GLENCOE MATHEMATICS

Algebra 2

Chapter 13
Resource Masters

Glencoe McGraw-Hill

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Consumable Workbooks

Many of the worksheets contained in the Chapter Resource Masters booklets are available as consumable workbooks.

Study Guide and Intervention Workbook	0-07-828029-X
Skills Practice Workbook	0-07-828023-0
Practice Workbook	0-07-828024-9

ANSWERS FOR WORKBOOKS The answers for Chapter 13 of these workbooks can be found in the back of this Chapter Resource Masters booklet.

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Algebra 2
Chapter 13 Resource Masters

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Contents

Teacher's Guide to Using the Chapter 13 Resource Masters

The ***Fast File*** Chapter Resource system allows you to conveniently file the resources you use most often. The *Chapter 13 Resource Masters* includes the core materials needed for Chapter 13. These materials include worksheets, extensions, and assessment options. The answers for these pages appear at the back of this booklet.

All of the materials found in this booklet are included for viewing and printing in the *Algebra 2 TeacherWorks* CD-ROM.

Vocabulary Builder Pages vii–viii include a student study tool that presents up to twenty of the key vocabulary terms from the chapter. Students are to record definitions and/or examples for each term. You may suggest that students highlight or star the terms with which they are not familiar.

WHEN TO USE Give these pages to students before beginning Lesson 13-1. Encourage them to add these pages to their Algebra 2 Study Notebook. Remind them to add definitions and examples as they complete each lesson.

Study Guide and Intervention

Each lesson in *Algebra 2* addresses two objectives. There is one Study Guide and Intervention master for each objective.

WHEN TO USE Use these masters as reteaching activities for students who need additional reinforcement. These pages can also be used in conjunction with the Student Edition as an instructional tool for students who have been absent.

Skills Practice There is one master for each lesson. These provide computational practice at a basic level.

WHEN TO USE These masters can be used with students who have weaker mathematics backgrounds or need additional reinforcement.

Practice There is one master for each lesson. These problems more closely follow the structure of the Practice and Apply section of the Student Edition exercises. These exercises are of average difficulty.

WHEN TO USE These provide additional practice options or may be used as homework for second day teaching of the lesson.

Reading to Learn Mathematics

One master is included for each lesson. The first section of each master asks questions about the opening paragraph of the lesson in the Student Edition. Additional questions ask students to interpret the context of and relationships among terms in the lesson. Finally, students are asked to summarize what they have learned using various representation techniques.

WHEN TO USE This master can be used as a study tool when presenting the lesson or as an informal reading assessment after presenting the lesson. It is also a helpful tool for ELL (English Language Learner) students.

Enrichment There is one extension master for each lesson. These activities may extend the concepts in the lesson, offer an historical or multicultural look at the concepts, or widen students' perspectives on the mathematics they are learning. These are not written exclusively for honors students, but are accessible for use with all levels of students.

WHEN TO USE These may be used as extra credit, short-term projects, or as activities for days when class periods are shortened.

Assessment Options

The assessment masters in the *Chapter 13 Resource Masters* offer a wide range of assessment tools for intermediate and final assessment. The following lists describe each assessment master and its intended use.

Chapter Assessment

CHAPTER TESTS

- *Form 1* contains multiple-choice questions and is intended for use with basic level students.
- *Forms 2A and 2B* contain multiple-choice questions aimed at the average level student. These tests are similar in format to offer comparable testing situations.
- *Forms 2C and 2D* are composed of free-response questions aimed at the average level student. These tests are similar in format to offer comparable testing situations. Grids with axes are provided for questions assessing graphing skills.
- *Form 3* is an advanced level test with free-response questions. Grids without axes are provided for questions assessing graphing skills.

  All of the above tests include a free-response Bonus question.
- The **Open-Ended Assessment** includes performance assessment tasks that are suitable for all students. A scoring rubric is included for evaluation guidelines. Sample answers are provided for assessment.
- A **Vocabulary Test**, suitable for all students, includes a list of the vocabulary words in the chapter and ten questions assessing students' knowledge of those terms. This can also be used in conjunction with one of the chapter tests or as a review worksheet.

Intermediate Assessment

- Four free-response **quizzes** are included to offer assessment at appropriate intervals in the chapter.
- A **Mid-Chapter Test** provides an option to assess the first half of the chapter. It is composed of both multiple-choice and free-response questions.

Continuing Assessment

- The **Cumulative Review** provides students an opportunity to reinforce and retain skills as they proceed through their study of Algebra 2. It can also be used as a test. This master includes free-response questions.
- The **Standardized Test Practice** offers continuing review of algebra concepts in various formats, which may appear on the standardized tests that they may encounter. This practice includes multiple-choice, grid-in, and quantitative-comparison questions. Bubble-in and grid-in answer sections are provided on the master.

Answers

- Page A1 is an answer sheet for the Standardized Test Practice questions that appear in the Student Edition on pages 758–759. This improves students' familiarity with the answer formats they may encounter in test taking.
- The answers for the lesson-by-lesson masters are provided as reduced pages with answers appearing in red.
- Full-size answer keys are provided for the assessment masters in this booklet.

NAME ____ DATE ____________ PERIOD _____

13 Reading to Learn Mathematics

Vocabulary Builder

This is an alphabetical list of the key vocabulary terms you will learn in Chapter 13. As you study the chapter, complete each term's definition or description. Remember to add the page number where you found the term. Add these pages to your Algebra Study Notebook to review vocabulary at the end of the chapter.

Vocabulary Term	Found on Page	Definition/Description/Example
angle of depression or elevation		
Arccosine function AHRK·KOH·SYN		
Arcsine function AHRK·SYN		
Arctangent function AHRK·TAN·juhnt		
cosecant KOH·SEE·KANT		
cosine		
coterminal angles		
cotangent		
Law of Cosines		
Law of Sines		

(continued on the next page)

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13

Reading to Learn Mathematics

Vocabulary Builder *(continued)*

Vocabulary Term	Found on Page	Definition/Description/Example
period		
principal values		
quadrantal angles kwah·DRAN·tuhl		
radian RAY·dee·uhn		
reference angle		
secant		
sine		
standard position		
tangent		
trigonometry TRIH·guh·NAH·muh·tree		

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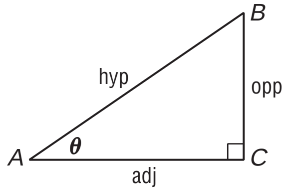
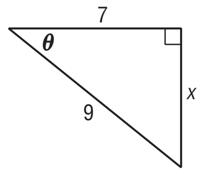
13-1 Study Guide and Intervention

Right Triangle Trigonometry

Trigonometric Values

Trigonometric Functions	If θ is the measure of an acute angle of a right triangle, *opp* is the measure of the leg opposite θ, *adj* is the measure of the leg adjacent to θ, and *hyp* is the measure of the hypotenuse, then the following are true.
	$\sin \theta = \frac{\text{opp}}{\text{hyp}}$ $\cos \theta = \frac{\text{adj}}{\text{hyp}}$ $\tan \theta = \frac{\text{opp}}{\text{adj}}$ $\csc \theta = \frac{\text{hyp}}{\text{opp}}$ $\sec \theta = \frac{\text{hyp}}{\text{adj}}$ $\cot \theta = \frac{\text{adj}}{\text{opp}}$

Example **Find the values of the six trigonometric functions for angle θ.**



Use the Pythagorean Theorem to find x, the measure of the leg opposite θ.

$x^2 + 7^2 = 9^2$ Pythagorean Theorem

$x^2 + 49 = 81$ Simplify.

$x^2 = 32$ Subtract 49 from each side.

$x = \sqrt{32}$ or $4\sqrt{2}$ Take the square root of each side.

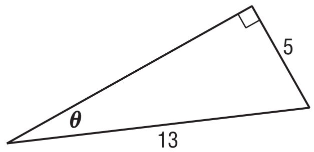
Use opp $= 4\sqrt{2}$, adj $= 7$, and hyp $= 9$ to write each trigonometric ratio.

$\sin \theta = \frac{4\sqrt{2}}{9}$ $\cos \theta = \frac{7}{9}$ $\tan \theta = \frac{4\sqrt{2}}{7}$ $\csc \theta = \frac{9\sqrt{2}}{8}$ $\sec \theta = \frac{9}{7}$ $\cot \theta = \frac{7\sqrt{2}}{8}$

Exercises

Find the values of the six trigonometric functions for angle θ.

1.



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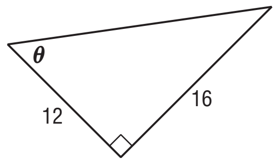

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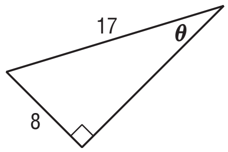

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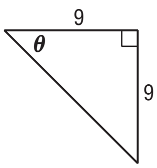

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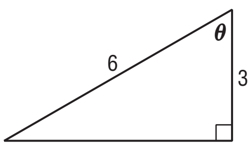

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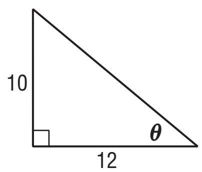


Lesson 13-1

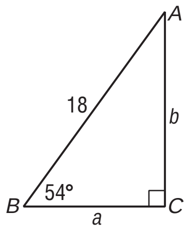
NAME ______________________________________ DATE __________ PERIOD _____

13-1 Study Guide and Intervention *(continued)*

Right Triangle Trigonometry

Right Triangle Problems

Example **Solve $\triangle ABC$. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.**



You know the measures of one side, one acute angle, and the right angle. You need to find a, b, and A.

Find a and b.

$\sin 54° = \frac{b}{18}$ $\qquad$ $\cos 54° = \frac{a}{18}$

$b = 18 \sin 54°$ $\qquad$ $a = 18 \cos 54°$

$b \approx 14.6$ $\qquad$ $a \approx 10.6$

Find A.

$54° + A = 90°$ $\quad$ Angles A and B are complementary.

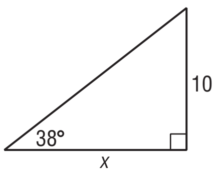
$A = 36°$ $\quad$ Solve for A.

Therefore $A = 36°$, $a \approx 10.6$, and $b \approx 14.6$.

Exercises

Write an equation involving sin, cos, or tan that can be used to find x. Then solve the equation. Round measures of sides to the nearest tenth.

1.



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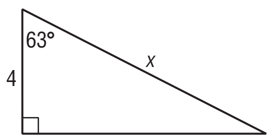

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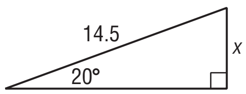
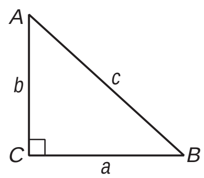


Solve $\triangle ABC$ by using the given measurements. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.



4. $A = 80°$, $b = 6$

5. $B = 25°$, $c = 20$

6. $b = 8$, $c = 14$

7. $a = 6$, $b = 7$

8. $a = 12$, $B = 42°$

9. $a = 15$, $A = 54°$

NAME ______________________________ DATE ____________ PERIOD _____

13-1 Skills Practice

Right Triangle Trigonometry

Find the values of the six trigonometric functions for angle θ.

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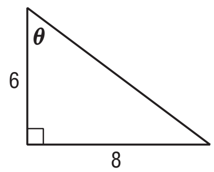

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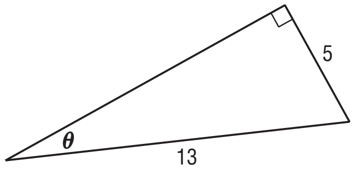

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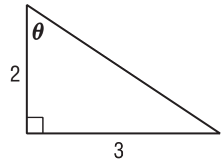
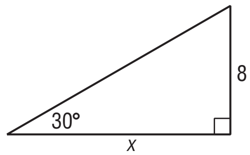


Write an equation involving sin, cos, or tan that can be used to find *x*. Then solve the equation. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.

4.



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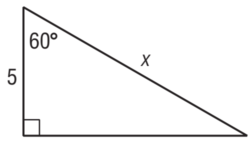

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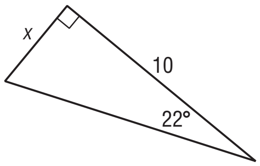

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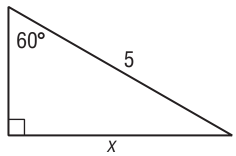

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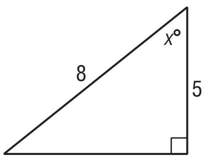

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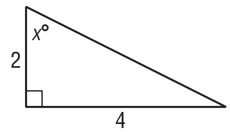
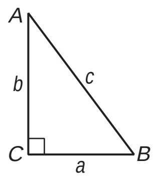


Solve $\triangle ABC$ by using the given measurements. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.



10. $A = 72°, c = 10$

11. $B = 20°, b = 15$

12. $A = 80°, a = 9$

13. $A = 58°, b = 12$

14. $b = 4, c = 9$

15. $a = 7, b = 5$

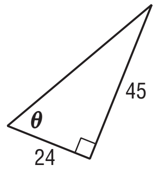
NAME ______________________________ DATE ____________ PERIOD _____

13-1 Practice

Right Triangle Trigonometry

Find the values of the six trigonometric functions for angle θ.

1.



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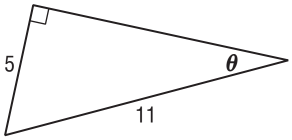

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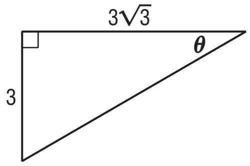
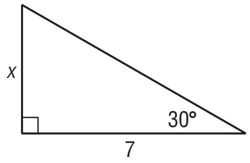


Write an equation involving sin, cos, or tan that can be used to find x. Then solve the equation. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.

4.



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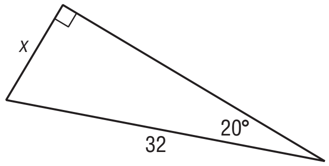

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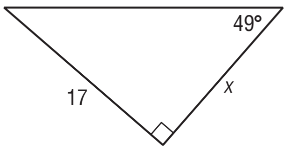

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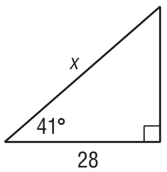

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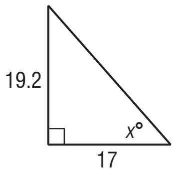

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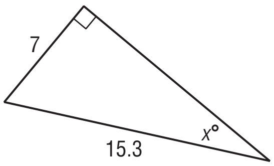
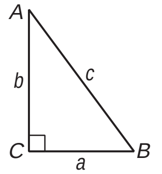


Solve $\triangle ABC$ by using the given measurements. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.



10. $A = 35°, a = 12$

11. $B = 71°, b = 25$

12. $B = 36°, c = 8$

13. $a = 4, b = 7$

14. $A = 17°, c = 3.2$

15. $b = 52, c = 95$

16. SURVEYING John stands 150 meters from a water tower and sights the top at an angle

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13-1 Reading to Learn Mathematics

Right Triangle Trigonometry

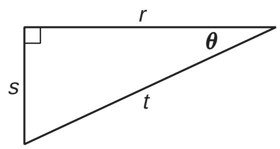
Pre-Activity **How is trigonometry used in building construction?**

Read the introduction to Lesson 13-1 at the top of page 701 in your textbook.

If a different ramp is built so that the angle shown in the figure has a tangent of $\frac{1}{14}$, will this ramp meet, exceed, or fail to meet ADA regulations?

Reading the Lesson

1. Refer to the triangle at the right. Match each trigonometric function with the correct ratio.



i. $\frac{r}{t}$ **ii.** $\frac{r}{s}$ **iii.** $\frac{t}{r}$ **iv.** $\frac{s}{t}$ **v.** $\frac{s}{r}$ **vi.** $\frac{t}{s}$

a. sin θ **b.** tan θ **c.** sec θ

d. cot θ **e.** cos θ **f.** csc θ

2. Refer to the Key Concept box on page 703 in your textbook. Use the drawings of the 30°-60°-90° triangle and 45°-45°-90° triangle and/or the table to complete the following.

a. The tangent of 45° and the _________________ of 45° are equal.

b. The sine of 30° is equal to the cosine of _________________.

c. The sine and _________________ of 45° are equal.

d. The reciprocal of the cosecant of 60° is the _________________ of 60°.

e. The reciprocal of the cosine of 30° is the _________________ of 60°.

f. The reciprocal of the tangent of 60° is the _________________ of 30°.

Helping You Remember

3. In studying trigonometry, it is important for you to know the relationships between the lengths of the sides of a 30°-60°-90° triangle. If you remember just one fact about this triangle, you will always be able to figure out the lengths of all the sides. What fact can you use, and why is it enough?

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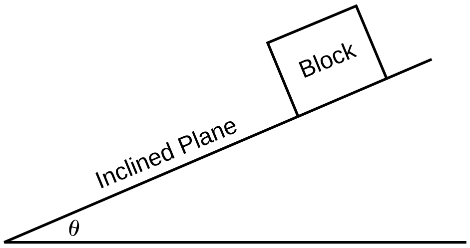
13-1 Enrichment

The Angle of Repose

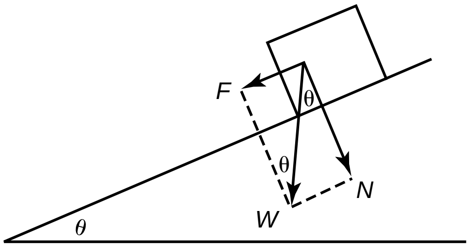
Suppose you place a block of wood on an inclined plane, as shown at the right. If the angle, θ, at which the plane is inclined from the horizontal is very small, the block will not move. If you increase the angle, the block will eventually overcome the force of friction and start to slide down the plane.



At the instant the block begins to slide, the angle formed by the plane is called the angle of friction, or the angle of repose.

For situations in which the block and plane are smooth but unlubricated, the angle of repose depends *only* on the types of materials in the block and the plane. The angle is independent of the area of contact between the two surfaces and of the weight of the block.

The drawing at the right shows how to use vectors to find a coefficient of friction. This coefficient varies with different materials and is denoted by the Greek leter mu, μ.



$$F = W \sin \theta \qquad N = W \cos$$

$$F = \mu N$$

$$\mu = \frac{\sin \theta}{\cos \theta} = \tan \theta$$

Solve each problem.

Material	Coefficient of Friction μ
Wood on wood	0.5
Wood on stone	0.5
Rubber tire on dry concrete	1.0
Rubber tire on wet concrete	0.7

1. A wooden chute is built so that wooden crates can slide down into the basement of a store. What angle should the chute make in order for the crates to slide down at a constant speed?

2. Will a 100-pound wooden crate slide down a stone ramp that makes an angle of 20° with the horizontal? Explain your answer.

3. If you increase the weight of the crate in Exercise 2 to 300 pounds, does it change your answer?

4. A car with rubber tires is being driven on dry concrete pavement. If the car tires spin without traction on a hill, how steep is the hill?

5. For Exercise 4, does it make a difference if it starts to rain? Explain your answer.

13-2 Study Guide and Intervention

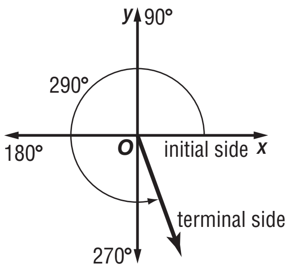
Angles and Angle Measurement

Angle Measurement An angle is determined by two rays. The degree measure of an angle is described by the amount and direction of rotation from the **initial side** along the positive *x*-axis to the **terminal side**. A counterclockwise rotation is associated with positive angle measure and a clockwise rotation is associated with negative angle measure. An angle can also be measured in **radians**.

Radian and Degree Measure	To rewrite the radian measure of an angle in degrees, multiply the number of radians by $\frac{180°}{\pi \text{ radians}}$. To rewrite the degree measure of an angle in radians, multiply the number of degrees by $\frac{\pi \text{ radians}}{180°}$.

Example 1 **Draw an angle with measure 290° in standard notation.**

The negative *y*-axis represents a positive rotation of 270°. To generate an angle of 290°, rotate the terminal side 20° more in the counterclockwise direction.



Example 2 **Rewrite the degree measure in radians and the radian measure in degrees.**

a. 45°

$$45° = 45°\left(\frac{\pi \text{ radians}}{180°}\right) = \frac{\pi}{4} \text{ radians}$$

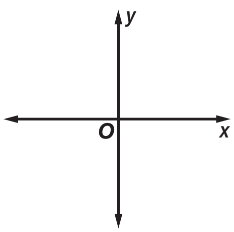
b. $\frac{5\pi}{3}$ radians

$$\frac{5\pi}{3} \text{ radians} = \frac{5\pi}{3}\left(\frac{180°}{\pi}\right) = 300°$$

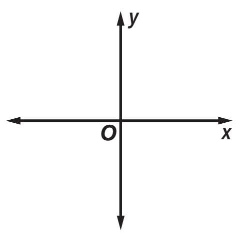
Exercises

Draw an angle with the given measure in standard position.

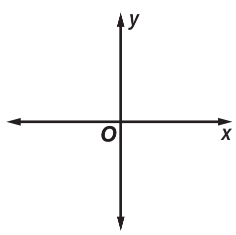
1. 160°



2. $-\frac{5\pi}{4}$



3. 400°



Rewrite each degree measure in radians and each radian measure in degrees.

4. 140°

5. −860°

6. $-\frac{3\pi}{5}$

7. $\frac{11\pi}{3}$

13-2 Study Guide and Intervention *(continued)*

Angles and Angle Measurement

Coterminal Angles When two angles in standard position have the same terminal sides, they are called **coterminal angles**. You can find an angle that is coterminal to a given angle by adding or subtracting a multiple of 360°. In radian measure, a coterminal angle is found by adding or subtracting a multiple of 2π.

Example **Find one angle with positive measure and one angle with negative measure coterminal with each angle.**

a. 250°

A positive angle is $250° + 360°$ or $610°$.

A negative angle is $250° - 360°$ or $-110°$.

b. $\mathbf{\frac{5\pi}{8}}$

A positive angle is $\frac{5\pi}{8} + 2\pi$ or $\frac{21\pi}{8}$.

A negative angle is $\frac{5\pi}{8} - 2\pi$ or $-\frac{11\pi}{8}$.

Exercises

Find one angle with a positive measure and one angle with a negative measure coterminal with each angle.

1. 65° **2.** −75° **3.** 230°

4. 420° **5.** 340° **6.** −130°

7. −290° **8.** 690° **9.** −420°

10. $\frac{\pi}{9}$ **11.** $\frac{3\pi}{8}$ **12.** $\frac{6\pi}{5}$

13. $\frac{-7\pi}{4}$ **14.** $\frac{15\pi}{4}$ **15.** $\frac{-13\pi}{6}$

16. $\frac{17\pi}{5}$ **17.** $\frac{-5\pi}{3}$ **18.** $\frac{-11\pi}{4}$

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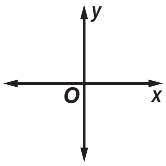
13-2 Skills Practice

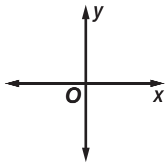
Angles and Angle Measure

Draw an angle with the given measure in standard position.

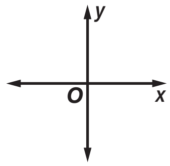
1. 185°



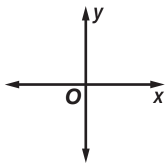
2. 810°



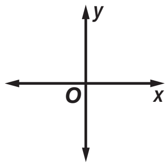
3. 390°



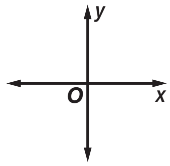
4. 495°



5. −50°



6. −420°



Rewrite each degree measure in radians and each radian measure in degrees.

7. 130°

8. 720°

9. 210°

10. 90°

11. −30°

12. −270°

13. $\frac{\pi}{3}$

14. $\frac{5\pi}{6}$

15. $\frac{2\pi}{3}$

16. $\frac{5\pi}{4}$

17. $-\frac{3\pi}{4}$

18. $-\frac{7\pi}{6}$

Find one angle with positive measure and one angle with negative measure coterminal with each angle.

19. 45°

20. 60°

21. 370°

22. −90°

23. $\frac{2\pi}{3}$

24. $\frac{5\pi}{2}$

25. $\frac{\pi}{6}$

26. $-\frac{3\pi}{4}$

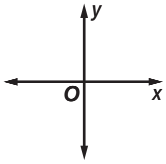
NAME ______________________________ DATE ____________ PERIOD _____

13-2 Practice

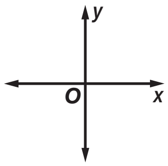
Angles and Angle Measure

Draw an angle with the given measure in standard position.

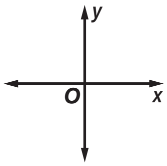
1. 210°



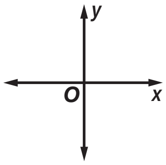
2. 305°



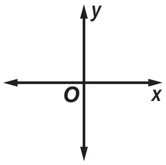
3. 580°



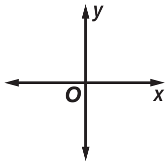
4. 135°



5. −450°



6. −560°



Rewrite each degree measure in radians and each radian measure in degrees.

7. 18° **8.** 6° **9.** 870° **10.** 347°

11. −72° **12.** −820° **13.** −250° **14.** −165°

15. 4π **16.** $\frac{5\pi}{2}$ **17.** $\frac{13\pi}{5}$ **18.** $\frac{13\pi}{30}$

19. $-\frac{9\pi}{2}$ **20.** $-\frac{7\pi}{12}$ **21.** $-\frac{3\pi}{8}$ **22.** $-\frac{3\pi}{16}$

Find one angle with positive measure and one angle with negative measure coterminal with each angle.

23. 65° **24.** 80° **25.** 285°

26. 110° **27.** −37° **28.** −93°

29. $\frac{2\pi}{5}$ **30.** $\frac{5\pi}{6}$ **31.** $\frac{17\pi}{6}$

32. $-\frac{3\pi}{2}$ **33.** $-\frac{\pi}{4}$ **34.** $-\frac{5\pi}{12}$

35. TIME Find both the degree and radian measures of the angle through which the hour hand on a clock rotates from 5 A.M. to 10 A.M.

36. ROTATION A truck with 16-inch radius wheels is driven at 77 feet per second (52.5 miles per hour). Find the measure of the angle through which a point on the outside of the wheel travels each second. Round to the nearest degree and nearest radian.

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13-2 Reading to Learn Mathematics

Angles and Angle Measure

Pre-Activity **How can angles be used to describe circular motion?**

Read the introduction to Lesson 13-2 at the top of page 709 in your textbook.

If a gondola revolves through a complete revolution in one minute, what is its angular velocity in degrees per second?

Reading the Lesson

1. Match each degree measure with the corresponding radian measure on the right.

a. 30°	**i.** $\frac{2\pi}{3}$
b. 90°	**ii.** $\frac{\pi}{2}$
c. 120°	**iii.** $\frac{7\pi}{6}$
d. 135°	**iv.** π
e. 180°	**v.** $\frac{\pi}{6}$
f. 210°	**vi.** $\frac{3\pi}{4}$

2. The sine of 30° is $\frac{1}{2}$ and the sine of 150° is also $\frac{1}{2}$. Does this mean that 30° and 150° are coterminal angles? Explain your reasoning.

3. Describe how to find two angles that are coterminal with an angle of 155°, one with positive measure and one with negative measure. (Do not actually calculate these angles.)

4. Describe how to find two angles that are coterminal with an angle of $\frac{5\pi}{3}$, one positive and one negative. (Do not actually calculate these angles.)

Helping You Remember

5. How can you use what you know about the circumference of a circle to remember how to convert between radian and degree measure?

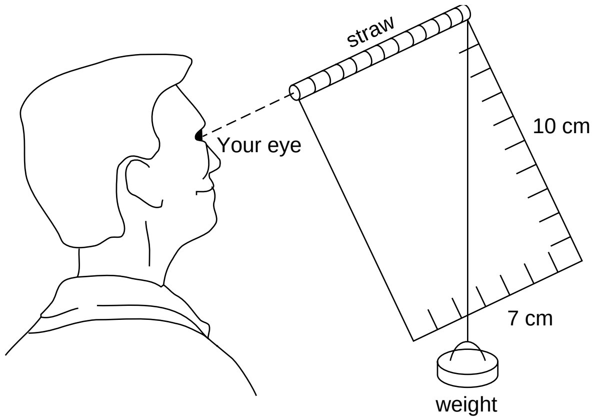
Lesson 13-2

13-2 Enrichment

Making and Using a Hypsometer

A **hypsometer** is a device that can be used to measure the height of an object. To construct your own hypsometer, you will need a rectangular piece of heavy cardboard that is at least 7 cm by 10 cm, a straw, transparent tape, a string about 20 cm long, and a small weight that can be attached to the string.

Mark off 1-cm increments along one short side and one long side of the cardboard. Tape the straw to the other short side. Then attach the weight to one end of the string, and attach the other end of the string to one corner of the cardboard, as shown in the figure below. The diagram below shows how your hypsometer should look.



To use the hypsometer, you will need to measure the distance from the base of the object whose height you are finding to where you stand when you use the hypsometer.

Sight the top of the object through the straw. Note where the free-hanging string crosses the bottom scale. Then use similar triangles to find the height of the object.

1. Draw a diagram to illustrate how you can use similar triangles and the hypsometer to find the height of a tall object.

Use your hypsometer to find the height of each of the following.

2. your school's flagpole

3. a tree on your school's property

4. the highest point on the front wall of your school building

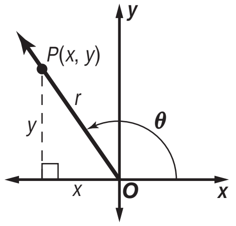
5. the goal posts on a football field

6. the hoop on a basketball court

13-3 Study Guide and Intervention

Trigonometric Functions of General Angles

Trigonometric Functions and General Angles

<table>
<tr>
<td>Trigonometric Functions, θ in Standard Position
</td>
<td>Let θ be an angle in standard position and let $P(x, y)$ be a point on the terminal side of θ. By the Pythagorean Theorem, the distance r from the origin is given by $r = \sqrt{x^2 + y^2}$. The trigonometric functions of an angle in standard position may be defined as follows.
$\sin \theta = \frac{y}{r}$ $\cos \theta = \frac{x}{r}$ $\tan \theta = \frac{y}{x}$
$\csc \theta = \frac{r}{y}$ $\sec \theta = \frac{r}{x}$ $\cot \theta = \frac{x}{y}$</td>
</tr>
</table>

Example **Find the exact values of the six trigonometric functions of θ if the terminal side of θ contains the point $(-5, 5\sqrt{2})$.**

You know that $x = -5$ and $y = 5\sqrt{2}$. You need to find r.

$r = \sqrt{x^2 + y^2}$ Pythagorean Theorem

$= \sqrt{(-5)^2 + (5\sqrt{2})^2}$ Replace x with -5 and y with $5\sqrt{2}$.

$= \sqrt{75}$ or $5\sqrt{3}$

Now use $x = -5$, $y = 5\sqrt{2}$, and $r = 5\sqrt{3}$ to write the ratios.

$\sin \theta = \frac{y}{r} = \frac{5\sqrt{2}}{5\sqrt{3}} = \frac{\sqrt{6}}{3}$ $\cos \theta = \frac{x}{r} = \frac{-5}{5\sqrt{3}} = -\frac{\sqrt{3}}{3}$ $\tan \theta = \frac{y}{x} = \frac{5\sqrt{2}}{-5} = -\sqrt{2}$

$\csc \theta = \frac{r}{y} = \frac{5\sqrt{3}}{5\sqrt{2}} = \frac{\sqrt{6}}{2}$ $\sec \theta = \frac{r}{x} = \frac{5\sqrt{3}}{-5} = -\sqrt{3}$ $\cot \theta = \frac{x}{y} = \frac{-5}{5\sqrt{2}} = -\frac{\sqrt{2}}{2}$

Exercises

Find the exact values of the six trigonometric functions of θ if the terminal side of θ in standard position contains the given point.

