GLENCOE MATHEMATICS

Algebra 2

Chapter 6
Resource Masters

Glencoe McGraw-Hill

New York, New York
Columbus, Ohio
Chicago, Illinois
Peoria, Illinois
Woodland Hills, California

Consumable Workbooks

Many of the worksheets contained in the Chapter Resource Masters booklets are available as consumable workbooks.

Study Guide and Intervention Workbook	0-07-828029-X
Skills Practice Workbook	0-07-828023-0
Practice Workbook	0-07-828024-9

ANSWERS FOR WORKBOOKS The answers for Chapter 6 of these workbooks can be found in the back of this Chapter Resource Masters booklet.

Glencoe/McGraw-Hill

A Division of The ***McGraw-Hill*** *Companies*

Send all inquiries to:
The McGraw-Hill Companies
8787 Orion Place
Columbus, OH 43240-4027

ISBN: 0-07-828009-5

Algebra 2
Chapter 6 Resource Masters

1 2 3 4 5 6 7 8 9 10 066 11 10 09 08 07 06 05 04 03 02

Contents

Teacher's Guide to Using the Chapter 6 Resource Masters

The ***Fast File*** Chapter Resource system allows you to conveniently file the resources you use most often. The *Chapter 6 Resource Masters* includes the core materials needed for Chapter 6. These materials include worksheets, extensions, and assessment options. The answers for these pages appear at the back of this booklet.

All of the materials found in this booklet are included for viewing and printing in the *Algebra 2 TeacherWorks* CD-ROM.

Vocabulary Builder Pages vii–viii include a student study tool that presents up to twenty of the key vocabulary terms from the chapter. Students are to record definitions and/or examples for each term. You may suggest that students highlight or star the terms with which they are not familiar.

WHEN TO USE Give these pages to students before beginning Lesson 6-1. Encourage them to add these pages to their Algebra 2 Study Notebook. Remind them to add definitions and examples as they complete each lesson.

Study Guide and Intervention

Each lesson in *Algebra 2* addresses two objectives. There is one Study Guide and Intervention master for each objective.

WHEN TO USE Use these masters as reteaching activities for students who need additional reinforcement. These pages can also be used in conjunction with the Student Edition as an instructional tool for students who have been absent.

Skills Practice There is one master for each lesson. These provide computational practice at a basic level.

WHEN TO USE These masters can be used with students who have weaker mathematics backgrounds or need additional reinforcement.

Practice There is one master for each lesson. These problems more closely follow the structure of the Practice and Apply section of the Student Edition exercises. These exercises are of average difficulty.

WHEN TO USE These provide additional practice options or may be used as homework for second day teaching of the lesson.

Reading to Learn Mathematics

One master is included for each lesson. The first section of each master asks questions about the opening paragraph of the lesson in the Student Edition. Additional questions ask students to interpret the context of and relationships among terms in the lesson. Finally, students are asked to summarize what they have learned using various representation techniques.

WHEN TO USE This master can be used as a study tool when presenting the lesson or as an informal reading assessment after presenting the lesson. It is also a helpful tool for ELL (English Language Learner) students.

Enrichment There is one extension master for each lesson. These activities may extend the concepts in the lesson, offer an historical or multicultural look at the concepts, or widen students' perspectives on the mathematics they are learning. These are not written exclusively for honors students, but are accessible for use with all levels of students.

WHEN TO USE These may be used as extra credit, short-term projects, or as activities for days when class periods are shortened.

Assessment Options

The assessment masters in the *Chapter 6 Resource Masters* offer a wide range of assessment tools for intermediate and final assessment. The following lists describe each assessment master and its intended use.

Chapter Assessment

CHAPTER TESTS

- *Form 1* contains multiple-choice questions and is intended for use with basic level students.
- *Forms 2A and 2B* contain multiple-choice questions aimed at the average level student. These tests are similar in format to offer comparable testing situations.
- *Forms 2C and 2D* are composed of free-response questions aimed at the average level student. These tests are similar in format to offer comparable testing situations. Grids with axes are provided for questions assessing graphing skills.
- *Form 3* is an advanced level test with free-response questions. Grids without axes are provided for questions assessing graphing skills.

  All of the above tests include a free-response Bonus question.
- The **Open-Ended Assessment** includes performance assessment tasks that are suitable for all students. A scoring rubric is included for evaluation guidelines. Sample answers are provided for assessment.
- A **Vocabulary Test**, suitable for all students, includes a list of the vocabulary words in the chapter and ten questions assessing students' knowledge of those terms. This can also be used in conjunction with one of the chapter tests or as a review worksheet.

Intermediate Assessment

- Four free-response **quizzes** are included to offer assessment at appropriate intervals in the chapter.
- A **Mid-Chapter Test** provides an option to assess the first half of the chapter. It is composed of both multiple-choice and free-response questions.

Continuing Assessment

- The **Cumulative Review** provides students an opportunity to reinforce and retain skills as they proceed through their study of Algebra 2. It can also be used as a test. This master includes free-response questions.
- The **Standardized Test Practice** offers continuing review of algebra concepts in various formats, which may appear on the standardized tests that they may encounter. This practice includes multiple-choice, grid-in, and quantitative-comparison questions. Bubble-in and grid-in answer sections are provided on the master.

Answers

- Page A1 is an answer sheet for the Standardized Test Practice questions that appear in the Student Edition on pages 342–343. This improves students' familiarity with the answer formats they may encounter in test taking.
- The answers for the lesson-by-lesson masters are provided as reduced pages with answers appearing in red.
- Full-size answer keys are provided for the assessment masters in this booklet.

NAME ____ DATE ____________ PERIOD _____

6 Reading to Learn Mathematics

Vocabulary Builder

This is an alphabetical list of the key vocabulary terms you will learn in Chapter 6. As you study the chapter, complete each term's definition or description. Remember to add the page number where you found the term. Add these pages to your Algebra Study Notebook to review vocabulary at the end of the chapter.

Vocabulary Term	Found on Page	Definition/Description/Example
axis of symmetry		
completing the square		
constant term		
discriminant dihs·KRIH·muh·nuhnt		
linear term		
maximum value		
minimum value		
parabola puh·RA·buh·luh		
quadratic equation kwah·DRA·tihk		
Quadratic Formula		

(continued on the next page)

6

Reading to Learn Mathematics

Vocabulary Builder *(continued)*

Vocabulary Term	Found on Page	Definition/Description/Example
quadratic function		
quadratic inequality		
quadratic term		
roots		
Square Root Property		
vertex		
vertex form		
Zero Product Property		
zeros		

NAME ______________________________ DATE ____________ PERIOD _____

6-1 Study Guide and Intervention

Graphing Quadratic Functions

Graph Quadratic Functions

Quadratic Function	A function defined by an equation of the form $f(x) = ax^2 + bx + c$, where $a \neq 0$
Graph of a Quadratic Function	A **parabola** with these characteristics: y intercept: c; axis of symmetry: $x = \frac{-b}{2a}$; x-coordinate of vertex: $\frac{-b}{2a}$

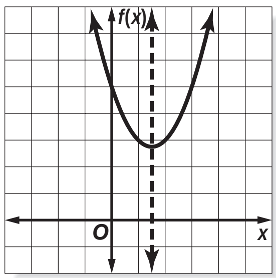
Lesson 6-1

Example **Find the y-intercept, the equation of the axis of symmetry, and the x-coordinate of the vertex for the graph of $f(x) = x^2 - 3x + 5$. Use this information to graph the function.**

$a = 1$, $b = -3$, and $c = 5$, so the y-intercept is 5. The equation of the axis of symmetry is $x = \frac{-(-3)}{2(1)}$ or $\frac{3}{2}$. The x-coordinate of the vertex is $\frac{3}{2}$.

Next make a table of values for x near $\frac{3}{2}$.

x	$x^2 - 3x + 5$	$f(x)$	$(x, f(x))$
0	$0^2 - 3(0) + 5$	5	(0, 5)
1	$1^2 - 3(1) + 5$	3	(1, 3)
$\frac{3}{2}$	$\left(\frac{3}{2}\right)^2 - 3\left(\frac{3}{2}\right) + 5$	$\frac{11}{4}$	$\left(\frac{3}{2}, \frac{11}{4}\right)$
2	$2^2 - 3(2) + 5$	3	(2, 3)
3	$3^2 - 3(3) + 5$	5	(3, 5)



Exercises

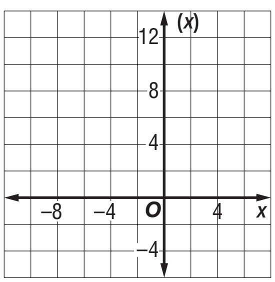
For Exercises 1–3, complete parts a–c for each quadratic function.

a. Find the y-intercept, the equation of the axis of symmetry, and the x-coordinate of the vertex.

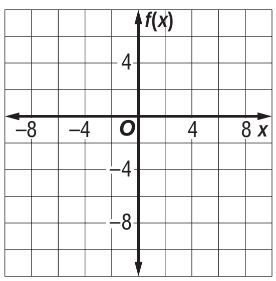
b. Make a table of values that includes the vertex.

c. Use this information to graph the function.

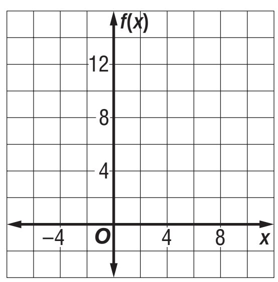
1. $f(x) = x^2 + 6x + 8$



2. $f(x) = -x^2 - 2x + 2$



3. $f(x) = 2x^2 - 4x + 3$



NAME ____ DATE ____________ PERIOD _____

6-1 Study Guide and Intervention *(continued)*

Graphing Quadratic Functions

Maximum and Minimum Values The y-coordinate of the vertex of a quadratic function is the maximum or minimum value of the function.

Maximum or Minimum Value of a Quadratic Function	The graph of $f(x) = ax^2 + bx + c$, where $a \neq 0$, opens up and has a minimum when $a > 0$. The graph opens down and has a maximum when $a < 0$.

Example **Determine whether each function has a maximum or minimum value. Then find the maximum or minimum value of each function.**

a. $f(x) = 3x^2 - 6x + 7$

For this function, $a = 3$ and $b = -6$.

Since $a > 0$, the graph opens up, and the function has a minimum value.

The minimum value is the y-coordinate of the vertex. The x-coordinate of the vertex is $\frac{-b}{2a} = -\frac{-6}{2(3)} = 1$.

Evaluate the function at $x = 1$ to find the minimum value.

$f(1) = 3(1)^2 - 6(1) + 7 = 4$, so the minimum value of the function is 4.

b. $f(x) = 100 - 2x - x^2$

For this function, $a = -1$ and $b = -2$.

Since $a < 0$, the graph opens down, and the function has a maximum value.

The maximum value is the y-coordinate of the vertex. The x-coordinate of the vertex is $\frac{-b}{2a} = -\frac{-2}{2(-1)} = -1$.

Evaluate the function at $x = -1$ to find the maximum value.

$f(-1) = 100 - 2(-1) - (-1)^2 = 101$, so the minimum value of the function is 101.

Exercises

Determine whether each function has a maximum or minimum value. Then find the maximum or minimum value of each function.

1. $f(x) = 2x^2 - x + 10$

2. $f(x) = x^2 + 4x - 7$

3. $f(x) = 3x^2 - 3x + 1$

4. $f(x) = 16 + 4x - x^2$

5. $f(x) = x^2 - 7x + 11$

6. $f(x) = -x^2 + 6x - 4$

7. $f(x) = x^2 + 5x + 2$

8. $f(x) = 20 + 6x - x^2$

9. $f(x) = 4x^2 + x + 3$

10. $f(x) = -x^2 - 4x + 10$

11. $f(x) = x^2 - 10x + 5$

12. $f(x) = -6x^2 + 12x + 21$

13. $f(x) = 25x^2 + 100x + 350$

14. $f(x) = 0.5x^2 + 0.3x - 1.4$

15. $f(x) = \frac{-x^2}{2} + \frac{x}{4} - 8$

NAME ______________________________ DATE ____________ PERIOD _____

6-1 Skills Practice

Graphing Quadratic Functions

For each quadratic function, find the *y*-intercept, the equation of the axis of symmetry, and the *x*-coordinate of the vertex.

1. $f(x) = 3x^2$

2. $f(x) = x^2 + 1$

3. $f(x) = -x^2 + 6x - 15$

4. $f(x) = 2x^2 - 11$

5. $f(x) = x^2 - 10x + 5$

6. $f(x) = -2x^2 + 8x + 7$

Complete parts a–c for each quadratic function.

a. Find the *y*-intercept, the equation of the axis of symmetry, and the *x*-coordinate of the vertex.

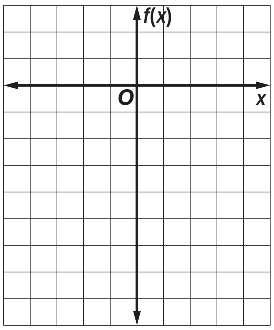
b. Make a table of values that includes the vertex.

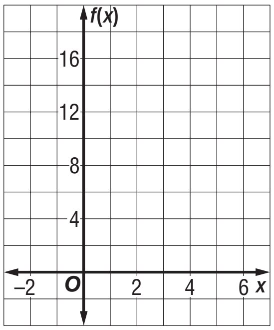
c. Use this information to graph the function.

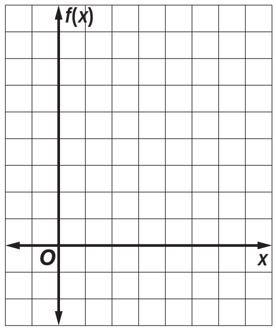
7. $f(x) = -2x^2$

8. $f(x) = x^2 - 4x + 4$

9. $f(x) = x^2 - 6x + 8$







Determine whether each function has a maximum or a minimum value. Then find the maximum or minimum value of each function.

10. $f(x) = 6x^2$

11. $f(x) = -8x^2$

12. $f(x) = x^2 + 2x$

13. $f(x) = x^2 + 2x + 15$

14. $f(x) = -x^2 + 4x - 1$

15. $f(x) = x^2 + 2x - 3$

16. $f(x) = -2x^2 + 4x - 3$

17. $f(x) = 3x^2 + 12x + 3$

18. $f(x) = 2x^2 + 4x + 1$

NAME ______________________________ DATE __________ PERIOD _____

6-1 Practice

Graphing Quadratic Functions

Complete parts a–c for each quadratic function.

a. Find the *y*-intercept, the equation of the axis of symmetry, and the *x*-coordinate of the vertex.

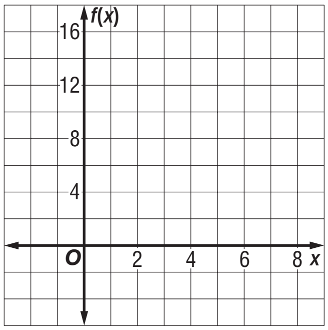
b. Make a table of values that includes the vertex.

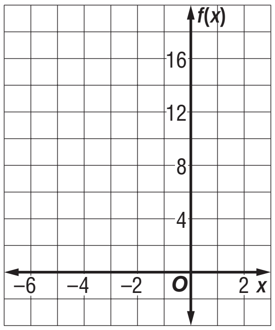
c. Use this information to graph the function.

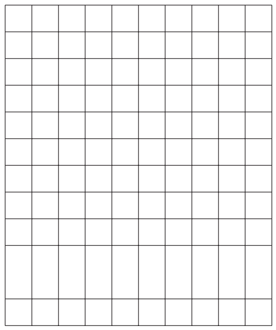
1. $f(x) = x^2 - 8x + 15$

2. $f(x) = -x^2 - 4x + 12$

3. $f(x) = 2x^2 - 2x + 1$







Determine whether each function has a maximum or a minimum value. Then find the maximum or minimum value of each function.

4. $f(x) = x^2 + 2x - 8$

5. $f(x) = x^2 - 6x + 14$

6. $v(x) = -x^2 + 14x - 57$

7. $f(x) = 2x^2 + 4x - 6$

8. $f(x) = -x^2 + 4x - 1$

9. $f(x) = -\frac{2}{3}x^2 + 8x - 24$

10. GRAVITATION From 4 feet above a swimming pool, Susan throws a ball upward with a velocity of 32 feet per second. The height $h(t)$ of the ball t seconds after Susan throws it is given by $h(t) = -16t^2 + 32t + 4$. Find the maximum height reached by the ball and the time that this height is reached.

11. HEALTH CLUBS Last year, the SportsTime Athletic Club charged $20 to participate in an aerobics class. Seventy people attended the classes. The club wants to increase the class price this year. They expect to lose one customer for each $1 increase in the price.

a. What price should the club charge to maximize the income from the aerobics classes?

b. What is the maximum income the SportsTime Athletic Club can expect to make?

6-1 Reading to Learn Mathematics

Graphing Quadratic Functions

Pre-Activity **How can income from a rock concert be maximized?**

Read the introduction to Lesson 6-1 at the top of page 286 in your textbook.

- Based on the graph in your textbook, for what ticket price is the income the greatest?
- Use the graph to estimate the maximum income.

Reading the Lesson

1. a. For the quadratic function $f(x) = 2x^2 + 5x + 3$, $2x^2$ is the ____________ term, $5x$ is the ____________ term, and 3 is the ____________ term.

b. For the quadratic function $f(x) = -4 + x - 3x^2$, $a =$ ________, $b =$ ________, and $c =$ ________.

2. Consider the quadratic function $f(x) = ax^2 + bx + c$, where $a \neq 0$.

a. The graph of this function is a ____________.

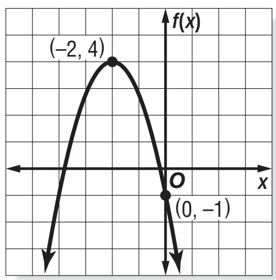
b. The y-intercept is ____________.

c. The axis of symmetry is the line ____________.

d. If $a > 0$, then the graph opens ____________ and the function has a ____________ value.

e. If $a < 0$, then the graph opens ____________ and the function has a ____________ value.

3. Refer to the graph at the right as you complete the following sentences.



a. The curve is called a ____________.

b. The line $x = -2$ is called the ____________.

c. The point $(-2, 4)$ is called the ____________.

d. Because the graph contains the point $(0, -1)$, -1 is the ____________.

Helping You Remember

4. How can you remember the way to use the x^2 term of a quadratic function to tell whether the function has a maximum or a minimum value?

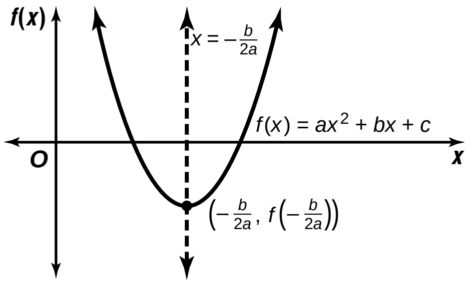
6-1 Enrichment

Finding the Axis of Symmetry of a Parabola

As you know, if $f(x) = ax^2 + bx + c$ is a quadratic function, the values of x that make $f(x)$ equal to zero are $\frac{-b + \sqrt{b^2 - 4ac}}{2a}$ and $\frac{-b - \sqrt{b^2 - 4ac}}{2a}$.

The average of these two number values is $-\frac{b}{2a}$.

The function $f(x)$ has its maximum or minimum value when $x = -\frac{b}{2a}$. Since the axis of symmetry of the graph of $f(x)$ passes through the point where the maximum or minimum occurs, the axis of symmetry has the equation $x = -\frac{b}{2a}$.



Example **Find the vertex and axis of symmetry for $f(x) = 5x^2 + 10x - 7$.**

Use $x = -\frac{b}{2a}$.

$x = -\frac{10}{2(5)} = -1$ The x-coordinate of the vertex is -1.

Substitute $x = -1$ in $f(x) = 5x^2 + 10x - 7$.

$f(-1) = 5(-1)^2 + 10(-1) - 7 = -12$

The vertex is $(-1, -12)$.

The axis of symmetry is $x = -\frac{b}{2a}$, or $x = -1$.

Find the vertex and axis of symmetry for the graph of each function using $x = -\frac{b}{2a}$.

1. $f(x) = x^2 - 4x - 8$

2. $g(x) = -4x^2 - 8x + 3$

3. $y = -x^2 + 8x + 3$

4. $f(x) = 2x^2 + 6x + 5$

5. $A(x) = x^2 + 12x + 36$

6. $k(x) = -2x^2 + 2x - 6$

NAME ______________________________ DATE ____________ PERIOD _____

6-2 Study Guide and Intervention

Solving Quadratic Equations by Graphing

Solve Quadratic Equations

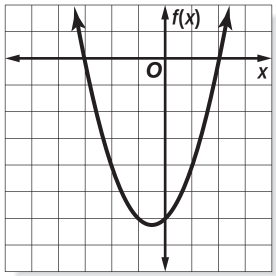
Quadratic Equation	A quadratic equation has the form $ax^2 + bx + c = 0$, where $a \neq 0$.
Roots of a Quadratic Equation	solution(s) of the equation, or the zero(s) of the related quadratic function

The zeros of a quadratic function are the x-intercepts of its graph. Therefore, finding the x-intercepts is one way of solving the related quadratic equation.

Example **Solve $x^2 + x - 6 = 0$ by graphing.**

Graph the related function $f(x) = x^2 + x - 6$.

The x-coordinate of the vertex is $\frac{-b}{2a} = -\frac{1}{2}$, and the equation of the axis of symmetry is $x = -\frac{1}{2}$.



Make a table of values using x-values around $-\frac{1}{2}$.

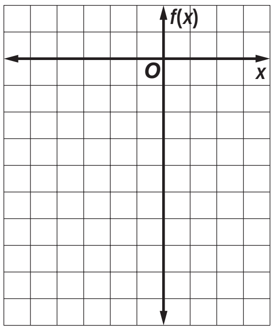
x	-1	$-\frac{1}{2}$	0	1	2
$f(x)$	-6	$-6\frac{1}{4}$	-6	-4	0

From the table and the graph, we can see that the zeros of the function are 2 and -3.

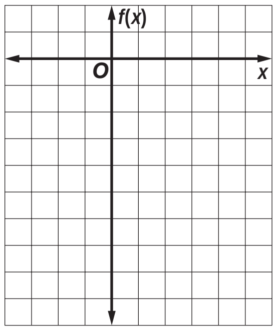
Exercises

Solve each equation by graphing.

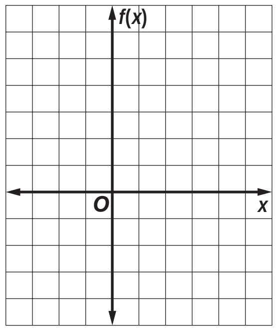
1. $x^2 + 2x - 8 = 0$



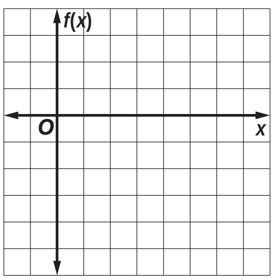
2. $x^2 - 4x - 5 = 0$



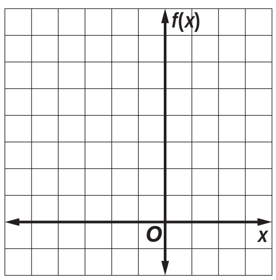
3. $x^2 - 5x + 4 = 0$



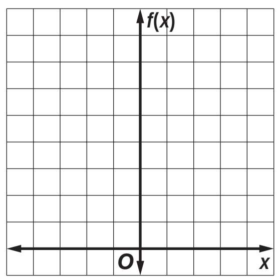
4. $x^2 - 10x + 21 = 0$



5. $x^2 + 4x + 6 = 0$



6. $4x^2 + 4x + 1 = 0$



Lesson 6-2

6-2 Study Guide and Intervention *(continued)*

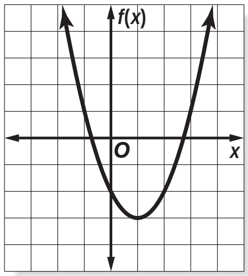
Solving Quadratic Equations by Graphing

Estimate Solutions Often, you may not be able to find exact solutions to quadratic equations by graphing. But you can use the graph to estimate solutions.

Example **Solve $x^2 - 2x - 2 = 0$ by graphing. If exact roots cannot be found, state the consecutive integers between which the roots are located.**

The equation of the axis of symmetry of the related function is $x = -\frac{-2}{2(1)} = 1$, so the vertex has x-coordinate 1. Make a table of values.

x	−1	0	1	2	3
$f(x)$	1	−2	−3	−2	1

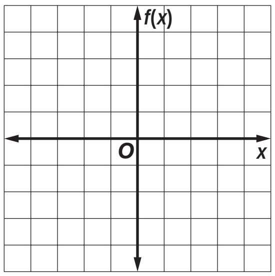


The x-intercepts of the graph are between 2 and 3 and between 0 and −1. So one solution is between 2 and 3, and the other solution is between 0 and −1.

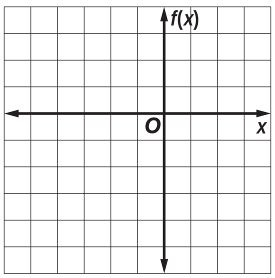
Exercises

Solve the equations by graphing. If exact roots cannot be found, state the consecutive integers between which the roots are located.

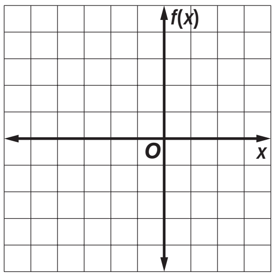
1. $x^2 - 4x + 2 = 0$



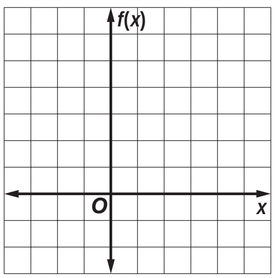
2. $x^2 + 6x + 6 = 0$



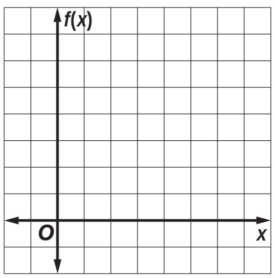
3. $x^2 + 4x + 2 = 0$



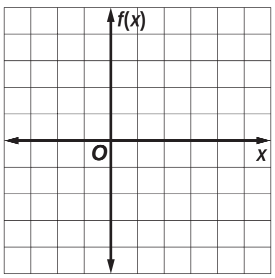
4. $-x^2 + 2x + 4 = 0$



5. $2x^2 - 12x + 17 = 0$



6. $-\frac{1}{2}x^2 + x + \frac{5}{2} = 0$



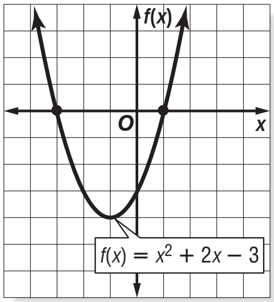
NAME ______________________________ DATE ____________ PERIOD _____

6-2 Skills Practice

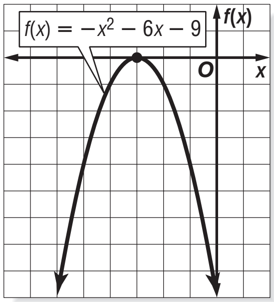
Solving Quadratic Equations By Graphing

Use the related graph of each equation to determine its solutions.

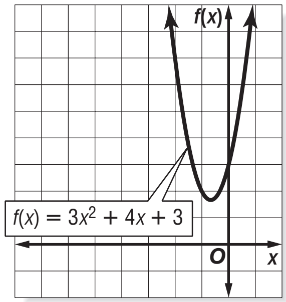
1. $x^2 + 2x - 3 = 0$



2. $-x^2 - 6x - 9 = 0$

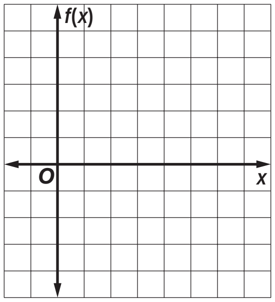


3. $3x^2 + 4x + 3 = 0$

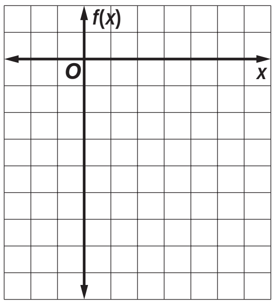


Solve each equation by graphing. If exact roots cannot be found, state the consecutive integers between which the roots are located.

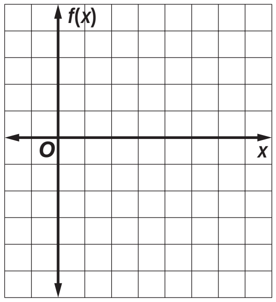
4. $x^2 - 6x + 5 = 0$



5. $-x^2 + 2x - 4 = 0$

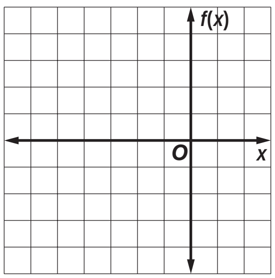


6. $x^2 - 6x + 4 = 0$

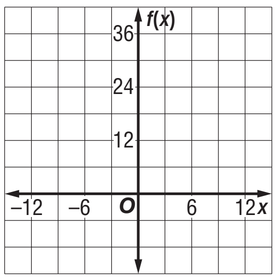


Use a quadratic equation to find two real numbers that satisfy each situation, or show that no such numbers exist.

7. Their sum is -4, and their product is 0.



8. Their sum is 0, and their product is -36.

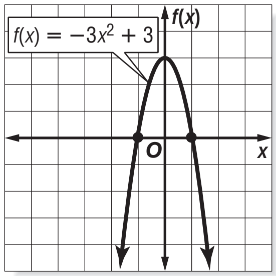


6-2 Practice

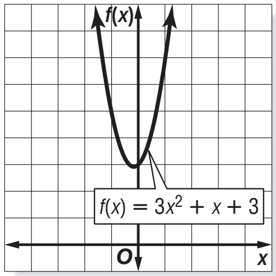
Solving Quadratic Equations By Graphing

Use the related graph of each equation to determine its solutions.

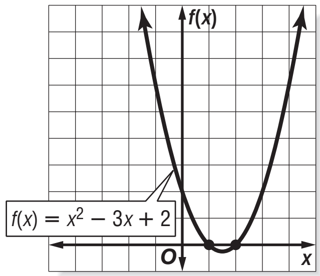
1. $-3x^2 + 3 = 0$



2. $3x^2 + x + 3 = 0$

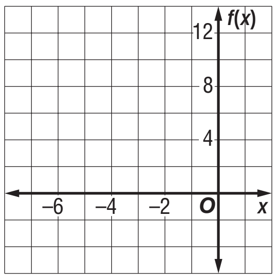


3. $x^2 - 3x + 2 = 0$

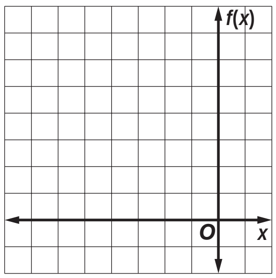


Solve each equation by graphing. If exact roots cannot be found, state the consecutive integers between which the roots are located.

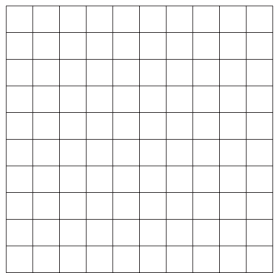
4. $-2x^2 - 6x + 5 = 0$



5. $x^2 + 10x + 24 = 0$

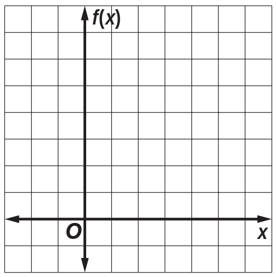


6. $2x^2 - x - 6 = 0$

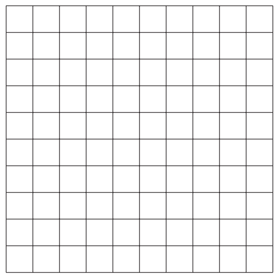


Use a quadratic equation to find two real numbers that satisfy each situation, or show that no such numbers exist.

7. Their sum is 1, and their product is -6.



8. Their sum is 5, and their product is 8.



For Exercises 9 and 10, use the formula $h(t) = v_0t - 16t^2$, where $h(t)$ is the height of an object in feet, v_0 is the object's initial velocity in feet per second, and t is the time in seconds.

9. BASEBALL Marta throws a baseball with an initial upward velocity of 60 feet per second. Ignoring Marta's height, how long after she releases the ball will it hit the ground?

10. VOLCANOES A volcanic eruption blasts a boulder upward with an initial velocity of 240 feet per second. How long will it take the boulder to hit the ground if it lands at the same elevation from which it was ejected?

NAME ____ DATE ____________ PERIOD _____

6-2 Reading to Learn Mathematics

Solving Quadratic Equations by Graphing

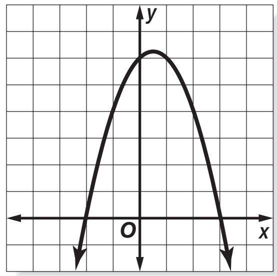
Pre-Activity **How does a quadratic function model a free-fall ride?**

Read the introduction to Lesson 6-2 at the top of page 294 in your textbook.

