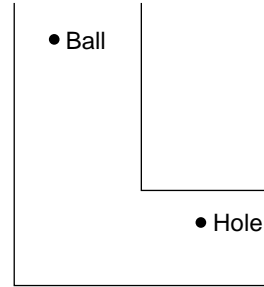
Helping You Remember

- A good way to remember something new is to relate it to something you already know. How are the vertical and horizontal line tests related?

7-8 Enrichment

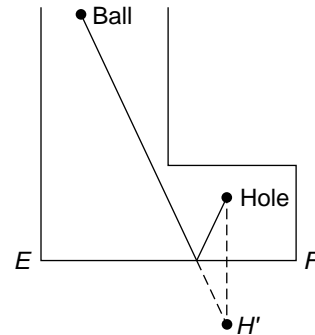
Miniature Golf

In miniature golf, the object of the game is to roll the golf ball into the hole in as few shots as possible. As in the diagram at the right, the hole is often placed so that a direct shot is impossible. Reflections can be used to help determine the direction that the ball should be rolled in order to score a hole-in-one.



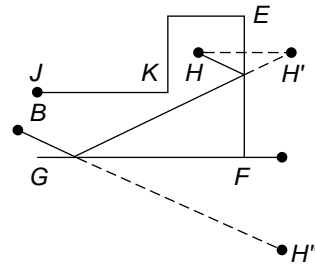
Example 1 Using wall \overline{EF} , find the path to use to score a hole-in-one.

Find the reflection image of the “hole” with respect to \overline{EF} and label it H' . The intersection of $\overline{BH'}$ with wall \overline{EF} is the point at which the shot should be directed.

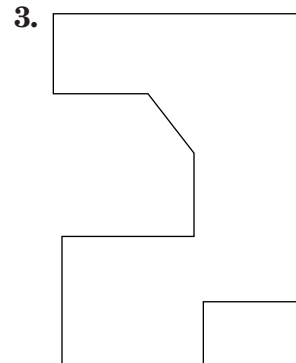
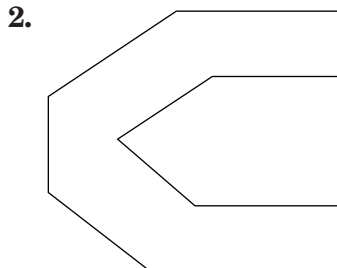
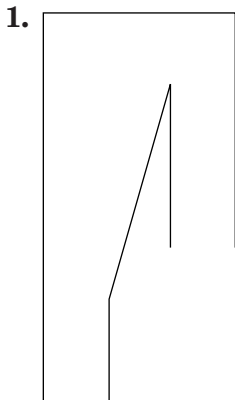


Example 2 For the hole at the right, find a path to score a hole-in-one.

Find the reflection image of H with respect to \overline{EF} and label it H' . In this case, $\overline{BH'}$ intersects \overline{JK} before intersecting \overline{EF} . Thus, this path cannot be used. To find a usable path, find the reflection image of H' with respect to \overline{GF} and label it H'' . Now, the intersection of $\overline{BH''}$ with wall \overline{GF} is the point at which the shot should be directed.



Copy each figure. Then, use reflections to determine a possible path for a hole-in-one.



7-9 Study Guide and Intervention

Square Root Functions and Inequalities

Square Root Functions A function that contains the square root of a variable expression is a **square root function**.

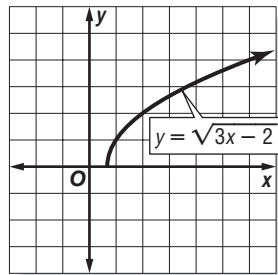
Example Graph $y = \sqrt{3x - 2}$. State its domain and range.

Since the radicand cannot be negative, $3x - 2 \geq 0$ or $x \geq \frac{2}{3}$.

The x -intercept is $\frac{2}{3}$. The range is $y \geq 0$.

Make a table of values and graph the function.

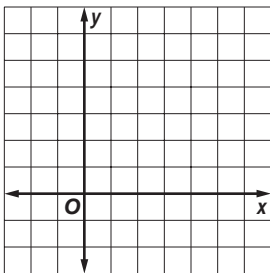
x	y
$\frac{2}{3}$	0
1	1
2	2
3	$\sqrt{7}$



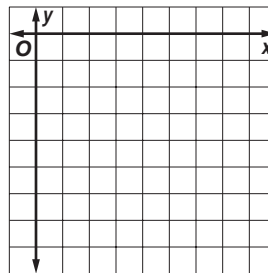
Exercises

Graph each function. State the domain and range of the function.

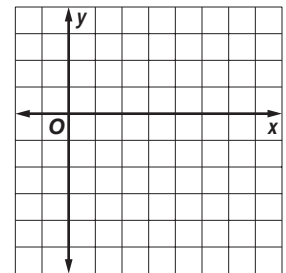
1. $y = \sqrt{2x}$



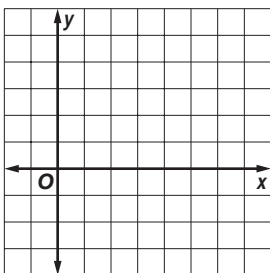
2. $y = -3\sqrt{x}$



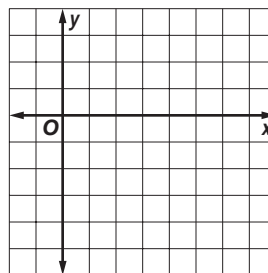
3. $y = -\sqrt{\frac{x}{2}}$



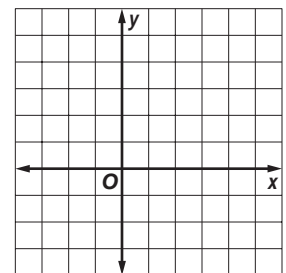
4. $y = 2\sqrt{x - 3}$



5. $y = -\sqrt{2x - 3}$



6. $y = \sqrt{2x + 5}$



7-9 Study Guide and Intervention *(continued)*

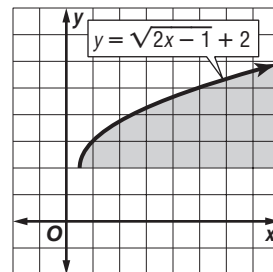
Square Root Functions and Inequalities

Square Root Inequalities A **square root inequality** is an inequality that contains the square root of a variable expression. Use what you know about graphing square root functions and quadratic inequalities to graph square root inequalities.

Example Graph $y \leq \sqrt{2x - 1} + 2$.

Graph the related equation $y = \sqrt{2x - 1} + 2$. Since the boundary should be included, the graph should be solid.

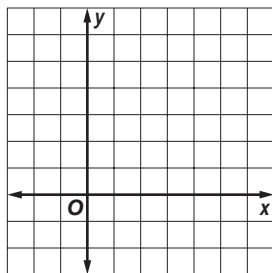
The domain includes values for $x \geq \frac{1}{2}$, so the graph is to the right of $x = \frac{1}{2}$. The range includes only numbers greater than 2, so the graph is above $y = 2$.



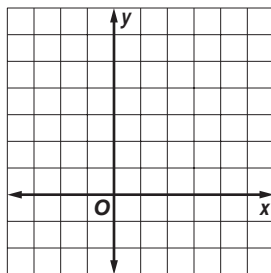
Exercises

Graph each inequality.

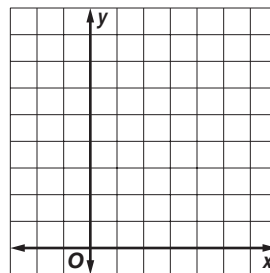
1. $y < 2\sqrt{x}$



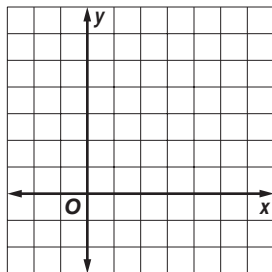
2. $y > \sqrt{x + 3}$



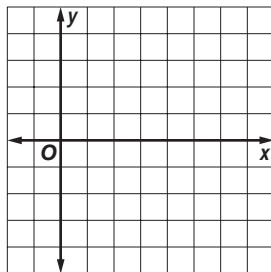
3. $y < 3\sqrt{2x - 1}$



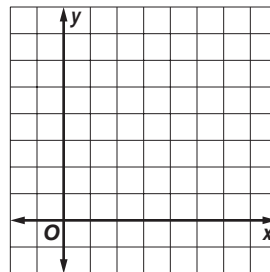
4. $y < \sqrt{3x - 4}$



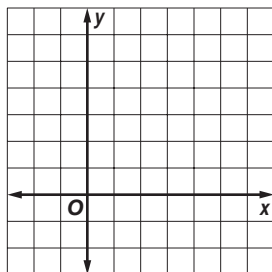
5. $y \geq \sqrt{x + 1} - 4$



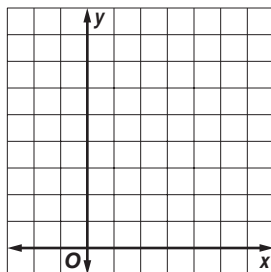
6. $y > 2\sqrt{2x - 3}$



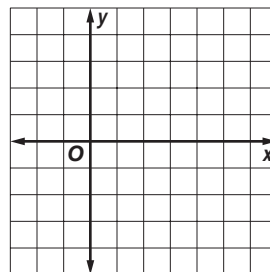
7. $y \geq \sqrt{3x + 1} - 2$



8. $y \leq \sqrt{4x - 2} + 1$



9. $y < 2\sqrt{2x - 1} - 4$

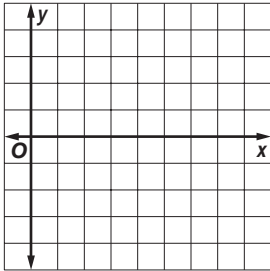


7-9 Skills Practice

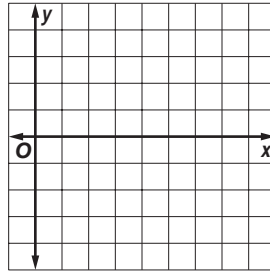
Square Root Functions and Inequalities

Graph each function. State the domain and range of each function.

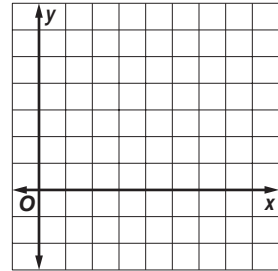
1. $y = \sqrt{2x}$



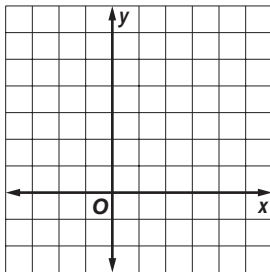
2. $y = -\sqrt{3x}$



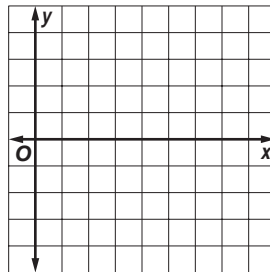
3. $y = 2\sqrt{x}$



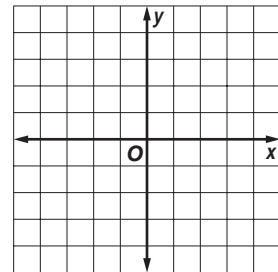
4. $y = \sqrt{x + 3}$



5. $y = -\sqrt{2x - 5}$

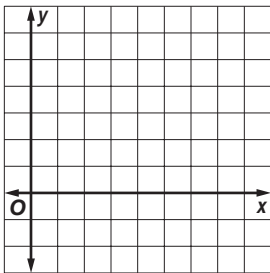


6. $y = \sqrt{x + 4} - 2$

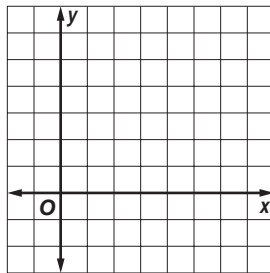


Graph each inequality.

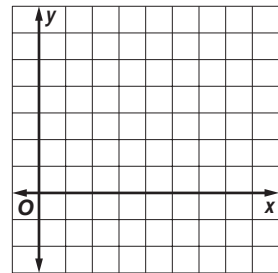
7. $y < \sqrt{4x}$



8. $y \geq \sqrt{x + 1}$



9. $y \leq \sqrt{4x - 3}$

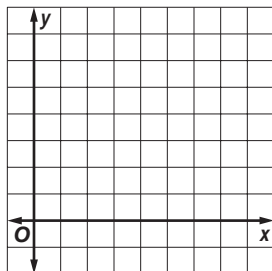


7-9 Practice

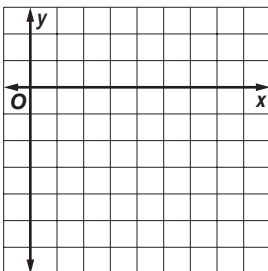
Square Root Functions and Inequalities

Graph each function. State the domain and range of each function.

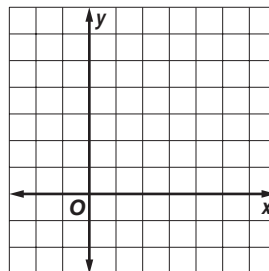
1. $y = \sqrt{5x}$



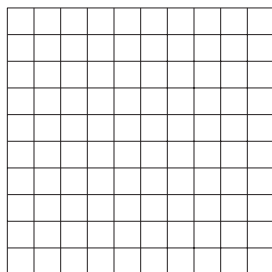
2. $y = -\sqrt{x - 1}$



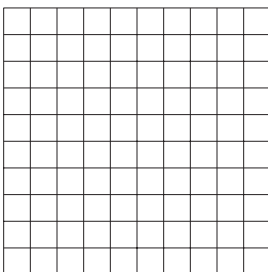
3. $y = 2\sqrt{x + 2}$



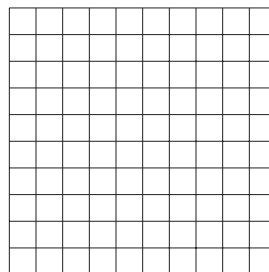
4. $y = \sqrt{3x - 4}$



5. $y = \sqrt{x + 7} - 4$

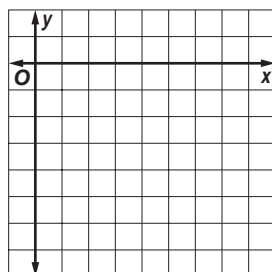


6. $y = 1 - \sqrt{2x + 3}$

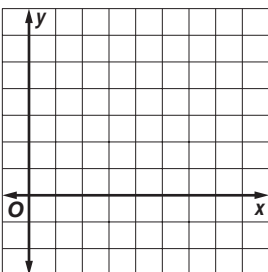


Graph each inequality.

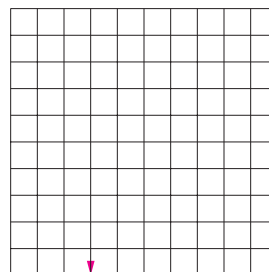
7. $y \geq -\sqrt{6x}$



8. $y \leq \sqrt{x - 5} + 3$



9. $y > -2\sqrt{3x + 2}$



10. ROLLER COASTERS The velocity of a roller coaster as it moves down a hill is

$v = \sqrt{v_0^2 + 64h}$, where v_0 is the initial velocity and h is the vertical drop in feet. If $v = 70$ feet per second and $v_0 = 8$ feet per second, find h .

11. WEIGHT Use the formula $d = \sqrt{\frac{3960^2 W_E}{W_s}} - 3960$, which relates distance from Earth d in miles to weight. If an astronaut's weight on Earth W_E is 148 pounds and in space W_s is 115 pounds, how far from Earth is the astronaut?

7-9

Reading to Learn Mathematics

Square Root Functions

Pre-Activity How are square root functions used in bridge design?

Read the introduction to Lesson 7-9 at the top of page 395 in your textbook.

If the weight to be supported by a steel cable is doubled, should the diameter of the support cable also be doubled? If not, by what number should the diameter be multiplied?

Reading the Lesson

1. Match each square root function from the list on the left with its domain and range from the list on the right.

a. $y = \sqrt{x}$

i. domain: $x \geq 0$; range: $y \geq 3$

b. $y = \sqrt{x + 3}$

ii. domain: $x \geq 0$; range: $y \leq 0$

c. $y = \sqrt{x} + 3$

iii. domain: $x \geq 0$; range: $y \leq -3$

d. $y = \sqrt{x - 3}$

iv. domain: $x \geq 0$; range: $y \geq 0$

e. $y = -\sqrt{x}$

v. domain: $x \geq 3$; range: $y \geq 0$

f. $y = -\sqrt{x - 3}$

vi. domain: $x \leq 3$; range: $y \geq 3$

g. $y = \sqrt{3 - x} + 3$

vii. domain: $x \geq 3$; range: $y \leq 0$

h. $y = -\sqrt{x} - 3$

viii. domain: $x \geq -3$; range: $y \geq 0$

2. The graph of the inequality $y \leq \sqrt{3x + 6}$ is a shaded region. Which of the following points lie inside this region?

(3, 0) (2, 4) (5, 2) (4, -2) (6, 6)

Helping You Remember

3. A good way to remember something is to explain it to someone else. Suppose you are studying this lesson with a classmate who thinks that you cannot have square root functions because every positive real number has two square roots. How would you explain the idea of square root functions to your classmate?

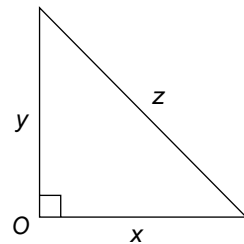
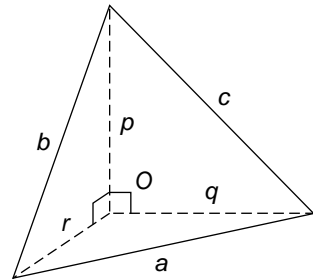
7-9 Enrichment

Reading Algebra

If two mathematical problems have basic structural similarities, they are said to be **analogous**. Using analogies is one way of discovering and proving new theorems.

The following numbered sentences discuss a three-dimensional analogy to the Pythagorean theorem.

- 01 Consider a tetrahedron with three perpendicular faces that meet at vertex O .
- 02 Suppose you want to know how the areas A , B , and C of the three faces that meet at vertex O are related to the area D of the face opposite vertex O .
- 03 It is natural to expect a formula analogous to the Pythagorean theorem $z^2 = x^2 + y^2$, which is true for a similar situation in two dimensions.
- 04 To explore the three-dimensional case, you might guess a formula and then try to prove it.
- 05 Two reasonable guesses are $D^3 = A^3 + B^3 + C^3$ and $D^2 = A^2 + B^2 + C^2$.



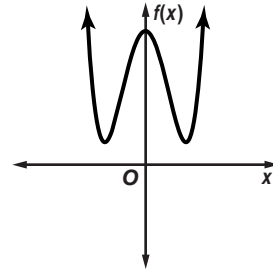
Refer to the numbered sentences to answer the questions.

1. Use sentence 01 and the top diagram. The prefix *tetra-* means four. Write an informal definition of tetrahedron.
2. Use sentence 02 and the top diagram. What are the lengths of the sides of each face of the tetrahedron?
3. Rewrite sentence 01 to state a two-dimensional analogue.
4. Refer to the top diagram and write expressions for the areas A , B , and C .
5. To explore the three-dimensional case, you might begin by expressing a , b , and c in terms of p , q , and r . Use the Pythagorean theorem to do this.
6. Which guess in sentence 05 seems more likely? Justify your answer.

7 Chapter 7 Test, Form 1

Write the letter for the correct answer in the blank at the right of each question.

1. Find $p(-3)$ if $p(x) = 4 - x$.
 A. 12 B. 4 C. 1 D. 7



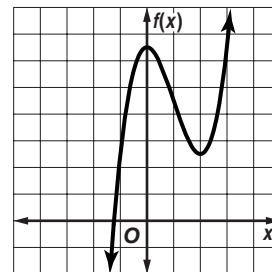
1. _____

2. State the number of real zeros for the function whose graph is shown at the right.
 A. 0 B. 1
 C. 2 D. 3

2. _____

For Questions 3 and 4, use the graph shown at the right.

3. Determine the values of x between which a real zero is located.
 A. between -1 and 0
 B. between 6 and 7
 C. between -2 and -1
 D. between 2 and 3



3. _____

4. Estimate the x -coordinate at which a relative minimum occurs.
 A. 3 B. 2 C. 0 D. -1

4. _____

5. Write the expression $x^4 + 5x^2 - 8$ in quadratic form, if possible.
 A. $(x^2)^2 + 5(x^2) - 8$ B. $(x^4)^2 + 5(x^4) - 8$
 C. $(x^2)^2 - 5(x^2) - 8$ D. not possible

5. _____

6. Solve $x^4 - 13x^2 + 36 = 0$.
 A. $-3, -2, 2, 3$ B. $-9, -4, 4, 9$ C. $2, 3, 2i, 3i$ D. $-2, -3, 2i, 3i$

6. _____

7. Use synthetic substitution to find $f(3)$ for $f(x) = x^2 - 9x + 5$.
 A. -23 B. -16 C. -13 D. 41

7. _____

8. One factor of $x^3 + 4x^2 - 11x - 30$ is $x + 2$. Find the remaining factors.
 A. $x - 5, x + 3$ B. $x - 3, x + 5$ C. $x - 6, x + 5$ D. $x - 5, x + 6$

8. _____

9. Which describes the number and type of roots of the equation $4x + 7 = 0$?
 A. 1 imaginary root B. 1 real root and 1 imaginary root
 C. 2 real roots D. 1 real root

9. _____

10. Which is *not* a root of the equation $x^3 - x^2 - 10x - 8 = 0$?
 A. 1 B. 4 C. -2 D. -1

10. _____

11. List all of the possible rational zeros of $f(x) = x^3 - 7x^2 + 8x - 6$.
 A. $\pm 1, \pm \frac{1}{2}, \pm \frac{1}{3}, \frac{1}{6}$ B. $0, \pm 1, \pm 2, \pm 3, \pm 6$
 C. $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6$ D. $\pm 1, \pm 2, \pm 3, \pm 6$

11. _____

7 Chapter 7 Test, Form 1 *(continued)*

12. Find all of the rational zeros of $p(x) = x^3 - 12x - 16$.
 A. -2, 4 B. 2, -4 C. 4 D. -2 12. _____

For Questions 13 and 14, use $f(x) = x + 5$ and $g(x) = 2x$.

13. Find $(f + g)(x)$.
 A. $3x + 5$ B. $x + 5$ C. $2x + 10$ D. $2x^2 + 5$ 13. _____

14. Find $(f \cdot g)(x)$.
 A. $2x^2 + 5$ B. $3x^2 + 10x$ C. $2x^2 + 10x$ D. $2x + 10$ 14. _____

15. If $f(x) = 3x + 7$ and $g(x) = 2x - 5$, find $g[f(-3)]$.
 A. -26 B. -9 C. -1 D. 10 15. _____

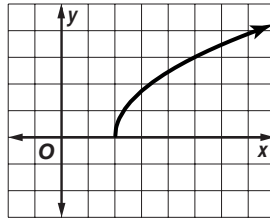
16. If $f(x) = x^2$ and $g(x) = 3x - 1$ find $[g \circ f](x)$.
 A. $x^2 + 3x - 1$ B. $9x^2 - 1$
 C. $9x^2 - 6x + 1$ D. $3x^2 - 1$ 16. _____

17. Find the inverse of $g(x) = -3x$.
 A. $g^{-1}(x) = x + 1$ B. $g^{-1}(x) = -3x - 3$
 C. $g^{-1}(x) = x - 1$ D. $g^{-1}(x) = -\frac{1}{3}x$ 17. _____

18. Determine which pair of functions are inverse functions.
 A. $f(x) = x - 4$ B. $f(x) = x - 4$
 $g(x) = x + 4$ $g(x) = 4x - 1$
 C. $f(x) = x - 4$ D. $f(x) = 4x - 1$ 18. _____
 $g(x) = \frac{x - 4}{4}$ $g(x) = 4x + 1$

19. State the domain and range of the function graphed.

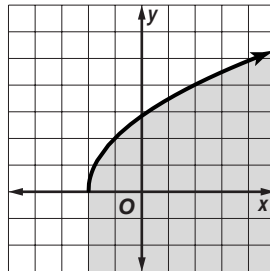
- A. D: $x > 2$, R: $y > 0$
 B. D: $x < 2$, R: $y > 0$
 C. D: $x \geq 2$, R: $y < 0$
 D. D: $x \geq 2$, R: $y \geq 0$



19. _____

20. Which inequality is graphed?

- A. $y \leq \sqrt{4x + 8}$
 B. $y > \sqrt{4x + 8}$
 C. $y < \sqrt{4x + 8}$
 D. $y \geq \sqrt{4x + 8}$



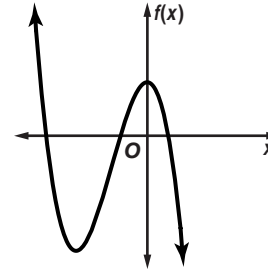
20. _____

Bonus If $g(x) = 2x + 1$, find $g[g(x)]$. **B:** _____

7 Chapter 7 Test, Form 2A

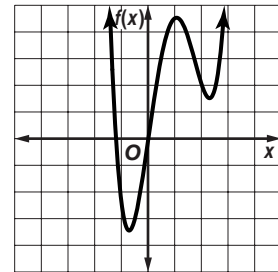
Write the letter for the correct answer in the blank at the right of each question.

1. Find $p(-4)$ if $p(x) = 3x^3 - 2x^2 + 6x - 4$.
 A. -188 B. -252 C. -140 D. -204 1. _____
2. If $r(x) = x^3 - 2x + 1$, find $r(2a^3)$.
 A. $8a^6 - 4a^3 + 1$ B. $4a^6 + 4a^3 + 1$
 C. $6a^6 - 4a^3 + 1$ D. $8a^9 - 4a^3 + 1$ 2. _____
3. State the number of real zeros for the function whose graph is shown at the right.
 A. 0 B. 2
 C. 3 D. 1 3. _____



For Questions 4 and 5, use the graph shown.

4. Determine the values of x between which a real zero is located.
 A. between 1 and 2
 B. between -4 and -3
 C. between -2 and -1
 D. between 2 and 3 4. _____
5. Estimate the x -coordinate at which a relative maximum occurs.
 A. 1 B. -1 C. 2 D. -2 5. _____
6. Write the expression $10x^8 - 6x^4 - 20$ in quadratic form, if possible.
 A. $10(x^4)^2 - 6(x^2)^2 - 20$ B. $10(x^4)^2 - 6(x^4) - 20$
 C. $10(x^2)^4 - 6(x^2)^2 - 20$ D. not possible 6. _____
7. Solve $x^4 - 6x^2 - 27 = 0$.
 A. $\sqrt{3}, 3, 3i, i\sqrt{3}$ B. $-3, -\sqrt{3}, \sqrt{3}, 3$
 C. $-3, 3, i\sqrt{3}, -i\sqrt{3}$ D. $-\sqrt{3}, 3, 3i, -3i$ 7. _____
8. Use synthetic substitution to find $f(-2)$ for $f(x) = 2x^4 - 3x^3 + x^2 - x + 5$.
 A. 15 B. 67 C. 63 D. 19 8. _____
9. One factor of $x^3 - 3x^2 - 4x + 12$ is $x + 2$. Find the remaining factors.
 A. $x + 2, x + 3$ B. $x + 2, x - 3$ C. $x - 2, x + 3$ D. $x - 2, x - 3$ 9. _____
10. Which describes the number and type of roots of the equation $x^4 - 64 = 0$?
 A. 2 real roots, 2 imaginary roots B. 4 real roots
 C. 3 real roots, 1 imaginary root D. 4 imaginary roots 10. _____
11. State the possible number of imaginary zeros of $f(x) = 7x^3 - x^2 + 10x - 4$.
 A. exactly 1 B. exactly 3 C. 3 or 1 D. 2 or 0 11. _____



7 Chapter 7 Test, Form 2A (continued)

12. Write a polynomial function of least degree with integral coefficients whose zeros include 4 and $2i$.

A. $f(x) = x^2 + 4$

B. $f(x) = x^3 - 4x^2 + 4x - 16$

C. $f(x) = x^3 + 4x^2 + 4x + 16$

D. $f(x) = x^3 - 4x^2 - 4x + 16$

12. _____

13. List all of the possible rational zeros of $f(x) = 3x^3 - 2x^2 + 7x + 6$.

A. $\pm 1, \pm 2, \pm 3, \pm 6$

B. $0, \pm 1, \pm 2, \pm 3, \pm 6, \pm \frac{1}{3}, \pm \frac{2}{3}$

C. $\pm 1, \pm 2, \pm 3, \pm 6, \pm \frac{1}{3}, \pm \frac{2}{3}$

D. $\pm 1, \pm 3, \pm \frac{1}{6}, \pm \frac{1}{3}, \pm \frac{1}{2}, \pm \frac{3}{2}$

13. _____

14. Find all of the rational zeros of $f(x) = 4x^3 - 3x^2 - 22x - 15$.

A. $\pm \frac{5}{2}, \pm 1, \pm 3$

B. $-1, 3$

C. $-1, 3$

D. $-5, -1, 3$

14. _____

15. Find $(f \cdot g)(x)$ for $f(x) = 3x^2$ and $g(x) = 5 - x$.

A. $3x^2 - x + 5$

B. $75 - 30x + 3x^2$

C. $3x^2 - 15x^2$

D. $15x^2 - 3x^3$

15. _____

16. If $f(x) = x^2 + 1$, and $g(x) = x - 2$, find $[f \circ g](x)$.

A. $x^2 - 4x + 5$

B. $x^2 - 1$

C. $x^2 - 3$

D. $x^3 - 2x^2 + x - 2$

16. _____

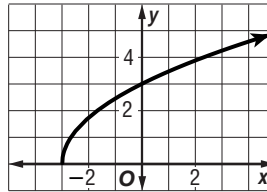
17. State the domain and range of the function graphed at the right.