1. $(8, 4)$

2. $(4, 4\sqrt{3})$

3. $(0, -4)$

4. $(6, 2)$

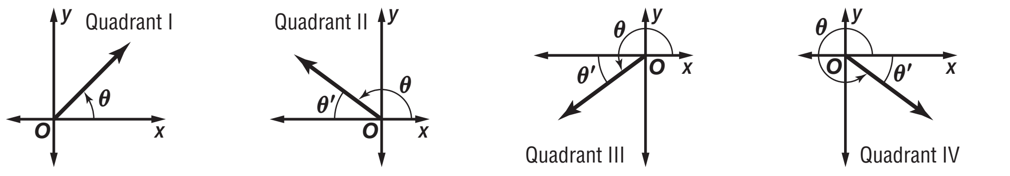
Lesson 13-3

13-3 Study Guide and Intervention *(continued)*

Trigonometric Functions of General Angles

Reference Angles If θ is a nonquadrantal angle in standard position, its reference angle θ' is defined as the acute angle formed by the terminal side of θ and the x-axis.

Reference Angle Rule



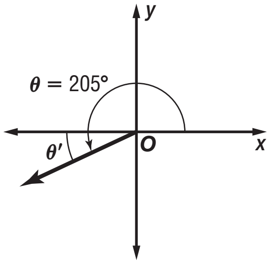
Quadrant I	Quadrant II	Quadrant III	Quadrant IV
$\theta' = \theta$	$\theta' = 180° - \theta$ ($\theta' = \pi - \theta$)	$\theta' = \theta - 180°$ ($\theta' = \theta - \pi$)	$\theta' = 360° - \theta$ ($\theta' = 2\pi - \theta$)

Signs of Trigonometric Functions

	Quadrant			
Function	**I**	**II**	**III**	**IV**
sin θ or csc θ	+	+	−	−
cosθ or sec θ	+	−	−	+
tan θ or cot θ	+	−	+	−

Example 1 **Sketch an angle of measure 205°. Then find its reference angle.**

Because the terminal side of 205° lies in Quadrant III, the reference angle θ' is $205° - 180°$ or 25°.



Example 2 **Use a reference angle to find the exact value of cos $\frac{3\pi}{4}$.**

Because the terminal side of $\frac{3\pi}{4}$ lies in Quadrant II, the reference angle θ' is $\pi - \frac{3\pi}{4}$ or $\frac{\pi}{4}$.

The cosine function is negative in Quadrant II.

$\cos \frac{3\pi}{4} = -\cos \frac{\pi}{4} = -\frac{\sqrt{2}}{2}$.

Exercises

Find the exact value of each trigonometric function.

1. $\tan(-510°)$

2. $\csc \frac{11\pi}{4}$

3. $\sin(-90°)$

4. $\cot 1665°$

5. $\cot 30°$

6. $\tan 315°$

7. $\csc \frac{\pi}{4}$

8. $\tan \frac{4\pi}{3}$

NAME ______________________________ DATE ____________ PERIOD _____

13-3 Skills Practice

Trigonometric Functions of General Angles

Find the exact values of the six trigonometric functions of θ if the terminal side of θ in standard position contains the given point.

1. (5, 12)

2. (3, 4)

3. (8, −15)

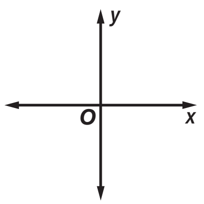
4. (−4, 3)

5. (−9, −40)

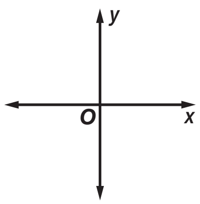
6. (1, 2)

Sketch each angle. Then find its reference angle.

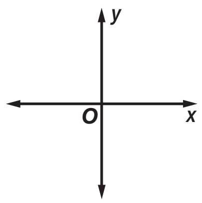
7. 135°



8. 200°



9. $\frac{5\pi}{3}$



Find the exact value of each trigonometric function.

10. sin 150°

11. cos 270°

12. cot 135°

13. tan (−30°)

14. $\tan \frac{\pi}{4}$

15. $\cos \frac{4\pi}{3}$

16. cot (−π)

17. $\sin \left(-\frac{3\pi}{4}\right)$

Suppose θ is an angle in standard position whose terminal side is in the given quadrant. For each function, find the exact values of the remaining five trigonometric functions of θ.

18. $\sin \theta = \frac{4}{5}$, Quadrant II

19. $\tan \theta = -\frac{12}{5}$, Quadrant IV

13-3 Practice

Trigonometric Functions of General Angles

Find the exact values of the six trigonometric functions of θ if the terminal side of θ in standard position contains the given point.

1. (6, 8) **2.** (−20, 21) **3.** (−2, −5)

Find the reference angle for the angle with the given measure.

4. 236° **5.** $\frac{13\pi}{8}$ **6.** −210° **7.** $-\frac{7\pi}{4}$

Find the exact value of each trigonometric function.

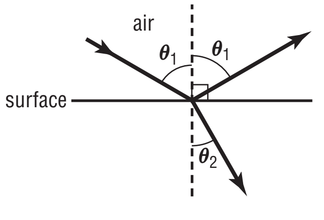
8. tan 135° **9.** cot 210° **10.** cot (−90°) **11.** cos 405°

12. $\tan \frac{5\pi}{3}$ **13.** $\csc \left(-\frac{3\pi}{4}\right)$ **14.** cot 2π **15.** $\tan \frac{13\pi}{6}$

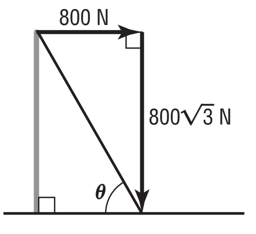
Suppose θ is an angle in standard position whose terminal side is in the given quadrant. For each function, find the exact values of the remaining five trigonometric functions of θ.

16. $\tan \theta = -\frac{12}{5}$, Quadrant IV **17.** $\sin \theta = \frac{2}{3}$, Quadrant III

18. LIGHT Light rays that "bounce off" a surface are *reflected* by the surface. If the surface is partially transparent, some of the light rays are bent or *refracted* as they pass from the air through the material. The angles of reflection θ_1 and of refraction θ_2 in the diagram at the right are related by the equation $\sin \theta_1 = n \sin \theta_2$. If $\theta_1 = 60°$ and $n = \sqrt{3}$, find the measure of θ_2.



19. FORCE A cable running from the top of a utility pole to the ground exerts a horizontal pull of 800 Newtons and a vertical pull of $800\sqrt{3}$ Newtons. What is the sine of the angle θ between the cable and the ground? What is the measure of this angle?



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13-3 Reading to Learn Mathematics

Trigonometric Functions of General Angles

Pre-Activity **How can you model the position of riders on a skycoaster?**

Read the introduction to Lesson 13-3 at the top of page 717 in your textbook.

- What does $t = 0$ represent in this application?
- Do negative values of t make sense in this application? Explain your answer.

Reading the Lesson

1. Suppose θ is an angle in standard position, $P(x, y)$ is a point on the terminal side of θ, and the distance from the origin to P is r. Determine whether each of the following statements is *true* or *false*.

a. The value of r can be found by using either the Pythagorean Theorem or the distance formula.

b. $\cos \theta = \frac{x}{r}$

c. $\csc \theta$ is defined if $y \neq 0$.

d. $\tan \theta$ is undefined if $y = 0$.

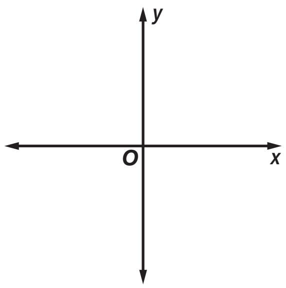
e. $\sin \theta$ is defined for every value of θ.

2. Let θ be an angle measured in degrees. Match the quadrant of θ from the first column with the description of how to find the reference angle for θ from the second column.

a. Quadrant III	**i.** Subtract θ from 360°.
b. Quadrant IV	**ii.** Subtract 180° from θ.
c. Quadrant II	**iii.** θ is its own reference angle.
d. Quadrant I	**iv.** Subtract θ from 180°.

Helping You Remember

3. The chart on page 719 in your textbook summarizes the signs of the six trigonometric functions in the four quadrants. Since reciprocals always have the same sign, you only need to remember where the sine, cosine, and tangent are positive. How can you remember this with a simple diagram?



Lesson 13-3

13-3 Enrichment

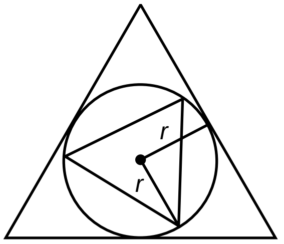
Areas of Polygons and Circles

A regular polygon has sides of equal length and angles of equal measure. A regular polygon can be inscribed in or circumscribed about a circle. For n-sided regular polygons, the following area formulas can be used.

Area of circle $A_C = \pi r^2$

Area of inscribed polygon $A_I = \frac{nr^2}{2} \times \sin \frac{360°}{n}$

Area of circumscribed polygon $A_C = nr^2 \times \tan \frac{180°}{n}$



Use a calculator to complete the chart below for a unit circle (a circle of radius 1).

	Number of Sides	Area of Inscribed Polygon	Area of Circle minus Area of Polygon	Area of Circumscribed Polygon	Area of Polygon minus Area of Circle
	3	1.2990381	1.8425545	5.1961524	2.054597
1.	4				
2.	8				
3.	12				
4.	20				
5.	24				
6.	28				
7.	32				
8.	1000				

9. What number do the areas of the circumscribed and inscribed polygons seem to be approaching?

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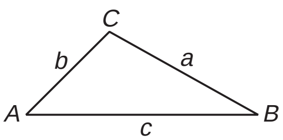
13-4 Study Guide and Intervention

Law of Sines

Law of Sines The area of any triangle is one half the product of the lengths of two sides and the sine of the included angle.

Area of a Triangle	area $= \frac{1}{2}bc \sin A$ area $= \frac{1}{2}ac \sin B$ area $= \frac{1}{2}ab \sin C$	

You can use the Law of Sines to solve any triangle if you know the measures of two angles and any side, or the measures of two sides and the angle opposite one of them.

Law of Sines	$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

Example 1 **Find the area of $\triangle ABC$ if $a = 10$, $b = 14$, and $C = 40°$.**

$\text{Area} = \frac{1}{2}ab \sin C$ Area formula

$= \frac{1}{2}(10)(14)\sin 40°$ Replace *a*, *b*, and *C*.

≈ 44.9951 Use a calculator.

The area of the triangle is approximately 45 square units.

Example 2 **If $a = 12$, $b = 9$, and $A = 28°$, find B.**

$\frac{\sin A}{a} = \frac{\sin B}{b}$ Law of Sines

$\frac{\sin 28°}{12} = \frac{\sin B}{9}$ Replace *A*, *a*, and *b*.

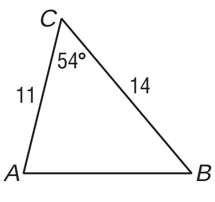
$\sin B = \frac{9 \sin 28°}{12}$ Solve for sin *B*.

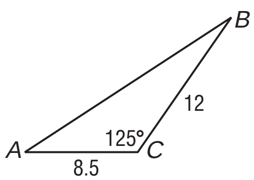
$\sin B \approx 0.3521$ Use a calculator.

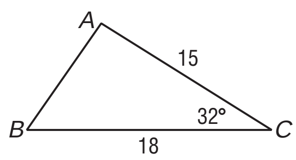
$B \approx 20.62°$ Use the $\sin^{-1}$ function.

Exercises

Find the area of $\triangle ABC$ to the nearest tenth.

1. 

2. 

3. 

Solve each triangle. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.

4. $B = 42°, C = 68°, a = 10$ **5.** $A = 40°, B = 14°, a = 52$ **6.** $A = 15°, B = 50°, b = 36$

Lesson 13-4

NAME ______________________________ DATE ____________ PERIOD _____

13-4 Study Guide and Intervention *(continued)*

Law of Sines

One, Two, or No Solutions

Possible Triangles Given Two Sides and One Opposite Angle	Suppose you are given *a*, *b*, and *A* for a triangle. If *a* is acute: $a < b \sin A$ ⇒ no solution $a = b \sin A$ ⇒ one solution $b > a > b \sin A$ ⇒ two solutions $a > b$ ⇒ one solution If *A* is right or obtuse: $a \le b$ ⇒ no solution $a > b$ ⇒ one solution

Example **Determine whether $\triangle ABC$ has no solutions, one solution, or two solutions. Then solve $\triangle ABC$.**

a. $A = 48°$, $a = 11$, and $b = 16$

Since A is acute, find $b \sin A$ and compare it with a.
$b \sin A = 16 \sin 48° \approx 11.89$
Since $11 < 11.89$, there is no solution.

b. $A = 34°$, $a = 6$, $b = 8$

Since A is acute, find $b \sin A$ and compare it with a; $b \sin A = 8 \sin 34° \approx 4.47$. Since $8 > 6 > 4.47$, there are two solutions. Thus there are two possible triangles to solve.

Acute B

First use the Law of Sines to find B.

$$\frac{\sin B}{8} = \frac{\sin 34°}{6}$$
$$\sin B \approx 0.7456$$
$$B \approx 48°$$

The measure of angle C is about $180° - (34° + 48°)$ or about 98°.

Use the Law of Sines again to find c.

$$\frac{\sin 98°}{c} \approx \frac{\sin 34°}{6}$$
$$c \approx \frac{6 \sin 98°}{\sin 34°}$$
$$c \approx 10.6$$

Obtuse B

To find B you need to find an obtuse angle whose sine is also 0.7456.

To do this, subtract the angle given by your calculator, 48°, from 180°. So B is approximately 132°.

The measure of angle C is about $180° - (34° + 132°)$ or about 14°.

Use the Law of Sines to find c.

$$\frac{\sin 14°}{c} \approx \frac{\sin 34°}{6}$$
$$c \approx \frac{6 \sin 14°}{\sin 34°}$$
$$c \approx 2.6$$

Exercises

Determine whether each triangle has no solutions, one solution, or two solutions. Then solve each triangle. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.

1. $A = 50°$, $a = 34$, $b = 40$ **2.** $A = 24°$, $a = 3$, $b = 8$ **3.** $A = 125°$, $a = 22$, $b = 15$

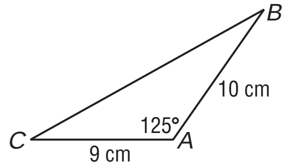
NAME ______________________________ DATE ____________ PERIOD _____

13-4 Skills Practice

Law of Sines

Find the area of $\triangle ABC$ to the nearest tenth.

1.



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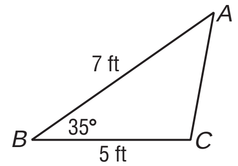


3. $A = 35°, b = 3 \text{ ft}, c = 7 \text{ ft}$

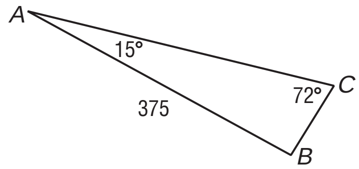
4. $C = 148°, a = 10 \text{ cm}, b = 7 \text{ cm}$

5. $C = 22°, a = 14 \text{ m}, b = 8 \text{ m}$

6. $B = 93°, c = 18 \text{ mi}, a = 42 \text{ mi}$

Solve each triangle. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.

7.



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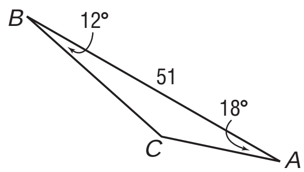

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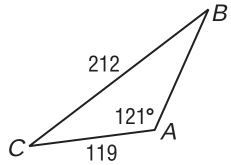

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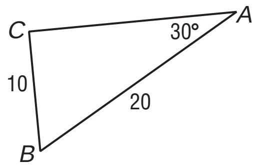

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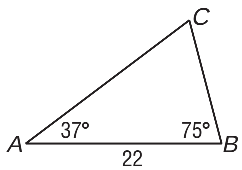

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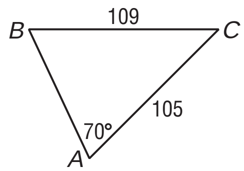


Determine whether each triangle has *no* solution, *one* solution, or *two* solutions. Then solve each triangle. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.

13. $A = 30°, a = 1, b = 4$

14. $A = 30°, a = 2, b = 4$

15. $A = 30°, a = 3, b = 4$

16. $A = 38°, a = 10, b = 9$

17. $A = 78°, a = 8, b = 5$

18. $A = 133°, a = 9, b = 7$

19. $A = 127°, a = 2, b = 6$

20. $A = 109°, a = 24, b = 13$

Lesson 13-4

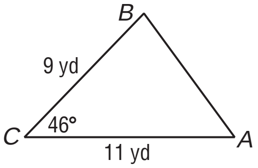
NAME ______________________________ DATE ____________ PERIOD _____

13-4 Practice

Law of Sines

Find the area of △*ABC* to the nearest tenth.

1.



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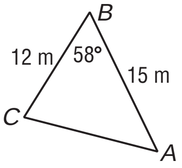

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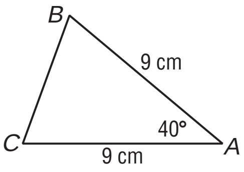

4. $C = 32°, a = 12.6$ m, $b = 8.9$ m

5. $B = 27°, a = 14.9$ cm, $c = 18.6$ cm

6. $A = 17.4°, b = 12$ km, $c = 14$ km

7. $A = 34°, b = 19.4$ ft, $c = 8.6$ ft

Solve each triangle. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.

8. $A = 50°, B = 30°, c = 9$

9. $A = 56°, B = 38°, a = 12$

10. $A = 80°, C = 14°, a = 40$

11. $B = 47°, C = 112°, b = 13$

12. $A = 72°, a = 8, c = 6$

13. $A = 25°, C = 107°, b = 12$

Determine whether each triangle has *no* solution, *one* solution, or *two* solutions. Then solve each triangle. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.

14. $A = 29°, a = 6, b = 13$

15. $A = 70°, a = 25, b = 20$

16. $A = 113°, a = 21, b = 25$

17. $A = 110°, a = 20, b = 8$

18. $A = 66°, a = 12, b = 7$

19. $A = 54°, a = 5, b = 8$

20. $A = 45°, a = 15, b = 18$

21. $A = 60°, a = 4\sqrt{3}, b = 8$

22. WILDLIFE Sarah Phillips, an officer for the Department of Fisheries and Wildlife, checks boaters on a lake to make sure they do not disturb two osprey nesting sites. She leaves a dock and heads due north in her boat to the first nesting site. From here, she turns 5° north of due west and travels an additional 2.14 miles to the second nesting site. She then travels 6.7 miles directly back to the dock. How far from the dock is the first osprey nesting site? Round to the nearest tenth.

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13-4 Reading to Learn Mathematics

Law of Sines

Pre-Activity **How can trigonometry be used to find the area of a triangle?**

Read the introduction to Lesson 13-4 at the top of page 725 in your textbook.

What happens when the formula Area $= \frac{1}{2}ab \sin C$ is applied to a right triangle in which C is the right angle?

Reading the Lesson

1. In each case below, the measures of three parts of a triangle are given. For each case, write the formula you would use to find the area of the triangle. Show the formulas with specific values substituted, but do not actually calculate the area. If there is not enough information provided to find the area of the triangle by using the area formulas on page 725 in your textbook and without finding other parts of the triangle first, explain why.

a. $A = 48°, b = 9, c = 5$

b. $a = 15, b = 15, C = 120°$

c. $b = 16, c = 10, B = 120°$

2. Tell whether the equation must be true based on the Law of Sines. Write *yes* or *no*.

a. $\frac{\sin A}{b} = \frac{\sin B}{a}$

b. $\frac{b}{\sin B} = \frac{c}{\sin C}$

c. $a \sin C = c \sin A$

d. $b = \frac{a \sin A}{\sin B}$

3. Determine whether $\triangle ABC$ has *no solution*, *one solution*, or *two solutions*. Do not try to solve the triangle.

a. $a = 20, A = 30°, B = 70°$

b. $A = 55°, b = 5, a = 3$ ($b \sin A \approx 4.1$)

c. $c = 12, A = 100°, a = 30$

d. $C = 27°, b = 23.5, c = 17.5$ ($b \sin C \approx 10.7$)

Helping You Remember

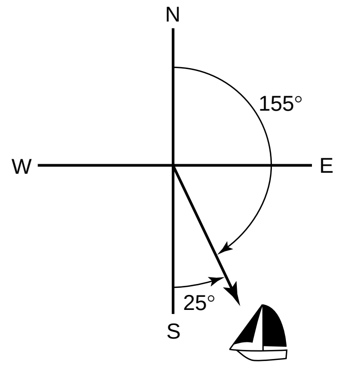
4. Suppose that you are taking a quiz and cannot remember whether the formula for the area of a triangle is Area $= \frac{1}{2}ab \cos C$ or Area $= \frac{1}{2}ab \sin C$. How can you quickly remember which of these is correct?

Lesson 13-4

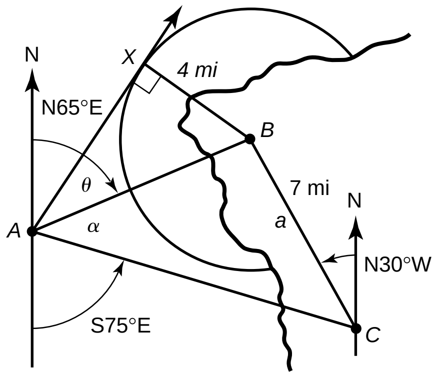
13-4 Enrichment

Navigation

The bearing of a boat is an angle showing the direction the boat is heading. Often, the angle is measured from north, but it can be measured from any of the four compass directions. At the right, the bearing of the boat is 155°. Or, it can be described as 25° east of south (S25°E).



Example **A boat A sights the lighthouse B in the direction N65°E and the spire of a church C in the direction S75°E. According to the map, B is 7 miles from C in the direction N30°W. In order for A to avoid running aground, find the bearing it should keep to pass B at 4 miles distance.**



In $\triangle ABC$, $\angle\alpha = 180° - 65° - 75°$ or $40°$

$\angle C = 180° - 30° - (180° - 75°)$

$= 45°$

$a = 7$ miles

With the Law of Sines,

$$AB = \frac{a \sin C}{\sin \alpha} = \frac{7(\sin 45°)}{\sin 40°} = 7.7 \text{ mi.}$$

The ray for the correct bearing for A must be tangent at X to circle B with radius $BX = 4$. Thus $\triangle ABX$ is a right triangle.

Then $\sin\theta = \frac{BX}{AB} = \frac{4}{7.7} \approx 0.519$. Therefore, $\angle\theta = 31°18'$.

The bearing of A should be $65° - 31°18'$ or $33°42'$.

Solve the following.

1. Suppose the lighthouse B in the example is sighted at S30°W by a ship P due north of the church C. Find the bearing P should keep to pass B at 4 miles distance.

2. In the fog, the lighthouse keeper determines by radar that a boat 18 miles away is heading to the shore. The direction of the boat from the lighthouse is S80°E. What bearing should the lighthouse keeper radio the boat to take to come ashore 4 miles south of the lighthouse?

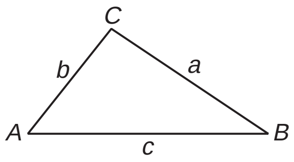
3. To avoid a rocky area along a shoreline, a ship at M travels 7 km to R, bearing $22°15'$, then 8 km to P, bearing $68°30'$, then 6 km to Q, bearing $109°15'$. Find the distance from M to Q.

NAME ______________________________ DATE ____________ PERIOD _____

13-5 Study Guide and Intervention

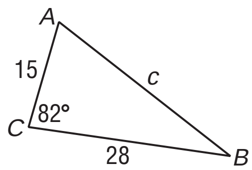
Law of Cosines

Law of Cosines

<table>
<tr>
<td>Law of Cosines

</td>
<td>Let $\triangle ABC$ be any triangle with a, b, and c representing the measures of the sides, and opposite angles with measures A, B, and C, respectively. Then the following equations are true.
$a^2 = b^2 + c^2 - 2bc \cos A$
$b^2 = a^2 + c^2 - 2ac \cos B$
$c^2 = a^2 + b^2 - 2ab \cos C$</td>
</tr>
</table>

You can use the Law of Cosines to solve any triangle if you know the measures of two sides and the included angle, or the measures of three sides.

Example **Solve $\triangle ABC$.**



You are given the measures of two sides and the included angle. Begin by using the Law of Cosines to determine c.

$c^2 = a^2 + b^2 - 2ab \cos C$

$c^2 = 28^2 + 15^2 - 2(28)(15)\cos 82°$

$c^2 \approx 892.09$

$c \approx 29.9$

Next you can use the Law of Sines to find the measure of angle A.

$\frac{\sin A}{a} = \frac{\sin C}{c}$

$\frac{\sin A}{28} \approx \frac{\sin 82°}{29.9}$

$\sin A \approx 0.9273$

$A \approx 68°$

The measure of B is about $180° - (82° + 68°)$ or about $30°$.

Exercises

Solve each triangle described below. Round measures of sides to the nearest tenth and angles to the nearest degree.

1. $a = 14, c = 20, B = 38°$

2. $A = 60°, c = 17, b = 12$

3. $a = 4, b = 6, c = 3$

4. $A = 103°, b = 31, c = 52$

5. $a = 15, b = 26, C = 132°$

6. $a = 31, b = 52, c = 43$

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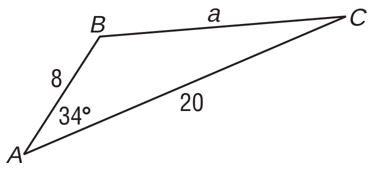
13-5 Study Guide and Intervention *(continued)*

Law of Cosines

Choose the Method

	Given	Begin by Using
Solving an Oblique Triangle	two angles and any side	Law of Sines
	two sides and a non-included angle	Law of Sines
	two sides and their included angle	Law of Cosines
	three sides	Law of Cosines

Example **Determine whether $\triangle ABC$ should be solved by beginning with the Law of Sines or Law of Cosines. Then solve the triangle. Round the measure of the side to the nearest tenth and measures of angles to the nearest degree.**



You are given the measures of two sides and their included angle, so use the Law of Cosines.

$a^2 = b^2 + c^2 - 2bc \cos A$ Law of Cosines

$a^2 = 20^2 + 8^2 - 2(20)(8) \cos 34°$ $b = 20$, $c = 8$, $A = 34°$

$a^2 \approx 198.71$ Use a calculator.

$a \approx 14.1$ Use a calculator.

Use the Law of Sines to find B.

$\frac{\sin B}{b} = \frac{\sin A}{a}$ Law of Sines

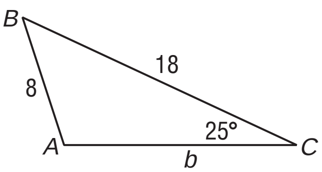
$\sin B \approx \frac{20 \sin 34°}{14.1}$ $b = 20$, $A = 34°$, $a \approx 14.1$

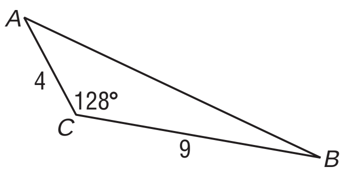
$B \approx 128°$ Use the $\sin^{-1}$ function.

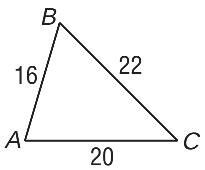
The measure of angle C is approximately $180° - (34° + 128°)$ or about 18°.

Exercises

Determine whether each triangle should be solved by beginning with the Law of Sines or Law of Cosines. Then solve each triangle. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.

1. 

2. 

3. 

4. $A = 58°$, $a = 12$, $b = 8.5$

5. $a = 28$, $b = 35$, $c = 20$

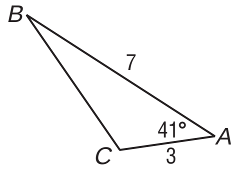
6. $A = 82°$, $B = 44°$, $b = 11$

NAME ____ DATE ____________ PERIOD _____

13-5 Skills Practice

Law of Cosines

Determine whether each triangle should be solved by beginning with the Law of Sines or Law of Cosines. Then solve each triangle. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.

1.


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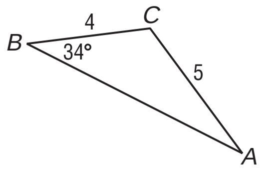

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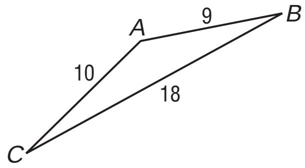

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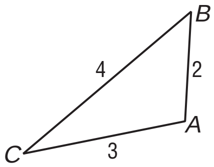

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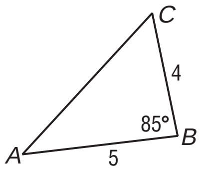

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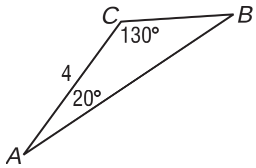


7. $C = 71°, a = 3, b = 4$

8. $A = 11°, C = 27°, c = 50$

9. $C = 35°, a = 5, b = 8$

10. $B = 47°, a = 20, c = 24$

11. $A = 71°, C = 62°, a = 20$

12. $a = 5, b = 12, c = 13$

13. $A = 51°, b = 7, c = 10$

14. $a = 13, A = 41°, B = 75°$

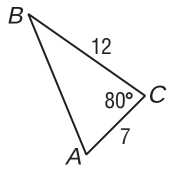
15. $B = 125°, a = 8, b = 14$

16. $a = 5, b = 6, c = 7$

13-5 Practice

Law of Cosines

Determine whether each triangle should be solved by beginning with the Law of Sines or Law of Cosines. Then solve each triangle. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.

1.


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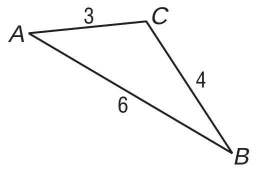

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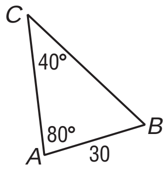


4. $a = 16, b = 20, C = 54°$

5. $B = 71°, c = 6, a = 11$

6. $A = 37°, a = 20, b = 18$

7. $C = 35°, a = 18, b = 24$

8. $a = 8, b = 6, c = 9$

9. $A = 23°, b = 10, c = 12$

10. $a = 4, b = 5, c = 8$

11. $B = 46.6°, C = 112°, b = 13$

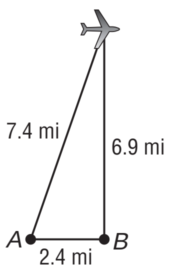
12. $A = 46.3°, a = 35, b = 30$

13. $a = 16.4, b = 21.1, c = 18.5$

14. $C = 43.5°, b = 8, c = 6$

15. $A = 78.3°, b = 7, c = 11$

16. SATELLITES Two radar stations 2.4 miles apart are tracking an airplane. The straight-line distance between Station *A* and the plane is 7.4 miles. The straight-line distance between Station *B* and the plane is 6.9 miles. What is the angle of elevation from Station *A* to the plane? Round to the nearest degree.



17. DRAFTING Marion is using a computer-aided drafting program to produce a drawing for a client. She begins a triangle by drawing a segment 4.2 inches long from point *A* to point *B*. From *B*, she moves 42° degrees counterclockwise from the segment connecting *A* and *B* and draws a second segment that is 6.4 inches long, ending at point *C*. To the nearest tenth, how long is the segment from *C* to *A*?