Write a quadratic function that describes the height of a ball t seconds after it is dropped from a height of 125 feet.

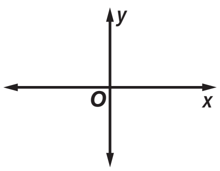
Reading the Lesson

1. The graph of the quadratic function $f(x) = -x^2 + x + 6$ is shown at the right. Use the graph to find the solutions of the quadratic equation $-x^2 + x + 6 = 0$.

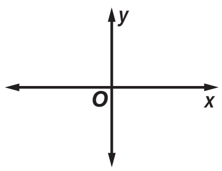


2. Sketch a graph to illustrate each situation.

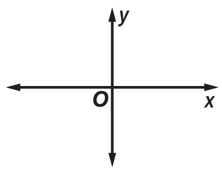
a. A parabola that opens downward and represents a quadratic function with two real zeros, both of which are negative numbers.



b. A parabola that opens upward and represents a quadratic function with exactly one real zero. The zero is a positive number.



c. A parabola that opens downward and represents a quadratic function with no real zeros.



Helping You Remember

3. Think of a memory aid that can help you recall what is meant by the *zeros* of a quadratic function.

NAME _______________ DATE _______ PERIOD _____

6-2 Enrichment

Graphing Absolute Value Equations

You can solve absolute value equations in much the same way you solved quadratic equations. Graph the related absolute value function for each equation using a graphing calculator. Then use the ZERO feature in the CALC menu to find its real solutions, if any. Recall that solutions are points where the graph intersects the x-axis.

For each equation, make a sketch of the related graph and find the solutions rounded to the nearest hundredth.

1. $|x + 5| = 0$

2. $|4x - 3| + 5 = 0$

3. $|x - 7| = 0$

4. $|x + 3| - 8 = 0$

5. $-|x + 3| + 6 = 0$

6. $|x - 2| - 3 = 0$

7. $|3x + 4| = 2$

8. $|x + 12| = 10$

9. $|x| - 3 = 0$

10. Explain how solving absolute value equations algebraically and finding zeros of absolute value functions graphically are related.

NAME ______________________________ DATE ____________ PERIOD _____

6-3 Study Guide and Intervention

Solving Quadratic Equations by Factoring

Solve Equations by Factoring When you use factoring to solve a quadratic equation, you use the following property.

Zero Product Property	For any real numbers a and b, if $ab = 0$, then either $a = 0$ or $b = 0$, or both a and $b = 0$.

Example **Solve each equation by factoring.**

a. $3x^2 = 15x$

$3x^2 = 15x$ Original equation

$3x^2 - 15x = 0$ Subtract $15x$ from both sides.

$3x(x - 5) = 0$ Factor the binomial.

$3x = 0$ or $x - 5 = 0$ Zero Product Property

$x = 0$ or $x = 5$ Solve each equation.

The solution set is $\{0, 5\}$.

b. $4x^2 - 5x = 21$

$4x^2 - 5x = 21$ Original equation

$4x^2 - 5x - 21 = 0$ Subtract 21 from both sides.

$(4x + 7)(x - 3) = 0$ Factor the trinomial.

$4x + 7 = 0$ or $x - 3 = 0$ Zero Product Property

$x = -\frac{7}{4}$ or $x = 3$ Solve each equation.

The solution set is $\left\{-\frac{7}{4}, 3\right\}$.

Exercises

Solve each equation by factoring.

1. $6x^2 - 2x = 0$

2. $x^2 = 7x$

3. $20x^2 = -25x$

4. $6x^2 = 7x$

5. $6x^2 - 27x = 0$

6. $12x^2 - 8x = 0$

7. $x^2 + x - 30 = 0$

8. $2x^2 - x - 3 = 0$

9. $x^2 + 14x + 33 = 0$

10. $4x^2 + 27x - 7 = 0$

11. $3x^2 + 29x - 10 = 0$

12. $6x^2 - 5x - 4 = 0$

13. $12x^2 - 8x + 1 = 0$

14. $5x^2 + 28x - 12 = 0$

15. $2x^2 - 250x + 5000 = 0$

16. $2x^2 - 11x - 40 = 0$

17. $2x^2 + 21x - 11 = 0$

18. $3x^2 + 2x - 21 = 0$

19. $8x^2 - 14x + 3 = 0$

20. $6x^2 + 11x - 2 = 0$

21. $5x^2 + 17x - 12 = 0$

22. $12x^2 + 25x + 12 = 0$

23. $12x^2 + 18x + 6 = 0$

24. $7x^2 - 36x + 5 = 0$

Lesson 6-3

6-3 Study Guide and Intervention *(continued)*

Solving Quadratic Equations by Factoring

Write Quadratic Equations To write a quadratic equation with roots p and q, let $(x - p)(x - q) = 0$. Then multiply using FOIL.

Example **Write a quadratic equation with the given roots. Write the equation in the form $ax^2 + bx + c = 0$.**

a. 3, −5

$(x - p)(x - q) = 0$	Write the pattern.
$(x - 3)[x - (-5)] = 0$	Replace p with 3, q with −5.
$(x - 3)(x + 5) = 0$	Simplify.
$x^2 + 2x - 15 = 0$	Use FOIL.

The equation $x^2 + 2x - 15 = 0$ has roots 3 and −5.

b. $-\frac{7}{8}, \frac{1}{3}$

$$(x - p)(x - q) = 0$$

$$\left[x - \left(-\frac{7}{8}\right)\right]\left(x - \frac{1}{3}\right) = 0$$

$$\left(x + \frac{7}{8}\right)\left(x - \frac{1}{3}\right) = 0$$

$$\frac{(8x + 7)}{8} \cdot \frac{(3x - 1)}{3} = 0$$

$$\frac{24 \cdot (8x + 7)(3x - 1)}{24} = 24 \cdot 0$$

$$24x^2 + 13x - 7 = 0$$

The equation $24x^2 + 13x - 7 = 0$ has roots $-\frac{7}{8}$ and $\frac{1}{3}$.

Exercises

Write a quadratic equation with the given roots. Write the equation in the form $ax^2 + bx + c = 0$.

1. 3, −4

2. −8, −2

3. 1, 9

4. −5

5. 10, 7

6. −2, 15

7. $-\frac{1}{3}, 5$

8. $2, \frac{2}{3}$

9. $-7, \frac{3}{4}$

10. $3, \frac{2}{5}$

11. $-\frac{4}{9}, -1$

12. $9, \frac{1}{6}$

13. $\frac{2}{3}, -\frac{2}{3}$

14. $\frac{5}{4}, -\frac{1}{2}$

15. $\frac{3}{7}, \frac{1}{5}$

16. $-\frac{7}{8}, \frac{7}{2}$

17. $\frac{1}{2}, \frac{3}{4}$

18. $\frac{1}{8}, \frac{1}{6}$

6-3

Skills Practice

Solving Quadratic Equations by Factoring

Solve each equation by factoring.

1. $x^2 = 64$

2. $x^2 - 100 = 0$

3. $x^2 - 3x + 2 = 0$

4. $x^2 - 4x + 3 = 0$

5. $x^2 + 2x - 3 = 0$

6. $x^2 - 3x - 10 = 0$

7. $x^2 - 6x + 5 = 0$

8. $x^2 - 9x = 0$

9. $-x^2 + 6x = 0$

10. $x^2 + 6x + 8 = 0$

11. $x^2 = -5x$

12. $x^2 - 14x + 49 = 0$

13. $x^2 + 6 = 5x$

14. $x^2 + 18x = -81$

15. $x^2 - 4x = 21$

16. $2x^2 + 5x - 3 = 0$

17. $4x^2 + 5x - 6 = 0$

18. $3x^2 - 13x - 10 = 0$

Write a quadratic equation with the given roots. Write the equation in the form $ax^2 + bx + c = 0$, where a, b, and c are integers.

19. 1, 4

20. 6, −9

21. −2, −5

22. 0, 7

23. $-\frac{1}{3}, -3$

24. $-\frac{1}{2}, \frac{3}{4}$

25. Find two consecutive integers whose product is 272.

NAME ______________________________ DATE ____________ PERIOD _____

6-3 Practice

Solving Quadratic Equations by Factoring

Solve each equation by factoring.

1. $x^2 - 4x - 12 = 0$

2. $x^2 - 16x + 64 = 0$

3. $x^2 - 20x + 100 = 0$

4. $x^2 - 6x + 8 = 0$

5. $x^2 + 3x + 2 = 0$

6. $x^2 - 9x + 14 = 0$

7. $x^2 - 4x = 0$

8. $7x^2 = 4x$

9. $x^2 + 25 = 10x$

10. $10x^2 = 9x$

11. $x^2 = 2x + 99$

12. $x^2 + 12x = -36$

13. $5x^2 - 35x + 60 = 0$

14. $36x^2 = 25$

15. $2x^2 - 8x - 90 = 0$

16. $3x^2 + 2x - 1 = 0$

17. $6x^2 = 9x$

18. $3x^2 + 24x + 45 = 0$

19. $15x^2 + 19x + 6 = 0$

20. $3x^2 - 8x = -4$

21. $6x^2 = 5x + 6$

Write a quadratic equation with the given roots. Write the equation in the form $ax^2 + bx + c = 0$, where a, b, and c are integers.

22. 7, 2

23. 0, 3

24. -5, 8

25. -7, -8

26. -6, -3

27. 3, -4

28. 1, $\frac{1}{2}$

29. $\frac{1}{3}$, 2

30. 0, $-\frac{7}{2}$

31. $\frac{1}{3}$, -3

32. 4, $\frac{1}{3}$

33. $-\frac{2}{3}$, $-\frac{4}{5}$

34. NUMBER THEORY Find two consecutive even positive integers whose product is 624.

35. NUMBER THEORY Find two consecutive odd positive integers whose product is 323.

36. GEOMETRY The length of a rectangle is 2 feet more than its width. Find the dimensions of the rectangle if its area is 63 square feet.

37. PHOTOGRAPHY The length and width of a 6-inch by 8-inch photograph are reduced by the same amount to make a new photograph whose area is half that of the original. By how many inches will the dimensions of the photograph have to be reduced?

NAME ____ DATE ____________ PERIOD _____

6-3 Reading to Learn Mathematics

Solving Quadratic Equations by Factoring

Pre-Activity **How is the Zero Product Property used in geometry?**

Read the introduction to Lesson 6-3 at the top of page 301 in your textbook.

What does the expression $x(x + 5)$ mean in this situation?

Reading the Lesson

1. The solution of a quadratic equation by factoring is shown below. Give the reason for each step of the solution.

$x^2 - 10x = -21$	Original equation
$x^2 - 10x + 21 = 0$	______________________
$(x - 3)(x - 7) = 0$	______________________
$x - 3 = 0$ or $x - 7 = 0$	______________________
$x = 3$ $\quad x = 7$	______________________

The solution set is ___________.

2. On an algebra quiz, students were asked to write a quadratic equation with −7 and 5 as its roots. The work that three students in the class wrote on their papers is shown below.

Marla	Rosa	Larry
$(x - 7)(x + 5) = 0$	$(x + 7)(x - 5) = 0$	$(x + 7)(x - 5) = 0$
$x^2 - 2x - 35 = 0$	$x^2 + 2x - 35 = 0$	$x^2 - 2x - 35 = 0$

Who is correct?

Explain the errors in the other two students' work.

Helping You Remember

3. A good way to remember a concept is to represent it in more than one way. Describe an algebraic way and a graphical way to recognize a quadratic equation that has a double root.

Lesson 6-3

NAME ______________________________________ DATE ____________ PERIOD _____

6-3 Enrichment

Euler's Formula for Prime Numbers

Many mathematicians have searched for a formula that would generate prime numbers. One such formula was proposed by Euler and uses a quadratic polynomial, $x^2 + x + 41$.

Find the values of $x^2 + x + 41$ for the given values of x. State whether each value of the polynomial is or is not a prime number.

1. $x = 0$

2. $x = 1$

3. $x = 2$

4. $x = 3$

5. $x = 4$

6. $x = 5$

7. $x = 6$

8. $x = 17$

9. $x = 28$

10. $x = 29$

11. $x = 30$

12. $x = 35$

13. Does the formula produce all prime numbers greater than 40? Give examples in your answer.

14. Euler's formula produces primes for many values of x, but it does not work for all of them. Find the first value of x for which the formula fails. (*Hint:* Try multiples of ten.)

NAME ______________________________ DATE ____________ PERIOD _____

6-4 Study Guide and Intervention

Completing the Square

Square Root Property Use the following property to solve a quadratic equation that is in the form "perfect square trinomial = constant."

Square Root Property	For any real number x if $x^2 = n$, then $x = \pm n$.

Example **Solve each equation by using the Square Root Property.**

a. $\mathbf{x^2 - 8x + 16 = 25}$

$x^2 - 8x + 16 = 25$

$(x - 4)^2 = 25$

$x - 4 = \sqrt{25}$ or $x - 4 = -\sqrt{25}$

$x = 5 + 4 = 9$ or $x = -5 + 4 = -1$

The solution set is $\{9, -1\}$.

b. $\mathbf{4x^2 - 20x + 25 = 32}$

$4x^2 - 20x + 25 = 32$

$(2x - 5)^2 = 32$

$2x - 5 = \sqrt{32}$ or $2x - 5 = -\sqrt{32}$

$2x - 5 = 4\sqrt{2}$ or $2x - 5 = -4\sqrt{2}$

$x = \dfrac{5 \pm 4\sqrt{2}}{2}$

The solution set is $\left\{\dfrac{5 \pm 4\sqrt{2}}{2}\right\}$.

Exercises

Solve each equation by using the Square Root Property.

1. $x^2 - 18x + 81 = 49$

2. $x^2 + 20x + 100 = 64$

3. $4x^2 + 4x + 1 = 16$

4. $36x^2 + 12x + 1 = 18$

5. $9x^2 - 12x + 4 = 4$

6. $25x^2 + 40x + 16 = 28$

7. $4x^2 - 28x + 49 = 64$

8. $16x^2 + 24x + 9 = 81$

9. $100x^2 - 60x + 9 = 121$

10. $25x^2 + 20x + 4 = 75$

11. $36x^2 + 48x + 16 = 12$

12. $25x^2 - 30x + 9 = 96$

Lesson 6-4

NAME ______________________________ DATE ____________ PERIOD _____

6-4 Study Guide and Intervention *(continued)*

Completing the Square

Complete the Square To complete the square for a quadratic expression of the form $x^2 + bx$, follow these steps.

1. Find $\frac{b}{2}$. → **2.** Square $\frac{b}{2}$. → **3.** Add $\left(\frac{b}{2}\right)^2$ to $x^2 + bx$.

Example 1 **Find the value of c that makes $x^2 + 22x + c$ a perfect square trinomial. Then write the trinomial as the square of a binomial.**

Step 1 $b = 22; \frac{b}{2} = 11$

Step 2 $11^2 = 121$

Step 3 $c = 121$

The trinomial is $x^2 + 22x + 121$, which can be written as $(x + 11)^2$.

Example 2 **Solve $2x^2 - 8x - 24 = 0$ by completing the square.**

$2x^2 - 8x - 24 = 0$	Original equation
$\frac{2x^2 - 8x - 24}{2} = \frac{0}{2}$	Divide each side by 2.
$x^2 - 4x - 12 = 0$	$x^2 - 4x - 12$ is not a perfect square.
$x^2 - 4x = 12$	Add 12 to each side.
$x^2 - 4x + 4 = 12 + 4$	Since $\left(-\frac{4}{2}\right)^2 = 4$, add 4 to each side.
$(x - 2)^2 = 16$	Factor the square.
$x - 2 = \pm 4$	Square Root Property
$x = 6$ or $x = -2$	Solve each equation.

The solution set is $\{6, -2\}$.

Exercises

Find the value of c that makes each trinomial a perfect square. Then write the trinomial as a perfect square.

1. $x^2 - 10x + c$

2. $x^2 + 60x + c$

3. $x^2 - 3x + c$

4. $x^2 + 3.2x + c$

5. $x^2 + \frac{1}{2}x + c$

6. $x^2 - 2.5x + c$

Solve each equation by completing the square.

7. $y^2 - 4y - 5 = 0$

8. $x^2 - 8x - 65 = 0$

9. $s^2 - 10s + 21 = 0$

10. $2x^2 - 3x + 1 = 0$

11. $2x^2 - 13x - 7 = 0$

12. $25x^2 + 40x - 9 = 0$

13. $x^2 + 4x + 1 = 0$

14. $y^2 + 12y + 4 = 0$

15. $t^2 + 3t - 8 = 0$

NAME ______________________________ DATE ____________ PERIOD _____

6-4 Skills Practice

Completing the Square

Solve each equation by using the Square Root Property.

1. $x^2 - 8x + 16 = 1$

2. $x^2 + 4x + 4 = 1$

3. $x^2 + 12x + 36 = 25$

4. $4x^2 - 4x + 1 = 9$

5. $x^2 + 4x + 4 = 2$

6. $x^2 - 2x + 1 = 5$

7. $x^2 - 6x + 9 = 7$

8. $x^2 + 16x + 64 = 15$

Find the value of *c* that makes each trinomial a perfect square. Then write the trinomial as a perfect square.

9. $x^2 + 10x + c$

10. $x^2 - 14x + c$

11. $x^2 + 24x + c$

12. $x^2 + 5x + c$

13. $x^2 - 9x + c$

14. $x^2 - x + c$

Solve each equation by completing the square.

15. $x^2 - 13x + 36 = 0$

16. $x^2 + 3x = 0$

17. $x^2 + x - 6 = 0$

18. $x^2 - 4x - 13 = 0$

19. $2x^2 + 7x - 4 = 0$

20. $3x^2 + 2x - 1 = 0$

21. $x^2 + 3x - 6 = 0$

22. $x^2 - x - 3 = 0$

23. $x^2 = -11$

24. $x^2 - 2x + 4 = 0$

NAME ______________________________ DATE ____________ PERIOD _____

6-4 Practice

Completing the Square

Solve each equation by using the Square Root Property.

1. $x^2 + 8x + 16 = 1$

2. $x^2 + 6x + 9 = 1$

3. $x^2 + 10x + 25 = 16$

4. $x^2 - 14x + 49 = 9$

5. $4x^2 + 12x + 9 = 4$

6. $x^2 - 8x + 16 = 8$

7. $x^2 - 6x + 9 = 5$

8. $x^2 - 2x + 1 = 2$

9. $9x^2 - 6x + 1 = 2$

Find the value of *c* that makes each trinomial a perfect square. Then write the trinomial as a perfect square.

10. $x^2 + 12x + c$

11. $x^2 - 20x + c$

12. $x^2 + 11x + c$

13. $x^2 + 0.8x + c$

14. $x^2 - 2.2x + c$

15. $x^2 - 0.36x + c$

16. $x^2 + \frac{5}{6}x + c$

17. $x^2 - \frac{1}{4}x + c$

18. $x^2 - \frac{5}{3}x + c$

Solve each equation by completing the square.

19. $x^2 + 6x + 8 = 0$

20. $3x^2 + x - 2 = 0$

21. $3x^2 - 5x + 2 = 0$

22. $x^2 + 18 = 9x$

23. $x^2 - 14x + 19 = 0$

24. $x^2 + 16x - 7 = 0$

25. $2x^2 + 8x - 3 = 0$

26. $x^2 + x - 5 = 0$

27. $2x^2 - 10x + 5 = 0$

28. $x^2 + 3x + 6 = 0$

29. $2x^2 + 5x + 6 = 0$

30. $7x^2 + 6x + 2 = 0$

31. GEOMETRY When the dimensions of a cube are reduced by 4 inches on each side, the surface area of the new cube is 864 square inches. What were the dimensions of the original cube?

32. INVESTMENTS The amount of money A in an account in which P dollars is invested for 2 years is given by the formula $A = P(1 + r)^2$, where r is the interest rate compounded annually. If an investment of \$800 in the account grows to \$882 in two years, at what interest rate was it invested?

NAME ______________________________ DATE ____________ PERIOD _____

6-4 Reading to Learn Mathematics

Completing the Square

Pre-Activity **How can you find the time it takes an accelerating race car to reach the finish line?**

Read the introduction to Lesson 6-4 at the top of page 306 in your textbook.

Explain what it means to say that the driver accelerates at a constant rate of 8 feet per second square.

Reading the Lesson

1. Give the reason for each step in the following solution of an equation by using the Square Root Property.

$x^2 - 12x + 36 = 81$	Original equation
$(x - 6)^2 = 81$	______________________
$x - 6 = \pm\sqrt{81}$	______________________
$x - 6 = \pm 9$	______________________
$x - 6 = 9$ or $x - 6 = -9$	______________________
$x = 15 \qquad x = -3$	______________________

2. Explain how to find the constant that must be added to make a binomial into a perfect square trinomial.

3. a. What is the first step in solving the equation $3x^2 + 6x = 5$ by completing the square?

b. What is the first step in solving the equation $x^2 + 5x - 12 = 0$ by completing the square?

Lesson 6-4

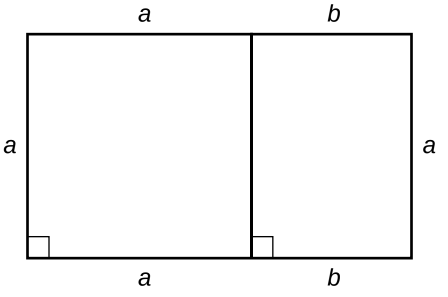
Helping You Remember

4. How can you use the rules for squaring a binomial to help you remember the procedure for changing a binomial into a perfect square trinomial?

6-4 Enrichment

The Golden Quadratic Equations

A **golden rectangle** has the property that its length can be written as $a + b$, where a is the width of the rectangle and $\frac{a + b}{a} = \frac{a}{b}$. Any golden rectangle can be divided into a square and a smaller golden rectangle, as shown.



The proportion used to define golden rectangles can be used to derive two quadratic equations. These are sometimes called *golden quadratic equations*.

Solve each problem.

1. In the proportion for the golden rectangle, let a equal 1. Write the resulting quadratic equation and solve for b.

2. In the proportion, let b equal 1. Write the resulting quadratic equation and solve for a.

3. Describe the difference between the two golden quadratic equations you found in exercises 1 and 2.

4. Show that the positive solutions of the two equations in exercises 1 and 2 are reciprocals.

5. Use the Pythagorean Theorem to find a radical expression for the diagonal of a golden rectangle when $a = 1$.

6. Find a radical expression for the diagonal of a golden rectangle when $b = 1$.

6-5 Study Guide and Intervention

The Quadratic Formula and the Discriminant

Quadratic Formula The **Quadratic Formula** can be used to solve *any* quadratic equation once it is written in the form $ax^2 + bx + c = 0$.

Quadratic Formula	The solutions of $ax^2 + bx + c = 0$, with $a \neq 0$, are given by $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Example **Solve $x^2 - 5x = 14$ by using the Quadratic Formula.**

Rewrite the equation as $x^2 - 5x - 14 = 0$.

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ Quadratic Formula

$= \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(-14)}}{2(1)}$ Replace *a* with 1, *b* with −5, and *c* with −14.

$= \frac{5 \pm \sqrt{81}}{2}$ Simplify.

$= \frac{5 \pm 9}{2}$

$= 7$ or -2

The solutions are −2 and 7.

Exercises

Solve each equation by using the Quadratic Formula.

1. $x^2 + 2x - 35 = 0$

2. $x^2 + 10x + 24 = 0$

3. $x^2 - 11x + 24 = 0$

4. $4x^2 + 19x - 5 = 0$

5. $14x^2 + 9x + 1 = 0$

6. $2x^2 - x - 15 = 0$

7. $3x^2 + 5x = 2$

8. $2y^2 + y - 15 = 0$

9. $3x^2 - 16x + 16 = 0$

10. $8x^2 + 6x - 9 = 0$

11. $r^2 - \frac{3r}{5} + \frac{2}{25} = 0$

12. $x^2 - 10x - 50 = 0$

13. $x^2 + 6x - 23 = 0$

14. $4x^2 - 12x - 63 = 0$

15. $x^2 - 6x + 21 = 0$

Lesson 6-5

6-5 Study Guide and Intervention *(continued)*

The Quadratic Formula and the Discriminant

Roots and the Discriminant

Discriminant	The expression under the radical sign, $b^2 - 4ac$, in the Quadratic Formula is called the **discriminant**.

Roots of a Quadratic Equation

Discriminant	Type and Number of Roots
$b^2 - 4ac > 0$ and a perfect square	2 rational roots
$b^2 - 4ac > 0$, but **not** a perfect square	2 irrational roots
$b^2 - 4ac = 0$	1 rational root
$b^2 - 4ac < 0$	2 complex roots

Example **Find the value of the discriminant for each equation. Then describe the number and types of roots for the equation.**

a. $2x^2 + 5x + 3$

The discriminant is
$b^2 - 4ac = 5^2 - 4(2)(3)$ or 1.
The discriminant is a perfect square, so the equation has 2 rational roots.

b. $3x^2 - 2x + 5$

The discriminant is
$b^2 - 4ac = (-2)^2 - 4(3)(5)$ or -56.
The discriminant is negative, so the equation has 2 complex roots.

Exercises

For Exercises 1–12, complete parts a–c for each quadratic equation.
a. Find the value of the discriminant.
b. Describe the number and type of roots.
c. Find the exact solutions by using the Quadratic Formula.

1. $p^2 + 12p = -4$

2. $9x^2 - 6x + 1 = 0$

3. $2x^2 - 7x - 4 = 0$

4. $x^2 + 4x - 4 = 0$

5. $5x^2 - 36x + 7 = 0$

6. $4x^2 - 4x + 11 = 0$

7. $x^2 - 7x + 6 = 0$

8. $m^2 - 8m = -14$

9. $25x^2 - 40x = -16$

10. $4x^2 + 20x + 29 = 0$

11. $6x^2 + 26x + 8 = 0$

12. $4x^2 - 4x - 11 = 0$

NAME ______________________________ DATE ____________ PERIOD _____

6-5 Skills Practice

The Quadratic Formula and the Discriminant

Complete parts a–c for each quadratic equation.

a. Find the value of the discriminant.

b. Describe the number and type of roots.

c. Find the exact solutions by using the Quadratic Formula.

1. $x^2 - 8x + 16 = 0$

2. $x^2 - 11x - 26 = 0$

3. $3x^2 - 2x = 0$

4. $20x^2 + 7x - 3 = 0$

5. $5x^2 - 6 = 0$

6. $x^2 - 6 = 0$

7. $x^2 + 8x + 13 = 0$

8. $5x^2 - x - 1 = 0$

9. $x^2 - 2x - 17 = 0$

10. $x^2 + 49 = 0$

11. $x^2 - x + 1 = 0$

12. $2x^2 - 3x = -2$

Solve each equation by using the method of your choice. Find exact solutions.

13. $x^2 = 64$

14. $x^2 - 30 = 0$

15. $x^2 - x = 30$

16. $16x^2 - 24x - 27 = 0$

17. $x^2 - 4x - 11 = 0$

18. $x^2 - 8x - 17 = 0$

19. $x^2 + 25 = 0$

20. $3x^2 + 36 = 0$

21. $2x^2 + 10x + 11 = 0$

22. $2x^2 - 7x + 4 = 0$

23. $8x^2 + 1 = 4x$

24. $2x^2 + 2x + 3 = 0$

25. **PARACHUTING** Ignoring wind resistance, the distance $d(t)$ in feet that a parachutist falls in t seconds can be estimated using the formula $d(t) = 16t^2$. If a parachutist jumps from an airplane and falls for 1100 feet before opening her parachute, how many seconds pass before she opens the parachute?

Lesson 6-5

NAME ____ DATE ____________ PERIOD _____

6-5 Practice

The Quadratic Formula and the Discriminant

Complete parts a–c for each quadratic equation.

a. Find the value of the discriminant.

b. Describe the number and type of roots.

c. Find the exact solutions by using the Quadratic Formula.

1. $x^2 - 16x + 64 = 0$

2. $x^2 = 3x$

3. $9x^2 - 24x + 16 = 0$

4. $x^2 - 3x = 40$

5. $3x^2 + 9x - 2 = 0$

6. $2x^2 + 7x = 0$

7. $5x^2 - 2x + 4 = 0$

8. $12x^2 - x - 6 = 0$

9. $7x^2 + 6x + 2 = 0$

10. $12x^2 + 2x - 4 = 0$

11. $6x^2 - 2x - 1 = 0$

12. $x^2 + 3x + 6 = 0$

13. $4x^2 - 3x^2 - 6 = 0$

14. $16x^2 - 8x + 1 = 0$

15. $2x^2 - 5x - 6 = 0$

Solve each equation by using the method of your choice. Find exact solutions.

16. $7x^2 - 5x = 0$

17. $4x^2 - 9 = 0$

18. $3x^2 + 8x = 3$

19. $x^2 - 21 = 4x$

20. $3x^2 - 13x + 4 = 0$

21. $15x^2 + 22x = -8$

22. $x^2 - 6x + 3 = 0$

23. $x^2 - 14x + 53 = 0$

24. $3x^2 = -54$

25. $25x^2 - 20x - 6 = 0$

26. $4x^2 - 4x + 17 = 0$

27. $8x - 1 = 4x^2$

28. $x^2 = 4x - 15$

29. $4x^2 - 12x + 7 = 0$

30. GRAVITATION The height $h(t)$ in feet of an object t seconds after it is propelled straight up from the ground with an initial velocity of 60 feet per second is modeled by the equation $h(t) = -16t^2 + 60t$. At what times will the object be at a height of 56 feet?

31. STOPPING DISTANCE The formula $d = 0.05s^2 + 1.1s$ estimates the minimum stopping distance d in feet for a car traveling s miles per hour. If a car stops in 200 feet, what is the fastest it could have been traveling when the driver applied the brakes?

NAME ______________________________ DATE ____________ PERIOD _____

6-5 Reading to Learn Mathematics

The Quadratic Formula and the Discriminant

Pre-Activity **How is blood pressure related to age?**

Read the introduction to Lesson 6-5 at the top of page 313 in your textbook.

Describe how you would calculate your normal blood pressure using one of the formulas in your textbook.

Reading the Lesson

1. a. Write the Quadratic Formula.

b. Identify the values of a, b, and c that you would use to solve $2x^2 - 5x = -7$, but do not actually solve the equation.

a = ____________ b = ____________ c = ____________

2. Suppose that you are solving four quadratic equations with rational coefficients and have found the value of the discriminant for each equation. In each case, give the number of roots and describe the type of roots that the equation will have.

Value of Discriminant	Number of Roots	Type of Roots
64		
−8		
21		
0		

Helping You Remember

3. How can looking at the Quadratic Formula help you remember the relationships between the value of the discriminant and the number of roots of a quadratic equation and whether the roots are real or complex?

Lesson 6-5

6-5 Enrichment

Sum and Product of Roots

Sometimes you may know the roots of a quadratic equation without knowing the equation itself. Using your knowledge of factoring to solve an equation, you can work backward to find the quadratic equation. The rule for finding the sum and product of roots is as follows:

Sum and Product of Roots	If the roots of $ax^2 + bx + c = 0$, with $a \neq 0$, are s_1 and s_2, then $s_1 + s_2 = -\frac{b}{a}$ and $s_1 \cdot s_2 = \frac{c}{a}$.

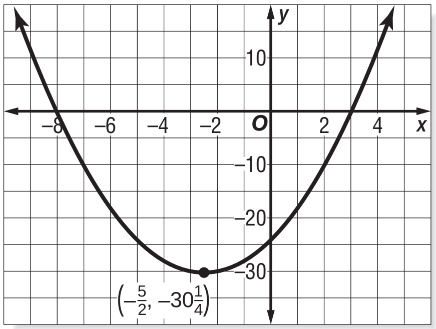
Example **A road with an initial gradient, or slope, of 3% can be represented by the formula $y = ax^2 + 0.03x + c$, where y is the elevation and x is the distance along the curve. Suppose the elevation of the road is 1105 feet at points 200 feet and 1000 feet along the curve. You can find the equation of the transition curve. Equations of transition curves are used by civil engineers to design smooth and safe roads.**

The roots are $x = 3$ and $x = -8$.

$3 + (-8) = -5$ Add the roots.

$3(-8) = -24$ Multiply the roots.

Equation: $x^2 + 5x - 24 = 0$



Write a quadratic equation that has the given roots.

1. 6, -9

2. 5, -1

3. 6, 6

4. $4 \pm \sqrt{3}$

6. $-\frac{2}{5}, \frac{2}{7}$

6. $\frac{-2 \pm 3\sqrt{5}}{7}$

Find k such that the number given is a root of the equation.

7. 7; $2x^2 + kx - 21 = 0$

8. -2; $x^2 - 13x + k = 0$

6-6 Study Guide and Intervention

Analyzing Graphs of Quadratic Functions

Analyze Quadratic Functions

Vertex Form of a Quadratic Function	The graph of $y = a(x - h)^2 + k$ has the following characteristics: • Vertex: (h, k) • Axis of symmetry: $x = h$ • Opens up if $a > 0$ • Opens down if $a < 0$ • Narrower than the graph of $y = x^2$ if $\lvert a \rvert > 1$ • Wider than the graph of $y = x^2$ if $\lvert a \rvert < 1$

Example **Identify the vertex, axis of symmetry, and direction of opening of each graph.**

a. $y = 2(x + 4)^2 - 11$

The vertex is at (h, k) or $(-4, -11)$, and the axis of symmetry is $x = -4$. The graph opens up, and is narrower than the graph of $y = x^2$.

a. $y = -\frac{1}{4}(x - 2)^2 + 10$

The vertex is at (h, k) or $(2, 10)$, and the axis of symmetry is $x = 2$. The graph opens down, and is wider than the graph of $y = x^2$.