A. D: $x > -3$, R: $y > 0$

B. D: $x > -3$, R: $y < 0$

C. D: $x \geq -3$, R: $y \geq 0$

D. D: $x \geq -3$, R: $y > 0$



17. _____

18. Find the inverse of $f(x) = 2x - 7$.

A. $f^{-1}(x) = 7x - 2$

B. $f^{-1}(x) = \frac{1}{2}x + 7$

C. $f^{-1}(x) = \frac{x+7}{2}$

D. $f^{-1}(x) = x + \frac{7}{2}$

18. _____

19. Determine which pair of functions are inverse functions.

A. $f(x) = 3x - 1$

B. $f(x) = 2x - 5$

C. $f(x) = 2x + 2$

D. $f(x) = 3x - 8$

19. _____

$g(x) = \frac{1}{3x-1}$

$g(x) = \frac{x+5}{2}$

$g(x) = 2x - 2$

$g(x) = \frac{1}{3}x + 8$

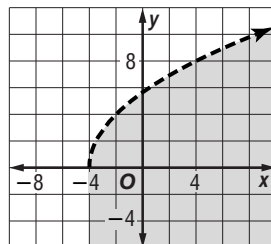
20. Which inequality is graphed at the right?

A. $y \leq \sqrt{x-4}$

B. $y < \sqrt{x+4}$

C. $y \geq \sqrt{x+4}$

D. $y > \sqrt{x-4}$



20. _____

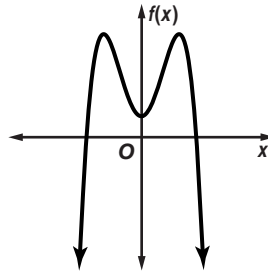
Bonus If $f(x) = 3x + 4$, solve $f[f(x)] = f(x)$ for x .

B: _____

7 Chapter 7 Test, Form 2B

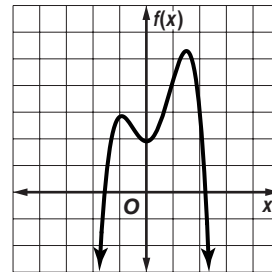
Write the letter for the correct answer in the blank at the right of each question.

1. Find $p(-3)$ if $p(x) = 4x^3 - 5x^2 + 7x - 10$.
 A. -94 B. 32 C. -184 D. -142 1. _____
2. If $r(x) = 4x^2 - 3x + 7$, find $r(3a^2)$.
 A. $36a^4 - 9a^2 + 7$ B. $144a^4 - 9a^2 + 7$
 C. $36a^4 + 9a^2 + 7$ D. $12a^4 - 9a^2 + 7$ 2. _____
3. State the number of real zeros for the function whose graph is shown.
 A. 1 B. 4
 C. 3 D. 2 3. _____



For Questions 4 and 5, use the graph shown.

4. Determine the values of x between which a real zero is located.
 A. between -2 and -1
 B. between -1 and 0
 C. between 0 and 1
 D. between -3 and -2 4. _____
5. Estimate the x -coordinate at which a relative minimum occurs.
 A. -1 B. 0 C. 1 D. 2 5. _____
6. Write the expression $9n^6 + 7n^3 - 6$ in quadratic form, if possible.
 A. $9(n^3)^3 + 7(n^3) - 6$ B. $9(n^2)^3 + 7(n^2) - 6$
 C. $9(n^3)^2 + 7(n^3) - 6$ D. not possible 6. _____
7. Solve $b^4 + 2b^2 - 24 = 0$.
 A. -2, $-\sqrt{6}$, $\sqrt{6}$, 2 B. $-\sqrt{6}$, 2, $2i$, $i\sqrt{6}$
 C. -2, 2, $-i\sqrt{6}$, $i\sqrt{6}$ D. $-2i$, $2i$, $-\sqrt{6}$, $\sqrt{6}$ 7. _____
8. Use synthetic substitution to find $f(-3)$ for $f(x) = x^4 - 4x^3 + 2x^2 - 4x + 6$.
 A. 9 B. 225 C. 201 D. -15 8. _____
9. One factor of $x^3 + 2x^2 - 11x - 12$ is $x + 4$. Find the remaining factors.
 A. $x + 1, x + 3$ B. $x - 1, x + 3$ C. $x + 1, x - 3$ D. $x - 1, x - 3$ 9. _____
10. Which describes the number and type of roots of the equation $x^3 + 121x = 0$?
 A. 1 real root, 2 imaginary roots B. 2 real roots, 1 imaginary root
 C. 3 real roots D. 3 imaginary roots 10. _____
11. State the possible number of imaginary zeros of $g(x) = x^4 + 3x^3 + 7x^2 - 6x - 13$.
 A. 3 or 1 B. 2 or 0 C. exactly 1 D. exactly 3 11. _____



7 Chapter 7 Test, Form 2B *(continued)*

12. Write a polynomial function of least degree with integral coefficients whose zeros include -3 and i .

- A. $g(x) = x^3 + 3x^2 - x - 3$ B. $g(x) = x^3 - 3x^2 + x - 3$
 C. $g(x) = x^2 + 1$ D. $g(x) = x^3 + 3x^2 + x + 3$ 12. _____

13. List all of the possible rational zeros of $p(x) = 2x^3 - 6x^2 + 7x - 6$.

- A. $\pm 1, \pm 2, \pm 3, \pm 6, \pm \frac{1}{2}, \pm \frac{3}{2}$ B. $\pm 1, \pm 2, \pm 3, \pm 6$
 C. $\pm 1, \pm 2, \pm \frac{1}{6}, \pm \frac{1}{3}, \pm \frac{1}{2}, \pm \frac{2}{3}$ D. $\pm 1, \pm 2, \pm 3, \pm 6, \pm \frac{1}{3}, \pm \frac{1}{2}, \pm \frac{2}{3}$ 13. _____

14. Find all of the rational zeros of $g(x) = 2x^3 - 11x^2 + 8x + 21$.

- A. $-1, 3, \frac{7}{2}$ B. $\pm 1, \pm 3, \pm \frac{7}{2}$ C. $-1, 3$ D. $-1, 3, 7$ 14. _____

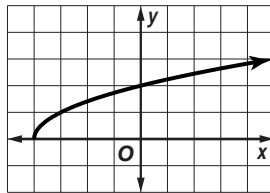
15. Find $(f - g)(x)$ for $f(x) = x^2 + 8x$ and $g(x) = 3x + 5$.

- A. $-x^2 - 5x + 5$ B. $x^2 + 5x + 5$ C. $x^2 + 5x - 5$ D. $x^2 + 11x + 5$ 15. _____

16. If $f(x) = x^2 - 3$, and $g(x) = 2x - 1$, find $[g \circ f](x)$.

- A. $2x^3 - x^2 - 6x + 3$ B. $4x^2 - 4x - 2$
 C. $x^2 + 2x - 4$ D. $2x^2 - 7$ 16. _____

17. State the domain and range of the function graphed at the right.



- A. D: $x > -4$, R: $y > 0$
 B. D: $x \geq -4$, R: $y \geq 0$
 C. D: $x \geq -4$, R: $y \leq 0$
 D. D: $x > -4$, R: $y < 0$ 17. _____

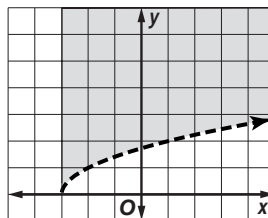
18. Find the inverse of $f(x) = 3 + 5x$.

- A. $f^{-1}(x) = 5 + 3x$ B. $f^{-1}(x) = \frac{x - 3}{5}$
 C. $f^{-1}(x) = \frac{3 + 5x}{5}$ D. $f^{-1}(x) = -3 + \frac{1}{5}x$ 18. _____

19. Determine which pair of functions are *not* inverse functions.

- A. $g(x) = 2x + 9$ B. $g(x) = x - 1$ C. $g(x) = 3x - 6$ D. $g(x) = 3x + 4$ 19. _____
 $h(x) = \frac{1}{2}x - 9$ $h(x) = x + 1$ $h(x) = \frac{1}{3}x + 2$ $h(x) = \frac{x - 4}{3}$

20. Which inequality is graphed at the right?



- A. $y \geq \sqrt{x - 3}$ B. $y < \sqrt{x + 3}$
 C. $y \leq \sqrt{x - 3}$ D. $y > \sqrt{x + 3}$ 20. _____

Bonus If $g(x) = 4x - 9$, solve $g[g(x)] = g(x)$ for x .

B: _____

7 Chapter 7 Test, Form 2C

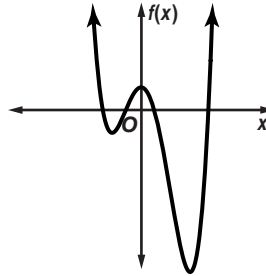
1. Find $p(-5)$ if $p(x) = x^3 - 2x^2 + x + 4$.

1. _____

2. Find $p(x + 1)$ if $p(x) = x^2 - 3x - 1$.

2. _____

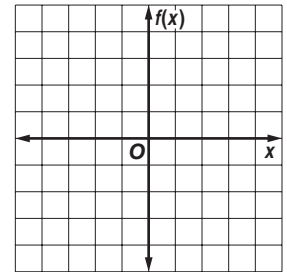
3. Determine whether the graph represents an odd-degree or an even-degree polynomial function. Then state the number of real zeros.



3. _____

4. Graph $f(x) = x^3 - 3x + 1$ by making a table of values. Then determine consecutive values of x between which each real zero is located.

4. _____



5. For the graph in Question 4, estimate the x -coordinates at which the relative maxima and relative minima occur.

5. _____

6. Write the expression $9n^6 - 36n^3$ in quadratic form, if possible.

6. _____

7. Solve $x^4 - 12x^2 - 45 = 0$.

7. _____

8. Use synthetic substitution to find $f(-4)$ for $f(x) = x^3 + 3x^2 - 5x - 7$.

8. _____

9. One factor of $x^3 + 2x^2 - 23x - 60$ is $x + 4$. Find the remaining factors.

9. _____

10. State the possible number of positive real zeros, negative real zeros, and imaginary zeros for $f(x) = 3x^4 - 2x^3 - 5x^2 + 6x - 2$.

10. _____

7 Chapter 7 Test, Form 2C *(continued)*

11. Find all the zeros of the function $h(x) = x^3 - 5x^2 + 4x - 20$. 11. _____

12. List all of the possible rational zeros of $f(x) = 2x^3 + x^2 - 4x + 8$. 12. _____

13. Find all of the rational zeros of $g(x) = 2x^3 - x^2 - 7x + 6$. 13. _____

14. Find $(f \cdot g)(x)$ for $f(x) = x^2 + 4$ and $g(x) = 7 - x$. 14. _____

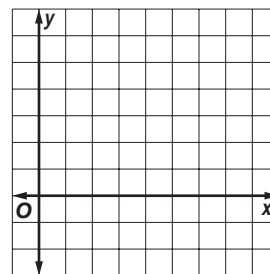
15. If $f(x) = x - 5$ and $g(x) = x^2 + 3$, find $f[g(-2)]$. 15. _____

16. If $f(x) = 2x + 5$ and $g(x) = x^2 - 3$, find $[f \circ g](x)$. 16. _____

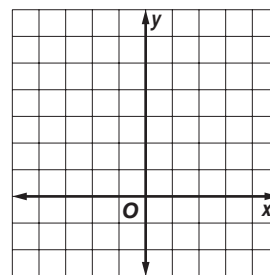
17. Find the inverse of $f(x) = 5x + 10$. 17. _____

18. Determine whether $f(x) = 5x - 3$ and $g(x) = \frac{x + 3}{5}$ are inverse functions. 18. _____

19. Graph $y = \sqrt{2x - 8}$. Then state the domain and range of the function. 19. _____



20. Graph $y < \sqrt{x + 2}$. 20. _____



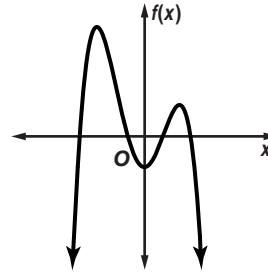
Bonus If $g(x) = 5x - 8$, solve $g[g(x)] = g(x)$ for x . **B:** _____

7 Chapter 7 Test, Form 2D

1. Find $p(-4)$ if $p(x) = x^3 - 3x^2 + 7x + 6$. 1. _____

2. Find $p(x + 1)$ if $p(x) = x^2 - 4x + 2$. 2. _____

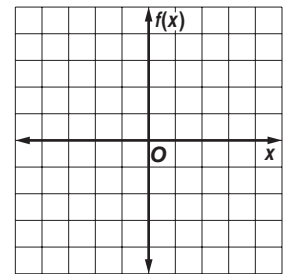
3. Determine whether the graph represents an odd-degree or an even-degree polynomial function. Then state the number of real zeros.



3. _____

4. Graph $f(x) = -x^3 + 3x + 1$ by making a table of values. Then determine consecutive values of x between which each real zero is located.

4. _____



5. For the graph in Question 4, estimate the x -coordinates at which the relative maxima and relative minima occur.

5. _____

6. Write the expression $5x^{10} - 4x^5 + 3$ in quadratic form, if possible.

6. _____

7. Solve $x^4 - 4x^2 - 12 = 0$.

7. _____

8. Use synthetic substitution to find $f(-4)$ for $f(x) = x^4 - 7x^2 - 12$.

8. _____

9. One factor of $g(x) = x^3 + x^2 - 9x - 9$ is $x - 3$. Find the remaining factors.

9. _____

10. State the number of positive real zeros, negative real zeros, and imaginary zeros for $f(x) = 2x^4 - 5x^3 - 3x^2 + x + 6$.

10. _____

7 Chapter 7 Test, Form 2D *(continued)*

11. Find all the zeros of the function $p(x) = x^3 - 2x^2 + 9x - 18$. **11.** _____

12. List all of the possible rational zeros of $g(x) = 2x^3 + 2x^2 - 7x - 14$. **12.** _____

13. Find all of the rational zeros of $h(x) = 3x^3 + 4x^2 - 13x + 6$. **13.** _____

14. Find $(f \cdot g)(x)$ for $f(x) = x^2 - 4$ and $g(x) = 6 - x$. **14.** _____

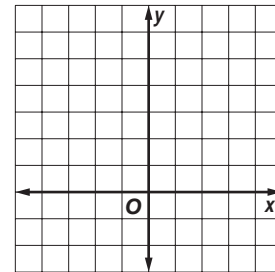
15. If $f(x) = 2x - 7$ and $g(x) = x^2 - 5$, find $g[f(5)]$. **15.** _____

16. If $f(x) = 3 - x$ and $g(x) = x^2 - 4$, find $[g \circ f](x)$. **16.** _____

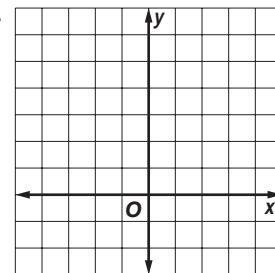
17. Find the inverse of $g(x) = -2x + 4$. **17.** _____

18. Determine whether $f(x) = 4x - 8$ and $g(x) = \frac{1}{4}x + 2$ are inverse functions. **18.** _____

19. Graph $y = \sqrt{3x + 6}$. Then state the domain and range of the function. **19.** _____



20. Graph $y \geq \sqrt{2x + 2}$. **20.** _____



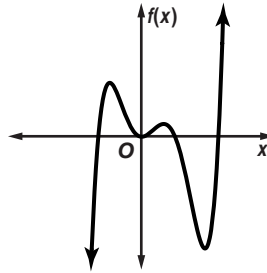
Bonus If $g(x) = 3x + 8$, solve $g[g(x)] = g(x)$ for x . **B:** _____

7 Chapter 7 Test, Form 3

1. Find $p(-2)$ if $p(x) = \frac{1}{8}x^3 + \frac{3}{4}x^2 - \frac{1}{2}x + \frac{4}{3}$. 1. _____

2. If $p(x) = 2x^2 - 3x + 1$ and $r(x) = x^2 - 5x$, find $r(x^2) - p(x + 1)$. 2. _____

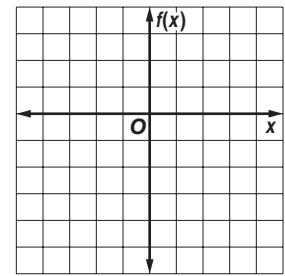
3. Describe the end behavior and determine whether the graph represents an odd-degree or an even-degree polynomial function. Then state the number of real zeros.



3. _____

4. Graph $f(x) = -x^4 + 3x^2 + x - 2$ by making a table of values. Then determine the values of x between which the real zeros are located.

4. _____



5. For the graph in Question 4, estimate the x -coordinates at which the relative maxima and relative minima occur.

5. _____

6. Write the expression $9b^5 + 3b^3 - 8b$ in quadratic form, if possible.

6. _____

7. Solve $x^{\frac{1}{2}} - 5x^{\frac{1}{4}} + 6 = 0$.

7. _____

8. Use synthetic substitution to find $f(-4)$ for $f(x) = 2x^6 - 4x^4 + 2x^3 + 5x - 6$.

8. _____

9. Find the value of k so that the remainder is 3 for $(x^2 + x - k) \div (x + 1)$.

9. _____

10. State the possible number of positive real zeros, negative real zeros, and imaginary zeros for $f(x) = 2x^{10} - 3x^8 + 4x^6 - x^4 + 3x^2 - 2$.

10. _____

7 Chapter 7 Test, Form 3 *(continued)*

11. Find all of the zeros of the function
 $q(x) = x^4 - 8x^3 + 22x^2 - 8x - 39$. 11. _____

12. List all of the possible rational zeros of
 $h(x) = 9x^6 - 12x^3 + 15$. 12. _____

13. Find all of the rational zeros of
 $h(x) = 24x^4 - 38x^3 - 23x^2 + 5x + 2$. 13. _____

14. Find $(f \cdot g)(x)$ for $f(x) = x^2 - 4$ and $g(x) = \frac{x}{x-2}$. 14. _____

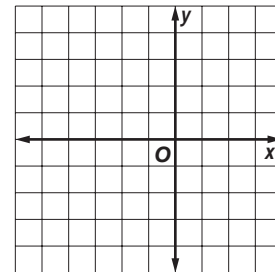
15. If $g(x) = 3x$ and $h(x) = x^3 - x^2 + x - 1$, find $[h \circ g](x)$. 15. _____

16. If $f(x) = 5x$, $g(x) = 2x - 1$, and $h(x) = x^2 - 1$, find
 $[h \circ (g \circ f)](-3)$. 16. _____

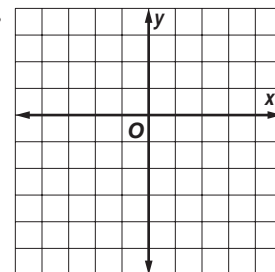
17. Find the inverse of $h(x) = \frac{2x + 6}{5}$. 17. _____

18. Determine whether $f(x) = \frac{1}{2}x - \frac{7}{3}$ and $g(x) = 2x + \frac{14}{3}$
 are inverse functions. 18. _____

19. Graph $y = \sqrt{x+4} - 2$. Then state the domain and
 range of the function. 19. _____



20. Graph $y \geq \sqrt{x+3} - 3$. 20. _____



Bonus If $f(-3) = -120$, for $f(x) = x^4 + x^3 - 19x^2 + kx + 30$,
 find $f(1)$. B: _____

Demonstrate your knowledge by giving a clear, concise solution to each problem. Be sure to include all relevant drawings and justify your answers. You may show your solutions in more than one way or investigate beyond the requirements of the problem.

1.
 - a. Sketch a graph of a polynomial function $f(x)$ of degree 5 that has the maximum number of real zeros possible for a function of its degree. Label the zeros z_1, z_2, \dots .
 - b. Label relative maximum points of the graph, if any, A_1, A_2, \dots and label relative minimum points of the graph, if any, B_1, B_2, \dots .
 - c. State the domain and range of the function.
 - d. Use the notation “As $x \rightarrow \underline{\hspace{1cm}}$, $f(x) \rightarrow \underline{\hspace{1cm}}$ ” to describe the end behavior of your graph.

2. The domain of a polynomial function $g(x)$ is all real numbers and the range of this function is $g(x) \geq 2$. What do you know about the degree, the leading coefficient, and the zeros of this function? Explain your reasoning.

3.
 - a. Write a fourth-degree polynomial $P(x)$ where no coefficient is zero, that is $a_n \neq 0$ for any n .
 - b. Find $P(-2)$ in two different ways.
 - c. Determine whether $x + 1$ is a factor of $P(x)$.
 - d. Explain what information Descartes’ Rule of Signs provides about $P(x)$.
 - e. Explain how to find, then list, all of the possible rational zeros of $P(x)$.
 - f. Explain how to find, then state, the rational zeros of $P(x)$.

4.
 - a. Write a first-degree function $g(x)$ and a second-degree function $h(x)$.
Find $g(2x + 3)$, $h(3a)$, $(g + h)(x)$, $(g - h)(x)$, $(g \cdot h)(x)$, $\left(\frac{h}{g}\right)(x)$, $(h \circ g)(x)$, $g[h(x)]$, $[h \circ (g \circ g)](2)$, and $g^{-1}(x)$.
 - b. Explain, then show, how to prove that $g(x)$ and $g^{-1}(x)$ are, in fact, inverse functions. Then explain the relationship between the graphs of these two functions.

Complex Conjugates Theorem	Factor Theorem	leading coefficient	Rational Zero Theorem
composition of functions	Fundamental Theorem of Algebra	Location Principle	relative maximum
degree of a polynomial	identity function	one-to-one	relative minimum
depressed polynomial	Integral Zero Theorem	polynomial function	Remainder Theorem
Descartes' Rule of Signs	inverse function	polynomial in one variable	square root function
end behavior	inverse relation	quadratic form	square root inequality
			synthetic substitution

Underline the correct word or phase that best completes each sentence.

1. (*End behavior, Composition of functions, Synthetic substitution*) is a method for evaluating a polynomial function $f(x)$ at a particular value of x .
2. If a function has an inverse that is also a function, then it must be a (*one-to-one function, square root function, power function*).
3. Writing the polynomial $2x^4 + 9x^2 - 15$ as $2(x^2)^2 - 9(x^2) - 15$ uses the idea of (*leading coefficients, quadratic form, end behavior*).
4. The (*rational zero theorem, remainder theorem, fundamental theorem of algebra*) says that every polynomial equation with degree one or greater has at least one root in the set of complex numbers.
5. $y = \sqrt{3x - 5}$ is a(n) (*square root, polynomial, inverse*) function.
6. If $f(x)$ is a polynomial function such that $f(-2) = 8$ and $f(-3) = -5$, then the (*depressed polynomial, Location Principle, relative minimum*) tells you that $f(x)$ has at least one real zero between -2 and -3 .
7. If a point is on a graph of a polynomial function and no other nearby points of the graph have a lesser y -coordinate, the point is a relative (*maximum, minimum*) of the function.
8. If $x^3 + 3x^2 - 4x - 12$ is divided by $x - 2$, the quotient will be $x^2 + 5x + 6$ and the remainder will be 0. In this case, $x^2 + 5x + 6$ is called the (*quadratic form, depressed polynomial, power function*).
9. The process of forming a new function from two given functions by performing the two functions in succession is called (*synthetic substitution, end behavior, composition of functions*).
10. The expression $4x^3 + 3x^2 - 5x + 6$ is a(n) (*polynomial function, inverse relation, polynomial in one variable*).

**In your own words—
Define each term.**

11. end behavior

12. Factor Theorem

7 Chapter 7 Quiz

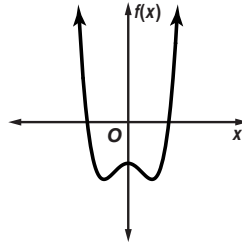
(Lessons 7-1 through 7-3)

SCORE _____

1. If $p(x) = 3x^2 - 2x + 1$, find $p(-4)$.

1. _____

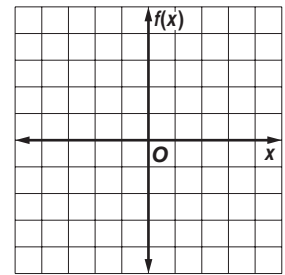
2. Determine whether the graph at the right represents an odd-degree polynomial or an even-degree polynomial function. Then state the number of real zeros.



2. _____

3. Graph $f(x) = x^3 - 5x^2 + 4x + 3$ by making a table of values. Then determine consecutive values of x between which each real zero is located. Estimate the x -coordinates at which the relative maxima and relative minima occur.

3. _____



Solve each equation.

4. $x^4 - 14x^2 + 45 = 0$

5. $a^4 = 49$

4. _____

5. _____

7 Chapter 7 Quiz

(Lessons 7-4 and 7-5)

SCORE _____

1. Use synthetic substitution to find $f(3)$ and $f(-4)$ for $f(x) = x^4 - 8x - 11$.

1. _____

2. One factor of $x^3 + x^2 - 14x - 24$ is $x - 4$. Find the remaining factors.

2. _____

3. State the possible number of positive real zeros, negative real zeros, and imaginary zeros for $g(x) = 3x^5 - 2x^3 - 4x^2 + 8x - 1$.

3. _____

4. Find all of the zeros of $f(x) = x^3 - 5x^2 + 8x - 6$.

4. _____

5. **Standardized Test Practice** Write a polynomial function of least degree with integral coefficients whose zeros include 4 and $1 - i$.

A. $f(x) = x^3 + 2x^2 - 6x + 8$

B. $f(x) = x^3 - 6x^2 + 10x - 8$

C. $f(x) = x^3 + 6x^2 + 10x - 8$

D. $f(x) = x^3 - 6x^2 + 10x + 8$

5. _____

7 Chapter 7 Quiz

(Lessons 7-6 and 7-7)

SCORE _____

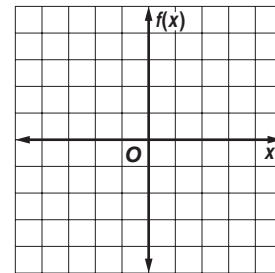
1. List all of the possible rational zeros of $h(x) = 2x^4 - 5x^3 + 3x^2 + 4x - 6$. Then find all of the rational zeros of the function. 1. _____
2. Find $(f + g)(x)$, $(f - g)(x)$, $(f \cdot g)(x)$, and $\left(\frac{f}{g}\right)(x)$ for $f(x) = x^2 - 3x + 2$ and $g(x) = 2x + 4$. 2. _____
3. For $f(x) = \{(2, 3), (4, 4), (5, 8)\}$ and $g(x) = \{(2, 4), (3, 5), (4, 2), (8, 4)\}$, find $f \circ g$ and $g \circ f$ if they exist. 3. _____
4. Find $[g \circ h](x)$ and $[h \circ g](x)$ for $g(x) = x^2 + 2x - 1$ and $h(x) = x - 4$. 4. _____
5. If $f(x) = 3x - 2$ and $g(x) = x^2 + 1$, find $f[g(-3)]$ and $g[f(-3)]$. 5. _____

7 Chapter 7 Quiz

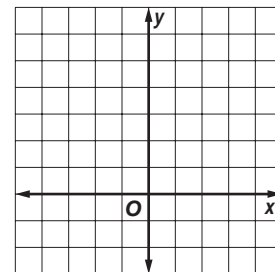
(Lessons 7-8 and 7-9)

SCORE _____

1. Find the inverse of the relation $\{(-2, 5), (0, 4), (1, -8), (4, 7)\}$. 1. _____
2. Find the inverse of the function $f(x) = 4x - 2$. Then graph the function and its inverse. 2. _____



3. Determine whether $g(x) = 3x - 6$ and $f(x) = \frac{1}{3}x + 2$ are inverse functions. 3. _____
4. Graph $y > \sqrt{3x + 9}$. Then state the domain and range of the function. 4. _____



7

Chapter 7 Mid-Chapter Test

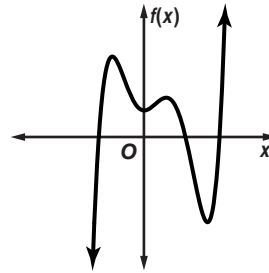
(Lessons 7-1 through 7-5)

SCORE _____

Part I Write the letter for the correct answer in the blank at the right of each question.

1. Find $p(-4)$ if $p(x) = 3x^2 - 4x + 7$.
 A. 7 B. 71 C. 57 D. 39 1. _____
2. State the degree of $2x^2 - 5x^3 + 7x^4 - 9$.
 A. 4 B. 7 C. -9 D. 3 2. _____

For Questions 3 and 4, use the graph shown.

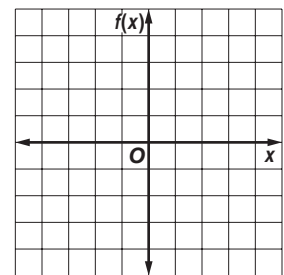


3. State the number of real zeros of the function.
 A. 2 B. 4 C. 1 D. 3 3. _____
4. As $x \rightarrow +\infty$, $f(x) \rightarrow ?$ describes the end behavior of the graph.
 A. $-\infty$ B. 0 C. $+\infty$ D. x 4. _____
5. Use synthetic substitution to find $f(-2)$ for $f(x) = x^3 - 6x^2 + 5x + 1$.
 A. -41 B. 7 C. -21 D. 27 5. _____
6. One factor of $x^3 - 6x^2 - x + 6$ is $x - 6$. Find the remaining factors.
 A. $x + 1, x + 1$ B. $x, x - 1$ C. $x - 1, x - 1$ D. $x - 1, x + 1$ 6. _____

Part II

7. Graph $f(x) = x^3 + 4x^2 - 5$ by using a table of values. Then determine consecutive values of x between which each real zero is located.