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13-5 Reading to Learn Mathematics

Law of Cosines

Pre-Activity **How can you determine the angle at which to install a satellite dish?**

Read the introduction to Lesson 13-5 at the top of page 733 in your textbook.

One side of the triangle in the figure is not labeled with a length. What does the length of this side represent? Is this length greater than or less than the distance from the satellite to the equator?

Reading the Lesson

1. Each of the following equations can be changed into a correct statement of the Law of Cosines by making one change. In each case, indicate what change should be made to make the statement correct.

a. $b^2 = a^2 + c^2 + 2ac \cos B$

b. $a^2 = b^2 + c^2 - 2bc \sin A$

c. $c = a^2 + b^2 - 2ab \cos C$

d. $a^2 = b^2 - c^2 - 2bc \cos A$

2. Suppose that you are asked to solve $\triangle ABC$ given the following information about the sides and angles of the triangle. In each case, indicate whether you would begin by using the *Law of Sines* or the *Law of Cosines*.

a. $a = 8, b = 7, c = 6$

b. $b = 9.5, A = 72°, B = 39°$

c. $C = 123°, b = 22.95, a = 34.35$

Helping You Remember

3. It is often easier to remember a complicated procedure if you can break it down into small steps. Describe in your own words how to use the Law of Cosines to find the length of one side of a triangle if you know the lengths of the other two sides and the measure of the included angle. Use numbered steps. (You may use mathematical terms, but do not use any mathematical symbols.)

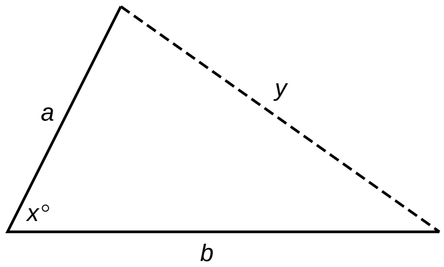
Lesson 13-5

13-5 Enrichment

The Law of Cosines and the Pythagorean Theorem

The law of cosines bears strong similarities to the Pythagorean theorem. According to the law of cosines, if two sides of a triangle have lengths a and b and if the angle between them has a measure of $x°$, then the length, y, of the third side of the triangle can be found by using the equation

$y^2 = a^2 + b^2 - 2ab \cos x°$.



Answer the following questions to clarify the relationship between the law of cosines and the Pythagorean theorem.

1. If the value of $x°$ becomes less and less, what number is $\cos x°$ close to?

2. If the value of $x°$ is very close to zero but then increases, what happens to $\cos x°$ as $x°$ approaches 90°?

3. If $x°$ equals 90°, what is the value of $\cos x°$? What does the equation of $y^2 = a^2 + b^2 - 2ab \cos x°$ simplify to if $x°$ equals 90°?

4. What happens to the value of $\cos x°$ as $x°$ increases beyond 90° and approaches 180°?

5. Consider some particular value of a and b, say 7 for a and 19 for b. Use a graphing calculator to graph the equation you get by solving $y^2 = 7^2 + 19^2 - 2(7)(19) \cos x°$ for y.

a. In view of the geometry of the situation, what range of values should you use for X?

b. Display the graph and use the TRACE function. What do the maximum and minimum values appear to be for the function?

c. How do the answers for part **b** relate to the lengths 7 and 19? Are the maximum and minimum values from part **b** ever actually attained in the geometric situation?

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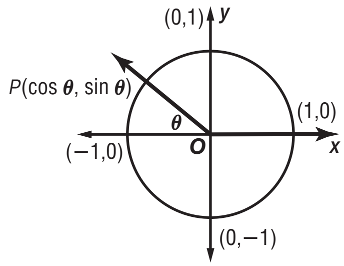
13-6 Study Guide and Intervention

Circular Functions

Unit Circle Definitions

Definition of Sine and Cosine	If the terminal side of an angle θ in standard position intersects the unit circle at $P(x, y)$, then $\cos \theta = x$ and $\sin \theta = y$. Therefore, the coordinates of P can be written as $P(\cos \theta, \sin \theta)$. 

Example **Given an angle θ in standard position, if $P\left(-\frac{5}{6}, \frac{\sqrt{11}}{6}\right)$ lies on the terminal side and on the unit circle, find $\sin \theta$ and $\cos \theta$.**

$P\left(-\frac{5}{6}, \frac{\sqrt{11}}{6}\right) = P(\cos \theta, \sin \theta)$, so $\sin \theta = \frac{\sqrt{11}}{6}$ and $\cos \theta = -\frac{5}{6}$.

Exercises

If θ is an angle in standard position and if the given point P is located on the terminal side of θ and on the unit circle, find $\sin \theta$ and $\cos \theta$.

1. $P\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

2. $P(0, -1)$

3. $P\left(-\frac{2}{3}, \frac{\sqrt{5}}{3}\right)$

4. $P\left(-\frac{4}{5}, -\frac{3}{5}\right)$

5. $P\left(\frac{1}{6}, -\frac{\sqrt{35}}{6}\right)$

6. $P\left(\frac{\sqrt{7}}{4}, \frac{3}{4}\right)$

7. P is on the terminal side of $\theta = 45°$.

8. P is on the terminal side of $\theta = 120°$.

9. P is on the terminal side of $\theta = 240°$.

10. P is on the terminal side of $\theta = 330°$.

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13-6 Study Guide and Intervention *(continued)*

Circular Functions

Periodic Functions

Periodic Functions	A function is called **periodic** if there is a number a such that $f(x) = f(x + a)$ for all x in the domain of the function. The least positive value of a for which $f(x) = f(x + a)$ is called the period of the function.

The sine and cosine functions are periodic; each has a period of 360° or 2π.

Example 1 **Find the exact value of each function.**

a. sin 855°

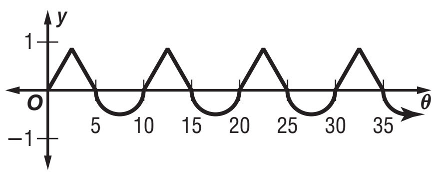
$\sin 855° = \sin(135° + 720°) = \sin 135° = \frac{\sqrt{2}}{2}$

b. $\mathbf{\cos\left(\frac{31\pi}{6}\right)}$

$\cos\left(\frac{31\pi}{6}\right) = \cos\left(\frac{7\pi}{6} + 4\pi\right)$

$= \cos\frac{7\pi}{6} = -\frac{\sqrt{3}}{2}$

Example 2 **Determine the period of the function graphed below.**



The pattern of the function repeats every 10 units, so the period of the function is 10.

Exercises

Find the exact value of each function.

1. $\cos(-240°)$ **2.** $\cos 2880°$ **3.** $\sin(-510°)$

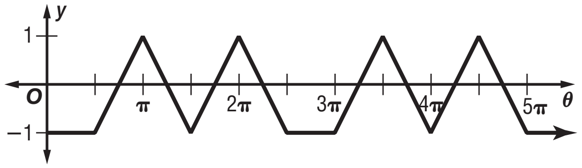
4. $\sin 495°$ **5.** $\cos\left(-\frac{5\pi}{2}\right)$ **6.** $\sin\left(\frac{5\pi}{3}\right)$

7. $\cos\left(\frac{11\pi}{4}\right)$ **8.** $\sin\left(-\frac{3\pi}{4}\right)$ **9.** $\cos 1440°$

10. $\sin(-750°)$ **11.** $\cos 870°$ **12.** $\cos 1980°$

13. $\sin 7\pi$ **14.** $\sin\left(-\frac{13\pi}{4}\right)$ **15.** $\cos\left(\frac{23\pi}{6}\right)$

16. Determine the period of the function.



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13-6 Skills Practice

Circular Functions

The given point *P* is located on the unit circle. Find sin *θ* and cos *θ*.

1. $P\left(\frac{3}{5}, \frac{4}{5}\right)$

2. $P\left(\frac{5}{13}, -\frac{12}{13}\right)$

3. $P\left(-\frac{9}{41}, -\frac{40}{41}\right)$

4. $P(0, 1)$

5. $P(-1, 0)$

6. $P\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

Find the exact value of each function.

7. $\cos 45°$

8. $\sin 210°$

9. $\sin 330°$

10. $\cos 330°$

11. $\cos (-60°)$

12. $\sin (-390°)$

13. $\sin 5\pi$

14. $\cos 3\pi$

15. $\sin \frac{5\pi}{2}$

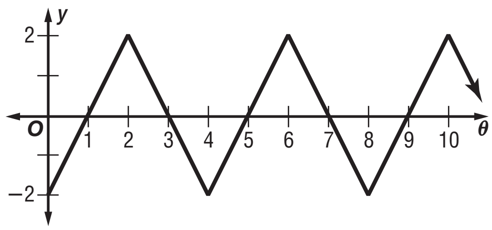
16. $\sin \frac{7\pi}{3}$

17. $\cos \left(-\frac{7\pi}{3}\right)$

18. $\cos \left(-\frac{5\pi}{6}\right)$

Determine the period of each function.

19.



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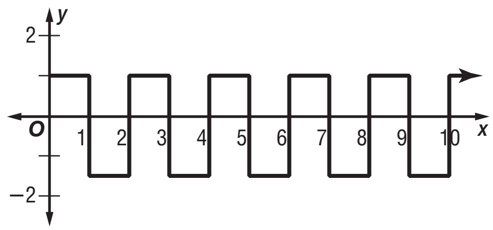

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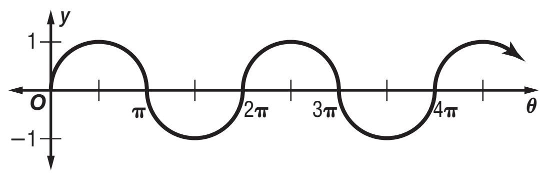

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13-6 Practice

Circular Functions

The given point P is located on the unit circle. Find sin θ and cos θ.

1. $P\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

2. $P\left(\frac{20}{29}, -\frac{21}{29}\right)$

3. $P(0.8, 0.6)$

4. $P(0, -1)$

5. $P\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

6. $P\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

Find the exact value of each function.

7. $\cos \frac{7\pi}{4}$

8. $\sin (-30°)$

9. $\sin \left(-\frac{2\pi}{3}\right)$

10. $\cos (-330°)$

11. $\cos 600°$

12. $\sin \frac{9\pi}{2}$

13. $\cos 7\pi$

14. $\cos \left(-\frac{11\pi}{4}\right)$

15. $\sin (-225°)$

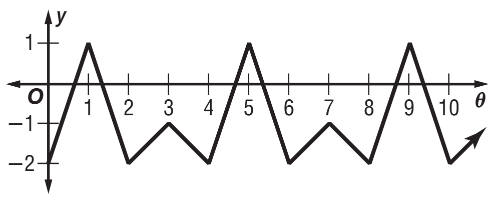
16. $\sin 585°$

17. $\cos \left(-\frac{10\pi}{3}\right)$

18. $\sin 840°$

Determine the period of each function.

19.



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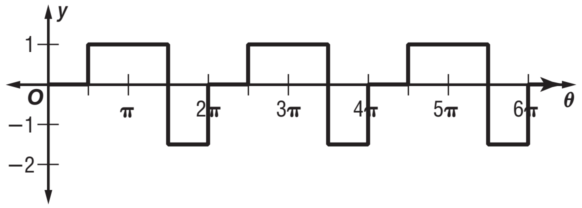


21. FERRIS WHEELS A Ferris wheel with a diameter of 100 feet completes 2.5 revolutions per minute. What is the period of the function that describes the height of a seat on the outside edge of the Ferris Wheel as a function of time?

13-6 Reading to Learn Mathematics

Circular Functions

Pre-Activity **How can you model annual temperature fluctuations?**

Read the introduction to Lesson 13-6 at the top of page 739 in your textbook.

- If the graph in your textbook is continued, what month will $x = 17$ represent?
- About what do you expect the average high temperature to be for that month?
- Will this be exactly the average high temperature for that month? Explain your answer.

Reading the Lesson

1. Use the unit circle on page 740 in your textbook to find the exact values of each expression.

a. cos 45° **b.** sin 150° **c.** sin 240°

d. sin 315° **e.** cos 270° **f.** sin 210°

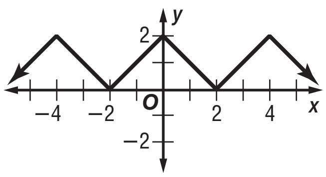
g. cos 0° **h.** sin 180° **i.** cos 330°

2. Tell whether each function is periodic. Write *yes* or *no*.

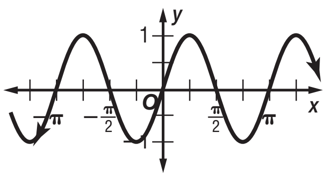
a. $y = 2x$ **b.** $y = x^2$ **c.** $y = \cos x$ **d.** $y = |x|$

3. Find the period of each function by examining its graph.

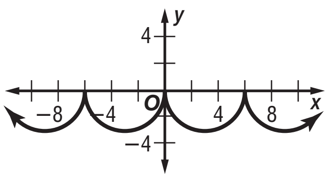
a.



b.



c.



Helping You Remember

4. What is an easy way to remember the periods of the sine and cosine functions in radian measure?

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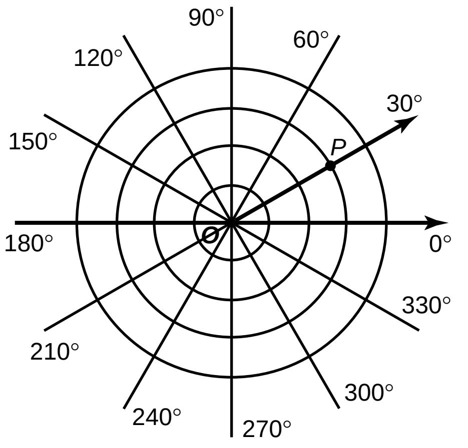
13-6 Enrichment

Polar Coordinates



Consider an angle in standard position with its vertex at a point O called the *pole*. Its initial side is on a coordinated axis called the *polar axis*. A point P on the terminal side of the angle is named by the *polar coordinates* (r, θ) where r is the directed distance of the point from O and θ is the measure of the angle.

Graphs in this system may be drawn on polar coordinate paper such as the kind shown at the right.

The polar coordinates of a point are not unique. For example, (3, 30°) names point P as well as (3, 390°). Another name for P is (−3, 210°). Can you see why? The coordinates of the pole are $(0, \theta)$ where θ may be any angle.

Example **Draw the graph of the function $r = \cos \theta$. Make a table of convenient values for θ and r. Then plot the points.**

θ	0°	30°	60°	90°	120°	150°	180°
r	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	−1

Since the period of the cosine function is 180°, values of r for $\theta > 180°$ are repeated.

Graph each function by making a table of values and plotting the values on polar coordinate paper.

1. $r = 4$

2. $r = 3 \sin \theta$

3. $r = 3 \cos 2\theta$

4. $r = 2(1 + \cos \theta)$

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13-7 Study Guide and Intervention

Inverse Trigonometric Functions

Solve Equations Using Inverses If the domains of trigonometric functions are restricted to their **principal values**, then their inverses are also functions.

Principal Values of Sine, Cosine, and Tangent	$y = \text{Sin } x$ if and only if $y = \sin x$ and $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$. $y = \text{Cos } x$ if and only if $y = \cos x$ and $0 \le x \le \pi$. $y = \text{Tan } x$ if and only if $y = \tan x$ and $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$.
Inverse Sine, Cosine, and Tangent	Given $y = \text{Sin } x$, the inverse Sine function is defined by $y = \text{Sin}^{-1} x$ or $y = \text{Arcsin } x$. Given $y = \text{Cos } x$, the inverse Cosine function is defined by $y = \text{Cos}^{-1} x$ or $y = \text{Arccos } x$. Given $y = \text{Tan } x$, the inverse Tangent function is given by $y = \text{Tan}^{-1} x$ or $y = \text{Arctan } x$.

Example 1 **Solve $x = \text{Sin}^{-1}\left(\frac{\sqrt{3}}{2}\right)$.**

If $x = \text{Sin}^{-1}\left(\frac{\sqrt{3}}{2}\right)$, then $\text{Sin } x = \frac{\sqrt{3}}{2}$ and $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$.

The only x that satisfies both criteria is $x = \frac{\pi}{3}$ or 60°.

Example 2 **Solve $\text{Arctan}\left(-\frac{\sqrt{3}}{3}\right) = x$.**

If $x = \text{Arctan}\left(-\frac{\sqrt{3}}{3}\right)$, then $\text{Tan } x = -\frac{\sqrt{3}}{3}$ and $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$.

The only x that satisfies both criteria is $-\frac{\pi}{6}$ or $-30°$.

Exercises

Solve each equation by finding the value of *x* to the nearest degree.

1. $\text{Cos}^{-1}\left(-\frac{\sqrt{3}}{2}\right) = x$

2. $x = \text{Sin}^{-1} \frac{\sqrt{3}}{2}$

3. $x = \text{Arccos } (-0.8)$

4. $x = \text{Arctan } \sqrt{3}$

5. $x = \text{Arccos}\left(-\frac{\sqrt{2}}{2}\right)$

6. $x = \text{Tan}^{-1} (-1)$

7. $\text{Sin}^{-1} 0.45 = x$

8. $x = \text{Arcsin}\left(-\frac{\sqrt{3}}{2}\right)$

9. $x = \text{Arccos}\left(-\frac{1}{2}\right)$

10. $\text{Cos}^{-1} (-0.2) = x$

11. $x = \text{Tan}^{-1} (-\sqrt{3})$

12. $x = \text{Arcsin } 0.3$

13. $x = \text{Tan}^{-1} (15)$

14. $x = \text{Cos}^{-1} 1$

15. $\text{Arctan}^{-1} (-3) = x$

16. $x = \text{Sin}^{-1} (-0.9)$

17. $\text{Arccos}^{-1} 0.15$

18. $x = \text{Tan}^{-1} 0.2$

Lesson 13-7

13-7 Study Guide and Intervention *(continued)*

Inverse Trigonometric Functions

Trigonometric Values You can use a calculator to find the values of trigonometric expressions.

Example **Find each value. Write angle measures in radians. Round to the nearest hundredth.**

a. Find $\tan\left(\text{Sin}^{-1} \frac{1}{2}\right)$.

Let $\theta = \text{Sin}^{-1} \frac{1}{2}$. Then $\text{Sin}\ \theta = \frac{1}{2}$ with $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$. The value $\theta = \frac{\pi}{6}$ satisfies both conditions. $\tan \frac{\pi}{6} = \frac{\sqrt{3}}{3}$ so $\tan\left(\text{Sin}^{-1} \frac{1}{2}\right) = \frac{\sqrt{3}}{3}$.

b. Find $\cos\ (\text{Tan}^{-1} 4.2)$.

KEYSTROKES: COS 2nd [$\tan^{-1}$] 4.2 ENTER .2316205273

Therefore $\cos\ (\text{Tan}^{-1} 4.2) \approx 0.23$.

Exercises

Find each value. Write angle measures in radians. Round to the nearest hundredth.

1. $\cot\ (\text{Tan}^{-1} 2)$

2. $\text{Arctan}(-1)$

3. $\cot^{-1} 1$

4. $\cos\left[\text{Sin}^{-1}\left(-\frac{\sqrt{2}}{2}\right)\right]$

5. $\text{Sin}^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

6. $\sin\left(\text{Arcsin}\ \frac{\sqrt{3}}{2}\right)$

7. $\tan\left[\text{Arcsin}\left(-\frac{5}{7}\right)\right]$

8. $\sin\left(\text{Tan}^{-1} \frac{5}{12}\right)$

9. $\sin\ [\text{Arctan}^{-1}(-\sqrt{2})]$

10. $\text{Arccos}\left(-\frac{\sqrt{3}}{2}\right)$

11. $\text{Arcsin}\left(\frac{\sqrt{3}}{2}\right)$

12. $\text{Arccot}\left(-\frac{\sqrt{3}}{3}\right)$

13. $\cos\ [\text{Arcsin}\ (-0.7)]$

14. $\tan\ (\text{Cos}^{-1} 0.28)$

15. $\cos\ (\text{Arctan}\ 5)$

16. $\text{Sin}^{-1}(-0.78)$

17. $\text{Cos}^{-1} 0.42$

18. $\text{Arctan}\ (-0.42)$

19. $\sin\ (\text{Cos}^{-1} 0.32)$

20. $\cos\ (\text{Arctan}\ 8)$

21. $\tan\ (\text{Cos}^{-1} 0.95)$

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13-7 Skills Practice

Inverse Trigonometric Functions

Write each equation in the form of an inverse function.

1. $\alpha = \cos \beta$

2. $\sin b = a$

3. $y = \tan x$

4. $\cos 45° = \frac{\sqrt{2}}{2}$

5. $b = \sin 150°$

6. $\tan y = \frac{4}{5}$

Solve each equation by finding the value of *x* to the nearest degree.

7. $x = \text{Cos}^{-1}(-1)$

8. $\text{Sin}^{-1}(-1) = x$

9. $\text{Tan}^{-1} 1 = x$

10. $x = \text{Arcsin}\left(-\frac{\sqrt{3}}{2}\right)$

11. $x = \text{Arctan } 0$

12. $x = \text{Arccos } \frac{1}{2}$

Find each value. Write angle measures in radians. Round to the nearest hundredth.

13. $\text{Sin}^{-1} \frac{\sqrt{2}}{2}$

14. $\text{Cos}^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

15. $\text{Tan}^{-1} \sqrt{3}$

16. $\text{Arctan}\left(-\frac{\sqrt{3}}{3}\right)$

17. $\text{Arccos}\left(-\frac{\sqrt{2}}{2}\right)$

18. $\text{Arcsin } 1$

19. $\sin(\text{Cos}^{-1} 1)$

20. $\sin\left(\text{Sin}^{-1} \frac{1}{2}\right)$

21. $\tan\left(\text{Arcsin } \frac{\sqrt{3}}{2}\right)$

22. $\cos(\text{Tan}^{-1} 3)$

23. $\sin[\text{Arctan}(-1)]$

24. $\sin\left[\text{Arccos}\left(-\frac{\sqrt{2}}{2}\right)\right]$

Lesson 13-7

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13-7 Practice

Inverse Trigonometric Functions

Write each equation in the form of an inverse function.

1. $\beta = \cos \alpha$

2. $\tan \beta = \alpha$

3. $y = \tan 120°$

4. $-\frac{1}{2} = \cos x$

5. $\sin \frac{2\pi}{3} = \frac{\sqrt{3}}{2}$

6. $\cos \frac{\pi}{3} = \frac{1}{2}$

Solve each equation by finding the value of *x* to the nearest degree.

7. $\text{Arcsin } 1 = x$

8. $\text{Cos}^{-1} \frac{\sqrt{3}}{2} = x$

9. $x = \tan^{-1} \left(-\frac{\sqrt{3}}{3}\right)$

10. $x = \text{Arccos } \frac{\sqrt{2}}{2}$

11. $x = \text{Arctan } \left(-\sqrt{3}\right)$

12. $\text{Sin}^{-1} \left(-\frac{1}{2}\right) = x$

Find each value. Write angle measures in radians. Round to the nearest hundredth.

13. $\text{Cos}^{-1} \left(-\frac{\sqrt{3}}{2}\right)$

14. $\text{Sin}^{-1} \left(-\frac{\sqrt{2}}{2}\right)$

15. $\text{Arctan } \left(-\frac{\sqrt{3}}{3}\right)$

16. $\tan \left(\text{Cos}^{-1} \frac{1}{2}\right)$

17. $\cos \left[\text{Sin}^{-1} \left(-\frac{3}{5}\right)\right]$

18. $\cos \left[\text{Arctan } (-1)\right]$

19. $\tan \left(\sin^{-1} \frac{12}{13}\right)$

20. $\sin \left(\text{Arctan } \frac{\sqrt{3}}{3}\right)$

21. $\text{Cos}^{-1} \left(\tan \frac{3\pi}{4}\right)$

22. $\text{Sin}^{-1} \left(\cos \frac{\pi}{3}\right)$

23. $\sin \left(2 \text{ Cos}^{-1} \frac{15}{17}\right)$

24. $\cos \left(2 \text{ Sin}^{-1} \frac{\sqrt{3}}{2}\right)$

25. PULLEYS The equation $x = \cos^{-1} 0.95$ describes the angle through which pulley A moves, and $y = \cos^{-1} 0.17$ describes the angle through which pulley B moves. Both angles are greater than 270° and less than 360°. Which pulley moves through a greater angle?

26. FLYWHEELS The equation $y = \text{Arctan } 1$ describes the counterclockwise angle through which a flywheel rotates in 1 millisecond. Through how many degrees has the flywheel rotated after 25 milliseconds?

13-7 Reading to Learn Mathematics

Inverse Trigonometric Functions

Pre-Activity **How are inverse trigonometric functions used in road design?**

Read the introduction to Lesson 13-7 at the top of page 746 in your textbook.

Suppose you are given specific values for v and r. What feature of your graphing calculator could you use to find the approximate measure of the banking angle θ?

Reading the Lesson

1. Indicate whether each statement is *true* or *false*.

a. The domain of the function $y = \sin x$ is the set of all real numbers.

b. The domain of the function $y = \text{Cos } x$ is $0 \leq x \leq \pi$.

c. The range of the function $y = \text{Tan } x$ is $-1 \leq y \leq 1$.

d. The domain of the function $y = \text{Cos}^{-1} x$ is $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$.

e. The domain of the function $y = \text{Tan}^{-1} x$ is the set of all real numbers.

f. The range of the function $y = \text{Arcsin } x$ is $0 \leq x \leq \pi$.

2. Answer each question in your own words.

a. What is the difference between the functions $y = \sin x$ and the function $y = \text{Sin } x$?

b. Why is it necessary to restrict the domains of the trigonometric functions in order to define their inverses?

Helping You Remember

3. What is a good way to remember the domains of the functions $y = \text{Sin } x$, $y = \text{Cos } x$, and $y = \text{Tan } x$, which are also the range of the functions $y = \text{Arcsin } x$, $y = \text{Arccos } x$, and $y = \text{Arctan } x$? (You may want to draw a diagram.)

13-7 Enrichment

Snell's Law

Snell's Law describes what happens to a ray of light that passes from air into water or some other substance. In the figure, the ray starts at the left and makes an angle of incidence θ with the surface.

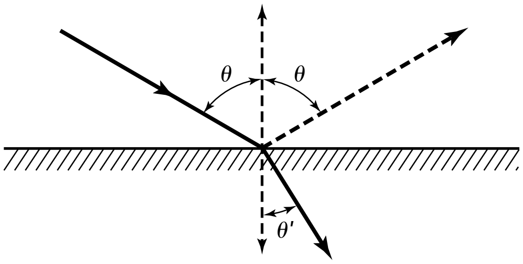
Part of the ray is reflected, creating an angle of reflection θ. The rest of the ray is bent, or refracted, as it passes through the other medium. This creates angle θ'.

The angle of incidence equals the angle of reflection.

The angles of incidence and refraction are related by Snell's Law:

$\sin \theta = k \sin \theta'$

The constant k is called the index of refraction.



k	Substance
1.33	Water
1.36	Ethyl alcohol
1.54	Rock salt and Quartz
1.46–1.96	Glass
2.42	Diamond

Use Snell's Law to solve the following. Round angle measures to the nearest tenth of a degree.

1. If the angle of incidence at which a ray of light strikes the surface of a window is 45° and $k = 1.6$, what is the measure of the angle of refraction?

2. If the angle of incidence of a ray of light that strikes the surface of water is 50°, what is the angle of refraction?

3. If the angle of refraction of a ray of light striking a quartz crystal is 24°, what is the angle of incidence?

4. The angles of incidence and refraction for rays of light were measured five times for a certain substance. The measurements (one of which was in error) are shown in the table. Was the substance glass, quartz, or diamond?

θ	15°	30°	40°	60°	80°
θ'	9.7°	16.1°	21.2°	28.6°	33.2°

5. If the angle of incidence at which a ray of light strikes the surface of ethyl alcohol is 60°, what is the angle of refraction?

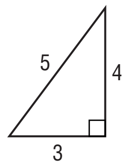
NAME ______________________ DATE __________ PERIOD ______

13 Chapter 13 Test, Form 1

SCORE ______

Write the letter for the correct answer in the blank at the right of each question.

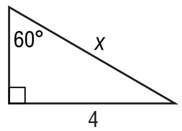
1. Find the value of tan θ.



A. $\frac{4}{3}$ **B.** $\frac{3}{4}$

C. $\frac{4}{5}$ **D.** $\frac{5}{3}$ **1.** ______

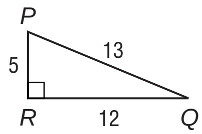
2. Which equation can be used to find x?



A. $\cos 60° = \frac{4}{x}$ **B.** $\tan 60° = \frac{x}{4}$

C. $\sin 60° = \frac{4}{x}$ **D.** $\cot 60° = \frac{4}{x}$ **2.** ______

3. Find P to the nearest degree.



A. 21° **B.** 23°

C. 67° **D.** 69° **3.** ______

4. Rewrite 90° in radian measure.

A. $\frac{\pi}{2}$ **B.** $\frac{\pi}{90}$ **C.** $\frac{\pi}{4}$ **D.** $\frac{2}{\pi}$ **4.** ______

5. Rewrite $\frac{\pi}{6}$ radians in degree measure.

A. $30\pi°$ **B.** 30° **C.** 120° **D.** 60° **5.** ______

6. Which angle is coterminal with a 90° angle in standard position?

A. 540° **B.** 450° **C.** $-90°$ **D.** 270° **6.** ______

7. Find the exact value of cos θ if the terminal side of θ in standard position contains the point (8, 15).

A. $\frac{17}{8}$ **B.** $\frac{8}{17}$ **C.** $\frac{8}{15}$ **D.** $\frac{15}{17}$ **7.** ______

8. What is the reference angle for 150°?

A. 150° **B.** 60° **C.** $-210°$ **D.** 30° **8.** ______

9. Find the exact value of sin 150°.

A. $-\frac{\sqrt{3}}{2}$ **B.** $\frac{\sqrt{3}}{2}$ **C.** $\frac{1}{2}$ **D.** $-\frac{1}{2}$ **9.** ______

10. Which formula can be used to find the area of $\triangle ABC$?

A. $\text{area} = \frac{1}{2}ac \sin C$ **B.** $\text{area} = \frac{1}{2}ab \sin A$

C. $\text{area} = \frac{1}{2}bc \sin A$ **D.** $\text{area} = \frac{1}{2}bc \sin B$ **10.** ______

13 Chapter 13 Test, Form 1 *(continued)*

11. In $\triangle ABC$, $A = 42°$, $C = 56°$, and $a = 12$. Find c.

A. 9.7 **B.** 21.6 **C.** 16.0 **D.** 14.9 **11.** ______

12. Determine the number of solutions for $\triangle ABC$ if $A = 139°$, $a = 12$, and $b = 19$.

A. no solution **B.** 1 solution **C.** 2 solutions **D.** 3 solutions **12.** ______

13. In $\triangle ABC$, find a if $b = 2$, $c = 6$, and $A = 35°$.

A. 20.3 **B.** 7.7 **C.** 5.5 **D.** 4.5 **13.** ______

14. Which triangle should be solved by beginning with the Law of Cosines?

A. $A = 20°, C = 50°, b = 3$ **B.** $a = 13, b = 24, c = 24$

C. $A = 30°, a = 5, b = 7$ **D.** $B = 45°, C = 25°, c = 10$ **14.** ______

15. $P\left(-\frac{4}{5}, -\frac{3}{5}\right)$ is located on the unit circle. Find $\cos\theta$.

A. $\frac{4}{5}$ **B.** $-\frac{4}{5}$ **C.** $-\frac{3}{5}$ **D.** $\frac{3}{4}$ **15.** ______

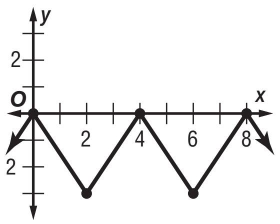
16. Find the exact value of sin 390°.