Exercises

Each quadratic function is given in vertex form. Identify the vertex, axis of symmetry, and direction of opening of the graph.

1. $y = (x - 2)^2 + 16$

2. $y = 4(x + 3)^2 - 7$

3. $y = \frac{1}{2}(x - 5)^2 + 3$

4. $y = -7(x + 1)^2 - 9$

5. $y = \frac{1}{5}(x - 4)^2 - 12$

6. $y = 6(x + 6)^2 + 6$

7. $y = \frac{2}{5}(x - 9)^2 + 12$

8. $y = 8(x - 3)^2 - 2$

9. $y = -3(x - 1)^2 - 2$

10. $y = -\frac{5}{2}(x + 5)^2 + 12$

11. $y = \frac{4}{3}(x - 7)^2 + 22$

12. $y = 16(x - 4)^2 + 1$

13. $y = 3(x - 1.2)^2 + 2.7$

14. $y = -0.4(x - 0.6)^2 - 0.2$

15. $y = 1.2(x + 0.8)^2 + 6.5$

6-6 Study Guide and Intervention *(continued)*

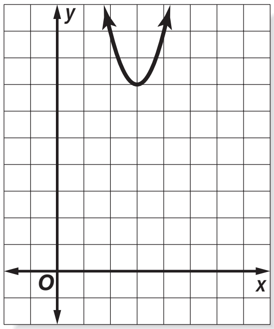
Analyzing Graphs of Quadratic Functions

Write Quadratic Functions in Vertex Form A quadratic function is easier to graph when it is in vertex form. You can write a quadratic function of the form $y = ax^2 + bx + c$ in vertex from by completing the square.

Example **Write $y = 2x^2 - 12x + 25$ in vertex form. Then graph the function.**

$y = 2x^2 - 12x + 25$
$y = 2(x^2 - 6x) + 25$
$y = 2(x^2 - 6x + 9) + 25 - 18$
$y = 2(x - 3)^2 + 7$

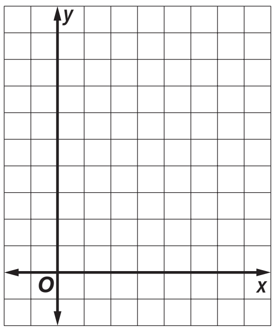
The vertex form of the equation is $y = 2(x - 3)^2 + 7$.



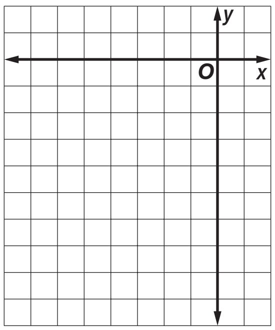
Exercises

Write each quadratic function in vertex form. Then graph the function.

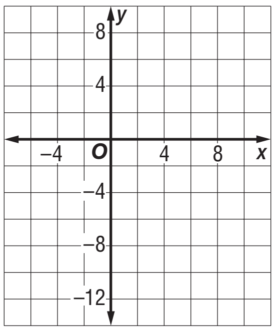
1. $y = x^2 - 10x + 32$



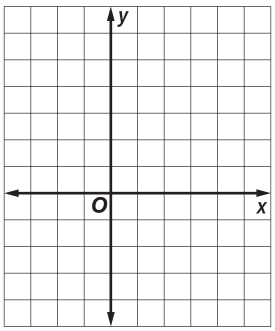
2. $y = x^2 + 6x$



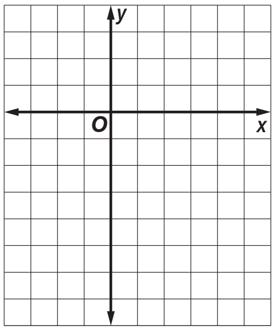
3. $y = x^2 - 8x + 6$



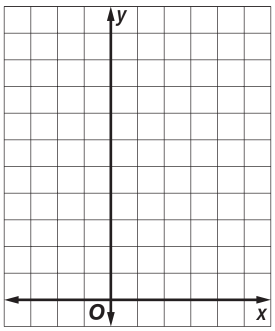
4. $y = -4x^2 + 16x - 11$



5. $y = 3x^2 - 12x + 5$



6. $y = 5x^2 - 10x + 9$



6-6 Skills Practice

Analyzing Graphs of Quadratic Functions

Write each quadratic function in vertex form, if not already in that form. Then identify the vertex, axis of symmetry, and direction of opening.

1. $y = (x - 2)^2$

2. $y = -x^2 + 4$

3. $y = x^2 - 6$

4. $y = -3(x + 5)^2$

5. $y = -5x^2 + 9$

6. $y = (x - 2)^2 - 18$

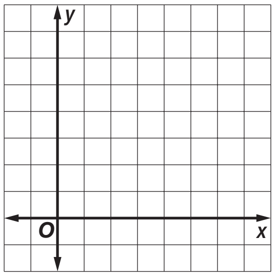
7. $y = x^2 - 2x - 5$

8. $y = x^2 + 6x + 2$

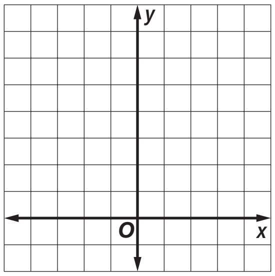
9. $y = -3x^2 + 24x$

Graph each function.

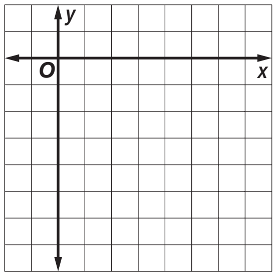
10. $y = (x - 3)^2 - 1$



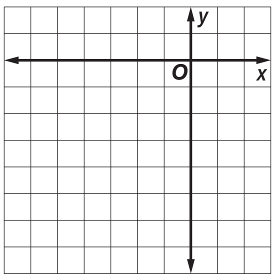
11. $y = (x + 1)^2 + 2$



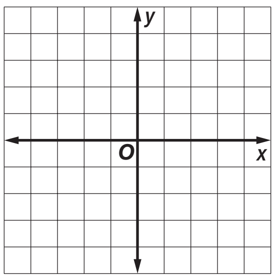
12. $y = -(x - 4)^2 - 4$



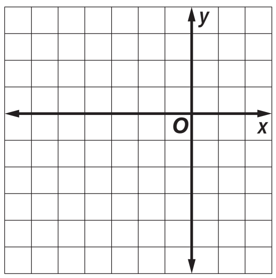
13. $y = -\frac{1}{2}(x + 2)^2$



14. $y = -3x^2 + 4$



15. $y = x^2 + 6x + 4$



Write an equation for the parabola with the given vertex that passes through the given point.

16. vertex: $(4, -36)$
point: $(0, -20)$

17. vertex: $(3, -1)$
point: $(2, 0)$

18. vertex: $(-2, 2)$
point: $(-1, 3)$

6-6 Practice

Analyzing Graphs of Quadratic Functions

Write each quadratic function in vertex form, if not already in that form. Then identify the vertex, axis of symmetry, and direction of opening.

1. $y = -6(x + 2)^2 - 1$

2. $y = 2x^2 + 2$

3. $y = -4x^2 + 8x$

4. $y = x^2 + 10x + 20$

5. $y = 2x^2 + 12x + 18$

6. $y = 3x^2 - 6x + 5$

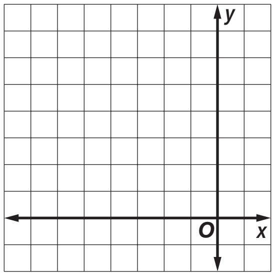
7. $y = -2x^2 - 16x - 32$

8. $y = -3x^2 + 18x - 21$

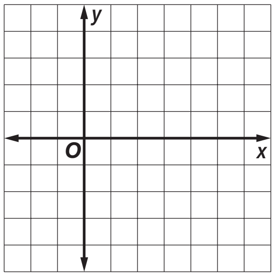
9. $y = 2x^2 + 16x + 29$

Graph each function.

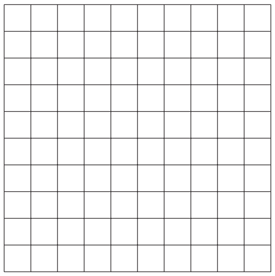
10. $y = (x + 3)^2 - 1$



11. $y = -x^2 + 6x - 5$



12. $y = 2x^2 - 2x + 1$



Write an equation for the parabola with the given vertex that passes through the given point.

13. vertex: (1, 3)
point: (−2, −15)

14. vertex: (−3, 0)
point: (3, 18)

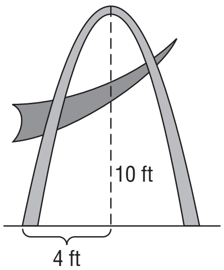
15. vertex: (10, −4)
point: (5, 6)

16. Write an equation for a parabola with vertex at (4, 4) and x-intercept 6.

17. Write an equation for a parabola with vertex at (−3, −1) and y-intercept 2.

18. BASEBALL The height h of a baseball t seconds after being hit is given by $h(t) = -16t^2 + 80t + 3$. What is the maximum height that the baseball reaches, and when does this occur?

19. SCULPTURE A modern sculpture in a park contains a parabolic arc that starts at the ground and reaches a maximum height of 10 feet after a horizontal distance of 4 feet. Write a quadratic function in vertex form that describes the shape of the outside of the arc, where y is the height of a point on the arc and x is its horizontal distance from the left-hand starting point of the arc.



NAME ______________________________ DATE __________ PERIOD _____

6-6 Reading to Learn Mathematics

Analyzing Graphs of Quadratic Equations

Pre-Activity **How can the graph of $y = x^2$ be used to graph any quadratic function?**

Read the introduction to Lesson 6-6 at the top of page 322 in your textbook.

- What does adding a positive number to x^2 do to the graph of $y = x^2$?

- What does subtracting a positive number to x before squaring do to the graph of $y = x^2$?

Reading the Lesson

1. Complete the following information about the graph of $y = a(x - h)^2 + k$.

a. What are the coordinates of the vertex?

b. What is the equation of the axis of symmetry?

c. In which direction does the graph open if $a > 0$? If $a < 0$?

d. What do you know about the graph if $|a| < 1$?

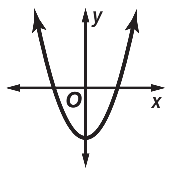
If $|a| > 1$?

2. Match each graph with the description of the constants in the equation in vertex form.

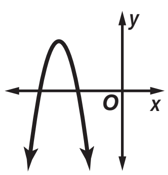
a. $a > 0, h > 0, k < 0$ **b.** $a < 0, h < 0, k < 0$

c. $a < 0, h < 0, k > 0$ **d.** $a > 0, h = 0, k < 0$

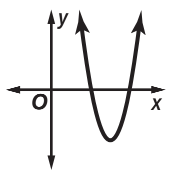
i.



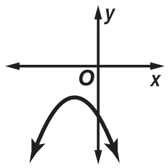
ii.



iii.



iv.



Helping You Remember

3. When graphing quadratic functions such as $y = (x + 4)^2$ and $y = (x - 5)^2$, many students have trouble remembering which represents a translation of the graph of $y = x^2$ to the left and which represents a translation to the right. What is an easy way to remember this?

NAME ______________________________________ DATE ____________ PERIOD _____

6-6 Enrichment

Patterns with Differences and Sums of Squares

Some whole numbers can be written as the difference of two squares and some cannot. Formulas can be developed to describe the sets of numbers algebraically.

If possible, write each number as the difference of two squares. Look for patterns.

1. 0 $0^2 - 0^2$ **2.** 1 $1^2 - 0^2$ **3.** 2 cannot **4.** 3 $2^2 - 1^2$

5. 4 $2^2 - 0^2$ **6.** 5 $3^2 - 2^2$ **7.** 6 cannot **8.** 7 $4^2 - 3^2$

9. 8 $3^2 - 1^2$ **10.** 9 $3^2 - 0^2$ **11.** 10 cannot **12.** 11 $6^2 - 5^2$

13. 12 $4^2 - 2^2$ **14.** 13 $7^2 - 6^2$ **15.** 14 cannot **16.** 15 $4^2 - 1^2$

Even numbers can be written as $2n$, where n is one of the numbers 0, 1, 2, 3, and so on. Odd numbers can be written $2n + 1$. Use these expressions for these problems.

17. Show that any odd number can be written as the difference of two squares.
$2n + 1 = (n + 1)^2 - n^2$

18. Show that the even numbers can be divided into two sets: those that can be written in the form $4n$ and those that can be written in the form $2 + 4n$.
Find $4n$ for $n = 0, 1, 2$, and so on. You get {0, 4, 8, 12, …}. For $2 + 4n$, you get {2, 6, 10, 12, …}. Together these sets include all even numbers.

19. Describe the even numbers that cannot be written as the difference of two squares. $2 + 4n$, for $n = 0, 1, 2, 3, \ldots$

20. Show that the other even numbers can be written as the difference of two squares. $4n = (n + 1)^2 - (n - 1)^2$

Every whole number can be written as the sum of squares. It is never necessary to use more than four squares. Show that this is true for the whole numbers from 0 through 15 by writing each one as the sum of the least number of squares.

21. 0 0^2 **22.** 1 1^2 **23.** 2 $1^2 + 1^2$

24. 3 $1^2 + 1^2 + 1^2$ **25.** 4 2^2 **26.** 5 $1^2 + 2^2$

27. 6 $1^2 + 1^2 + 2^2$ **28.** 7 $1^2 + 1^2 + 1^2 + 2^2$ **29.** 8 $2^2 + 2^2$

30. 9 3^2 **31.** 10 $1^2 + 3^2$ **32.** 11 $1^2 + 1^2 + 3^2$

33. 12 $1^2 + 1^2 + 1^2 + 3^2$ **34.** 13 $2^2 + 3^2$ **35.** 14 $1^2 + 2^2 + 3^2$

36. 15 $1^2 + 1^2 + 2^2 + 3^2$

NAME ______________________________ DATE __________ PERIOD _____

6-7 Study Guide and Intervention

Graphing and Solving Quadratic Inequalities

Lesson 6-7

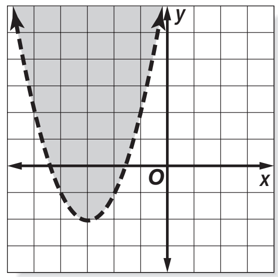
Graph Quadratic Inequalities To graph a quadratic inequality in two variables, use the following steps:

1. Graph the related quadratic equation, $y = ax^2 + bx + c$. Use a dashed line for $<$ or $>$; use a solid line for $\leq$ or $\geq$.
2. Test a point inside the parabola. If it satisfies the inequality, shade the region inside the parabola; otherwise, shade the region outside the parabola.

Example **Graph the inequality $y > x^2 + 6x + 7$.**

First graph the equation $y = x^2 + 6x + 7$. By completing the square, you get the vertex form of the equation $y = (x + 3)^2 - 2$, so the vertex is $(-3, -2)$. Make a table of values around $x = -3$, and graph. Since the inequality includes $>$, use a dashed line.

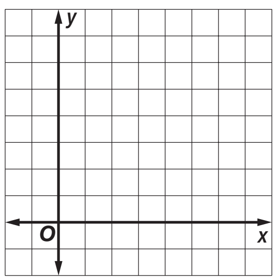
Test the point $(-3, 0)$, which is inside the parabola. Since $(-3)^2 + 6(-3) + 7 = -2$, and $0 > -2$, $(-3, 0)$ satisfies the inequality. Therefore, shade the region inside the parabola.



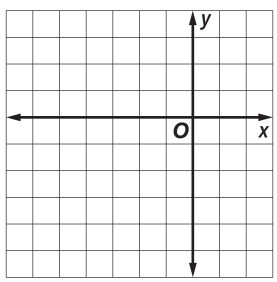
Exercises

Graph each inequality.

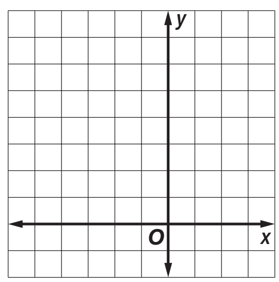
1. $y > x^2 - 8x + 17$



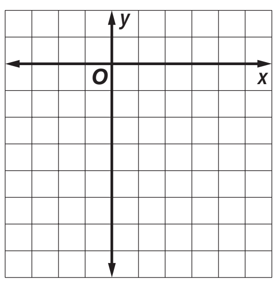
2. $y \leq x^2 + 6x + 4$



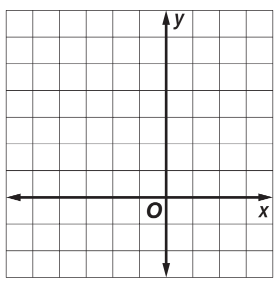
3. $y \geq x^2 + 2x + 2$



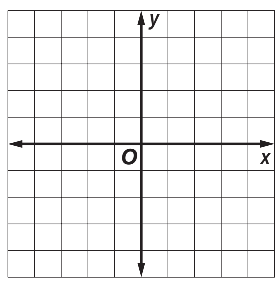
4. $y < -x^2 + 4x - 6$



5. $y \geq 2x^2 + 4x$



6. $y > -2x^2 - 4x + 2$



6-7 Study Guide and Intervention *(continued)*

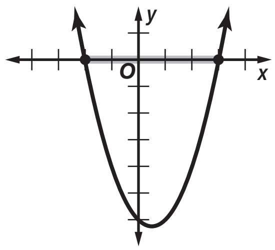
Graphing and Solving Quadratic Inequalities

Solve Quadratic Inequalities Quadratic inequalities in one variable can be solved graphically or algebraically.

Graphical Method	To solve $ax^2 + bx + c < 0$: First graph $y = ax^2 + bx + c$. The solution consists of the x-values for which the graph is **below** the x-axis. To solve $ax^2 + bx + c > 0$: First graph $y = ax^2 + bx + c$. The solution consists the x-values for which the graph is **above** the x-axis.
Algebraic Method	Find the roots of the related quadratic equation by factoring, completing the square, or using the Quadratic Formula. 2 roots divide the number line into 3 intervals. Test a value in each interval to see which intervals are solutions.

If the inequality involves $\leq$ or $\geq$, the roots of the related equation are included in the solution set.

Example **Solve the inequality $x^2 - x - 6 \leq 0$.**



First find the roots of the related equation $x^2 - x - 6 = 0$. The equation factors as $(x - 3)(x + 2) = 0$, so the roots are 3 and -2. The graph opens up with x-intercepts 3 and -2, so it must be on or below the x-axis for $-2 \leq x \leq 3$. Therefore the solution set is $\{x \mid -2 \leq x \leq 3\}$.

Exercises

Solve each inequality.

1. $x^2 + 2x < 0$

2. $x^2 - 16 < 0$

3. $0 < 6x - x^2 - 5$

4. $c^2 \leq 4$

5. $2m^2 - m < 1$

6. $y^2 < -8$

7. $x^2 - 4x - 12 < 0$

8. $x^2 + 9x + 14 > 0$

9. $-x^2 + 7x - 10 \geq 0$

10. $2x^2 + 5x - 3 \leq 0$

11. $4x^2 - 23x + 15 > 0$

12. $-6x^2 - 11x + 2 < 0$

13. $2x^2 - 11x + 12 \geq 0$

14. $x^2 - 4x + 5 < 0$

15. $3x^2 - 16x + 5 < 0$

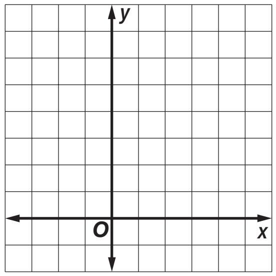
NAME ______________________________ DATE __________ PERIOD _____

6-7 Skills Practice

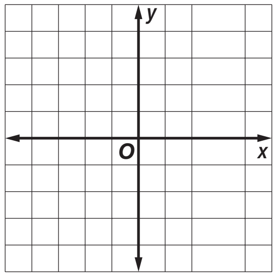
Graphing and Solving Quadratic Inequalities

Graph each inequality.

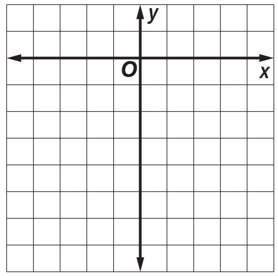
1. $y \geq x^2 - 4x + 4$



2. $y \leq x^2 - 4$

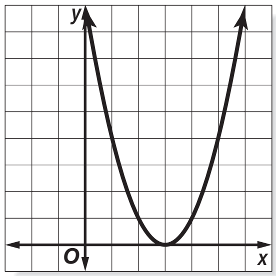


3. $y > x^2 + 2x - 5$

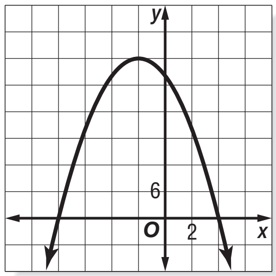


Use the graph of its related function to write the solutions of each inequality.

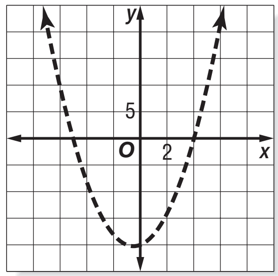
4. $x^2 - 6x + 9 \leq 0$



5. $-x^2 - 4x + 32 \geq 0$



6. $x^2 + x - 20 > 0$



Solve each inequality algebraically.

7. $x^2 - 3x - 10 < 0$

8. $x^2 + 2x - 35 \geq 0$

9. $x^2 - 18x + 81 \leq 0$

10. $x^2 \leq 36$

11. $x^2 - 7x > 0$

12. $x^2 + 7x + 6 < 0$

13. $x^2 + x - 12 > 0$

14. $x^2 + 9x + 18 \leq 0$

15. $x^2 - 10x + 25 \geq 0$

16. $-x^2 - 2x + 15 \geq 0$

17. $x^2 + 3x > 0$

18. $2x^2 + 2x > 4$

19. $-x^2 - 64 \leq -16x$

20. $9x^2 + 12x + 9 < 0$

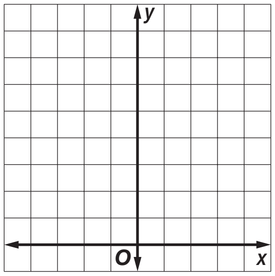
NAME ______________________________ DATE __________ PERIOD _____

6-7 Practice

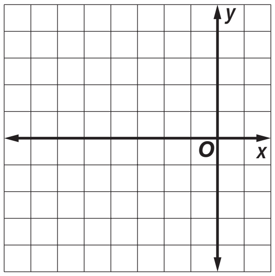
Graphing and Solving Quadratic Inequalities

Graph each inequality.

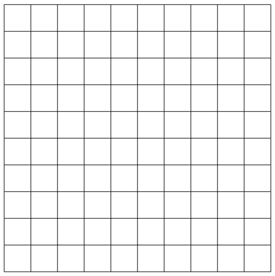
1. $y \leq x^2 + 4$



2. $y > x^2 + 6x + 6$

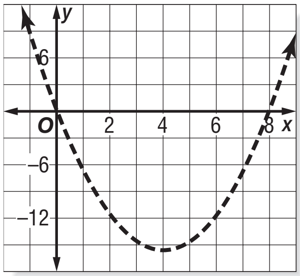


3. $y < 2x^2 - 4x - 2$

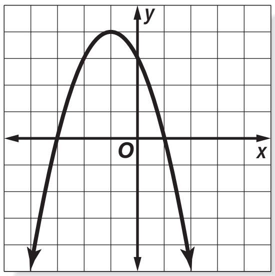


Use the graph of its related function to write the solutions of each inequality.

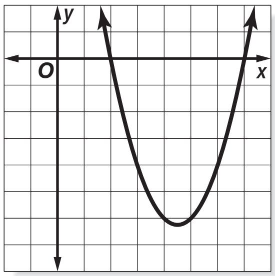
4. $x^2 - 8x > 0$



5. $-x^2 - 2x + 3 \geq 0$



6. $x^2 - 9x + 14 \leq 0$



Solve each inequality algebraically.

7. $x^2 - x - 20 > 0$

8. $x^2 - 10x + 16 < 0$

9. $x^2 + 4x + 5 \leq 0$

10. $x^2 + 14x + 49 \geq 0$

11. $x^2 - 5x > 14$

12. $-x^2 - 15 \geq 8x$

13. $-x^2 + 5x - 7 \leq 0$

14. $9x^2 + 36x + 36 \leq 0$

15. $9x \leq 12x^2$

16. $4x^2 + 4x + 1 > 0$

17. $5x^2 + 10 \geq 27x$

18. $9x^2 + 31x + 12 \leq 0$

19. FENCING Vanessa has 180 feet of fencing that she intends to use to build a rectangular play area for her dog. She wants the play area to enclose at least 1800 square feet. What are the possible widths of the play area?

20. BUSINESS A bicycle maker sold 300 bicycles last year at a profit of \$300 each. The maker wants to increase the profit margin this year, but predicts that each \$20 increase in profit will reduce the number of bicycles sold by 10. How many \$20 increases in profit can the maker add in and expect to make a total profit of at least \$100,000?

NAME ______________________________ DATE ____________ PERIOD _____

6-7 Reading to Learn Mathematics

Graphing and Solving Quadratic Inequalities

Pre-Activity **How can you find the time a trampolinist spends above a certain height?**

Read the introduction to Lesson 6-7 at the top of page 329 in your textbook.

- How far above the ground is the trampoline surface?
- Using the quadratic function given in the introduction, write a quadratic inequality that describes the times at which the trampolinist is more than 20 feet above the ground.

Lesson 6-7

Reading the Lesson

1. Answer the following questions about how you would graph the inequality $y \geq x^2 + x - 6$.

a. What is the related quadratic equation?

b. Should the parabola be solid or dashed? How do you know?

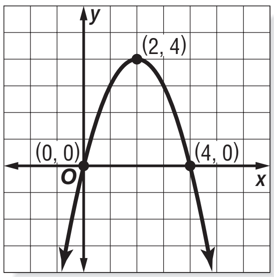
c. The point (0, 2) is inside the parabola. To use this as a test point, substitute

__________ for x and __________ for y in the quadratic inequality.

d. Is the statement $2 \geq 0^2 + 0 - 6$ true or false?

e. Should the region inside or outside the parabola be shaded?

2. The graph of $y = -x^2 + 4x$ is shown at the right. Match each of the following related inequalities with its solution set.



a. $-x^2 + 4x > 0$ **i.** $\{x \mid x < 0 \text{ or } x > 4\}$

b. $-x^2 + 4x \leq 0$ **ii.** $\{x \mid 0 < x < 4\}$

c. $-x^2 + 4x \geq 0$ **iii.** $\{x \mid x \leq 0 \text{ or } x \geq 4\}$

d. $-x^2 + 4x < 0$ **iv.** $\{x \mid 0 \leq x \leq 4\}$

Helping You Remember

3. A quadratic inequality in two variables may have the form $y > ax^2 + bx + c$, $y < ax^2 + bx + c$, $y \geq ax^2 + bx + c$, or $y \leq ax^2 + bx + c$. Describe a way to remember which region to shade by looking at the inequality symbol and without using a test point.

NAME ____ DATE ____________ PERIOD _____

6-7 Enrichment

Graphing Absolute Value Inequalities

You can solve absolute value inequalities by graphing in much the same manner you graphed quadratic inequalities. Graph the related absolute function for each inequality by using a graphing calculator. For $>$ and $\geq$, identify the x-values, if any, for which the graph lies *below* the x-axis. For $<$ and $\leq$, identify the x values, if any, for which the graph lies *above* the x-axis.

For each inequality, make a sketch of the related graph and find the solutions rounded to the nearest hundredth.

1. $|x - 3| > 0$

2. $|x| - 6 < 0$

3. $-|x + 4| + 8 < 0$

4. $2|x + 6| - 2 \geq 0$

5. $|3x - 3| \geq 0$

6. $|x - 7| < 5$

7. $|7x - 1| > 13$

8. $|x - 3.6| \leq 4.2$

9. $|2x + 5| \leq 7$

NAME ______________________ DATE ____________ PERIOD ______

6 Chapter 6 Test, Form 1

SCORE ______

Write the letter for the correct answer in the blank at the right of each question.

1. Find the y-intercept for $f(x) = -(x + 1)^2$.

A. 1 **B.** -1 **C.** x **D.** 0 **1.** ______

2. What is the equation of the axis of symmetry of $y = -3(x + 6)^2 + 12$?

A. $x = 2$ **B.** $x = -6$ **C.** $x = 6$ **D.** $x = -18$ **2.** ______

3. Find the minimum value of $f(x) = x^2 - 6x$.

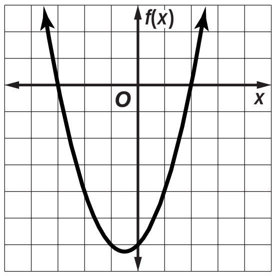
A. 3 **B.** -6 **C.** -9 **D.** 27 **3.** ______

4. The graph of $f(x) = -2x^2 + x$ opens ______ and has a ______ value.

A. down; maximum **B.** down; minimum

C. up; maximum **D.** up; minimum **4.** ______

5. The related graph of a quadratic equation is shown at the right. Use the graph to determine the solutions of the equation.



A. $-2, 3$ **B.** $-3, 2$

C. $0, -6$ **D.** $0, 2$ **5.** ______

6. The quadratic function $f(x) = x^2$ has ______.

A. no zeros **B.** exactly one zero

C. exactly two zeros **D.** more than two zeros **6.** ______

For Questions 7 and 8, solve each equation by factoring.

7. $x^2 - 3x - 10 = 0$

A. $\{-5, 2\}$ **B.** $(-2, 5)$ **C.** $\{-2, 5\}$ **D.** $\{-10, 1\}$ **7.** ______

8. $2x^2 - 6x = 0$

A. $\{-3, 0\}$ **B.** $\{0, 3\}$ **C.** $\{0, 6\}$ **D.** $\{-3, 3\}$ **8.** ______

9. Which quadratic equation has roots -2 and 3?

A. $x^2 + x + 6 = 0$ **B.** $x^2 - x - 6 = 0$

C. $x^2 - 6x + 1 = 0$ **D.** $x^2 + x - 6 = 0$ **9.** ______

10. To solve $x^2 + 8x + 16 = 25$ by using the Square Root Property, you would first rewrite the equation as ______.

A. $(x + 4)^2 = 25$ **B.** $x^2 + 8x - 9 = 0$

C. $(x + 4)^2 = 5$ **D.** $x^2 + 8x = 9$ **10.** ______

11. Find the value of c that makes $x^2 + 10x + c$ a perfect square.

A. 100 **B.** 25 **C.** 10 **D.** 50 **11.** ______

Assessment

Chapter 6 Test, Form 1 *(continued)*

12. The quadratic equation $x^2 + 6x = 1$ is to be solved by completing the square. Which equation would be the first step in that solution?

A. $x^2 + 6x - 1 = 0$ **B.** $x^2 + 6x + 36 = 1 + 36$

C. $x(x + 6) = 1$ **D.** $x^2 + 6x + 9 = 1 + 9$ **12.** _____

13. Find the exact solutions to $x^2 - 3x + 1 = 0$ by using the Quadratic Formula.

A. $\frac{-3 \pm \sqrt{5}}{2}$ **B.** $\frac{3 \pm \sqrt{13}}{2}$ **C.** $\frac{-3 \pm \sqrt{13}}{2}$ **D.** $\frac{3 \pm \sqrt{5}}{2}$ **13.** _____

For Questions 14 and 15, use the value of the discriminant to determine the number and type of roots for each equation.

14. $x^2 - 3x + 7 = 0$

A. 2 complex roots **B.** 2 real, irrational roots

C. 2 real, rational roots **D.** 1 real, rational root **14.** _____

15. $x^2 = 4x - 4$

A. 2 real, rational roots **B.** 2 real, irrational roots

C. 1 real, rational root **D.** no real roots **15.** _____

16. What is the vertex of $y = 2(x - 3)^2 + 6$?

A. $(-3, -6)$ **B.** $(3, -6)$ **C.** $(-3, 6)$ **D.** $(3, 6)$ **16.** _____

17. What is the equation of the axis of symmetry of $y = -3(x + 6)^2 + 1$?

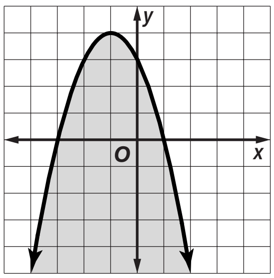
A. $x = 2$ **B.** $x = -6$ **C.** $x = -3$ **D.** $x = 6$ **17.** _____

18. Which quadratic function has its vertex at (2, 3) and passes through (1, 0)?

A. $y = 2(x - 2)^2 + 3$ **B.** $y = -3(x + 2)^2 + 3$

C. $y = -3(x - 2)^2 + 3$ **D.** $y = 2(x - 2)^2 - 3$ **18.** _____

19. Which quadratic inequality is graphed at the right?



A. $y \geq (x + 1)^2 + 4$

B. $y \leq -(x + 1)^2 + 4$

C. $y \leq -(x - 1)^2 + 4$

D. $y \leq -(x - 1)^2 - 4$ **19.** _____

20. Solve $(x - 4)(x + 2) \leq 0$.

A. $\{x \mid x \leq -2 \text{ or } x \geq 4\}$ **B.** $\{x \mid -4 \leq x \leq 2\}$

C. $\{x \mid -2 \leq x \leq 4\}$ **D.** $\{x \mid x = -2 \text{ or } x = 4\}$ **20.** _____

Bonus Find the x-intercepts and the y-intercept of the graph of $y = 2(x - 4)^2 - 18$. **B:** ____________

NAME ______________________________ DATE ____________ PERIOD _____

6 Chapter 6 Test, Form 2A

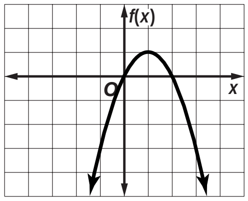
SCORE _____

Write the letter for the correct answer in the blank at the right of each question.