7. _____



8. Solve $t^5 - 81t = 0$. 8. _____
9. State the possible number of positive real zeros, negative real zeros, and imaginary zeros of $f(x) = x^4 - 3x^3 + 2x^2 + x + 1$. 9. _____
10. Write a polynomial function of least degree with integral coefficients whose zeros include 3 and $3i$. 10. _____
11. Find all the zeros of the function $f(x) = x^3 + x^2 + 16x + 16$. 11. _____

Assessment

7 Chapter 7 Cumulative Review

(Chapters 1–7)

1. Define a variable and write an inequality. Then solve. 1. _____
 Marlea received an inheritance of \$10,000. She plans to invest some in a stock that pays 7% interest annually. She will deposit the remainder in a savings account that pays 5% interest annually. What is the least amount that Marlea can invest in stock if she wants to earn at least \$550 on her investments for the year? (Lesson 1-5)

2. Describe the system of equations as *consistent* and *independent*, *consistent* and *dependent*, or *inconsistent*. 2. _____
 $6x + 2y = 4$
 $9x + 3y = 6$ (Lesson 3-1)

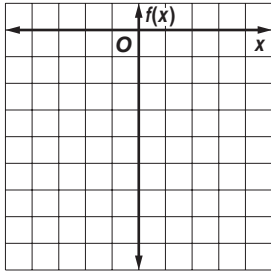
3. Triangle ABC with vertices at $A(-1, -3)$, $B(2, 3)$, and $C(-4, 1)$ is translated 5 units right and 3 units down. Find the coordinates of A' , B' , and C' . (Lesson 4-4) 3. _____

4. Use Cramer's Rule to solve the system of equations. 4. _____
 $2x + y = -1$
 $-3x - y = 4$ (Lesson 4-6)

5. Factor $4n^2 + 20n + 25$ completely. If the polynomial is not factorable, write *prime*. (Lesson 5-4) 5. _____

6. Solve $\sqrt{2x + 10} - 1 > 5$. (Lesson 5-8) 6. _____

7. Graph the quadratic function $f(x) = x^2 + 2x - 8$, labeling the y -intercept, vertex, and axis of symmetry. (Lesson 6-1) 7.



8. Write a quadratic equation with 3 and -2 as its roots. Write the equation in the form $ax^2 + bx + c = 0$, where a , b , and c are integers. (Lesson 6-3) 8. _____

9. Find $p(-3)$ if $p(x) = x^4 - 8x^3 + 5x - 4$. (Lesson 7-1) 9. _____

10. State the number of possible positive real zeros, negative real zeros, and imaginary zeros for $h(x) = x^4 - 2x^3 + 5x + 4$. (Lesson 7-5) 10. _____

11. Find all of the rational zeros of $f(x) = 2x^3 + x^2 - 13x + 6$. (Lesson 7-6) 11. _____

12. Find the inverse of $f(x) = 5x - 4$. (Lesson 7-8) 12. _____

7

Standardized Test Practice

(Chapters 1–7)

Part 1: Multiple Choice

Instructions: Fill in the appropriate oval for the best answer.

1. If $r^2 + 1 = -2r$, then $\left(r + \frac{1}{2}\right)^2 = \underline{\hspace{1cm}}$.
 A. $-\frac{1}{4}$ B. $\frac{1}{4}$
 C. 1 D. cannot be determined 1. (A) (B) (C) (D)

2. How many fourths is $26\frac{2}{3}\%$?
 E. $\frac{1}{4}$ F. 4 G. $\frac{16}{15}$ H. $\frac{4}{15}$ 2. (E) (F) (G) (H)

3. Find p in terms of m if $\frac{m}{p} = q$, $q = p$, $p \neq 0$, and $m \geq 0$.
 A. $\pm\sqrt{m}$ B. $\pm\sqrt{mq}$ C. m D. $\pm\sqrt{p}$ 3. (A) (B) (C) (D)

4. Find the average of $\frac{a}{3}$, $\frac{a}{6}$, and $\frac{a}{9}$.
 E. $\frac{11a}{54}$ F. $\frac{11a}{18}$ G. $\frac{a}{54}$ H. $\frac{11a}{6}$ 4. (E) (F) (G) (H)

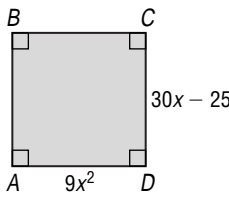
5. What is the ones digit in 3^{50} ?
 A. 1 B. 3 C. 7 D. 9 5. (A) (B) (C) (D)

6. What is the value of $a^2 - b^2$ if $a + b = 6$ and $a - b = -3$?
 E. -18 F. 3 G. 9 H. 18 6. (E) (F) (G) (H)

7. If \sqrt{n} is an irrational number, which of the following must be irrational?
 A. $\sqrt{n^2}$ B. $2\sqrt{n}$ C. $\sqrt{\frac{n}{2}}$ D. $\sqrt{2n}$ 7. (A) (B) (C) (D)

8. Evaluate $4m^3 - 3m^2 + 2m - 2$ if $m = -1$.
 E. 1 F. -11 G. -1 H. 7 8. (E) (F) (G) (H)

9. If the slope of the line through $A(-7, 4)$ and $B(5, y)$ is $-\frac{1}{4}$, what is the value of y ?
 A. -1 B. 7 C. $\frac{9}{2}$ D. 1 9. (A) (B) (C) (D)

10. Find the area of square $ABCD$.
 E. $\frac{5}{3}$ units² F. 25 units²
 G. 625 units² H. $\frac{25}{9}$ units²
 10. (E) (F) (G) (H)

Standardized Test Practice *(continued)*

Part 2: Grid In

Instructions: Enter your answer by writing each digit of the answer in a column box and then shading in the appropriate oval that corresponds to that entry.

11. Of the 24 socks in a drawer, there are three times as many black socks as brown socks. Some of the black socks are plain and some are patterned. There are five times as many plain socks as there are patterned socks. What is the probability that, without looking, you select a plain black sock from the drawer?

11.

.	.	.	.
0	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

12.

.	.	.	.
0	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

12. Let $d\blacklozenge$ and d^* be defined for any positive integer d as follows: $d\blacklozenge$ is the number obtained by dividing d by its first digit and d^* is the sum of the digits of d . What is the value of $\frac{354\blacklozenge}{354^*}$?

13. Point X lies between points P and Q on a number line. If $XQ = 15$ and $PQ = 24$, then $PX = \underline{\hspace{1cm}}$.

13.

.	.	.	.
0	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

14.

.	.	.	.
0	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

14. If $\frac{1}{8} = \frac{n}{0.4}$, what is the value of n ?

Part 3: Quantitative Comparison

Instructions: Compare the quantities in columns A and B. Shade in
 (A) if the quantity in column A is greater;
 (B) if the quantity in column B is greater;
 (C) if the quantities are equal; or
 (D) if the relationship cannot be determined from the information given.

Column A

Column B

15. the number of vertices of a pentagon

15. the number of faces of a cube

15. (A) (B) (C) (D)

16. $8 \div 6 \cdot 9 - 3$

16. $8 \div 6 \cdot (9 - 3)$

16. (A) (B) (C) (D)

17. $(x + 2)^2$

17. $(x - 1)^2$

17. (A) (B) (C) (D)

Unit 2 Test

(Chapters 5–7)

SCORE _____

For Questions 1–7, simplify. Assume that no denominator equals 0.

1. $(7x^2 + 3x - 9) - (-x^2 + 8x - 3)$

2. $5x^3(7x)^2$

3. $(2x - 3)^2$

4. $\frac{8y^3 + 27}{2xy - 10y + 3x - 15}$

5. $\sqrt{16x^2y^4}$

6. $\sqrt{12} - \sqrt{18} + 3\sqrt{50} + \sqrt{75}$

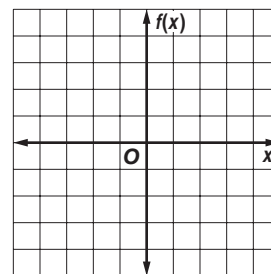
7. $\frac{2 + i}{1 - 3i}$

8. Use synthetic division to find $(2x^3 - 5x^2 + 7x - 1) \div (x - 1)$.

9. Write the expression $m^{\frac{7}{9}}$ in radical form.

10. Solve $\sqrt{3x + 6} + 4 \leq 7$.

11. Graph $f(x) = -x^2 + 4x - 3$, labeling the y -intercept, vertex, and axis of symmetry.



12. The shape of a supporting arch can be modeled by $h(x) = -0.03x^2 + 3x$, where $h(x)$ represents the height of the arch and x represents the horizontal distance from one end of the base of the arch in meters. Find the maximum height of the arch.

13. Solve $2x^2 = 3x + 2$ by graphing. If exact roots cannot be found, state the consecutive integers between which the roots are located.

14. Solve $x^2 - 2x = 24$ by factoring.

15. Write a quadratic equation with $-\frac{3}{4}$ and 4 as its roots. Write the equation in the form $ax^2 + bx + c = 0$, where a , b , and c are integers.

16. Find the exact solutions to $6x^2 + x + 4 = 0$ by using the Quadratic Formula.

17. Find the value of the discriminant for $9x^2 + 1 = 6x$. Then describe the number and type of roots for the equation.

1. _____

2. _____

3. _____

4. _____

5. _____

6. _____

7. _____

8. _____

9. _____

10. _____

11. _____

12. _____

13. _____

14. _____

15. _____

16. _____

17. _____

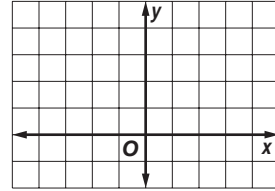
Unit 2 Test *(continued)*

(Chapters 5–7)

18. Identify the vertex, axis of symmetry, and direction of opening for $y = 2(x + 3)^2 - 5$. 18. _____

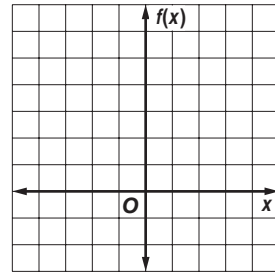
19. Write $y = -4x^2 + 8x - 1$ in vertex form. 19. _____

20. Graph $y > x^2 - 2x + 1$. 20. _____



21. Find $p(-3)$ if $p(x) = x^5 + 3x^2$. 21. _____

22. Graph $f(x) = (-x)^4 + 4x^2 - 2x$ by making a table of values. Then estimate the x -coordinates at which the relative maxima and relative minima occur. 22. _____



23. Solve $-x^4 + 200 = 102x^2$. 23. _____

24. Use synthetic substitution to find $f(-3)$ for $f(x) = 2x^3 - 6x^2 - 5x + 7$. 24. _____

25. One factor of $f(x) = x^3 + x^2 - 22x - 40$ is $x + 4$. Find the other factors. 25. _____

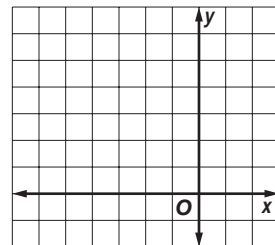
26. State the number of positive real zeros, negative real zeros, and imaginary zeros for $g(x) = 9x^3 - 7x^2 + 10x - 4$. 26. _____

27. List all of the possible rational zeros of $f(x) = 3x^5 - 7x^3 + 2x - 15$. 27. _____

28. If $f(x) = 3x$ and $g(x) = 4x - 3$, find $f[g(5)]$ and $g[f(5)]$. 28. _____

29. Find the inverse of $f(x) = 7x - 2$. 29. _____

30. Graph $y \geq \sqrt{3x + 12}$. 30. _____



First Semester Test

(Chapters 1–7)

For Questions 1–18, write the letter for the correct answer in the blank at the right of each question.

- Name the sets of numbers to which $\sqrt{14}$ belongs.

A. rational numbers	B. irrational numbers	
C. rational numbers, real numbers	D. irrational numbers, real numbers	1. _____
- Find the range of the relation $\{(-3, 0), (-2, 0), (1, 4)\}$. Then determine whether the relation is a function.

A. $\{0, 4\}$; function	B. $\{0, 4\}$; not a function	
C. $\{-3, -2, 1\}$; function	D. $\{-3, -2, 1\}$; not a function	2. _____
- The graph of which equation is a line with undefined slope that passes through $(5, 1)$?

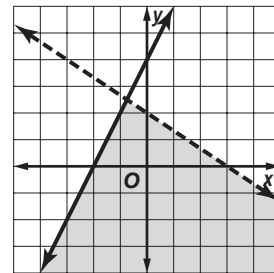
A. $y = 1$	B. $y = 5$	C. $x = 1$	D. $x = 5$	3. _____
------------	------------	------------	------------	----------
- Which point does not satisfy the inequality $y < |2x - 3|$?

A. $(0, 2)$	B. $(-1, -3)$	C. $(1, 3)$	D. $(2, 0)$	4. _____
-------------	---------------	-------------	-------------	----------
- To solve the system of equations $3x - y = 5$ and $2x + 3y = 18$, which expression could be substituted for y into the second equation?

A. $5 - 3x$	B. $3x - 5$	C. $6 - \frac{2}{3}x$	D. $18 - 2x$	5. _____
-------------	-------------	-----------------------	--------------	----------

6. Which system of inequalities is graphed?

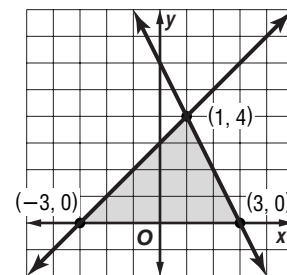
- | | |
|-------------------------|-------------------------|
| A. $y \geq 2x + 4$ | B. $y \geq 2x + 4$ |
| $y > -\frac{2}{3}x + 2$ | $y < -\frac{2}{3}x + 2$ |
| C. $y \leq 2x + 4$ | D. $y \leq 2x + 4$ |
| $y > -\frac{2}{3}x + 2$ | $y < -\frac{2}{3}x + 2$ |



6. _____

7. Find the maximum value of the function $f(x, y) = 2x + 3y$ for the feasible region shown.

- | | |
|-------|-------|
| A. 14 | B. 6 |
| C. 11 | D. -6 |



7. _____

8. Which statement is *not* true for the matrices

$A = \begin{bmatrix} 2 & -3 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 7 \\ -6 & 3 \end{bmatrix}$, $C = \begin{bmatrix} 7 & 5 \\ 9 & 1 \end{bmatrix}$, and the constant $c = -2$?

- | | | |
|--------------------------|--------------------------|----------|
| A. $A(B + C) = (B + C)A$ | B. $A(B + C) = A(C + B)$ | |
| C. $c(A + B) = (A + B)c$ | D. $A(B + C) = AB + AC$ | 8. _____ |

First Semester Test *(continued)*

(Chapters 1–7)

9. Find the value of $\begin{vmatrix} 7 & 1 \\ 12 & 5 \end{vmatrix}$.

- A. 58 B. 47 C. 23 D. -1 9. _____

10. Cramer's Rule is used to solve the system of equations at the right. Which determinant represents the numerator for y ?

$$\begin{aligned} 3x - y + 2z &= 1 \\ 4x + 2y + z &= -2 \\ 2x + 3y + 3z &= 4 \end{aligned}$$

- A. $\begin{vmatrix} 3 & -1 & 2 \\ 4 & 2 & 1 \\ 2 & 3 & 3 \end{vmatrix}$ B. $\begin{vmatrix} 3 & 1 & 2 \\ 4 & -2 & 1 \\ 2 & 4 & 3 \end{vmatrix}$ C. $\begin{vmatrix} 3 & -1 & 1 \\ 4 & 2 & -2 \\ 2 & 3 & 4 \end{vmatrix}$ D. $\begin{vmatrix} 3 & 2 & 1 \\ 4 & 1 & -2 \\ 2 & 3 & 4 \end{vmatrix}$ 10. _____

11. Evaluate $\frac{6 \times 10^{-2}}{20 \times 10^{-5}}$. Express the result in scientific notation.

- A. 0.3×10 B. 3×10^2 C. 0.3×10^{-3} D. 3×10^{-4} 11. _____

12. Simplify $(5 + 2\sqrt{3})(2 - 4\sqrt{3})$.

- A. $10 - 8\sqrt{3}$ B. $-62 - 16\sqrt{3}$ C. -14 D. $-14 - 16\sqrt{3}$ 12. _____

13. Solve $\sqrt[3]{y - 3} - 6 = 4$.

- A. 1003 B. 103 C. -5 D. 11 13. _____

14. The quadratic equation $9x^2 + 6x + 1 = 9$ is to be solved by completing the square. Which equation would *not* be a step in that solution?

- A. $(x + \frac{1}{3})^2 = 1$ B. $x = -\frac{1}{3} \pm 1$
 C. $9x^2 + 6x - 8 = 0$ D. $x^2 + \frac{2}{3}x + \frac{1}{9} = 1$ 14. _____

15. Solve the inequality $-x^2 + 25 < 0$.

- A. $\{x \mid x < -5 \text{ or } x > 5\}$ B. $\{x \mid x = -5 \text{ or } x = 5\}$
 C. $\{x \mid -5 < x < 5\}$ D. \emptyset 15. _____

16. Write the expression $2n^{\frac{2}{3}} - 3n^{\frac{1}{3}} + 5$ in quadratic form, if possible.

- A. $(2n^{\frac{1}{3}})^2 - 3(n^{\frac{1}{3}}) + 5$ B. $2(n^{\frac{1}{3}})^2 - 3(n^{\frac{1}{3}}) + 5$
 C. $2(n^2)^{\frac{1}{3}} - 3(n)^{\frac{1}{3}} + 5$ D. not possible 16. _____

17. Find all of the rational zeros of $f(x) = x^5 + 2x^3 - 24x$.

- A. $0, \pm 2, \pm \sqrt{6}$ B. $0, \pm 2$ C. $-6, 0, 4$
 D. $0, \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24$ 17. _____

18. Determine which pair of functions are *not* inverses.

- A. $g(x) = 3x - 5$ B. $g(x) = \frac{4}{3}x$ C. $g(x) = 2x - 8$ D. $g(x) = \frac{x}{3} + 1$ 18. _____
 $h(x) = \frac{x + 5}{3}$ $h(x) = \frac{3}{4}x$ $h(x) = \frac{x}{2} + 8$ $h(x) = 3x - 3$

First Semester Test *(continued)*

(Chapters 1–7)

19. The formula for the area A of a circle with diameter d is $A = \pi\left(\frac{d}{2}\right)^2$. Find the area of a circle with a diameter of 40 centimeters. Use 3.14 for π .

19. _____

20. Simplify $7(4 - x) - 5(x + 1)$.

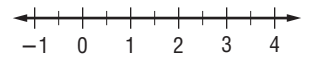
20. _____

21. Define a variable and write an inequality. Then solve. A shelf in a lumber yard will safely hold up to 1000 pounds. A crate on the shelf is marked 270 pounds. What is the greatest number of sheets of plywood, each weighing 7 pounds, that may safely be stacked on the shelf?

21. _____

22. Solve $10 \leq 9 + 2(1 + m) < 19$. Describe the solution set using set builder or interval notation. Then graph the solution set on a number line.

22. _____



23. If $f(x) = 4(x - 3) + x^2$, find $f(2a)$.

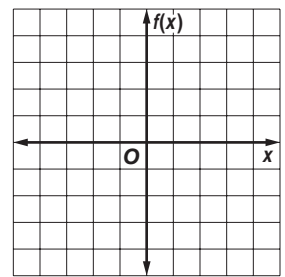
23. _____

24. What is the slope of a line that is perpendicular to the graph of the line passing through $(2, 1)$ and $(3, 5)$?

24. _____

25. Graph the piecewise function $f(x) = \begin{cases} 3x & \text{if } x \leq 1 \\ -2x + 2 & \text{if } x > 1 \end{cases}$. Identify the domain and range.

25. _____



26. Solve the system of equations $7x + 3y = -1$ and $2x - y = 9$ by using elimination.

26. _____

At a school-sponsored car wash, the fees charged were: \$5 per car, \$8 per pickup truck, \$10 per full-size van. Twice as many cars were washed as pickup trucks. The amount collected for washing cars and pickup trucks was \$360. A total of \$410 was collected at the car wash.

27. Let c represent the number of cars washed, t represent the number of pickup trucks washed, and v represent the number of vans washed. Write a system of three equations that represents the number of vehicles washed.

27. _____

28. Find the number of cars washed.

28. _____

For Questions 29 and 30, perform the indicated matrix operations. If the matrix does not exist, write *impossible*.

29. _____

29. $-4\begin{bmatrix} 6 & -4 \\ 2 & 3 \end{bmatrix} + 3\begin{bmatrix} 7 & -4 \\ 1 & 4 \end{bmatrix}$ 30. $\begin{bmatrix} 12 & 4 & -9 \\ -5 & 0 & 3 \end{bmatrix} \cdot \begin{bmatrix} 2 & 0 \\ -3 & -4 \\ 1 & 5 \end{bmatrix}$

30. _____

First Semester Test *(continued)*

(Chapters 1–7)

31. Evaluate $\begin{vmatrix} 3 & -4 & 5 \\ 1 & -1 & -2 \\ 0 & 2 & 3 \end{vmatrix}$ using diagonals. 31. _____

32. Find the inverse of $A = \begin{bmatrix} 8 & 2 \\ -2 & 1 \end{bmatrix}$, if it exists. 32. _____

For Questions 33–35, simplify. Assume that no variable equals 0. 33. _____

33. $(3x^2)^2(4y^5)(2x^0y^{-2})$ 34. $(2x - 3y)(x + 5y)$ 34. _____

35. $2i(3 - 4i) - (-1 + i)$ 35. _____

36. Use a calculator to approximate $\sqrt[3]{287}$ to three decimal places. 36. _____

37. Determine whether the function $f(x) = 3x^2 + 6x - 11$ has a minimum or a maximum value and find that value. 37. _____

38. Solve the equation $x^2 + 2x - 63 = 0$ by factoring. 38. _____

39. Solve the equation $x^2 - 4x + 13 = 0$ by completing the square. 39. _____

40. Find the exact solutions to $7x^2 + 6x = 1$ by using the Quadratic Formula. 40. _____

41. Write an equation for the parabola with vertex at $(-2, 3)$ and y -intercept -1 . 41. _____

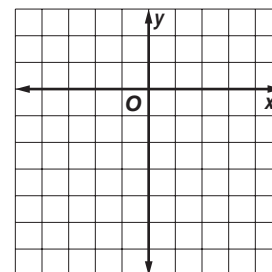
42. One factor of $x^3 - 5x^2 - 8x + 12$ is $x + 2$. Find the remaining factors. 42. _____

43. Find all the zeros of the function $f(x) = x^3 + 6x^2 + 16x + 96$. 43. _____

44. Find $(f - g)(x)$ and $(f \cdot g)(x)$ for $f(x) = 4x - 9$ and $g(x) = 3x^2$. 44. _____

45. Find the inverse of the function $p(x) = 4x - 8$. 45. _____

46. Graph $y = -\sqrt{2x + 5}$. Then state the domain and range of the function. 46. _____



7

Standardized Test Practice

Student Record Sheet (Use with pages 406–407 of the Student Edition.)

Part 1 Multiple Choice

Select the best answer from the choices given and fill in the corresponding oval.

1 (A) (B) (C) (D)

4 (A) (B) (C) (D)

7 (A) (B) (C) (D)

10 (A) (B) (C) (D)

2 (A) (B) (C) (D)

5 (A) (B) (C) (D)

8 (A) (B) (C) (D)

11 (A) (B) (C) (D)

3 (A) (B) (C) (D)

6 (A) (B) (C) (D)

9 (A) (B) (C) (D)

12 (A) (B) (C) (D)

Part 2 Short Response/Grid In

Solve the problem and write your answer in the blank.

Also enter your answer by writing each number or symbol in a box. Then fill in the corresponding oval for that number or symbol.

13 _____

.	/	/	
0	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

15 _____

.	/	/	
0	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

17 _____

.	/	/	
0	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

19 _____

.	/	/	
0	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

14 _____

.	/	/	
0	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

16 _____

.	/	/	
0	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

18 _____

.	/	/	
0	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

Part 3 Quantitative Comparison

Select the best answer from the choices given and fill in the corresponding oval.

20 (A) (B) (C) (D)

22 (A) (B) (C) (D)

24 (A) (B) (C) (D)

21 (A) (B) (C) (D)

23 (A) (B) (C) (D)

NAME _____

DATE _____

PERIOD _____

7-1

Study Guide and Intervention

Polynomial Functions

Polynomial in One Variable

A polynomial of degree n in one variable x is an expression of the form $a_0x^n + a_1x^{n-1} + \dots + a_{n-1}x^2 + a_{n-1}x + a_n$ where the coefficients $a_0, a_1, a_2, \dots, a_n$ represent real numbers, a_0 is not zero, and n represents a nonnegative integer.

The **degree of a polynomial** in one variable is the greatest exponent of its variable. The **leading coefficient** is the coefficient of the term with the highest degree.

Polynomial Function

A polynomial function of degree n can be described by an equation of the form $f(x) = a_0x^n + a_1x^{n-1} + \dots + a_{n-1}x^2 + a_{n-1}x + a_n$ where the coefficients $a_0, a_1, a_2, \dots, a_n$ represent real numbers, a_0 is not zero, and n represents a nonnegative integer.

Example 1

What are the **degree** and **leading coefficient** of $3x^2 - 2x^4 - 7 + x^3$?
 Rewrite the expression so the powers of x are in decreasing order.

$-2x^4 + x^3 + 3x^2 - 7$
 This is a polynomial in one variable. The degree is 4, and the leading coefficient is -2 .

Example 2

Find $f(-5)$ if $f(x) = x^3 + 2x^2 - 10x + 20$.

$f(x) = x^3 + 2x^2 - 10x + 20$ Original function
 $f(-5) = (-5)^3 + 2(-5)^2 - 10(-5) + 20$ Replace x with -5 .
 $= -125 + 50 + 50 + 20$ Evaluate.
 $= -5$ Simplify.

Example 3

Find $g(a^2 - 1)$ if $g(x) = x^2 + 3x - 4$.

$g(x) = x^2 + 3x - 4$ Original function
 $g(a^2 - 1) = (a^2 - 1)^2 + 3(a^2 - 1) - 4$ Replace x with $a^2 - 1$.
 $= a^4 - 2a^2 + 1 + 3a^2 - 3 - 4$ Evaluate.
 $= a^4 + a^2 - 6$ Simplify.

Exercises

State the **degree** and **leading coefficient** of each polynomial in one variable. If it is not a polynomial in one variable, explain why.

1. $3x^4 + 6x^3 - x^2 + 12$ **4; 3**
2. $100 - 5x^3 + 10x$ **7; 10**
3. $4x^6 + 6x^4 + 8x^8 - 10x^2 + 20$ **8; 8**
4. $4x^2 - 3xy + 16y^2$ **not a polynomial in one variable; contains two variables**
5. $8x^3 - 9x^5 + 4x^2 - 36$ **5; -9**
6. $\frac{x^2}{18} - \frac{x^6}{25} + \frac{x^3}{36} - \frac{1}{72}$ **6; $-\frac{25}{18}$**

Find $f(2)$ and $f(-5)$ for each function.

7. $f(x) = x^2 - 9$ **-5; 16**
8. $f(x) = 4x^3 - 3x^2 + 2x - 1$ **9; $f(x) = 9x^3 - 4x^2 + 5x + 7$
23; -586**

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Glencoe Algebra 2

NAME _____

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7-1

Study Guide and Intervention

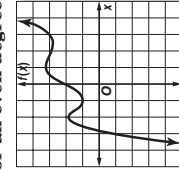
Polynomial Functions

Graphs of Polynomial Functions

<p>End Behavior of Polynomial Functions</p>	<p>If the degree is even and the leading coefficient is positive, then $f(x) \rightarrow +\infty$ as $x \rightarrow -\infty$ $f(x) \rightarrow +\infty$ as $x \rightarrow +\infty$</p> <p>If the degree is even and the leading coefficient is negative, then $f(x) \rightarrow -\infty$ as $x \rightarrow -\infty$ $f(x) \rightarrow -\infty$ as $x \rightarrow +\infty$</p> <p>If the degree is odd and the leading coefficient is positive, then $f(x) \rightarrow -\infty$ as $x \rightarrow -\infty$ $f(x) \rightarrow +\infty$ as $x \rightarrow +\infty$</p> <p>If the degree is odd and the leading coefficient is negative, then $f(x) \rightarrow +\infty$ as $x \rightarrow -\infty$ $f(x) \rightarrow -\infty$ as $x \rightarrow +\infty$</p>
<p>Real Zeros of a Polynomial Function</p>	<p>The maximum number of zeros of a polynomial function is equal to the degree of the polynomial. A zero of a function is a point at which the graph intersects the x-axis. On a graph, count the number of real zeros of the function by counting the number of times the graph crosses or touches the x-axis.</p>

Example

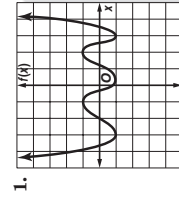
Determine whether the graph represents an odd-degree polynomial or an even-degree polynomial. Then state the number of real zeros.



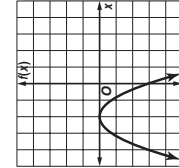
As $x \rightarrow -\infty, f(x) \rightarrow -\infty$ and as $x \rightarrow +\infty, f(x) \rightarrow +\infty$, so it is an odd-degree polynomial function. The graph intersects the x -axis at 3 points, so the function has 3 real zeros.

Exercises

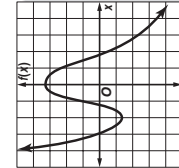
Determine whether each graph represents an odd-degree polynomial or an even-degree polynomial. Then state the number of real zeros.



1. **even; 6**



2. **even; 1 double zero**



3. **odd; 3**

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Glencoe Algebra 2

NAME _____ DATE _____ PERIOD _____

7-1 Skills Practice

Polynomial Functions

State the degree and leading coefficient of each polynomial in one variable. If it is not a polynomial in one variable, explain why.

- $a + 8$ **1; 1**
- $(2x - 1)(4x^2 + 3)$ **3; 8**
- $-5x^5 + 3x^3 - 8$ **5; -5**
- $4t^3 + 4t^2v^2 + v^4$

No, this polynomial contains two variables, u and v .

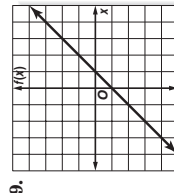
No, this is not a polynomial because $\frac{1}{r^2}$ cannot be written in the form r^n , where n is a nonnegative integer.

Find $p(-1)$ and $p(2)$ for each function.