A. $\frac{1}{2}$ **B.** $-\frac{1}{2}$ **C.** 0 **D.** 1 **16.** ______

17. Determine the period of the function.

A. 2 **B.** 8

C. 3 **D.** 4



17. ______

18. Solve $y = \text{Sin}^{-1}\frac{\sqrt{3}}{2}$.

A. 30° **B.** 60° **C.** 45° **D.** 90° **18.** ______

19. Find the value of $\text{Sin}^{-1}(-1)$.

A. 30° **B.** −45° **C.** 180° **D.** −90° **19.** ______

20. Find the value of $\cos(\text{Cos}^{-1} 1)$.

A. 1 **B.** −1 **C.** $\frac{1}{2}$ **D.** $-\frac{1}{2}$ **20.** ______

Bonus Find the perimeter of $\triangle ABC$ to the nearest tenth if $A = 25°$, $C = 90°$, and $c = 10$ meters. **B:** ______________

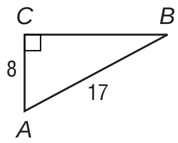
NAME ______________________ DATE __________ PERIOD ______

13 Chapter 13 Test, Form 2A

SCORE ______

Write the letter for the correct answer in the blank at the right of each question.

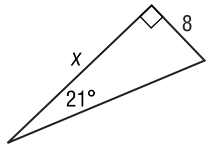
1. Find the value of csc A.



A. $\frac{8}{17}$ **B.** $\frac{17}{15}$

C. $\frac{17}{8}$ **D.** $\frac{15}{17}$ **1.** ______

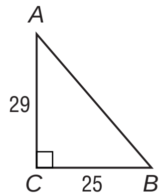
2. Which equation can be used to find x?



A. $\sin 21° = \frac{8}{x}$ **B.** $\tan 21° = \frac{x}{8}$

C. $\tan 21° = \frac{8}{x}$ **D.** $\sin 21° = \frac{x}{8}$ **2.** ______

3. Find A to the nearest degree.



A. 49° **B.** 37°

C. 41° **D.** 53° **3.** ______

4. Rewrite $\frac{2\pi}{9}$ radians in degree measure.

A. 20° **B.** 80° **C.** 40° **D.** $\frac{40°}{\pi}$ **4.** ______

5. Which angle is coterminal with an angle in standard position measuring $-\frac{5\pi}{9}$?

A. $\frac{13\pi}{9}$ **B.** $\frac{5\pi}{9}$ **C.** $\frac{23\pi}{9}$ **D.** $\frac{10\pi}{9}$ **5.** ______

6. Find the exact value of $\sin \theta$ if the terminal side of θ in standard position contains the point $(-4, -3)$.

A. $-\frac{4}{5}$ **B.** $-\frac{3}{5}$ **C.** $\frac{3}{5}$ **D.** $\frac{4}{5}$ **6.** ______

7. Find the exact value of cot 450°.

A. 0 **B.** undefined **C.** 1 **D.** −1 **7.** ______

8. Find the exact value of $\cos\left(-\frac{\pi}{4}\right)$.

A. $\frac{\sqrt{2}}{2}$ **B.** $-\frac{\sqrt{2}}{2}$ **C.** $\frac{\sqrt{3}}{2}$ **D.** $-\frac{\sqrt{3}}{2}$ **8.** ______

9. In $\triangle ABC$, $A = 40°$, $B = 60°$, and $a = 5$. Find b.

A. 6.4 **B.** 7.5 **C.** 6.7 **D.** 3.7 **9.** ______

10. Find the area of $\triangle ABC$ if $A = 72°$, $b = 9$ feet and $c = 10$ feet.

A. 85.6 ft^2 **B.** 42.8 ft^2 **C.** 45.0 ft^2 **D.** 13.9 ft^2 **10.** ______

13 Chapter 13 Test, Form 2A *(continued)*

11. Which triangle has two solutions?

A. $A = 130°, a = 19, b = 11$ **B.** $A = 45°, a = 4\sqrt{2}, b = 8$
C. $A = 32°, a = 16, b = 21$ **D.** $A = 90°, a = 25, c = 15$ **11.** ______

12. In $\triangle ABC$, $C = 60°$, $a = 12$, and $b = 5$. Find c.

A. 109.0 **B.** 10.4 **C.** 11.8 **D.** 15.1 **12.** ______

13. Which triangle should be solved by beginning with the Law of Cosines?

A. $A = 115°, a = 19, b = 13$ **B.** $A = 62°, B = 15°, b = 10$
C. $B = 48°, a = 22, b = 5$ **D.** $A = 50°, b = 20, c = 18$ **13.** ______

14. $P\left(-\frac{9}{41}, \frac{40}{41}\right)$ is located on the unit circle. Find $\sin \theta$.

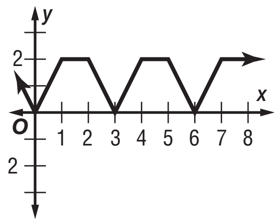
A. $\frac{40}{41}$ **B.** $-\frac{9}{41}$ **C.** $-\frac{9}{40}$ **D.** $-\frac{40}{9}$ **14.** ______

15. Find the exact value of $\cos(-420)°$.

A. $-\frac{1}{2}$ **B.** $\frac{1}{2}$ **C.** $\frac{\sqrt{3}}{2}$ **D.** $-\frac{\sqrt{3}}{2}$ **15.** ______

16. Determine the period of the function.

A. 2 **B.** 3
C. 6 **D.** 1 **16.** ______



17. Write the equation $\sin y = x$ in the form of an inverse function.

A. $y = \text{Sin}^{-1} x$ **B.** $x = \text{Sin}^{-1} y$ **C.** $y = \sin^{-1} x$ **D.** $y = \text{Sin } x$ **17.** ______

18. Solve $y = \text{Arcsin } \frac{1}{2}$.

A. $-\frac{5\pi}{6}$ **B.** $\frac{5\pi}{6}$ **C.** $-\frac{\pi}{6}$ **D.** $\frac{\pi}{6}$ **18.** ______

19. Find the value of $\text{Sin}^{-1}\left(-\frac{1}{2}\right)$.

A. $-30°$ **B.** $30°$ **C.** $150°$ **D.** $330°$ **19.** ______

20. Find the value of $\tan\left(\text{Tan}^{-1} \frac{1}{2}\right)$.

A. -1 **B.** 1 **C.** $\frac{1}{2}$ **D.** $-\frac{1}{2}$ **20.** ______

Bonus From one point on the ground, the angle of elevation to the top of a building is 35°, while 100 feet closer, the angle of elevation is 45°. Find the height of the building to the nearest foot. **B:** ______________

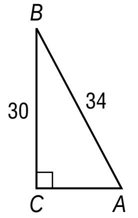
NAME ______________________ DATE ____________ PERIOD ______

13 Chapter 13 Test, Form 2B

SCORE ______

Write the letter for the correct answer in the blank at the right of each question.

1. Find the value of sec A.

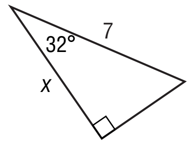


A. $\frac{17}{8}$ **B.** $\frac{8}{17}$

C. $\frac{15}{17}$ **D.** $\frac{17}{15}$

1. ______

2. Which equation can be used to find x?

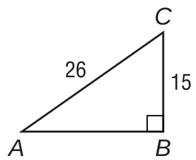


A. $\sin 32° = \frac{x}{7}$ **B.** $\cot 32° = \frac{7}{x}$

C. $\tan 32° = \frac{x}{7}$ **D.** $\cos 32° = \frac{x}{7}$

2. ______

3. Find A to the nearest degree.



A. 55° **B.** 30°

C. 35° **D.** 60°

3. ______

4. Rewrite $\frac{5\pi}{4}$ radians in degree measure.

A. 450° **B.** 225° **C.** 225π° **D.** 112.5°

4. ______

5. Which angle is coterminal with a −400° angle in standard position?

A. 40° **B.** 80° **C.** 320° **D.** 400°

5. ______

6. Find the exact value of $\cos \theta$ if the terminal side of θ in standard position contains the point (6, −8).

A. $-\frac{4}{5}$ **B.** $\frac{3}{5}$ **C.** $\frac{4}{5}$ **D.** $-\frac{3}{5}$

6. ______

7. Find the exact value of cot (−315°).

A. 1 **B.** $\sqrt{2}$ **C.** $\frac{\sqrt{2}}{2}$ **D.** 2

7. ______

8. Find the exact value of $\sin\left(-\frac{\pi}{6}\right)$.

A. $-\frac{1}{2}$ **B.** $-\frac{\sqrt{3}}{2}$ **C.** $-\frac{\sqrt{2}}{2}$ **D.** $\frac{\sqrt{2}}{2}$

8. ______

9. In $\triangle ABC$, $C = 30°$, $c = 22$, and $b = 42$. Find B.

A. 73° **B.** 107° **C.** 77° **D.** 15°

9. ______

10. Find the area of $\triangle ABC$ if $A = 55°$, $b = 8$ meters and $c = 14$ meters.

A. 91.7 m^2 **B.** 32.1 m^2 **C.** 45.9 m^2 **D.** 56.0 m^2

10. ______

NAME ______________ DATE ________ PERIOD ______

13 Chapter 13 Test, Form 2B *(continued)*

11. Which triangle has no solution?

A. $A = 45°, a = 3, b = 4$
B. $A = 135°, a = 15, b = 9$
C. $A = 15°, a = 3, b = 19$
D. $A = 69°, a = 12, b = 6$

11. ______

12. In $\triangle ABC, A = 15°, b = 19$, and $c = 12$. Find a.

A. 64.5 **B.** 16.9 **C.** 30.7 **D.** 8.0

12. ______

13. Which triangle should be solved by beginning with the Law of Sines?

A. $A = 125°, B = 16°, a = 10$
B. $A = 85°, b = 31, c = 24$
C. $B = 72°, a = 5, c = 17$
D. $a = 13, b = 9, c = 15$

13. ______

14. $P\left(-\frac{40}{41}, -\frac{9}{41}\right)$ is located on the unit circle. Find $\cos \theta$.

A. $-\frac{9}{41}$ **B.** $-\frac{40}{41}$ **C.** $\frac{40}{9}$ **D.** $-\frac{9}{40}$

14. ______

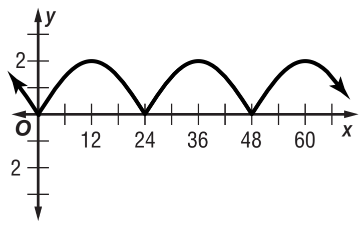
15. Find the exact value of $\sin 870°$.

A. $-\frac{1}{2}$ **B.** $\frac{1}{2}$ **C.** $-\frac{\sqrt{3}}{2}$ **D.** $\frac{\sqrt{3}}{2}$

15. ______

16. Determine the period of the function.

A. 60 **B.** 48
C. 2 **D.** 24



16. ______

17. Write the equation $\tan b = c$ in the form of an inverse function.

A. $b = \tan c$ **B.** $c = \text{Tan}^{-1} b$ **C.** $b = \text{Tan}^{-1} c$ **D.** $\tan = \frac{b}{c}$

17. ______

18. Solve $y = \text{Cos}^{-1} \frac{\sqrt{2}}{2}$.

A. $-135°$ **B.** $-45°$ **C.** $45°$ **D.** $135°$

18. ______

19. Find the value of $\text{Tan}^{-1} \sqrt{3}$.

A. $\frac{\pi}{3}$ **B.** $\frac{2\pi}{3}$ **C.** $-\frac{\pi}{3}$ **D.** $-\frac{2\pi}{3}$

19. ______

20. Find the value of $\tan\left(\text{Arccos } \frac{1}{2}\right)$.

A. 1.36 **B.** 0.58 **C.** 1.73 **D.** 0.02

20. ______

Bonus From one point on the ground, the angle of elevation to the top of a building is 34°, while 100 feet closer, the angle of elevation is 48°. Find the height of the building to the nearest foot.

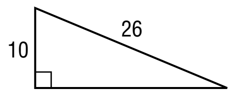
B: ______________

NAME ______________________ DATE ____________ PERIOD ______

13 Chapter 13 Test, Form 2C

SCORE ______

1. Find the values of the six trigonometric functions for angle θ.

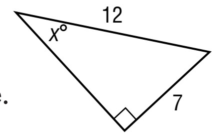


1. ______________

2. Solve $\triangle ABC$ if $a = 3$, $c = 7$, and $C = 90°$. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.

2. ______________

3. Write an equation involving sin, cos, or tan that can be used to find x. Then solve the equation, rounding to the nearest degree.



3. ______________

4. Rewrite $-75°$ in radian measure.

4. ______________

5. Rewrite $\frac{5\pi}{3}$ radians in degree measure.

5. ______________

6. Find one angle with positive measure and one angle with negative measure coterminal with an angle in standard position measuring $\frac{5\pi}{4}$.

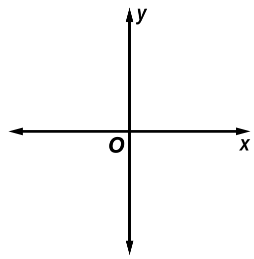
6. ______________

7. Find the exact values of the six trigonometric functions of θ if the terminal side of θ in standard position contains the point $(-4, -6)$.

7. ______________

8. Sketch the angle with measure $\frac{5\pi}{3}$ radians. Then find its reference angle.

8. ______________



For Questions 9 and 10, find the exact value of each trigonometric function.

9. $\sin\left(-\frac{\pi}{3}\right)$

9. ______________

10. $\cos 810°$

10. ______________

11. Find the area of $\triangle ABC$ if $C = 74°$, $a = 21$ miles, and $b = 63$ miles. Round to the nearest tenth.

11. ______________

Assessment

13 Chapter 13 Test, Form 2C *(continued)*

Determine whether each triangle has *no* solution, *one* solution, or *two* solutions. Then solve each triangle. Round to the nearest tenth.

12. $A = 58°, a = 17, b = 12$ **12.** __________

13. $A = 110°, a = 6, b = 15$ **13.** __________

For Questions 14 and 15, determine whether each triangle should be solved by beginning with the Law of Sines or Law of Cosines. Then solve each triangle. Round to the nearest tenth.

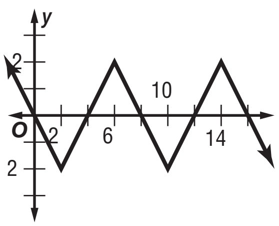
14. $A = 70°, B = 80°, a = 9$ **14.** __________

15. $C = 114.6°, a = 5, b = 7$ **15.** __________

16. Given an angle θ in standard position, if $P\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$ lies on the terminal side and on the unit circle, find $\sin \theta$ and $\cos \theta$. **16.** __________

17. Find the exact value of $\sin\left(-\frac{10\pi}{3}\right)$. **17.** __________

18. Determine the period of the function. **18.** __________



19. Solve $x = \tan^{-1}(-1)$. **19.** __________

20. Find the value of $\sin\left(\text{Arctan } \frac{\sqrt{3}}{3}\right)$. Round to the nearest hundredth. **20.** __________

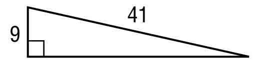
Bonus A tree is observed on the opposite bank of a river. At that point, the river is known to be 140 feet wide. The angle of elevation from a point 5 feet off the ground to the top of the tree is 20°. Find the height of the tree to the nearest foot. **B:** __________

NAME ________________________ DATE ____________ PERIOD ________

13 Chapter 13 Test, Form 2D

SCORE ________

1. Find the values of the six trigonometric functions for angle θ.

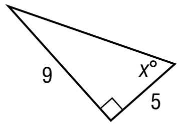


1. ________________

2. Solve $\triangle ABC$ if $c = 8$, $a = 5$, and $C = 90°$. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.

2. ________________

3. Write an equation involving sin, cos, or tan that can be used to find x. Then solve the equation, rounding to the nearest degree.



3. ________________

4. Rewrite 330° in radian measure.

4. ________________

5. Rewrite $-\frac{7\pi}{4}$ radians in degree measure.

5. ________________

6. Find one angle with positive measure and one angle with negative measure coterminal with an angle in standard position measuring $-120°$.

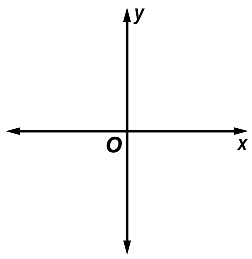
6. ________________

7. Find the exact values of the six trigonometric functions of θ if the terminal side of θ in standard position contains the point $(12, -8)$.

7. ________________

8. Sketch the angle with measure $-\frac{7\pi}{4}$ radians. Then find its reference angle.

8. ________________



For Questions 9 and 10, find the exact value of each trigonometric function.

9. $\cos \frac{2\pi}{3}$

9. ________________

10. $\sin -630°$

10. ________________

11. Find the area of $\triangle ABC$ if $C = 62°$, $a = 12$ yards, and $b = 9$ yards. Round to the nearest tenth.

11. ________________

Assessment

13 Chapter 13 Test, Form 2D *(continued)*

Determine whether each triangle has *no* solution, *one* solution, or *two* solutions. Then solve each triangle. Round to the nearest tenth.

12. $A = 29°, a = 5, b = 14$ **12.** __________

13. $A = 60°, a = 9, b = 6$ **13.** __________

For Questions 14 and 15, determine whether each triangle should be solved by beginning with the Law of Sines or Law of Cosines. Then solve each triangle. Round to the nearest tenth.

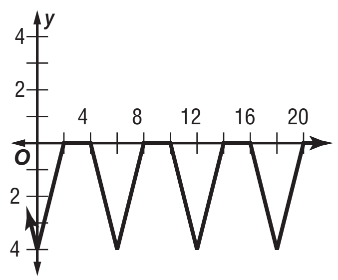
14. $A = 19°, a = 10, b = 8$ **14.** __________

15. $C = 45°, a = 4, b = 9$ **15.** __________

16. Given an angle θ in standard position, if $P\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$ lies on the terminal side and on the unit circle, find $\sin \theta$ and $\cos \theta$. **16.** __________

17. Find the exact value of $\cos\left(-\frac{13\pi}{3}\right)$. **17.** __________

18. Determine the period of the function. **18.** __________



19. Solve $x = \text{Arcsin}\left(\frac{1}{2}\right)$. **19.** __________

20. Find the value of $\tan\left(\text{Tan}^{-1} \frac{3}{8}\right)$. Round to the nearest hundredth. **20.** __________

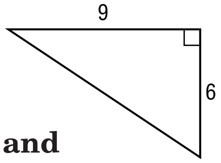
Bonus A tree is observed on the opposite bank of a river. At that point, the river is known to be 120 feet wide. The angle of elevation from a point 4 feet off the ground to the top of the tree is 25°. Find the height of the tree to the nearest foot. **B:** __________

NAME ________________ DATE ________ PERIOD ______

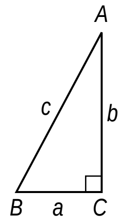
13 Chapter 13 Test, Form 3

SCORE ______

1. Find the values of the six trigonometric functions for angle θ. **1.** ________



Solve $\triangle ABC$ using the diagram at the right and the given measurements. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.



2. $B = 25°$, $a = \sqrt{7}$ **2.** ________

3. $\cos A = \frac{3}{5}$, $b = 6$ **3.** ________

For Questions 4 and 5, rewrite each degree measure in radians and each radian measure in degrees.

4. $-315°$ **4.** ________

5. -5 **5.** ________

6. Find one angle with positive measure and one angle with negative measure coterminal with 723°. **6.** ________

7. Find the exact values of the six trigonometric functions of θ if the terminal side of θ in standard position contains the point $(-\sqrt{3}, 1)$. **7.** ________

For Questions 8 and 9, find the exact value of each trigonometric function.

8. $\cos(-300°)$ **8.** ________

9. $\cot \frac{9\pi}{4}$ **9.** ________

10. In $\triangle ABC$, $a = 12$ meters, $b = 9$ meters, and $c = 6$ meters. Find the area of $\triangle ABC$. Round to the nearest tenth. **10.** ________

Determine whether each triangle has *no* solution, *one* solution, or *two* solutions. Then solve each triangle. Round to the nearest tenth.

11. $A = 42°$, $a = 9$, $b = 12$ **11.** ________

12. $A = 59°$, $a = 10$, $b = 7$ **12.** ________

Assessment

NAME ______________________________ DATE ____________ PERIOD _____

13 Chapter 13 Test, Form 3 *(continued)*

For Questions 13 and 14, determine whether each triangle should be solved by beginning with the Law of Sines or Law of Cosines. Then solve each triangle. Round to the nearest tenth.

13. $C = 40.1°, a = 3, b = 8.2$ **13.** ______________

14. $C = 132°, a = 15, c = 26$ **14.** ______________

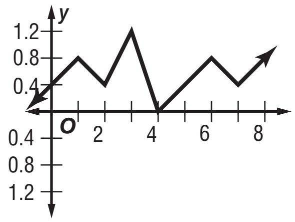
15. Given an angle θ in standard position, if $P\left(-\frac{2\sqrt{7}}{7}, \frac{\sqrt{21}}{7}\right)$ lies on the terminal side and on the unit circle, find $\sin \theta$ and $\cos \theta$. **15.** ______________

For Questions 16 and 17, find the exact value of each function.

16. $\sin \frac{19\pi}{6}$ **16.** ______________

17. $3(\sin 120°)(\cos 120°)$ **17.** ______________

18. Determine the period of the function. **18.** ______________



19. Solve $\text{Arccos}\left(-\frac{\sqrt{2}}{2}\right) = x$. **19.** ______________

20. Find the value of $\cos\left(2 \text{ Sin}^{-1} \frac{4}{5}\right)$. Round to the nearest hundredth. **20.** ______________

Bonus Given $\triangle ABC$ with $A = 27°$. Point X lies on $\overline{AC}$ such that $BX = 8$ meters and $\angle BXC$ has measure 142°. The area of $\triangle BXC$ is 21.9 square meters. Find the perimeter of $\triangle ABC$ to the nearest tenth. **B:** ______________

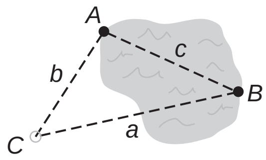
NAME ______________________ DATE ____________ PERIOD ______

13 Chapter 13 Open-Ended Assessment

SCORE ______

Demonstrate your knowledge by giving a clear, concise solution to each problem. Be sure to include all relevant drawings and justify your answers. You may show your solution in more than one way or investigate beyond the requirements of the problem.

1. Stakes driven at points A and B in the diagram indicate where a new bridge will be built over the body of water shown. Monica, a surveyor, must determine the length c of the new bridge. She drives a third stake at point C, then uses a transit to determine the measures of angles A, B, and C.



a. Explain why Monica does not yet have enough information to find c.

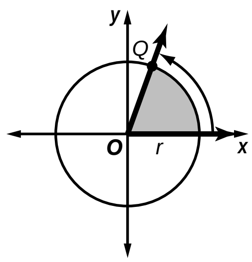
b. What additional information can she determine to help her find c?

c. Select reasonable measures for angles A, B, and C, and for the information you suggested in part b. Then determine the length of the bridge to the nearest whole unit. Explain your method.

2. For $\triangle XYZ$ with $X = 24°$, $Z = 90°$, $y = 13.7$, and $z = 15$, show three distinctly different ways to find the length x of the third side of the triangle. Round to the nearest tenth.

3. Select any point P in Quadrant III. Explain how to find the measure of θ if the terminal side of θ in standard position contains your point P. Round to the nearest degree.

4. The area of a sector with radius r and central angle θ is given by $A = \frac{1}{2}r^2\theta$, where θ is measured in radians. Select any point Q in Quadrant I. Explain how to find the area of the sector bounded by θ, whose terminal side contains your point Q, and the arc intercepted by θ (the area shaded in the figure). Round to the nearest tenth.



5. a. Explain how to find the area of $\triangle ABC$ with $C = 45°$, $b = 18$ inches, and $c = 9\sqrt{2}$ inches using the formula area $= \frac{1}{2}bc \sin A$ or $\frac{1}{2}ac \sin B$ or $\frac{1}{2}ab \sin C$. Determine the exact area.

b. Is it possible to find this area using the formula area $= \frac{1}{2}$(base)(height)? Explain your reasoning.

c. Explain the relationship, if any, between the two formulas.

Assessment

NAME ______________________________ DATE ____________ PERIOD _______

13 Chapter 13 Vocabulary Test/Review

SCORE _______

angle of depression	cosine	periodic	solve a right triangle
angle of elevation	cotangent	principal values	standard position
Arccosine function	coterminal angles	quadrantal angles	tangent
Arcsine function	initial side	radian	terminal side
Arctangent function	Law of Cosines	reference angle	trigonometric functions
circular function	Law of Sines	secant	trigonometry
cosecant	period	sine	unit circle

Choose the letter of the term that best matches each phrase.

_____ **1.** the acute angle formed by the terminal side of a nonquadrantal angle and the x-axis

_____ **2.** the ratio of the length of the side adjacent to an acute angle of a right triangle to the length of the hypotenuse

_____ **3.** the formula that is used to solve a triangle when two angles and one side are known

_____ **4.** the inverse of the function $y = \text{Sin } x$, which is the sine function with a restricted domain

_____ **5.** the angle between a line parallel to the ground and the line of sight of an object

_____ **6.** the side of an angle that is a ray fixed along the positive x-axis when the angle is in standard position

_____ **7.** the formula that is used to find the third side of a triangle when two sides and the included angle are known

_____ **8.** the ratio of the length of the side opposite an acute angle of a right triangle to the length of the adjacent side

_____ **9.** the side of an angle that is a ray that can rotate around the origin

_____ **10.** the reciprocal of the sine function

a. Law of Cosines
b. tangent
c. Arcsine function
d. terminal side
e. reference angle
f. cosecant
g. Law of Sines
h. cosine
i. initial side
j. angle of elevation

In your own words—
Define each term.

11. standard position

12. unit circle

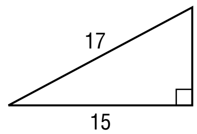
NAME ______________________ DATE __________ PERIOD ______

13 Chapter 13 Quiz

(Lessons 13–1 and 13–2)

SCORE ______

1. Find the values of the six trigonometric functions for angle θ.



1. ______

2. **Standardized Test Practice**

   If $\sin A = \frac{7}{10}$, find the value of $\cos A$.

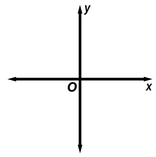
   A. $\frac{7\sqrt{149}}{149}$ **B.** $\frac{\sqrt{51}}{10}$ **C.** $\frac{10}{7}$ **D.** $\frac{\sqrt{51}}{7}$

2. ______

3. Solve $\triangle ABC$ if $A = 20°$, $C = 90°$, and $b = 10$. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.

3. ______

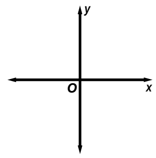
Draw an angle with the given measure in standard position. Find one angle with positive measure and one angle with negative measure coterminal with each angle.



4. ______

4. $-225°$

5. $\frac{\pi}{3}$



5. ______

NAME ______________________ DATE __________ PERIOD ______

13 Chapter 13 Quiz

(Lessons 13–3 and 13–4)

SCORE ______

1. Find the exact values of the six trigonometric functions of θ if the terminal side of θ in standard position contains the point $(-3, 1)$.

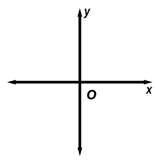
1. ______

2. Find the exact value of the trigonometric function $\sin(-135°)$.

2. ______

3. Sketch the angle $-\frac{8\pi}{3}$. Then find its reference angle.

3. ______



4. Find the area of $\triangle ABC$ if $A = 110°$, $b = 10$ inches, and $c = 19$ inches. Round to the nearest tenth.

4. ______

5. Determine whether $\triangle ABC$ has *no* solution, *one* solution, or *two* solutions if $A = 15°$, $a = 12$, and $b = 15$. Then solve. Round to the nearest tenth.

5. ______

NAME ____________ DATE ________ PERIOD ______

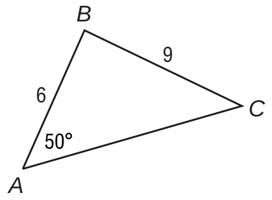
Chapter 13 Quiz

(Lessons 13–5 and 13–6)

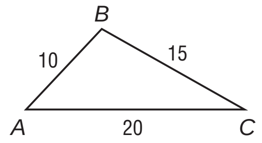
SCORE ______

Determine whether each triangle should be solved by beginning with the Law of Sines or Law of Cosines. Then solve each triangle. Round to the nearest tenth.

1.



2.



3. $A = 36°, b = 6, c = 12$

4. $a = 14, b = 8, c = 5$

$P\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$ **is located on the unit circle.**

5. Find $\sin \theta$.

6. Find $\cos \theta$.

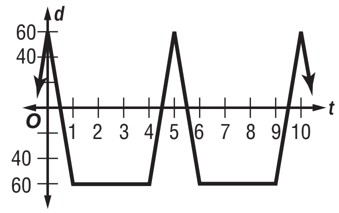
Find the exact value of each function.

7. $\cos -540°$

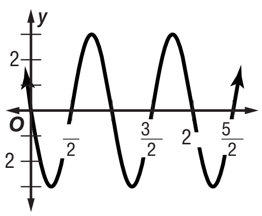
8. $\sin\left(\frac{13\pi}{4}\right)$

Determine the period of each function.

9.



10.



1. ____________
2. ____________
3. ____________
4. ____________
5. ____________
6. ____________
7. ____________
8. ____________
9. ____________
10. ____________

NAME ____________ DATE ________ PERIOD ______

Chapter 13 Quiz

(Lesson 13–7)

SCORE ______

1. Write the equation $\tan \theta = y$ in the form of an inverse function.

2. Solve $y = \text{Arctan } \sqrt{3}$.

Find each value. Round to the nearest hundredth.

3. $\text{Cos}^{-1}\ 1$

4. $\cos\left[\text{Sin}^{-1}\left(-\frac{3}{5}\right)\right]$

5. $\cot\left(\text{Arccos } \frac{1}{6}\right)$

1. ____________
2. ____________
3. ____________
4. ____________
5. ____________

NAME ______________________ DATE __________ PERIOD ______

13 Chapter 13 Mid-Chapter Test

SCORE ______

(Lessons 13–1 through 13–4)

Part I ***Write the letter for the correct answer in the blank at the right of each question.***

1. If $\sin A = \frac{3}{5}$, find $\cos A$.

A. $\frac{3}{4}$ **B.** $\frac{4}{5}$ **C.** $\frac{5}{3}$ **D.** $\frac{4}{3}$ **1.** ______

2. Rewrite 75° in radian measure.

A. $\frac{5\pi}{6}$ **B.** $\frac{5\pi}{12}$ **C.** $\frac{5}{12}$ **D.** $\frac{\pi}{5}$ **2.** ______

3. Rewrite $\frac{3\pi}{4}$ radians in degree measure.

A. 135° **B.** 540° **C.** 270° **D.** 240° **3.** ______

4. Which angle is coterminal with 590°?

A. 130° **B.** 50° **C.** 230° **D.** −140° **4.** ______

5. Which trigonometric function has a value of 0?

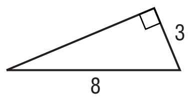
A. $\tan \frac{\pi}{2}$ **B.** sin 180° **C.** $\cos \pi$ **D.** cot 0° **5.** ______

6. Find the exact value of sin 240°.

A. $-\sqrt{3}$ **B.** $-\frac{\sqrt{3}}{2}$ **C.** $-\frac{1}{2}$ **D.** $\frac{1}{\sqrt{3}}$ **6.** ______

Part II

7. Find the values of the six trigonometric functions for angle θ. **7.** ______________



8. Solve $\triangle ABC$ if $A = 40°$, $C = 90°$, and $b = 10$. Round measures of sides to the nearest tenth and measures of angles to the nearest degree. **8.** ______________

9. Find the exact values of the six trigonometric functions of θ if the terminal side of θ in standard position contains the point $(-3, 6)$. **9.** ______________

10. Find the area of $\triangle ABC$ if $A = 98°$, $b = 45$ feet, and $c = 61$ feet. Round to the nearest tenth. **10.** ______________

Determine whether each triangle has *no* solution, *one* solution, or *two* solutions. Then solve each triangle. Round to the nearest tenth.