1. Identify the y-intercept and the axis of symmetry for the graph of $f(x) = 10x^2 + 40x + 42$.

A. 42; $x = 4$ **B.** 0; $x = -4$ **C.** 42; $x = -2$ **D.** -42; $x = 2$ **1.** _____

2. Identify the quadratic function graphed at the right.



A. $f(x) = -x^2 - 2x$

B. $f(x) = -x^2 + 2x$

C. $f(x) = x^2 - 2x$

D. $f(x) = -(x + 2)^2$ **2.** _____

3. Determine whether $f(x) = 4x^2 - 16x + 6$ has a maximum or a minimum value and find that value.

A. minimum; -10 **B.** minimum; 2 **C.** maximum; -10 **D.** maximum; 2 **3.** _____

Solve each equation by graphing. If exact roots cannot be found, state the consecutive integers between which the roots are located.

4. $-x^2 = 4x$

A. 4, 0 **B.** -4, 0

C. between -4 and 4 **D.** -2, 4 **4.** _____

5. $x^2 + 2x = 5$

A. between -4 and -3; between 1 and 2 **B.** between -2 and -1; between 3 and 4

C. no real solutions **D.** -1, -6 **5.** _____

For Questions 6 and 7, solve each equation by factoring.

6. $x^2 - 3x = 18$

A. $\{6\}$ **B.** $\{-6, 3\}$ **C.** $\{-9, 2\}$ **D.** $\{-3, 6\}$ **6.** _____

7. $3x^2 = 20 - 7x$

A. $\{-10, 2\}$ **B.** $\left\{-5, \frac{4}{3}\right\}$ **C.** $\left\{-4, \frac{5}{3}\right\}$ **D.** $\left\{-20, \frac{1}{3}\right\}$ **7.** _____

8. Which quadratic equation has roots -2 and $\frac{1}{5}$?

A. $x^2 + 4x + 4 = 0$ **B.** $5x^2 - 9x - 2 = 0$

C. $5x^2 + 9x - 2 = 0$ **D.** $5x^2 - 11x + 2 = 0$ **8.** _____

9. To solve $9x^2 - 12x + 4 = 49$ by using the Square Root Property, you would first rewrite the equation as _____.

A. $9x^2 - 12x - 45 = 0$ **B.** $(3x - 2)^2 = \pm 49$

C. $(3x - 2)^2 = 7$ **D.** $(3x - 2)^2 = 49$ **9.** _____

10. Find the value of c that makes $x^2 - 9x + c$ a perfect square.

A. $\frac{81}{4}$ **B.** $\frac{9}{2}$ **C.** $-\frac{81}{4}$ **D.** 81 **10.** _____

11. The quadratic equation $x^2 - 8x = -20$ is to be solved by completing the square. Which equation would be a step in that solution?

A. $(x - 4)^2 = 4$ **B.** $x - 4 = \pm 2\boldsymbol{i}$

C. $x^2 - 8x + 20 = 0$ **D.** $x^2 - 8x + 16 = -20$ **11.** ______

12. Find the exact solutions to $3x^2 = 5x - 1$ by using the Quadratic Formula.

A. $\frac{-5 \pm \sqrt{13}}{6}$ **B.** $\frac{5 \pm \sqrt{13}}{3}$ **C.** $\frac{5 \pm \sqrt{37}}{6}$ **D.** $\frac{5 \pm \sqrt{13}}{6}$ **12.** ______

For Questions 13 and 14, use the value of the discriminant to determine the number and type of roots for each equation.

13. $2x^2 - 7x + 9 = 0$

A. 2 real, rational **B.** 2 real, irrational

C. 2 complex **D.** 1 real, rational **13.** ______

14. $x^2 + 20 = 12x - 16$

A. 1 real, irrational **B.** 2 real, rational

C. no real **D.** 1 real, rational **14.** ______

15. Identify the vertex, axis of symmetry, and direction of opening for $y = \frac{1}{2}(x - 8)^2 + 2$.

A. $(-8, 2)$; $x = -8$; up **B.** $(-8, -2)$; $x = -8$; down

C. $(8, -2)$; $x = 8$; up **D.** $(8, 2)$; $x = 8$; up **15.** ______

16. Which quadratic function has its vertex at $(-2, 7)$ and opens down?

A. $y = -3(x + 2)^2 + 7$ **B.** $y = (x - 2)^2 + 7$

C. $y = -12(x + 2)^2 - 7$ **D.** $y = -2(x - 2)^2 + 7$ **16.** ______

17. Write $y = x^2 + 4x - 1$ in vertex form.

A. $y = (x - 2)^2 + 5$ **B.** $y = (x + 2)^2 - 5$

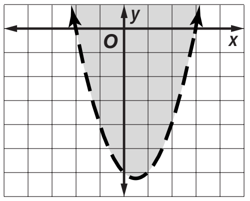
C. $y = (x + 2)^2 - 1$ **D.** $y = (x + 2)^2 + 3$ **17.** ______

18. Write an equation for the parabola whose vertex is at $(-8, 4)$ and passes through $(-6, -2)$.

A. $y = -\frac{3}{2}(x + 8)^2 + 4$ **B.** $y = -\frac{1}{4}(x + 8)^2 + 4$

C. $y = \frac{3}{2}(x + 6)^2 - 2$ **D.** $y = -\frac{3}{2}(x - 8)^2 + 4$ **18.** ______

19. Which quadratic inequality is graphed at the right?



A. $y \geq (x - 2)(x + 3)$ **B.** $y > (x - 2)(x + 3)$

C. $y > (x + 2)(x - 3)$ **D.** $y < (x + 2)(x - 3)$ **19.** ______

20. Solve $x^2 \geq 2x + 24$.

A. $\{x \mid -4 \leq x \leq 6\}$ **B.** $\{x \mid -6 \leq x \leq 4\}$

C. $\{x \mid x \leq -6 \text{ or } x \geq 4\}$ **D.** $\{x \mid x \leq -4 \text{ or } x \geq 6\}$ **20.** ______

Bonus Write a quadratic equation with roots $\pm\frac{\boldsymbol{i}\sqrt{3}}{4}$. **B:** ______________

NAME ______________________ DATE ____________ PERIOD ______

6 Chapter 6 Test, Form 2B

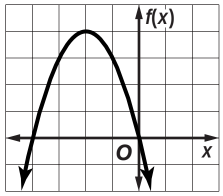
SCORE ______

Write the letter for the correct answer in the blank at the right of each question.

1. Identify the y-intercept and the axis of symmetry for the graph of $f(x) = -3x^2 + 6x + 12$.

A. $2; x = -12$ **B.** $12; x = 1$ **C.** $-2; x = 0$ **D.** $-12; x = -1$ **1.** ______

2. Identify the quadratic function graphed at the right.



A. $f(x) = x^2 - 4x$

B. $f(x) = -x^2 + 4x$

C. $f(x) = -x^2 - 4x$

D. $f(x) = -(x + 4)^2$ **2.** ______

3. Determine whether $f(x) = -5x^2 - 10x + 6$ has a maximum or a minimum value and find that value.

A. minimum; -1 **B.** maximum; 11 **C.** maximum; -1 **D.** minimum; 11 **3.** ______

Solve each equation by graphing. If exact roots cannot be found, state the consecutive integers between which the roots are located.

4. $x^2 = 4x$

A. $-4, 0$ **B.** between -4 and 4

C. $2, -4$ **D.** $0, 4$ **4.** ______

5. $x^2 + 2x = -2$

A. between -3 and -2; between 0 and 1 **B.** between -1 and 0; between 2 and 3

C. no real solutions **D.** $-1, 1$ **5.** ______

For Questions 6 and 7, solve each equation by factoring.

6. $x^2 - 3x = 28$

A. $\{-4, 7\}$ **B.** $\{-14, 2\}$ **C.** $\{-7, 4\}$ **D.** $\{-2, 14\}$ **6.** ______

7. $5x^2 = 4 - 19x$

A. $\left\{-2, \frac{2}{5}\right\}$ **B.** $\left\{-\frac{2}{5}, 2\right\}$ **C.** $\left\{-\frac{1}{5}, 4\right\}$ **D.** $\left\{-4, \frac{1}{5}\right\}$ **7.** ______

8. Which quadratic equation has roots 7 and $-\frac{2}{3}$?

A. $2x^2 - 11x - 21 = 0$ **B.** $3x^2 - 19x - 14 = 0$

C. $3x^2 + 23x + 14 = 0$ **D.** $2x^2 + 11x - 21 = 0$ **8.** ______

9. To solve $4x^2 - 28x + 49 = 25$ by using the Square Root Property, you would first rewrite the equation as _____.

A. $(2x - 7)^2 = 25$ **B.** $(2x - 7)^2 = 5$

C. $(2x - 7)^2 = \pm 5$ **D.** $4x^2 - 28x + 24 = 0$ **9.** ______

10. Find the value of c that makes $x^2 + 5x + c$ a perfect square trinomial.

A. $\frac{25}{16}$ **B.** $\frac{5}{4}$ **C.** $\frac{25}{4}$ **D.** $\frac{5}{2}$ **10.** ______

6 Chapter 6 Test, Form 2B *(continued)*

11. The quadratic equation $x^2 - 18x = -106$ is to be solved by completing the square. Which equation would be a step in that solution?

A. $(x - 9)^2 = 25$ **B.** $x^2 - 18x + 106 = 0$

C. $x - 9 = \pm 5\boldsymbol{i}$ **D.** $x^2 - 18x + 81 = -106$ **11.** ______

12. Find the exact solutions to $2x^2 = 5x - 1$ by using the Quadratic Formula.

A. $\frac{-5 \pm \sqrt{17}}{4}$ **B.** $\frac{5 \pm \sqrt{17}}{4}$ **C.** $\frac{5 \pm \sqrt{33}}{4}$ **D.** $\frac{5 \pm \sqrt{17}}{2}$ **12.** ______

For Questions 13 and 14, use the value of the discriminant to determine the number and type of roots for each equation.

13. $3x^2 - x - 12 = 0$

A. 2 complex roots **B.** 1 real, rational root

C. 2 real, rational roots **D.** 2 real, irrational roots **13.** ______

14. $x^2 + 10 = 3x - 3$

A. 2 complex roots **B.** 2 real, irrational roots

C. 1 real, rational root **D.** 2 real, rational roots **14.** ______

15. Identify the vertex, axis of symmetry, and direction of opening for $y = -8(x + 2)^2$.

A. $(-8, -2)$; $x = -8$ up **B.** $(-2, 0)$; $x = -2$; down

C. $(2, 0)$; $x = 2$; down **D.** $(-2, -8)$; $x = -2$; down **15.** ______

16. Which quadratic function has its vertex at $(-3, 5)$ and opens down?

A. $y = (x - 3)^2 + 5$ **B.** $y = (x + 3)^2 - 5$

C. $y = -(x + 3)^2 + 5$ **D.** $y = -(x - 3)^2 + 5$ **16.** ______

17. Write $y = x^2 - 18x + 52$ in vertex form.

A. $y = (x - 9)^2 + 113$ **B.** $y = (x + 9)^2 - 29$

C. $y = (x - 9)^2 + 52$ **D.** $y = (x - 9)^2 - 29$ **17.** ______

18. Write an equation for the parabola whose vertex is at $(-5, 7)$ and passes through $(-3, -1)$.

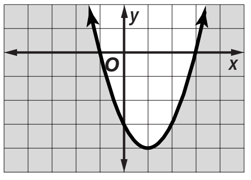
A. $y = -\frac{1}{11}(x + 5)^2 + 7$ **B.** $y = -2(x + 5)^2 + 7$

C. $y = -\frac{1}{2}(x + 5)^2 + 7$ **D.** $y = -\frac{1}{2}(x - 5)^2 + 7$ **18.** ______

19. Which quadratic inequality is graphed at the right?

A. $y \leq (x - 3)(x + 1)$ **B.** $y > (x - 3)(x + 1)$

C. $y \geq (x + 3)(x - 1)$ **D.** $y < (x + 3)(x - 1)$ **19.** ______



20. Solve $2x + 3 \geq x^2$.

A. $\{x \mid -1 \leq x \leq 3\}$ **B.** $\{x \mid -3 \leq x \leq 1\}$

C. $\{x \mid x \leq -1 \text{ or } x \geq 3\}$ **D.** $\{x \mid x \leq -3 \text{ or } x \geq 1\}$ **20.** ______

Bonus Write a quadratic equation with roots $\pm\frac{\boldsymbol{i}\sqrt{2}}{3}$. **B:** ______________

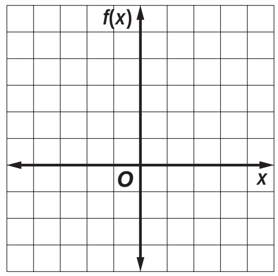
NAME ______________________ DATE ____________ PERIOD ______

6 Chapter 6 Test, Form 2C

SCORE ______

1. Graph $f(x) = -5x^2 + 10x$, labeling the y-intercept, vertex, and axis of symmetry.

1.



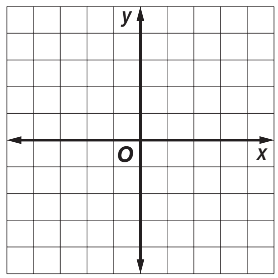
2. Determine whether $f(x) = -3x^2 + 6x + 1$ has a maximum or a minimum value and find that value.

2. ______________

For Questions 3 and 4, solve each equation by graphing. If exact roots cannot be found, state the consecutive integers between which the roots are located.

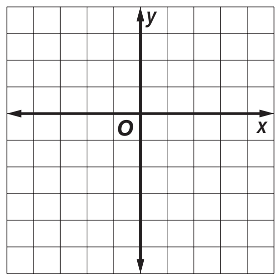
3. $x^2 = 6x - 8$

3. ______________



4. $x^2 + x - 5 = 0$

4. ______________



5. Solve $5x^2 + 13x = 6$ by factoring.

5. ______________

6. **GEOMETRY** The length of a rectangle is 7 inches longer than its width. If the area of the rectangle is 144 square inches, what are its dimensions?

6. ______________

7. Write a quadratic equation with -6 and $\frac{3}{4}$ as its roots. Write the equation in the form $ax^2 + bx + c = 0$, where a, b, and c are integers.

7. ______________

Solve each equation by using the Square Root Property.

8. $x^2 + 6x + 9 = 25$

8. ______________

9. $4x^2 - 20x + 25 = 7$

9. ______________

Assessment

6 Chapter 6 Test, Form 2C *(continued)*

For Questions 10 and 11, solve each equation by completing the square.

10. $x^2 + 4x - 9 = 0$ **10.** ________

11. $2x^2 + 3x - 2 = 0$ **11.** ________

12. Find the exact solutions to $5x^2 = 3x - 2$ by using the Quadratic Formula. **12.** ________

For Questions 13 and 14, find the value of the discriminant for each quadratic equation. Then describe the number and type of roots for the equation.

13. $9x^2 - 12x + 4 = 0$ **13.** ________

14. $4x^2 + 1 = 9x - 2$ **14.** ________

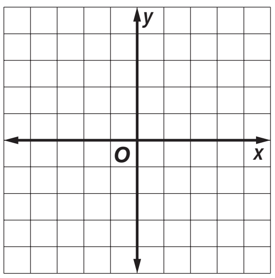
15. Identify the vertex, axis of symmetry, and direction of opening for $y = -\frac{2}{3}(x + 5)^2 - 7$. **15.** ________

16. Write an equation for the parabola with vertex at $(2, -1)$ and y-intercept 5. **16.** ________

17. Write $y = x^2 - 6x + 8$ in vertex form. **17.** ________

18. **PHYSICS** The height h (in feet) of a certain rocket t seconds after it leaves the ground is modeled by $h(t) = -16t^2 + 48t + 15$. Write the function in vertex form and find the maximum height reached by the rocket. **18.** ________

19. Graph $y < x^2 + 6x + 9$. **19.**



20. Solve $2x^2 - 5x - 3 \geq 0$ algebraically. **20.** ________

Bonus Write a quadratic equation with roots $\pm\frac{\sqrt{7}}{3}$. Write the equation in the form $ax^2 + bx + c = 0$, where a, b, and c are integers. **B:** ________

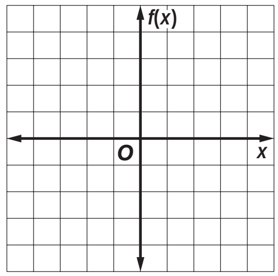
NAME ______________________ DATE ____________ PERIOD ______

6 Chapter 6 Test, Form 2D

SCORE ______

1. Graph $f(x) = x^2 - 4x + 3$, labeling the y-intercept, vertex, and axis of symmetry.

1.



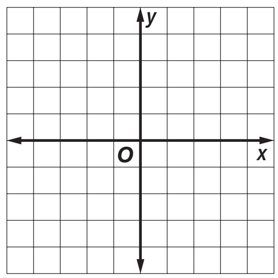
2. Determine whether $f(x) = 5x^2 - 20x + 3$ has a maximum or a minimum value and find that value.

2. ______________________

For Questions 3 and 4, solve each equation by graphing. If exact roots cannot be found, state the consecutive integers between which the roots are located.

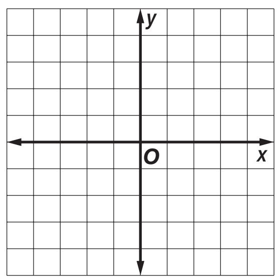
3. $x^2 + 2x - 3 = 0$

3. ______________________



4. $2x^2 - 2x - 3 = 0$

4. ______________________



5. Solve $3x^2 - x = 4$ by factoring.

5. ______________________

6. **GEOMETRY** The length of a rectangle is 10 inches longer than its width. If the area of the rectangle is 144 square inches, what are its dimensions?

6. ______________________

7. Write a quadratic equation with -4 and $\frac{3}{2}$ as its roots. Write the equation in the form $ax^2 + bx + c = 0$, where a, b, and c are integers.

7. ______________________

6 Chapter 6 Test, Form 2D *(continued)*

Solve each equation by using the Square Root Property.

8. $x^2 - 14x + 49 = 16$ **8.** ____________

9. $9x^2 + 12x + 4 = 6$ **9.** ____________

Solve each equation by completing the square.

10. $x^2 - 8x + 14 = 0$ **10.** ____________

11. $3x^2 + x - 2 = 0$ **11.** ____________

12. Find the exact solutions to $2x^2 = 9x - 5$ by using the Quadratic Formula. **12.** ____________

For Questions 13 and 14, find the value of the discriminant for each quadratic equation. Then describe the number and type of roots for the equation.

13. $25x^2 - 20x + 4 = 0$ **13.** ____________

14. $2x^2 + 10x + 9 = 2x$ **14.** ____________

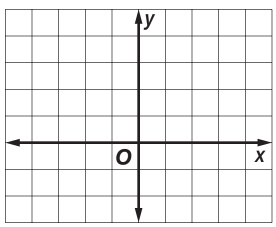
15. Identify the vertex, axis of symmetry, and direction of opening for $y = -(x - 6)^2 - 5$. **15.** ____________

16. Write an equation for the parabola with vertex at $(-4, 2)$ and y-intercept -2. **16.** ____________

17. Write $y = x^2 + 4x + 8$ in vertex form. **17.** ____________

18. **PHYSICS** The height h (in feet) of a certain rocket t seconds after it leaves the ground is modeled by $h(t) = -16t^2 + 64t + 12$. Write the function in vertex form and find the maximum height reached by the rocket. **18.** ____________

19. Graph $y \geq x^2 - 4x + 4$. **19.**



20. Solve $2x^2 - 7x - 15 < 0$ algebraically. **20.** ____________

Bonus Write a quadratic equation with roots $\pm\frac{\sqrt{5}}{4}$. Write the equation in the form $ax^2 + bx + c = 0$, where a, b, and c are integers. **B:** ____________

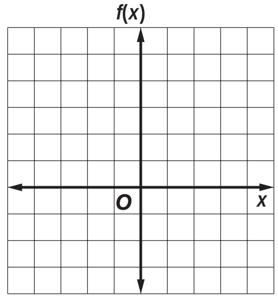
NAME ______________________ DATE ____________ PERIOD ______

6 Chapter 6 Test, Form 3

SCORE ______

1. Graph $f(x) = 3 + 3x^2 + 2x$, labeling the y-intercept, vertex, and axis of symmetry.

1.



2. Determine whether $f(x) = 1 - \frac{3}{5}x + \frac{3}{4}x^2$ has a maximum or a minimum value and find that value.

3. **BUSINESS** Khalid charges \$10 for a one-year subscription to his on-line newsletter. Khalid currently has 600 subscribers and he estimates that for each \$1 decrease in the subscription price, he would gain 100 new subscribers. What subscription price will maximize Khalid's income? If he charges this price, how much income should Khalid expect?

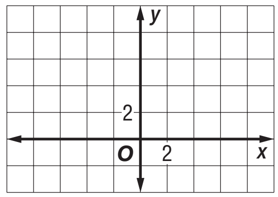
2. ______________

3. ______________

For Questions 4–6, solve each equation by graphing. If exact roots cannot be found, state the consecutive integers between which the roots are located.

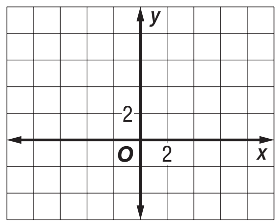
4. $0.5x^2 + 9 = 4.5x$

4. ______________



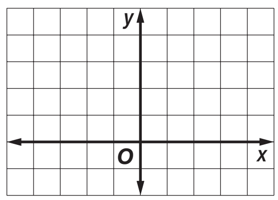
5. $\frac{2}{3}x + 3 = \frac{1}{3}x^2$

5. ______________



6. $4x(x - 3) = -9$

6. ______________



7. Solve $18x^2 + 15 = 39x$ by factoring.

7. ______________

8. Write a quadratic equation with $-\frac{2}{3}$ and 1.75 as its roots. Write the equation in the form $ax^2 + bx + c = 0$, where a, b, and c are integers.

8. ______________

9. If the roots of an equation are −5 and 3, what is the equation of the axis of symmetry?

9. ______________

Assessment

NAME ________________________ DATE ____________ PERIOD ______

6

Chapter 6 Test, Form 3 *(continued)*

10. Solve $4x^2 - 2x + 0.25 = 1.44$ by using the Square Root Property. **10.** ________

For Questions 11 and 12, solve each equation by completing the square.

11. $2x^2 - \frac{5}{2}x + 2 = 0$ **11.** ________

12. $x^2 + 2.5x - 3 = 0.5$ **12.** ________

13. Find the exact solutions to $\frac{1}{4}x^2 - 3x + 1 = 0$ by using the Quadratic Formula. **13.** ________

14. Find the value of the discriminant for $3x(0.2x - 0.4) + 1 = 0.9$. Then describe the number and type of roots for the equation. **14.** ________

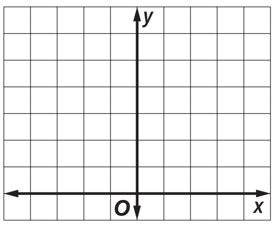
15. Find all values of k such that $x^2 + kx + 1 = 0$ has two complex roots. **15.** ________

16. Write an equation of the parabola with equation $y = -\frac{3}{5}\left(x - \frac{1}{2}\right)^2 - \frac{5}{2}$, translated 4 units left and 2 units up. Then identify the vertex, axis of symmetry, and direction of opening of your function. **16.** ________

17. **PHYSICS** The height h (in feet) of a certain aircraft t seconds after it leaves the ground is modeled by $h(t) = -9.1t^2 + 591.5t + 20{,}388.125$. Write the function in vertex form and find the maximum height reached by the aircraft. **17.** ________

18. Write an equation for the parabola that has the same vertex as $y = \frac{1}{3}x^2 + 6x + \frac{83}{2}$ and x-intercept 1. **18.** ________

19. Graph $y < -(x^2 + 2x) + 5.25$. **19.**



20. Solve $\left(x + \frac{7}{2}\right)(x - 1)^2 \leq 0$. **20.** ________

Bonus Write a quadratic equation with roots $\frac{-3 \pm 2i\sqrt{5}}{4}$. Write the equation in the form $ax^2 + bx + c = 0$, where a, b, and c are integers. **B:** ________

6

Chapter 6 Open-Ended Assessment

SCORE _____

Demonstrate your knowledge by giving a clear, concise solution to each problem. Be sure to include all relevant drawings and justify your answers. You may show your solution in more than one way or investigate beyond the requirements of the problem.

1. Mr. Moseley asked the students in his Algebra class to work in groups to solve $(x - 3)^2 = 25$, stating that each student in the first group to solve the equation correctly would earn five bonus points on the next quiz. Mi-Ling's group solved the equation using the Square Root Property. Emilia's group used the Quadratic Formula to find the solutions. In which group would you prefer to be? Explain your reasoning.

2. The next day, Mr. Moseley had his students work in pairs to review for their chapter exam. He asked each student to write a practice problem for his or her partner. Len wrote the following problem for his partner, Jocelyn: *Write an equation for the parabola whose vertex is (−3, −4), that passes through (−1, 0), and that opens downward.*

a. Jocelyn had trouble solving Len's problem. Explain why.

b. How would you change Len's problem?

c. Make the change you suggested in part **b** and complete the problem.

3. a. Write a quadratic function in vertex form whose maximum value is 8.

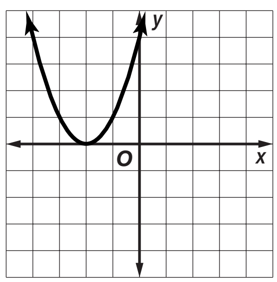
b. Write a quadratic function that transforms the graph of your function from part **a** so that it is shifted horizontally. Explain the change you made and describe the transformation that results from this change.

4. When asked to write $f(x) = 2x^2 + 12x - 5$ in vertex form, Joseph wrote:

$f(x) = 2x^2 + 12x - 5$
Step 1 $f(x) = 2(x^2 + 6x) - 5$
Step 2 $f(x) = 2(x^2 + 6x + 9) - 5 + 9$
Step 3 $f(x) = 2(x + 3)^2 + 4$

Is Joseph's answer correct? Explain your reasoning.

5. The graph of $y = x^2 + 4x + 4$ is shown. Susan used this graph to solve three quadratic inequalities. Her three solutions are given below. Replace each ● with an inequality symbol (<, >, ≤, ≥) so that each solution is correct. Explain your reasoning for each.



a. The solution of $x^2 + 4x + 4$ ● 0 is $\{x \mid x < -2 \text{ or } x > -2\}$.

b. The solution of $x^2 + 4x + 4$ ● 0 is $\varnothing$.

c. The solution of $x^2 + 4x + 4$ ● 0 is all real numbers.

Assessment

NAME ______________________ DATE ____________ PERIOD ________

6 Chapter 6 Vocabulary Test/Review

SCORE ________

axis of symmetry	maximum value	quadratic function	vertex
completing the square	minimum value	quadratic inequality	vertex form
constant term	parabola	quadratic term	Zero Product Property
discriminant	quadratic equation	roots	zeros
linear term	Quadratic Formula	Square Root Property	

Write whether each sentence is *true* or *false*. If false, replace the underlined word or words to make a true sentence.

1. The Square Root Property is used when a quadratic equation is solved by factoring. **1.** ____________

2. In $f(x) = 3x^2 - 2x + 5$, the linear term is 5. **2.** ____________

3. $2x^2 + 3x - 4 \leq 0$ is an example of a quadratic equation. **3.** ____________

4. The solutions of a quadratic equation are called its zeros. **4.** ____________

5. The quadratic function $y = 2(x + 3)^2 - 1$ is written in vertex form. **5.** ____________

6. If a parabola opens upward, the y-coordinate of the vertex is the maximum value. **6.** ____________

7. In $f(x) = -x^2 + 2x - 1$, the constant term is $-x^2$. **7.** ____________

8. It is necessary to identify the values of a, b, and c in order to solve a quadratic equation by completing the square. **8.** ____________

9. The highest or lowest point on a parabola is called the vertex. **9.** ____________

10. In the Quadratic Formula, the expression $b^2 - 4ac$ is called the quadratic term. **10.** ____________

In your own words—
Define each term.

11. parabola

12. axis of symmetry

NAME ______________________ DATE ____________ PERIOD ______

6 Chapter 6 Quiz

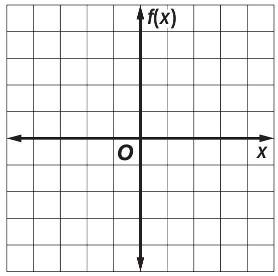
(Lessons 6–1 and 6–2)

SCORE ______

For Questions 1 and 2, consider $f(x) = x^2 + 2x - 3$.

1. Find the y-intercept, the equation of the axis of symmetry, and the x-coordinate of the vertex. — **1.** ____________

2. Graph the function, labeling the y-intercept, vertex, and axis of symmetry. — **2.**



3. Determine whether $f(x) = 2x^2 - 8x + 9$ has a maximum or a minimum value and find that value. — **3.** ____________

Solve each equation. If exact roots cannot be found, state the consecutive integers between which the roots are located.

4. $x^2 - 2x = 3$ — **4.** ____________

5. $x^2 + 4x - 7 = 0$ — **5.** ____________

NAME ______________________ DATE ____________ PERIOD ______

6 Chapter 6 Quiz

(Lessons 6–3 and 6–4)

SCORE ______

For Questions 1 and 2, solve each equation by factoring.

1. $3x^2 = 10 - 13x$

2. $x^2 + 4x = 45$

3. **STANDARDIZED TEST PRACTICE** What is the integer solution of $6x^2 + 9 = 21x$?

A. -3 **B.** 3 **C.** $\frac{1}{2}$ **D.** 2

Write a quadratic equation with the given roots. Write the equation in the form $ax^2 + bx + c = 0$, where a, b, and c are integers.

4. -6 and 2

5. $\frac{2}{3}$ and -4

Solve each equation by using the Square Root Property.

6. $x^2 + 8x + 16 = 36$

7. $x^2 - 2x + 1 = 45$

8. $25x^2 + 20x + 4 = 3$

Solve each equation by completing the square.

9. $x^2 - 10x = 11$

10. $x^2 - 4x + 29 = 11$

1. ____________
2. ____________
3. ____________
4. ____________
5. ____________
6. ____________
7. ____________
8. ____________
9. ____________
10. ____________

NAME ______________________ DATE __________ PERIOD ______

Chapter 6 Quiz

SCORE ______

(Lessons 6–5 and 6–6)

1. Solve $x^2 - 4x = 1$ by using the Quadratic Formula. Find exact solutions.

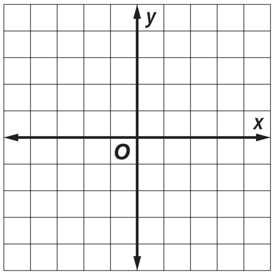
1. ____________

2. Find the value of the discriminant for $3x^2 = 6x - 11$. Then describe the number and type of roots for the equation.

2. ____________

3. Graph $y = -(x - 2)^2 - 1$. Show and label the vertex and axis of symmetry.

3.



4. Write $y = -3x^2 + 12x - 6$ in vertex form.

4. ____________

5. Write an equation for the parabola whose vertex is at $(-5, 0)$ and passes through $(0, 50)$.

5. ____________

NAME ______________________ DATE __________ PERIOD ______

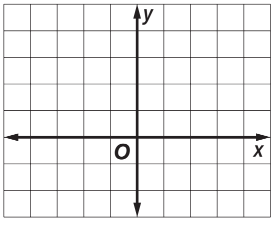
Chapter 6 Quiz

SCORE ______

(Lesson 6–7)

1. Graph $y \le -\frac{1}{3}(x + 2)^2 + 3$.

1.

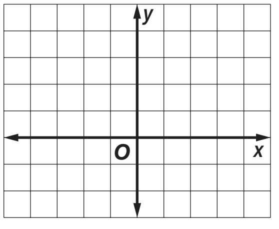


2. Use the graph of its related function to write the solutions of $-x^2 + 6x - 5 > 0$.

2. ____________

3. Solve $0 \le x^2 - 4x + 3$ by graphing.

3.