- $p(x) = 4 - 3x$ **7; -2**
- $p(x) = 3x + x^2 - 2$ **10**
- $p(x) = 2x^2 - 4x + 1$ **7; 1**
- $p(x) = -2x^3 + 5x + 3$ **0; -3**
- $p(x) = x^4 + 8x^2 - 10$ **-1; 38**
- $p(x) = \frac{1}{3}x^2 - \frac{2}{3}x + 2$ **3; 2**
- $p(a^2) = 4a^2 - 3$
- $r(a) = 1 + 3a$, find each value.
 - $r(a) = 1 + 6a$
 - $-4p(a) - 16a^2 + 12$
 - $r(x + 2) = 7 + 3x$

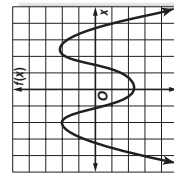
For each graph,

- describe the end behavior,
- determine whether it represents an odd-degree or an even-degree polynomial function, and
- state the number of real zeroes.



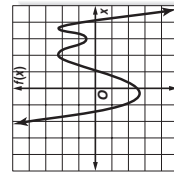
19.

$f(x) \rightarrow +\infty$ as $x \rightarrow +\infty$,
 $f(x) \rightarrow -\infty$ as $x \rightarrow -\infty$;
 odd; 1



20.

$f(x) \rightarrow -\infty$ as $x \rightarrow +\infty$,
 $f(x) \rightarrow +\infty$ as $x \rightarrow -\infty$;
 even; 4



21.

$f(x) \rightarrow -\infty$ as $x \rightarrow +\infty$,
 $f(x) \rightarrow +\infty$ as $x \rightarrow -\infty$;
 odd; 3

7-1 Practice (Average)

Polynomial Functions

State the degree and leading coefficient of each polynomial in one variable. If it is not a polynomial in one variable, explain why.

- $(3x^2 + 1)(2x^2 - 9)$ **4; 6**
- $\frac{1}{5}a^3 - \frac{3}{5}a^2 + \frac{4}{5}a$ **3; $\frac{1}{5}$**
- $\frac{2}{m^2} + 3m - 12$ **Not a polynomial;**

$\frac{2}{m^2}$ cannot be written in the form m^n for a nonnegative integer n .

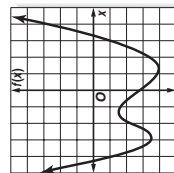
No, this polynomial contains two variables, x and y .

Find $p(-2)$ and $p(3)$ for each function.

- $p(x) = x^3 - x^5$ **24; -216**
- $p(x) = -7x^2 + 5x + 9$ **-29; -39**
- $p(x) = x^3 - x^2 + 2x - 5$ **-37; 73**
- $p(x) = 3x^3 - x^2 + 2x - 5$ **13; 93**
- $p(x) = x^4 + \frac{1}{2}x^3 - \frac{1}{2}x$ **13; 93**
- $p(x) = -x^5 + 4x^3 + 3x$ **0; -135**
- $p(x) = 3x^2 - 4$ and $r(x) = 2x^2 - 5x + 1$, find each value.
 - $r(3a)$ **$192a^2 - 4$**
 - $r(a^2)$ **$2a^4 - 5a^2 + 1$**
 - $r(x + 2)$ **$3x^2 + 3x - 1$**
 - $p(x^2 - 1)$ **$5[x^2(x + 2)]$**
- $r(x) = x^4 + \frac{1}{2}x^3 - \frac{1}{2}x$ **$15x^2 + 60x + 40$**
- $r(a^2)$ **$-40a^2 + 50a - 5$**

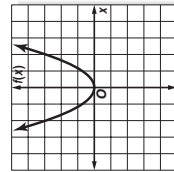
For each graph,

- describe the end behavior,
- determine whether it represents an odd-degree or an even-degree polynomial function, and
- state the number of real zeroes.



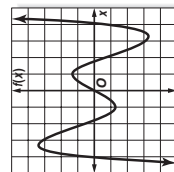
17.

$f(x) \rightarrow +\infty$ as $x \rightarrow +\infty$,
 $f(x) \rightarrow +\infty$ as $x \rightarrow -\infty$;
 even; 2



18.

$f(x) \rightarrow +\infty$ as $x \rightarrow +\infty$,
 $f(x) \rightarrow +\infty$ as $x \rightarrow -\infty$;
 even; 1



19.

$f(x) \rightarrow +\infty$ as $x \rightarrow +\infty$,
 $f(x) \rightarrow -\infty$ as $x \rightarrow -\infty$;
 odd; 5

20. WIND CHILL The function $C(s) = 0.013s^2 - s - 7$ estimates the wind chill temperature $C(s)$ at 0°F for wind speeds s from 5 to 30 miles per hour. Estimate the wind chill temperature at 0°F if the wind speed is 20 miles per hour. **about -22°F**

Lesson 7-1

NAME _____ DATE _____ PERIOD _____

7-1 Practice (Average)

Polynomial Functions

State the degree and leading coefficient of each polynomial in one variable. If it is not a polynomial in one variable, explain why.

- $(3x^2 + 1)(2x^2 - 9)$ **4; 6**
- $\frac{1}{5}a^3 - \frac{3}{5}a^2 + \frac{4}{5}a$ **3; $\frac{1}{5}$**
- $\frac{2}{m^2} + 3m - 12$ **Not a polynomial;**

$\frac{2}{m^2}$ cannot be written in the form m^n for a nonnegative integer n .

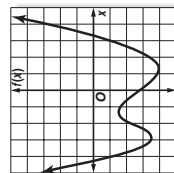
No, this polynomial contains two variables, x and y .

Find $p(-2)$ and $p(3)$ for each function.

- $p(x) = x^3 - x^5$ **24; -216**
- $p(x) = -7x^2 + 5x + 9$ **-29; -39**
- $p(x) = 3x^3 - x^2 + 2x - 5$ **-37; 73**
- $p(x) = 3x^3 - x^2 + 2x - 5$ **13; 93**
- $p(x) = x^4 + \frac{1}{2}x^3 - \frac{1}{2}x$ **13; 93**
- $p(x) = -x^5 + 4x^3 + 3x$ **0; -135**
- $p(x) = 3x^2 - 4$ and $r(x) = 2x^2 - 5x + 1$, find each value.
 - $r(3a)$ **$192a^2 - 4$**
 - $r(a^2)$ **$2a^4 - 5a^2 + 1$**
 - $r(x + 2)$ **$3x^2 + 3x - 1$**
 - $p(x^2 - 1)$ **$5[x^2(x + 2)]$**
- $r(x) = x^4 + \frac{1}{2}x^3 - \frac{1}{2}x$ **$15x^2 + 60x + 40$**
- $r(a^2)$ **$-40a^2 + 50a - 5$**

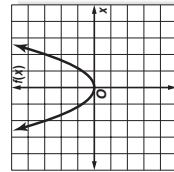
For each graph,

- describe the end behavior,
- determine whether it represents an odd-degree or an even-degree polynomial function, and
- state the number of real zeroes.



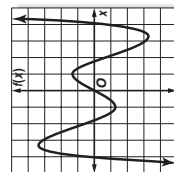
17.

$f(x) \rightarrow +\infty$ as $x \rightarrow +\infty$,
 $f(x) \rightarrow +\infty$ as $x \rightarrow -\infty$;
 even; 2



18.

$f(x) \rightarrow +\infty$ as $x \rightarrow +\infty$,
 $f(x) \rightarrow +\infty$ as $x \rightarrow -\infty$;
 even; 1



19.

$f(x) \rightarrow +\infty$ as $x \rightarrow +\infty$,
 $f(x) \rightarrow -\infty$ as $x \rightarrow -\infty$;
 odd; 5

20. WIND CHILL The function $C(s) = 0.013s^2 - s - 7$ estimates the wind chill temperature $C(s)$ at 0°F for wind speeds s from 5 to 30 miles per hour. Estimate the wind chill temperature at 0°F if the wind speed is 20 miles per hour. **about -22°F**

NAME _____ DATE _____ PERIOD _____

7-1 Enrichment

Approximation by Means of Polynomials

Many scientific experiments produce pairs of numbers $[x, f(x)]$ that can be related by a formula. If the pairs form a function, you can fit a polynomial to the pairs in exactly one way. Consider the pairs given by the following table.

x	1	2	4	7
$f(x)$	6	11	39	-54

We will assume the polynomial is of degree three. Substitute the given values into this expression.

$$f(x) = A + B(x - x_0) + C(x - x_0)(x - x_1) + D(x - x_0)(x - x_1)(x - x_2)$$

You will get the system of equations shown below. You can solve this system and use the values for A , B , C , and D to find the desired polynomial.

$$6 = A$$

$$11 = A + B(2 - 1) = A + B$$

$$39 = A + B(4 - 1) + C(4 - 1)(4 - 2) = A + 3B + 6C$$

$$-54 = A + B(7 - 1) + C(7 - 1)(7 - 2) + D(7 - 1)(7 - 2)(7 - 4) = A + 6B + 30C + 90D$$

Solve.

- Solve the system of equations for the values A , B , C , and D .
 $A = 6, B = 5, C = 3, D = -2$
- Find the polynomial that represents the four ordered pairs. Write your answer in the form $y = a + bx + cx^2 + dx^3$.
 $y = -2x^3 + 17x^2 - 32x + 23$
- Find the polynomial that gives the following values.

x	8	12	15	20
$f(x)$	-207	169	976	3801

 $A = -207, B = 94, C = 25, D = 1; y = x^3 - 10x^2 - 10x + 1$
- A scientist measured the volume $f(x)$ of carbon dioxide gas that can be absorbed by one cubic centimeter of charcoal at pressure x . Find the values for A , B , C , and D .

x	120	340	534	698
$f(x)$	3.1	5.5	7.1	8.3

 $A = 3.1, B = 0.01091, C = -0.000000643, D = 0.0000000066$

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7-1 Reading to Learn Mathematics

Pre-Activity Where are polynomial functions found in nature?

Read the introduction to Lesson 7-1 at the top of page 346 in your textbook.

- In the honeycomb cross section shown in your textbook, there is 1 hexagon in the center, 6 hexagons in the second ring, and 12 hexagons in the third ring. How many hexagons will there be in the fourth, fifth, and sixth rings?
18; 24; 30
- There is 1 hexagon in a honeycomb with 1 ring. There are 7 hexagons in a honeycomb with 2 rings. How many hexagons are there in honeycombs with 3 rings, 4 rings, 5 rings, and 6 rings?
19; 37; 61; 91

Reading the Lesson

1. Give the degree and leading coefficient of each polynomial in one variable.

a. $10x^3 + 3x^2 - x + 7$	degree	leading coefficient
b. $7y^2 - 2y^5 + y - 4y^3$	3	10
c. 100	5	-2
	0	100

2. Match each description of a polynomial function from the list on the left with the corresponding end behavior from the list on the right.

a. even degree, negative leading coefficient	iii	i. $f(x) \rightarrow +\infty$ as $x \rightarrow +\infty$; $f(x) \rightarrow +\infty$ as $x \rightarrow -\infty$
b. odd degree, positive leading coefficient	iv	ii. $f(x) \rightarrow -\infty$ as $x \rightarrow +\infty$; $f(x) \rightarrow +\infty$ as $x \rightarrow -\infty$
c. odd degree, negative leading coefficient	ii	iii. $f(x) \rightarrow -\infty$ as $x \rightarrow +\infty$; $f(x) \rightarrow -\infty$ as $x \rightarrow -\infty$
d. even degree, positive leading coefficient	i	iv. $f(x) \rightarrow +\infty$ as $x \rightarrow +\infty$; $f(x) \rightarrow -\infty$ as $x \rightarrow -\infty$

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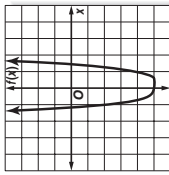
7-2 Study Guide and Intervention

Graphing Polynomial Functions

Graph Polynomial Functions

Location Principle Suppose $y = f(x)$ represents a polynomial function and a and b are two numbers such that $f(a) < 0$ and $f(b) > 0$. Then the function has at least one real zero between a and b .

Example Determine the values of x between which each real zero of the function $f(x) = 2x^4 - x^3 - 5$ is located. Then draw the graph. Make a table of values. Look at the values of $f(x)$ to locate the zeros. Then use the points to sketch a graph of the function.

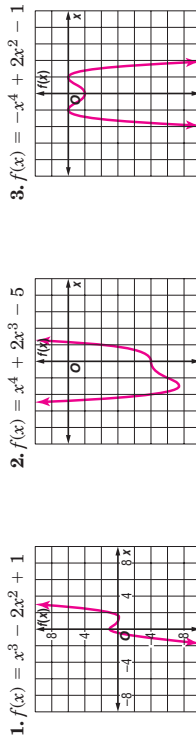


The changes in sign indicate that there are zeros between $x = -2$ and $x = -1$ and between $x = 1$ and $x = 2$.

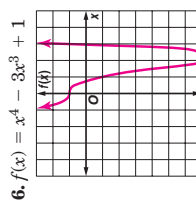
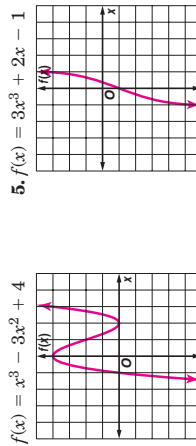
x	$f(x)$
-2	35
-1	-2
0	-5
1	-4
2	19

Exercises

Graph each function by making a table of values. Determine the values of x at which or between which each real zero is located.



4. $f(x) = x^3 - 3x^2 + 4$ **between 0 and -1; at 1; between 1 and 2**



5. $f(x) = 3x^3 + 2x - 1$ **between 0 and 1**

6. $f(x) = x^4 - 3x^2 + 1$ **between 0 and 1; between 2 and 3**

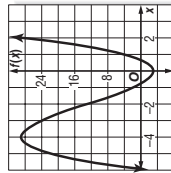
NAME _____ DATE _____ PERIOD _____

7-2 Study Guide and Intervention

Graphing Polynomial Functions

Maximum and Minimum Points A quadratic function has either a maximum or a minimum point on its graph. For higher degree polynomial functions, you can find *turning points*, which represent **relative maximum** or **relative minimum** points.

Example Graph $f(x) = x^3 + 6x^2 - 3$. Estimate the x -coordinates at which the relative maxima and minima occur. Make a table of values and graph the function.



A relative maximum occurs at $x = -4$ and a relative minimum occurs at $x = 0$.

x	$f(x)$
-5	22
-4	29
-3	24
-2	13
-1	2
0	-3
1	4
2	29

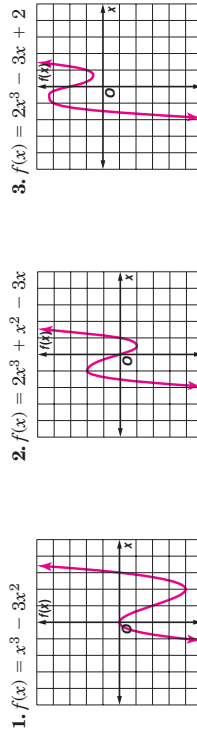
← indicates a relative maximum

← zero between $x = -1$, $x = 0$

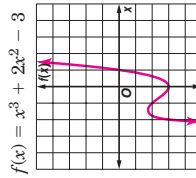
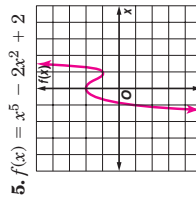
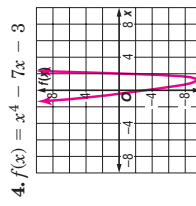
← indicates a relative minimum

Exercises

Graph each function by making a table of values. Estimate the x -coordinates at which the relative maxima and minima occur.



4. $f(x) = x^4 - 7x - 3$ **min. about 1**



5. $f(x) = x^5 - 2x^2 + 2$ **max. at 0, min. about 1**

6. $f(x) = x^3 + 2x^2 - 3$ **max. about -1, min. at 0**

NAME _____

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7-2

Skills Practice

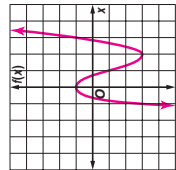
Graphing Polynomial Functions

Complete each of the following.

- Graph each function by making a table of values.
- Determine consecutive values of x between which each real zero is located.
- Estimate the x -coordinates at which the relative maxima and minima occur.

1. $f(x) = x^3 - 3x^2 + 1$

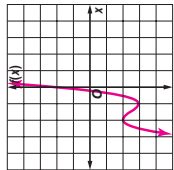
x	f(x)
-2	-19
-1	-3
0	1
1	-1
2	-3
3	1
4	17



- zeros between -1 and 0 , 0 and 1 , and 2 and 3 ; rel. max. at $x = 0$, rel. min. at $x = 2$

3. $f(x) = 2x^3 + 9x^2 + 12x + 2$

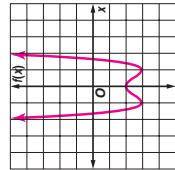
x	f(x)
-3	-7
-2	-2
-1	-3
0	2
1	25



- zero between -1 and 0 ; rel. max. at $x = -2$, rel. min. at $x = -1$

5. $f(x) = x^4 - 2x^2 - 2$

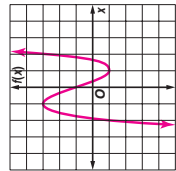
x	f(x)
-3	61
-2	6
-1	-3
0	-2
1	-3
2	6
3	61



- zeros between -2 and -1 , and 1 and 2 ; rel. max. at $x = 0$, rel. min. at $x = -1$ and $x = 1$

2. $f(x) = x^3 - 3x + 1$

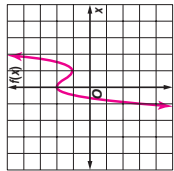
x	f(x)
-3	-17
-2	-1
-1	3
0	1
1	-1
2	3
3	19



- zeros between -2 and -1 , 0 and 1 , and 1 and 2 ; rel. max. at $x = -1$, rel. min. at $x = 1$

4. $f(x) = 2x^3 - 3x^2 + 2$

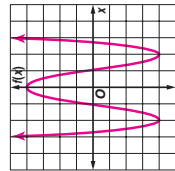
x	f(x)
-1	-3
0	2
1	1
2	6
3	29



- zero between -1 and 0 ; rel. min. at $x = 1$, rel. max. at $x = 0$

6. $f(x) = 0.5x^4 - 4x^2 + 4$

x	f(x)
-3	8.5
-2	-4
-1	0.5
0	4
1	0.5
2	-4
3	8.5



- zeros between -1 and -2 , -2 and -3 , 1 and 2 , and 2 and 3 ; rel. max. at $x = 0$, rel. min. at $x = -2$ and $x = 2$

NAME _____

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7-2

Practice (Average)

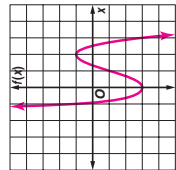
Graphing Polynomial Functions

Complete each of the following.

- Graph each function by making a table of values.
- Determine consecutive values of x between which each real zero is located.
- Estimate the x -coordinates at which the relative and relative minima occur.

1. $f(x) = -x^3 + 3x^2 - 3$

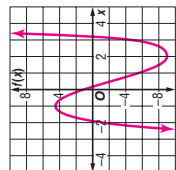
x	f(x)
-2	17
-1	1
0	-3
1	-1
2	1
3	-3
4	-19



- zeros between -1 and 0 , 1 and 2 , and 2 and 3 ; rel. max. at $x = 2$, rel. min. at $x = 0$

2. $f(x) = x^3 - 1.5x^2 - 6x + 1$

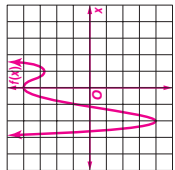
x	f(x)
-2	-1
-1	4.5
0	1
1	-5.5
2	-9
3	-3.5
4	17



- zeros between -2 and -1 , 0 and 1 , and 3 and 4 ; rel. max. at $x = -1$, rel. min. at $x = 2$

3. $f(x) = 0.75x^4 + x^3 - 3x^2 + 4$

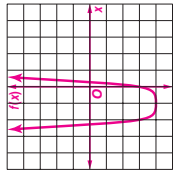
x	f(x)
-3	10.75
-2	-4
-1	0.75
0	4
1	2.75
2	12



- zeros between -3 and -2 , and -2 and -1 ; rel. max. at $x = 0$, rel. min. at $x = -2$ and $x = 1$

4. $f(x) = x^4 + 4x^3 + 6x^2 + 4x - 3$

x	f(x)
-3	12
-2	-3
-1	-4
0	-3
1	12
2	77

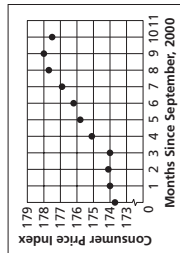


- zeros between -3 and -2 , and 0 and 1 ; rel. min. at $x = -1$

PRICES For Exercises 5 and 6, use the following information.

The Consumer Price Index (CPI) gives the relative price for a fixed set of goods and services. The CPI from September, 2000 to July, 2001 is shown in the graph.

Source: U.S. Bureau of Labor Statistics



- Describe the turning points of the graph.
rel. max. in Nov. and June; rel. min. in Dec.
- If the graph were modeled by a polynomial equation, what is the least degree the equation could have? **4**

- LABOR** A town's jobless rate can be modeled by $(1, 3.3)$, $(2, 4.9)$, $(3, 5.3)$, $(4, 6.4)$, $(5, 4.5)$, $(6, 5.6)$, $(7, 2.5)$, $(8, 2.7)$. How many turning points would the graph of a polynomial function through these points have? **4: 2 rel. max. and 2 rel. min.**

7-2 Reading to Learn Mathematics

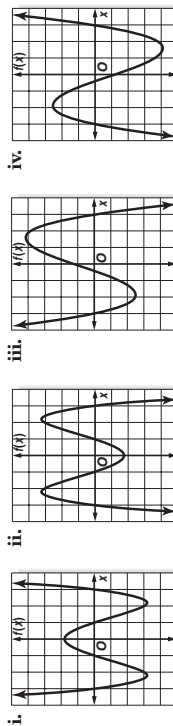
Graphing Polynomial Functions

Pre-Activity How can graphs of polynomial functions show trends in data?

Read the introduction to Lesson 7-2 at the top of page 353 in your textbook. Three points on the graph shown in your textbook are (0, 1.4), (70, 3.78), and (100, 9). Give the real-world meaning of the coordinates of these points.
Sample answer: In 1900, 1.4% of the U. S. population was foreign born. In 1970, 3.78% of the population was foreign born. In 2000, 9% of the population was foreign born.

Reading the Lesson

- Suppose that $f(x)$ is a third-degree polynomial function and that c and d are real numbers, with $d > c$. Indicate whether each statement is *true* or *false*. (Remember that *true* means *always* true.)
 - If $f(c) > 0$ and $f(d) < 0$, there is exactly one real zero between c and d . **false**
 - If $f(c) = f(d) \neq 0$, there are no real zeros between c and d . **false**
 - If $f(c) < 0$ and $f(d) > 0$, there is at least one real zero between c and d . **true**
- Match each graph with its description.
 - third-degree polynomial with one relative maximum and one relative minimum; leading coefficient negative **iii**
 - fourth-degree polynomial with two relative minima and one relative maximum **i**
 - third-degree polynomial with one relative maximum and one relative minimum; leading coefficient positive **iv**
 - fourth-degree polynomial with two relative maxima and one relative minimum **ii**



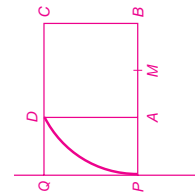
Helping You Remember

- The origins of words can help you to remember their meaning and to distinguish between similar words. Look up *maximum* and *minimum* in a dictionary and describe their origins (original language and meaning). **Sample answer:** *Maximum* comes from the Latin word *maximus*, meaning *greatest*. *Minimum* comes from the Latin word *minimus*, meaning *least*.

7-2 Enrichment

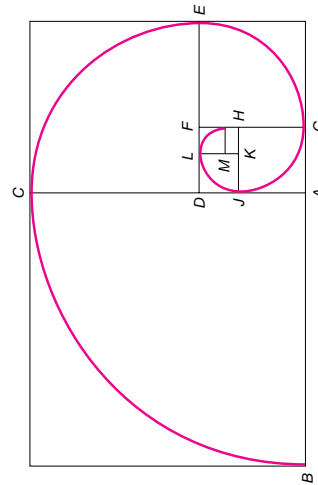
Golden Rectangles

Use a straightedge, a compass, and the instructions below to construct a golden rectangle.



- Construct square $ABCD$ with sides of 2 centimeters.
- Construct the midpoint of \overline{AB} . Call the midpoint M .
- Using M as the center, set your compass opening at MC . Construct an arc with center M that intersects \overline{AD} . Call the point of intersection P .
- Construct a line through P that is perpendicular to \overline{AB} .
- Extend \overline{DC} so that it intersects the perpendicular. Call the intersection point Q . $APQD$ is a golden rectangle. Check this conclusion by finding the value of $\frac{QP}{AP}$. **0.62**

A figure consisting of similar golden rectangles is shown below. Use a compass and the instructions below to draw quarter-circle arcs that form a spiral like that found in the shell of a chambered nautilus.



- Using A as a center, draw an arc that passes through B and C .
- Using D as a center, draw an arc that passes through C and E .
- Using F as a center, draw an arc that passes through E and G .
- Continue drawing arcs, using H , K , and M as the centers.

7-3 Study Guide and Intervention (continued)
Solving Equations Using Quadratic Techniques

Solve Equations Using Quadratic Form If a polynomial expression can be written in quadratic form, then you can use what you know about solving quadratic equations to solve the related polynomial equation.

Example 1 Solve $x^4 - 40x^2 + 144 = 0$.

$x^4 - 40x^2 + 144 = 0$ Original equation
 $(x^2)^2 - 40(x^2) + 144 = 0$ Write the expression on the left in quadratic form.
 $(x^2 - 4)(x^2 - 36) = 0$ Factor.
 $x^2 - 4 = 0$ or $x^2 - 36 = 0$ Zero Product Property
 $(x - 2)(x + 2) = 0$ or $(x - 6)(x + 6) = 0$ Factor.
 $x - 2 = 0$ or $x + 2 = 0$ or $x - 6 = 0$ or $x + 6 = 0$ Zero Product Property
 $x = 2$ or $x = -2$ or $x = 6$ or $x = -6$ Simplify.

The solutions are ± 2 and ± 6 .

Example 2 Solve $2x + \sqrt{x} - 15 = 0$.

$2x + \sqrt{x} - 15 = 0$ Original equation
 $2(\sqrt{x})^2 + \sqrt{x} - 15 = 0$ Write the expression on the left in quadratic form.
 $(2\sqrt{x} - 5)(\sqrt{x} + 3) = 0$ Factor.
 $2\sqrt{x} - 5 = 0$ or $\sqrt{x} + 3 = 0$ Zero Product Property
 $\sqrt{x} = \frac{5}{2}$ or $\sqrt{x} = -3$ Simplify.

Since the principal square root of a number cannot be negative, $\sqrt{x} = -3$ has no solution. The solution is $\frac{25}{4}$ or $6\frac{1}{4}$.

Exercises

Solve each equation.

- $x^4 = 49$ $\pm\sqrt{7}, \pm i\sqrt{7}$
- $x^4 - 6x^2 = -8$ $\pm 2, \pm\sqrt{2}$
- $x^4 - 3x^2 = 54$ $\pm 3, \pm i\sqrt{6}$
- $3t^6 - 48t^2 = 0$ $0, \pm 2, \pm 2i$
- $m^6 - 16m^3 + 64 = 0$ $2, -1 \pm i\sqrt{3}$
- $y^4 - 5y^2 + 4 = 0$ $\pm 1, \pm 2$
- $x^4 - 29x^2 + 100 = 0$ $4x^4 - 73x^2 + 144 = 0$ $9, \frac{1}{x^2} - \frac{7}{x} + 12 = 0$
 $\pm 5, \pm 2$ $\pm 4, \pm 2$ $\frac{1}{3}, \frac{1}{4}$
- $x - 5\sqrt{x} + 6 = 0$ $11, x - 10\sqrt{x} + 21 = 0$ $12, x^3 - 5x^3 + 6 = 0$
 $4, 9$ $9, 49$ $27, 8$

7-3 Study Guide and Intervention
Solving Equations Using Quadratic Techniques

Quadratic Form Certain polynomial expressions in x can be written in the quadratic form $au^2 + bu + c$ for any numbers a , b , and c , $a \neq 0$, where u is an expression in x .

Example Write each polynomial in quadratic form, if possible.

- $3a^6 - 9a^3 + 12$
 Let $u = a^3$.
 $3a^6 - 9a^3 + 12 = 3(a^3)^2 - 9(a^3) + 12$
- $101b - 49\sqrt{b} + 42$
 Let $u = \sqrt{b}$.
 $101b - 49\sqrt{b} + 42 = 101(\sqrt{b})^2 - 49(\sqrt{b}) + 42$
- $24a^5 + 12a^3 + 18$
 This expression cannot be written in quadratic form, since $a^5 \neq (a^3)^2$.

Exercises

Write each polynomial in quadratic form, if possible.

- $x^4 + 6x^2 - 8$ $4p^2 + 6p^2 + 8$
- $(x^2)^2 + 6(x^2) - 8$ $4(p^2)^2 + 6(p^2) + 8$
- $x^8 + 2x^4 + 1$ $\frac{1}{(x^{16})^2} + 2(\frac{1}{x^{16}}) + 1$
- $x^8 + 2x^4 + 1$ $(x^4)^2 + 2(x^4) + 1$
- $6x^4 + 3x^2 + 18$ $12x^4 + 10x^2 - 4$
- $12x^4 + 10x^2 - 4$ $12(x^2)^2 + 10(x^2) - 4$
- $18x^6 - 2x^3 + 12$ $18(x^3)^2 - 2(x^3) + 12$
- $18(x^3)^2 - 2(x^3) + 12$ $100(x^2)^2 - 9(x^2) - 15$
- $100x^4 - 9x^2 - 15$ $100(x^2)^2 - 9(x^2) - 15$
- $48x^6 - 32x^3 + 20$ $48(x^2)^2 - 32(x^3) + 20$
- $48(x^2)^2 - 32(x^3) + 20$ $32(x^5)^2 + 14(x^5) - 143$
- $32x^{10} + 14x^5 - 143$ $32(x^2)^2 - 15(x^2) - 18$
- $32(x^5)^2 + 14(x^5) - 143$ $16, 10x^{10} - 7x^5 - 7$
- $60x^6 - 7x^3 + 3$ $10(x^5)^2 - 7(x^5) - 7$

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7-3 Skills Practice

Solving Equations Using Quadratic Techniques

Write each expression in quadratic form, if possible.