11. $A = 52°$, $a = 7$, $b = 3$ **11.** ______________

12. $A = 137°$, $a = 10$, $b = 15$ **12.** ______________

Assessment

NAME ______________________ DATE ____________ PERIOD ______

13 Chapter 13 Cumulative Review

(Chapters 1–13)

1. Write an equation in slope-intercept form for the line that passes through $(-6, 5)$ and is perpendicular to the line whose equation is $3x - 2y = 8$. (Lesson 2–4)

1. ____________

Solve. Round to four decimal places, if necessary.

2. $\begin{bmatrix} 3x + 2y \\ 4x - y \end{bmatrix} = \begin{bmatrix} 4 \\ -13 \end{bmatrix}$ (Lesson 4–1)

3. $\ln 4x = 6$ (Lesson 10–5)

2. ____________

3. ____________

For Questions 4 and 5, simplify.

4. $(2 + 7i)(5 - 6i)$ (Lesson 5–9)

5. $\frac{x}{x^2 - 9} + \frac{3}{5x - 15}$ (Lesson 9–2)

4. ____________

5. ____________

6. Write the quadratic function $y = 4x^2 - 24x + 20$ in vertex form. Then identify the vertex, axis of symmetry, and direction of opening of the graph. (Lesson 6–6)

6. ____________

7. Write $13n^4 = 52n^2$ in quadratic form, if possible. Then solve. (Lesson 7–3)

7. ____________

8. Find the coordinates of the vertices and foci and the equations of the asymptotes for the hyperbola with equation $y^2 - 4x^2 = 16$. (Lesson 8–5)

8. ____________

9. Find the sum of the arithmetic series $14 + 11 + 8 + \cdots + (-10)$. (Lesson 11–2)

9. ____________

10. Use Pascal's triangle to expand $(m + 3)^5$. (Lesson 11–7)

10. ____________

11. How many four-digit codes are possible if no digit can be used more than once? (Lesson 12–1)

11. ____________

12. Find the mean, median, mode, and standard deviation of the data set {26, 11, 5, 24, 12}. Round to the nearest hundredth, if necessary. (Lesson 12–6)

12. ____________

For Questions 13 and 14, solve $\triangle ABC$ using the given measurements. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.

13. $B = 49°, C = 90°, a = 9$ (Lesson 13–1)

14. $a = 16, b = 7, c = 12$ (Lessons 13–4 and 13–5)

13. ____________

14. ____________

15. Rewrite $\frac{7\pi}{4}$ radians in degree measure. (Lesson 13–2)

15. ____________

16. Find the value of $\text{Cos}^{-1} \frac{\sqrt{2}}{2}$. (Lesson 13–7)

16. ____________

NAME ______________________________ DATE ____________ PERIOD _____

13 Standardized Test Practice

(Chapters 1–13)

Part 1: Multiple Choice

Instructions: Fill in the appropriate oval for the best answer.

1. Which real number is irrational?

A. $\frac{27}{5}$ **B.** -0.257 **C.** $0.\overline{257}$ **D.** $0.2573\ldots$

1. (A) (B) (C) (D)

2. Which expression is the greatest in value?

E. $27 \div \frac{1}{3}$ **F.** $27 + \frac{1}{3}$ **G.** $27 \cdot \frac{1}{3}$ **H.** $27^{\frac{1}{3}}$

2. (E) (F) (G) (H)

3. Point B lies between points A and C such that the lengths AB and BC are in the ratio 2:5. If AB is 30 units in length, what is the length of AC?

A. 75 **B.** 105 **C.** 150 **D.** 42

3. (A) (B) (C) (D)

4. What is the sum of the integer factors of 36?

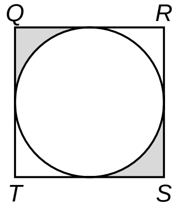
E. 97 **F.** 91 **G.** 85 **H.** 54

4. (E) (F) (G) (H)

5. What is the area of the shaded region in the figure if the perimeter of square $QRST$ is 48 units?

A. $72 - 18\pi$ **B.** $72 - 6\pi$

C. $144 - 36\pi$ **D.** $72 - 72\pi$



5. (A) (B) (C) (D)

6. If (x) is defined, for all positive integers x, to be $(x) = 4\sqrt[3]{x}$, what is the value of $(a^6) + (64)$?

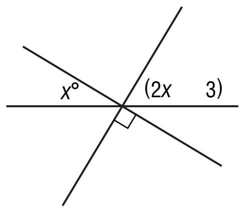
E. $4a^3 + 32$ **F.** $4a^2 + 4$ **G.** $4a^2 + 16$ **H.** $4a^3 + 16$

6. (E) (F) (G) (H)

7. In the figure, what is the value of x?

A. 3 **B.** 45

C. 61 **D.** 31



7. (A) (B) (C) (D)

8. What is the sum of the squares of the roots of the equation $x^2 + 2x = 80$?

E. -36 **F.** 164 **G.** 4 **H.** 416

8. (E) (F) (G) (H)

9. What is the length of the diameter of the base of a cylinder if its volume is 768π in^3 and its height is 12 in.?

A. 16 in. **B.** 8 in. **C.** 8π in. **D.** 64 in.

9. (A) (B) (C) (D)

10. Five people are to be seated on the stage during a graduation ceremony. In how many different ways can the people be arranged?

E. 5 **F.** 15 **G.** 24 **H.** 120

10. (E) (F) (G) (H)

Assessment

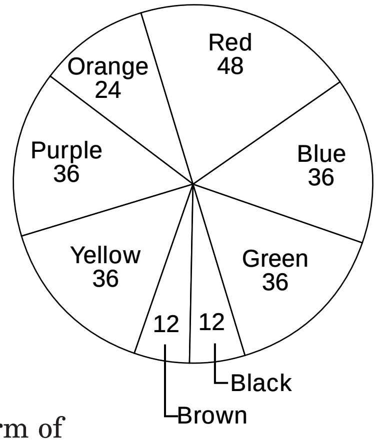
NAME ______________________________ DATE ____________ PERIOD ______

13 Standardized Test Practice *(continued)*

Part 2: Grid In

Instructions: Enter your answer by writing each digit of the answer in a column box and then shading in the appropriate oval that corresponds to that entry.

11. The circle graph shows the results of a survey of elementary school students who were asked to select their favorite color. What percent of the students selected orange?

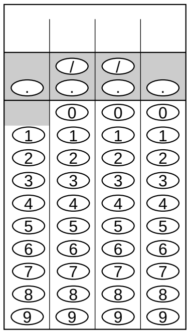


12. What is the 14th term of the sequence 1, 4, 9, 16, 25, …?

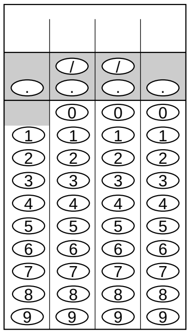
13. If one decasecond is equivalent to 10 seconds, how many decaseconds are equivalent to 2 hours?

14. By how much does twice the sum of 50 and 20 exceed the quotient of 80 and 20?

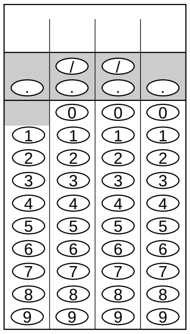
11.



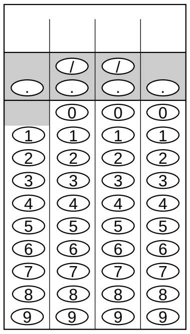
12.



13.



14.



Part 3: Quantitative Comparison

Instructions: Compare the quantities in columns A and B. Shade in
- (A) if the quantity in column A is greater;
- (B) if the quantity in column B is greater;
- (C) if the quantities are equal; or
- (D) if the relationship cannot be determined from the information given.

	Column A	Column B	
15.	30% of a where $a > 0$	$\frac{3a}{10}$	**15.** (A) (B) (C) (D)
16.	z where $\frac{1}{z} < 0$	z^2	**16.** (A) (B) (C) (D)
17.	$(r + t)^2$ where $r < 0$, $t < 0$	$(r - t)^2$	**17.** (A) (B) (C) (D)
18.	The probability of selecting a multiple of 3 when a number is randomly chosen from {5, 6, 7, 8, 9}	The probability of selecting an even integer when a number is randomly chosen from {5, 6, 7, 8, 9}	**18.** (A) (B) (C) (D)

NAME ______________________ DATE ____________ PERIOD ______

13

Standardized Test Practice

Student Record Sheet *(Use with pages 758–759 of the Student Edition.)*

Part 1 Multiple Choice

Select the best answer from the choices given and fill in the corresponding oval.

1 A B C D

2 A B C D

3 A B C D

4 A B C D

5 A B C D

6 A B C D

7 A B C D

8 A B C D

9 A B C D

10 A B C D

Part 2 Short Response/Grid In

Solve the problem and write your answer in the blank.

For Questions 12–17, also enter your answer by writing each number or symbol in a box. Then fill in the corresponding oval for that number or symbol.

11 ____________

12 ____________

13 ____________

14 ____________

15 ____________

16 ____________

17 ____________

Part 3 Quantitative Comparison

Select the best answer from the choices given and fill in the corresponding oval.

18 A B C D

19 A B C D

20 A B C D

21 A B C D

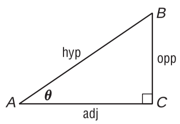
Answers

NAME ______________________ DATE ____________ PERIOD _____

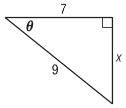
13-1 Study Guide and Intervention

Right Triangle Trigonometry

Trigonometric Values

Trigonometric Functions	If θ is the measure of an acute angle of a right triangle, *opp* is the measure of the leg opposite θ, *adj* is the measure of the leg adjacent to θ, and *hyp* is the measure of the hypotenuse, then the following are true.
	$\sin\theta = \frac{opp}{hyp}$ $\cos\theta = \frac{adj}{hyp}$ $\tan\theta = \frac{opp}{adj}$ $\csc\theta = \frac{hyp}{opp}$ $\sec\theta = \frac{hyp}{adj}$ $\cot\theta = \frac{adj}{opp}$

Example **Find the values of the six trigonometric functions for angle θ.**



Use the Pythagorean Theorem to find x, the measure of the leg opposite θ.

$x^2 + 7^2 = 9^2$ — Pythagorean Theorem
$x^2 + 49 = 81$ — Simplify.
$x^2 = 32$ — Subtract 49 from each side.
$x = \sqrt{32}$ or $4\sqrt{2}$ — Take the square root of each side.

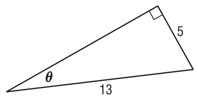
Use opp $= 4\sqrt{2}$, adj $= 7$, and hyp $= 9$ to write each trigonometric ratio.

$\sin\theta = \frac{4\sqrt{2}}{9}$ $\cos\theta = \frac{7}{9}$ $\tan\theta = \frac{4\sqrt{2}}{7}$ $\csc\theta = \frac{9\sqrt{2}}{8}$ $\sec\theta = \frac{9}{7}$ $\cot\theta = \frac{7\sqrt{2}}{8}$

Exercises

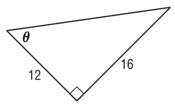
Find the values of the six trigonometric functions for angle θ.

1.



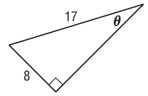
$\sin\theta = \frac{5}{13}$; $\cos\theta = \frac{12}{13}$; $\tan\theta = \frac{5}{12}$; $\csc\theta = \frac{13}{5}$; $\sec\theta = \frac{13}{12}$; $\cot\theta = \frac{12}{5}$

2.



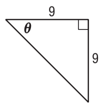
$\sin\theta = \frac{4}{5}$; $\cos\theta = \frac{3}{5}$; $\tan\theta = \frac{4}{3}$; $\csc\theta = \frac{5}{4}$; $\sec\theta = \frac{5}{3}$; $\cot\theta = \frac{3}{4}$

3.



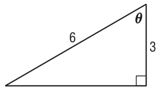
$\sin\theta = \frac{8}{17}$; $\cos\theta = \frac{15}{17}$; $\tan\theta = \frac{8}{15}$; $\csc\theta = \frac{17}{8}$; $\sec\theta = \frac{17}{15}$; $\cot\theta = \frac{15}{8}$

4.



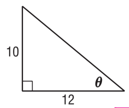
$\sin\theta = \frac{\sqrt{2}}{2}$; $\cos\theta = \frac{\sqrt{2}}{2}$; $\tan\theta = 1$; $\csc\theta = \sqrt{2}$; $\sec\theta = \sqrt{2}$; $\cot\theta = 1$

5.



$\sin\theta = \frac{\sqrt{3}}{2}$; $\cos\theta = \frac{1}{2}$; $\tan\theta = \sqrt{3}$; $\csc\theta = \frac{2\sqrt{3}}{3}$; $\sec\theta = 2$; $\cot\theta = \frac{\sqrt{3}}{3}$

6.



$\sin\theta = \frac{5\sqrt{61}}{61}$; $\cos\theta = \frac{6\sqrt{61}}{61}$; $\tan\theta = \frac{5}{6}$; $\csc\theta = \frac{\sqrt{61}}{5}$; $\sec\theta = \frac{\sqrt{61}}{6}$; $\cot\theta = \frac{6}{5}$

Lesson 13-1

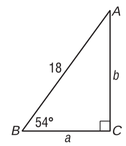
NAME ______________________ DATE ____________ PERIOD _____

13-1 Study Guide and Intervention *(continued)*

Right Triangle Trigonometry

Right Triangle Problems

Example **Solve $\triangle ABC$. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.**



You know the measures of one side, one acute angle, and the right angle. You need to find a, b, and A.

Find a and b.

$\sin 54° = \frac{b}{18}$ $\qquad$ $\cos 54° = \frac{a}{18}$
$b = 18 \sin 54°$ $\qquad$ $a = 18 \cos 54°$
$b \approx 14.6$ $\qquad$ $a \approx 10.6$

Find A.

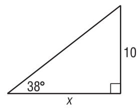
$54° + A = 90°$ — Angles A and B are complementary.
$A = 36°$ — Solve for A.

Therefore $A = 36°$, $a \approx 10.6$, and $b \approx 14.6$.

Exercises

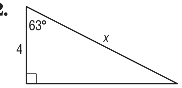
Write an equation involving sin, cos, or tan that can be used to find x. Then solve the equation. Round measures of sides to the nearest tenth.

1.



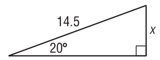
$\tan 38° = \frac{10}{x}$; 12.8

2.



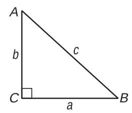
$\cos 63° = \frac{4}{x}$; 8.8

3.



$\sin 20° = \frac{x}{14.5}$; 5.0

Solve $\triangle ABC$ by using the given measurements. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.



4. $A = 80°$, $b = 6$
$a \approx 34.0$, $c \approx 34.6$, $B = 10°$

5. $B = 25°$, $c = 20$
$a \approx 18.1$, $b \approx 8.5$, $A = 65°$

6. $b = 8$, $c = 14$
$a \approx 11.5$, $B \approx 35°$, $C \approx 55°$

7. $a = 6$, $b = 7$
$c \approx 9.2$, $A \approx 41°$, $B \approx 49°$

8. $a = 12$, $B = 42°$
$b \approx 10.8$, $c \approx 16.1$, $A = 48°$

9. $a = 15$, $A = 54°$
$b \approx 10.9$, $c \approx 18.5$, $B = 36°$

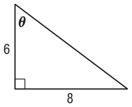
NAME ______________________ DATE __________ PERIOD _____

13-1 Skills Practice

Right Triangle Trigonometry

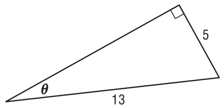
Find the values of the six trigonometric functions for angle θ.

1.



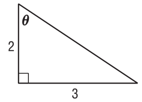
$\sin\theta = \frac{4}{5}, \cos\theta = \frac{3}{5}, \tan\theta = \frac{4}{3}, \csc\theta = \frac{5}{4}, \sec\theta = \frac{5}{3}, \cot\theta = \frac{3}{4}$

2.



$\sin\theta = \frac{5}{13}, \cos\theta = \frac{12}{13}, \tan\theta = \frac{5}{12}, \csc\theta = \frac{13}{5}, \sec\theta = \frac{13}{12}, \cot\theta = \frac{12}{5}$

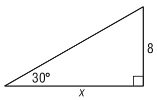
3.



$\sin\theta = \frac{3\sqrt{13}}{13}, \cos\theta = \frac{2\sqrt{13}}{13}, \tan\theta = \frac{3}{2}, \csc\theta = \frac{\sqrt{13}}{3}, \sec\theta = \frac{\sqrt{13}}{2}, \cot\theta = \frac{2}{3}$

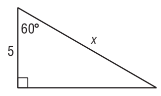
Write an equation involving sin, cos, or tan that can be used to find x. Then solve the equation. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.

4.



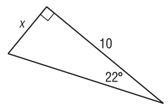
$\tan 30° = \frac{8}{x}, x \approx 13.9$

5.



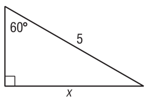
$\cos 60° = \frac{5}{x}, x = 10$

6.



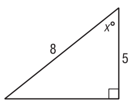
$\tan 22° = \frac{x}{10}, x \approx 4.0$

7.



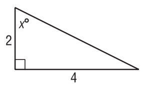
$\sin 60° = \frac{x}{5}, x \approx 4.3$

8.



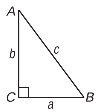
$\cos x° = \frac{5}{8}, x \approx 51$

9.



$\tan x° = \frac{4}{2}, x \approx 63$

Solve $\triangle ABC$ by using the given measurements. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.



10. $A = 72°, c = 10$
$a \approx 9.5, b \approx 3.1, B = 18°$

11. $B = 20°, b = 15$
$a \approx 41.2, c \approx 43.9, A = 70°$

12. $A = 80°, a = 9$
$b \approx 1.6, c \approx 9.1, B = 10°$

13. $A = 58°, b = 12$
$a \approx 19.2, c \approx 22.6, B = 32°$

14. $b = 4, c = 9$
$a \approx 8.1, A \approx 64°, B \approx 26°$

15. $a = 7, b = 5$
$c \approx 8.6, A \approx 54°, B \approx 36°$

Lesson 13-1

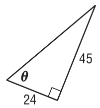
NAME ______________________ DATE __________ PERIOD _____

13-1 Practice (Average)

Right Triangle Trigonometry

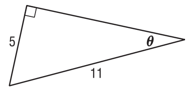
Find the values of the six trigonometric functions for angle θ.

1.



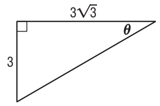
$\sin\theta = \frac{15}{17}, \cos\theta = \frac{8}{17}, \tan\theta = \frac{15}{8}, \csc\theta = \frac{17}{15}, \sec\theta = \frac{17}{8}, \cot\theta = \frac{8}{15}$

2.



$\sin\theta = \frac{5}{11}, \cos\theta = \frac{4\sqrt{6}}{11}, \tan\theta = \frac{5\sqrt{6}}{24}, \csc\theta = \frac{11}{5}, \sec\theta = \frac{11\sqrt{6}}{24}, \cot\theta = \frac{4\sqrt{6}}{5}$

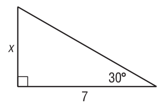
3.



$\sin\theta = \frac{1}{2}, \cos\theta = \frac{\sqrt{3}}{2}, \tan\theta = \frac{\sqrt{3}}{3}, \csc\theta = 2, \sec\theta = \frac{2\sqrt{3}}{3}, \cot\theta = \sqrt{3}$

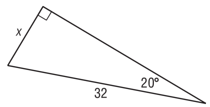
Write an equation involving sin, cos, or tan that can be used to find x. Then solve the equation. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.

4.



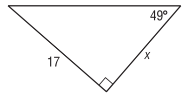
$\tan 30° = \frac{x}{7}, x \approx 4.0$

5.



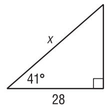
$\sin 20° = \frac{x}{32}, x \approx 10.9$

6.



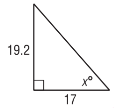
$\tan 49° = \frac{17}{x}, x \approx 14.8$

7.



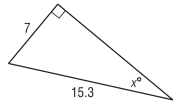
$\cos 41° = \frac{28}{x}, x \approx 37.1$

8.



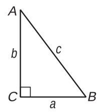
$\tan x° = \frac{19.2}{17}, x \approx 48$

9.



$\sin x° = \frac{7}{15.3}, x \approx 27$

Solve $\triangle ABC$ by using the given measurements. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.



10. $A = 35°, a = 12$
$b \approx 17.1, c \approx 20.9, B = 55°$

11. $B = 71°, b = 25$
$a \approx 8.6, c \approx 26.4, A = 19°$

12. $B = 36°, c = 8$
$a \approx 6.5, b \approx 4.7, A = 54°$

13. $a = 4, b = 7$
$c \approx 8.1, A \approx 30°, B \approx 60°$

14. $A = 17°, c = 3.2$
$a \approx 0.9, b \approx 3.1, B = 73°$

15. $b = 52, c = 95$
$a \approx 79.5, A \approx 33°, B \approx 57°$

16. SURVEYING John stands 150 meters from a water tower and sights the top at an angle of elevation of 36°. How tall is the tower? Round to the nearest meter. **109 m**

Answers

NAME ______________________ DATE __________ PERIOD _____

13-1 Reading to Learn Mathematics

Right Triangle Trigonometry

Pre-Activity **How is trigonometry used in building construction?**

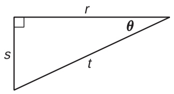
Read the introduction to Lesson 13-1 at the top of page 701 in your textbook.

If a different ramp is built so that the angle shown in the figure has a tangent of $\frac{1}{14}$, will this ramp meet, exceed, or fail to meet ADA regulations?

exceed

Reading the Lesson

1. Refer to the triangle at the right. Match each trigonometric function with the correct ratio.



i. $\frac{r}{t}$ **ii.** $\frac{r}{s}$ **iii.** $\frac{t}{r}$ **iv.** $\frac{s}{t}$ **v.** $\frac{s}{r}$ **vi.** $\frac{t}{s}$

a. $\sin\theta$ **iv** **b.** $\tan\theta$ **v** **c.** $\sec\theta$ **iii**

d. $\cot\theta$ **ii** **e.** $\cos\theta$ **i** **f.** $\csc\theta$ **vi**

2. Refer to the Key Concept box on page 703 in your textbook. Use the drawings of the 30°-60°-90° triangle and 45°-45°-90° triangle and/or the table to complete the following.

a. The tangent of 45° and the **cotangent** of 45° are equal.

b. The sine of 30° is equal to the cosine of **60°**.

c. The sine and **cosine** of 45° are equal.

d. The reciprocal of the cosecant of 60° is the **sine** of 60°.

e. The reciprocal of the cosine of 30° is the **cosecant** of 60°.

f. The reciprocal of the tangent of 60° is the **tangent** of 30°.

Helping You Remember

3. In studying trigonometry, it is important for you to know the relationships between the lengths of the sides of a 30°-60°-90° triangle. If you remember just one fact about this triangle, you will always be able to figure out the lengths of all the sides. What fact can you use, and why is it enough?

Sample answer: The shorter leg is half as long as the hypotenuse. You can use the Pythagorean Theorem to find the length of the longer leg.

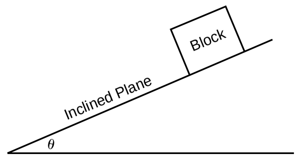
Lesson 13-1

NAME ______________________ DATE __________ PERIOD _____

13-1 Enrichment

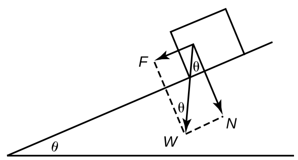
The Angle of Repose

Suppose you place a block of wood on an inclined plane, as shown at the right. If the angle, θ, at which the plane is inclined from the horizontal is very small, the block will not move. If you increase the angle, the block will eventually overcome the force of friction and start to slide down the plane.



At the instant the block begins to slide, the angle formed by the plane is called the angle of friction, or the angle of repose.

For situations in which the block and plane are smooth but unlubricated, the angle of repose depends *only* on the types of materials in the block and the plane. The angle is independent of the area of contact between the two surfaces and of the weight of the block.



The drawing at the right shows how to use vectors to find a coefficient of friction. This coefficient varies with different materials and is denoted by the Greek leter mu, μ.

$F = W\sin\theta$ $N = W\cos$

$F = \mu N$

$\mu = \frac{\sin\theta}{\cos\theta} = \tan\theta$

Material	Coefficient of Friction μ
Wood on wood	0.5
Wood on stone	0.5
Rubber tire on dry concrete	1.0
Rubber tire on wet concrete	0.7

Solve each problem.

1. A wooden chute is built so that wooden crates can slide down into the basement of a store. What angle should the chute make in order for the crates to slide down at a constant speed?
27°

2. Will a 100-pound wooden crate slide down a stone ramp that makes an angle of 20° with the horizontal? Explain your answer.
No, the angle must be at least 27°.

3. If you increase the weight of the crate in Exercise 2 to 300 pounds, does it change your answer?
No, the weight does not affect the angle.

4. A car with rubber tires is being driven on dry concrete pavement. If the car tires spin without traction on a hill, how steep is the hill?
at least 45°

5. For Exercise 4, does it make a difference if it starts to rain? Explain your answer.
Yes, the street needs to be only 35° for the car tires to spin.

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13-2 Study Guide and Intervention

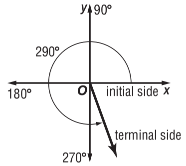
Angles and Angle Measurement

Angle Measurement An angle is determined by two rays. The degree measure of an angle is described by the amount and direction of rotation from the **initial side** along the positive x-axis to the **terminal side**. A counterclockwise rotation is associated with positive angle measure and a clockwise rotation is associated with negative angle measure. An angle can also be measured in **radians**.

Radian and Degree Measure	To rewrite the radian measure of an angle in degrees, multiply the number of radians by $\frac{180°}{\pi \text{ radians}}$. To rewrite the degree measure of an angle in radians, multiply the number of degrees by $\frac{\pi \text{ radians}}{180°}$.

Example 1 **Draw an angle with measure 290° in standard notation.**

The negative y-axis represents a positive rotation of 270°. To generate an angle of 290°, rotate the terminal side 20° more in the counterclockwise direction.



Example 2 **Rewrite the degree measure in radians and the radian measure in degrees.**

a. 45°

$45° = 45°\left(\frac{\pi \text{ radians}}{180°}\right) = \frac{\pi}{4}$ radians

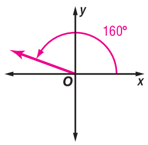
b. $\frac{5\pi}{3}$ radians

$\frac{5\pi}{3}$ radians $= \frac{5\pi}{3}\left(\frac{180°}{\pi}\right) = 300°$

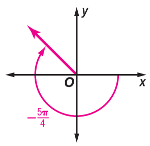
Exercises

Draw an angle with the given measure in standard position.

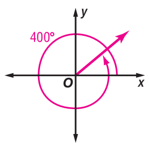
1. 160°



2. $-\frac{5\pi}{4}$



3. 400°



Rewrite each degree measure in radians and each radian measure in degrees.

4. 140° $\frac{7\pi}{9}$

5. −860° $-\frac{43\pi}{9}$

6. $-\frac{3\pi}{5}$ −108°

7. $\frac{11\pi}{3}$ 660°

NAME ______________________ DATE __________ PERIOD _____

13-2 Study Guide and Intervention *(continued)*

Angles and Angle Measurement

Coterminal Angles When two angles in standard position have the same terminal sides, they are called **coterminal angles**. You can find an angle that is coterminal to a given angle by adding or subtracting a multiple of 360°. In radian measure, a coterminal angle is found by adding or subtracting a multiple of 2π.

Example **Find one angle with positive measure and one angle with negative measure coterminal with each angle.**

a. 250°

A positive angle is 250° + 360° or 610°.

A negative angle is 250° − 360° or −110°.

b. $\frac{5\pi}{8}$

A positive angle is $\frac{5\pi}{8} + 2\pi$ or $\frac{21\pi}{8}$.

A negative angle is $\frac{5\pi}{8} - 2\pi$ or $-\frac{11\pi}{8}$.

Exercises

Find one angle with a positive measure and one angle with a negative measure coterminal with each angle. 1–18 Sample answers are given.

1. 65° 425°, −295°

2. −75° 285°, −435°

3. 230° 590°, −130°

4. 420° 60°, −300°

5. 340° 700°, −20°

6. −130° 230°, −490°

7. −290° 70°, −650°

8. 690° 330°, −30°

9. −420° 300°, −60°

10. $\frac{\pi}{9}$ $\frac{19\pi}{9}, -\frac{17\pi}{9}$

11. $\frac{3\pi}{8}$ $\frac{19\pi}{8}, -\frac{13\pi}{8}$

12. $\frac{6\pi}{5}$ $\frac{16\pi}{5}, -\frac{4\pi}{5}$

13. $\frac{-7\pi}{4}$ $\frac{\pi}{4}, -\frac{15\pi}{4}$

14. $\frac{15\pi}{4}$ $\frac{7\pi}{4}, -\frac{\pi}{4}$

15. $\frac{-13\pi}{6}$ $\frac{11\pi}{6}, -\frac{\pi}{6}$

16. $\frac{17\pi}{5}$ $\frac{7\pi}{5}, -\frac{3\pi}{5}$

17. $\frac{-5\pi}{3}$ $\frac{\pi}{3}, -\frac{11\pi}{3}$

18. $\frac{-11\pi}{4}$ $\frac{5\pi}{4}, -\frac{3\pi}{4}$

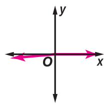
NAME ______________________ DATE __________ PERIOD _____

13-2 Skills Practice

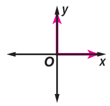
Angles and Angle Measure

Draw an angle with the given measure in standard position.

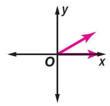
1. 185°



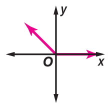
2. 810°



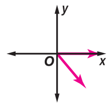
3. 390°



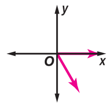
4. 495°



5. −50°



6. −420°



Rewrite each degree measure in radians and each radian measure in degrees.

7. 130° $\frac{13\pi}{18}$

8. 720° 4π

9. 210° $\frac{7\pi}{6}$

10. 90° $\frac{\pi}{2}$

11. −30° $-\frac{\pi}{6}$

12. −270° $-\frac{3\pi}{2}$

13. $\frac{\pi}{3}$ 60°

14. $\frac{5\pi}{6}$ 150°

15. $\frac{2\pi}{3}$ 120°

16. $\frac{5\pi}{4}$ 225°

17. $-\frac{3\pi}{4}$ −135°

18. $-\frac{7\pi}{6}$ −210°

Find one angle with positive measure and one angle with negative measure coterminal with each angle. 19–26. Sample answers are given.

19. 45° 405°, −315°

20. 60° 420°, −300°

21. 370° 10°, −350°

22. −90° 270°, −450°

23. $\frac{2\pi}{3}$ $\frac{8\pi}{3}$, $-\frac{4\pi}{3}$

24. $\frac{5\pi}{2}$ $\frac{9\pi}{2}$, $-\frac{\pi}{2}$

25. $\frac{\pi}{6}$ $\frac{13\pi}{6}$, $-\frac{11\pi}{6}$

26. $-\frac{3\pi}{4}$ $\frac{5\pi}{4}$, $-\frac{3\pi}{2}$

Lesson 13-2

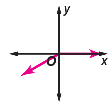
NAME ______________________ DATE __________ PERIOD _____

13-2 Practice (Average)

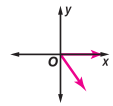
Angles and Angle Measure

Draw an angle with the given measure in standard position.