____

4. Solve $4x^2 + 1 \ge 4x$ algebraically.

4. ____________

NAME ______________________ DATE ____________ PERIOD ______

6 Chapter 6 Mid-Chapter Test

(Lessons 6–1 through 6–4)

SCORE ______

Part I ***Write the letter for the correct answer in the blank at the right of each question.***

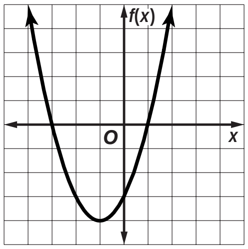
1. Which function is graphed?

A. $f(x) = x^2 - 2x - 3$

B. $f(x) = x^2 + 2x - 3$

C. $f(x) = x^2 + x - 3$

D. $f(x) = (x - 3)^2$



1. ______

2. By the Zero Product Property, if $(2x - 1)(x - 5) = 0$, then ______.

A. $x = 1$ or $x = 5$ **B.** $x = \frac{1}{2}$ or $x = 5$

C. $x = -\frac{1}{2}$ or $x = -5$ **D.** $x = -1$ or $x = -5$ **2.** ______

3. Write a quadratic equation with 7 and $\frac{2}{5}$ as its roots. Write the equation in the form $ax^2 + bx + c = 0$, where a, b, and c are integers.

A. $y = 5x^2 - 37x + 14$ **B.** $y = 2x^2 + 9x - 35$

C. $y = 5x^2 + 37x + 14$ **D.** $y = 2x^2 - 9x - 35$ **3.** ______

4. The quadratic equation $x^2 + 4x = 16$ is to be solved by completing the square. Which equation would be a step in that solution?

A. $(x + 2)^2 = \pm 20$ **B.** $x^2 + 4x - 16 = 0$

C. $(x + 2)^2 = 20$ **D.** $(x + 2)^2 = 4$ **4.** ______

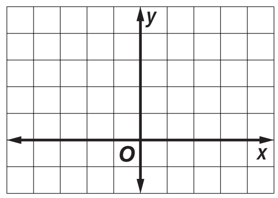
5. Solve $x^2 + 6x = -6$. If exact roots cannot be found, state the consecutive integers between which the roots are located.

A. $-2, -3$ **B.** between -4 and -3; between -2 and -1

C. -3 **D.** between -5 and -4; between -2 and -1 **5.** ______

Part II

6. Solve $x^2 - 4x + 3 = 0$ by graphing. **6.** ______________



7. Determine whether $f(x) = \frac{1}{2}x^2 - x - 9$ has a maximum or a minimum value and find that value. **7.** ______________

For Questions 8 and 9, solve each equation by factoring.

8. $x^2 - 7x = 18$ **9.** $4x^2 = x$

8. ______________

9. ______________

10. Solve $9x^2 + 6x + 1 = 5$ by using the Square Root Property. **10.** ______________

NAME ________________________ DATE ____________ PERIOD ________

6

Chapter 6 Cumulative Review

(Chapters 1–6)

1. Find the value of $12 + 36 \div 4 - (5 - 7)^2$. (Lesson 1-1)

1. ____________________

2. Find the slope of the line that is parallel to the line with equation $3x + 4y = 10$. (Lesson 2-3)

2. ____________________

3. Describe the system $2x - 3y = 21$ and $y - 5 = \frac{2}{3}x$ as *consistent* and *independent*, *consistent* and *dependent*, or *inconsistent*. (Lesson 3-1)

3. ____________________

4. Find the coordinates of the vertices of the figure formed by the system of inequalities. (Lesson 3-3)

$x \geq -2$ $\qquad$ $x + y \leq 6$

$y \geq -2$ $\qquad$ $x + y \geq -2$

4. ____________________

5. Find the value of $\begin{vmatrix} 5 & 12 \\ -6 & 4 \end{vmatrix}$. (Lesson 4-5)

5. ____________________

6. Solve $\begin{bmatrix} 4 & -1 \\ -2 & 3 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 11 \\ -13 \end{bmatrix}$ by using inverse matrices. (Lesson 4-8)

6. ____________________

7. Use synthetic division to find $(2x^4 + 5x^3 - x^2 + 10x + 4) \div (x + 3)$. (Lesson 5-3)

7. ____________________

8. Use a calculator to approximate $\sqrt[4]{983}$ to three decimal places. (Lesson 5-5)

8. ____________________

9. Solve $\sqrt{x - 2} + 1 = 8$. (Lesson 5-8)

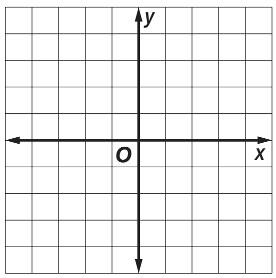
9. ____________________

10. **PHYSICS** An object is thrown straight up from the top of a 100-foot platform at a velocity of 48 feet per second. The height $h(t)$ of the object t seconds after being thrown is given by $h(t) = -16t^2 + 48t + 100$. Find the maximum height reached by the object and the time it takes to achieve this height. (Lesson 6-1)

10. ____________________

11. Solve $x^2 = 2x + 3$ by graphing. (Lesson 6-2)

11. ____________________



12. Solve $4x^2 - 4x = 24$ by factoring. (Lesson 6-3)

12. ____________________

13. Find the value of the discriminant for $7x^2 + 5x + 1 = 0$. Then describe the number and type of roots for the equation. (Lesson 6-5)

13. ____________________

14. Write $y = x^2 - 7x + 5$ in vertex form. (Lesson 6-6)

14. ____________________

NAME ______________________ DATE ____________ PERIOD ______

6 Standardized Test Practice

(Chapters 1–6)

Part 1: Multiple Choice

Instructions: Fill in the appropriate oval for the best answer.

1. If $\frac{a}{b} = \frac{3}{2}$, then $8a$ equals which of the following?

A. $16b$ **B.** $12b$ **C.** $\frac{3b}{2}$ **D.** $\frac{8}{3}b$

1. (A) (B) (C) (D)

2. 20% of 3 yards is how many fifths of 9 feet?

E. 1 **F.** 6 **G.** 10 **H.** 15

2. (E) (F) (G) (H)

3. If $u > v$ and $t > 0$, which of the following are true?

I. $ut > vt$ **II.** $u + t > v + t$ **III.** $u - t > v - t$

A. I only **B.** III only

C. I and II only **D.** I, II, and III

3. (A) (B) (C) (D)

4. Which of the following is the greatest?

E. $\frac{2}{3}$ **F.** $\frac{7}{9}$ **G.** $\frac{10}{15}$ **H.** $\frac{8}{11}$

4. (E) (F) (G) (H)

5. If $2a + 3b$ represents the perimeter of a rectangle and $a - 2b$ represents its width, the length is ______.

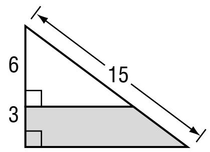
A. $7b$ **B.** b **C.** $\frac{7b}{2}$ **D.** $14b$

5. (A) (B) (C) (D)

6. In the figure, what is the area of the shaded region?

E. 30 **F.** 36

G. 54 **H.** 27



6. (E) (F) (G) (H)

7. Mr. Salazár rented a car for d days. The rental agency charged x dollars per day plus c cents per mile for the model he selected. When Mr. Salazár returned the car, he paid a total of T dollars. In terms of d, x, c, and T, how many miles did he drive?

A. $T - (xd + c)$ **B.** $T - \frac{xd}{c}$ **C.** $\frac{T}{xd + c}$ **D.** $\frac{T - xd}{c}$

7. (A) (B) (C) (D)

8. If $P(3, 2)$ and $Q(7, 10)$ are the endpoints of the diameter of a circle, what is the area of the circle?

E. $2\sqrt{5}\pi$ **F.** 80π **G.** $4\sqrt{5}\pi$ **H.** 20π

8. (E) (F) (G) (H)

9. If $(x - y)^2 = 100$ and $xy = 20$, what is the value of $x^2 + y^2$?

A. 120 **B.** 140 **C.** 80 **D.** 60

9. (A) (B) (C) (D)

10. The tenth term in the sequence 7, 12, 19, 28, … is ______.

E. 124 **F.** 103 **G.** 57 **H.** 147

10. (E) (F) (G) (H)

6 Standardized Test Practice *(continued)*

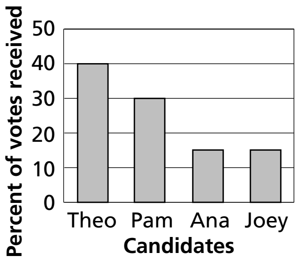
Part 2: Grid In

Instructions: Enter your answer by writing each digit of the answer in a column box and then shading in the appropriate oval that corresponds to that entry.

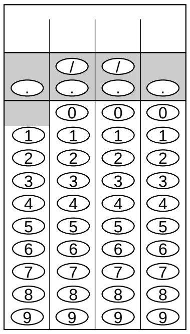
11. If $t^2 + 6t = -9$, what is the value of $\left(t - \frac{1}{2}\right)^2$?

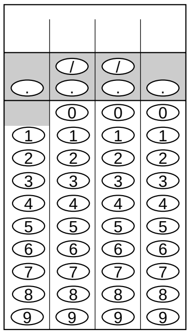
12. All four walls of a rectangular room that is 14 feet wide, 20 feet long, and 8 feet high, are to be painted. What is the minimum cost of paint if one gallon covers at most 130 square feet and the paint costs $22 per gallon?

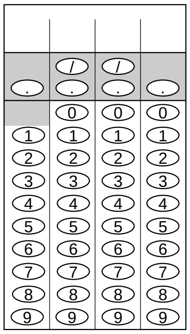
13. The bar graph shows the distribution of votes among the candidates for senior class president. If 220 seniors voted in all, how many students voted for either Theo or Pam?

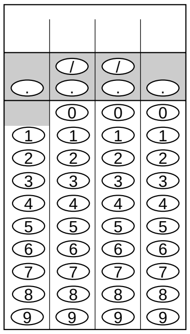


14. Find the median of x, $2x + 1$, $\frac{x}{2} - 13$, 45, and $x + 22$ if the mean of this set of numbers is 83.

11.


12.


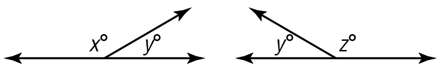
13.


14.


Part 3: Quantitative Comparison

Instructions: Compare the quantities in columns A and B. Shade in
- (A) if the quantity in column A is greater;
- (B) if the quantity in column B is greater;
- (C) if the quantities are equal; or
- (D) if the relationship cannot be determined from the information given.

	Column A	Column B	
15.			**15.** (A) (B) (C) (D)



	Column A	Column B
15.	x	z

16. $1 < a < c$ — **16.** (A) (B) (C) (D)

	Column A	Column B
16.	$\frac{a}{c}$	$\frac{c}{a}$

17. $x^2 = 25$, $y^2 = 16$ — **17.** (A) (B) (C) (D)

	Column A	Column B
17.	x	y

NAME ______________________ DATE ____________ PERIOD ______

6 Standardized Test Practice

Student Record Sheet *(Use with pages 342–343 of the Student Edition.)*

Part 1 *Multiple Choice*

Select the best answer from the choices given and fill in the corresponding oval.

1 (A) (B) (C) (D)
2 (A) (B) (C) (D)
3 (A) (B) (C) (D)
4 (A) (B) (C) (D)
5 (A) (B) (C) (D)
6 (A) (B) (C) (D)
7 (A) (B) (C) (D)
8 (A) (B) (C) (D)
9 (A) (B) (C) (D)
10 (A) (B) (C) (D)

Part 2 *Short Response/Grid In*

Solve the problem and write your answer in the blank.

For Questions 14–20, also enter your answer by writing each number or symbol in a box. Then fill in the corresponding oval for that number or symbol.

11 ____________

12 ____________

13 ____________

14 ____________

15 ____________

16 ____________

17 ____________

18 ____________

19 ____________

20 ____________

Part 3 *Quantitative Comparison*

Select the best answer from the choices given and fill in the corresponding oval.

21 (A) (B) (C) (D)
22 (A) (B) (C) (D)
23 (A) (B) (C) (D)
24 (A) (B) (C) (D)
25 (A) (B) (C) (D)
26 (A) (B) (C) (D)
27 (A) (B) (C) (D)
28 (A) (B) (C) (D)

Answers

NAME ______________________ DATE ________ PERIOD _____

6-1 Study Guide and Intervention

Graphing Quadratic Functions

Graph Quadratic Functions

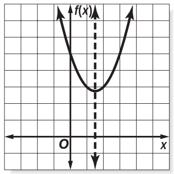
Quadratic Function	A function defined by an equation of the form $f(x) = ax^2 + bx + c$, where $a \neq 0$
Graph of a Quadratic Function	A **parabola** with these characteristics: y intercept: c; axis of symmetry: $x = \frac{-b}{2a}$; x-coordinate of vertex: $\frac{-b}{2a}$

Example **Find the y-intercept, the equation of the axis of symmetry, and the x-coordinate of the vertex for the graph of $f(x) = x^2 - 3x + 5$. Use this information to graph the function.**

$a = 1$, $b = -3$, and $c = 5$, so the y-intercept is 5. The equation of the axis of symmetry is $x = \frac{-(-3)}{2(1)}$ or $\frac{3}{2}$. The x-coordinate of the vertex is $\frac{3}{2}$.

Next make a table of values for x near $\frac{3}{2}$.

x	$x^2 - 3x + 5$	$f(x)$	$(x, f(x))$
0	$0^2 - 3(0) + 5$	5	(0, 5)
1	$1^2 - 3(1) + 5$	3	(1, 3)
$\frac{3}{2}$	$\left(\frac{3}{2}\right)^2 - 3\left(\frac{3}{2}\right) + 5$	$\frac{11}{4}$	$\left(\frac{3}{2}, \frac{11}{4}\right)$
2	$2^2 - 3(2) + 5$	3	(2, 3)
3	$3^2 - 3(3) + 5$	5	(3, 5)



Exercises

For Exercises 1–3, complete parts a–c for each quadratic function.

a. Find the y-intercept, the equation of the axis of symmetry, and the x-coordinate of the vertex.

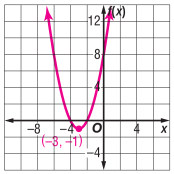
b. Make a table of values that includes the vertex.

c. Use this information to graph the function.

1. $f(x) = x^2 + 6x + 8$

8, $x = -3$, -3

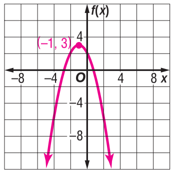
x	−3	−2	−1	−4
$f(x)$	−1	0	3	0



2. $f(x) = -x^2 - 2x + 2$

2, $x = -1$, -1

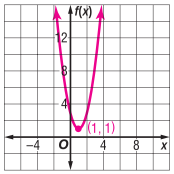
x	−1	0	−2	1
$f(x)$	3	2	2	−1



3. $f(x) = 2x^2 - 4x + 3$

3, $x = 1$, 1

x	1	0	2	3
$f(x)$	1	3	3	9



Lesson 6-1

NAME ______________________ DATE ________ PERIOD _____

6-1 Study Guide and Intervention *(continued)*

Graphing Quadratic Functions

Maximum and Minimum Values The y-coordinate of the vertex of a quadratic function is the maximum or minimum value of the function.

Maximum or Minimum Value of a Quadratic Function	The graph of $f(x) = ax^2 + bx + c$, where $a \neq 0$, opens up and has a minimum when $a > 0$. The graph opens down and has a maximum when $a < 0$.

Example **Determine whether each function has a maximum or minimum value. Then find the maximum or minimum value of each function.**

a. $f(x) = 3x^2 - 6x + 7$

For this function, $a = 3$ and $b = -6$.

Since $a > 0$, the graph opens up, and the function has a minimum value.

The minimum value is the y-coordinate of the vertex. The x-coordinate of the vertex is $\frac{-b}{2a} = -\frac{-6}{2(3)} = 1$.

Evaluate the function at $x = 1$ to find the minimum value.

$f(1) = 3(1)^2 - 6(1) + 7 = 4$, so the minimum value of the function is 4.

b. $f(x) = 100 - 2x - x^2$

For this function, $a = -1$ and $b = -2$.

Since $a < 0$, the graph opens down, and the function has a maximum value.

The maximum value is the y-coordinate of the vertex. The x-coordinate of the vertex is $\frac{-b}{2a} = -\frac{-2}{2(-1)} = -1$.

Evaluate the function at $x = -1$ to find the maximum value.

$f(-1) = 100 - 2(-1) - (-1)^2 = 101$, so the minimum value of the function is 101.

Exercises

Determine whether each function has a maximum or minimum value. Then find the maximum or minimum value of each function.

1. $f(x) = 2x^2 - x + 10$ **min., $9\frac{7}{8}$**

2. $f(x) = x^2 + 4x - 7$ **min., −11**

3. $f(x) = 3x^2 - 3x + 1$ **min., $\frac{1}{4}$**

4. $f(x) = 16 + 4x - x^2$ **max., 20**

5. $f(x) = x^2 - 7x + 11$ **min., $-\frac{5}{4}$**

6. $f(x) = -x^2 + 6x - 4$ **max., 5**

7. $f(x) = x^2 + 5x + 2$ **min., $-\frac{17}{4}$**

8. $f(x) = 20 + 6x - x^2$ **max., 29**

9. $f(x) = 4x^2 + x + 3$ **min., $2\frac{15}{16}$**

10. $f(x) = -x^2 - 4x + 10$ **max., 14**

11. $f(x) = x^2 - 10x + 5$ **min., −20**

12. $f(x) = -6x^2 + 12x + 21$ **max., 27**

13. $f(x) = 25x^2 + 100x + 350$ **min., 250**

14. $f(x) = 0.5x^2 + 0.3x - 1.4$ **min., −1.445**

15. $f(x) = \frac{-x^2}{2} + \frac{x}{4} - 8$ **max., $-7\frac{31}{32}$**

NAME ______________________ DATE __________ PERIOD _____

6-1 Skills Practice

Graphing Quadratic Functions

For each quadratic function, find the y-intercept, the equation of the axis of symmetry, and the x-coordinate of the vertex.

1. $f(x) = 3x^2$
   0; $x = 0$; 0
2. $f(x) = x^2 + 1$
   1; $x = 0$; 0
3. $f(x) = -x^2 + 6x - 15$
   −15; $x = 3$; 3
4. $f(x) = 2x^2 - 11$
   −11; $x = 0$; 0
5. $f(x) = x^2 - 10x + 5$
   5; $x = 5$; 5
6. $f(x) = -2x^2 + 8x + 7$
   7; $x = 2$; 2

Complete parts a–c for each quadratic function.

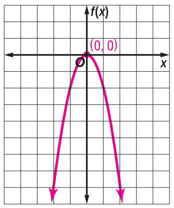
a. Find the y-intercept, the equation of the axis of symmetry, and the x-coordinate of the vertex.

b. Make a table of values that includes the vertex.

c. Use this information to graph the function.

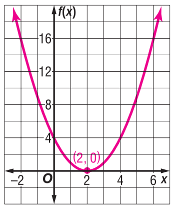
7. $f(x) = -2x^2$
   0; $x = 0$; 0

x	−2	−1	0	1	2
f(x)	−8	−2	0	−2	−8



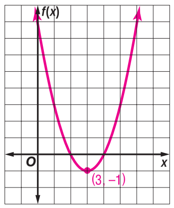
8. $f(x) = x^2 - 4x + 4$
   4; $x = 2$; 2

x	−2	0	2	4	6
f(x)	16	4	0	4	16



9. $f(x) = x^2 - 6x + 8$
   8; $x = 3$; 3

x	0	2	3	4	6
f(x)	8	0	−1	0	8



Determine whether each function has a maximum or a minimum value. Then find the maximum or minimum value of each function.

10. $f(x) = 6x^2$
    min.; 0
11. $f(x) = -8x^2$
    max.; 0
12. $f(x) = x^2 + 2x$
    min.; −1
13. $f(x) = x^2 + 2x + 15$
    min.; 14
14. $f(x) = -x^2 + 4x - 1$
    max.; 3
15. $f(x) = x^2 + 2x - 3$
    min.; −4
16. $f(x) = -2x^2 + 4x - 3$
    max.; −1
17. $f(x) = 3x^2 + 12x + 3$
    min.; −9
18. $f(x) = 2x^2 + 4x + 1$
    min.; −1

Lesson 6-1

NAME ______________________ DATE __________ PERIOD _____

6-1 Practice (Average)

Graphing Quadratic Functions

Complete parts a–c for each quadratic function.

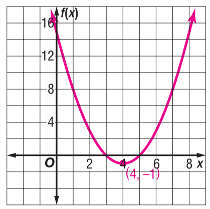
a. Find the y-intercept, the equation of the axis of symmetry, and the x-coordinate of the vertex.

b. Make a table of values that includes the vertex.

c. Use this information to graph the function.

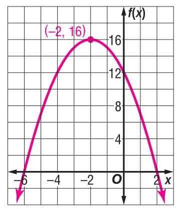
1. $f(x) = x^2 - 8x + 15$
   15; $x = 4$; 4

x	0	2	4	6	8
f(x)	15	3	−1	3	15



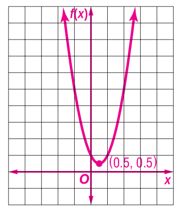
2. $f(x) = -x^2 - 4x + 12$
   12; $x = -2$; −2

x	−6	−4	−2	0	2
f(x)	0	12	16	12	0



3. $f(x) = 2x^2 - 2x + 1$
   1; $x = 0.5$; 0.5

x	−1	0	0.5	1	2
f(x)	5	1	0.5	1	5



Determine whether each function has a maximum or a minimum value. Then find the maximum or minimum value of each function.

4. $f(x) = x^2 + 2x - 8$
   min.; −9
5. $f(x) = x^2 - 6x + 14$
   min.; 5
6. $v(x) = -x^2 + 14x - 57$
   max.; −8
7. $f(x) = 2x^2 + 4x - 6$
   min.; −8
8. $f(x) = -x^2 + 4x - 1$
   max.; 3
9. $f(x) = -\frac{2}{3}x^2 + 8x - 24$
   max.; 0

10. **GRAVITATION** From 4 feet above a swimming pool, Susan throws a ball upward with a velocity of 32 feet per second. The height $h(t)$ of the ball t seconds after Susan throws it is given by $h(t) = -16t^2 + 32t + 4$. Find the maximum height reached by the ball and the time that this height is reached. **20 ft; 1 s**

11. **HEALTH CLUBS** Last year, the SportsTime Athletic Club charged $20 to participate in an aerobics class. Seventy people attended the classes. The club wants to increase the class price this year. They expect to lose one customer for each $1 increase in the price.

    a. What price should the club charge to maximize the income from the aerobics classes?
    $45

    b. What is the maximum income the SportsTime Athletic Club can expect to make?
    $2025

Answers

NAME ______________________ DATE __________ PERIOD _____

6-1 Reading to Learn Mathematics

Graphing Quadratic Functions

Pre-Activity **How can income from a rock concert be maximized?**

Read the introduction to Lesson 6-1 at the top of page 286 in your textbook.

- Based on the graph in your textbook, for what ticket price is the income the greatest? **$40**
- Use the graph to estimate the maximum income. **about $72,000**

Reading the Lesson

1. a. For the quadratic function $f(x) = 2x^2 + 5x + 3$, $2x^2$ is the **quadratic** term, $5x$ is the **linear** term, and 3 is the **constant** term.

b. For the quadratic function $f(x) = -4 + x - 3x^2$, $a =$ **−3**, $b =$ **1**, and $c =$ **−4**.

2. Consider the quadratic function $f(x) = ax^2 + bx + c$, where $a \neq 0$.

a. The graph of this function is a **parabola**.

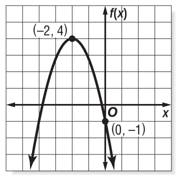
b. The y-intercept is **c**.

c. The axis of symmetry is the line $x = -\frac{b}{2a}$.

d. If $a > 0$, then the graph opens **upward** and the function has a **minimum** value.

e. If $a < 0$, then the graph opens **downward** and the function has a **maximum** value.

3. Refer to the graph at the right as you complete the following sentences.



a. The curve is called a **parabola**.

b. The line $x = -2$ is called the **axis of symmetry**.

c. The point $(-2, 4)$ is called the **vertex**.

d. Because the graph contains the point $(0, -1)$, -1 is the **y-intercept**.

Helping You Remember

4. How can you remember the way to use the x^2 term of a quadratic function to tell whether the function has a maximum or a minimum value? **Sample answer: Remember that the graph of $f(x) = x^2$ (with $a > 0$) is a U-shaped curve that opens up and has a minimum. The graph of $g(x) = -x^2$ (with $a < 0$) is just the opposite. It opens down and has a maximum.**

Lesson 6-1

NAME ______________________ DATE __________ PERIOD _____

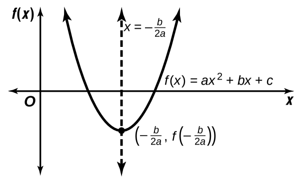
6-1 Enrichment

Finding the Axis of Symmetry of a Parabola

As you know, if $f(x) = ax^2 + bx + c$ is a quadratic function, the values of x that make $f(x)$ equal to zero are $\frac{-b + \sqrt{b^2 - 4ac}}{2a}$ and $\frac{-b - \sqrt{b^2 - 4ac}}{2a}$.

The average of these two number values is $-\frac{b}{2a}$.

The function $f(x)$ has its maximum or minimum value when $x = -\frac{b}{2a}$. Since the axis of symmetry of the graph of $f(x)$ passes through the point where the maximum or minimum occurs, the axis of symmetry has the equation $x = -\frac{b}{2a}$.



Example **Find the vertex and axis of symmetry for $f(x) = 5x^2 + 10x - 7$.**

Use $x = -\frac{b}{2a}$.

$x = -\frac{10}{2(5)} = -1$ The x-coordinate of the vertex is -1.

Substitute $x = -1$ in $f(x) = 5x^2 + 10x - 7$.

$f(-1) = 5(-1)^2 + 10(-1) - 7 = -12$

The vertex is $(-1, -12)$.

The axis of symmetry is $x = -\frac{b}{2a}$, or $x = -1$.

Find the vertex and axis of symmetry for the graph of each function using $x = -\frac{b}{2a}$.

1. $f(x) = x^2 - 4x - 8$ **(2, −12); x = 2**

2. $g(x) = -4x^2 - 8x + 3$ **(−1, 7); x = −1**

3. $y = -x^2 + 8x + 3$ **(4, 19); x = 4**

4. $f(x) = 2x^2 + 6x + 5$ $\left(-\frac{3}{2}, \frac{1}{2}\right)$; $x = -\frac{3}{2}$

5. $A(x) = x^2 + 12x + 36$ **(−6, 0); x = −6**

6. $k(x) = -2x^2 + 2x - 6$ $\left(\frac{1}{2}, -5\frac{1}{2}\right)$; $x = \frac{1}{2}$

NAME ______________________ DATE __________ PERIOD _____

6-2 Study Guide and Intervention

Solving Quadratic Equations by Graphing

Solve Quadratic Equations

Quadratic Equation	A quadratic equation has the form $ax^2 + bx + c = 0$, where $a \neq 0$.
Roots of a Quadratic Equation	solution(s) of the equation, or the zero(s) of the related quadratic function

The zeros of a quadratic function are the x-intercepts of its graph. Therefore, finding the x-intercepts is one way of solving the related quadratic equation.

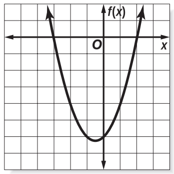
Example **Solve $x^2 + x - 6 = 0$ by graphing.**

Graph the related function $f(x) = x^2 + x - 6$.

The x-coordinate of the vertex is $\frac{-b}{2a} = -\frac{1}{2}$, and the equation of the axis of symmetry is $x = -\frac{1}{2}$.

Make a table of values using x-values around $-\frac{1}{2}$.

x	−1	$-\frac{1}{2}$	0	1	2
f(x)	−6	$-6\frac{1}{4}$	−6	−4	0

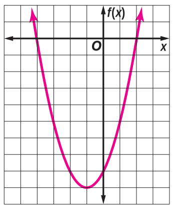


From the table and the graph, we can see that the zeros of the function are 2 and −3.

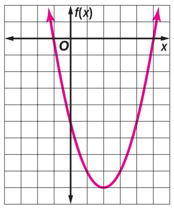
Exercises

Solve each equation by graphing.

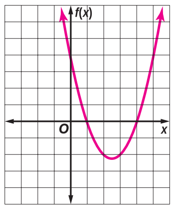
1. $x^2 + 2x - 8 = 0$ **2, −4**



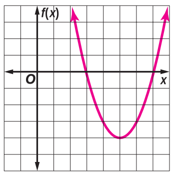
2. $x^2 - 4x - 5 = 0$ **5, −1**



3. $x^2 - 5x + 4 = 0$ **1, 4**

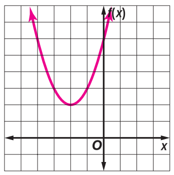


4. $x^2 - 10x + 21 = 0$



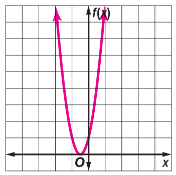
3, 7

5. $x^2 + 4x + 6 = 0$



no real solutions

6. $4x^2 + 4x + 1 = 0$



$-\frac{1}{2}$

NAME ______________________ DATE __________ PERIOD _____

6-2 Study Guide and Intervention *(continued)*

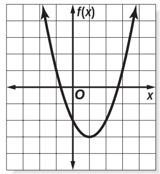
Solving Quadratic Equations by Graphing

Estimate Solutions Often, you may not be able to find exact solutions to quadratic equations by graphing. But you can use the graph to estimate solutions.

Example **Solve $x^2 - 2x - 2 = 0$ by graphing. If exact roots cannot be found, state the consecutive integers between which the roots are located.**

The equation of the axis of symmetry of the related function is $x = -\frac{-2}{2(1)} = 1$, so the vertex has x-coordinate 1. Make a table of values.

x	−1	0	1	2	3
f(x)	1	−2	−3	−2	1



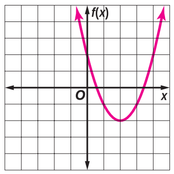
The x-intercepts of the graph are between 2 and 3 and between 0 and −1. So one solution is between 2 and 3, and the other solution is between 0 and −1.

Exercises

Solve the equations by graphing. If exact roots cannot be found, state the consecutive integers between which the roots are located.

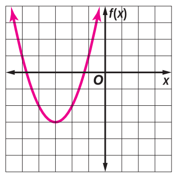
1. $x^2 - 4x + 2 = 0$

between 0 and 1; between 3 and 4



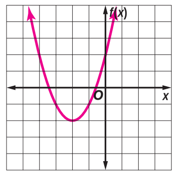
2. $x^2 + 6x + 6 = 0$

between −2 and −1; between −5 and −4



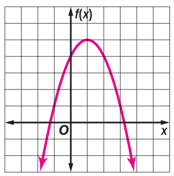
3. $x^2 + 4x + 2 = 0$

between −1 and 0; between −4 and −3



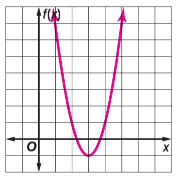
4. $-x^2 + 2x + 4 = 0$

between 3 and 4; between −2 and −1



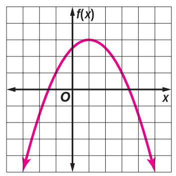
5. $2x^2 - 12x + 17 = 0$

between 2 and 3; between 3 and 4



6. $-\frac{1}{2}x^2 + x + \frac{5}{2} = 0$

between −2 and −1; between 3 and 4



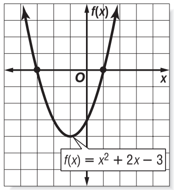
NAME ______________________ DATE ____________ PERIOD _____

6-2 Skills Practice

Solving Quadratic Equations By Graphing

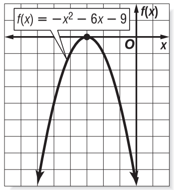
Use the related graph of each equation to determine its solutions.

1. $x^2 + 2x - 3 = 0$



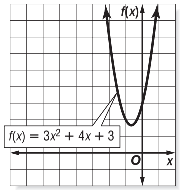
−3, 1

2. $-x^2 - 6x - 9 = 0$



−3

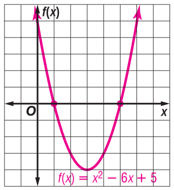
3. $3x^2 + 4x + 3 = 0$



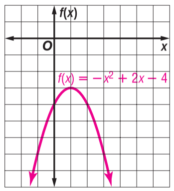
no real solutions

Solve each equation by graphing. If exact roots cannot be found, state the consecutive integers between which the roots are located.

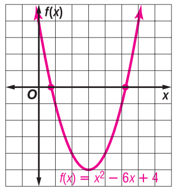
4. $x^2 - 6x + 5 = 0$ **1, 5**



5. $-x^2 + 2x - 4 = 0$ **no real solutions**



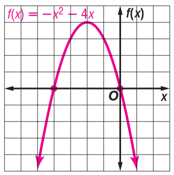
6. $x^2 - 6x + 4 = 0$ **between 0 and 1; between 5 and 6**



Use a quadratic equation to find two real numbers that satisfy each situation, or show that no such numbers exist.

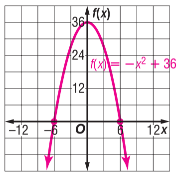
7. Their sum is −4, and their product is 0.