- $5x^4 + 2x^2 - 8$ **$5(x^2)^2 + 2(x^2) - 8$**
- $3y^8 - 4y^2 + 3$ **not possible**
- $100a^6 + a^3$ **$100(a^3)^2 + a^3$**
- $x^8 + 4x^4 + 9$ **$(x^4)^2 + 4(x^4) + 9$**
- $12x^4 - 7x^2$ **$12(x^2)^2 - 7(x^2)$**
- $6b^5 + 3b^3 - 1$ **not possible**
- $15v^6 - 8v^3 + 9$ **$15(v^3)^2 - 8(v^3) + 9$**

Solve each equation.

- $9a^3 - 9a^2 + 14a = 0$ **0, 7, 2**
- $t^4 - 3t^3 - 40t^2 = 0$ **0, -5, 8**
- $m^4 = 4$ **$-\sqrt{2}, \sqrt{2}, -i\sqrt{2}, i\sqrt{2}$**
- $m^4 - 18m^2 = -81$ **-3, 3**
- $h^4 - 10h^2 = -9$ **-1, 1, -3, 3**
- $y^4 - 7y^2 + 12 = 0$ **2, -2, $\sqrt{3}$, $-\sqrt{3}$**
- $x^5 - 7x^3 + 6x = 0$ **0, -1, 1, $\sqrt{6}$, $-\sqrt{6}$**
- $z - 5\sqrt{z} = -6$ **4, 9**

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7-3 Practice (Average)

Solving Equations Using Quadratic Techniques

Write each expression in quadratic form, if possible.

- $10b^4 + 3b^2 - 11$
- $-5x^8 + x^2 + 6$
- $28d^6 + 25d^3$
- $10(b^2)^2 + 3(b^2) - 11$ **not possible**
- $28(d^3)^2 + 25(d^3)$
- $4s^8 + 4s^4 + 7$
- $500x^4 - x^2$
- $8b^5 - 8b^3 - 1$
- $4(s^4)^2 + 4(s^4) + 7$ **500(x^2)^2 - x^2** **not possible**
- $32w^5 - 56w^3 + 8w$
- $e^{\frac{2}{3}} + 7e^{\frac{1}{3}} - 10$
- $x^{\frac{1}{5}} + 29x^{\frac{1}{10}} + 2$
- $8w[4(w^2)^2 - 7(w^2) + 1] + 1$ **$(e^{\frac{1}{3}})^2 + 7(e^{\frac{1}{3}}) - 10$**
- $(e^{\frac{1}{3}})^2 + 29(x^{\frac{1}{10}}) + 2$

Solve each equation.

- $y^4 - 7y^3 - 18y^2 = 0$ **-2, 0, 9**
- $m^4 - 625 = 0$ **-5, 5, -5i, 5i**
- $x^4 - 50x^2 + 49 = 0$ **-1, 1, -7, 7**
- $4t^6 - 9t^4 = 0$ **0, $\frac{3}{2}$, $-\frac{3}{2}$**
- $d^4 = 16d^3 - 48$ **-2, 2, $-2\sqrt{3}$, $2\sqrt{3}$**
- $t^3 - 343 = 0$ **7, $\frac{-7 - 7i\sqrt{3}}{2}$, $\frac{-7 + 7i\sqrt{3}}{2}$**
- $x^3 - 28y^{\frac{3}{2}} + 27 = 0$ **1, 9**
- $w - 12\sqrt{w} + 27 = 0$ **9, 81**
- $x - 2\sqrt{x} - 80 = 0$ **100**

26. PHYSICS A proton in a magnetic field follows a path on a coordinate grid modeled by the function $f(x) = x^4 - 2x^2 - 15$. What are the x -coordinates of the points on the grid where the proton crosses the x -axis? **$-\sqrt{5}, \sqrt{5}$**

27. SURVEYING Vista county is setting aside a large parcel of land to preserve it as open space. The county has hired Meghan's surveying firm to survey the parcel, which is in the shape of a right triangle. The longer leg of the triangle measures 5 miles less than the square of the shorter leg, and the hypotenuse of the triangle measures 13 miles less than twice the square of the shorter leg. The length of each boundary is a whole number. Find the length of each boundary. **3 mi, 4 mi, 5 mi**

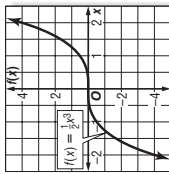
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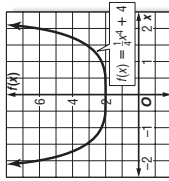
7-3 Enrichment

Odd and Even Polynomial Functions

Functions whose graphs are symmetric with respect to the origin are called **odd** functions. If $f(-x) = -f(x)$ for all x in the domain of $f(x)$, then $f(x)$ is odd.



Functions whose graphs are symmetric with respect to the y -axis are called **even** functions. If $f(-x) = f(x)$ for all x in the domain of $f(x)$, then $f(x)$ is even.



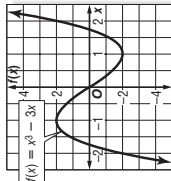
Example

Determine whether $f(x) = x^3 - 3x$ is odd, even, or neither.

$$\begin{aligned} f(x) &= x^3 - 3x \\ f(-x) &= (-x)^3 - 3(-x) && \text{Replace } x \text{ with } -x. \\ &= -x^3 + 3x && \text{Simplify.} \\ &= -(x^3 - 3x) && \text{Factor out } -1. \\ &= -f(x) && \text{Substitute.} \end{aligned}$$

Therefore, $f(x)$ is odd.

The graph at the right verifies that $f(x)$ is odd. The graph of the function is symmetric with respect to the origin.



Determine whether each function is **odd**, **even**, or **neither** by graphing or by applying the rules for odd and even functions.

1. $f(x) = 4x^2$ **even**
2. $f(x) = -7x^4$ **even**
3. $f(x) = x^7$ **odd**
4. $f(x) = x^3 - x^2$ **neither**
5. $f(x) = 3x^3 + 1$ **neither**
6. $f(x) = x^8 - x^5 - 6$ **neither**
7. $f(x) = -8x^5 - 2x^3 + 6x$ **odd**
8. $f(x) = x^4 - 3x^3 + 2x^2 - 6x + 1$ **neither**
9. $f(x) = x^4 + 3x^2 + 11$ **even**
10. $f(x) = x^7 - 6x^5 + 2x^3 + x$ **odd**

11. Complete the following definitions: A polynomial function is odd if and only if all the terms are of **odd** degrees. A polynomial function is even if and only if all the terms are of **even** degrees.

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7-3 Reading to Learn Mathematics

Solving Equations Using Quadratic Techniques

Pre-Activity How can solving polynomial equations help you to find dimensions?

Read the introduction to Lesson 7-3 at the top of page 360 in your textbook. Explain how the formula given for the volume of the box can be obtained from the dimensions shown in the figure.

Sample answer: The volume of a rectangular box is given by the formula $V = \ell wh$. Substitute $50 - 2x$ for ℓ , $32 - 2x$ for w , and x for h to get $V(x) = (50 - 2x)(32 - 2x)(x) = 4x^3 - 164x^2 + 1600x$.

Reading the Lesson

1. Which of the following expressions can be written in quadratic form? **b, c, d, f, g, h, i**

- a. $x^3 + 6x^2 + 9$
- b. $x^4 - 7x^2 + 6$
- c. $m^6 + 4m^3 + 4$
- d. $y - 2y^{\frac{1}{2}} - 15$
- e. $x^5 + x^3 + 1$
- f. $r^4 + 6 - r^8$
- g. $p^{\frac{1}{4}} + 8p^{\frac{1}{2}} + 12$
- h. $r^{\frac{1}{3}} + 2r^{\frac{1}{6}} - 3$
- i. $5\sqrt{z} + 2z - 3$

2. Match each expression from the list on the left with its factorization from the list on the right.

- | | |
|-----------------------------------|------------------------------------|
| a. $x^4 - 3x^2 - 40$ vi | i. $(x^3 + 3)(x^3 - 3)$ |
| b. $x^4 - 10x^2 + 25$ v | ii. $(\sqrt{x} + 3)(\sqrt{x} - 3)$ |
| c. $x^6 - 9$ i | iii. $(\sqrt{x} + 3)^2$ |
| d. $x - 9$ ii | iv. $(x^2 + 1)(x^4 - x^2 + 1)$ |
| e. $x^6 + 1$ iv | v. $(x^2 - 5)^2$ |
| f. $x + 6\sqrt{x} + 9$ iii | vi. $(x^2 + 5)(x^2 - 8)$ |

Helping You Remember

3. What is an easy way to tell whether a trinomial in one variable containing one constant term can be written in quadratic form?

Sample answer: Look at the two terms that are not constants and compare the exponents on the variable. If one of the exponents is twice the other, the trinomial can be written in quadratic form.

Lesson 7-3

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7-4 Study Guide and Intervention

The Remainder and Factor Theorems

Synthetic Substitution

Remainder Theorem

The remainder, when you divide the polynomial $f(x)$ by $(x - a)$, is the constant $f(a)$. $f(x) = q(x) \cdot (x - a) + f(a)$, where $q(x)$ is a polynomial with degree one less than the degree of $f(x)$.

Example 1 If $f(x) = 3x^4 + 2x^3 - 5x^2 + x - 2$, find $f(-2)$.

Method 1 Synthetic Substitution

By the Remainder Theorem, $f(-2)$ should

be the remainder when you divide the

$$\begin{array}{r|rrrrr} -2 & 3 & 2 & -5 & 1 & -2 \\ & & -6 & 8 & -6 & 10 \\ \hline & 3 & -4 & 3 & -5 & 8 \end{array}$$

The remainder is 8, so $f(-2) = 8$.

Example 2 If $f(x) = 5x^3 + 2x - 1$, find $f(3)$.

Again, by the Remainder Theorem, $f(3)$ should be the remainder when you divide the polynomial by $x - 3$.

$$\begin{array}{r|rrrr} 3 & 5 & 0 & 2 & -1 \\ & & 15 & 45 & 141 \\ \hline & 5 & 15 & 47 & 140 \end{array}$$

The remainder is 140, so $f(3) = 140$.

Exercises

Use synthetic substitution to find $f(-5)$ and $f(\frac{1}{2})$ for each function.

- $f(x) = -3x^2 + 5x - 1$ **-101; $\frac{3}{4}$**
- $f(x) = 4x^2 + 6x - 7$ **63; -3**
- $f(x) = -x^3 + 3x^2 - 5$ **195; $-\frac{35}{8}$**
- $f(x) = x^4 + 11x^2 - 1$ **899; $\frac{29}{16}$**

Use synthetic substitution to find $f(4)$ and $f(-3)$ for each function.

- $f(x) = 2x^3 + x^2 - 5x + 3$ **127; -27**
- $f(x) = 5x^3 - 4x^2 + 2$ **258; -169**
- $f(x) = 5x^4 + 3x^3 - 4x^2 - 2x + 4$ **1404; 298**
- $f(x) = 2x^4 - 4x^3 - x^2 - 6x + 3$ **219; 282**
- $f(x) = 3x^3 - 4x^2 - 2x^3 - x^2 + x - 6$ **178; -67**
- $f(x) = 2x^4 - 4x^3 + 3x^2 + x - 6$ **302; 288**
- $f(x) = 3x^4 - 2x^3 - x^2 + 2x - 5$ **627; 277**
- $f(x) = 4x^4 - 4x^3 + 3x^2 - 2x - 3$ **805; 462**

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Glencoe Algebra 2

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7-4 Study Guide and Intervention

The Remainder and Factor Theorems

Factors of Polynomials The Factor Theorem can help you find all the factors of a polynomial.

Factor Theorem

The binomial $x - a$ is a factor of the polynomial $f(x)$ if and only if $f(a) = 0$.

Example Show that $x + 5$ is a factor of $x^3 + 2x^2 - 13x + 10$. Then find the remaining factors of the polynomial.

By the Factor Theorem, the binomial $x + 5$ is a factor of the polynomial if -5 is a zero of the polynomial function. To check this, use synthetic substitution.

$$\begin{array}{r|rrrr} -5 & 1 & 2 & -13 & 10 \\ & & -5 & 15 & -10 \\ \hline & 1 & -3 & 2 & 0 \end{array}$$

Since the remainder is 0, $x + 5$ is a factor of the polynomial. The polynomial $x^3 + 2x^2 - 13x + 10$ can be factored as $(x + 5)(x^2 - 3x + 2)$. The depressed polynomial $x^2 - 3x + 2$ can be factored as $(x - 2)(x - 1)$.

So $x^3 + 2x^2 - 13x + 10 = (x + 5)(x - 2)(x - 1)$.

Exercises

Given a polynomial and one of its factors, find the remaining factors of the polynomial. Some factors may not be binomials.

- $x^3 + x^2 - 10x + 8; x - 2$ **$(x + 4)(x - 1)$**
- $x^3 - 4x^2 - 11x + 30; x + 3$ **$(x - 5)(x - 2)$**
- $x^3 + 15x^2 + 71x + 105; x + 7$ **$(x + 3)(x + 5)$**
- $x^3 - 7x^2 - 26x + 72; x + 4$ **$(x - 2)(x - 9)$**
- $2x^3 - x^2 - 7x + 6; x - 1$ **$(2x - 3)(x + 2)$**
- $3x^3 - x^2 - 62x - 40; x + 4$ **$(3x + 2)(x - 5)$**
- $12x^3 - 71x^2 + 57x - 10; x - 5$ **$(4x - 1)(3x - 2)$**
- $14x^3 + x^2 - 24x + 9; x - 1$ **$(7x - 3)(2x + 3)$**
- $x^3 + x + 10; x + 2$ **$(x^2 - 2x + 5)$**
- $2x^3 - x^2 - 11x^2 + 19x - 28; x - 4$ **$(2x^2 - 3x + 7)$**
- $3x^3 - 13x^2 - 34x + 24; x - 6$ **$(3x^2 + 5x - 4)$**
- $3x^4 + x^3 - 11x^2 - 9x + 18; x - 1$ **$(x + 2)(x + 3)(x - 3)$**

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7-4 Skills Practice

The Remainder and Factor Theorems

Use synthetic substitution to find $f(2)$ and $f(-1)$ for each function.

1. $f(x) = x^2 + 6x + 5$ **21, 0**
2. $f(x) = x^2 - x + 1$ **3, 3**
3. $f(x) = x^2 - 2x - 2$ **-2, 1**
4. $f(x) = x^3 + 2x^2 + 5$ **21, 6**
5. $f(x) = x^3 - x^2 - 2x + 3$ **3, 3**
6. $f(x) = x^3 + 6x^2 + x - 4$ **30, 0**
7. $f(x) = x^3 - 3x^2 + x - 2$ **-4, -7**
8. $f(x) = x^3 - 5x^2 - x + 6$ **-8, 1**
9. $f(x) = x^4 + 2x^2 - 9$ **15, -6**
10. $f(x) = x^4 - 3x^3 + 2x^2 - 2x + 6$ **2, 14**
11. $f(x) = x^5 - 7x^3 - 4x + 10$
-22, 20
12. $f(x) = x^6 - 2x^5 + x^4 + x^3 - 9x^2 - 20$
-32, -26

Given a polynomial and one of its factors, find the remaining factors of the polynomial. Some factors may not be binomials.

13. $x^3 + 2x^2 - x - 2; x + 1$
 $x - 1, x + 2$
14. $x^3 + x^2 - 5x + 3; x - 1$
 $x - 1, x + 3$
15. $x^3 + 3x^2 - 4x - 12; x + 3$
 $x - 2, x + 2$
16. $x^3 - 6x^2 + 11x - 6; x - 3$
 $x - 1, x - 2$
17. $x^3 + 2x^2 - 33x - 90; x + 5$
 $x + 3, x - 6$
18. $x^3 - 6x^2 + 32; x - 4$
 $x - 4, x + 2$
19. $x^3 - x^2 - 10x - 8; x + 2$
 $x + 1, x - 4$
20. $x^3 - 19x + 30; x - 2$
 $x + 5, x - 3$
21. $2x^3 + x^2 - 2x - 1; x + 1$
 $2x + 1, x - 1$
22. $2x^3 + x^2 - 5x + 2; x + 2$
 $x - 1, 2x - 1$
23. $3x^3 + 4x^2 - 5x - 2; 3x + 1$
 $x - 1, x + 2$
24. $3x^3 + x^2 - 5x - 2; 3x + 1$
 $x^2 + x + 1$

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Glencoe Algebra 2

7-4 Practice (Average)

The Remainder and Factor Theorems

Use synthetic substitution to find $f(-3)$ and $f(4)$ for each function.

1. $f(x) = x^2 + 2x + 3$ **6, 27**
2. $f(x) = x^2 - 5x + 10$ **34, 6**
3. $f(x) = x^2 - 5x - 4$ **20, -8**
4. $f(x) = x^3 - x^2 - 2x + 3$ **-27, 43**
5. $f(x) = x^3 + 2x^2 + 5$ **-4, 101**
6. $f(x) = x^3 - 6x^2 + 2x$ **-87, -24**
7. $f(x) = x^3 - 2x^2 - 2x + 8$ **-31, 32**
8. $f(x) = x^3 - x^2 + 4x - 4$ **-52, 60**
9. $f(x) = x^3 + 3x^2 + 2x - 50$ **-56, 70**
10. $f(x) = x^4 + x^3 - 3x^2 - x + 12$ **42, 280**
11. $f(x) = x^4 - 2x^2 - x + 7$ **73, 227**
12. $f(x) = 2x^4 - 3x^3 + 4x^2 - 2x + 1$ **286, 377**
13. $f(x) = 2x^4 - x^3 + 2x^2 - 26$ **181, 454**
14. $f(x) = 3x^4 - 4x^3 + 3x^2 - 5x - 3$ **390, 537**
15. $f(x) = x^5 + 7x^3 - 4x - 10$
-430, 1446
16. $f(x) = x^6 + 2x^5 - x^4 + x^3 - 9x^2 + 20$
74, 5828

Given a polynomial and one of its factors, find the remaining factors of the polynomial. Some factors may not be binomials.

17. $x^3 + 3x^2 - 6x - 8; x - 2$
 $x + 1, x + 4$
18. $x^3 + 7x^2 + 7x - 15; x - 1$
 $x + 3, x + 5$
19. $x^3 - 9x^2 + 27x - 27; x - 3$
 $x - 3, x - 3$
20. $x^3 - x^2 - 8x + 12; x + 3$
 $x - 2, x - 2$
21. $x^3 + 5x^2 - 2x - 24; x - 2$
 $x + 3, x + 4$
22. $x^3 - x^2 - 14x + 24; x + 4$
 $x - 3, x - 2$
23. $3x^3 - 4x^2 - 17x + 6; x + 2$
 $x - 3, 3x - 1$
24. $4x^3 - 12x^2 - x + 3; x - 3$
 $2x - 1, 2x + 1$
25. $18x^3 + 9x^2 - 2x - 1; 2x + 1$
 $3x + 1, 3x - 1$
26. $6x^3 + 5x^2 - 3x - 2; 3x - 2$
 $2x + 1, x + 1$
27. $x^5 + x^4 - 5x^3 - 5x^2 + 4x + 4; x + 1$
 $x - 1, x + 1, x - 2, x + 2$
28. $x^5 - 2x^4 + 4x^3 - 8x^2 - 5x + 10; x - 2$
 $x - 1, x + 1, x^2 + 5$
29. **POPULATION** The projected population in thousands for a city over the next several years can be estimated by the function $P(x) = x^3 + 2x^2 - 8x + 520$, where x is the number of years since 2000. Use synthetic substitution to estimate the population for 2005. **655,000**

30. VOLUME The volume of water in a rectangular swimming pool can be modeled by the polynomial $2x^3 - 9x^2 + 7x + 6$. If the depth of the pool is given by the polynomial $2x + 1$, what polynomials express the length and width of the pool? **$x - 3$ and $x - 2$**

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Lesson 7-4

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7-4 Reading to Learn Mathematics

The Remainder and Factor Theorems

Pre-Activity How can you use the Remainder Theorem to evaluate polynomials?

Read the introduction to Lesson 7-4 at the top of page 365 in your textbook. Show how you would use the model in the introduction to estimate the number of international travelers (in millions) to the United States in the year 2000. (Show how you would substitute numbers, but do not actually calculate the result.)

Sample answer: $0.02(14)^3 - 0.6(14)^2 + 6(14) + 25.9$

Reading the Lesson

1. Consider the following synthetic division.

$$\begin{array}{r|rrrr} 1 & 3 & 2 & -6 & 4 \\ & & 3 & 5 & -1 \\ \hline & 3 & 5 & -1 & 3 \end{array}$$

- a. Using the division symbol \div , write the division problem that is represented by this synthetic division. (Do not include the answer.) $(3x^3 + 2x^2 - 6x + 4) \div (x - 1)$
- b. Identify each of the following for this division.
- | | | | |
|----------|------------------------|-----------|---------|
| dividend | $3x^3 + 2x^2 - 6x + 4$ | divisor | $x - 1$ |
| quotient | $3x^2 + 5x - 1$ | remainder | 3 |

c. If $f(x) = 3x^3 + 2x^2 - 6x + 4$, what is $f(1)$? 3

2. Consider the following synthetic division.

$$\begin{array}{r|rrrr} -3 & 1 & 0 & 0 & 27 \\ & & -3 & 9 & -27 \\ \hline & 1 & -3 & 9 & 0 \end{array}$$

- a. This division shows that $x + 3$ is a factor of $x^3 + 27$.
- b. The division shows that -3 is a zero of the polynomial function $f(x) = x^3 + 27$.
- c. The division shows that the point $(-3, 0)$ is on the graph of the polynomial function $f(x) = x^3 + 27$.

Helping You Remember

3. Think of a mnemonic for remembering the sentence, "Dividend equals quotient times divisor plus remainder."

Sample answer: **Definitely every quiet teacher deserves proper rewards.**

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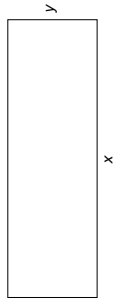
7-4 Enrichment

Using Maximum Values

Many times maximum solutions are needed for different situations. For instance, what is the area of the largest rectangular field that can be enclosed with 2000 feet of fencing?

Let x and y denote the length and width of the field, respectively.

Perimeter: $2x + 2y = 2000 \rightarrow y = 1000 - x$
 Area: $A = xy = x(1000 - x) = -x^2 + 1000x$

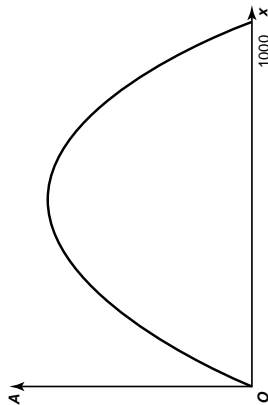


This problem is equivalent to finding the highest point on the graph of $A(x) = -x^2 + 1000x$ shown on the right.

Complete the square for $-x^2 + 1000x$.

$$A = -(x^2 - 1000x + 500^2) + 500^2 = -(x - 500)^2 + 500^2$$

Because the term $-(x - 500)^2$ is either negative or 0, the greatest value of A is 500^2 . The maximum area enclosed is 500^2 or 250,000 square feet.



Solve each problem.

- Find the area of the largest rectangular garden that can be enclosed by 300 feet of fence.
5625 ft²
- A farmer will make a rectangular pen with 100 feet of fence using part of his barn for one side of the pen. What is the largest area he can enclose?
1250 ft²
- An area along a straight stone wall is to be fenced. There are 600 meters of fencing available. What is the greatest rectangular area that can be enclosed?
45,000 m²

Lesson 7-4

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7-5 Study Guide and Intervention Roots and Zeros

Types of Roots The following statements are equivalent for any polynomial function $f(x)$.

- c is a zero of the polynomial function $f(x)$.
 - $(x - c)$ is a factor of the polynomial $f(x)$.
 - c is a root or solution of the polynomial equation $f(x) = 0$.
- If c is real, then $(c, 0)$ is an intercept of the graph of $f(x)$.

Fundamental Theorem of Algebra	Every polynomial equation with degree greater than zero has at least one root in the set of complex numbers.
Corollary to the Fundamental Theorem of Algebra	A polynomial equation of the form $P(x) = 0$ of degree n with complex coefficients has exactly n roots in the set of complex numbers.
Descartes' Rule of Signs	If $P(x)$ is a polynomial with real coefficients whose terms are arranged in descending powers of the variable, <ul style="list-style-type: none"> the number of positive real zeros of $y = P(x)$ is the same as the number of changes in sign of the coefficients of the terms, or is less than this by an even number, and the number of negative real zeros of $y = P(x)$ is the same as the number of changes in sign of the coefficients of the terms of $P(-x)$, or is less than this number by an even number.

Example 1

Solve the equation $6x^3 + 3x = 0$ and state the number and type of roots.

$$6x^3 + 3x = 0$$

$$3x(2x^2 + 1) = 0$$

Use the Zero Product Property.

$$3x = 0 \text{ or } 2x^2 + 1 = 0$$

$$x = 0 \text{ or } 2x^2 = -1$$

$$x = \pm \frac{i\sqrt{2}}{2}$$

The equation has one real root, 0, and two imaginary roots, $\pm \frac{i\sqrt{2}}{2}$.

Exercises

Solve each equation and state the number and type of roots.

1. $x^2 + 4x - 21 = 0$

2. $2x^3 - 50x = 0$

3, -7; 2 real

0, $\pm \frac{5i\sqrt{3}}{3}$; 1 real, 2 imaginary

State the number of positive real zeros, negative real zeros, and imaginary zeros for each function.

4. $f(x) = 3x^3 + x^2 - 8x - 12$ **1; 2 or 0; 0 or 2**

5. $f(x) = 2x^4 - x^3 - 3x + 7$ **2 or 0; 0; 2 or 4**

6. $f(x) = 3x^5 - x^4 - x^3 + 6x^2 - 5$ **3 or 1; 2 or 0; 0, 2, or 4**

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7-5 Study Guide and Intervention Roots and Zeros

Find Zeros

Complex Conjugate Theorem Suppose a and b are real numbers with $b \neq 0$. If $a + bi$ is a zero of a polynomial function with real coefficients, then $a - bi$ is also a zero of the function.

Example Find all of the zeros of $f(x) = x^4 - 15x^2 + 38x - 60$.

Since $f(x)$ has degree 4, the function has 4 zeros.

$$f(x) = x^4 - 15x^2 + 38x - 60 \quad f(-x) = x^4 - 15x^2 - 38x - 60$$

Since there are 3 sign changes for the coefficients of $f(x)$, the function has 3 or 1 positive real zeros. Since there is 1 sign change for the coefficients of $f(-x)$, the function has 1 negative real zero. Use synthetic substitution to test some possible zeros.

2	1	0	-15	38	-60
	2	4	-22	32	
	1	2	-11	16	-28

3	1	0	-15	38	-60
	3	9	-18	60	
	1	3	-6	20	0

So 3 is a zero of the polynomial function. Now try synthetic substitution again to find a zero of the depressed polynomial.

-2	1	3	-6	20
	-2	-2	2	16
	1	1	-8	36

-4	1	3	-6	20
	-4	4	8	
	1	-1	-2	28

-5	1	3	-6	20
	-5	10	-20	
	1	-2	4	0

So -5 is another zero. Use the Quadratic Formula on the depressed polynomial $x^2 - 2x + 4$ to find the other 2 zeros, $1 \pm i\sqrt{3}$.

The function has two real zeros at 3 and -5 and two imaginary zeros at $1 \pm i\sqrt{3}$.

Exercises

Find all of the zeros of each function.