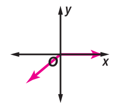
1. 210°



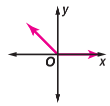
2. 305°



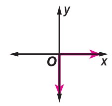
3. 580°



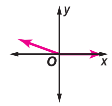
4. 135°



5. −450°



6. −560°



Rewrite each degree measure in radians and each radian measure in degrees.

7. 18° $\frac{\pi}{10}$

8. 6° $\frac{\pi}{30}$

9. 870° $\frac{29\pi}{6}$

10. 347° $\frac{347\pi}{180}$

11. −72° $-\frac{2\pi}{5}$

12. −820° $-\frac{41\pi}{9}$

13. −250° $-\frac{25\pi}{18}$

14. −165° $-\frac{11\pi}{12}$

15. 4π 720°

16. $\frac{5\pi}{2}$ 450°

17. $\frac{13\pi}{5}$ 468°

18. $\frac{13\pi}{30}$ 78°

19. $-\frac{9\pi}{2}$ −810°

20. $-\frac{7\pi}{12}$ −105°

21. $-\frac{3\pi}{8}$ −67.5°

22. $-\frac{3\pi}{16}$ −33.75°

Find one angle with positive measure and one angle with negative measure coterminal with each angle. 23–34. Sample answers are given.

23. 65° 425°, −295°

24. 80° 440°, −280°

25. 285° 645°, −75°

26. 110° 470°, −250°

27. −37° 323°, −397°

28. −93° 267°, −453°

29. $\frac{2\pi}{5}$ $\frac{12\pi}{5}$, $-\frac{8\pi}{5}$

30. $\frac{5\pi}{6}$ $\frac{17\pi}{6}$, $-\frac{7\pi}{6}$

31. $\frac{17\pi}{6}$ $\frac{29\pi}{6}$, $-\frac{7\pi}{6}$

32. $-\frac{3\pi}{2}$ $\frac{\pi}{2}$, $-\frac{7\pi}{2}$

33. $-\frac{\pi}{4}$ $\frac{7\pi}{4}$, $-\frac{9\pi}{4}$

34. $-\frac{5\pi}{12}$ $\frac{19\pi}{12}$, $-\frac{29\pi}{12}$

35. TIME Find both the degree and radian measures of the angle through which the hour hand on a clock rotates from 5 A.M. to 10 A.M. −150°; $-\frac{5\pi}{6}$

36. ROTATION A truck with 16-inch radius wheels is driven at 77 feet per second (52.5 miles per hour). Find the measure of the angle through which a point on the outside of the wheel travels each second. Round to the nearest degree and nearest radian. 3309°/s; 58 radians/s

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13-2 Reading to Learn Mathematics

Angles and Angle Measure

Pre-Activity How can angles be used to describe circular motion?

Read the introduction to Lesson 13-2 at the top of page 709 in your textbook.

If a gondola revolves through a complete revolution in one minute, what is its angular velocity in degrees per second? **6° per second**

Reading the Lesson

1. Match each degree measure with the corresponding radian measure on the right.

a. 30° **v**	**i.** $\frac{2\pi}{3}$
b. 90° **ii**	**ii.** $\frac{\pi}{2}$
c. 120° **i**	**iii.** $\frac{7\pi}{6}$
d. 135° **vi**	**iv.** π
e. 180° **iv**	**v.** $\frac{\pi}{6}$
f. 210° **iii**	**vi.** $\frac{3\pi}{4}$

2. The sine of 30° is $\frac{1}{2}$ and the sine of 150° is also $\frac{1}{2}$. Does this mean that 30° and 150° are coterminal angles? Explain your reasoning. **Sample answer: No; the terminal side of a 30° angle is in Quadrant I, while the terminal side of a 150° angle is in Quadrant II.**

3. Describe how to find two angles that are coterminal with an angle of 155°, one with positive measure and one with negative measure. (Do not actually calculate these angles.) **Sample answer: Positive angle: Add 360° to 155°. Negative angle: Subtract 360° from 155°.**

4. Describe how to find two angles that are coterminal with an angle of $\frac{5\pi}{3}$, one positive and one negative. (Do not actually calculate these angles.) **Sample answer: Positive angle: Add 2π to $\frac{5\pi}{3}$. Negative angle: Subtract 2π from $\frac{5\pi}{3}$.**

Helping You Remember

5. How can you use what you know about the circumference of a circle to remember how to convert between radian and degree measure? **Sample answer: The circumference of a circle is given by the formula $C = 2\pi r$, so the circumference of a circle with radius 1 is 2π. In degree measure, one complete circle is 360°. So 2π radians = 360° and π radians = 180°.**

Lesson 13-2

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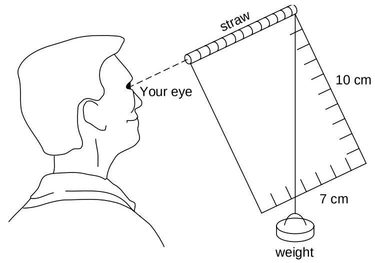
13-2 Enrichment

Making and Using a Hypsometer

A **hypsometer** is a device that can be used to measure the height of an object. To construct your own hypsometer, you will need a rectangular piece of heavy cardboard that is at least 7 cm by 10 cm, a straw, transparent tape, a string about 20 cm long, and a small weight that can be attached to the string.

Mark off 1-cm increments along one short side and one long side of the cardboard. Tape the straw to the other short side. Then attach the weight to one end of the string, and attach the other end of the string to one corner of the cardboard, as shown in the figure below. The diagram below shows how your hypsometer should look.



To use the hypsometer, you will need to measure the distance from the base of the object whose height you are finding to where you stand when you use the hypsometer.

Sight the top of the object through the straw. Note where the free-hanging string crosses the bottom scale. Then use similar triangles to find the height of the object.

1. Draw a diagram to illustrate how you can use similar triangles and the hypsometer to find the height of a tall object. **See students' diagrams.**

Use your hypsometer to find the height of each of the following. **See students' work.**

2. your school's flagpole

3. a tree on your school's property

4. the highest point on the front wall of your school building

5. the goal posts on a football field

6. the hoop on a basketball court

Answers

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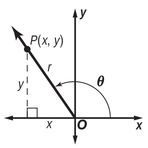
13-3 Study Guide and Intervention

Trigonometric Functions of General Angles

Trigonometric Functions and General Angles

Trigonometric Functions, θ in Standard Position	Let θ be an angle in standard position and let $P(x, y)$ be a point on the terminal side of θ. By the Pythagorean Theorem, the distance r from the origin is given by $r = \sqrt{x^2 + y^2}$. The trigonometric functions of an angle in standard position may be defined as follows.
	$\sin\theta = \frac{y}{r}$ $\cos\theta = \frac{x}{r}$ $\tan\theta = \frac{y}{x}$ $\csc\theta = \frac{r}{y}$ $\sec\theta = \frac{r}{x}$ $\cot\theta = \frac{x}{y}$

Example **Find the exact values of the six trigonometric functions of θ if the terminal side of θ contains the point $(-5, 5\sqrt{2})$.**

You know that $x = -5$ and $y = 5\sqrt{2}$. You need to find r.

$r = \sqrt{x^2 + y^2}$ Pythagorean Theorem

$= \sqrt{(-5)^2 + (5\sqrt{2})^2}$ Replace x with -5 and y with $5\sqrt{2}$.

$= \sqrt{75}$ or $5\sqrt{3}$

Now use $x = -5$, $y = 5\sqrt{2}$, and $r = 5\sqrt{3}$ to write the ratios.

$\sin\theta = \frac{y}{r} = \frac{5\sqrt{2}}{5\sqrt{3}} = \frac{\sqrt{6}}{3}$ $\cos\theta = \frac{x}{r} = \frac{-5}{5\sqrt{3}} = -\frac{\sqrt{3}}{3}$ $\tan\theta = \frac{y}{x} = \frac{5\sqrt{2}}{-5} = -\sqrt{2}$

$\csc\theta = \frac{r}{y} = \frac{5\sqrt{3}}{5\sqrt{2}} = \frac{\sqrt{6}}{2}$ $\sec\theta = \frac{r}{x} = \frac{5\sqrt{3}}{-5} = -\sqrt{3}$ $\cot\theta = \frac{x}{y} = \frac{-5}{5\sqrt{2}} = -\frac{\sqrt{2}}{2}$

Exercises

Find the exact values of the six trigonometric functions of θ if the terminal side of θ in standard position contains the given point.

1. $(8, 4)$

$\sin\theta = \frac{\sqrt{5}}{5}$, $\cos\theta = \frac{2\sqrt{5}}{5}$, $\tan\theta = \frac{1}{2}$, $\csc\theta = \sqrt{5}$, $\sec\theta = \frac{\sqrt{5}}{2}$, $\cot\theta = 2$

2. $(4, 4\sqrt{3})$

$\sin\theta = \frac{\sqrt{3}}{2}$, $\cos\theta = \frac{1}{2}$, $\tan\theta = \sqrt{3}$, $\csc\theta = \frac{2\sqrt{3}}{3}$, $\sec\theta = 2$, $\cot\theta = \frac{\sqrt{3}}{3}$

3. $(0, -4)$

$\sin\theta = -1$, $\cos\theta = 0$, $\tan\theta$ undefined, $\csc\theta = -1$, $\sec\theta$ undefined , $\cot\theta = 0$

4. $(6, 2)$

$\sin\theta = \frac{\sqrt{10}}{10}$, $\cos\theta = \frac{3\sqrt{10}}{10}$, $\tan\theta = \frac{1}{3}$, $\csc\theta = \sqrt{10}$, $\sec\theta = \frac{\sqrt{10}}{3}$, $\cot\theta = 3$

Lesson 13-3

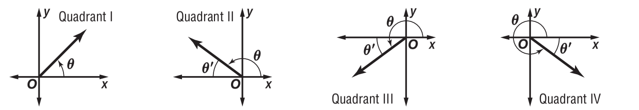
NAME ______________________ DATE ________ PERIOD _____

13-3 Study Guide and Intervention *(continued)*

Trigonometric Functions of General Angles

Reference Angles If θ is a nonquadrantal angle in standard position, its reference angle θ' is defined as the acute angle formed by the terminal side of θ and the x-axis.

Reference Angle Rule



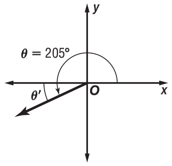
Quadrant I	Quadrant II	Quadrant III	Quadrant IV
$\theta' = \theta$	$\theta' = 180° - \theta$ $(\theta' = \pi - \theta)$	$\theta' = \theta - 180°$ $(\theta' = \theta - \pi)$	$\theta' = 360° - \theta$ $(\theta' = 2\pi - \theta)$

Signs of Trigonometric Functions

Function	I	II	III	IV
$\sin\theta$ or $\csc\theta$	+	+	–	–
$\cos\theta$ or $\sec\theta$	+	–	–	+
$\tan\theta$ or $\cot\theta$	+	–	+	–

Example 1 **Sketch an angle of measure 205°. Then find its reference angle.**

Because the terminal side of 205° lies in Quadrant III, the reference angle θ' is $205° - 180°$ or $25°$.



Example 2 **Use a reference angle to find the exact value of $\cos\frac{3\pi}{4}$.**

Because the terminal side of $\frac{3\pi}{4}$ lies in Quadrant II, the reference angle θ' is $\pi - \frac{3\pi}{4}$ or $\frac{\pi}{4}$.

The cosine function is negative in Quadrant II.

$\cos\frac{3\pi}{4} = -\cos\frac{\pi}{4} = -\frac{\sqrt{2}}{2}$.

Exercises

Find the exact value of each trigonometric function.

1. $\tan(-510°)$ **$\frac{\sqrt{3}}{3}$**

2. $\csc\frac{11\pi}{4}$ **$\sqrt{2}$**

3. $\sin(-90°)$ **-1**

4. $\cot 1665°$ **1**

5. $\cot 30°$ **$\sqrt{3}$**

6. $\tan 315°$ **-1**

7. $\csc\frac{\pi}{4}$ **$\sqrt{2}$**

8. $\tan\frac{4\pi}{3}$ **$\sqrt{3}$**

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13-3 Skills Practice

Trigonometric Functions of General Angles

Find the exact values of the six trigonometric functions of θ if the terminal side of θ in standard position contains the given point.

1. (5, 12)
$\sin\theta = \frac{12}{13}$, $\cos\theta = \frac{5}{13}$, $\tan\theta = \frac{12}{5}$,
$\csc\theta = \frac{13}{12}$, $\sec\theta = \frac{13}{5}$, $\cot\theta = \frac{5}{12}$

2. (3, 4)
$\sin\theta = \frac{4}{5}$, $\cos\theta = \frac{3}{5}$, $\tan\theta = \frac{4}{3}$,
$\csc\theta = \frac{5}{4}$, $\sec\theta = \frac{5}{3}$, $\cot\theta = \frac{3}{4}$

3. (8, −15)
$\sin\theta = -\frac{15}{17}$, $\cos\theta = \frac{8}{17}$, $\tan\theta = -\frac{15}{8}$,
$\csc\theta = -\frac{17}{15}$, $\sec\theta = \frac{17}{8}$, $\cot\theta = -\frac{8}{15}$

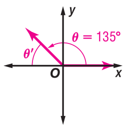
4. (−4, 3)
$\sin\theta = \frac{3}{5}$, $\cos\theta = -\frac{4}{5}$, $\tan\theta = -\frac{3}{4}$,
$\csc\theta = \frac{5}{3}$, $\sec\theta = -\frac{5}{4}$, $\cot\theta = -\frac{4}{3}$

5. (−9, −40)
$\sin\theta = -\frac{40}{41}$, $\cos\theta = -\frac{9}{41}$, $\tan\theta = \frac{40}{9}$,
$\csc\theta = -\frac{41}{40}$, $\sec\theta = -\frac{41}{9}$, $\cot\theta = \frac{9}{40}$

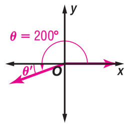
6. (1, 2)
$\sin\theta = \frac{2\sqrt{5}}{5}$, $\cos\theta = \frac{\sqrt{5}}{5}$, $\tan\theta = 2$,
$\csc\theta = \frac{\sqrt{5}}{2}$, $\sec\theta = \sqrt{5}$, $\cot\theta = \frac{1}{2}$

Sketch each angle. Then find its reference angle.

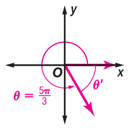
7. 135° **45°**



8. 200° **20°**



9. $\frac{5\pi}{3}$ $\frac{\pi}{3}$



Find the exact value of each trigonometric function.

10. sin 150° $\frac{1}{2}$ **11.** cos 270° **0** **12.** cot 135° **−1** **13.** tan (−30°) $-\frac{\sqrt{3}}{3}$

14. $\tan\frac{\pi}{4}$ **1** **15.** $\cos\frac{4\pi}{3}$ $-\frac{1}{2}$ **16.** $\cot(-\pi)$ **undefined** **17.** $\sin\left(-\frac{3\pi}{4}\right)$ $-\frac{\sqrt{2}}{2}$

Suppose θ is an angle in standard position whose terminal side is in the given quadrant. For each function, find the exact values of the remaining five trigonometric functions of θ.

18. $\sin\theta = \frac{4}{5}$, Quadrant II
$\cos\theta = -\frac{3}{5}$, $\tan\theta = -\frac{4}{3}$, $\csc\theta = \frac{5}{4}$,
$\sec\theta = -\frac{5}{3}$, $\cot\theta = -\frac{3}{4}$

19. $\tan\theta = -\frac{12}{5}$, Quadrant IV
$\sin\theta = -\frac{12}{13}$, $\cos\theta = \frac{5}{13}$, $\csc\theta = -\frac{13}{12}$,
$\sec\theta = \frac{13}{5}$, $\cot\theta = -\frac{5}{12}$

Lesson 13-3

NAME ______________________ DATE ________ PERIOD ____

13-3 Practice (Average)

Trigonometric Functions of General Angles

Find the exact values of the six trigonometric functions of θ if the terminal side of θ in standard position contains the given point.

1. (6, 8)
$\sin\theta = \frac{4}{5}$, $\cos\theta = \frac{3}{5}$,
$\tan\theta = \frac{4}{3}$, $\csc\theta = \frac{5}{4}$,
$\sec\theta = \frac{5}{3}$, $\cot\theta = \frac{3}{4}$

2. (−20, 21)
$\sin\theta = \frac{21}{29}$, $\cos\theta = -\frac{20}{29}$,
$\tan\theta = -\frac{21}{20}$, $\csc\theta = \frac{29}{21}$,
$\sec\theta = -\frac{29}{20}$, $\cot\theta = -\frac{20}{21}$

3. (−2, −5) $\sin\theta = -\frac{5\sqrt{29}}{29}$,
$\cos\theta = -\frac{2\sqrt{29}}{29}$,
$\tan\theta = \frac{5}{2}$, $\csc\theta = -\frac{\sqrt{29}}{5}$,
$\sec\theta = -\frac{\sqrt{29}}{2}$, $\cot\theta = \frac{2}{5}$

Find the reference angle for the angle with the given measure.

4. 236° **56°** **5.** $\frac{13\pi}{8}$ $\frac{3\pi}{8}$ **6.** −210° **30°** **7.** $-\frac{7\pi}{4}$ $\frac{\pi}{4}$

Find the exact value of each trigonometric function.

8. tan 135° **−1** **9.** cot 210° $\sqrt{3}$ **10.** cot (−90°) **0** **11.** cos 405° $\frac{\sqrt{2}}{2}$

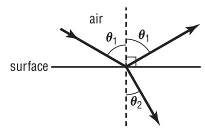
12. $\tan\frac{5\pi}{3}$ $-\sqrt{3}$ **13.** $\csc\left(-\frac{3\pi}{4}\right)$ $-\sqrt{2}$ **14.** $\cot 2\pi$ **undefined** **15.** $\tan\frac{13\pi}{6}$ $\frac{\sqrt{3}}{3}$

Suppose θ is an angle in standard position whose terminal side is in the given quadrant. For each function, find the exact values of the remaining five trigonometric functions of θ.

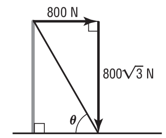
16. $\tan\theta = -\frac{12}{5}$, Quadrant IV
$\sin\theta = -\frac{12}{13}$, $\cos\theta = \frac{5}{13}$, $\csc\theta = -\frac{13}{12}$,
$\sec\theta = \frac{13}{5}$, $\cot\theta = -\frac{5}{12}$

17. $\sin\theta = \frac{2}{3}$, Quadrant III $\cos\theta = -\frac{\sqrt{5}}{3}$,
$\tan\theta = -\frac{2\sqrt{5}}{5}$, $\csc\theta = \frac{3}{2}$,
$\sec\theta = -\frac{3\sqrt{5}}{5}$, $\cot\theta = -\frac{\sqrt{5}}{2}$

18. LIGHT Light rays that "bounce off" a surface are *reflected* by the surface. If the surface is partially transparent, some of the light rays are bent or *refracted* as they pass from the air through the material. The angles of reflection θ_1 and of refraction θ_2 in the diagram at the right are related by the equation $\sin\theta_1 = n \sin\theta_2$. If $\theta_1 = 60°$ and $n = \sqrt{3}$, find the measure of θ_2. **30°**



19. FORCE A cable running from the top of a utility pole to the ground exerts a horizontal pull of 800 Newtons and a vertical pull of $800\sqrt{3}$ Newtons. What is the sine of the angle θ between the cable and the ground? What is the measure of this angle? $\frac{\sqrt{3}}{2}$; **60°**



Answers

NAME ______________________ DATE __________ PERIOD _____

13-3 Reading to Learn Mathematics

Trigonometric Functions of General Angles

Pre-Activity **How can you model the position of riders on a skycoaster?**

Read the introduction to Lesson 13-3 at the top of page 717 in your textbook.

- What does $t = 0$ represent in this application? **Sample answer: the time when the riders leave the bottom of their swing**
- Do negative values of t make sense in this application? Explain your answer. **Sample answer: No; $t = 0$ represents the starting time, so the value of t cannot be less than 0.**

Reading the Lesson

1. Suppose θ is an angle in standard position, $P(x, y)$ is a point on the terminal side of θ, and the distance from the origin to P is r. Determine whether each of the following statements is *true* or *false*.

a. The value of r can be found by using either the Pythagorean Theorem or the distance formula. **true**

b. $\cos \theta = \frac{x}{r}$ **true**

c. $\csc \theta$ is defined if $y \neq 0$. **true**

d. $\tan \theta$ is undefined if $y = 0$. **false**

e. $\sin \theta$ is defined for every value of θ. **true**

2. Let θ be an angle measured in degrees. Match the quadrant of θ from the first column with the description of how to find the reference angle for θ from the second column.

a. Quadrant III **ii**

b. Quadrant IV **i**

c. Quadrant II **iv**

d. Quadrant I **iii**

i. Subtract θ from 360°.

ii. Subtract 180° from θ.

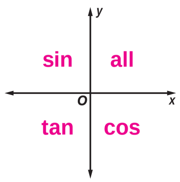
iii. θ is its own reference angle.

iv. Subtract θ from 180°.

Helping You Remember

3. The chart on page 719 in your textbook summarizes the signs of the six trigonometric functions in the four quadrants. Since reciprocals always have the same sign, you only need to remember where the sine, cosine, and tangent are positive. How can you remember this with a simple diagram?

Sample answer:



NAME ______________________ DATE __________ PERIOD _____

13-3 Enrichment

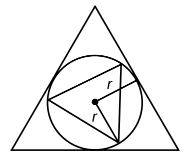
Areas of Polygons and Circles

A regular polygon has sides of equal length and angles of equal measure. A regular polygon can be inscribed in or circumscribed about a circle. For n-sided regular polygons, the following area formulas can be used.

Area of circle $A_C = \pi r^2$

Area of inscribed polygon $A_I = \frac{nr^2}{2} \times \sin \frac{360°}{n}$

Area of circumscribed polygon $A_C = nr^2 \times \tan \frac{180°}{n}$



Use a calculator to complete the chart below for a unit circle (a circle of radius 1).

	Number of Sides	Area of Inscribed Polygon	Area of Circle minus Area of Polygon	Area of Circumscribed Polygon	Area of Polygon minus Area of Circle
	3	1.2990381	1.8425545	5.1961524	2.054597
1.	4	**2**	**1.1415927**	**4**	**0.8584073**
2.	8	**2.8284271**	**0.3131655**	**3.3137085**	**0.1721158**
3.	12	**3**	**0.1415926**	**3.2153903**	**0.0737977**
4.	20	**3.0901699**	**0.0514227**	**3.1676888**	**0.0260961**
5.	24	**3.1058285**	**0.0357641**	**3.1596599**	**0.0180672**
6.	28	**3.1152931**	**0.0262996**	**3.1548423**	**0.0132496**
7.	32	**3.1214452**	**0.0201475**	**3.1517249**	**0.0101322**
8.	1000	**3.1415720**	**0.0000206**	**3.1416030**	**0.0000103**

9. What number do the areas of the circumscribed and inscribed polygons seem to be approaching? **π**

Lesson 13-3

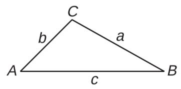
NAME ______________________ DATE ________ PERIOD _____

13-4 Study Guide and Intervention

Law of Sines

Law of Sines The area of any triangle is one half the product of the lengths of two sides and the sine of the included angle.

Area of a Triangle	area = $\frac{1}{2}bc \sin A$ area = $\frac{1}{2}ac \sin B$ area = $\frac{1}{2}ab \sin C$



You can use the Law of Sines to solve any triangle if you know the measures of two angles and any side, or the measures of two sides and the angle opposite one of them.

Law of Sines	$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

Example 1 **Find the area of $\triangle ABC$ if $a = 10$, $b = 14$, and $C = 40°$.**

Area $= \frac{1}{2}ab \sin C$ — Area formula

$= \frac{1}{2}(10)(14)\sin 40°$ — Replace *a*, *b*, and *C*.

≈ 44.9951 — Use a calculator.

The area of the triangle is approximately 45 square units.

Example 2 **If $a = 12$, $b = 9$, and $A = 28°$, find B.**

$\frac{\sin A}{a} = \frac{\sin B}{b}$ — Law of Sines

$\frac{\sin 28°}{12} = \frac{\sin B}{9}$ — Replace *A*, *a*, and *b*.

$\sin B = \frac{9 \sin 28°}{12}$ — Solve for sin *B*.

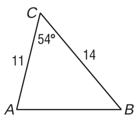
$\sin B \approx 0.3521$ — Use a calculator.

$B \approx 20.62°$ — Use the $\sin^{-1}$ function.

Exercises

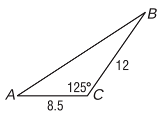
Find the area of $\triangle ABC$ to the nearest tenth.

1.



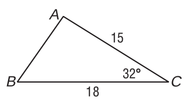
62.3 units^2

2.



41.8 units^2

3.



71.5 units^2

Solve each triangle. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.

4. $B = 42°$, $C = 68°$, $a = 10$
$A = 70°$, $b \approx 7.1$, $c \approx 9.9$

5. $A = 40°$, $B = 14°$, $a = 52$
$b \approx 19.6$, $c \approx 65.4$, $C = 126°$

6. $A = 15°$, $B = 50°$, $b = 36$
$C = 115°$, $a \approx 12.2$, $c \approx 42.6$

Lesson 13-4

NAME ______________________ DATE ________ PERIOD _____

13-4 Study Guide and Intervention *(continued)*

Law of Sines

One, Two, or No Solutions

Possible Triangles Given Two Sides and One Opposite Angle	Suppose you are given *a*, *b*, and *A* for a triangle. If *a* is acute: $a < b \sin A$ ⇒ no solution $a = b \sin A$ ⇒ one solution $b > a > b \sin A$ ⇒ two solutions $a > b$ ⇒ one solution If *A* is right or obtuse: $a \leq b$ ⇒ no solution $a > b$ ⇒ one solution

Example **Determine whether $\triangle ABC$ has no solutions, one solution, or two solutions. Then solve $\triangle ABC$.**

a. $A = 48°$, $a = 11$, and $b = 16$

Since A is acute, find $b \sin A$ and compare it with a.
$b \sin A = 16 \sin 48° \approx 11.89$
Since $11 < 11.89$, there is no solution.

b. $A = 34°$, $a = 6$, $b = 8$

Since A is acute, find $b \sin A$ and compare it with a; $b \sin A = 8 \sin 34° \approx 4.47$. Since $8 > 6 > 4.47$, there are two solutions. Thus there are two possible triangles to solve.

Acute *B*

First use the Law of Sines to find B.

$\frac{\sin B}{8} = \frac{\sin 34°}{6}$

$\sin B \approx 0.7456$

$B \approx 48°$

The measure of angle C is about $180° - (34° + 48°)$ or about $98°$.

Use the Law of Sines again to find c.

$\frac{\sin 98°}{c} \approx \frac{\sin 34°}{6}$

$c \approx \frac{6 \sin 98°}{\sin 34°}$

$c \approx 10.6$

Obtuse *B*

To find B you need to find an obtuse angle whose sine is also 0.7456.

To do this, subtract the angle given by your calculator, 48°, from 180°. So B is approximately 132°.

The measure of angle C is about $180° - (34° + 132°)$ or about $14°$.

Use the Law of Sines to find c.

$\frac{\sin 14°}{c} \approx \frac{\sin 34°}{6}$

$c \approx \frac{6 \sin 14°}{\sin 34°}$

$c \approx 2.6$

Exercises

Determine whether each triangle has no solutions, one solution, or two solutions. Then solve each triangle. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.

1. $A = 50°$, $a = 34$, $b = 40$
two solutions; $B \approx 64°$, $C \approx 66°$, $c \approx 47.6$; $B \approx 116°$, $C \approx 14°$, $c \approx 12.9$

2. $A = 24°$, $a = 3$, $b = 8$
no solutions

3. $A = 125°$, $a = 22$, $b = 15$
one solution; $B \approx 34°$, $C \approx 21°$, $c \approx 9.6$

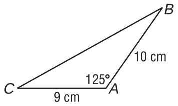
Answers

NAME ________________________ DATE ____________ PERIOD _____

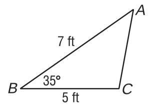
13-4 Skills Practice

Law of Sines

Find the area of △*ABC* to the nearest tenth.

1. 

36.9 cm^2

2. 

10.0 ft^2

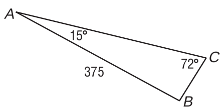
3. $A = 35°, b = 3$ ft, $c = 7$ ft **6.0 ft^2**

4. $C = 148°, a = 10$ cm, $b = 7$ cm **18.5 cm^2**

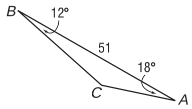
5. $C = 22°, a = 14$ m, $b = 8$ m **21.0 m^2**

6. $B = 93°, c = 18$ mi, $a = 42$ mi **377.5 mi^2**

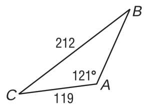
Solve each triangle. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.

7. 

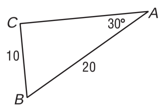
***B* = 93°, *a* ≈ 102.1, *b* ≈ 393.8**

8. 

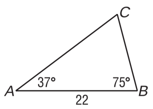
***C* = 150°, *a* ≈ 31.5, *b* ≈ 21.2**

9. 

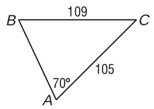
***B* ≈ 29°, *C* ≈ 30°, *c* ≈ 124.6**

10. 

***B* = 60°, *C* = 90°, *b* ≈ 17.3**

11. 

***C* = 68°, *a* ≈ 14.3, *b* ≈ 22.9**

12. 

***B* ≈ 65°, *C* ≈ 45°, *c* ≈ 82.2**

Determine whether each triangle has *no* solution, *one* solution, or *two* solutions. Then solve each triangle. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.

13. $A = 30°, a = 1, b = 4$
no solution

14. $A = 30°, a = 2, b = 4$ **one solution;**
***B* = 90°, *C* = 60°, *c* ≈ 3.5**

15. $A = 30°, a = 3, b = 4$ **two solutions;**
***B* ≈ 42°, *C* ≈ 108°, *c* ≈ 5.7;**
***B* ≈ 138°, *C* ≈ 12°, *c* ≈ 1.2**

16. $A = 38°, a = 10, b = 9$ **one solution;**
***B* ≈ 34°, *C* ≈ 108°, *c* ≈ 15.4**

17. $A = 78°, a = 8, b = 5$ **one solution;**
***B* ≈ 38°, *C* ≈ 64°, *c* ≈ 7.4**

18. $A = 133°, a = 9, b = 7$ **one solution;**
***B* ≈ 35°, *C* ≈ 12°, *c* ≈ 2.6**

19. $A = 127°, a = 2, b = 6$ **no solution**

20. $A = 109°, a = 24, b = 13$ **one solution;**
***B* ≈ 31°, *C* ≈ 40°, *c* ≈ 16.4**

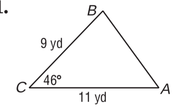
Lesson 13-4

NAME ________________________ DATE ____________ PERIOD _____

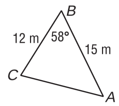
13-4 Practice (Average)

Law of Sines

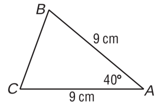
Find the area of △*ABC* to the nearest tenth.

1. 

35.6 yd^2

2. 

76.3 m^2

3. 