$-x^2 - 4x = 0$; 0, −4



8. Their sum is 0, and their product is −36.

$-x^2 + 36 = 0$; −6, 6



Lesson 6-2

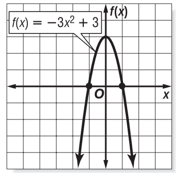
NAME ______________________ DATE ____________ PERIOD _____

6-2 Practice (Average)

Solving Quadratic Equations By Graphing

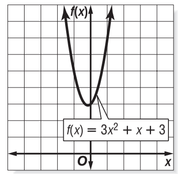
Use the related graph of each equation to determine its solutions.

1. $-3x^2 + 3 = 0$



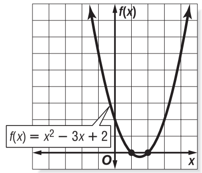
−1, 1

2. $3x^2 + x + 3 = 0$



no real solutions

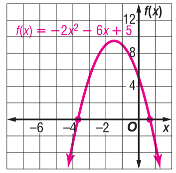
3. $x^2 - 3x + 2 = 0$



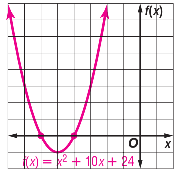
1, 2

Solve each equation by graphing. If exact roots cannot be found, state the consecutive integers between which the roots are located.

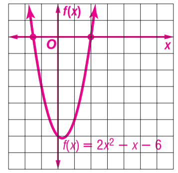
4. $-2x^2 - 6x + 5 = 0$ **between 0 and 1; between −4 and −3**



5. $x^2 + 10x + 24 = 0$ **−6, −4**

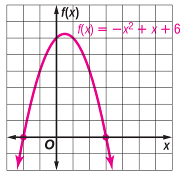


6. $2x^2 - x - 6 = 0$ **between −2 and −1, 2**



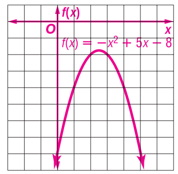
Use a quadratic equation to find two real numbers that satisfy each situation, or show that no such numbers exist.

7. Their sum is 1, and their product is −6.



$-x^2 + x + 6 = 0$; 3, −2

8. Their sum is 5, and their product is 8.



$-x^2 + 5x - 8 = 0$; no such real numbers exist

For Exercises 9 and 10, use the formula $h(t) = v_0t - 16t^2$, where $h(t)$ is the height of an object in feet, v_0 is the object's initial velocity in feet per second, and t is the time in seconds.

9. **BASEBALL** Marta throws a baseball with an initial upward velocity of 60 feet per second. Ignoring Marta's height, how long after she releases the ball will it hit the ground? **3.75 s**

10. **VOLCANOES** A volcanic eruption blasts a boulder upward with an initial velocity of 240 feet per second. How long will it take the boulder to hit the ground if it lands at the same elevation from which it was ejected? **15 s**

NAME ______________________ DATE __________ PERIOD _____

6-2 Reading to Learn Mathematics

Solving Quadratic Equations by Graphing

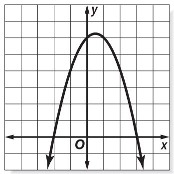
Pre-Activity **How does a quadratic function model a free-fall ride?**

Read the introduction to Lesson 6-2 at the top of page 294 in your textbook.

Write a quadratic function that describes the height of a ball t seconds after it is dropped from a height of 125 feet. **$h(t) = -16t^2 + 125$**

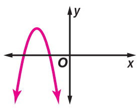
Reading the Lesson

1. The graph of the quadratic function $f(x) = -x^2 + x + 6$ is shown at the right. Use the graph to find the solutions of the quadratic equation $-x^2 + x + 6 = 0$. **−2 and 3**

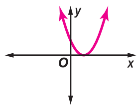


2. Sketch a graph to illustrate each situation.

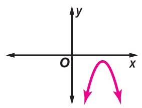
a. A parabola that opens downward and represents a quadratic function with two real zeros, both of which are negative numbers.



b. A parabola that opens upward and represents a quadratic function with exactly one real zero. The zero is a positive number.



c. A parabola that opens downward and represents a quadratic function with no real zeros.



Helping You Remember

3. Think of a memory aid that can help you recall what is meant by the *zeros* of a quadratic function.

Sample answer: The basic facts about a subject are sometimes called the ABCs. In the case of zeros, the ABCs are the XYZs, because the zeros are the <u>x</u>-values that make the <u>y</u>-values equal to <u>z</u>ero.

Lesson 6-2

NAME ______________________ DATE __________ PERIOD _____

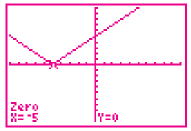
6-2 Enrichment

Graphing Absolute Value Equations

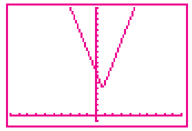
You can solve absolute value equations in much the same way you solved quadratic equations. Graph the related absolute value function for each equation using a graphing calculator. Then use the ZERO feature in the CALC menu to find its real solutions, if any. Recall that solutions are points where the graph intersects the x-axis.

For each equation, make a sketch of the related graph and find the solutions rounded to the nearest hundredth.

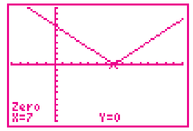
1. $|x + 5| = 0$ **−5**



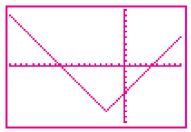
2. $|4x - 3| + 5 = 0$ **No solutions**



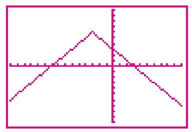
3. $|x - 7| = 0$ **7**



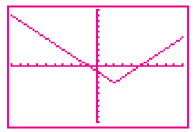
4. $|x + 3| - 8 = 0$ **−11, 5**



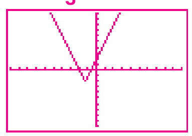
5. $-|x + 3| + 6 = 0$ **−9, 3**



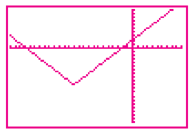
6. $|x - 2| - 3 = 0$ **−1, 5**



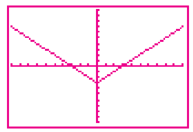
7. $|3x + 4| = 2$ **$-2, -\frac{2}{3}$**



8. $|x + 12| = 10$ **−22, −2**



9. $|x| - 3 = 0$ **−3, 3**



10. Explain how solving absolute value equations algebraically and finding zeros of absolute value functions graphically are related.

Sample answer: values of x when solving algebraically are the x-intercepts (or zeros) of the function when graphed.

Answers

NAME ______________ DATE ________ PERIOD _____

6-3 Study Guide and Intervention

Solving Quadratic Equations by Factoring

Solve Equations by Factoring When you use factoring to solve a quadratic equation, you use the following property.

Zero Product Property	For any real numbers a and b, if $ab = 0$, then either $a = 0$ or $b = 0$, or both a and $b = 0$.

Example **Solve each equation by factoring.**

a. $3x^2 = 15x$

$3x^2 = 15x$ Original equation
$3x^2 - 15x = 0$ Subtract 15x from both sides.
$3x(x - 5) = 0$ Factor the binomial.
$3x = 0$ or $x - 5 = 0$ Zero Product Property
$x = 0$ or $x = 5$ Solve each equation.

The solution set is {0, 5}.

b. $4x^2 - 5x = 21$

$4x^2 - 5x = 21$ Original equation
$4x^2 - 5x - 21 = 0$ Subtract 21 from both sides.
$(4x + 7)(x - 3) = 0$ Factor the trinomial.
$4x + 7 = 0$ or $x - 3 = 0$ Zero Product Property
$x = -\frac{7}{4}$ or $x = 3$ Solve each equation.

The solution set is $\left\{-\frac{7}{4}, 3\right\}$.

Exercises

Solve each equation by factoring.

1. $6x^2 - 2x = 0$ $\left\{0, \frac{1}{3}\right\}$
2. $x^2 = 7x$ $\{0, 7\}$
3. $20x^2 = -25x$ $\left\{0, -\frac{5}{4}\right\}$
4. $6x^2 = 7x$ $\left\{0, \frac{7}{6}\right\}$
5. $6x^2 - 27x = 0$ $\left\{0, \frac{9}{2}\right\}$
6. $12x^2 - 8x = 0$ $\left\{0, \frac{2}{3}\right\}$
7. $x^2 + x - 30 = 0$ $\{5, -6\}$
8. $2x^2 - x - 3 = 0$ $\left\{\frac{3}{2}, -1\right\}$
9. $x^2 + 14x + 33 = 0$ $\{-11, -3\}$
10. $4x^2 + 27x - 7 = 0$ $\left\{\frac{1}{4}, -7\right\}$
11. $3x^2 + 29x - 10 = 0$ $\left\{-10, \frac{1}{3}\right\}$
12. $6x^2 - 5x - 4 = 0$ $\left\{-\frac{1}{2}, \frac{4}{3}\right\}$
13. $12x^2 - 8x + 1 = 0$ $\left\{\frac{1}{6}, \frac{1}{2}\right\}$
14. $5x^2 + 28x - 12 = 0$ $\left\{\frac{2}{5}, -6\right\}$
15. $2x^2 - 250x + 5000 = 0$ $\{100, 25\}$
16. $2x^2 - 11x - 40 = 0$ $\left\{8, -\frac{5}{2}\right\}$
17. $2x^2 + 21x - 11 = 0$ $\left\{-11, \frac{1}{2}\right\}$
18. $3x^2 + 2x - 21 = 0$ $\left\{\frac{7}{3}, -3\right\}$
19. $8x^2 - 14x + 3 = 0$ $\left\{\frac{3}{2}, \frac{1}{4}\right\}$
20. $6x^2 + 11x - 2 = 0$ $\left\{-2, \frac{1}{6}\right\}$
21. $5x^2 + 17x - 12 = 0$ $\left\{\frac{3}{5}, -4\right\}$
22. $12x^2 + 25x + 12 = 0$ $\left\{-\frac{4}{3}, -\frac{3}{4}\right\}$
23. $12x^2 + 18x + 6 = 0$ $\left\{-\frac{1}{2}, -1\right\}$
24. $7x^2 - 36x + 5 = 0$ $\left\{\frac{1}{7}, 5\right\}$

NAME ______________ DATE ________ PERIOD _____

6-3 Study Guide and Intervention *(continued)*

Solving Quadratic Equations by Factoring

Write Quadratic Equations To write a quadratic equation with roots p and q, let $(x - p)(x - q) = 0$. Then multiply using FOIL.

Example **Write a quadratic equation with the given roots. Write the equation in the form $ax^2 + bx + c = 0$.**

a. 3, −5

$(x - p)(x - q) = 0$ Write the pattern.
$(x - 3)[x - (-5)] = 0$ Replace p with 3, q with −5.
$(x - 3)(x + 5) = 0$ Simplify.
$x^2 + 2x - 15 = 0$ Use FOIL.

The equation $x^2 + 2x - 15 = 0$ has roots 3 and −5.

b. $-\frac{7}{8}, \frac{1}{3}$

$(x - p)(x - q) = 0$
$\left[x - \left(-\frac{7}{8}\right)\right]\left(x - \frac{1}{3}\right) = 0$
$\left(x + \frac{7}{8}\right)\left(x - \frac{1}{3}\right) = 0$
$\frac{(8x + 7)}{8} \cdot \frac{(3x - 1)}{3} = 0$
$\frac{24 \cdot (8x + 7)(3x - 1)}{24} = 24 \cdot 0$
$24x^2 + 13x - 7 = 0$

The equation $24x^2 + 13x - 7 = 0$ has roots $-\frac{7}{8}$ and $\frac{1}{3}$.

Exercises

Write a quadratic equation with the given roots. Write the equation in the form $ax^2 + bx + c = 0$.

1. 3, −4 $x^2 + x - 12 = 0$
2. −8, −2 $x^2 + 10x + 16 = 0$
3. 1, 9 $x^2 - 10x + 9 = 0$
4. −5 $x^2 + 10x + 25 = 0$
5. 10, 7 $x^2 - 17x + 70 = 0$
6. −2, 15 $x^2 - 13x - 30 = 0$
7. $-\frac{1}{3}, 5$ $3x^2 - 14x - 5 = 0$
8. $2, \frac{2}{3}$ $3x^2 - 8x + 4 = 0$
9. $-7, \frac{3}{4}$ $4x^2 + 25x - 21 = 0$
10. $3, \frac{2}{5}$ $5x^2 - 17x + 6 = 0$
11. $-\frac{4}{9}, -1$ $9x^2 + 13x + 4 = 0$
12. $9, \frac{1}{6}$ $6x^2 - 55x + 9 = 0$
13. $\frac{2}{3}, -\frac{2}{3}$ $9x^2 - 4 = 0$
14. $\frac{5}{4}, -\frac{1}{2}$ $8x^2 - 6x - 5 = 0$
15. $\frac{3}{7}, \frac{1}{5}$ $35x^2 - 22x + 3 = 0$
16. $-\frac{7}{8}, \frac{7}{2}$ $16x^2 - 42x - 49$
17. $\frac{1}{2}, \frac{3}{4}$ $8x^2 - 10x + 3 = 0$
18. $\frac{1}{8}, \frac{1}{6}$ $48x^2 - 14x + 1 = 0$

NAME ______________________ DATE __________ PERIOD _____

6-3 Skills Practice

Solving Quadratic Equations by Factoring

Solve each equation by factoring.

1. $x^2 = 64$ **{−8, 8}**

2. $x^2 - 100 = 0$ **{10, −10}**

3. $x^2 - 3x + 2 = 0$ **{1, 2}**

4. $x^2 - 4x + 3 = 0$ **{1, 3}**

5. $x^2 + 2x - 3 = 0$ **{1, −3}**

6. $x^2 - 3x - 10 = 0$ **{5, −2}**

7. $x^2 - 6x + 5 = 0$ **{1, 5}**

8. $x^2 - 9x = 0$ **{0, 9}**

9. $-x^2 + 6x = 0$ **{0, 6}**

10. $x^2 + 6x + 8 = 0$ **{−2, −4}**

11. $x^2 = -5x$ **{0, −5}**

12. $x^2 - 14x + 49 = 0$ **{7}**

13. $x^2 + 6 = 5x$ **{2, 3}**

14. $x^2 + 18x = -81$ **{−9}**

15. $x^2 - 4x = 21$ **{−3, 7}**

16. $2x^2 + 5x - 3 = 0$ $\left\{\frac{1}{2}, -3\right\}$

17. $4x^2 + 5x - 6 = 0$ $\left\{\frac{3}{4}, -2\right\}$

18. $3x^2 - 13x - 10 = 0$ $\left\{-\frac{2}{3}, 5\right\}$

Write a quadratic equation with the given roots. Write the equation in the form $ax^2 + bx + c = 0$, where a, b, and c are integers.

19. 1, 4 $x^2 - 5x + 4 = 0$

20. 6, −9 $x^2 + 3x - 54 = 0$

21. −2, −5 $x^2 + 7x + 10 = 0$

22. 0, 7 $x^2 - 7x = 0$

23. $-\frac{1}{3}, -3$ $3x^2 + 10x + 3 = 0$

24. $-\frac{1}{2}, \frac{3}{4}$ $8x^2 - 2x - 3 = 0$

25. Find two consecutive integers whose product is 272. **16, 17**

Lesson 6-3

NAME ______________________ DATE __________ PERIOD _____

6-3 Practice (Average)

Solving Quadratic Equations by Factoring

Solve each equation by factoring.

1. $x^2 - 4x - 12 = 0$ **{6, −2}**

2. $x^2 - 16x + 64 = 0$ **{8}**

3. $x^2 - 20x + 100 = 0$ **{10}**

4. $x^2 - 6x + 8 = 0$ **{2, 4}**

5. $x^2 + 3x + 2 = 0$ **{−2, −1}**

6. $x^2 - 9x + 14 = 0$ **{2, 7}**

7. $x^2 - 4x = 0$ **{0, 4}**

8. $7x^2 = 4x$ $\left\{0, \frac{4}{7}\right\}$

9. $x^2 + 25 = 10x$ **{5}**

10. $10x^2 = 9x$ $\left\{0, \frac{9}{10}\right\}$

11. $x^2 = 2x + 99$ **{−9, 11}**

12. $x^2 + 12x = -36$ **{−6}**

13. $5x^2 - 35x + 60 = 0$ **{3, 4}**

14. $36x^2 = 25$ $\left\{\frac{5}{6}, -\frac{5}{6}\right\}$

15. $2x^2 - 8x - 90 = 0$ **{9, −5}**

16. $3x^2 + 2x - 1 = 0$ $\left\{\frac{1}{3}, -1\right\}$

17. $6x^2 = 9x$ $\left\{0, \frac{3}{2}\right\}$

18. $3x^2 + 24x + 45 = 0$ **{−5, −3}**

19. $15x^2 + 19x + 6 = 0$ $\left\{-\frac{3}{5}, -\frac{2}{3}\right\}$

20. $3x^2 - 8x = -4$ $\left\{2, \frac{2}{3}\right\}$

21. $6x^2 = 5x + 6$ $\left\{\frac{3}{2}, -\frac{2}{3}\right\}$

Write a quadratic equation with the given roots. Write the equation in the form $ax^2 + bx + c = 0$, where a, b, and c are integers.

22. 7, 2 $x^2 - 9x + 14 = 0$

23. 0, 3 $x^2 - 3x = 0$

24. −5, 8 $x^2 - 3x - 40 = 0$

25. −7, −8 $x^2 + 15x + 56 = 0$

26. −6, −3 $x^2 + 9x + 18 = 0$

27. 3, −4 $x^2 + x - 12 = 0$

28. $1, \frac{1}{2}$ $2x^2 - 3x + 1 = 0$

29. $\frac{1}{3}, 2$ $3x^2 - 7x + 2 = 0$

30. $0, -\frac{7}{2}$ $2x^2 + 7x = 0$

31. $\frac{1}{3}, -3$ $3x^2 + 8x - 3 = 0$

32. $4, \frac{1}{3}$ $3x^2 - 13x + 4 = 0$

33. $-\frac{2}{3}, -\frac{4}{5}$ $15x^2 + 22x + 8 = 0$

34. NUMBER THEORY Find two consecutive even positive integers whose product is 624. **24, 26**

35. NUMBER THEORY Find two consecutive odd positive integers whose product is 323. **17, 19**

36. GEOMETRY The length of a rectangle is 2 feet more than its width. Find the dimensions of the rectangle if its area is 63 square feet. **7 ft by 9 ft**

37. PHOTOGRAPHY The length and width of a 6-inch by 8-inch photograph are reduced by the same amount to make a new photograph whose area is half that of the original. By how many inches will the dimensions of the photograph have to be reduced? **2 in.**

NAME ______________________ DATE ________ PERIOD _____

6-3 Reading to Learn Mathematics

Solving Quadratic Equations by Factoring

Pre-Activity **How is the Zero Product Property used in geometry?**

Read the introduction to Lesson 6-3 at the top of page 301 in your textbook.

What does the expression $x(x + 5)$ mean in this situation?

It represents the area of the rectangle, since the area is the product of the width and length.

Reading the Lesson

1. The solution of a quadratic equation by factoring is shown below. Give the reason for each step of the solution.

$x^2 - 10x = -21$	Original equation
$x^2 - 10x + 21 = 0$	**Add 21 to each side.**
$(x - 3)(x - 7) = 0$	**Factor the trinomial.**
$x - 3 = 0$ or $x - 7 = 0$	**Zero Product Property**
$x = 3$ $\quad$ $x = 7$	**Solve each equation.**

The solution set is **{3, 7}**.

2. On an algebra quiz, students were asked to write a quadratic equation with −7 and 5 as its roots. The work that three students in the class wrote on their papers is shown below.

Marla	Rosa	Larry
$(x - 7)(x + 5) = 0$	$(x + 7)(x - 5) = 0$	$(x + 7)(x - 5) = 0$
$x^2 - 2x - 35 = 0$	$x^2 + 2x - 35 = 0$	$x^2 - 2x - 35 = 0$

Who is correct? **Rosa**

Explain the errors in the other two students' work.

Sample answer: Marla used the wrong factors. Larry used the correct factors but multiplied them incorrectly.

Helping You Remember

3. A good way to remember a concept is to represent it in more than one way. Describe an algebraic way and a graphical way to recognize a quadratic equation that has a double root.

Sample answer: Algebraic: Write the equation in the standard form $ax^2 + bx + c = 0$ and examine the trinomial. If it is a perfect square trinomial, the quadratic function has a double root. Graphical: Graph the related quadratic function. If the parabola has exactly one x-intercept, then the equation has a double root.

NAME ______________________ DATE ________ PERIOD _____

6-3 Enrichment

Euler's Formula for Prime Numbers

Many mathematicians have searched for a formula that would generate prime numbers. One such formula was proposed by Euler and uses a quadratic polynomial, $x^2 + x + 41$.

Find the values of $x^2 + x + 41$ for the given values of x. State whether each value of the polynomial is or is not a prime number.

1. $x = 0$ **41, prime**

2. $x = 1$ **43, prime**

3. $x = 2$ **47, prime**

4. $x = 3$ **53, prime**

5. $x = 4$ **61, prime**

6. $x = 5$ **71, prime**

7. $x = 6$ **83, prime**

8. $x = 17$ **347, prime**

9. $x = 28$ **853, prime**

10. $x = 29$ **911, prime**

11. $x = 30$ **971, prime**

12. $x = 35$ **1301, prime**

13. Does the formula produce all prime numbers greater than 40? Give examples in your answer.

No. Among the primes omitted are 59, 67, 73, 79, 89, 101, 103, 107, 109, and 127.

14. Euler's formula produces primes for many values of x, but it does not work for all of them. Find the first value of x for which the formula fails. (*Hint:* Try multiples of ten.)

$x = 40$ gives 1681, which equals 41^2.

NAME ______________ DATE ________ PERIOD _____

6-4 Study Guide and Intervention

Completing the Square

Square Root Property Use the following property to solve a quadratic equation that is in the form "perfect square trinomial = constant."

Square Root Property	For any real number x if $x^2 = n$, then $x = \pm n$.

Example **Solve each equation by using the Square Root Property.**

a. $\mathbf{x^2 - 8x + 16 = 25}$

$x^2 - 8x + 16 = 25$

$(x - 4)^2 = 25$

$x - 4 = \sqrt{25}$ or $x - 4 = -\sqrt{25}$

$x = 5 + 4 = 9$ or $x = -5 + 4 = -1$

The solution set is $\{9, -1\}$.

b. $\mathbf{4x^2 - 20x + 25 = 32}$

$4x^2 - 20x + 25 = 32$

$(2x - 5)^2 = 32$

$2x - 5 = \sqrt{32}$ or $2x - 5 = -\sqrt{32}$

$2x - 5 = 4\sqrt{2}$ or $2x - 5 = -4\sqrt{2}$

$x = \frac{5 \pm 4\sqrt{2}}{2}$

The solution set is $\left\{\frac{5 \pm 4\sqrt{2}}{2}\right\}$.

Exercises

Solve each equation by using the Square Root Property.

1. $x^2 - 18x + 81 = 49$ $\{2, 16\}$

2. $x^2 + 20x + 100 = 64$ $\{-2, -18\}$

3. $4x^2 + 4x + 1 = 16$ $\left\{\frac{3}{2}, -\frac{5}{2}\right\}$

4. $36x^2 + 12x + 1 = 18$ $\left\{\frac{-1 \pm 3\sqrt{2}}{6}\right\}$

5. $9x^2 - 12x + 4 = 4$ $\left\{0, \frac{4}{3}\right\}$

6. $25x^2 + 40x + 16 = 28$ $\left\{\frac{-4 \pm 2\sqrt{7}}{5}\right\}$

7. $4x^2 - 28x + 49 = 64$ $\left\{\frac{15}{2}, -\frac{1}{2}\right\}$

8. $16x^2 + 24x + 9 = 81$ $\left\{\frac{3}{2}, -3\right\}$

9. $100x^2 - 60x + 9 = 121$ $\{-0.8, 1.4\}$

10. $25x^2 + 20x + 4 = 75$ $\left\{\frac{-2 \pm 5\sqrt{3}}{5}\right\}$

11. $36x^2 + 48x + 16 = 12$ $\left\{\frac{-2 \pm \sqrt{3}}{3}\right\}$

12. $25x^2 - 30x + 9 = 96$ $\left\{\frac{3 \pm 4\sqrt{6}}{5}\right\}$

Lesson 6-4

NAME ______________ DATE ________ PERIOD _____

6-4 Study Guide and Intervention *(continued)*

Completing the Square

Complete the Square To complete the square for a quadratic expression of the form $x^2 + bx$, follow these steps.

1. Find $\frac{b}{2}$. → **2.** Square $\frac{b}{2}$. → **3.** Add $\left(\frac{b}{2}\right)^2$ to $x^2 + bx$.

Example 1 **Find the value of c that makes $x^2 + 22x + c$ a perfect square trinomial. Then write the trinomial as the square of a binomial.**

Step 1 $b = 22$; $\frac{b}{2} = 11$

Step 2 $11^2 = 121$

Step 3 $c = 121$

The trinomial is $x^2 + 22x + 121$, which can be written as $(x + 11)^2$.

Example 2 **Solve $2x^2 - 8x - 24 = 0$ by completing the square.**

$2x^2 - 8x - 24 = 0$	Original equation
$\frac{2x^2 - 8x - 24}{2} = \frac{0}{2}$	Divide each side by 2.
$x^2 - 4x - 12 = 0$	$x^2 - 4x - 12$ is not a perfect square.
$x^2 - 4x = 12$	Add 12 to each side.
$x^2 - 4x + 4 = 12 + 4$	Since $\left(-\frac{4}{2}\right)^2 = 4$, add 4 to each side.
$(x - 2)^2 = 16$	Factor the square.
$x - 2 = \pm 4$	Square Root Property
$x = 6$ or $x = -2$	Solve each equation.

The solution set is $\{6, -2\}$.

Exercises

Find the value of c that makes each trinomial a perfect square. Then write the trinomial as a perfect square.

1. $x^2 - 10x + c$ $25; (x - 5)^2$

2. $x^2 + 60x + c$ $900; (x + 30)^2$

3. $x^2 - 3x + c$ $\frac{9}{4}; \left(x - \frac{3}{2}\right)^2$

4. $x^2 + 3.2x + c$ $2.56; (x + 1.6)^2$

5. $x^2 + \frac{1}{2}x + c$ $\frac{1}{16}; \left(x + \frac{1}{4}\right)^2$

6. $x^2 - 2.5x + c$ $1.5625; (x - 1.25)^2$

Solve each equation by completing the square.

7. $y^2 - 4y - 5 = 0$ $-1, 5$

8. $x^2 - 8x - 65 = 0$ $-5, 13$

9. $s^2 - 10s + 21 = 0$ $3, 7$

10. $2x^2 - 3x + 1 = 0$ $1, \frac{1}{2}$

11. $2x^2 - 13x - 7 = 0$ $-\frac{1}{2}, 7$

12. $25x^2 + 40x - 9 = 0$ $\frac{1}{5}, -\frac{9}{5}$

13. $x^2 + 4x + 1 = 0$ $-2 \pm \sqrt{3}$

14. $y^2 + 12y + 4 = 0$ $-6 \pm 4\sqrt{2}$

15. $t^2 + 3t - 8 = 0$ $\frac{-3 \pm \sqrt{41}}{2}$

Answers

NAME ______________________ DATE __________ PERIOD _____

6-4 Skills Practice

Completing the Square

Solve each equation by using the Square Root Property.

1. $x^2 - 8x + 16 = 1$ **3, 5**
2. $x^2 + 4x + 4 = 1$ **−1, −3**
3. $x^2 + 12x + 36 = 25$ **−1, −11**
4. $4x^2 - 4x + 1 = 9$ **−1, 2**
5. $x^2 + 4x + 4 = 2$ $-2 \pm \sqrt{2}$
6. $x^2 - 2x + 1 = 5$ $1 \pm \sqrt{5}$
7. $x^2 - 6x + 9 = 7$ $3 \pm \sqrt{7}$
8. $x^2 + 16x + 64 = 15$ $-8 \pm \sqrt{15}$

Find the value of *c* that makes each trinomial a perfect square. Then write the trinomial as a perfect square.

9. $x^2 + 10x + c$ $25; (x + 5)^2$
10. $x^2 - 14x + c$ $49; (x - 7)^2$
11. $x^2 + 24x + c$ $144; (x + 12)^2$
12. $x^2 + 5x + c$ $\frac{25}{4}; \left(x + \frac{5}{2}\right)^2$
13. $x^2 - 9x + c$ $\frac{81}{4}; \left(x - \frac{9}{2}\right)^2$
14. $x^2 - x + c$ $\frac{1}{4}; \left(x - \frac{1}{2}\right)^2$

Solve each equation by completing the square.

15. $x^2 - 13x + 36 = 0$ **4, 9**
16. $x^2 + 3x = 0$ **0, −3**
17. $x^2 + x - 6 = 0$ **2, −3**
18. $x^2 - 4x - 13 = 0$ $2 \pm \sqrt{17}$
19. $2x^2 + 7x - 4 = 0$ $-4, \frac{1}{2}$
20. $3x^2 + 2x - 1 = 0$ $\frac{1}{3}, -1$
21. $x^2 + 3x - 6 = 0$ $\frac{-3 \pm \sqrt{33}}{2}$
22. $x^2 - x - 3 = 0$ $\frac{1 \pm \sqrt{13}}{2}$
23. $x^2 = -11$ $\pm i\sqrt{11}$
24. $x^2 - 2x + 4 = 0$ $1 \pm i\sqrt{3}$

NAME ______________________ DATE __________ PERIOD _____

6-4 Practice (Average)

Completing the Square

Solve each equation by using the Square Root Property.

1. $x^2 + 8x + 16 = 1$ **−5, −3**
2. $x^2 + 6x + 9 = 1$ **−4, −2**
3. $x^2 + 10x + 25 = 16$ **−9, −1**
4. $x^2 - 14x + 49 = 9$ **4, 10**
5. $4x^2 + 12x + 9 = 4$ $-\frac{1}{2}, -\frac{5}{2}$
6. $x^2 - 8x + 16 = 8$ $4 \pm 2\sqrt{2}$
7. $x^2 - 6x + 9 = 5$ $3 \pm \sqrt{5}$
8. $x^2 - 2x + 1 = 2$ $1 \pm \sqrt{2}$
9. $9x^2 - 6x + 1 = 2$ $\frac{1 \pm \sqrt{2}}{3}$

Find the value of *c* that makes each trinomial a perfect square. Then write the trinomial as a perfect square.

10. $x^2 + 12x + c$ $36; (x + 6)^2$
11. $x^2 - 20x + c$ $100; (x - 10)^2$
12. $x^2 + 11x + c$ $\frac{121}{4}; \left(x + \frac{11}{2}\right)^2$
13. $x^2 + 0.8x + c$ $0.16; (x + 0.4)^2$
14. $x^2 - 2.2x + c$ $1.21; (x - 1.1)^2$
15. $x^2 - 0.36x + c$ $0.0324; (x - 0.18)^2$
16. $x^2 + \frac{5}{6}x + c$ $\frac{25}{144}; \left(x + \frac{5}{12}\right)^2$
17. $x^2 - \frac{1}{4}x + c$ $\frac{1}{64}; \left(x - \frac{1}{8}\right)^2$
18. $x^2 - \frac{5}{3}x + c$ $\frac{25}{36}; \left(x - \frac{5}{6}\right)^2$

Solve each equation by completing the square.

19. $x^2 + 6x + 8 = 0$ **−4, −2**
20. $3x^2 + x - 2 = 0$ $\frac{2}{3}, -1$
21. $3x^2 - 5x + 2 = 0$ $1, \frac{2}{3}$
22. $x^2 + 18 = 9x$ **6, 3**
23. $x^2 - 14x + 19 = 0$ $7 \pm \sqrt{30}$
24. $x^2 + 16x - 7 = 0$ $-8 \pm \sqrt{71}$
25. $2x^2 + 8x - 3 = 0$ $\frac{-4 \pm \sqrt{22}}{2}$
26. $x^2 + x - 5 = 0$ $\frac{-1 \pm \sqrt{21}}{2}$
27. $2x^2 - 10x + 5 = 0$ $\frac{5 \pm \sqrt{15}}{2}$
28. $x^2 + 3x + 6 = 0$ $\frac{-3 \pm i\sqrt{15}}{2}$
29. $2x^2 + 5x + 6 = 0$ $\frac{-5 \pm i\sqrt{23}}{4}$
30. $7x^2 + 6x + 2 = 0$ $\frac{-3 \pm i\sqrt{5}}{7}$

31. **GEOMETRY** When the dimensions of a cube are reduced by 4 inches on each side, the surface area of the new cube is 864 square inches. What were the dimensions of the original cube? **16 in. by 16 in. by 16 in.**

32. **INVESTMENTS** The amount of money A in an account in which P dollars is invested for 2 years is given by the formula $A = P(1 + r)^2$, where r is the interest rate compounded annually. If an investment of \$800 in the account grows to \$882 in two years, at what interest rate was it invested? **5%**

NAME ______________________ DATE __________ PERIOD _____

6-4 Reading to Learn Mathematics

Completing the Square

Pre-Activity **How can you find the time it takes an accelerating race car to reach the finish line?**

Read the introduction to Lesson 6-4 at the top of page 306 in your textbook.

Explain what it means to say that the driver accelerates at a constant rate of 8 feet per second square.