1. $f(x) = x^3 + x^2 + 9x + 9$ **-1, $\pm 3i$**

2. $f(x) = x^3 - 3x^2 + 4x - 12$ **3, $\pm 2i$**

3. $p(a) = a^3 - 10a^2 + 34a - 40$ **4, 3 $\pm i$**

4. $p(x) = x^3 - 5x^2 + 11x - 15$ **3, 1 $\pm 2i$**

5. $f(x) = x^3 + 6x + 20$ **-2, 1 $\pm 3i$**

6. $f(x) = x^4 - 3x^3 + 21x^2 - 75x - 100$ **-1, 4, $\pm 5i$**

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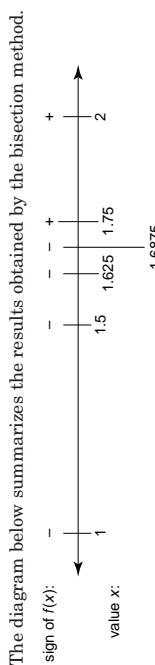
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<p style="text-align: center;">7-5 Skills Practice Roots and Zeros</p> <p>Solve each equation. State the number and type of roots.</p> <p>1. $5x + 12 = 0$ $-\frac{12}{5}$; 1 real</p> <p>3. $x^5 + 4x^3 = 0$ 0, 0, 0, 2i, -2i; 3 real, 2 imaginary</p> <p>5. $4x^2 - 4x - 1 = 0$ $1 \pm \sqrt{2}$; 2 real</p> <p>State the possible number of positive real zeros, negative real zeros, and imaginary zeros of each function.</p> <p>7. $g(x) = 3x^3 - 4x^2 - 17x + 6$ 2 or 0; 1; 2 or 0</p> <p>9. $f(x) = x^3 - 8x^2 + 2x - 4$ 3 or 1; 0; 2 or 0</p> <p>11. $q(x) = x^4 + 7x^2 + 3x - 9$ 1; 1; 2</p> <p>Find all the zeros of each function.</p> <p>13. $h(x) = x^3 - 5x^2 + 5x + 3$ 3, 1 + $\sqrt{2}$, 1 - $\sqrt{2}$</p> <p>15. $h(x) = x^3 + 4x^2 + x - 6$ 1, -2, -3</p> <p>17. $g(x) = x^4 - 3x^3 - 5x^2 + 3x + 4$ -1, -1, 1, 4</p> <p>Write a polynomial function of least degree with integral coefficients that has the given zeros.</p> <p>19. -3, -5, 1 $f(x) = x^3 + 7x^2 + 7x - 15$</p> <p>21. -5 + i $f(x) = x^2 + 10x + 26$</p> <p>23. i, 5i $f(x) = x^4 + 26x^2 + 25$</p>	<p style="text-align: center;">7-5 Practice (Average) Roots and Zeros</p> <p>Solve each equation. State the number and type of roots.</p> <p>1. $-9x - 15 = 0$ $-\frac{5}{3}$; 1 real</p> <p>3. $x^5 = 81x$ 0, -3, 3, -3i, 3i; 3 real, 2 imaginary</p> <p>5. $x^3 + 6x + 20 = 0$ -2, 1 \pm 3i; 1 real, 2 imaginary</p> <p>State the possible number of positive real zeros, negative real zeros, and imaginary zeros of each function.</p> <p>7. $f(x) = 4x^3 - 2x^2 + x + 3$ 2 or 0; 1; 2 or 0</p> <p>9. $q(x) = 3x^4 + x^3 - 3x^2 + 7x + 5$ 2 or 0; 2 or 0; 4, 2, or 0</p> <p>11. $h(x) = 2x^3 + 3x^2 - 65x + 84$ -7, 2, 4</p> <p>13. $h(x) = x^3 - 7x^2 + 17x - 15$ 3, 2 + i, 2 - i</p> <p>15. $g(x) = x^4 + 4x^3 - 3x^2 - 14x - 8$ -1, -1, 2, -4</p> <p>Write a polynomial function of least degree with integral coefficients that has the given zeros.</p> <p>17. -5, 3i $f(x) = x^3 + 5x^2 + 9x + 45$</p> <p>19. -1, 4, 3i $f(x) = x^4 - 3x^3 + 5x^2 - 27x - 36$</p> <p>21. CRAFTS Stephan has a set of plans to build a wooden box. He wants to reduce the volume of the box to 105 cubic inches. He would like to reduce the length of each dimension in the plan by the same amount. The plans call for the box to be 10 inches by 8 inches by 6 inches. Write and solve a polynomial equation to find out how much Stephen should take from each dimension. $(10 - x)(8 - x)(6 - x) = 105$; 3 in.</p>
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7-5 Enrichment

The Bisection Method for Approximating Real Zeros

The **bisection method** can be used to approximate zeros of polynomial functions like $f(x) = x^3 + x^2 - 3x - 3$. Since $f(1) = -4$ and $f(2) = 3$, there is at least one real zero between 1 and 2. The midpoint of this interval is $\frac{1+2}{2} = 1.5$. Since $f(1.5) = -1.875$, the zero is between 1.5 and 2. The midpoint of this interval is $\frac{1.5+2}{2} = 1.75$. Since $f(1.75)$ is about 0.172, the zero is between 1.5 and 1.75. The midpoint of this interval is $\frac{1.5+1.75}{2} = 1.625$ and $f(1.625)$ is about -0.94 . The zero is between 1.625 and 1.75. The midpoint of this interval is $\frac{1.625+1.75}{2} = 1.6875$. Since $f(1.6875)$ is about -0.41 , the zero is between 1.6875 and 1.75. Therefore, the zero is 1.7 to the nearest tenth.



Using the bisection method, approximate to the nearest tenth the zero between the two integral values of x for each function.

1. $f(x) = x^3 - 4x^2 - 11x + 2$, $f(0) = 2$, $f(1) = -12$ **0.2**
2. $f(x) = 2x^4 + x^2 - 15$, $f(1) = -12$, $f(2) = 21$ **1.6**
3. $f(x) = x^5 - 2x^3 - 12$, $f(1) = -13$, $f(2) = 4$ **1.9**
4. $f(x) = 4x^3 - 2x + 7$, $f(-2) = -21$, $f(-1) = 5$ **-1.3**
5. $f(x) = 3x^3 - 14x^2 - 27x + 126$, $f(4) = -14$, $f(5) = 16$ **4.7**

7-5 Reading to Learn Mathematics

Roots and Zeros

Pre-Activity How can the roots of an equation be used in pharmacology? Read the introduction to Lesson 7-5 at the top of page 371 in your textbook. Using the model given in the introduction, write a polynomial equation with 0 on one side that can be solved to find the time or times at which there is 100 milligrams of medication in a patient's bloodstream. **$0.5t^4 + 3.5t^3 - 100t^2 + 350t - 100 = 0$**

Reading the Lesson

1. Indicate whether each statement is *true* or *false*.
 - a. Every polynomial equation of degree greater than one has at least one root in the set of real numbers. **false**
 - b. If c is a root of the polynomial equation $f(x) = 0$, then $(x - c)$ is a factor of the polynomial $f(x)$. **true**
 - c. If $(x + c)$ is a factor of the polynomial $f(x)$, then c is a zero of the polynomial function f . **false**
 - d. A polynomial function f of degree n has exactly $(n - 1)$ complex zeros. **false**
2. Let $f(x) = x^6 - 2x^5 + 3x^4 - 4x^3 + 5x^2 + 6x - 7$.
 - a. What are the possible numbers of positive real zeros of f ? **5, 3, or 1**
 - b. Write $f(-x)$ in simplified form (with no parentheses). **$x^6 + 2x^5 + 3x^4 + 4x^3 + 5x^2 - 6x - 7$**
3. What are the possible numbers of negative real zeros of f ? **1**
4. Complete the following chart to show the possible combinations of positive real zeros, negative real zeros, and imaginary zeros of the polynomial function f .

Number of Positive Real Zeros	Number of Negative Real Zeros	Number of Imaginary Zeros	Total Number of Zeros
5	1	0	6
3	1	2	6
1	1	4	6

Helping You Remember

3. It is easier to remember mathematical concepts and results if you relate them to each other. How can the Complex Conjugates Theorem help you remember the part of Descartes' Rule of Signs that says, "or is less than this number by an even number." **Sample answer: For a polynomial function in which the polynomial has real coefficients, imaginary zeros come in conjugate pairs. Therefore, there must be an even number of imaginary zeros. For each pair of imaginary zeros, the number of positive or negative zeros decreases by 2.**

7-6 Study Guide and Intervention

Rational Zero Theorem

Identify Rational Zeros

Rational Zero Theorem	Let $f(x) = a_nx^n + a_{n-1}x^{n-1} + \dots + a_1x + a_0$ represent a polynomial function with integral coefficients. If $\frac{p}{q}$ is a rational number in simplest form and is a zero of $y = f(x)$, then p is a factor of a_0 , and q is a factor of a_n .
Corollary (Integral Zero Theorem)	If the coefficients of a polynomial are integers such that $a_0 = 1$ and $a_n \neq 0$, any rational zeros of the function must be factors of a_0 .

Example List all of the possible rational zeros of each function.

a. $f(x) = 3x^4 - 2x^2 + 6x - 10$

If $\frac{p}{q}$ is a rational root, then p is a factor of -10 and q is a factor of 3 . The possible values for p are $\pm 1, \pm 2, \pm 5$, and ± 10 . The possible values for q are ± 1 and ± 3 . So all of the possible rational zeros are $\frac{p}{q} = \pm 1, \pm 2, \pm 5, \pm 10, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{5}{3},$ and $\pm \frac{10}{3}$.

b. $g(x) = x^3 - 10x^2 + 14x - 36$

Since the coefficient of x^3 is 1 , the possible rational zeros must be the factors of the constant term -36 . So the possible rational zeros are $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 9, \pm 12, \pm 18$, and ± 36 .

Exercises

List all of the possible rational zeros of each function.

- $f(x) = x^3 + 3x^2 - x + 8$
 $\pm 1, \pm 2, \pm 4, \pm 8$
- $g(x) = x^5 - 7x^4 + 3x^2 + x - 20$
 $\pm 1, \pm 2, \pm 4, \pm 5, \pm 10, \pm 20$
- $h(x) = x^4 - 7x^3 - 4x^2 + x - 49$
 $\pm 1, \pm 7, \pm 49$
- $q(x) = 3x^4 - 5x^3 + 10x + 12$
 $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12,$
 $\pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{4}{3}$
- $f(x) = x^7 - 6x^5 - 3x^4 + x^3 + 4x^2 - 120$
 $\pm 1, \pm 2, \pm 3, \pm 4, \pm 5, \pm 6, \pm 8, \pm 10, \pm 12,$
 $\pm 15, \pm 20, \pm 24, \pm 30, \pm 40, \pm 60, \pm 120$
- $r(x) = 4x^5 - 2x + 18$
 $\pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18,$
 $\pm \frac{1}{4}, \pm \frac{3}{4}, \pm \frac{9}{4}, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{9}{2}, \pm \frac{1}{6}, \pm \frac{1}{3}, \pm \frac{1}{2}, \pm \frac{1}{4}, \pm \frac{1}{6}$
- $g(x) = 5x^6 - 3x^4 + 5x^3 + 2x^2 - 15$
 $\pm 1, \pm 3, \pm 5, \pm 15, \pm \frac{1}{5}, \pm \frac{3}{5}, \pm \frac{1}{3}, \pm \frac{5}{3}$
- $p(x) = 2x^7 - 3x^6 + 11x^5 - 20x^2 + 11$
 $\pm 1, \pm 11, \pm \frac{1}{2}, \pm \frac{11}{2}$
- $h(x) = 6x^5 - 3x^4 + 12x^3 + 18x^2 - 9x + 21$
 $\pm 1, \pm 3, \pm 7, \pm 21, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{7}{2}, \pm \frac{21}{2},$
 $\pm \frac{1}{3}, \pm \frac{7}{3}, \pm \frac{1}{6}, \pm \frac{7}{6}$

7-6 Study Guide and Intervention

Rational Zero Theorem

Find Rational Zeros

Example 1

Find all of the rational zeros of $f(x) = 5x^3 + 12x^2 - 29x + 12$. From the corollary to the Fundamental Theorem of Algebra, we know that there are exactly 3 complex roots. According to Descartes' Rule of Signs there are 2 or 0 positive real roots and 1 negative real root. The possible rational zeros are $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12, \pm \frac{1}{5}, \pm \frac{2}{5}, \pm \frac{3}{5}, \pm \frac{4}{5}, \pm \frac{6}{5}, \pm \frac{12}{5}$. Make a table and test some possible rational zeros.

$\frac{p}{q}$	5	12	-29	12
q	1	5	17	-12
				0

Since $f(1) = 0$, you know that $x = 1$ is a zero. The depressed polynomial is $5x^2 + 17x - 12$, which can be factored as $(5x - 3)(x + 4)$. By the Zero Product Property, this expression equals 0 when $x = \frac{3}{5}$ or $x = -4$. The rational zeros of this function are $1, \frac{3}{5}$, and -4 .

Example 2

Find all of the zeros of $f(x) = 8x^4 + 2x^3 + 5x^2 + 2x - 3$. There are 4 complex roots, with 1 positive real root and 3 or 1 negative real roots. The possible rational zeros are $\pm 1, \pm 3, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{1}{4}, \pm \frac{3}{4}, \pm \frac{1}{8}, \pm \frac{3}{8}$, and $\pm \frac{1}{8}$.

Make a table and test some possible values.

$\frac{p}{q}$	8	2	5	2	-3
q	1	8	10	15	17
	2	8	18	41	84
	1	2	8	6	8
					0

Since $f(\frac{1}{2}) = 0$, we know that $x = \frac{1}{2}$ is a root.

The depressed polynomial is $8x^3 + 6x^2 + 8x + 6$. Try synthetic substitution again. Any remaining rational roots must be negative.

$\frac{p}{q}$	8	6	8	6
q	-1	-4	8	4
	-3	-4	8	0
				0

$x = -\frac{3}{4}$ is another rational root. The depressed polynomial is $8x^2 + 8 = 0$, which has roots $\pm i$.

The zeros of this function are $\frac{1}{2}, -\frac{3}{4}$, and $\pm i$.

Exercises

Find all of the rational zeros of each function.

- $f(x) = x^3 + 4x^2 - 25x - 28$ $-1, 4, -7$
- $f(x) = x^3 + 6x^2 + 4x + 24$ -6
- Find all of the zeros of each function.
 - $f(x) = x^4 + 2x^3 - 11x^2 + 8x - 60$
 - $f(x) = 4x^4 + 5x^3 + 30x^2 + 45x - 54$
 $\frac{3}{4}, -2, \pm 3i$

<div style="text-align: center; border-bottom: 1px solid black; padding-bottom: 5px;"> Lesson 7-6 </div> <div style="display: flex; justify-content: space-between; border-bottom: 1px solid black; padding-bottom: 5px;"> NAME _____ DATE _____ PERIOD _____ </div> <div style="text-align: center; border-bottom: 1px solid black; padding-bottom: 5px;"> <h2 style="margin: 0;">7-6 Skills Practice</h2> <h3 style="margin: 0;">Rational Zero Theorem</h3> </div> <p>List all of the possible rational zeros of each function.</p> <ol style="list-style-type: none"> 1. $n(x) = x^2 + 5x + 3$ $\pm 1, \pm 3$ 2. $h(x) = x^2 - 2x - 5$ $\pm 1, \pm 5$ 3. $w(x) = x^2 - 5x + 12$ $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$ 4. $f(x) = 2x^2 + 5x + 3$ $\pm \frac{1}{2}, \pm \frac{3}{2}, \pm 1, \pm 3$ 5. $q(x) = 6x^3 + x^2 - x + 2$ $\pm \frac{1}{6}, \pm \frac{1}{3}, \pm \frac{1}{2}, \pm 1, \pm 2, \pm 3, \pm 6$ 6. $g(x) = 9x^4 + 3x^3 + 3x^2 - x + 27$ $\pm \frac{1}{9}, \pm \frac{1}{3}, \pm 1, \pm 3, \pm 9, \pm 27$ <p>Find all of the rational zeros of each function.</p> <ol style="list-style-type: none"> 7. $f(x) = x^3 - 2x^2 + 5x - 4 = 0$ 1 8. $g(x) = x^3 - 3x^2 - 4x + 12$ $-2, 2, 3$ 9. $p(x) = x^3 - x^2 + x - 1$ 1 10. $z(x) = x^3 - 4x^2 + 6x - 4$ 2 11. $h(x) = x^3 - x^2 + 4x - 4$ 1 12. $g(x) = 3x^3 - 9x^2 - 10x - 8$ 4 13. $g(x) = 2x^3 + 7x^2 - 7x - 12$ $-4, -1, \frac{3}{2}$ 14. $h(x) = 2x^3 - 5x^2 - 4x + 3$ $-1, \frac{1}{2}, 3$ 15. $p(x) = 3x^3 - 5x^2 - 14x - 4 = 0$ $-\frac{1}{3}$ 16. $q(x) = 3x^3 + 2x^2 + 27x + 18$ $-\frac{2}{3}$ 17. $q(x) = 3x^3 - 7x^2 + 4$ $-\frac{2}{3}, 1, 2$ 18. $f(x) = x^4 - 2x^3 - 13x^2 + 14x + 24$ $-3, -1, 2, 4$ 19. $p(x) = x^4 - 5x^3 - 9x^2 - 25x - 70$ $-2, 7$ 20. $n(x) = 16x^4 - 32x^3 - 13x^2 + 29x - 6$ $-\frac{1}{4}, \frac{3}{4}, 2$ 21. $f(x) = x^3 + 5x^2 + 11x + 15$ $-3, -1 + 2i, -1 - 2i$ 22. $q(x) = x^3 - 10x^2 + 18x - 4$ $2, 4 + \sqrt{14}, 4 - \sqrt{14}$ 23. $m(x) = 6x^4 - 17x^3 + 8x^2 + 8x - 3$ $\frac{1}{3}, \frac{2}{3}, \frac{1 + \sqrt{5}}{2}, \frac{1 - \sqrt{5}}{2}$ 	<div style="text-align: center; border-bottom: 1px solid black; padding-bottom: 5px;"> Lesson 7-6 </div> <div style="display: flex; justify-content: space-between; border-bottom: 1px solid black; padding-bottom: 5px;"> NAME _____ DATE _____ PERIOD _____ </div> <div style="text-align: center; border-bottom: 1px solid black; padding-bottom: 5px;"> <h2 style="margin: 0;">7-6 Practice (Average)</h2> <h3 style="margin: 0;">Rational Zero Theorem</h3> </div> <p>List all of the possible rational zeros of each function.</p> <ol style="list-style-type: none"> 1. $h(x) = x^3 - 5x^2 + 2x + 12$ $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$ 2. $s(x) = x^4 - 8x^3 + 7x - 14$ $\pm 1, \pm 2, \pm 7, \pm 14$ 3. $f(x) = 3x^5 - 5x^4 + x + 6$ $\pm \frac{1}{3}, \pm \frac{2}{3}, \pm 1, \pm 2, \pm 3, \pm 6$ 4. $p(x) = 3x^2 + x + 7$ $\pm \frac{1}{3}, \pm \frac{7}{3}, \pm 1, \pm 7$ 5. $g(x) = 5x^3 + x^2 - x + 8$ $\pm \frac{1}{5}, \pm \frac{2}{5}, \pm \frac{4}{5}, \pm 1, \pm 2, \pm 4, \pm 8$ 6. $q(x) = 6x^5 + x^3 - 3$ $\pm \frac{1}{6}, \pm \frac{1}{3}, \pm \frac{1}{2}, \pm 1, \pm 3$ <p>Find all of the rational zeros of each function.</p> <ol style="list-style-type: none"> 7. $q(x) = x^3 + 3x^2 - 6x - 8 = 0$ $-4, -1, 2$ 8. $v(x) = x^3 - 9x^2 + 27x - 27 = 3$ 3 9. $c(x) = x^3 - x^2 - 8x + 12$ $-3, 2$ 10. $f(x) = x^4 - 49x^2 = 0$ $-7, 7$ 11. $h(x) = x^3 - 7x^2 + 17x - 15 = 3$ 3 12. $b(x) = x^3 + 6x + 20 = 2$ 2 13. $f(x) = x^3 - 6x^2 + 4x - 24 = 6$ 6 14. $g(x) = 2x^3 + 3x^2 - 4x - 4 = 2$ 2 15. $h(x) = 2x^3 - 7x^2 - 21x + 54 = 0$ $-3, 2, \frac{9}{2}$ 16. $z(x) = x^4 - 3x^3 + 5x^2 - 27x - 36 = -1, 4$ $-1, 4$ 17. $d(x) = x^4 + x^3 + 16$ no rational zeros 18. $n(x) = x^4 - 2x^3 - 3 = 3$ 3 19. $p(x) = 2x^4 - 7x^3 + 4x^2 + 7x - 6$ $-1, 1, \frac{3}{2}, 2$ 20. $q(x) = 6x^4 + 29x^3 + 40x^2 + 7x - 12$ $-\frac{3}{2}, -\frac{3}{4}$ 21. $f(x) = 2x^4 + 7x^3 - 2x^2 - 19x - 12$ $-1, -3, \frac{1 + \sqrt{33}}{4}, \frac{1 - \sqrt{33}}{4}$ 22. $q(x) = x^4 - 4x^3 + x^2 + 16x - 20$ $-2, 2, 2 + i, 2 - i$ 23. $h(x) = x^6 - 8x^3$ $0, 2, -1 + i\sqrt{3}, -1 - i\sqrt{3}$ 24. $g(x) = x^6 - 1 = 1, 1, \frac{-1 + i\sqrt{3}}{2}, \frac{-1 - i\sqrt{3}}{2}, \frac{1 + i\sqrt{3}}{2}, \frac{1 - i\sqrt{3}}{2}$ <p>Find all of the zeros of each function.</p> <ol style="list-style-type: none"> 21. $f(x) = 2x^4 + 7x^3 - 2x^2 - 19x - 12$ $-1, -3, \frac{1 + \sqrt{33}}{4}, \frac{1 - \sqrt{33}}{4}$ 22. $q(x) = x^4 - 4x^3 + x^2 + 16x - 20$ $-2, 2, 2 + i, 2 - i$ 23. $h(x) = x^6 - 8x^3$ $0, 2, -1 + i\sqrt{3}, -1 - i\sqrt{3}$ 24. $g(x) = x^6 - 1 = 1, 1, \frac{-1 + i\sqrt{3}}{2}, \frac{-1 - i\sqrt{3}}{2}, \frac{1 + i\sqrt{3}}{2}, \frac{1 - i\sqrt{3}}{2}$ <p>25. TRAVEL The height of a box that Joan is shipping is 3 inches less than the width of the box. The length is 2 inches more than twice the width. The volume of the box is 1540 in.³. What are the dimensions of the box? 22 in. by 10 in. by 7 in.</p> <p>26. GEOMETRY The height of a square pyramid is 3 meters shorter than the side of its base. If the volume of the pyramid is 432 m³, how tall is it? Use the formula $V = \frac{1}{3}Bh$. 9 m</p>
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7-6 Reading to Learn Mathematics

Rational Zero Theorem

Pre-Activity How can the Rational Zero Theorem solve problems involving large numbers?

Read the introduction to Lesson 7-6 at the top of page 378 in your textbook.

Rewrite the polynomial equation $w(w + 8)(w - 5) = 2772$ in the form $f(x) = 0$, where $f(x)$ is a polynomial written in descending powers of x .

$$w^3 + 3w^2 - 40w - 2772 = 0$$

Reading the Lesson

1. For each of the following polynomial functions, list all the possible values of p , all the possible values of q , and all the possible rational zeros $\frac{p}{q}$.

a. $f(x) = x^3 - 2x^2 - 11x + 12$

possible values of p : $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

possible values of q : ± 1

possible values of $\frac{p}{q}$: $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

b. $f(x) = 2x^4 + 9x^3 - 23x^2 - 81x + 45$

possible values of p : $\pm 1, \pm 3, \pm 5, \pm 9, \pm 15, \pm 45$

possible values of q : $\pm 1, \pm 2$

possible values of $\frac{p}{q}$: $\pm 1, \pm 3, \pm 5, \pm 9, \pm 15, \pm 45, \pm \frac{3}{2}, \pm \frac{5}{2}, \pm \frac{9}{2}, \pm \frac{15}{2}, \pm \frac{45}{2}$

2. Explain in your own words how Descartes' Rule of Signs, the Rational Zero Theorem, and synthetic division can be used together to find all of the rational zeros of a polynomial function with integer coefficients.

Sample answer: Use Descartes' Rule to find the possible numbers of positive and negative real zeros. Use the Rational Zero Theorem to list all possible rational zeros. Use synthetic division to test which of the numbers on the list of possible rational zeros are actually zeros of the polynomial function. (Descartes' Rule may help you to limit the possibilities.)

Helping You Remember

3. Some students have trouble remembering which numbers go in the numerators and which go in the denominators when forming a list of possible rational zeros of a polynomial function. How can you use the linear polynomial equation $ax + b = 0$, where a and b are nonzero integers, to remember this?

Sample answer: The solution of the equation is $-\frac{b}{a}$. The numerator a is a factor of the leading coefficient in $ax + b$. The denominator a is a factor of the leading coefficient in $ax + b$.

7-6 Enrichment

Infinite Continued Fractions

Some infinite expressions are actually equal to real numbers! The infinite continued fraction at the right is one example.

$$x = 1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \dots}}}}$$

If you use x to stand for the infinite fraction, then the entire denominator of the first fraction on the right is also equal to x . This observation leads to the following equation:

$$x = 1 + \frac{1}{x}$$

Write a decimal for each continued fraction.

1. $1 + \frac{1}{2}$

2. $1 + \frac{1}{1 + \frac{1}{1}}$

3. $1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1}}}$

1.5

1.666

4. $1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1}}}}$

5. $1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1}}}}}$

1.625

6. The more terms you add to the fractions above, the closer their value approaches the value of the infinite continued fraction. What value do the fractions seem to be approaching? **about 1.6**

7. Rewrite $x = 1 + \frac{1}{x}$ as a quadratic equation and solve for x .

$$x^2 - x - 1 = 0; x = \frac{1 \pm \sqrt{5}}{2}, x \approx 1.618 \text{ or } -0.618$$

(The positive root is the value of the infinite fraction, because the original fraction is clearly not negative.)

8. Find the value of the following infinite continued fraction.

$$3 + \frac{1}{3 + \frac{1}{3 + \frac{1}{3 + \frac{1}{3 + \dots}}}}$$

$x = 3 + \frac{1}{x}; x = \frac{3 + \sqrt{13}}{2}$ or about 3.30

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7-7

Study Guide and Intervention

Operations on Functions

Arithmetic Operations

Sum	$(f + g)(x) = f(x) + g(x)$
Difference	$(f - g)(x) = f(x) - g(x)$
Product	$(f \cdot g)(x) = f(x) \cdot g(x)$
Quotient	$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, g(x) \neq 0$

Example Find $(f + g)(x)$, $(f - g)(x)$, $(f \cdot g)(x)$, and $\left(\frac{f}{g}\right)(x)$ for $f(x) = x^2 + 3x - 4$ and $g(x) = 3x - 2$.

$(f + g)(x) = f(x) + g(x)$
 $= (x^2 + 3x - 4) + (3x - 2)$
 $= x^2 + 6x - 6$

$(f - g)(x) = f(x) - g(x)$
 $= (x^2 + 3x - 4) - (3x - 2)$
 $= x^2 - 2$

$(f \cdot g)(x) = f(x) \cdot g(x)$
 $= (x^2 + 3x - 4)(3x - 2)$
 $= x^2(3x - 2) + 3x(3x - 2) - 4(3x - 2)$
 $= 3x^3 - 2x^2 + 9x^2 - 6x - 12x + 8$
 $= 3x^3 + 7x^2 - 18x + 8$

$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$
 $= \frac{x^2 + 3x - 4}{3x - 2}, x \neq \frac{2}{3}$
 $f(x) = x^2 + 3x - 4$ and $g(x) = 3x - 2$

Exercises

Find $(f + g)(x)$, $(f - g)(x)$, $(f \cdot g)(x)$, and $\left(\frac{f}{g}\right)(x)$ for each $f(x)$ and $g(x)$.

- $f(x) = 8x - 3$; $g(x) = 4x + 5$
 $12x + 2$; $4x - 8$; $32x^2 + 28x - 15$;
 $\frac{8x - 3}{4x + 5}, x \neq -\frac{5}{4}$;
 $x^3 - x^2 - 8x + 12$; $x + 3, x \neq 2$
- $f(x) = 2x - 1$; $g(x) = 3x^2 + 11x - 4$
 $3x^2 + x + 2$; $3x^2 - 3x + 8$;
 $6x^3 - 11x^2 + 13x - 15$;
 $\frac{3x^2 - x + 5}{2x - 3}, x \neq \frac{3}{2}$
- $f(x) = x^2 - 1$; $g(x) = \frac{1}{x + 1}$
 $x^2 - 1 + \frac{1}{x + 1}$; $x^2 - 1 - \frac{1}{x + 1}$; $x - 1$; $x^3 + x^2 - x - 1, x \neq -1$

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7-7

Study Guide and Intervention

Operations on Functions

Composition of Functions

Composition of Functions Suppose f and g are functions such that the range of g is a subset of the domain of f . Then the composite function $f \circ g$ can be described by the equation $(f \circ g)(x) = f(g(x))$.

Example 1 For $f = \{(1, 2), (3, 3), (2, 4), (4, 1)\}$ and $g = \{(1, 3), (3, 4), (2, 2), (4, 1)\}$, find $f \circ g$ and $g \circ f$ if they exist.
 $f(g(1)) = f(3) = 3$ $f(g(2)) = f(2) = 4$ $f(g(3)) = f(4) = 1$ $f(g(4)) = f(1) = 2$
 $f \circ g = \{(1, 3), (2, 4), (3, 1), (4, 2)\}$
 $g(f(1)) = g(2) = 2$ $g(f(2)) = g(4) = 1$ $g(f(3)) = g(3) = 4$ $g(f(4)) = g(1) = 3$
 $g \circ f = \{(1, 2), (2, 1), (3, 4), (4, 3)\}$

Example 2 Find $(g \circ h)(x)$ and $(h \circ g)(x)$ for $g(x) = 3x - 4$ and $h(x) = x^2 - 1$.
 $(g \circ h)(x) = g(h(x))$
 $= g(x^2 - 1)$
 $= 3(x^2 - 1) - 4$
 $= 3x^2 - 7$
 $(h \circ g)(x) = h(g(x))$
 $= h(3x - 4)$
 $= (3x - 4)^2 - 1$
 $= 9x^2 - 24x + 16 - 1$
 $= 9x^2 - 24x + 15$

Exercises

For each set of ordered pairs, find $f \circ g$ and $g \circ f$ if they exist.

- $f = \{(-1, 2), (5, 6), (0, 9)\}$,
 $g = \{(6, 0), (2, -1), (9, 5)\}$
 $f \circ g = \{(2, 2), (6, 9), (9, 6)\}$;
 $g \circ f = \{(-1, -1), (0, 5), (5, 0)\}$
- $f = \{(5, -2), (9, 8), (-4, 3), (0, 4)\}$,
 $g = \{(3, 7), (-2, 6), (4, -2), (8, 10)\}$
 $f \circ g$ does not exist;
 $g \circ f = \{(-4, 7), (0, -2), (5, 6), (9, 10)\}$
- $f(x) = 2x + 7$; $g(x) = -5x - 1$
 $(f \circ g)(x) = -10x + 5$,
 $(g \circ f)(x) = -10x - 36$
- $f(x) = x^2 + 2x$; $g(x) = x - 9$
 $(f \circ g)(x) = x^2 - 16x + 63$,
 $(g \circ f)(x) = x^2 + 2x - 9$
- $f(x) = x^2 - 1$; $g(x) = -4x^2$
 $(f \circ g)(x) = 16x^4 - 1$,
 $(g \circ f)(x) = -4x^4 + 8x^2 - 4$
- $f(x) = 5x + 4$; $g(x) = 3 - x$
 $(f \circ g)(x) = 19 - 5x$,
 $(g \circ f)(x) = -1 - 5x$

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Glencoe Algebra 2

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7-7 Skills Practice

Operations on Functions

Find $(f + g)(x)$, $(f - g)(x)$, $(f \cdot g)(x)$, and $(\frac{f}{g})(x)$ for each $f(x)$ and $g(x)$.