26.0 cm^2

4. $C = 32°, a = 12.6$ m, $b = 8.9$ m
29.7 m^2

5. $B = 27°, a = 14.9$ cm, $c = 18.6$ cm
62.9 cm^2

6. $A = 17.4°, b = 12$ km, $c = 14$ km
25.1 km^2

7. $A = 34°, b = 19.4$ ft, $c = 8.6$ ft
46.6 ft^2

Solve each triangle. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.

8. $A = 50°, B = 30°, c = 9$
***C* = 100°, *a* ≈ 7.0, *b* ≈ 4.6**

9. $A = 56°, B = 38°, a = 12$
***C* = 86°, *b* ≈ 8.9, *c* ≈ 14.4**

10. $A = 80°, C = 14°, a = 40$
***B* = 86°, *b* ≈ 40.5, *c* ≈ 9.8**

11. $B = 47°, C = 112°, b = 13$
***A* = 21°, *a* ≈ 6.4, *c* ≈ 16.5**

12. $A = 72°, a = 8, c = 6$
***B* ≈ 62°, *C* ≈ 46°, *b* ≈ 7.5**

13. $A = 25°, C = 107°, b = 12$
***B* = 48°, *a* ≈ 6.8, *c* ≈ 15.4**

Determine whether each triangle has *no* solution, *one* solution, or *two* solutions. Then solve each triangle. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.

14. $A = 29°, a = 6, b = 13$ **no solution**

15. $A = 70°, a = 25, b = 20$ **one solution;**
***B* ≈ 49°, *C* ≈ 61°, *c* ≈ 23.3**

16. $A = 113°, a = 21, b = 25$ **no solution**

17. $A = 110°, a = 20, b = 8$ **one solution;**
***B* ≈ 22°, *C* ≈ 48°, *c* ≈ 15.8**

18. $A = 66°, a = 12, b = 7$ **one solution;**
***B* ≈ 32°, *C* ≈ 82°, *c* ≈ 13.0**

19. $A = 54°, a = 5, b = 8$ **no solution**

20. $A = 45°, a = 15, b = 18$ **two solutions;**
***B* ≈ 58°, *C* ≈ 77°, *c* ≈ 20.7;**
***B* ≈ 122°, *C* ≈ 13°, *c* ≈ 4.8**

21. $A = 60°, a = 4\sqrt{3}, b = 8$ **one solution;**
***B* = 90°, *C* = 30°, *c* ≈ 23.3**

22. **WILDLIFE** Sarah Phillips, an officer for the Department of Fisheries and Wildlife, checks boaters on a lake to make sure they do not disturb two osprey nesting sites. She leaves a dock and heads due north in her boat to the first nesting site. From here, she turns 5° north of due west and travels an additional 2.14 miles to the second nesting site. She then travels 6.7 miles directly back to the dock. How far from the dock is the first osprey nesting site? Round to the nearest tenth. **6.2 mi**

NAME ______________________ DATE __________ PERIOD _____

13-4 Reading to Learn Mathematics

Law of Sines

Pre-Activity **How can trigonometry be used to find the area of a triangle?**

Read the introduction to Lesson 13-4 at the top of page 725 in your textbook.

What happens when the formula Area $= \frac{1}{2}ab \sin C$ is applied to a right triangle in which C is the right angle? **Sample answer: The formula gives Area $= \frac{1}{2}ab \sin 90° = \frac{1}{2}ab \cdot 1 = \frac{1}{2}ab$, which is the same as the result from using the formula Area $= \frac{1}{2}$(base)(height).**

Reading the Lesson

1. In each case below, the measures of three parts of a triangle are given. For each case, write the formula you would use to find the area of the triangle. Show the formulas with specific values substituted, but do not actually calculate the area. If there is not enough information provided to find the area of the triangle by using the area formulas on page 725 in your textbook and without finding other parts of the triangle first, explain why.

a. $A = 48°, b = 9, c = 5$ **$\frac{1}{2}(9)(5) \sin 48°$**

b. $a = 15, b = 15, C = 120°$ **$\frac{1}{2}(15)(15) \sin 120°$**

c. $b = 16, c = 10, B = 120°$ **Not enough information; *B* is not the included angle between the two given sides.**

2. Tell whether the equation must be true based on the Law of Sines. Write *yes* or *no*.

a. $\frac{\sin A}{b} = \frac{\sin B}{a}$ **no**

b. $\frac{b}{\sin B} = \frac{c}{\sin C}$ **yes**

c. $a \sin C = c \sin A$ **yes**

d. $b = \frac{a \sin A}{\sin B}$ **no**

3. Determine whether $\triangle ABC$ has *no solution*, *one solution*, or *two solutions*. Do not try to solve the triangle.

a. $a = 20, A = 30°, B = 70°$ **one solution**

b. $A = 55°, b = 5, a = 3$ ($b \sin A \approx 4.1$) **no solution**

c. $c = 12, A = 100°, a = 30$ **one solution**

d. $C = 27°, b = 23.5, c = 17.5$ ($b \sin C \approx 10.7$) **two solutions**

Helping You Remember

4. Suppose that you are taking a quiz and cannot remember whether the formula for the area of a triangle is Area $= \frac{1}{2}ab \cos C$ or Area $= \frac{1}{2}ab \sin C$. How can you quickly remember which of these is correct? **Sample answer: The formula has to work when *C* is a right angle. The formula cannot contain cos *C* because cos 90° = 0 and this would make the area of a right triangle be 0.**

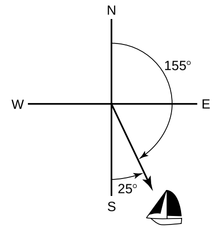
Lesson 13-4

NAME ______________________ DATE __________ PERIOD _____

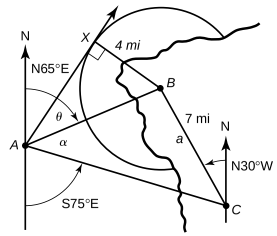
13-4 Enrichment

Navigation

The bearing of a boat is an angle showing the direction the boat is heading. Often, the angle is measured from north, but it can be measured from any of the four compass directions. At the right, the bearing of the boat is 155°. Or, it can be described as 25° east of south (S25°E).



Example **A boat *A* sights the lighthouse *B* in the direction N65°E and the spire of a church *C* in the direction S75°E. According to the map, *B* is 7 miles from *C* in the direction N30°W. In order for *A* to avoid running aground, find the bearing it should keep to pass *B* at 4 miles distance.**



In $\triangle ABC$, $\angle\alpha = 180° - 65° - 75°$ or $40°$

$\angle C = 180° - 30° - (180° - 75°)$

$= 45°$

$a = 7$ miles

With the Law of Sines,

$$AB = \frac{a \sin C}{\sin \alpha} = \frac{7(\sin 45°)}{\sin 40°} = 7.7 \text{ mi.}$$

The ray for the correct bearing for A must be tangent at X to circle B with radius $BX = 4$. Thus $\triangle ABX$ is a right triangle.

Then $\sin \theta = \frac{BX}{AB} = \frac{4}{7.7} \approx 0.519$. Therefore, $\angle\theta = 31°18'$.

The bearing of A should be $65° - 31°18'$ or $33°42'$.

Solve the following.

1. Suppose the lighthouse B in the example is sighted at S30°W by a ship P due north of the church C. Find the bearing P should keep to pass B at 4 miles distance. **S64°51′ W**

2. In the fog, the lighthouse keeper determines by radar that a boat 18 miles away is heading to the shore. The direction of the boat from the lighthouse is S80°E. What bearing should the lighthouse keeper radio the boat to take to come ashore 4 miles south of the lighthouse? **S87.2°E**

3. To avoid a rocky area along a shoreline, a ship at M travels 7 km to R, bearing $22°15'$, then 8 km to P, bearing $68°30'$, then 6 km to Q, bearing $109°15'$. Find the distance from M to Q. **17.4 km**

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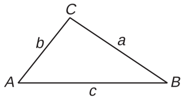
13-5 Study Guide and Intervention

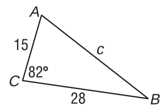
Law of Cosines

Law of Cosines

Law of Cosines	
	Let △*ABC* be any triangle with *a*, *b*, and *c* representing the measures of the sides, and opposite angles with measures *A*, *B*, and *C*, respectively. Then the following equations are true. $a^2 = b^2 + c^2 - 2bc \cos A$ $b^2 = a^2 + c^2 - 2ac \cos B$ $c^2 = a^2 + b^2 - 2ab \cos C$

You can use the Law of Cosines to solve any triangle if you know the measures of two sides and the included angle, or the measures of three sides.

Example **Solve △*ABC*.**



You are given the measures of two sides and the included angle. Begin by using the Law of Cosines to determine *c*.

$c^2 = a^2 + b^2 - 2ab \cos C$

$c^2 = 28^2 + 15^2 - 2(28)(15)\cos 82°$

$c^2 \approx 892.09$

$c \approx 29.9$

Next you can use the Law of Sines to find the measure of angle *A*.

$\frac{\sin A}{a} = \frac{\sin C}{c}$

$\frac{\sin A}{28} \approx \frac{\sin 82°}{29.9}$

$\sin A \approx 0.9273$

$A \approx 68°$

The measure of *B* is about 180° − (82° + 68°) or about 30°.

Exercises

Solve each triangle described below. Round measures of sides to the nearest tenth and angles to the nearest degree.

1. $a = 14, c = 20, B = 38°$
b ≈ 12.4, A ≈ 44°, C ≈ 98°

2. $A = 60°, c = 17, b = 12$
a ≈ 15.1, B ≈ 43°, C ≈ 77°

3. $a = 4, b = 6, c = 3$
A ≈ 36°, B ≈ 118°, C ≈ 26°

4. $A = 103°, b = 31, c = 52$
a ≈ 66, B ≈ 27°, C ≈ 50°

5. $a = 15, b = 26, C = 132°$
c ≈ 38, A ≈ 17°, B ≈ 31°

6. $a = 31, b = 52, c = 43$
A ≈ 36°, B ≈ 88°, C ≈ 56°

Lesson 13-5

NAME ______________________________ DATE __________ PERIOD _____

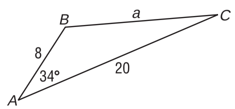
13-5 Study Guide and Intervention *(continued)*

Law of Cosines

Choose the Method

	Given	Begin by Using
Solving an Oblique Triangle	two angles and any side	Law of Sines
	two sides and a non-included angle	Law of Sines
	two sides and their included angle	Law of Cosines
	three sides	Law of Cosines

Example **Determine whether △*ABC* should be solved by beginning with the Law of Sines or Law of Cosines. Then solve the triangle. Round the measure of the side to the nearest tenth and measures of angles to the nearest degree.**



You are given the measures of two sides and their included angle, so use the Law of Cosines.

$a^2 = b^2 + c^2 - 2bc \cos A$ — Law of Cosines

$a^2 = 20^2 + 8^2 - 2(20)(8) \cos 34°$ — $b = 20, c = 8, A = 34°$

$a^2 \approx 198.71$ — Use a calculator.

$a \approx 14.1$ — Use a calculator.

Use the Law of Sines to find *B*.

$\frac{\sin B}{b} = \frac{\sin A}{a}$ — Law of Sines

$\sin B \approx \frac{20 \sin 34°}{14.1}$ — $b = 20, A = 34°, a \approx 14.1$

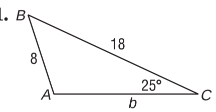
$B \approx 128°$ — Use the $\sin^{-1}$ function.

The measure of angle *C* is approximately 180° − (34° + 128°) or about 18°.

Exercises

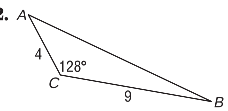
Determine whether each triangle should be solved by beginning with the Law of Sines or Law of Cosines. Then solve each triangle. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.

1.



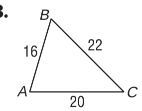
Law of Sines; A ≈ 108°, B ≈ 47°, b ≈ 13.8

2.



Law of Cosines; c ≈ 11.9, B ≈ 15°, A ≈ 37°

3.



Law of Cosines; A ≈ 74°, B ≈ 61°, C ≈ 45°

4. $A = 58°, a = 12, b = 8.5$
Law of Sines; B ≈ 37°, C ≈ 85°, c ≈ 14.1

5. $a = 28, b = 35, c = 20$
Law of Cosines; A ≈ 53°, B ≈ 92°, C ≈ 35°

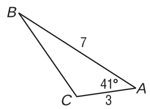
6. $A = 82°, B = 44°, b = 11$
Law of Sines; a ≈ 15.7, c ≈ 12.8, C ≈ 54°

NAME ______________________ DATE ____________ PERIOD _____

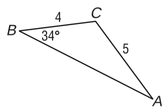
13-5 Skills Practice

Law of Cosines

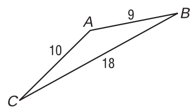
Determine whether each triangle should be solved by beginning with the Law of Sines or Law of Cosines. Then solve each triangle. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.

1. 

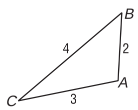
cosines; $B \approx 23°$, $C \approx 116°$, $a \approx 5.1$

2. 

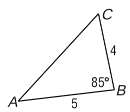
sines; $A \approx 27°$, $C \approx 119°$, $c \approx 7.9$

3. 

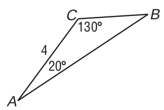
cosines; $A \approx 143°$, $B \approx 20°$, $C \approx 18°$

4. 

cosines; $A \approx 104°$, $B \approx 47°$, $C \approx 29°$

5. 

cosines; $A \approx 41°$, $C \approx 54°$, $b \approx 6.1$

6. 

sines; $B = 30°$, $a \approx 2.7$, $c \approx 6.1$

7. $C = 71°, a = 3, b = 4$
cosines; $A \approx 43°$, $B \approx 66°$, $c \approx 4.1$

8. $A = 11°, C = 27°, c = 50$
sines; $B = 142°$, $a \approx 21.0$, $b \approx 67.8$

9. $C = 35°, a = 5, b = 8$
cosines; $A \approx 37°$, $B \approx 108°$, $c \approx 4.8$

10. $B = 47°, a = 20, c = 24$
cosines; $A = 55°$, $C = 78°$, $b \approx 17.9$

11. $A = 71°, C = 62°, a = 20$
sines; $B = 47°$, $b \approx 15.5$, $c \approx 18.7$

12. $a = 5, b = 12, c = 13$
cosines; $A \approx 23°$, $B \approx 67°$, $C = 90°$

13. $A = 51°, b = 7, c = 10$
cosines; $B \approx 44°$, $C \approx 85°$, $a \approx 7.8$

14. $a = 13, A = 41°, B = 75°$
sines; $C = 64°$, $b \approx 19.1$, $c = 17.8$

15. $B = 125°, a = 8, b = 14$
sines; $A \approx 28°$, $C \approx 27°$, $c \approx 7.8$

16. $a = 5, b = 6, c = 7$
cosines; $A \approx 44°$, $B \approx 57°$, $C \approx 78°$

Lesson 13-5

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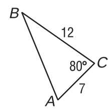
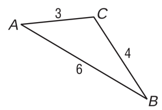
13-5 Practice (Average)

Law of Cosines

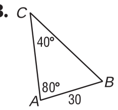
Determine whether each triangle should be solved by beginning with the Law of Sines or Law of Cosines. Then solve each triangle. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.

1. 

cosines; $c \approx 12.8$, $A \approx 67°$, $B \approx 33°$

2. 

cosines; $A \approx 36°$, $B \approx 26°$, $C \approx 117°$

3. 

sines; $B = 60°$, $a \approx 46.0$, $b \approx 40.4$

4. $a = 16, b = 20, C = 54°$
cosines; $A \approx 51°$, $B \approx 75°$, $c \approx 16.7$

5. $B = 71°, c = 6, a = 11$
cosines; $A \approx 77°$, $C \approx 32°$, $b \approx 10.7$

6. $A = 37°, a = 20, b = 18$
sines; $B \approx 33°$, $C \approx 110°$, $c \approx 31.2$

7. $C = 35°, a = 18, b = 24$
cosines; $A \approx 48°$, $B \approx 97°$, $c \approx 13.9$

8. $a = 8, b = 6, c = 9$
cosines; $A \approx 61°$, $B \approx 41°$, $C \approx 79°$

9. $A = 23°, b = 10, c = 12$
cosines; $B \approx 54°$, $C \approx 103°$, $a \approx 4.8$

10. $a = 4, b = 5, c = 8$
cosines; $A \approx 24°$, $B \approx 31°$, $C \approx 125°$

11. $B = 46.6°, C = 112°, b = 13$
sines; $A = 21°$, $a \approx 6.5$, $c \approx 16.6$

12. $A = 46.3°, a = 35, b = 30$
sines; $B \approx 38°$, $C \approx 95°$, $c \approx 48.2$

13. $a = 16.4, b = 21.1, c = 18.5$
cosines; $A \approx 48°$, $B \approx 74°$, $C \approx 57°$

14. $C = 43.5°, b = 8, c = 6$
sines; $A \approx 70°$, $B \approx 67°$, $a \approx 8.2$

15. $A = 78.3°, b = 7, c = 11$
cosines; $B \approx 36°$, $C \approx 66°$, $a \approx 11.8$

16. **SATELLITES** Two radar stations 2.4 miles apart are tracking an airplane. The straight-line distance between Station A and the plane is 7.4 miles. The straight-line distance between Station B and the plane is 6.9 miles. What is the angle of elevation from Station A to the plane? Round to the nearest degree. **69°**

17. **DRAFTING** Marion is using a computer-aided drafting program to produce a drawing for a client. She begins a triangle by drawing a segment 4.2 inches long from point A to point B. From B, she moves 42° degrees counterclockwise from the segment connecting A and B and draws a second segment that is 6.4 inches long, ending at point C. To the nearest tenth, how long is the segment from C to A? **9.9 in.**

Answers

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13-5 Reading to Learn Mathematics

Law of Cosines

Pre-Activity **How can you determine the angle at which to install a satellite dish?**

Read the introduction to Lesson 13-5 at the top of page 733 in your textbook.

One side of the triangle in the figure is not labeled with a length. What does the length of this side represent? Is this length greater than or less than the distance from the satellite to the equator?

the distance from the satellite to Valparaiso; greater than

Reading the Lesson

1. Each of the following equations can be changed into a correct statement of the Law of Cosines by making one change. In each case, indicate what change should be made to make the statement correct.

a. $b^2 = a^2 + c^2 + 2ac \cos B$ **Change the second + to −.**

b. $a^2 = b^2 + c^2 - 2bc \sin A$ **Change sin *A* to cos *A*.**

c. $c = a^2 + b^2 - 2ab \cos C$ **Change *c* to c^2.**

d. $a^2 = b^2 - c^2 - 2bc \cos A$ **Change the first − to +.**

2. Suppose that you are asked to solve $\triangle ABC$ given the following information about the sides and angles of the triangle. In each case, indicate whether you would begin by using the *Law of Sines* or the *Law of Cosines*.

a. $a = 8, b = 7, c = 6$ **Law of Cosines**

b. $b = 9.5, A = 72°, B = 39°$ **Law of Sines**

c. $C = 123°, b = 22.95, a = 34.35$ **Law of Cosines**

Helping You Remember

3. It is often easier to remember a complicated procedure if you can break it down into small steps. Describe in your own words how to use the Law of Cosines to find the length of one side of a triangle if you know the lengths of the other two sides and the measure of the included angle. Use numbered steps. (You may use mathematical terms, but do not use any mathematical symbols.)

Sample answer: 1. Square each of the lengths of the two known sides. 2. Add these squares. 3. Find the cosine of the included angle. 4. Multiply this cosine by two times the product of the lengths of the two known sides. 5. Subtract the product from the sum. 6. Take the positive square root of the result.

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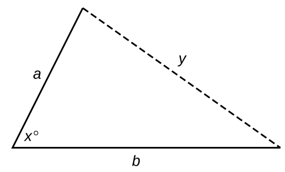
13-5 Enrichment

The Law of Cosines and the Pythagorean Theorem

The law of cosines bears strong similarities to the Pythagorean theorem. According to the law of cosines, if two sides of a triangle have lengths a and b and if the angle between them has a measure of $x°$, then the length, y, of the third side of the triangle can be found by using the equation



$y^2 = a^2 + b^2 - 2ab \cos x°$.

Answer the following questions to clarify the relationship between the law of cosines and the Pythagorean theorem.

1. If the value of $x°$ becomes less and less, what number is $\cos x°$ close to? **1**

2. If the value of $x°$ is very close to zero but then increases, what happens to $\cos x°$ as $x°$ approaches 90°? **decreases, approaches 0**

3. If $x°$ equals 90°, what is the value of $\cos x°$? What does the equation of $y^2 = a^2 + b^2 - 2ab \cos x°$ simplify to if $x°$ equals 90°? **0, $y^2 = a^2 + b^2$**

4. What happens to the value of $\cos x°$ as $x°$ increases beyond 90° and approaches 180°? **decreases to −1**

5. Consider some particular value of a and b, say 7 for a and 19 for b. Use a graphing calculator to graph the equation you get by solving $y^2 = 7^2 + 19^2 - 2(7)(19) \cos x°$ for y. **See students' graphs.**

a. In view of the geometry of the situation, what range of values should you use for X? **X min = 0°; X max = 180°**

b. Display the graph and use the TRACE function. What do the maximum and minimum values appear to be for the function? **See students' graphs.**

c. How do the answers for part **b** relate to the lengths 7 and 19? Are the maximum and minimum values from part **b** ever actually attained in the geometric situation? **min = 19 − 7; max = 19 + 7; no**

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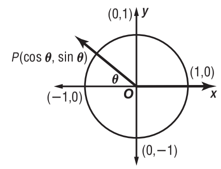
13-6 Study Guide and Intervention

Circular Functions

Unit Circle Definitions

Definition of Sine and Cosine	If the terminal side of an angle θ in standard position intersects the unit circle at $P(x, y)$, then $\cos \theta = x$ and $\sin \theta = y$. Therefore, the coordinates of P can be written as $P(\cos \theta, \sin \theta)$.



Example **Given an angle θ in standard position, if $P\left(-\frac{5}{6}, \frac{\sqrt{11}}{6}\right)$ lies on the terminal side and on the unit circle, find $\sin \theta$ and $\cos \theta$.**

$P\left(-\frac{5}{6}, \frac{\sqrt{11}}{6}\right) = P(\cos \theta, \sin \theta)$, so $\sin \theta = \frac{\sqrt{11}}{6}$ and $\cos \theta = -\frac{5}{6}$.

Exercises

If θ is an angle in standard position and if the given point P is located on the terminal side of θ and on the unit circle, find $\sin \theta$ and $\cos \theta$.

1. $P\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$
$\sin \theta = \frac{1}{2}, \cos \theta = -\frac{\sqrt{3}}{2}$

2. $P(0, -1)$
$\sin \theta = -1, \cos \theta = 0$

3. $P\left(-\frac{2}{3}, \frac{\sqrt{5}}{3}\right)$
$\sin \theta = \frac{\sqrt{5}}{3}, \cos \theta = -\frac{2}{3}$

4. $P\left(-\frac{4}{5}, -\frac{3}{5}\right)$
$\sin \theta = -\frac{3}{5}, \cos \theta = -\frac{4}{5}$

5. $P\left(\frac{1}{6}, -\frac{\sqrt{35}}{6}\right)$
$\sin \theta = -\frac{\sqrt{35}}{6}, \cos \theta = \frac{1}{6}$

6. $P\left(\frac{\sqrt{7}}{4}, \frac{3}{4}\right)$
$\sin \theta = \frac{3}{4}, \cos \theta = \frac{\sqrt{7}}{4}$

7. P is on the terminal side of $\theta = 45°$.
$\sin \theta = \frac{\sqrt{2}}{2}, \cos \theta = \frac{\sqrt{2}}{2}$

8. P is on the terminal side of $\theta = 120°$.
$\sin \theta = \frac{\sqrt{3}}{2}, \cos \theta = -\frac{1}{2}$

9. P is on the terminal side of $\theta = 240°$.
$\sin \theta = -\frac{\sqrt{3}}{2}, \cos \theta = -\frac{1}{2}$

10. P is on the terminal side of $\theta = 330°$.
$\sin \theta = -\frac{1}{2}, \cos \theta = \frac{\sqrt{3}}{2}$

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13-6 Study Guide and Intervention *(continued)*

Circular Functions

Periodic Functions

Periodic Functions	A function is called **periodic** if there is a number a such that $f(x) = f(x + a)$ for all x in the domain of the function. The least positive value of a for which $f(x) = f(x + a)$ is called the period of the function.

The sine and cosine functions are periodic; each has a period of 360° or 2π.

Example 1 **Find the exact value of each function.**

a. sin 855°

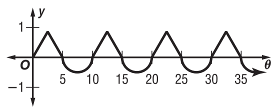
$\sin 855° = \sin(135° + 720°) = \sin 135° = \frac{\sqrt{2}}{2}$

b. $\cos\left(\frac{31\pi}{6}\right)$

$\cos\left(\frac{31\pi}{6}\right) = \cos\left(\frac{7\pi}{6} + 4\pi\right)$

$= \cos \frac{7\pi}{6} = -\frac{\sqrt{3}}{2}$

Example 2 **Determine the period of the function graphed below.**



The pattern of the function repeats every 10 units, so the period of the function is 10.

Exercises

Find the exact value of each function.

1. $\cos(-240°)$ $-\frac{1}{2}$

2. $\cos 2880°$ 1

3. $\sin(-510°)$ $-\frac{1}{2}$

4. $\sin 495°$ $\frac{\sqrt{2}}{2}$

5. $\cos\left(-\frac{5\pi}{2}\right)$ 0

6. $\sin\left(\frac{5\pi}{3}\right)$ $-\frac{\sqrt{3}}{2}$

7. $\cos\left(\frac{11\pi}{4}\right)$ $-\frac{\sqrt{2}}{2}$

8. $\sin\left(-\frac{3\pi}{4}\right)$ $-\frac{\sqrt{2}}{2}$

9. $\cos 1440°$ 1

10. $\sin(-750°)$ $-\frac{1}{2}$

11. $\cos 870°$ $-\frac{\sqrt{3}}{2}$

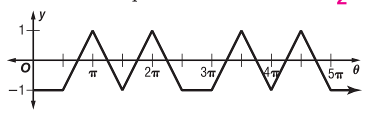
12. $\cos 1980°$ -1

13. $\sin 7\pi$ 0

14. $\sin\left(-\frac{13\pi}{4}\right)$ $\frac{\sqrt{2}}{2}$

15. $\cos\left(\frac{23\pi}{6}\right)$ $\frac{\sqrt{3}}{2}$

16. Determine the period of the function. $\frac{5\pi}{2}$



Answers

NAME ______________________ DATE __________ PERIOD _____

13-6 Skills Practice

Circular Functions

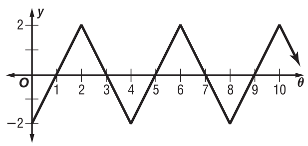
The given point P is located on the unit circle. Find $\sin\theta$ and $\cos\theta$.

1. $P\left(\frac{3}{5}, \frac{4}{5}\right)$ $\sin\theta = \frac{4}{5}$, $\cos\theta = \frac{3}{5}$
2. $P\left(\frac{5}{13}, -\frac{12}{13}\right)$ $\sin\theta = -\frac{12}{13}$, $\cos\theta = \frac{5}{13}$
3. $P\left(-\frac{9}{41}, -\frac{40}{41}\right)$ $\sin\theta = -\frac{40}{41}$, $\cos\theta = -\frac{9}{41}$
4. $P(0, 1)$ $\sin\theta = 1$, $\cos\theta = 0$
5. $P(-1, 0)$ $\sin\theta = 0$, $\cos\theta = -1$
6. $P\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$ $\sin\theta = -\frac{\sqrt{3}}{2}$, $\cos\theta = \frac{1}{2}$

Find the exact value of each function.

7. $\cos 45°$ $\frac{\sqrt{2}}{2}$
8. $\sin 210°$ $-\frac{1}{2}$
9. $\sin 330°$ $-\frac{1}{2}$
10. $\cos 330°$ $\frac{\sqrt{3}}{2}$
11. $\cos(-60°)$ $\frac{1}{2}$
12. $\sin(-390°)$ $-\frac{1}{2}$
13. $\sin 5\pi$ 0
14. $\cos 3\pi$ -1
15. $\sin\frac{5\pi}{2}$ 1
16. $\sin\frac{7\pi}{3}$ $\frac{\sqrt{3}}{2}$
17. $\cos\left(-\frac{7\pi}{3}\right)$ $\frac{1}{2}$
18. $\cos\left(-\frac{5\pi}{6}\right)$ $-\frac{\sqrt{3}}{2}$

Determine the period of each function.

19. 

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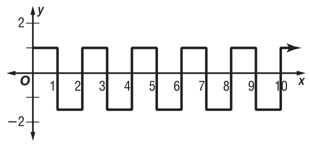
20. 

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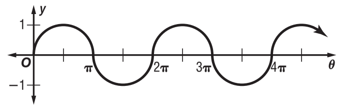
21. 

2π

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13-6 Practice (Average)

Circular Functions

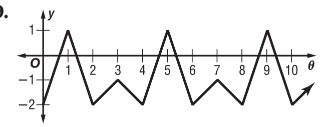
The given point P is located on the unit circle. Find $\sin\theta$ and $\cos\theta$.

1. $P\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ $\sin\theta = \frac{\sqrt{3}}{2}$, $\cos\theta = -\frac{1}{2}$
2. $P\left(\frac{20}{29}, -\frac{21}{29}\right)$ $\sin\theta = -\frac{21}{29}$, $\cos\theta = \frac{20}{29}$
3. $P(0.8, 0.6)$ $\sin\theta = 0.6$, $\cos\theta = 0.8$
4. $P(0, -1)$ $\sin\theta = -1$, $\cos\theta = 0$
5. $P\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$ $\sin\theta = -\frac{\sqrt{2}}{2}$, $\cos\theta = -\frac{\sqrt{2}}{2}$
6. $P\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ $\sin\theta = \frac{1}{2}$, $\cos\theta = \frac{\sqrt{3}}{2}$

Find the exact value of each function.

7. $\cos\frac{7\pi}{4}$ $\frac{\sqrt{2}}{2}$
8. $\sin(-30°)$ $-\frac{1}{2}$
9. $\sin\left(-\frac{2\pi}{3}\right)$ $-\frac{\sqrt{3}}{2}$
10. $\cos(-330°)$ $\frac{\sqrt{3}}{2}$
11. $\cos 600°$ $-\frac{1}{2}$
12. $\sin\frac{9\pi}{2}$ 1
13. $\cos 7\pi$ -1
14. $\cos\left(-\frac{11\pi}{4}\right)$ $-\frac{\sqrt{2}}{2}$
15. $\sin(-225°)$ $\frac{\sqrt{2}}{2}$
16. $\sin 585°$ $-\frac{\sqrt{2}}{2}$
17. $\cos\left(-\frac{10\pi}{3}\right)$ $-\frac{1}{2}$
18. $\sin 840°$ $\frac{\sqrt{3}}{2}$

Determine the period of each function.

19. 

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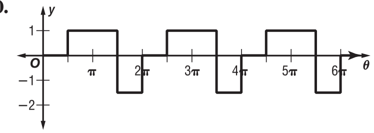
20. 

2π

21. **FERRIS WHEELS** A Ferris wheel with a diameter of 100 feet completes 2.5 revolutions per minute. What is the period of the function that describes the height of a seat on the outside edge of the Ferris Wheel as a function of time? **24 s**

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13-6 Reading to Learn Mathematics

Circular Functions

Pre-Activity **How can you model annual temperature fluctuations?**

Read the introduction to Lesson 13-6 at the top of page 739 in your textbook.