If the driver is traveling at a certain speed at a particular moment, then one second later, the driver is traveling 8 feet per second faster.

Reading the Lesson

1. Give the reason for each step in the following solution of an equation by using the Square Root Property.

$x^2 - 12x + 36 = 81$	Original equation
$(x - 6)^2 = 81$	**Factor the perfect square trinomial.**
$x - 6 = \pm\sqrt{81}$	**Square Root Property**
$x - 6 = \pm 9$	**$\sqrt{81} = 9$**
$x - 6 = 9$ or $x - 6 = -9$	**Rewrite as two equations.**
$x = 15 \qquad x = -3$	**Solve each equation.**

2. Explain how to find the constant that must be added to make a binomial into a perfect square trinomial.

Sample answer: Find half of the coefficient of the linear term and square it.

3. a. What is the first step in solving the equation $3x^2 + 6x = 5$ by completing the square? **Divide the equation by 3.**

b. What is the first step in solving the equation $x^2 + 5x - 12 = 0$ by completing the square? **Add 12 to each side.**

Helping You Remember

4. How can you use the rules for squaring a binomial to help you remember the procedure for changing a binomial into a perfect square trinomial?

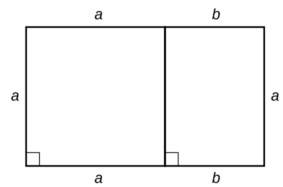
One of the rules for squaring a binomial is $(x + y)^2 = x^2 + 2xy + y^2$. In completing the square, you are starting with $x^2 + bx$ and need to find y^2. This shows you that $b = 2y$, so $y = \frac{b}{2}$. That is why you must take half of the coefficient and square it to get the constant that must be added to complete the square.

NAME ______________________ DATE __________ PERIOD _____

6-4 Enrichment

The Golden Quadratic Equations

A **golden rectangle** has the property that its length can be written as $a + b$, where a is the width of the rectangle and $\frac{a+b}{a} = \frac{a}{b}$. Any golden rectangle can be divided into a square and a smaller golden rectangle, as shown.



The proportion used to define golden rectangles can be used to derive two quadratic equations. These are sometimes called *golden quadratic equations.*

Solve each problem.

1. In the proportion for the golden rectangle, let a equal 1. Write the resulting quadratic equation and solve for b.

$b^2 + b - 1 = 0$

$b = \frac{-1 + \sqrt{5}}{2}$

2. In the proportion, let b equal 1. Write the resulting quadratic equation and solve for a.

$a^2 - a - 1 = 0$

$a = \frac{1 + \sqrt{5}}{2}$

3. Describe the difference between the two golden quadratic equations you found in exercises 1 and 2.

The signs of the first-degree terms are opposite.

4. Show that the positive solutions of the two equations in exercises 1 and 2 are reciprocals.

$\left(\frac{-1 + \sqrt{5}}{2}\right)\left(\frac{1 + \sqrt{5}}{2}\right) = \frac{-(1^2) + (\sqrt{5})^2}{4} = \frac{-1 + 5}{4} = 1$

5. Use the Pythagorean Theorem to find a radical expression for the diagonal of a golden rectangle when $a = 1$.

$d = \frac{\sqrt{10 - 2\sqrt{5}}}{2}$

6. Find a radical expression for the diagonal of a golden rectangle when $b = 1$.

$d = \frac{\sqrt{10 + 2\sqrt{5}}}{2}$

NAME ______________ DATE ________ PERIOD _____

6-5 Study Guide and Intervention

The Quadratic Formula and the Discriminant

Quadratic Formula The **Quadratic Formula** can be used to solve *any* quadratic equation once it is written in the form $ax^2 + bx + c = 0$.

Quadratic Formula	The solutions of $ax^2 + bx + c = 0$, with $a \neq 0$, are given by $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Example **Solve $x^2 - 5x = 14$ by using the Quadratic Formula.**

Rewrite the equation as $x^2 - 5x - 14 = 0$.

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ — Quadratic Formula

$= \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(-14)}}{2(1)}$ — Replace *a* with 1, *b* with −5, and *c* with −14.

$= \frac{5 \pm \sqrt{81}}{2}$ — Simplify.

$= \frac{5 \pm 9}{2}$

$= 7$ or -2

The solutions are −2 and 7.

Exercises

Solve each equation by using the Quadratic Formula.

1. $x^2 + 2x - 35 = 0$ **5, −7**
2. $x^2 + 10x + 24 = 0$ **−4, −6**
3. $x^2 - 11x + 24 = 0$ **3, 8**
4. $4x^2 + 19x - 5 = 0$ **$\frac{1}{4}$, −5**
5. $14x^2 + 9x + 1 = 0$ **$-\frac{1}{2}, -\frac{1}{7}$**
6. $2x^2 - x - 15 = 0$ **$3, -\frac{5}{2}$**
7. $3x^2 + 5x = 2$ **$-2, \frac{1}{3}$**
8. $2y^2 + y - 15 = 0$ **$\frac{5}{2}, -3$**
9. $3x^2 - 16x + 16 = 0$ **$4, \frac{4}{3}$**
10. $8x^2 + 6x - 9 = 0$ **$-\frac{3}{2}, \frac{3}{4}$**
11. $r^2 - \frac{3r}{5} + \frac{2}{25} = 0$ **$\frac{2}{5}, \frac{1}{5}$**
12. $x^2 - 10x - 50 = 0$ **$5 \pm 5\sqrt{3}$**
13. $x^2 + 6x - 23 = 0$ **$-3 \pm 4\sqrt{2}$**
14. $4x^2 - 12x - 63 = 0$ **$\frac{3 \pm 6\sqrt{2}}{2}$**
15. $x^2 - 6x + 21 = 0$ **$3 \pm 2i\sqrt{3}$**

NAME ______________ DATE ________ PERIOD _____

6-5 Study Guide and Intervention *(continued)*

The Quadratic Formula and the Discriminant

Roots and the Discriminant

Discriminant	The expression under the radical sign, $b^2 - 4ac$, in the Quadratic Formula is called the **discriminant**.

Roots of a Quadratic Equation

Discriminant	Type and Number of Roots
$b^2 - 4ac > 0$ and a perfect square	2 rational roots
$b^2 - 4ac > 0$, but **not** a perfect square	2 irrational roots
$b^2 - 4ac = 0$	1 rational root
$b^2 - 4ac < 0$	2 complex roots

Example **Find the value of the discriminant for each equation. Then describe the number and types of roots for the equation.**

a. $2x^2 + 5x + 3$
The discriminant is
$b^2 - 4ac = 5^2 - 4(2)(3)$ or 1.
The discriminant is a perfect square, so the equation has 2 rational roots.

b. $3x^2 - 2x + 5$
The discriminant is
$b^2 - 4ac = (-2)^2 - 4(3)(5)$ or −56.
The discriminant is negative, so the equation has 2 complex roots.

Exercises

For Exercises 1–12, complete parts a–c for each quadratic equation.
a. Find the value of the discriminant.
b. Describe the number and type of roots.
c. Find the exact solutions by using the Quadratic Formula.

1. $p^2 + 12p = -4$ **128; two irrational roots; $-6 \pm 4\sqrt{2}$**
2. $9x^2 - 6x + 1 = 0$ **0; one rational root; $\frac{1}{3}$**
3. $2x^2 - 7x - 4 = 0$ **81; 2 rational roots; $-\frac{1}{2}$, 4**
4. $x^2 + 4x - 4 = 0$ **32; 2 irrational roots; $-2 \pm 2\sqrt{2}$**
5. $5x^2 - 36x + 7 = 0$ **1156; 2 rational roots; $\frac{1}{5}$, 7**
6. $4x^2 - 4x + 11 = 0$ **−160; 2 complex roots; $\frac{1 \pm i\sqrt{10}}{2}$**
7. $x^2 - 7x + 6 = 0$ **25; 2 rational roots; 1, 6**
8. $m^2 - 8m = -14$ **8; 2 irrational roots; $4 \pm \sqrt{2}$**
9. $25x^2 - 40x = -16$ **0; 1 rational root; $\frac{4}{5}$**
10. $4x^2 + 20x + 29 = 0$ **−64; 2 complex roots;**
11. $6x^2 + 26x + 8 = 0$ **484; 2 rational roots;**
12. $4x^2 - 4x - 11 = 0$ **192; 2 irrational roots;**

NAME ______________________ DATE ____________ PERIOD _____

6-5 Skills Practice

The Quadratic Formula and the Discriminant

Complete parts a–c for each quadratic equation.
a. Find the value of the discriminant.
b. Describe the number and type of roots.
c. Find the exact solutions by using the Quadratic Formula.

1. $x^2 - 8x + 16 = 0$
0; 1 rational root; 4

2. $x^2 - 11x - 26 = 0$
225; 2 rational roots; −2, 13

3. $3x^2 - 2x = 0$
4; 2 rational roots; 0, $\frac{2}{3}$

4. $20x^2 + 7x - 3 = 0$
289; 2 rational roots; $-\frac{3}{5}, \frac{1}{4}$

5. $5x^2 - 6 = 0$
120; 2 irrational roots; $\pm\frac{\sqrt{30}}{5}$

6. $x^2 - 6 = 0$
24; 2 irrational roots; $\pm\sqrt{6}$

7. $x^2 + 8x + 13 = 0$
12; 2 irrational roots; $-4 \pm \sqrt{3}$

8. $5x^2 - x - 1 = 0$
21; 2 irrational roots; $\frac{1 \pm \sqrt{21}}{10}$

9. $x^2 - 2x - 17 = 0$
72; 2 irrational roots; $1 \pm 3\sqrt{2}$

10. $x^2 + 49 = 0$
−196; 2 complex roots; $\pm 7i$

11. $x^2 - x + 1 = 0$
−3; 2 complex roots; $\frac{1 \pm i\sqrt{3}}{2}$

12. $2x^2 - 3x = -2$
−7; 2 complex roots; $\frac{3 \pm i\sqrt{7}}{4}$

Solve each equation by using the method of your choice. Find exact solutions.

13. $x^2 = 64$ **± 8**

14. $x^2 - 30 = 0$ **$\pm\sqrt{30}$**

15. $x^2 - x = 30$ **−5, 6**

16. $16x^2 - 24x - 27 = 0$ **$\frac{9}{4}, -\frac{3}{4}$**

17. $x^2 - 4x - 11 = 0$ **$2 \pm \sqrt{15}$**

18. $x^2 - 8x - 17 = 0$ **$4 \pm \sqrt{33}$**

19. $x^2 + 25 = 0$ **$\pm 5i$**

20. $3x^2 + 36 = 0$ **$\pm 2i\sqrt{3}$**

21. $2x^2 + 10x + 11 = 0$ **$\frac{-5 \pm \sqrt{3}}{2}$**

22. $2x^2 - 7x + 4 = 0$ **$\frac{7 \pm \sqrt{17}}{4}$**

23. $8x^2 + 1 = 4x$ **$\frac{1 \pm i}{4}$**

24. $2x^2 + 2x + 3 = 0$ **$\frac{-1 \pm i\sqrt{5}}{2}$**

25. PARACHUTING Ignoring wind resistance, the distance $d(t)$ in feet that a parachutist falls in t seconds can be estimated using the formula $d(t) = 16t^2$. If a parachutist jumps from an airplane and falls for 1100 feet before opening her parachute, how many seconds pass before she opens the parachute? **about 8.3 s**

NAME ______________________ DATE ____________ PERIOD _____

6-5 Practice (Average)

The Quadratic Formula and the Discriminant

Complete parts a–c for each quadratic equation.
a. Find the value of the discriminant.
b. Describe the number and type of roots.
c. Find the exact solutions by using the Quadratic Formula.

1. $x^2 - 16x + 64 = 0$
0; 1 rational; 8

2. $x^2 = 3x$
9; 2 rational; 0, 3

3. $9x^2 - 24x + 16 = 0$
0; 1 rational; $\frac{4}{3}$

4. $x^2 - 3x = 40$
169; 2 rational; −5, 8

5. $3x^2 + 9x - 2 = 0$ **105; 2 irrational; $\frac{-9 \pm \sqrt{105}}{6}$**

6. $2x^2 + 7x = 0$
49; 2 rational; 0, $-\frac{7}{2}$

7. $5x^2 - 2x + 4 = 0$ **−76; 2 complex; $\frac{1 \pm i\sqrt{19}}{5}$**

8. $12x^2 - x - 6 = 0$ **289; 2 rational; $\frac{3}{4}, -\frac{2}{3}$**

9. $7x^2 + 6x + 2 = 0$ **−20; 2 complex; $\frac{-3 \pm i\sqrt{5}}{7}$**

10. $12x^2 + 2x - 4 = 0$ **196; 2 rational; $\frac{1}{2}, -\frac{2}{3}$**

11. $6x^2 - 2x - 1 = 0$ **28; 2 irrational; $\frac{1 \pm \sqrt{7}}{6}$**

12. $x^2 + 3x + 6 = 0$ **−15; 2 complex; $\frac{-3 \pm i\sqrt{15}}{2}$**

13. $4x^2 - 3x^2 - 6 = 0$ **105; 2 irrational; $\frac{3 \pm \sqrt{105}}{8}$**

14. $16x^2 - 8x + 1 = 0$
0; 1 rational; $\frac{1}{4}$

15. $2x^2 - 5x - 6 = 0$ **73; 2 irrational; $\frac{5 \pm \sqrt{73}}{4}$**

Solve each equation by using the method of your choice. Find exact solutions.

16. $7x^2 - 5x = 0$ **0, $\frac{5}{7}$**

17. $4x^2 - 9 = 0$ **$\pm\frac{3}{2}$**

18. $3x^2 + 8x = 3$ **$\frac{1}{3}$, −3**

19. $x^2 - 21 = 4x$ **−3, 7**

20. $3x^2 - 13x + 4 = 0$ **$\frac{1}{3}$, 4**

21. $15x^2 + 22x = -8$ **$-\frac{2}{3}, -\frac{4}{5}$**

22. $x^2 - 6x + 3 = 0$ **$3 \pm \sqrt{6}$**

23. $x^2 - 14x + 53 = 0$ **$7 \pm 2i$**

24. $3x^2 = -54$ **$\pm 3i\sqrt{2}$**

25. $25x^2 - 20x - 6 = 0$ **$\frac{2 \pm \sqrt{10}}{5}$**

26. $4x^2 - 4x + 17 = 0$ **$\frac{1 \pm 4i}{2}$**

27. $8x - 1 = 4x^2$ **$\frac{2 \pm \sqrt{3}}{2}$**

28. $x^2 = 4x - 15$ **$2 \pm i\sqrt{11}$**

29. $4x^2 - 12x + 7 = 0$ **$\frac{3 \pm \sqrt{2}}{2}$**

30. GRAVITATION The height $h(t)$ in feet of an object t seconds after it is propelled straight up from the ground with an initial velocity of 60 feet per second is modeled by the equation $h(t) = -16t^2 + 60t$. At what times will the object be at a height of 56 feet? **1.75 s, 2 s**

31. STOPPING DISTANCE The formula $d = 0.05s^2 + 1.1s$ estimates the minimum stopping distance d in feet for a car traveling s miles per hour. If a car stops in 200 feet, what is the fastest it could have been traveling when the driver applied the brakes? **about 53.2 mi/h**

NAME ______________________ DATE ________ PERIOD _____

6-5 Reading to Learn Mathematics

The Quadratic Formula and the Discriminant

Pre-Activity **How is blood pressure related to age?**

Read the introduction to Lesson 6-5 at the top of page 313 in your textbook.

Describe how you would calculate your normal blood pressure using one of the formulas in your textbook.

Sample answer: Substitute your age for *A* in the appropriate formula (for females or males) and evaluate the expression.

Reading the Lesson

1. a. Write the Quadratic Formula. $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

b. Identify the values of a, b, and c that you would use to solve $2x^2 - 5x = -7$, but do not actually solve the equation.

$a =$ **2** $\quad b =$ **−5** $\quad c =$ **7**

2. Suppose that you are solving four quadratic equations with rational coefficients and have found the value of the discriminant for each equation. In each case, give the number of roots and describe the type of roots that the equation will have.

Value of Discriminant	Number of Roots	Type of Roots
64	**2**	**real, rational**
−8	**2**	**complex**
21	**2**	**real, irrational**
0	**1**	**real, rational**

Helping You Remember

3. How can looking at the Quadratic Formula help you remember the relationships between the value of the discriminant and the number of roots of a quadratic equation and whether the roots are real or complex?

Sample answer: The discriminant is the expression under the radical in the Quadratic Formula. Look at the Quadratic Formula and consider what happens when you take the principal square root of $b^2 - 4ac$ and apply ± in front of the result. If $b^2 - 4ac$ is positive, its principal square root will be a positive number and applying ± will give two different real solutions, which may be rational or irrational. If $b^2 - 4ac = 0$, its principal square root is 0, so applying ± in the Quadratic Formula will only lead to one solution, which will be rational (assuming *a*, *b*, and *c* are integers). If $b^2 - 4ac$ is negative, since the square roots of negative numbers are not real numbers, you will get two complex roots, corresponding to the + and − in the ± symbol.

Lesson 6-5

NAME ______________________ DATE ________ PERIOD _____

6-5 Enrichment

Sum and Product of Roots

Sometimes you may know the roots of a quadratic equation without knowing the equation itself. Using your knowledge of factoring to solve an equation, you can work backward to find the quadratic equation. The rule for finding the sum and product of roots is as follows:

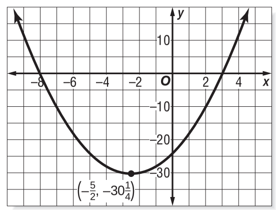
Sum and Product of Roots	If the roots of $ax^2 + bx + c = 0$, with $a \neq 0$, are s_1 and s_2, then $s_1 + s_2 = -\frac{b}{a}$ and $s_1 \cdot s_2 = \frac{c}{a}$.

Example **A road with an initial gradient, or slope, of 3% can be represented by the formula $y = ax^2 + 0.03x + c$, where y is the elevation and x is the distance along the curve. Suppose the elevation of the road is 1105 feet at points 200 feet and 1000 feet along the curve. You can find the equation of the transition curve. Equations of transition curves are used by civil engineers to design smooth and safe roads.**

The roots are $x = 3$ and $x = -8$.

$3 + (-8) = -5$ Add the roots.
$3(-8) = -24$ Multiply the roots.

Equation: $x^2 + 5x - 24 = 0$



Write a quadratic equation that has the given roots.

1. 6, −9
$x^2 + 3x - 54 = 0$

2. 5, −1
$x^2 - 4x - 5 = 0$

3. 6, 6
$x^2 - 12x + 36 = 0$

4. $4 \pm \sqrt{3}$
$x^2 - 8x + 13 = 0$

6. $-\frac{2}{5}, \frac{2}{7}$
$35x^2 + 4x - 4 = 0$

6. $\frac{-2 \pm 3\sqrt{5}}{7}$
$49x^2 - 42x + 205 = 0$

Find *k* such that the number given is a root of the equation.

7. 7; $2x^2 + kx - 21 = 0$
−11

8. −2; $x^2 - 13x + k = 0$
−30

NAME ______________________ DATE __________ PERIOD _____

6-6 Study Guide and Intervention

Analyzing Graphs of Quadratic Functions

Analyze Quadratic Functions

Vertex Form of a Quadratic Function	The graph of $y = a(x - h)^2 + k$ has the following characteristics: • Vertex: (h, k) • Axis of symmetry: $x = h$ • Opens up if $a > 0$ • Opens down if $a < 0$ • Narrower than the graph of $y = x^2$ if $\lvert a \rvert > 1$ • Wider than the graph of $y = x^2$ if $\lvert a \rvert < 1$

Example **Identify the vertex, axis of symmetry, and direction of opening of each graph.**

a. $\mathbf{y = 2(x + 4)^2 - 11}$

The vertex is at (h, k) or $(-4, -11)$, and the axis of symmetry is $x = -4$. The graph opens up, and is narrower than the graph of $y = x^2$.

a. $\mathbf{y = -\frac{1}{4}(x - 2)^2 + 10}$

The vertex is at (h, k) or $(2, 10)$, and the axis of symmetry is $x = 2$. The graph opens down, and is wider than the graph of $y = x^2$.

Exercises

Each quadratic function is given in vertex form. Identify the vertex, axis of symmetry, and direction of opening of the graph.

1. $y = (x - 2)^2 + 16$
(2, 16); $x = 2$; up

2. $y = 4(x + 3)^2 - 7$
(−3, −7); $x = -3$; up

3. $y = \frac{1}{2}(x - 5)^2 + 3$
(5, 3); $x = 5$; up

4. $y = -7(x + 1)^2 - 9$
(−1, −9); $x = -1$; down

5. $y = \frac{1}{5}(x - 4)^2 - 12$
(4, −12); $x = 4$; up

6. $y = 6(x + 6)^2 + 6$
(−6, 6); $x = -6$; up

7. $y = \frac{2}{5}(x - 9)^2 + 12$
(9, 12); $x = 9$; up

8. $y = 8(x - 3)^2 - 2$
(3, −2); $x = 3$; up

9. $y = -3(x - 1)^2 - 2$
(1, −2); $x = 1$; down

10. $y = -\frac{5}{2}(x + 5)^2 + 12$
(−5, 12); $x = -5$; down

11. $y = \frac{4}{3}(x - 7)^2 + 22$
(7, 22); $x = 7$; up

12. $y = 16(x - 4)^2 + 1$
(4, 1); $x = 4$; up

13. $y = 3(x - 1.2)^2 + 2.7$
(1.2, 2.7); $x = 1.2$; up

14. $y = -0.4(x - 0.6)^2 - 0.2$
(0.6, −0.2); $x = 0.6$; down

15. $y = 1.2(x + 0.8)^2 + 6.5$
(−0.8, 6.5); $x = -0.8$; up

Lesson 6-6

NAME ______________________ DATE __________ PERIOD _____

6-6 Study Guide and Intervention *(continued)*

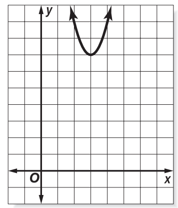
Analyzing Graphs of Quadratic Functions

Write Quadratic Functions in Vertex Form A quadratic function is easier to graph when it is in vertex form. You can write a quadratic function of the form $y = ax^2 + bx + c$ in vertex from by completing the square.

Example **Write $y = 2x^2 - 12x + 25$ in vertex form. Then graph the function.**

$y = 2x^2 - 12x + 25$
$y = 2(x^2 - 6x) + 25$
$y = 2(x^2 - 6x + 9) + 25 - 18$
$y = 2(x - 3)^2 + 7$

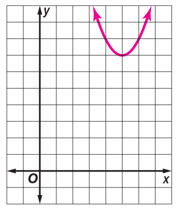
The vertex form of the equation is $y = 2(x - 3)^2 + 7$.



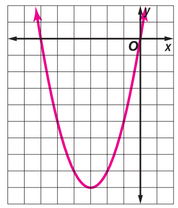
Exercises

Write each quadratic function in vertex form. Then graph the function.

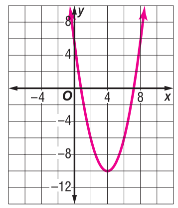
1. $y = x^2 - 10x + 32$
$y = (x - 5)^2 + 7$



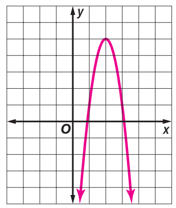
2. $y = x^2 + 6x$
$y = (x + 3)^2 - 9$



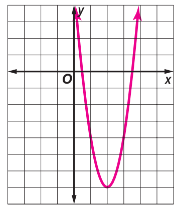
3. $y = x^2 - 8x + 6$
$y = (x - 4)^2 - 10$



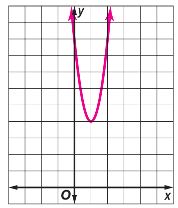
4. $y = -4x^2 + 16x - 11$
$y = -4(x - 2)^2 + 5$



5. $y = 3x^2 - 12x + 5$
$y = 3(x - 2)^2 - 7$



6. $y = 5x^2 - 10x + 9$
$y = 5(x - 1)^2 + 4$



Answers

Answers (Lesson 6-6)

NAME ______________________ DATE __________ PERIOD _____

6-6 Skills Practice

Analyzing Graphs of Quadratic Functions

Write each quadratic function in vertex form, if not already in that form. Then identify the vertex, axis of symmetry, and direction of opening.

1. $y = (x - 2)^2$
$y = (x - 2)^2 + 0$; $(2, 0)$; $x = 2$; up

2. $y = -x^2 + 4$
$y = -(x - 0)^2 + 4$; $(0, 4)$; $x = 0$; down

3. $y = x^2 - 6$
$y = (x - 0)^2 - 6$; $(0, -6)$; $x = 0$; up

4. $y = -3(x + 5)^2$
$y = -3(x + 5)^2 + 0$; $(-5, 0)$; $x = -5$; down

5. $y = -5x^2 + 9$
$y = -5(x - 0)^2 + 9$; $(0, 9)$; $x = 0$; down

6. $y = (x - 2)^2 - 18$
$y = (x - 2)^2 - 18$; $(2, -18)$; $x = 2$; up

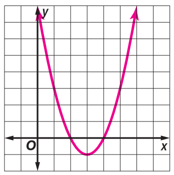
7. $y = x^2 - 2x - 5$
$y = (x - 1)^2 - 6$; $(1, -6)$; $x = 1$; up

8. $y = x^2 + 6x + 2$
$y = (x + 3)^2 - 7$; $(-3, -7)$; $x = -3$; up

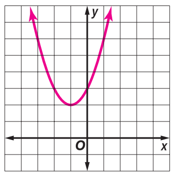
9. $y = -3x^2 + 24x$
$y = -3(x - 4)^2 + 48$; $(4, 48)$; $x = 4$; down

Graph each function.

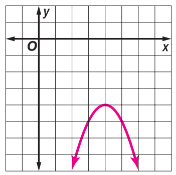
10. $y = (x - 3)^2 - 1$



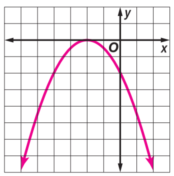
11. $y = (x + 1)^2 + 2$



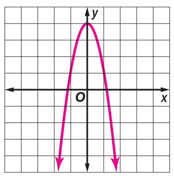
12. $y = -(x - 4)^2 - 4$



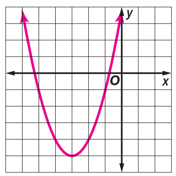
13. $y = -\frac{1}{2}(x + 2)^2$



14. $y = -3x^2 + 4$



15. $y = x^2 + 6x + 4$



Write an equation for the parabola with the given vertex that passes through the given point.

16. vertex: $(4, -36)$
point: $(0, -20)$
$y = (x - 4)^2 - 36$

17. vertex: $(3, -1)$
point: $(2, 0)$
$y = (x - 3)^2 - 1$

18. vertex: $(-2, 2)$
point: $(-1, 3)$
$y = (x + 2)^2 + 2$

NAME ______________________ DATE __________ PERIOD _____

6-6 Practice (Average)

Analyzing Graphs of Quadratic Functions

Write each quadratic function in vertex form, if not already in that form. Then identify the vertex, axis of symmetry, and direction of opening.

1. $y = -6(x + 2)^2 - 1$
$y = -6(x + 2)^2 - 1$; $(-2, -1)$; $x = -2$; down

2. $y = 2x^2 + 2$
$y = 2(x + 0)^2 + 2$; $(0, 2)$; $x = 0$; up

3. $y = -4x^2 + 8x$
$y = -4(x - 1)^2 + 4$; $(1, 4)$; $x = 1$; down

4. $y = x^2 + 10x + 20$
$y = (x + 5)^2 - 5$; $(-5, -5)$; $x = -5$; up

5. $y = 2x^2 + 12x + 18$
$y = 2(x + 3)^2$; $(-3, 0)$; $x = -3$; up

6. $y = 3x^2 - 6x + 5$
$y = 3(x - 1)^2 + 2$; $(1, 2)$; $x = 1$; up

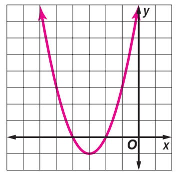
7. $y = -2x^2 - 16x - 32$
$y = -2(x + 4)^2$; $(-4, 0)$; $x = -4$; down

8. $y = -3x^2 + 18x - 21$
$y = -3(x - 3)^2 + 6$; $(3, 6)$; $x = 3$; down

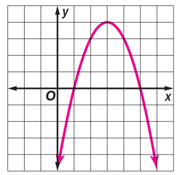
9. $y = 2x^2 + 16x + 29$
$y = 2(x + 4)^2 - 3$; $(-4, -3)$; $x = -4$; up

Graph each function.

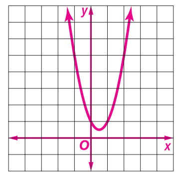
10. $y = (x + 3)^2 - 1$



11. $y = -x^2 + 6x - 5$



12. $y = 2x^2 - 2x + 1$



Write an equation for the parabola with the given vertex that passes through the given point.

13. vertex: $(1, 3)$
point: $(-2, -15)$
$y = -2(x - 1)^2 + 3$

14. vertex: $(-3, 0)$
point: $(3, 18)$
$y = \frac{1}{2}(x + 3)^2$

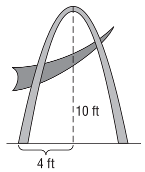
15. vertex: $(10, -4)$
point: $(5, 6)$
$y = \frac{2}{5}(x - 10)^2 - 4$

16. Write an equation for a parabola with vertex at $(4, 4)$ and x-intercept 6.
$y = -(x - 4)^2 + 4$

17. Write an equation for a parabola with vertex at $(-3, -1)$ and y-intercept 2.
$y = \frac{1}{3}(x + 3)^2 - 1$

18. BASEBALL The height h of a baseball t seconds after being hit is given by $h(t) = -16t^2 + 80t + 3$. What is the maximum height that the baseball reaches, and when does this occur? 103 ft; 2.5 s

19. SCULPTURE A modern sculpture in a park contains a parabolic arc that starts at the ground and reaches a maximum height of 10 feet after a horizontal distance of 4 feet. Write a quadratic function in vertex form that describes the shape of the outside of the arc, where y is the height of a point on the arc and x is its horizontal distance from the left-hand starting point of the arc. $y = -\frac{5}{8}(x - 4)^2 + 10$



NAME ______________________ DATE __________ PERIOD _____

6-6 Reading to Learn Mathematics

Analyzing Graphs of Quadratic Equations

Pre-Activity **How can the graph of $y = x^2$ be used to graph any quadratic function?**

Read the introduction to Lesson 6-6 at the top of page 322 in your textbook.

- What does adding a positive number to x^2 do to the graph of $y = x^2$? **It moves the graph up.**
- What does subtracting a positive number to x before squaring do to the graph of $y = x^2$? **It moves the graph to the right.**

Reading the Lesson

1. Complete the following information about the graph of $y = a(x - h)^2 + k$.

a. What are the coordinates of the vertex? **(h, k)**

b. What is the equation of the axis of symmetry? **$x = h$**

c. In which direction does the graph open if $a > 0$? If $a < 0$? **up; down**

d. What do you know about the graph if $|a| < 1$? **It is wider than the graph of $y = x^2$.**

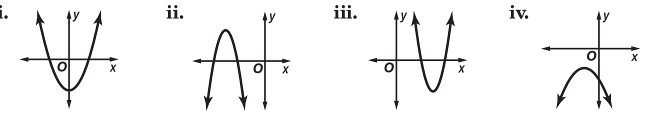
If $|a| > 1$? **It is narrower than the graph of $y = x^2$.**

2. Match each graph with the description of the constants in the equation in vertex form.

a. $a > 0, h > 0, k < 0$ **iii** **b.** $a < 0, h < 0, k < 0$ **iv**

c. $a < 0, h < 0, k > 0$ **ii** **d.** $a > 0, h = 0, k < 0$ **i**

i. **ii.** **iii.** **iv.**



Helping You Remember

3. When graphing quadratic functions such as $y = (x + 4)^2$ and $y = (x - 5)^2$, many students have trouble remembering which represents a translation of the graph of $y = x^2$ to the left and which represents a translation to the right. What is an easy way to remember this?
Sample answer: In functions like $y = (x + 4)^2$, the plus sign puts the graph "ahead" so that the vertex comes "sooner" than the origin and the translation is to the left. In functions like $y = (x - 5)^2$, the minus puts the graph "behind" so that the vertex comes "later" than the origin and the translation is to the right.

NAME ______________________ DATE __________ PERIOD _____

6-6 Enrichment

Patterns with Differences and Sums of Squares

Some whole numbers can be written as the difference of two squares and some cannot. Formulas can be developed to describe the sets of numbers algebraically.

If possible, write each number as the difference of two squares. Look for patterns.

1. 0 **$0^2 - 0^2$**	**2.** 1 **$1^2 - 0^2$**	**3.** 2 **cannot**	**4.** 3 **$2^2 - 1^2$**
5. 4 **$2^2 - 0^2$**	**6.** 5 **$3^2 - 2^2$**	**7.** 6 **cannot**	**8.** 7 **$4^2 - 3^2$**
9. 8 **$3^2 - 1^2$**	**10.** 9 **$3^2 - 0^2$**	**11.** 10 **cannot**	**12.** 11 **$6^2 - 5^2$**
13. 12 **$4^2 - 2^2$**	**14.** 13 **$7^2 - 6^2$**	**15.** 14 **cannot**	**16.** 15 **$4^2 - 1^2$**

Even numbers can be written as $2n$, where n is one of the numbers 0, 1, 2, 3, and so on. Odd numbers can be written $2n + 1$. Use these expressions for these problems.