- $f(x) = x + 5$; $g(x) = x - 4$
 $2x + 1$; 9 ;
 $x^2 + x - 20$;
 $\frac{x + 5}{x - 4}$, $x \neq 4$
- $f(x) = 3x + 1$; $g(x) = 2x - 3$
 $5x - 2$; $x + 4$; $6x^2 - 7x - 3$;
 $\frac{3x + 1}{2x - 3}$, $x \neq \frac{3}{2}$
- $f(x) = x^2 - x + 4$; $g(x) = x^2 - x - 4$
 $2x^2 - x + 4$; $x^2 + x - 4$;
 $4x^2 - x^3$; $\frac{x^2}{4 - x}$, $x \neq 4$
- $f(x) = 3x^2 + 5$; $g(x) = \frac{3x^3 - 5}{x}$, $x \neq 0$;
 $15x$, $x \neq 0$;
 $\frac{3x^3 - 5}{x}$, $x \neq 0$;
 $\frac{3x^3 - 5}{5}$, $x \neq 0$

For each set of ordered pairs, find $f \circ g$ and $g \circ f$ if they exist.

- $f = \{(0, 0), (4, -2)\}$
 $g = \{(0, 4), (-2, 0), (5, 0)\}$
 $\{(0, -2), (-2, 0), (5, 0)\}$;
 $\{(0, 4), (4, 0)\}$
- $f = \{(1, 3), (2, 1), (3, 1)\}$
 $g = \{(1, -4), (2, -1), (3, -1)\}$
 $\{(1, 3), (2, 1), (3, 1)\}$;
 $\{(-4, -1), (-1, -4), (2, -1)\}$
- $f(x) = x - 6$; $g(x) = x + 6$
 $2x - 6$; x ;
 $2x^2 + x - 1$; $25x^2 + 5x - 1$
- $f(x) = 3x$; $g(x) = x + 4$, and $h(x) = x^2 - 1$, find each value.
 16 ; $g[h(0)]$ **3**;
 17 ; $g[f(-1)]$ **1**
- $h[f(5)]$ **224**;
 19 ; $g[h(-3)]$ **12**;
 21 ; $f[h(8)]$ **189**;
 22 ; $f \circ (h \circ g)(1)$ **72**;
 23 ; $f \circ (g \circ h)(-2)$ **1**
- $g(x) = -3x$; $h(x) = -12x + 3$; $-12x - 1$
 $h(x) = 4x - 1$
- $g(x) = x - 3$; $h(x) = x^2 - 3$; $x^2 - 6x + 9$
 $h(x) = x^2$
- $g(x) = x + 2$; $h(x) = 2x^2 - 3$
 $2x^2 + 5x - 5$;
 $2x^2 + x - 1$

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7-7 Practice (Average)

Operations on Functions

Find $(f + g)(x)$, $(f - g)(x)$, $(f \cdot g)(x)$, and $(\frac{f}{g})(x)$ for each $f(x)$ and $g(x)$.

- $f(x) = 2x + 1$; $g(x) = x - 3$
 $3x - 2$; $x + 4$;
 $2x^2 - 5x - 3$;
 $\frac{2x + 1}{x - 3}$, $x \neq 3$
- $f(x) = 8x^2$; $g(x) = \frac{1}{x^2}$
 $8x^4 + 1$; $x \neq 0$;
 $\frac{8x^4 - 1}{x^2}$, $x \neq 0$;
 8 , $x \neq 0$; $8x^4$, $x \neq 0$
- $f(x) = x^2 + 7x + 12$; $g(x) = x^2 - 9$
 $2x^2 + 7x + 3$; $7x + 21$;
 $x^4 + 7x^3 + 3x^2 - 63x - 108$;
 $\frac{x + 4}{x - 3}$, $x \neq \pm 3$

For each set of ordered pairs, find $f \circ g$ and $g \circ f$ if they exist.

- $f = \{(-9, -1), (-1, 0), (3, 4)\}$
 $g = \{(0, -9), (-1, 3), (4, -1)\}$
 $\{(0, -1), (-1, 4), (4, 0)\}$;
 $\{(-9, 3), (-1, -9), (3, -1)\}$
- $f = \{(6, 1), (-5, 0), (3, -4)\}$
 $g = \{(6, 1), (-5, 0), (3, -4)\}$
 $\{(6, 6), (-5, 3), (3, -5)\}$;
 $\{(-4, 0), (0, -4), (1, 1)\}$
- $f(x) = x - 5$; $g(x) = x + 3$
 $2x^2 + 3$;
 $2x^2 + 12x + 18$
- $f(x) = x^2$; $g(x) = 5x$, and $h(x) = x + 4$, find each value.
 14 ; $f[g(1)]$ **25**;
 17 ; $f[h(-9)]$ **25**;
 20 ; $h[f(20)]$ **404**;
 21 ; $f \circ (h \circ g)(-1)$ **1**
- $f = \{(-4, 3), (0, -2), (1, -2)\}$
 $g = \{(-2, 0), (3, 1)\}$
 $\{(-2, -2), (3, -2)\}$;
 $\{(-4, 1), (0, 0), (1, 0)\}$
- $f = \{(0, -3), (0, -3), (6, 8)\}$
 $g = \{(8, 2), (-3, 0), (-3, 1)\}$
does not exist;
 $\{(0, 0), (1, 0), (6, 2)\}$
- $g(x) = 3x$; $h(x) = x - 4$
 $3x - 12$; $3x - 4$
- $g(x) = -2x$; $h(x) = x^2 + 3x + 2$
 $-2x^2 - 6x - 4$;
 $4x^2 - 6x + 2$
- $g(x) = x + 6$; $h(x) = 3x^2 + 6$
 $3x^2 + 3x^2 + 6$;
 $3x^2 + 36x + 108$
- $g(x) = x - 2$; $h(x) = 3x^2 + 1$
 $3x^2 - 1$;
 $3x^2 - 12x + 13$
- $f[g(1)]$ **25**;
 15 ; $g[h(-2)]$ **10**;
 18 ; $h[g(-3)]$ **-11**;
 22 ; $[f \circ (g \circ h)](4)$ **1600**

23. BUSINESS The function $f(x) = 1000 - 0.01x^2$ models the manufacturing cost per item when x items are produced, and $g(x) = 150 - 0.001x^2$ models the service cost per item. Write a function $C(x)$ for the total manufacturing and service cost per item.
 $C(x) = 1150 - 0.011x^2$

24. MEASUREMENT The formula $f = \frac{n}{12}$ converts inches n to feet f , and $m = \frac{5280}{f}$ converts feet to miles m . Write a composition of functions that converts inches to miles.
 $[m \circ f]n = 63.360$

Lesson 7-7

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7-7

Reading to Learn Mathematics
Operations on Functions

Pre-Activity

Why is it important to combine functions in business?

Read the introduction to Lesson 7-7 at the top of page 383 in your textbook.

Describe two ways to calculate Ms. Coffino's profit from the sale of 50 birdhouses. (Do not actually calculate her profit.)

- Sample answer:**
 1. Find the revenue by substituting 50 for x in the expression $125x$. Next, find the cost by substituting 50 for x in the expression $65x + 5400$. Finally, subtract the cost from the revenue to find the profit.
 2. Form the profit function $p(x) = r(x) - c(x) = 125x - (65x + 5400) = 60x - 5400$. Substitute 50 for x in the expression $60x - 5400$.

Reading the Lesson

- Determine whether each statement is *true* or *false*. (Remember that *true* means *always true*.)
 - If f and g are polynomial functions, then $f + g$ is a polynomial function. **true**
 - If f and g are polynomial functions, then $\frac{f}{g}$ is a polynomial function. **false**
 - If f and g are polynomial functions, the domain of the function $f \cdot g$ is the set of all real numbers. **true**
 - If $f(x) = 3x + 2$ and $g(x) = x - 4$, the domain of the function $\frac{f}{g}$ is the set of all real numbers. **false**
 - If f and g are polynomial functions, then $(f \circ g)(x) = (g \circ f)(x)$. **false**
 - If f and g are polynomial functions, then $(f \cdot g)(x) = (g \cdot f)(x)$. **true**
- Let $f(x) = 2x - 5$ and $g(x) = x^2 + 1$.
 - Explain in words how you would find $(f \circ g)(-3)$. (Do not actually do any calculations.)
Sample answer: Square -3 and add 1. Take the number you get, multiply it by 2, and subtract 5.
 - Explain in words how you would find $(g \circ f)(-3)$. (Do not actually do any calculations.)
Sample answer: Multiply -3 by 2 and subtract 5. Take the number you get, square it, and add 1.

Helping You Remember

- Some students have trouble remembering the correct order in which to apply the two original functions when evaluating a composite function. Write three sentences, each of which explains how to do this in a slightly different way. (Hint: Use the word *closest* in the first sentence, the words *inside* and *outside* in the second, and the words *left* and *right* in the third.)
Sample answer: 1. The function that is written closest to the variable is applied first. 2. Work from the inside to the outside. 3. Work from right to left.

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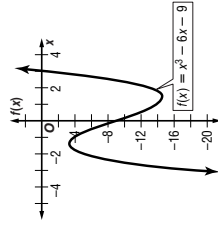
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7-7

Enrichment

Relative Maximum Values

The graph of $f(x) = x^3 - 6x - 9$ shows a relative maximum value somewhere between $f(-2)$ and $f(-1)$. You can obtain a closer approximation by comparing values such as those shown in the table.

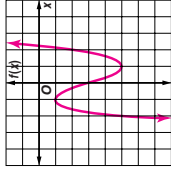
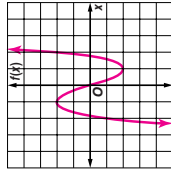


x	$f(x)$
-2	-5
-1.5	-3.375
-1.4	-3.344
-1.3	-3.397
-1	-4

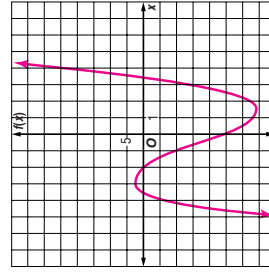
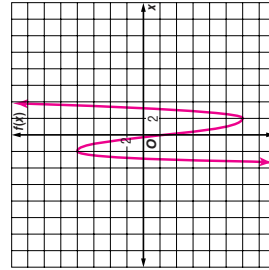
To the nearest tenth a relative maximum value for $f(x)$ is -3.3 .

Using a calculator to find points, graph each function. To the nearest tenth, find a relative maximum value of the function.

- $f(x) = x(x^2 - 3)$ **rel. max. of 2.0**
- $f(x) = x^3 - 3x - 3$ **rel. max. of -1.0**



- $f(x) = x^3 - 9x - 2$ **rel. max. of 8.4**
- $f(x) = x^3 + 2x^2 - 12x - 24$ **rel. max. of 3.3**



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7-8 Study Guide and Intervention

Inverse Functions and Relations

Find Inverses

Inverse Relations	Two relations are inverse relations if and only if whenever one relation contains the element (a, b), the other relation contains the element (b, a).
Property of Inverse Functions	Suppose f and f^{-1} are inverse functions. Then $f(a) = b$ if and only if $f^{-1}(b) = a$.

Example Find the inverse of the function $f(x) = \frac{2}{5}x - \frac{1}{5}$. Then graph the function and its inverse.

Step 1 Replace $f(x)$ with y in the original equation.

$$f(x) = \frac{2}{5}x - \frac{1}{5} \rightarrow y = \frac{2}{5}x - \frac{1}{5}$$

Step 2 Interchange x and y .

$$x = \frac{2}{5}y - \frac{1}{5}$$

Step 3 Solve for y .

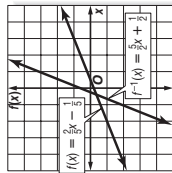
$$x = \frac{2}{5}y - \frac{1}{5}$$

$$5x = 2y - 1$$

$$5x + 1 = 2y$$

$$\frac{1}{2}(5x + 1) = y$$

The inverse of $f(x) = \frac{2}{5}x - \frac{1}{5}$ is $f^{-1}(x) = \frac{1}{2}(5x + 1)$.



Inverse

Multiply each side by 5.

Add 1 to each side.

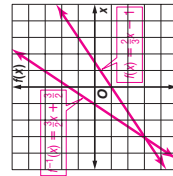
Divide each side by 2.

Examples

Find the inverse of each function. Then graph the function and its inverse.

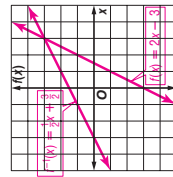
1. $f(x) = \frac{2}{3}x - 1$

$$f^{-1}(x) = \frac{3}{2}x + \frac{3}{2}$$



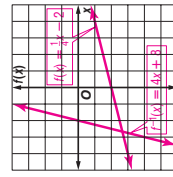
2. $f(x) = 2x - 3$

$$f^{-1}(x) = \frac{1}{2}x + \frac{3}{2}$$



3. $f(x) = \frac{1}{4}x - 2$

$$f^{-1}(x) = 4x + 8$$



7-8

Study Guide and Intervention

(continued)

Inverse Functions and Relations

Inverses of Relations and Functions

Inverse Functions Two functions f and g are inverse functions if and only if $[f \circ g](x) = x$ and $[g \circ f](x) = x$.

Example 1 Determine whether $f(x) = 2x - 7$ and $g(x) = \frac{1}{2}(x + 7)$ are inverse functions.

$$\begin{aligned} [f \circ g](x) &= f[g(x)] \\ &= f\left[\frac{1}{2}(x + 7)\right] \\ &= 2\left[\frac{1}{2}(x + 7)\right] - 7 \\ &= x + 7 - 7 \\ &= x \end{aligned}$$

$$\begin{aligned} [g \circ f](x) &= g[f(x)] \\ &= g(2x - 7) \\ &= \frac{1}{2}(2x - 7 + 7) \\ &= x \end{aligned}$$

The functions are inverses since both $[f \circ g](x) = x$ and $[g \circ f](x) = x$.

Example 2 Determine whether $f(x) = 4x + \frac{1}{3}$ and $g(x) = \frac{1}{4}x - 3$ are inverse functions.

$$\begin{aligned} [f \circ g](x) &= f[g(x)] \\ &= f\left[\frac{1}{4}x - 3\right] \\ &= 4\left[\frac{1}{4}x - 3\right] + \frac{1}{3} \\ &= x - 12 + \frac{1}{3} \\ &= x - 11\frac{2}{3} \end{aligned}$$

Since $[f \circ g](x) \neq x$, the functions are not inverses.

Examples

Determine whether each pair of functions are inverse functions.

1. $f(x) = 3x - 1$

$g(x) = \frac{1}{3}x + \frac{1}{3}$ **yes**

2. $f(x) = \frac{1}{4}x + 5$

$g(x) = 4x - 20$ **yes**

3. $f(x) = \frac{1}{2}x - 10$

$g(x) = 2x + \frac{1}{10}$ **no**

4. $f(x) = 2x + 5$

$g(x) = 5x + 2$ **no**

5. $f(x) = 8x - 12$

$g(x) = \frac{1}{8}x + 12$ **no**

6. $f(x) = -2x + 3$

$g(x) = -\frac{1}{2}x + \frac{3}{2}$ **yes**

7. $f(x) = 4x - \frac{1}{2}$

$g(x) = \frac{1}{4}x + \frac{1}{8}$ **yes**

8. $f(x) = 2x - \frac{3}{5}$

$g(x) = \frac{1}{10}(5x + 3)$ **yes**

9. $f(x) = 4x + \frac{1}{2}$

$g(x) = \frac{1}{2}x - \frac{2}{2}$ **no**

10. $f(x) = 10 - \frac{x}{2}$

$g(x) = 20 - 2x$ **yes**

11. $f(x) = 4x - \frac{4}{5}$

$g(x) = \frac{x}{4} + \frac{1}{5}$ **yes**

12. $f(x) = 9 + \frac{3}{2}x$

$g(x) = \frac{2}{3}x - 6$ **yes**

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7-8

Skills Practice

Inverse Functions and Relations

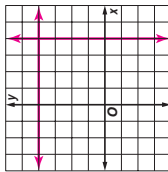
Find the inverse of each relation.

- $\{(3, 1), (4, -3), (8, -3)\}$
 $\{(1, 3), (-3, 4), (-3, 8)\}$
- $\{(-7, 1), (0, 5), (5, -1)\}$
 $\{(1, -7), (5, 0), (-1, 5)\}$
- $\{(-10, -2), (-7, 6), (-4, -2), (-4, 0)\}$
 $\{(-2, -10), (6, -7), (-2, -4), (0, -4)\}$ $\{(-9, 0), (-3, 5), (6, 6), (-3, 8)\}$
- $\{(-4, 12), (0, 7), (9, -1), (10, -5)\}$
 $\{(12, -4), (7, 0), (-1, 9), (-5, 10)\}$
- $\{(-4, 1), (-4, 3), (0, -8), (8, -9)\}$
 $\{(1, -4), (3, -4), (-8, 0), (-9, 8)\}$

Find the inverse of each function. Then graph the function and its inverse.

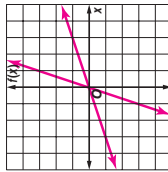
7. $y = 4$

$x = 4$



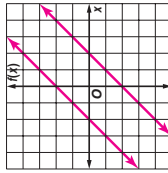
8. $f(x) = 3x$

$f^{-1}(x) = \frac{1}{3}x$



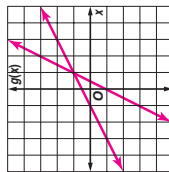
9. $f(x) = x + 2$

$f^{-1}(x) = x - 2$



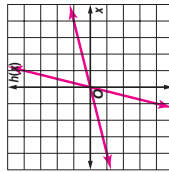
10. $g(x) = 2x - 1$

$g^{-1}(x) = \frac{x+1}{2}$



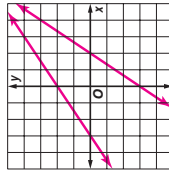
11. $h(x) = \frac{1}{4}x$

$h^{-1}(x) = 4x$



12. $y = \frac{2}{3}x + 2$

$y = \frac{3}{2}x - 3$



13. $f(x) = x - 1$ **no**

$g(x) = 1 - x$

16. $f(x) = 2x$ **yes**
 $g(x) = \frac{1}{2}x$

17. $h(x) = 6x - 2$ **no**
 $g(x) = \frac{1}{6}x + 3$

Determine whether each pair of functions are inverse functions.

14. $f(x) = 2x + 3$ **yes**
 $g(x) = \frac{1}{2}(x - 3)$

15. $f(x) = 5x - 5$ **yes**
 $g(x) = \frac{1}{5}x + 1$

18. $f(x) = 8x - 10$ **yes**
 $g(x) = \frac{1}{8}x + \frac{5}{4}$

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7-8

Practice (Average)

Inverse Functions and Relations

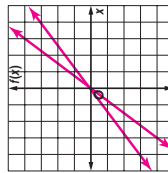
Find the inverse of each relation.

- $\{(0, 3), (4, 2), (5, -6)\}$
 $\{(3, 0), (2, 4), (-6, 5)\}$
- $\{(-5, 1), (-5, -1), (-5, 8)\}$
 $\{(1, -5), (-1, -5), (8, -5)\}$
- $\{(-3, -7), (0, -1), (5, 9), (7, 13)\}$
 $\{(-7, -3), (-1, 0), (9, 5), (13, 7)\}$
- $\{(8, -2), (10, 5), (12, 6), (14, 7)\}$
 $\{(-2, 8), (5, 10), (6, 12), (7, 14)\}$
- $\{(-5, -4), (1, 2), (3, 4), (7, 8)\}$
 $\{(-4, -5), (2, 1), (4, 3), (8, 7)\}$
- $\{(-3, 9), (-2, 4), (0, 0), (1, 1)\}$
 $\{(9, -3), (4, -2), (0, 0), (1, 1)\}$

Find the inverse of each function. Then graph the function and its inverse.

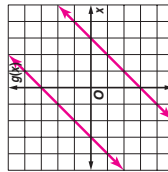
7. $f(x) = \frac{3}{4}x$

$f^{-1}(x) = \frac{4}{3}x$



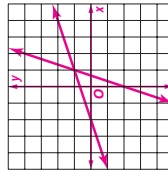
8. $g(x) = 3 + x$

$g^{-1}(x) = x - 3$



9. $y = 3x - 2$

$y = \frac{x+2}{3}$



Determine whether each pair of functions are inverse functions.

10. $f(x) = x + 6$ **yes**

$g(x) = x - 6$

11. $f(x) = -4x + 1$ **yes**

$g(x) = \frac{1}{4}(1 - x)$

12. $g(x) = 13x - 13$ **no**

$h(x) = \frac{1}{13}x - 1$

13. $f(x) = 2x$ **no**

$g(x) = -2x$

14. $f(x) = \frac{6}{7}x$ **yes**

$g(x) = \frac{7}{6}x$

15. $g(x) = 2x - 8$ **yes**

$h(x) = \frac{1}{2}x + 4$

16. **MEASUREMENT** The points (63, 121), (71, 180), (67, 140), (65, 108), and (72, 165) give the weight in pounds as a function of height in inches for 5 students in a class. Give the points for these students that represent height as a function of weight.
(121, 63), (180, 71), (140, 67), (108, 65), (165, 72)

REMODELING For Exercises 17 and 18, use the following information.

The Clearlys are replacing the flooring in their 15 foot by 18 foot kitchen. The new flooring costs \$17.99 per square yard. The formula $f(x) = 9x$ converts square yards to square feet.

17. Find the inverse $f^{-1}(x)$. What is the significance of $f^{-1}(x)$ for the Clearlys? $f^{-1}(x) = \frac{x}{9}$; **it will allow them to convert the square footage of their kitchen floor to square yards, so they can then calculate the cost of the new flooring.**

18. What will the new flooring cost the Clearlys? **\$539.70**

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Glencoe Algebra 2

7-8 Reading to Learn Mathematics

Inverse Functions and Relations

Pre-Activity How are inverse functions related to measurement conversions?

Read the introduction to Lesson 7-8 at the top of page 390 in your textbook. A function multiplies a number by 3 and then adds 5 to the result. What does the inverse function do, and in what order? **Sample answer: It first subtracts 5 from the number and then divides the result by 3.**

Reading the Lesson

- Complete each statement.
 - If two relations are inverses, the domain of one relation is the **range** of the other.
 - Suppose that $g(x)$ is a relation and that the point $(4, -2)$ is on its graph. Then a point on the graph of $g^{-1}(x)$ is **$(-2, 4)$** .
 - The **horizontal line** test can be used on the graph of a function to determine whether the function has an inverse function.
 - If you are given the graph of a function, you can find the graph of its inverse by reflecting the original graph over the line with equation **$y = x$** .
 - If f and g are inverse functions, then $(f \circ g)(x) = \underline{\hspace{2cm}}$ **x** and $(g \circ f)(x) = \underline{\hspace{2cm}}$ **x** .
 - A function has an inverse that is also a function only if the given function is **one-to-one**.
 - Suppose that $h(x)$ is a function whose inverse is also a function. If $h(5) = 12$, then $h^{-1}(12) = \underline{\hspace{2cm}}$ **5**.
- Assume that $f(x)$ is a one-to-one function defined by an algebraic equation. Write the four steps you would follow in order to find the equation for $f^{-1}(x)$.
 - Replace $f(x)$ with y in the original equation.** _____
 - Interchange x and y .** _____
 - Solve for y .** _____
 - Replace y with $f^{-1}(x)$.** _____

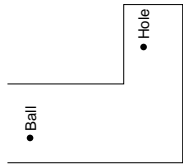
Helping You Remember

- A good way to remember something new is to relate it to something you already know. How are the vertical and horizontal line tests related? **Sample answer: The vertical line test determines whether a relation is a function because the ordered pairs in a function can have no repeated x -values. The horizontal line test determines whether a function is one-to-one because a one-to-one function cannot have any repeated y -values.**

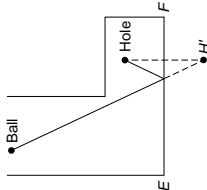
7-8 Enrichment

Miniature Golf

In miniature golf, the object of the game is to roll the golf ball into the hole in as few shots as possible. As in the diagram at the right, the hole is often placed so that a direct shot is impossible. Reflections can be used to help determine the direction that the ball should be rolled in order to score a hole-in-one.

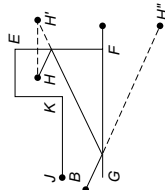


Example 1 Using wall \overline{EF} , find the path to use to score a hole-in-one.



Find the reflection image of the "hole" with respect to \overline{EF} and label it H' . The intersection of $\overline{BH'}$ with wall \overline{EF} is the point at which the shot should be directed.

Example 2 For the hole at the right, find a path to score a hole-in-one.



Find the reflection image of H with respect to \overline{EF} and label it H' . In this case, $\overline{BH'}$ intersects \overline{JK} before intersecting \overline{EF} . Thus, this path cannot be used. To find a usable path, find the reflection image of H' with respect to \overline{GF} and label it H'' . Now, the intersection of $\overline{BH''}$ with wall \overline{GF} is the point at which the shot should be directed.

Copy each figure. Then, use reflections to determine a possible path for a hole-in-one.

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7-9 Study Guide and Intervention (continued)

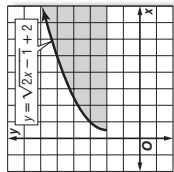
Square Root Functions and Inequalities

Square Root Inequalities A square root inequality is an inequality that contains the square root of a variable expression. Use what you know about graphing square root functions and quadratic inequalities to graph square root inequalities.

Example Graph $y \leq \sqrt{2x - 1} + 2$.

Graph the related equation $y = \sqrt{2x - 1} + 2$. Since the boundary should be included, the graph should be solid.

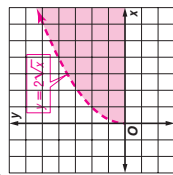
The domain includes values for $x \geq \frac{1}{2}$, so the graph is to the right of $x = \frac{1}{2}$. The range includes only numbers greater than 2, so the graph is above $y = 2$.



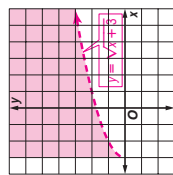
Exercises

Graph each inequality.

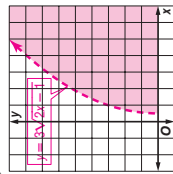
1. $y < 2\sqrt{x}$



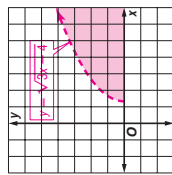
2. $y > \sqrt{x + 3}$



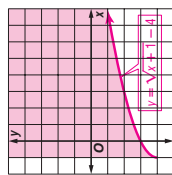
3. $y < 3\sqrt{2x - 1}$



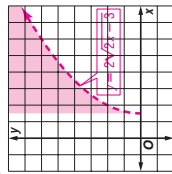
4. $y < \sqrt{3x - 4}$



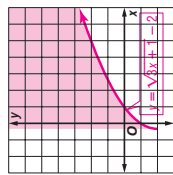
5. $y \geq \sqrt{x + 1} - 4$



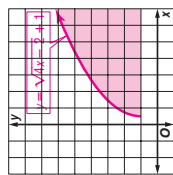
6. $y > 2\sqrt{2x - 3}$



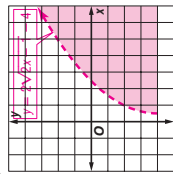
7. $y \geq \sqrt{3x + 1} - 2$



8. $y \leq \sqrt{4x - 2} + 1$



9. $y < 2\sqrt{2x - 1} - 4$



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7-9 Study Guide and Intervention

Square Root Functions and Inequalities

Square Root Functions A function that contains the square root of a variable expression is a square root function.

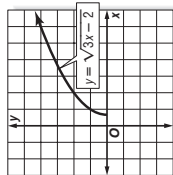
Example Graph $y = \sqrt{3x - 2}$. State its domain and range.

Since the radicand cannot be negative, $3x - 2 \geq 0$ or $x \geq \frac{2}{3}$.

The x-intercept is $\frac{2}{3}$. The range is $y \geq 0$.

Make a table of values and graph the function.

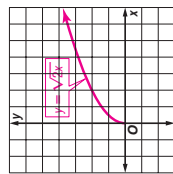
x	y
$\frac{2}{3}$	0
1	1
2	2
3	$\sqrt{7}$



Exercises

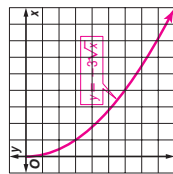
Graph each function. State the domain and range of the function.

1. $y = \sqrt{2x}$



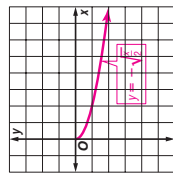
D: $x \geq 0$; **R:** $y \geq 0$

2. $y = -3\sqrt{x}$



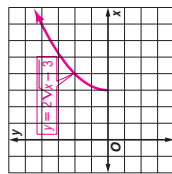
D: $x \geq 0$; **R:** $y \leq 0$

3. $y = -\sqrt{\frac{x}{2}}$



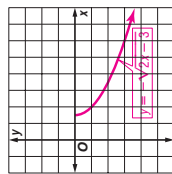
D: $x \geq 0$; **R:** $y \leq 0$

4. $y = 2\sqrt{x - 3}$



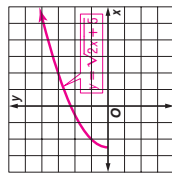
D: $x \geq 3$; **R:** $y \geq 0$

5. $y = -\sqrt{2x - 3}$



D: $x \geq \frac{3}{2}$; **R:** $y \leq 0$

6. $y = \sqrt{2x} + 5$



D: $x \geq -\frac{5}{2}$; **R:** $y \geq 0$

Lesson 7-9

NAME _____ DATE _____ PERIOD _____

7-9 Skills Practice

Square Root Functions and Inequalities

Graph each function. State the domain and range of each function.

1. $y = \sqrt{2x}$

D: $x \geq 0$, R: $y \geq 0$

2. $y = -\sqrt{3x}$

D: $x \geq 0$, R: $y \leq 0$

3. $y = 2\sqrt{x}$

D: $x \geq 0$, R: $y \geq 0$

4. $y = \sqrt{x+3}$

D: $x \geq -3$, R: $y \geq 0$

5. $y = -\sqrt{2x-5}$

D: $x \geq 2.5$, R: $y \leq 0$

6. $y = \sqrt{x+4} - 2$

D: $x \geq -4$, R: $y \geq -2$

NAME _____ DATE _____ PERIOD _____

7-9 Practice (Average)

Square Root Functions and Inequalities

Graph each function. State the domain and range of each function.