17. Show that any odd number can be written as the difference of two squares.
$2n + 1 = (n + 1)^2 - n^2$

18. Show that the even numbers can be divided into two sets: those that can be written in the form $4n$ and those that can be written in the form $2 + 4n$.
Find $4n$ for $n = 0, 1, 2$, and so on. You get {0, 4, 8, 12, …}. For $2 + 4n$, you get {2, 6, 10, 12, …}. Together these sets include all even numbers.

19. Describe the even numbers that cannot be written as the difference of two squares. **$2 + 4n$, for $n = 0, 1, 2, 3, \ldots$**

20. Show that the other even numbers can be written as the difference of two squares. **$4n = (n + 1)^2 - (n - 1)^2$**

Every whole number can be written as the sum of squares. It is never necessary to use more than four squares. Show that this is true for the whole numbers from 0 through 15 by writing each one as the sum of the least number of squares.

21. 0 **0^2**	**22.** 1 **1^2**	**23.** 2 **$1^2 + 1^2$**
24. 3 **$1^2 + 1^2 + 1^2$**	**25.** 4 **2^2**	**26.** 5 **$1^2 + 2^2$**
27. 6 **$1^2 + 1^2 + 2^2$**	**28.** 7 **$1^2 + 1^2 + 1^2 + 2^2$**	**29.** 8 **$2^2 + 2^2$**
30. 9 **3^2**	**31.** 10 **$1^2 + 3^2$**	**32.** 11 **$1^2 + 1^2 + 3^2$**
33. 12 **$1^2 + 1^2 + 1^2 + 3^2$**	**34.** 13 **$2^2 + 3^2$**	**35.** 14 **$1^2 + 2^2 + 3^2$**
36. 15 **$1^2 + 1^2 + 2^2 + 3^2$**		

NAME ______________________ DATE __________ PERIOD _____

6-7 Study Guide and Intervention

Graphing and Solving Quadratic Inequalities

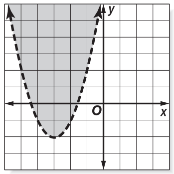
Graph Quadratic Inequalities To graph a quadratic inequality in two variables, use the following steps:

1. Graph the related quadratic equation, $y = ax^2 + bx + c$. Use a dashed line for $<$ or $>$; use a solid line for $\leq$ or $\geq$.
2. Test a point inside the parabola. If it satisfies the inequality, shade the region inside the parabola; otherwise, shade the region outside the parabola.

Example **Graph the inequality $y > x^2 + 6x + 7$.**

First graph the equation $y = x^2 + 6x + 7$. By completing the square, you get the vertex form of the equation $y = (x + 3)^2 - 2$, so the vertex is $(-3, -2)$. Make a table of values around $x = -3$, and graph. Since the inequality includes $>$, use a dashed line.

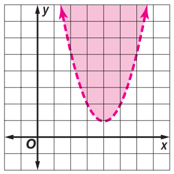
Test the point $(-3, 0)$, which is inside the parabola. Since $(-3)^2 + 6(-3) + 7 = -2$, and $0 > -2$, $(-3, 0)$ satisfies the inequality. Therefore, shade the region inside the parabola.



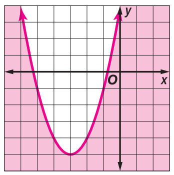
Exercises

Graph each inequality.

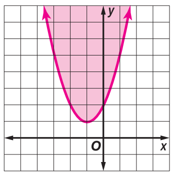
1. $y > x^2 - 8x + 17$



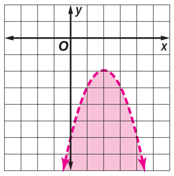
2. $y \leq x^2 + 6x + 4$



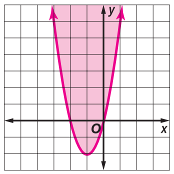
3. $y \geq x^2 + 2x + 2$



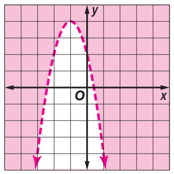
4. $y < -x^2 + 4x - 6$



5. $y \geq 2x^2 + 4x$



6. $y > -2x^2 - 4x + 2$



Lesson 6-7

NAME ______________________ DATE __________ PERIOD _____

6-7 Study Guide and Intervention *(continued)*

Graphing and Solving Quadratic Inequalities

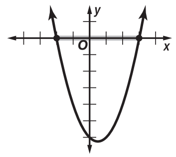
Solve Quadratic Inequalities Quadratic inequalities in one variable can be solved graphically or algebraically.

Graphical Method	To solve $ax^2 + bx + c < 0$: First graph $y = ax^2 + bx + c$. The solution consists of the x-values for which the graph is **below** the x-axis. To solve $ax^2 + bx + c > 0$: First graph $y = ax^2 + bx + c$. The solution consists the x-values for which the graph is **above** the x-axis.
Algebraic Method	Find the roots of the related quadratic equation by factoring, completing the square, or using the Quadratic Formula. 2 roots divide the number line into 3 intervals. Test a value in each interval to see which intervals are solutions.

If the inequality involves $\leq$ or $\geq$, the roots of the related equation are included in the solution set.

Example **Solve the inequality $x^2 - x - 6 \leq 0$.**

First find the roots of the related equation $x^2 - x - 6 = 0$. The equation factors as $(x - 3)(x + 2) = 0$, so the roots are 3 and -2. The graph opens up with x-intercepts 3 and -2, so it must be on or below the x-axis for $-2 \leq x \leq 3$. Therefore the solution set is $\{x \mid -2 \leq x \leq 3\}$.



Exercises

Solve each inequality.

1. $x^2 + 2x < 0$ $\{x \mid -2 < x < 0\}$

2. $x^2 - 16 < 0$ $\{x \mid -4 < x < 4\}$

3. $0 < 6x - x^2 - 5$ $\{x \mid 1 < x < 5\}$

4. $c^2 \leq 4$ $\{c \mid -2 \leq c \leq 2\}$

5. $2m^2 - m < 1$ $\left\{m \mid -\frac{1}{2} < m < 1\right\}$

6. $y^2 < -8$ $\varnothing$

7. $x^2 - 4x - 12 < 0$ $\{x \mid -2 < x < 6\}$

8. $x^2 + 9x + 14 > 0$ $\{x \mid x < -7 \text{ or } x > -2\}$

9. $-x^2 + 7x - 10 \geq 0$ $\{x \mid 2 \leq x \leq 5\}$

10. $2x^2 + 5x - 3 \leq 0$ $\left\{x \mid -3 \leq x \leq \frac{1}{2}\right\}$

11. $4x^2 - 23x + 15 > 0$ $\left\{x \mid x < \frac{3}{4} \text{ or } x > 5\right\}$

12. $-6x^2 - 11x + 2 < 0$ $\left\{x \mid x < -2 \text{ or } x > \frac{1}{6}\right\}$

13. $2x^2 - 11x + 12 \geq 0$ $\left\{x \mid x < \frac{3}{2} \text{ or } x > 4\right\}$

14. $x^2 - 4x + 5 < 0$ $\varnothing$

15. $3x^2 - 16x + 5 < 0$ $\left\{x \mid \frac{1}{3} < x < 5\right\}$

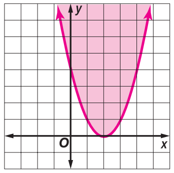
NAME ____________________ DATE __________ PERIOD _____

6-7 Skills Practice

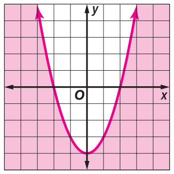
Graphing and Solving Quadratic Inequalities

Graph each inequality.

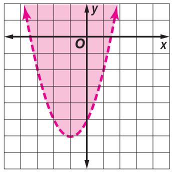
1. $y \geq x^2 - 4x + 4$



2. $y \leq x^2 - 4$

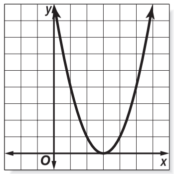


3. $y > x^2 + 2x - 5$



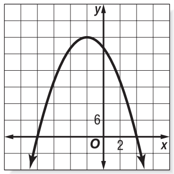
Use the graph of its related function to write the solutions of each inequality.

4. $x^2 - 6x + 9 \leq 0$



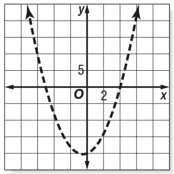
3

5. $-x^2 - 4x + 32 \geq 0$



$-8 \leq x \leq 4$

6. $x^2 + x - 20 > 0$



$x < -5$ or $x > 4$

Solve each inequality algebraically.

7. $x^2 - 3x - 10 < 0$ **$\{x \mid -2 < x < 5\}$**
8. $x^2 + 2x - 35 \geq 0$ **$\{x \mid x \leq -7 \text{ or } x \geq 5\}$**
9. $x^2 - 18x + 81 \leq 0$ **$\{x \mid x = 9\}$**
10. $x^2 \leq 36$ **$\{x \mid -6 \leq x \leq 6\}$**
11. $x^2 - 7x > 0$ **$\{x \mid x < 0 \text{ or } x > 7\}$**
12. $x^2 + 7x + 6 < 0$ **$\{x \mid -6 < x < -1\}$**
13. $x^2 + x - 12 > 0$ **$\{x \mid x < -4 \text{ or } x > 3\}$**
14. $x^2 + 9x + 18 \leq 0$ **$\{x \mid -6 \leq x \leq -3\}$**
15. $x^2 - 10x + 25 \geq 0$ **all reals**
16. $-x^2 - 2x + 15 \geq 0$ **$\{x \mid -5 \leq x \leq 3\}$**
17. $x^2 + 3x > 0$ **$\{x \mid x < -3 \text{ or } x > 0\}$**
18. $2x^2 + 2x > 4$ **$\{x \mid x < -2 \text{ or } x > 1\}$**
19. $-x^2 - 64 \leq -16x$ **all reals**
20. $9x^2 + 12x + 9 < 0$ **$\varnothing$**

Lesson 6-7

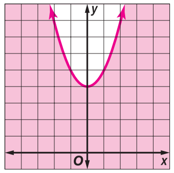
NAME ____________________ DATE __________ PERIOD _____

6-7 Practice (Average)

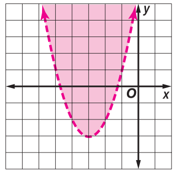
Graphing and Solving Quadratic Inequalities

Graph each inequality.

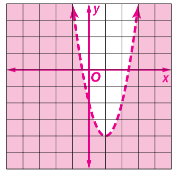
1. $y \leq x^2 + 4$



2. $y > x^2 + 6x + 6$

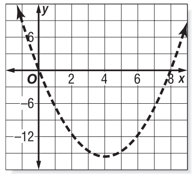


3. $y < 2x^2 - 4x - 2$



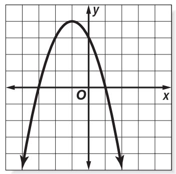
Use the graph of its related function to write the solutions of each inequality.

4. $x^2 - 8x > 0$



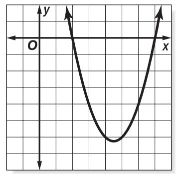
$x < 0$ or $x > 8$

5. $-x^2 - 2x + 3 \geq 0$



$-3 \leq x \leq 1$

6. $x^2 - 9x + 14 \leq 0$



$2 \leq x \leq 7$

Solve each inequality algebraically.

7. $x^2 - x - 20 > 0$ **$\{x \mid x < -4 \text{ or } x > 5\}$**
8. $x^2 - 10x + 16 < 0$ **$\{x \mid 2 < x < 8\}$**
9. $x^2 + 4x + 5 \leq 0$ **$\varnothing$**
10. $x^2 + 14x + 49 \geq 0$ **all reals**
11. $x^2 - 5x > 14$ **$\{x \mid x < -2 \text{ or } x > 7\}$**
12. $-x^2 - 15 \geq 8x$ **$\{x \mid -5 \leq x \leq -3\}$**
13. $-x^2 + 5x - 7 \leq 0$ **all reals**
14. $9x^2 + 36x + 36 \leq 0$ **$\{x \mid x = -2\}$**
15. $9x \leq 12x^2$ **$\left\{x \mid x \leq 0 \text{ or } x \geq \frac{3}{4}\right\}$**
16. $4x^2 + 4x + 1 > 0$ **$\left\{x \mid x \neq -\frac{1}{2}\right\}$**
17. $5x^2 + 10 \geq 27x$ **$\left\{x \mid x \leq \frac{2}{5} \text{ or } x \geq 5\right\}$**
18. $9x^2 + 31x + 12 \leq 0$ **$\left\{x \mid -3 \leq x \leq -\frac{4}{9}\right\}$**

19. **FENCING** Vanessa has 180 feet of fencing that she intends to use to build a rectangular play area for her dog. She wants the play area to enclose at least 1800 square feet. What are the possible widths of the play area? **30 ft to 60 ft**

20. **BUSINESS** A bicycle maker sold 300 bicycles last year at a profit of \$300 each. The maker wants to increase the profit margin this year, but predicts that each \$20 increase in profit will reduce the number of bicycles sold by 10. How many \$20 increases in profit can the maker add in and expect to make a total profit of at least \$100,000? **from 5 to 10**

Answers

NAME ______________________ DATE ________ PERIOD _____

6-7 Reading to Learn Mathematics

Graphing and Solving Quadratic Inequalities

Pre-Activity **How can you find the time a trampolinist spends above a certain height?**

Read the introduction to Lesson 6-7 at the top of page 329 in your textbook.

- How far above the ground is the trampoline surface? **3.75 feet**
- Using the quadratic function given in the introduction, write a quadratic inequality that describes the times at which the trampolinist is more than 20 feet above the ground. $-16t^2 + 42t + 3.75 > 20$

Reading the Lesson

1. Answer the following questions about how you would graph the inequality $y \geq x^2 + x - 6$.

a. What is the related quadratic equation? $y = x^2 + x - 6$

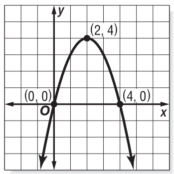
b. Should the parabola be solid or dashed? How do you know? **solid; The inequality symbol is ≥.**

c. The point (0, 2) is inside the parabola. To use this as a test point, substitute **0** for x and **2** for y in the quadratic inequality.

d. Is the statement $2 \geq 0^2 + 0 - 6$ true or false? **true**

e. Should the region inside or outside the parabola be shaded? **inside**

2. The graph of $y = -x^2 + 4x$ is shown at the right. Match each of the following related inequalities with its solution set.



a. $-x^2 + 4x > 0$ **ii** **i.** $\{x \mid x < 0 \text{ or } x > 4\}$

b. $-x^2 + 4x \leq 0$ **iii** **ii.** $\{x \mid 0 < x < 4\}$

c. $-x^2 + 4x \geq 0$ **iv** **iii.** $\{x \mid x \leq 0 \text{ or } x \geq 4\}$

d. $-x^2 + 4x < 0$ **i** **iv.** $\{x \mid 0 \leq x \leq 4\}$

Helping You Remember

3. A quadratic inequality in two variables may have the form $y > ax^2 + bx + c$, $y < ax^2 + bx + c$, $y \geq ax^2 + bx + c$, or $y \leq ax^2 + bx + c$. Describe a way to remember which region to shade by looking at the inequality symbol and without using a test point. **Sample answer: If the symbol is > or ≥, shade the region above the parabola. If the symbol is < or ≤, shade the region below the parabola.**

Lesson 6-7

NAME ______________________ DATE ________ PERIOD _____

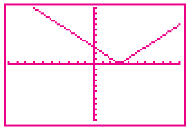
6-7 Enrichment

Graphing Absolute Value Inequalities

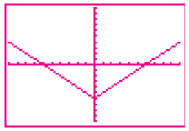
You can solve absolute value inequalities by graphing in much the same manner you graphed quadratic inequalities. Graph the related absolute function for each inequality by using a graphing calculator. For > and ≥, identify the x-values, if any, for which the graph lies *below* the x-axis. For < and ≤, identify the x values, if any, for which the graph lies *above* the x-axis.

For each inequality, make a sketch of the related graph and find the solutions rounded to the nearest hundredth.

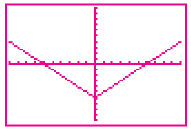
1. $|x - 3| > 0$
$x > 3$ or $x < 3$



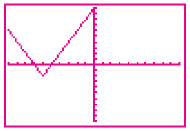
2. $|x| - 6 < 0$
$-6 < x < 6$



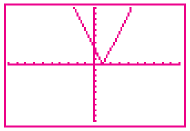
3. $-|x + 4| + 8 < 0$
$-12 < x < 4$



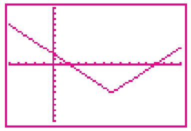
4. $2|x + 6| - 2 \geq 0$
$x \leq -7$ or $x \geq -5$



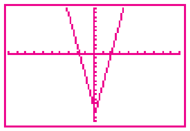
5. $|3x - 3| \geq 0$
all real numbers



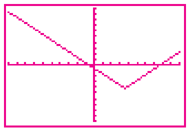
6. $|x - 7| < 5$
$2 < x < 12$



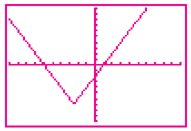
7. $|7x - 1| > 13$
$x < -1.71$ or $x > 2$



8. $|x - 3.6| \leq 4.2$
$-0.6 \leq x \leq 7.8$



9. $|2x + 5| \leq 7$
$-6 \leq x \leq 1$



Chapter 6 Assessment Answer Key

Form 1

Page 355

1. B
2. B
3. C
4. A
5. B
6. B
7. C
8. B
9. B
10. A
11. B

Page 356

12. D
13. D
14. A
15. C
16. D
17. B
18. C
19. B
20. C

B: 1 and 7; 14

Form 2A

Page 357

1. C
2. B
3. A
4. B
5. A
6. D
7. C
8. C
9. D
10. A

(continued on the next page)

Answers

Chapter 6 Assessment Answer Key

Form 2A *(continued)*
Page 358

11. B
12. D
13. C
14. D
15. D
16. A
17. B
18. A
19. C
20. D

B: Sample answer: $16x^2 + 3 = 0$

Form 2B
Page 359

1. B
2. C
3. B
4. D
5. C
6. A
7. D
8. B
9. A
10. C

Page 360

11. C
12. B
13. D
14. A
15. B
16. C
17. D
18. B
19. A
20. A

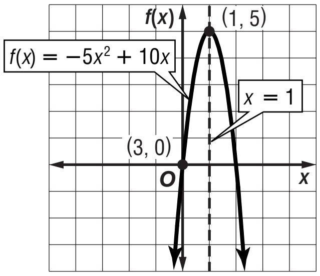
B: Sample answer: $9x^2 + 2 = 0$

Chapter 6 Assessment Answer Key

Form 2C

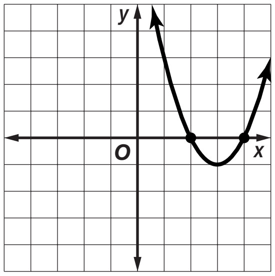
Page 361

1.

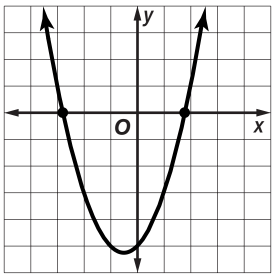


2. maximum; 4

3. 2, 4



4. between −2 and −1; between 1 and 2



5. $\left\{-3, \frac{2}{5}\right\}$

6. 9 in. by 16 in.

7. $4x^2 + 21x - 18 = 0$

8. $\{-8, 2\}$

9. $\left\{\frac{5 \pm \sqrt{7}}{2}\right\}$

Page 362

10. $\{-2 \pm \sqrt{13}\}$

11. $\left\{-2, \frac{1}{2}\right\}$

12. $\frac{3 \pm i\sqrt{31}}{10}$

13. 0; 1 real, rational root

14. 33; 2 real, irrational roots

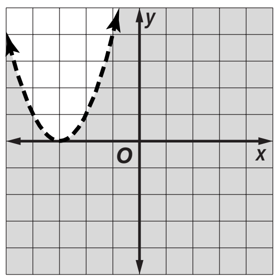
15. $(-5, -7)$; $x = -5$; down

16. $y = \frac{3}{2}(x - 2)^2 - 1$

17. $y = (x - 3)^2 - 1$

18. $h(t) = -16(t - 1.5)^2 + 51$; 51 ft

19.



20. $\left\{x \mid x \leq -\frac{1}{2} \text{ or } x \geq 3\right\}$

B: $9x^2 - 7 = 0$

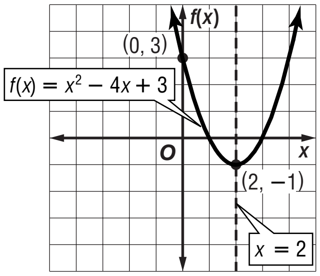
Answers

Chapter 6 Assessment Answer Key

Form 2D

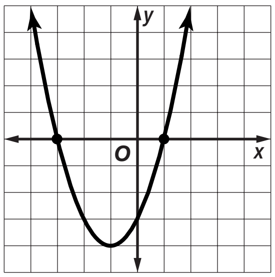
Page 363

1.

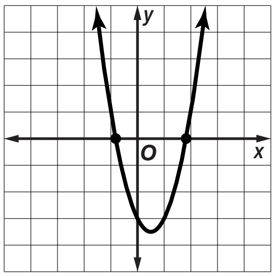


2. minimum; -17

3. $1, -3$



4. between -1 and 0; between 1 and 2



5. $\left\{-1, \frac{4}{3}\right\}$

6. 8 in. by 18 in.

7. $2x^2 + 5x - 12$

Page 364

8. $\{3, 11\}$

9. $\left\{\frac{-2 \pm \sqrt{6}}{3}\right\}$

10. $\{4 \pm \sqrt{2}\}$

11. $\left\{-1, \frac{2}{3}\right\}$

12. $\frac{9 \pm \sqrt{41}}{4}$

13. 0; 1 real, rational root

14. -8; 2 complex roots

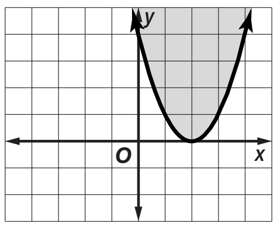
15. $(6, -5)$; $x = 6$; down

16. $y = -\frac{1}{4}(x + 4)^2 + 2$

17. $y = (x + 2)^2 + 4$

18. $h(t) = -16(t - 2)^2 + 76$; 76 ft

19.



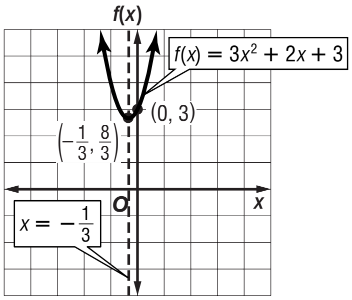
20. $\left\{x \mid -\frac{3}{2} < x < 5\right\}$

B: $16x^2 - 5 = 0$

Chapter 6 Assessment Answer Key

Form 3

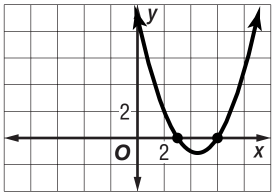
Page 365

1. 

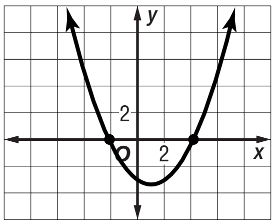
2. minimum; $\frac{22}{25}$

3. \$8.00; \$6400

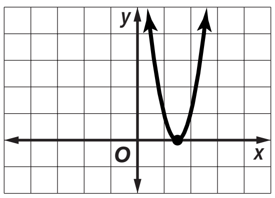
4. 3, 6



5. between −3 and −2; between 4 and 5



6. between 1 and 2



7. $\left\{\frac{1}{2}, \frac{5}{3}\right\}$

8. $12x^2 - 13x - 14 = 0$

9. $x = -1$

Page 366

10. $\{-0.35, 0.85\}$

11. $\left\{\frac{5 \pm i\sqrt{39}}{8}\right\}$

12. $\{-3.5, 1\}$

13. $6 \pm 4\sqrt{2}$

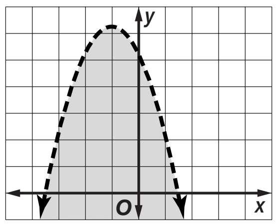
14. 1.2; two real, irrational roots

15. $-2 < k < 2$

16. $y = -\frac{3}{5}\left(x + \frac{7}{2}\right)^2 - \frac{1}{2}$; $\left(-\frac{7}{2}, -\frac{1}{2}\right)$; $x = -\frac{7}{2}$; down

17. $h(t) = -9.1(t - 32.5)^2 + 30{,}000$; 30,000 ft

18. $y = -\frac{29}{200}(x + 9)^2 + \frac{29}{2}$

19. 

20. $\left\{x \mid x \leq -\frac{7}{2} \text{ or } x = 1\right\}$

B: $16x^2 + 24x + 29 = 0$

Answers

Chapter 6 Assessment Answer Key

Page 367, Open-Ended Assessment
Scoring Rubric

Score	General Description	Specific Criteria
4	**Superior** A correct solution that is supported by well-developed, accurate explanations	• Shows thorough understanding of the concepts of *graphing, analyzing, and finding the maximum and minimum values of quadratic functions; solving quadratic equations;* and *solving inequalities.* • Uses appropriate strategies to solve problems. • Computations are correct. • Written explanations are exemplary. • Goes beyond requirements of some or all problems.
3	**Satisfactory** A generally correct solution, but may contain minor flaws in reasoning or computation	• Shows an understanding of the concepts of *graphing, analyzing, and finding the maximum and minimum values of quadratic functions; solving quadratic equations;* and *solving inequalities.* • Uses appropriate strategies to solve problems. • Computations are mostly correct. • Written explanations are effective. • Satisfies all requirements of problems.
2	**Nearly Satisfactory** A partially correct interpretation and/or solution to the problem	• Shows an understanding of most of the concepts of *graphing, analyzing, and finding the maximum and minimum values of quadratic functions; solving quadratic equations;* and *solving inequalities.* • May not use appropriate strategies to solve problems. • Computations are mostly correct. • Written explanations are satisfactory. • Satisfies the requirements of most of the problems.
1	**Nearly Unsatisfactory** A correct solution with no supporting evidence or explanation	• Final computation is correct. • No written explanations or work is shown to substantiate the final computation. • Satisfies minimal requirements of some of the problems.
0	**Unsatisfactory** An incorrect solution indicating no mathematical understanding of the concept or task, or no solution is given	• Shows little or no understanding of most of the concepts of *graphing, analyzing, and finding the maximum and minimum values of quadratic functions; solving quadratic equations;* and *solving inequalities.* • Does not use appropriate strategies to solve problems. • Computations are incorrect. • Written explanations are unsatisfactory. • Does not satisfy requirements of problems. • No answer may be given.

Chapter 6 Assessment Answer Key

Page 367, Open-Ended Assessment
Sample Answers

In addition to the scoring rubric found on page A28, the following sample answers may be used as guidance in evaluating open-ended assessment items.

1. Student responses should indicate that using the Square Root Property, as Mi-Ling's group did, would take less time than the other two methods since the equation is already set up as a perfect square set equal to a constant. To solve using either of the other two methods, the binomial would need to be expanded and the constant on the right brought to the left side of the equal sign.

2a. Jocelyn had trouble because the problem is impossible. No such parabola exists.

2b. Student responses will vary. One of the three conditions must be omitted or modified. Sample answer: Delete "...and passes through $(-1, 0)$."

2c. Answers will vary and depend on the answer for part **b**. For example, for the sample answer in part **b** above, a possible equation is: $y = -2(x + 3)^2 - 4$.

3a. Answer must be of the form $y = a(x - h)^2 + 8$ where h is any real number and $a < 0$.

3b. Answers must be of the form $y = a[x - (h + n)]^2 + 8$ where h and a represent the same values as in part **a**. The student choice is for the value of n. The student should indicate that the graph will shift to the left n units if his or her value of n is negative, but will shift the graph to the right n units if the chosen value of n is positive.

4. Students should indicate that Joseph's answer is not correct. In Step 2, when he completed the square by inserting $+9$ inside the parentheses, he actually added $2(9) = 18$ to the right side of the equation, so he must subtract 18 from the constant on the same side, rather than add 9, to keep the statements equivalent. The correct solution is $f(x) = 2(x + 3)^2 - 23$.

5a. $>$; The graph is strictly above the x-axis for all values of x other than 2.

5b. $<$; The graph is never below the x-axis.

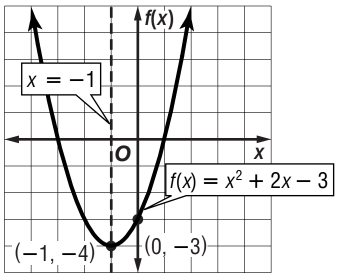
5c. $\leq$; The graph is always on or above the x-axis.

Chapter 6 Assessment Answer Key

Vocabulary Test/Review
Page 368

1. false; Zero Product Property
2. false; constant term
3. false; quadratic inequality
4. false; roots
5. true
6. false; minimum value
7. false; quadratic term
8. false; (the) Quadratic Formula
9. true
10. false; discriminant
11. Sample answer: A parabola is a smooth curve that is the graph of a quadratic function.
12. Sample answer: An axis of symmetry is a line along which you can fold a graph and get matching parts on both sides of the line.

Quiz (Lessons 6–1 and 6–2)
Page 369

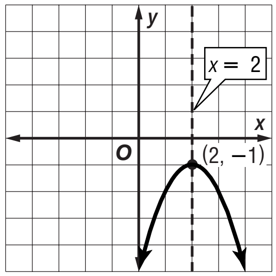
1. -3; $x = -1$; -1
2. 

3. minimum; 1
4. 3, -1
5. between 1 and 2; between -6 and -5

Quiz (Lessons 6–3 and 6–4)
Page 369

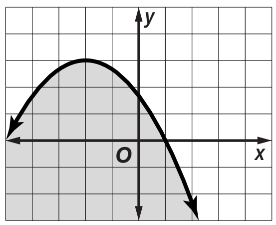
1. $\left\{-5, \frac{2}{3}\right\}$
2. $\{-9, 5\}$
3. B
4. $x^2 + 4x - 12 = 0$
5. $3x^2 + 10x - 8 = 0$
6. $\{-10, 2\}$
7. $\{1 \pm 3\sqrt{5}\}$
8. $\left\{\frac{-2 \pm \sqrt{3}}{5}\right\}$
9. $\{-1, 11\}$
10. $\{2 \pm i\sqrt{14}\}$

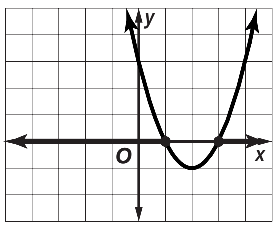
Quiz (Lessons 6–5 and 6–6)
Page 370

1. $2 \pm \sqrt{5}$
2. -96; 2 complex roots
3. 

4. $y = -3(x - 2)^2 + 6$
5. $y = 2(x + 5)^2$

Quiz (Lesson 6–7)
Page 370

1. 

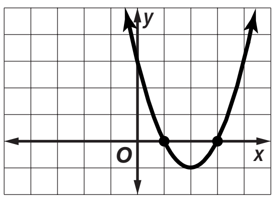
2. $\{x \mid 1 < x < 5\}$
3. 

$\{x \mid x \leq 1 \text{ or } x \geq 3\}$
4. all reals

Chapter 6 Assessment Answer Key

Mid-Chapter Test
Page 371

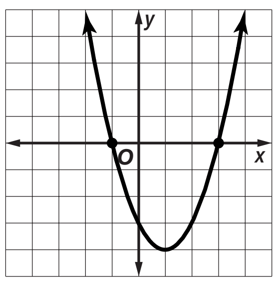
1. B
2. B
3. A
4. C
5. D
6. 1, 3



7. minimum; $-9\frac{1}{2}$
8. $\{-2, 9\}$
9. $\left\{0, \frac{1}{4}\right\}$
10. $\left\{\frac{-1 \pm \sqrt{5}}{3}\right\}$

Cumulative Review
Page 372

1. 17
2. $-\frac{3}{4}$
3. inconsistent
4. (−2, 0), (−2, 8), (0, −2), (8, −2)
5. 92
6. (2, −3)
7. $2x^3 - x^2 + 2x + 4 - \frac{8}{x + 3}$
8. 5.599
9. 51
10. 136 ft; 1.5 s
11. −1, 3



12. $\{-2, 3\}$
13. −3, 2 complex roots
14. $y = \left(x - \frac{7}{2}\right)^2 - \frac{29}{4}$

Chapter 6 Assessment Answer Key

Standardized Test Practice

Page 373

1. A **B** C D

2. **E** F G H

3. A B C **D**

4. E **F** G H

5. A B **C** D

6. **E** F G H

7. A B C **D**

8. E F G **H**

9. A **B** C D

10. **E** F G H

Page 374

11. 49/4

12. 110

13. 154

14. 80

15. A B **C** D

16. A **B** C D

17. A B C **D**