1. $y = \sqrt{5x}$

D: $x \geq 0$, R: $y \geq 0$

2. $y = -\sqrt{x-1}$

D: $x \geq 1$, R: $y \leq 0$

3. $y = 2\sqrt{x+2}$

D: $x \geq -2$, R: $y \geq 0$

4. $y = \sqrt{3x-4}$

D: $x \geq \frac{4}{3}$, R: $y \geq 0$

5. $y = \sqrt{x+7} - 4$

D: $x \geq -7$, R: $y \geq -4$

6. $y = 1 - \sqrt{2x+3}$

D: $x \geq -\frac{3}{2}$, R: $y \leq 1$

Lesson 7-9

7. $y \geq -\sqrt{6x}$

Graph each inequality.

8. $y \leq \sqrt{x-5} + 3$

9. $y > -2\sqrt{3x+2}$

NAME _____ DATE _____ PERIOD _____

7-9 Skills Practice

Square Root Functions and Inequalities

Graph each function. State the domain and range of each function.

1. $y = \sqrt{2x}$

D: $x \geq 0$, R: $y \geq 0$

2. $y = -\sqrt{3x}$

D: $x \geq 0$, R: $y \leq 0$

3. $y = 2\sqrt{x}$

D: $x \geq 0$, R: $y \geq 0$

4. $y = \sqrt{x+3}$

D: $x \geq -3$, R: $y \geq 0$

5. $y = -\sqrt{2x-5}$

D: $x \geq 2.5$, R: $y \leq 0$

6. $y = \sqrt{x+4} - 2$

D: $x \geq -4$, R: $y \geq -2$

NAME _____ DATE _____ PERIOD _____

7-9 Practice (Average)

Square Root Functions and Inequalities

Graph each function. State the domain and range of each function.

1. $y = \sqrt{5x}$

D: $x \geq 0$, R: $y \geq 0$

2. $y = -\sqrt{x-1}$

D: $x \geq 1$, R: $y \leq 0$

3. $y = 2\sqrt{x+2}$

D: $x \geq -2$, R: $y \geq 0$

4. $y = \sqrt{3x-4}$

D: $x \geq \frac{4}{3}$, R: $y \geq 0$

5. $y = \sqrt{x+7} - 4$

D: $x \geq -7$, R: $y \geq -4$

6. $y = 1 - \sqrt{2x+3}$

D: $x \geq -\frac{3}{2}$, R: $y \leq 1$

Lesson 7-9

7. $y < \sqrt{4x}$

Graph each inequality.

8. $y \geq \sqrt{x+1}$

9. $y \leq \sqrt{4x-3}$

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Answers

10. ROLLER COASTERS The velocity of a roller coaster as it moves down a hill is $v = \sqrt{v_0^2 + 64h}$, where v_0 is the initial velocity and h is the vertical drop in feet. If $v = 70$ feet per second and $t_0 = 8$ feet per second, find h . **about 75.6 ft**

11. WEIGHT Use the formula $d = \sqrt{\frac{3960^2 W_E}{W_S}} - 3960$, which relates distance from Earth d in miles to weight. If an astronaut's weight on Earth W_E is 148 pounds and in space W_S is 115 pounds, how far from Earth is the astronaut? **about 532 mi**

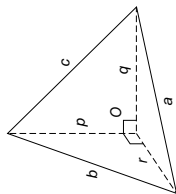
NAME _____ DATE _____ PERIOD _____

7-9 Enrichment

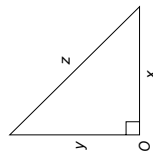
Reading Algebra

If two mathematical problems have basic structural similarities, they are said to be **analogous**. Using analogies is one way of discovering and proving new theorems.

The following numbered sentences discuss a three-dimensional analogy to the Pythagorean theorem.



- 01 Consider a tetrahedron with three perpendicular faces that meet at vertex O .
- 02 Suppose you want to know how the areas A , B , and C of the three faces that meet at vertex O are related to the area D of the face opposite vertex O .
- 03 It is natural to expect a formula analogous to the Pythagorean theorem $z^2 = x^2 + y^2$, which is true for a similar situation in two dimensions.
- 04 To explore the three-dimensional case, you might guess a formula and then try to prove it.
- 05 Two reasonable guesses are $D^3 = A^3 + B^3 + C^3$ and $D^2 = A^2 + B^2 + C^2$.



Refer to the numbered sentences to answer the questions.

1. Use sentence 01 and the top diagram. The prefix *tetra-* means four. Write an informal definition of tetrahedron.
a three-dimensional figure with four faces
2. Use sentence 02 and the top diagram. What are the lengths of the sides of each face of the tetrahedron?
 a , b , and c ; a , q , and r ; b , p , and r ; c , p , and q
3. Rewrite sentence 01 to state a two-dimensional analogue.
Consider a triangle with two perpendicular sides that meet at vertex C .
4. Refer to the top diagram and write expressions for the areas A , B , and C mentioned in sentence 02.
Possible answer: $A = \frac{1}{2}pr$, $B = \frac{1}{2}pq$, $C = \frac{1}{2}rq$
5. To explore the three-dimensional case, you might begin by expressing a , b , and c in terms of p , q , and r . Use the Pythagorean theorem to do this.
 $a^2 = q^2 + r^2$, $b^2 = r^2 + p^2$, $c^2 = p^2 + q^2$
6. Which guess in sentence 05 seems more likely? Justify your answer.
See students' explanations.

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Glencoe Algebra 2

NAME _____ DATE _____ PERIOD _____

7-9 Reading to Learn Mathematics

Square Root Functions

Pre-Activity How are square root functions used in bridge design?

Read the introduction to Lesson 7-9 at the top of page 395 in your textbook.

If the weight to be supported by a steel cable is doubled, should the diameter of the support cable also be doubled? If not, by what number should the diameter be multiplied?

no; $\sqrt{2}$

Reading the Lesson

1. Match each square root function from the list on the left with its domain and range from the list on the right.

- | | |
|---------------------------------|---|
| a. $y = \sqrt{x}$ iv | i. domain: $x \geq 0$; range: $y \geq 3$ |
| b. $y = \sqrt{x+3}$ viii | ii. domain: $x \geq 0$; range: $y \leq 0$ |
| c. $y = \sqrt{x+3}$ i | iii. domain: $x \geq 0$; range: $y \leq -3$ |
| d. $y = \sqrt{x-3}$ v | iv. domain: $x \geq 0$; range: $y \geq 0$ |
| e. $y = -\sqrt{x}$ ii | v. domain: $x \geq 3$; range: $y \geq 0$ |
| f. $y = -\sqrt{x-3}$ vii | vi. domain: $x \leq 3$; range: $y \geq 3$ |
| g. $y = \sqrt{3-x+3}$ vi | vii. domain: $x \geq 3$; range: $y \leq 0$ |
| h. $y = -\sqrt{x-3}$ iii | viii. domain: $x \geq -3$; range: $y \geq 0$ |

2. The graph of the inequality $y \leq \sqrt{3x+6}$ is a shaded region. Which of the following points lie inside this region?

- | | | | | |
|--------|--------|--------|---------|--------|
| (3, 0) | (2, 4) | (5, 2) | (4, -2) | (6, 6) |
|--------|--------|--------|---------|--------|
- (3, 0), (5, 2), (4, -2)**

Helping You Remember

3. A good way to remember something is to explain it to someone else. Suppose you are studying this lesson with a classmate who thinks that you cannot have square root functions because every positive real number has two square roots. How would you explain the idea of square root functions to your classmate?

Sample answer: To form a square root function, choose either the positive or negative square root. For example, $y = \sqrt{x}$ and $y = -\sqrt{x}$ are two separate functions.

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Chapter 7 Assessment Answer Key

Form 1
Page 429

1. D
2. A
3. C
4. B
5. A
6. A
7. C
8. B
9. D
10. A
11. D

Page 430

12. A
 13. A
 14. C
 15. B
 16. D
 17. D
 18. A
 19. D
 20. A
- B: 4x + 3

Form 2A
Page 431

1. B
2. D
3. C
4. C
5. A
6. B
7. C
8. B
9. D
10. A
11. D

(continued on the next page)

Chapter 7 Assessment Answer Key

Form 2A (continued)

Page 432

12. B

13. C

14. B

15. D

16. A

17. C

18. C

19. B

20. B

B: -2

Form 2B

Page 433

1. C

2. A

3. D

4. A

5. B

6. C

7. C

8. B

9. C

10. A

11. B

Page 434

12. D

13. A

14. A

15. C

16. D

17. B

18. B

19. A

20. D

B: 3

Chapter 7 Assessment Answer Key

Form 2C

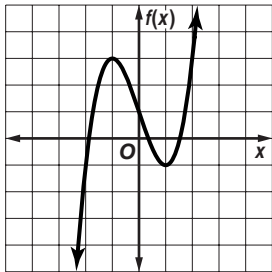
Page 435

1. -176

2. $x^2 - x - 3$

3. even; 4

4. between -2 and -1,
 between 0 and 1,
 between 1 and 2



5. Sample answer:
 rel. max. at $x = -1$,
 rel. min. at $x = 1$

6. $9(n^3)^2 - 36(n^3)$

7. $\sqrt{15}, -\sqrt{15},$
 $i\sqrt{3}, -i\sqrt{3}$

8. -3

9. $x + 3, x - 5$

10. 3 or 1; 1; 2 or 0

Page 436

11. 5, -2i, 2i

12. $\pm 1, \pm 2, \pm 4, \pm 8, \pm \frac{1}{2}$

13. -2, 1, $\frac{3}{2}$

14. $-x^3 + 7x^2 - 4x + 28$

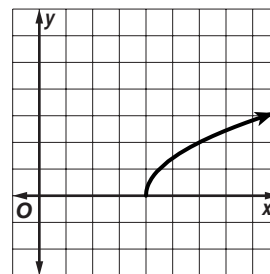
15. 2

16. $2x^2 - 1$

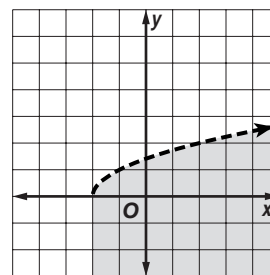
17. $f^{-1}(x) = \frac{1}{5}x - 2$

18. yes

19.



20.



B: 2

Chapter 7 Assessment Answer Key

Form 2D

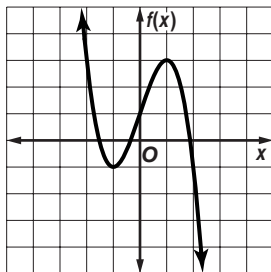
Page 437

1. -134

2. $x^2 - 2x - 1$

3. even; 4

4. between -2 and -1,
 between -1 and 0,
 between 1 and 2



5. Sample answer:
 rel. max. at $x = 1$,
 rel. min. at $x = -1$

6. $5(x^5)^2 - 4(x^5) + 3$

7. $-\sqrt{6}, \sqrt{6},$
 $i\sqrt{2}, -i\sqrt{2}$

8. 132

9. $x + 3, x + 1$

10. 2 or 0; 2 or 0; 4, 2, or 0

Page 438

11. 2, -3i, 3i

12. $\pm 1, \pm 2, \pm 7, \pm 14, \pm \frac{1}{2}, \pm \frac{7}{2}$

13. $-3, \frac{2}{3}, 1$

14. $-x^3 + 6x^2 + 4x - 24$

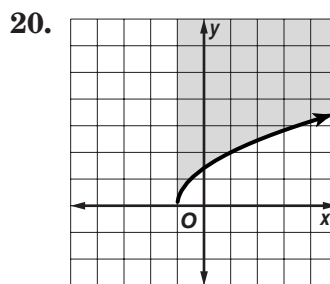
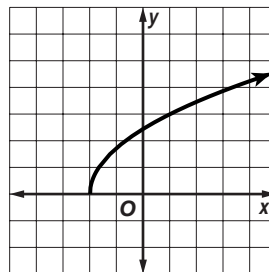
15. 4

16. $x^2 - 6x + 5$

17. $g^{-1}(x) = -\frac{1}{2}x + 2$

18. yes

19. D: $x \geq -2$, R: $y \geq 0$



B: -4

Chapter 7 Assessment Answer Key

Form 3

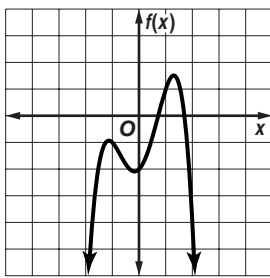
Page 439

1. $\frac{13}{3}$

2. $x^4 - 7x^2 - x$

3. $f(x) \rightarrow -\infty$ as $x \rightarrow -\infty$,
 $f(x) \rightarrow +\infty$ as $x \rightarrow +\infty$; odd; 4

4. between 0 and 1,
 between 1 and 2



Sample answer: rel. max. at $x = -1$ and $x = 1$, rel. min. at $x = 0$

5. $b[9(b^2)^2 + 3(b^2) - 8]$

6. 16, 81

7. 7014

8. -3

9. 5, 3, or 1; 5, 3, or 1; 10, 8, 6, 4, 2, or 0

Page 440

11. $-1, 3, 3 - 2i, 3 + 2i$

12. $\pm 1, \pm 3, \pm 5, \pm 15,$
 $\pm \frac{1}{9}, \pm \frac{1}{3}, \pm \frac{5}{9}, \pm \frac{5}{3}$

13. $-\frac{1}{4}, -\frac{1}{2}, \frac{1}{3}, 2$

14. $x^2 + 2x$

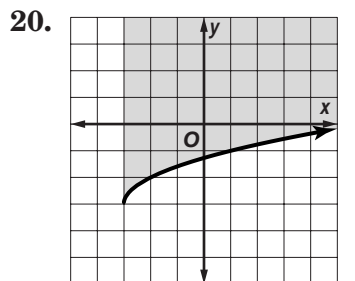
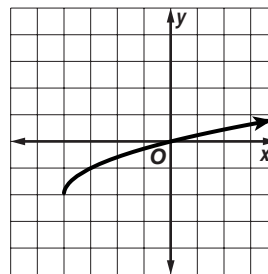
15. $27x^3 - 9x^2 + 3x - 1$

16. 960

17. $h^{-1}(x) = \frac{5x - 6}{2}$

18. yes

19. D: $x \geq -4$, R: $y \geq -2$



20. B: 24

Chapter 7 Assessment Answer Key

Page 441, Open-Ended Assessment Scoring Rubric

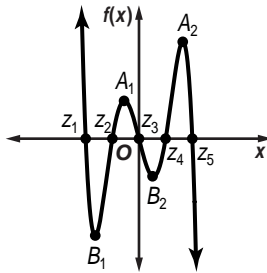
Score	General Description	Specific Criteria
4	Superior A correct solution that is supported by well-developed, accurate explanations	<ul style="list-style-type: none"> Shows thorough understanding of the concepts of <i>polynomial functions; graphing polynomial functions; determining number and type of roots of a polynomial equation; finding rational zeros of a polynomial function; operations with functions; and finding inverse of a function.</i> Uses appropriate strategies to solve problems. Computations are correct. Written explanations are exemplary. Graphs are accurate and appropriate. Goes beyond requirements of some or all problems.
3	Satisfactory A generally correct solution, but may contain minor flaws in reasoning or computation	<ul style="list-style-type: none"> Shows an understanding of the concepts of <i>polynomial functions; graphing polynomial functions; determining number and type of roots of a polynomial equation; finding rational zeros of a polynomial function; operations with functions; and finding inverse of a function.</i> Uses appropriate strategies to solve problems. Computations are mostly correct. Written explanations are effective. Graphs are mostly accurate and appropriate. Satisfies all requirements of problems.
2	Nearly Satisfactory A partially correct interpretation and/or solution to the problem	<ul style="list-style-type: none"> Shows an understanding of most of the concepts of <i>polynomial functions; graphing polynomial functions; determining number and type of roots of a polynomial equation; finding rational zeros of a polynomial function; operations with functions; and finding inverse of a function.</i> May not use appropriate strategies to solve problems. Computations are mostly correct. Written explanations are satisfactory. Graphs are mostly accurate. Satisfies the requirements of most of the problems.
1	Nearly Unsatisfactory A correct solution with no supporting evidence or explanation	<ul style="list-style-type: none"> Final computation is correct. No written explanations or work is shown to substantiate the final computation. Graphs may be accurate but lack detail or explanation. Satisfies minimal requirements of some of the problems.
0	Unsatisfactory An incorrect solution indicating no mathematical understanding of the concept or task, or no solution is given	<ul style="list-style-type: none"> Shows little or no understanding of most of the concepts of <i>polynomial functions; graphing polynomial functions; determining number and type of roots of a polynomial equation; finding rational zeros of a polynomial function; operations with functions; and finding inverse of a function.</i> Does not use appropriate strategies to solve problems. Computations are incorrect. Written explanations are unsatisfactory. Graphs are inaccurate or inappropriate. Does not satisfy requirements of problems. No answer may be given.

Chapter 7 Assessment Answer Key

Page 441, Open-Ended Assessment Sample Answers

In addition to the scoring rubric found on page A34, the following sample answers may be used as guidance in evaluating open-ended assessment items.

- 1a. and b.** Students must sketch a polynomial function having exactly 5 zeros, opposite end behavior, 2 relative maxima, and 2 relative minima, labeled as shown.



- 1c.** Regardless of the function sketched,
D: all real numbers and
R: all real numbers.
- 1d.** Students must state either “As $x \rightarrow +\infty, f(x) \rightarrow -\infty$ and as $x \rightarrow -\infty, f(x) \rightarrow +\infty$ ” (as for the sample function shown) or “As $x \rightarrow +\infty, f(x) \rightarrow +\infty$ and as $x \rightarrow -\infty, f(x) \rightarrow -\infty$.”
- 2.** Since the range is $g(x) \geq 2$, the graph of $g(x)$ lies entirely on or above the horizontal line $g(x) = 2$. Students should indicate that this would require that the degree of $g(x)$ must be even, that the leading coefficient must be positive, and that the function has no zeros.
- 3a.** Answers must be of the form $P(x) = a_0x^4 + a_1x^3 + a_2x^2 + a_3x + a_4$, where $a_n \neq 0$ for any n . Sample answer: $P(x) = x^4 + x^3 + x^2 + 2x + 3$.
- 3b.** Students should show by direct substitution, and by synthetic substitution, how to find $P(-2)$. For the sample function in **a**, $P(-2) = 11$.
- 3c.** Students should indicate that $x + 1$ is a factor of $P(x)$ if and only if $P(-1) = 0$. For the sample function in **a**, $P(-1) = 2 \neq 0$, so $x + 1$ is not a factor of $P(x)$.
- 3d.** Students should use Descartes’ Rule of Signs to determine the number of positive and negative real zeros of $P(x)$. For the sample function in **a**, $P(x)$ has no positive real zeros and has 4, 2 or 0 negative real zeros.
- 3e.** Students must explain that any rational zeros of $P(x)$ must be of the form $\frac{p}{q}$, where p is a factor of a_4 and q is a factor of a_0 . For the sample function in **a**, $a_0 = 1$ and $a_4 = 3$, so the only possible rational zeros are ± 1 and ± 3 .
- 3f.** For each of the possible rational zeros z_n found in part **e**, students must show whether $x - z_n$ is a factor of $P(x)$. For the sample function in **a**, there are no rational zeros.
- 4a.** Sample answers: For $g(x) = x + 1$ and $h(x) = x^2$, the answers would be: $2x + 4$; $9a^2$; $x^2 + x + 1$; $1 + x - x^2$; $x^3 + x^2$; $\frac{x^2}{x + 1}$ for $x \neq -1$; $x^2 + 2x + 1$; $x^2 + 1$; 16 ; $x - 1$.
- 4b.** Students should show that, for their functions $g(x)$ and $g^{-1}(x)$, $g^{-1} \circ g(x) = x$. Students should then indicate that the graphs of inverse functions are reflections of one another over the line $y = x$.

Chapter 7 Assessment Answer Key

Vocabulary Test/Review Page 442

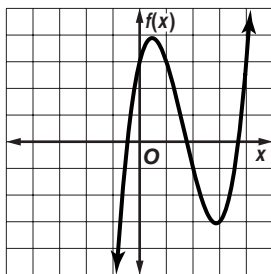
- synthetic substitution
- one-to-one function
- quadratic form
- fundamental theorem of algebra
- square root
- location principle
- minimum
- depressed polynomial
- composition of functions
- polynomial in one variable
- Sample answer: The end behavior of a graph is a description of how the graph behaves when the value of x becomes very small or very large.
- Sample answer: The Factor Theorem is the theorem that states that the binomial $x - a$ is a factor of the polynomial $f(x)$ if and only if $f(a) = 0$.

Quiz (Lessons 7-1 through 7-3) Page 443

1. 57

2. even; 2

3. between -1 and 0,
between 1 and 2,
between 3 and 4



4. $-3, -\sqrt{5}, \sqrt{5}, 3$

5. $-\sqrt{7}, \sqrt{7},$
 $-i\sqrt{7}, i\sqrt{7}$

Quiz (Lessons 7-4 and 7-5) Page 443

1. 46, 277

2. $x + 2, x + 3$

3. 3 or 1; 2 or 0; 4, 2, or 0

4. $3, 1 - i, 1 + i$

5. B

Quiz (Lessons 7-6 and 7-7) Page 444

1. $\pm 1, \pm 2, \pm 3, \pm 6,$
 $\pm \frac{1}{2}, \pm \frac{3}{2}; -1, \frac{3}{2}$

2. $x^2 - x + 6; x^2 - 5x - 2;$
 $2x^3 - 2x^2 - 8x + 8;$
 $\frac{x^2 - 3x + 2}{2x + 4}, x \neq -2$

3. $\{(2, 4), (3, 8), (4, 3), (8, 4)\};$
 $\{(2, 5), (4, 2), (5, 4)\}$

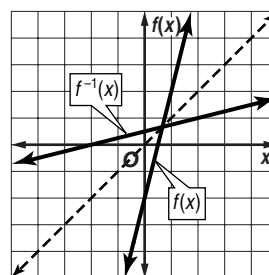
4. $x^2 - 6x + 7; x^2 + 2x - 5$

5. 28; 122

Quiz (Lessons 7-8 and 7-9) Page 444

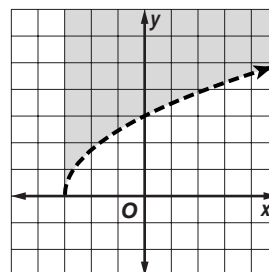
1. $\{(5, -2), (4, 0), (-8, 1),$
 $(7, 4)\}$

2. $f^{-1}(x) = \frac{x + 2}{4}$



3. yes

4. D: $x > -3$, R: $y \geq 0$



Chapter 7 Assessment Answer Key

Mid-Chapter Test

Page 445

1. B

2. A

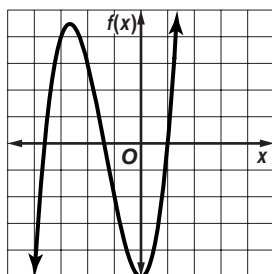
3. D

4. C

5. A

6. D

7. at $x = 1$, between -4 and -3 , between -2 and -1



8. $-3, 0, 3, -3i, 3i$

9. 2 or 0; 2 or 0; 4, 2, or 0

10. $f(x) = x^3 - 3x^2 + 9x - 27$

11. $-1, -4i, 4i$

Cumulative Review

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1. $s =$ amount invested in stock:
 $0.07s + 0.05(10,000 - s) \geq 550$; at least \$2500

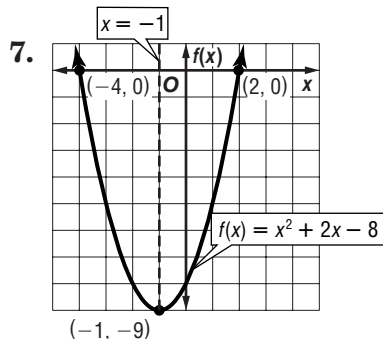
2. consistent and dependent

3. $A'(4, -6), B'(7, 0), C'(1, -2)$

4. $(-3, 5)$

5. $(2n + 5)^2$

6. $x > 13$



8. $x^2 - x - 6 = 0$

9. 278

10. 2 or 0; 2 or 0; 4, 2, or 0

11. $-3, \frac{1}{2}, 2$

12. $f^{-1}(x) = \frac{x + 4}{5}$

Chapter 7 Assessment Answer Key

Standardized Test Practice

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1. A B C D

2. E F G H

3. A B C D

4. E F G H

5. A B C D

6. E F G H

7. A B C D

8. E F G H

9. A B C D

10. E F G H

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11.

5	/	8	
.	.	.	.
0	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

12.

5	9	/	6
.	.	.	.
0	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

13.

9			
.	.	.	.
0	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

14.

.	0	5	
.	.	.	.
0	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

15. A B C D

16. A B C D

17. A B C D

Chapter 7 Assessment Answer Key

Unit 2 Test

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1. $\underline{8x^2 - 5x - 6}$

2. $\underline{245x^5}$

3. $\underline{4x^2 - 12x + 9}$

4. $\underline{\frac{4y^2 - 6y + 9}{x - 5}}$

5. $\underline{4|x|y^2}$

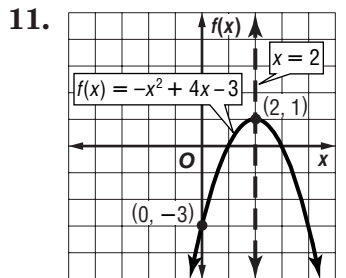
6. $\underline{7\sqrt{3} + 12\sqrt{2}}$

7. $\underline{-\frac{1}{10} + \frac{7}{10}i}$

8. $\underline{2x^2 - 3x + 4 + \frac{3}{x-1}}$

9. $\underline{\sqrt[9]{m^7}}$

10. $\underline{-2 \leq x \leq 1}$



12. $\underline{75 \text{ m}}$

13. $\underline{\text{between } -1 \text{ and } 0; 2}$

14. $\underline{\{-4, 6\}}$

15. $\underline{4x^2 - 13x - 12 = 0}$

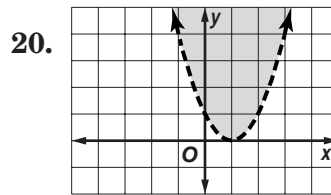
16. $\underline{\frac{-1 \pm i\sqrt{95}}{12}}$

17. $\underline{0; 1 \text{ real root}}$

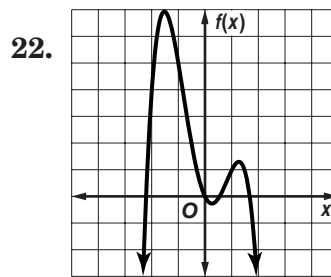
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18. $\underline{(-3, -5); x = -3; \text{ up}}$

19. $\underline{y = -4(x - 1)^2 + 3}$



21. $\underline{-216}$



Sample answer: rel. max. at $x = -2$ and $x = 1$, rel. min. at $x = 0$

23. $\underline{10, -10, \sqrt{2}, -\sqrt{2}}$

24. $\underline{-86}$

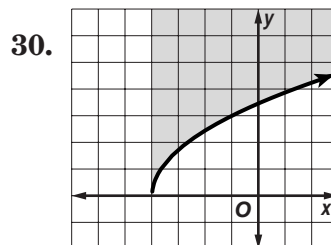
25. $\underline{x + 2; x - 5}$

26. $\underline{3 \text{ or } 1; 0; 2 \text{ or } 0}$

27. $\underline{\pm 1, \pm 3, \pm 5, \pm 15, \pm \frac{1}{3}, \pm \frac{5}{3}}$

28. $\underline{51; 57}$

29. $\underline{f^{-1}(x) = \frac{x + 2}{7}}$



Semester Test

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1. \underline{D}

2. \underline{A}

3. \underline{D}

4. \underline{C}

5. \underline{B}

6. \underline{D}

7. \underline{A}

8. \underline{A}

(continued on the next page)

Chapter 7 Assessment Answer Key

Semester Test (continued)

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9. C

10. B

11. B

12. D

13. A

14. C

15. A

16. B

17. B

18. C

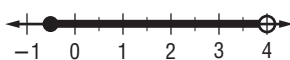
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19. 1256 cm²

20.

21. $p = \text{the number of sheets of plywood;}$
 $270 + 7p \leq 1000;$ no more than 104 sheets of plywood

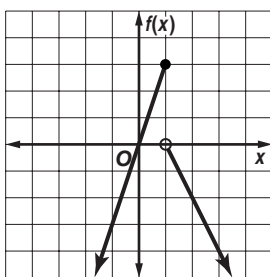
22. $\left\{ m \mid -\frac{1}{2} \leq m < 4 \right\}$
 or $\left[-\frac{1}{2}, 4 \right)$



23. $4a^2 + 8a - 12$

24. $-\frac{1}{4}$

25. **D = all reals;**
R = $\{y \mid y \leq 3\}$



26. $(2, -5)$

27. $c = 2t;$
 $5c + 8t = 360;$
 $5c + 8t + 10v = 410$

28. 40 cars

29. $\begin{bmatrix} -3 & 4 \\ -5 & 0 \end{bmatrix}$

30. $\begin{bmatrix} 3 & -61 \\ -7 & 15 \end{bmatrix}$

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31. 25

32. $\frac{1}{12} \begin{bmatrix} 1 & -2 \\ 2 & 8 \end{bmatrix}$

33. $72x^4y^3$

34. $2x^2 + 7xy - 15y^2$

35.

36. 6.596

37. minimum; -14

38. $\{-9, 7\}$

39.

40. $\left\{ -1, \frac{1}{7} \right\}$

41. $y = -(x + 2)^2 + 3$

42.

43. $-6, -4i, 4i$

44. $-3x^2 + 4x - 9;$
 $12x^3 - 27x^2$

45. $p^{-1}(x) = \frac{x + 8}{4}$

46. **D: $x \geq -\frac{5}{2}$; R: $y \leq 0$**