- If the graph in your textbook is continued, what month will $x = 17$ represent? **May of the following year**
- About what do you expect the average high temperature to be for that month? **24.2°F**
- Will this be exactly the average high temperature for that month? Explain your answer. **Sample answer: No; temperatures vary from year to year.**

Reading the Lesson

1. Use the unit circle on page 740 in your textbook to find the exact values of each expression.

a. $\cos 45°$ $\frac{\sqrt{2}}{2}$ b. $\sin 150°$ $\frac{1}{2}$ c. $\sin 240°$ $-\frac{\sqrt{3}}{2}$

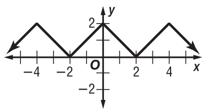
d. $\sin 315°$ $-\frac{\sqrt{2}}{2}$ e. $\cos 270°$ 0 f. $\sin 210°$ $-\frac{1}{2}$

g. $\cos 0°$ 1 h. $\sin 180°$ 0 i. $\cos 330°$ $\frac{\sqrt{3}}{2}$

2. Tell whether each function is periodic. Write *yes* or *no*.

a. $y = 2x$ **no** b. $y = x^2$ **no** c. $y = \cos x$ **yes** d. $y = |x|$ **no**

3. Find the period of each function by examining its graph.

a. 

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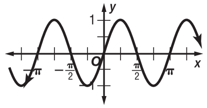
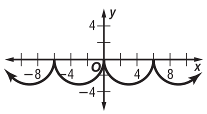
b. 

π

c. 

6

Helping You Remember

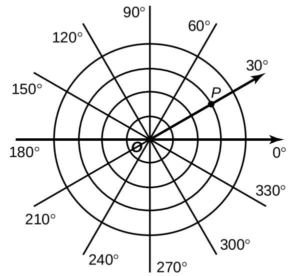
4. What is an easy way to remember the periods of the sine and cosine functions in radian measure? **Sample answer: The period of both functions is 2π, which is the circumference of the unit circle.**

Lesson 13-6

NAME ______________________ DATE ________ PERIOD _____

13-6 Enrichment

Polar Coordinates



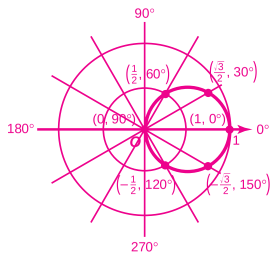
Consider an angle in standard position with its vertex at a point O called the *pole*. Its initial side is on a coordinated axis called the *polar axis*. A point P on the terminal side of the angle is named by the *polar coordinates* (r, θ) where r is the directed distance of the point from O and θ is the measure of the angle.

Graphs in this system may be drawn on polar coordinate paper such as the kind shown at the right.

The polar coordinates of a point are not unique. For example, (3, 30°) names point P as well as (3, 390°). Another name for P is (−3, 210°). Can you see why? The coordinates of the pole are $(0, \theta)$ where θ may be any angle.

Example **Draw the graph of the function $r = \cos \theta$. Make a table of convenient values for θ and r. Then plot the points.**

θ	0°	30°	60°	90°	120°	150°	180°
r	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	−1



Since the period of the cosine function is 180°, values of r for $\theta > 180°$ are repeated.

Graph each function by making a table of values and plotting the values on polar coordinate paper.

1. $r = 4$
$r = 4$ for all values of θ. Graph should be a circle with radius 4 and center at the pole.

2. $r = 3 \sin \theta$
Graph is circle of radius $\frac{3}{2}$ with center at $\left(\frac{3}{2}, 90°\right)$.

3. $r = 3 \cos 2\theta$
Graph looks like flower with 4 petals, points of petals are at (3, 0°), (3, 90°), (3, 180°), (3, 270°). All petals meet at pole.

4. $r = 2(1 + \cos \theta)$
Graph is heart-shaped curve, symmetric with respect to polar axis.

Answers

NAME ______________________ DATE ________ PERIOD _____

13-7 Study Guide and Intervention

Inverse Trigonometric Functions

Solve Equations Using Inverses If the domains of trigonometric functions are restricted to their **principal values**, then their inverses are also functions.

Principal Values of Sine, Cosine, and Tangent	$y = \text{Sin } x$ if and only if $y = \sin x$ and $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$. $y = \text{Cos } x$ if and only if $y = \cos x$ and $0 \le x \le \pi$. $y = \text{Tan } x$ if and only if $y = \tan x$ and $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$.
Inverse Sine, Cosine, and Tangent	Given $y = \text{Sin } x$, the inverse Sine function is defined by $y = \text{Sin}^{-1} x$ or $y = \text{Arcsin } x$. Given $y = \text{Cos } x$, the inverse Cosine function is defined by $y = \text{Cos}^{-1} x$ or $y = \text{Arccos } x$. Given $y = \text{Tan } x$, the inverse Tangent function is given by $y = \text{Tan}^{-1} x$ or $y = \text{Arctan } x$.

Example 1 **Solve $x = \text{Sin}^{-1}\left(\frac{\sqrt{3}}{2}\right)$.**

If $x = \text{Sin}^{-1}\left(\frac{\sqrt{3}}{2}\right)$, then $\text{Sin } x = \frac{\sqrt{3}}{2}$ and $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$.

The only x that satisfies both criteria is $x = \frac{\pi}{3}$ or 60°.

Example 2 **Solve $\text{Arctan}\left(-\frac{\sqrt{3}}{3}\right) = x$.**

If $x = \text{Arctan}\left(-\frac{\sqrt{3}}{3}\right)$, then $\text{Tan } x = -\frac{\sqrt{3}}{3}$ and $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$.

The only x that satisfies both criteria is $-\frac{\pi}{6}$ or −30°.

Exercises

Solve each equation by finding the value of x to the nearest degree.

1. $\text{Cos}^{-1}\left(-\frac{\sqrt{3}}{2}\right) = x$ **150°**
2. $x = \text{Sin}^{-1}\frac{\sqrt{3}}{2}$ **60°**
3. $x = \text{Arccos }(-0.8)$ **143°**
4. $x = \text{Arctan }\sqrt{3}$ **60°**
5. $x = \text{Arccos}\left(-\frac{\sqrt{2}}{2}\right)$ **135°**
6. $x = \text{Tan}^{-1}(-1)$ **−45°**
7. $\text{Sin}^{-1} 0.45 = x$ **27°**
8. $x = \text{Arcsin}\left(-\frac{\sqrt{3}}{2}\right)$ **−60°**
9. $x = \text{Arccos}\left(-\frac{1}{2}\right)$ **120°**
10. $\text{Cos}^{-1}(-0.2) = x$ **102°**
11. $x = \text{Tan}^{-1}(-\sqrt{3})$ **−60°**
12. $x = \text{Arcsin } 0.3$ **17°**
13. $x = \text{Tan}^{-1}(15)$ **86°**
14. $x = \text{Cos}^{-1} 1$ **0**
15. $\text{Arctan}^{-1}(-3) = x$ **−72°**
16. $x = \text{Sin}^{-1}(-0.9)$ **−64°**
17. $\text{Arccos}^{-1} 0.15$ **81°**
18. $x = \text{Tan}^{-1} 0.2$ **11°**

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13-7 Study Guide and Intervention *(continued)*

Inverse Trigonometric Functions

Trigonometric Values You can use a calculator to find the values of trigonometric expressions.

Example **Find each value. Write angle measures in radians. Round to the nearest hundredth.**

a. Find $\tan\left(\text{Sin}^{-1}\frac{1}{2}\right)$.

Let $\theta = \text{Sin}^{-1}\frac{1}{2}$. Then $\text{Sin }\theta = \frac{1}{2}$ with $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$. The value $\theta = \frac{\pi}{6}$ satisfies both conditions. $\tan\frac{\pi}{6} = \frac{\sqrt{3}}{3}$ so $\tan\left(\text{Sin}^{-1}\frac{1}{2}\right) = \frac{\sqrt{3}}{3}$.

b. Find $\cos(\text{Tan}^{-1} 4.2)$.

KEYSTROKES: [COS] [2nd] [tan⁻¹] 4.2 [ENTER] .2316205273

Therefore $\cos(\text{Tan}^{-1} 4.2) \approx 0.23$.

Exercises

Find each value. Write angle measures in radians. Round to the nearest hundredth.

1. $\cot(\text{Tan}^{-1} 2)$ **$\frac{1}{2}$**
2. $\text{Arctan}(-1)$ **−0.79**
3. $\cot^{-1} 1$ **1.27**
4. $\cos\left[\text{Sin}^{-1}\left(-\frac{\sqrt{2}}{2}\right)\right]$ **0.71**
5. $\text{Sin}^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ **−1.05**
6. $\sin\left(\text{Arcsin}\frac{\sqrt{3}}{2}\right)$ **0.87**
7. $\tan\left[\text{Arcsin}\left(-\frac{5}{7}\right)\right]$ **−1.02**
8. $\sin\left(\text{Tan}^{-1}\frac{5}{12}\right)$ **0.38**
9. $\sin[\text{Arctan}^{-1}(-\sqrt{2})]$ **−0.82**
10. $\text{Arccos}\left(-\frac{\sqrt{3}}{2}\right)$ **2.62**
11. $\text{Arcsin}\left(\frac{\sqrt{3}}{2}\right)$ **1.05**
12. $\text{Arccot}\left(-\frac{\sqrt{3}}{3}\right)$ **−1.91**
13. $\cos[\text{Arcsin}(-0.7)]$ **0.71**
14. $\tan(\text{Cos}^{-1} 0.28)$ **3.43**
15. $\cos(\text{Arctan } 5)$ **0.20**
16. $\text{Sin}^{-1}(-0.78)$ **−0.89**
17. $\text{Cos}^{-1} 0.42$ **1.14**
18. $\text{Arctan}(-0.42)$ **−0.40**
19. $\sin(\text{Cos}^{-1} 0.32)$ **0.95**
20. $\cos(\text{Arctan } 8)$ **0.12**
21. $\tan(\text{Cos}^{-1} 0.95)$ **0.33**

NAME ______________________ DATE __________ PERIOD _____

13-7 Skills Practice

Inverse Trigonometric Functions

Write each equation in the form of an inverse function.

1. $\alpha = \cos \beta$ **$\beta = \cos^{-1} \alpha$**

2. $\sin b = a$ **$\sin^{-1} a = b$**

3. $y = \tan x$ **$x = \tan^{-1} y$**

4. $\cos 45° = \frac{\sqrt{2}}{2}$ **$\cos^{-1} \frac{\sqrt{2}}{2} = 45°$**

5. $b = \sin 150°$ **$150° = \sin^{-1} b$**

6. $\tan y = \frac{4}{5}$ **$\tan^{-1} \frac{4}{5} = y$**

Solve each equation by finding the value of x to the nearest degree.

7. $x = \text{Cos}^{-1}(-1)$ **180°**

8. $\text{Sin}^{-1}(-1) = x$ **−90°**

9. $\text{Tan}^{-1} 1 = x$ **45°**

10. $x = \text{Arcsin}\left(-\frac{\sqrt{3}}{2}\right)$ **−60°**

11. $x = \text{Arctan } 0$ **0°**

12. $x = \text{Arccos } \frac{1}{2}$ **60°**

Find each value. Write angle measures in radians. Round to the nearest hundredth.

13. $\text{Sin}^{-1} \frac{\sqrt{2}}{2}$ **0.79 radians**

14. $\text{Cos}^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ **2.62 radians**

15. $\text{Tan}^{-1} \sqrt{3}$ **1.05 radians**

16. $\text{Arctan}\left(-\frac{\sqrt{3}}{3}\right)$ **−0.52 radians**

17. $\text{Arccos}\left(-\frac{\sqrt{2}}{2}\right)$ **2.36 radians**

18. $\text{Arcsin } 1$ **1.57 radians**

19. $\sin(\text{Cos}^{-1} 1)$ **0**

20. $\sin\left(\text{Sin}^{-1} \frac{1}{2}\right)$ **0.5**

21. $\tan\left(\text{Arcsin} \frac{\sqrt{3}}{2}\right)$ **1.73**

22. $\cos(\text{Tan}^{-1} 3)$ **0.32**

23. $\sin[\text{Arctan}(-1)]$ **−0.71**

24. $\sin\left[\text{Arccos}\left(-\frac{\sqrt{2}}{2}\right)\right]$ **0.71**

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13-7 Practice (Average)

Inverse Trigonometric Functions

Write each equation in the form of an inverse function.

1. $\beta = \cos \alpha$ **$\alpha = \cos^{-1} \beta$**

2. $\tan \beta = \alpha$ **$\beta = \tan^{-1} \alpha$**

3. $y = \tan 120°$ **$120° = \tan^{-1} y$**

4. $-\frac{1}{2} = \cos x$ **$x = \cos^{-1}\left(-\frac{1}{2}\right)$**

5. $\sin \frac{2\pi}{3} = \frac{\sqrt{3}}{2}$ **$\sin^{-1} \frac{\sqrt{3}}{2} = \frac{2\pi}{3}$**

6. $\cos \frac{\pi}{3} = \frac{1}{2}$ **$\cos^{-1} \frac{1}{2} = \frac{\pi}{3}$**

Solve each equation by finding the value of x to the nearest degree.

7. $\text{Arcsin } 1 = x$ **90°**

8. $\text{Cos}^{-1} \frac{\sqrt{3}}{2} = x$ **30°**

9. $x = \tan^{-1}\left(-\frac{\sqrt{3}}{3}\right)$ **−30°**

10. $x = \text{Arccos} \frac{\sqrt{2}}{2}$ **45°**

11. $x = \text{Arctan}(-\sqrt{3})$ **−60°**

12. $\text{Sin}^{-1}\left(-\frac{1}{2}\right) = x$ **−30°**

Find each value. Write angle measures in radians. Round to the nearest hundredth.

13. $\text{Cos}^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ **2.62 radians**

14. $\text{Sin}^{-1}\left(-\frac{\sqrt{2}}{2}\right)$ **−0.79 radians**

15. $\text{Arctan}\left(-\frac{\sqrt{3}}{3}\right)$ **−0.52 radians**

16. $\tan\left(\text{Cos}^{-1} \frac{1}{2}\right)$ **1.73**

17. $\cos\left[\text{Sin}^{-1}\left(-\frac{3}{5}\right)\right]$ **0.8**

18. $\cos[\text{Arctan}(-1)]$ **0.71**

19. $\tan\left(\sin^{-1} \frac{12}{13}\right)$ **2.4**

20. $\sin\left(\text{Arctan} \frac{\sqrt{3}}{3}\right)$ **0.5**

21. $\text{Cos}^{-1}\left(\tan \frac{3\pi}{4}\right)$ **3.14 radians**

22. $\text{Sin}^{-1}\left(\cos \frac{\pi}{3}\right)$ **0.52 radians**

23. $\sin\left(2\ \text{Cos}^{-1} \frac{15}{17}\right)$ **0.83**

24. $\cos\left(2\ \text{Sin}^{-1} \frac{\sqrt{3}}{2}\right)$ **−0.5**

25. PULLEYS The equation $x = \cos^{-1} 0.95$ describes the angle through which pulley A moves, and $y = \cos^{-1} 0.17$ describes the angle through which pulley B moves. Both angles are greater than 270° and less than 360°. Which pulley moves through a greater angle? **pulley A**

26. FLYWHEELS The equation $y = \text{Arctan } 1$ describes the counterclockwise angle through which a flywheel rotates in 1 millisecond. Through how many degrees has the flywheel rotated after 25 milliseconds? **1125°**

NAME ______________ DATE ________ PERIOD _____

13-7 Reading to Learn Mathematics

Inverse Trigonometric Functions

Pre-Activity **How are inverse trigonometric functions used in road design?**

Read the introduction to Lesson 13-7 at the top of page 746 in your textbook.

Suppose you are given specific values for v and r. What feature of your graphing calculator could you use to find the approximate measure of the banking angle θ? **Sample answer: the TABLE feature**

Reading the Lesson

1. Indicate whether each statement is *true* or *false*.

a. The domain of the function $y = \sin x$ is the set of all real numbers. **true**

b. The domain of the function $y = \text{Cos } x$ is $0 \le x \le \pi$. **true**

c. The range of the function $y = \text{Tan } x$ is $-1 \le y \le 1$. **false**

d. The domain of the function $y = \text{Cos}^{-1} x$ is $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$. **false**

e. The domain of the function $y = \text{Tan}^{-1} x$ is the set of all real numbers. **true**

f. The range of the function $y = \text{Arcsin } x$ is $0 \le x \le \pi$. **false**

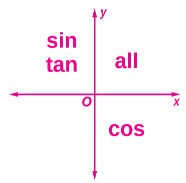
2. Answer each question in your own words.

a. What is the difference between the functions $y = \sin x$ and the function $y = \text{Sin } x$? **Sample answer: The domain of $y = \sin x$ is the set of all real numbers, while the domain of $y = \text{Sin } x$ is restricted to $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$.**

b. Why is it necessary to restrict the domains of the trigonometric functions in order to define their inverses? **Sample answer: Only one-to-one functions have inverses. None of the six basic trigonometric functions is one-to-one, but related one-to-one functions can be formed if the domains are restricted in certain ways.**

Helping You Remember

3. What is a good way to remember the domains of the functions $y = \text{Sin } x$, $y = \text{Cos } x$, and $y = \text{Tan } x$, which are also the range of the functions $y = \text{Arcsin } x$, $y = \text{Arccos } x$, and $y = \text{Arctan } x$? (You may want to draw a diagram.) **Sample answer: Each restricted domain must include an interval of numbers for which the function values are positive and one for which they are negative.**



Lesson 13-7

NAME ______________ DATE ________ PERIOD _____

13-7 Enrichment

Snell's Law

Snell's Law describes what happens to a ray of light that passes from air into water or some other substance. In the figure, the ray starts at the left and makes an angle of incidence θ with the surface.

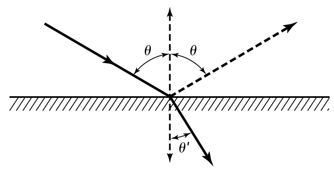
Part of the ray is reflected, creating an angle of reflection θ. The rest of the ray is bent, or refracted, as it passes through the other medium. This creates angle θ'.

The angle of incidence equals the angle of reflection.

The angles of incidence and refraction are related by Snell's Law:

$\sin \theta = k \sin \theta'$

The constant k is called the index of refraction.



k	Substance
1.33	Water
1.36	Ethyl alcohol
1.54	Rock salt and Quartz
1.46–1.96	Glass
2.42	Diamond

Use Snell's Law to solve the following. Round angle measures to the nearest tenth of a degree.

1. If the angle of incidence at which a ray of light strikes the surface of a window is 45° and $k = 1.6$, what is the measure of the angle of refraction? **26.2°**

2. If the angle of incidence of a ray of light that strikes the surface of water is 50°, what is the angle of refraction? **35.2°**

3. If the angle of refraction of a ray of light striking a quartz crystal is 24°, what is the angle of incidence? **38.8°**

4. The angles of incidence and refraction for rays of light were measured five times for a certain substance. The measurements (one of which was in error) are shown in the table. Was the substance glass, quartz, or diamond? **glass**

θ	15°	30°	40°	60°	80°
θ'	9.7°	16.1°	21.2°	28.6°	33.2°

5. If the angle of incidence at which a ray of light strikes the surface of ethyl alcohol is 60°, what is the angle of refraction? **39.6°**

Chapter 13 Assessment Answer Key

Form 1

Page 817

1. A
2. C
3. C
4. A
5. B
6. B
7. B
8. D
9. C
10. C

Page 818

11. D
12. A
13. D
14. B
15. B
16. A
17. D
18. B
19. D
20. A

B: 23.3 m

Form 2A

Page 819

1. B
2. C
3. C
4. C
5. A
6. B
7. A
8. A
9. C
10. B

(continued on the next page)

Answers

Chapter 13 Assessment Answer Key

Form 2A *(continued)*
Page 820

11. C
12. B
13. D
14. A
15. B
16. B
17. A
18. D
19. A
20. C

B: 234 ft

Form 2B
Page 821

1. A
2. D
3. C
4. B
5. C
6. B
7. A
8. A
9. A
10. C

Page 822

11. C
12. D
13. A
14. B
15. B
16. D
17. C
18. C
19. A
20. C

B: 172 ft

Chapter 13 Assessment Answer Key

Form 2C

Page 823

1. $\sin \theta = \frac{5}{13}$; $\cos \theta = \frac{12}{13}$; $\tan \theta = \frac{5}{12}$; $\csc \theta = \frac{13}{5}$; $\sec \theta = \frac{13}{12}$; $\cot \theta = \frac{12}{5}$

2. $A \approx 25°$; $B \approx 65°$; $b \approx 6.3$

3. $\sin x° = \frac{7}{12}$; $x \approx 36$

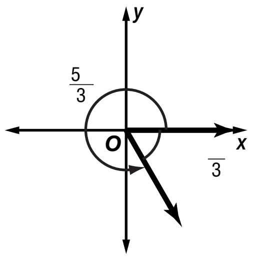
4. $-\frac{5\pi}{12}$

5. 300°

6. Sample answers: $\frac{13\pi}{4}$, $-\frac{3\pi}{4}$

7. $\sin \theta = -\frac{3\sqrt{13}}{13}$; $\cos \theta = -\frac{2\sqrt{13}}{13}$; $\tan \theta = \frac{3}{2}$; $\csc \theta = -\frac{\sqrt{13}}{3}$; $\sec \theta = -\frac{\sqrt{13}}{2}$; $\cot \theta = \frac{2}{3}$

8. $\frac{\pi}{3}$



9. $-\frac{\sqrt{3}}{2}$

10. 0

11. 635.9 mi^2

Page 824

12. one; $B \approx 36.8°$, $C \approx 85.2°$, $c \approx 20.0$

13. no solution

14. Law of Sines; $C = 30°$, $b \approx 9.4$, $c \approx 4.8$

15. Law of Cosines; $A \approx 26.8°$, $B \approx 38.6°$, $c \approx 10.2$

16. $\sin \theta = -\frac{\sqrt{3}}{2}$, $\cos \theta = -\frac{1}{2}$

17. $\frac{\sqrt{3}}{2}$

18. 8π

19. $-45°$ or $-\frac{\pi}{4}$

20. 0.50

B: 56 ft

Answers

Chapter 13 Assessment Answer Key

Form 2D

Page 825

1. $\sin\theta = \frac{40}{41}$; $\cos\theta = \frac{9}{41}$; $\tan\theta = \frac{40}{9}$; $\csc\theta = \frac{41}{40}$; $\sec\theta = \frac{41}{9}$; $\cot\theta = \frac{9}{40}$

2. $A \approx 39°$, $B \approx 51°$, $b \approx 6.2$

3. $\tan x° = \frac{9}{5}$; $x \approx 61$

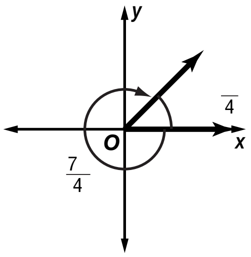
4. $\frac{11\pi}{6}$

5. $-315°$

6. Sample answers: $240°$, $-480°$

7. $\sin\theta = -\frac{2\sqrt{13}}{13}$; $\cos\theta = \frac{3\sqrt{13}}{13}$; $\tan\theta = -\frac{2}{3}$; $\csc\theta = -\frac{\sqrt{13}}{2}$; $\sec\theta = \frac{\sqrt{13}}{3}$; $\cot\theta = -\frac{3}{2}$

8. $\frac{\pi}{4}$



9. $-\frac{1}{2}$

10. 1

11. 47.7 yd^2

Page 826

12. no solution

13. one; $B \approx 35.3°$, $C \approx 84.7°$, $c \approx 10.3$

14. Law of Sines; $B \approx 15.1°$, $C \approx 145.9°$, $c \approx 17.2$

15. Law of Cosines; $A \approx 24.6°$, $B \approx 110.4°$, $c \approx 6.8$

16. $\sin\theta = -\frac{1}{2}$, $\cos\theta = \frac{\sqrt{3}}{2}$

17. $\frac{1}{2}$

18. 6π

19. $30°$ or $\frac{\pi}{6}$

20. 0.38

B: 60 ft

Chapter 13 Assessment Answer Key

Form 3

Page 827

1. $\sin\theta = \frac{2\sqrt{13}}{13}$; $\cos\theta = \frac{3\sqrt{13}}{13}$; $\tan\theta = \frac{2}{3}$; $\csc\theta = \frac{\sqrt{13}}{2}$; $\sec\theta = \frac{\sqrt{13}}{3}$; $\cot\theta = \frac{3}{2}$

2. $A = 65°$; $b \approx 1.2$, $c \approx 2.9$

3. $A \approx 53°$, $B \approx 37°$, $a = 8$, $c = 10$

4. $-\frac{7\pi}{4}$

5. $\left(\frac{-900}{\pi}\right)^\circ \approx -286.5°$

6. Sample answers: 3°, −357°

7. $\sin\theta = \frac{1}{2}$; $\cos\theta = -\frac{\sqrt{3}}{2}$; $\tan\theta = -\frac{\sqrt{3}}{3}$; $\csc\theta = 2$; $\sec\theta = -\frac{2\sqrt{3}}{3}$; $\cot\theta = -\sqrt{3}$

8. $\frac{1}{2}$

9. 1

10. 26.1 m^2

11. two; $B \approx 63.1°$, $C \approx 74.9°$, $c \approx 13.0$; $B \approx 116.9°$, $C \approx 21.1°$, $c \approx 4.8$

12. one; $B \approx 36.9°$, $C \approx 84.1°$, $c \approx 11.6$

Page 828

13. Law of Cosines; $A \approx 18.2°$, $B \approx 121.7°$, $c \approx 6.2$

14. Law of Sines; $A \approx 25.4°$, $B \approx 22.6°$, $b = 13.4$

15. $\sin\theta = \frac{\sqrt{21}}{7}$, $\cos\theta = -\frac{2\sqrt{7}}{7}$

16. $-\frac{1}{2}$

17. $-\frac{3\sqrt{3}}{4}$

18. 5π

19. 135° or $\frac{3\pi}{4}$

20. −0.28

B: 51.7 m

Answers

Chapter 13 Assessment Answer Key

Page 829, Open-Ended Assessment
Scoring Rubric

Score	General Description	Specific Criteria
4	**Superior** A correct solution that is supported by well-developed, accurate explanations	• Shows thorough understanding of the concepts of *solving problems involving right triangles, finding values of trigonometric functions for general angles, using reference angles, applying the Laws of Sines and Cosines,* and *solving equations using inverse trigonometric functions.* • Uses appropriate strategies to solve problems. • Computations are correct. • Written explanations are exemplary. • Goes beyond requirements of some or all problems.
3	**Satisfactory** A generally correct solution, but may contain minor flaws in reasoning or computation	• Shows an understanding of the concepts of *solving problems involving right triangles, finding values of trigonometric functions for general angles, using reference angles, applying the Laws of Sines and Cosines,* and *solving equations using inverse trigonometric functions.* • Uses appropriate strategies to solve problems. • Computations are mostly correct. • Written explanations are effective. • Satisfies all requirements of problems.
2	**Nearly Satisfactory** A partially correct interpretation and/or solution to the problem	• Shows an understanding of most of the concepts of *solving problems involving right triangles, finding values of trigonometric functions for general angles, using reference angles, applying the Laws of Sines and Cosines,* and *solving equations using inverse trigonometric functions.* • May not use appropriate strategies to solve problems. • Computations are mostly correct. • Written explanations are satisfactory. • Satisfies the requirements of most of the problems.
1	**Nearly Unsatisfactory** A correct solution with no supporting evidence or explanation	• Final computation is correct. • No written explanations or work is shown to substantiate the final computation. • Satisfies minimal requirements of some of the problems.
0	**Unsatisfactory** An incorrect solution indicating no mathematical understanding of the concept or task, or no solution is given	• Shows little or no understanding of most of the concepts of *solving problems involving right triangles, finding values of trigonometric functions for general angles, using reference angles, applying the Laws of Sines and Cosines,* and *solving equations using inverse trigonometric functions.* • Does not use appropriate strategies to solve problems. • Computations are incorrect. • Written explanations are unsatisfactory. • Does not satisfy requirements of problems. • No answer may be given.

Chapter 13 Assessment Answer Key

Page 829, Open-Ended Assessment
Sample Answers

In addition to the scoring rubric found on page A28, the following sample answers may be used as guidance in evaluating open-ended assessment items.

1a. Students should indicate that knowing the measures of the angles of a triangle gives no information about the lengths of its sides.

1b. Students should explain that Monica can determine the length b since it does not involve measuring across the body of water.

1c. Sample answer: For $A = 115°$, $B = 25°$, $C = 40°$, and $b = 1000$ yd, the Law of Sines gives
$\frac{\sin 25°}{1000} = \frac{\sin 40°}{c}$, so $c \approx 1521$ yd.

2. Ideally, students should apply three of the following methods: the Pythagorean Theorem, the Law of Sines, the Law of Cosines, a right triangle trigonometry formula/definition, to find $x \approx 6.1$. (Students may, however, apply two different right triangle formulas and the Pythagorean Theorem as their three methods, for example.)
Sample answers:
By the Pythagorean Theorem, $x^2 + 13.7^2 = 15^2$.
By the Law of Sines, $\frac{\sin 24°}{x} = \frac{\sin 90°}{15}$.
By the Law of Cosines, $x^2 = 15^2 + 13.7^2 - 2(15)(13.7)\cos 24°$.
By right triangle trigonometry, $\sin 24° = \frac{x}{15}$.

3. Sample answer: For $P(-3, -4)$, $x = -3$ and $y = -4$, so $\tan \theta = \frac{4}{3}$. This means that $\theta' = \tan^{-1}\left(\frac{4}{3}\right) \approx 53°$ is the reference angle for the angle θ in Quadrant III. Thus, $\theta \approx 180° + 53° = 233°$.

4. For any point $Q(x, y)$ chosen, students should use the relationship $r = \sqrt{x^2 + y^2}$, or the distance formula, to find the radius of the sector r. Then, students should use an inverse trigonometric function to find θ in radians. Finally, students should substitute these values for r and θ into the given formula.
Sample answer: For $Q(3, 4)$,
$r = \sqrt{3^2 + 4^2} = 5$,
$\theta = \tan^{-1}\left(\frac{4}{3}\right) \approx 0.9273$, so
$A \approx \frac{1}{2}(5^2)(0.9273) \approx 11.6$ square units.

5a. Students should explain that they must find the length of another side of the triangle to be able to apply the given formula. They must apply the Law of Sines to determine that $B = 90°$. This gives $A = 45°$ and $a = 9\sqrt{2}$ in. Applying the given formula, area $= \frac{1}{2}(9\sqrt{2})(9\sqrt{2})\sin 90°$ or area $= \frac{1}{2}(9\sqrt{2})(18)\sin 45°$, so area $= 81$ in^2.

5b. Since $\triangle ABC$ is a right triangle, it *is* possible to apply the formula, so
area $= \frac{1}{2}$(base)(height)
$= \frac{1}{2}(9\sqrt{2})(9\sqrt{2}) = 81$ in^2.

5c. The formula area $= \frac{1}{2}$(base)(height) is a special case of the formula area $= \frac{1}{2}ab \sin C$, where C is a right angle, so $\sin C = 1$.

Answers

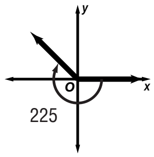
Chapter 13 Assessment Answer Key

Vocabulary Test/Review
Page 830

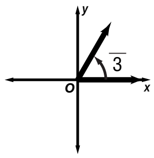
1. e
2. h
3. g
4. c
5. j
6. i
7. a
8. b
9. d
10. f
11. Sample answer: The position of an angle if its vertex is at the origin and its initial side is on the positive *x*-axis is its standard position.
12. Sample answer: A unit circle is a circle with center at the origin and radius 1.

Quiz (Lessons 13–1 and 13–2)
Page 831

1. $\sin \theta = \frac{8}{17}$; $\cos \theta = \frac{15}{17}$; $\tan \theta = \frac{8}{15}$; $\csc \theta = \frac{17}{8}$; $\sec \theta = \frac{17}{15}$; $\cot \theta = \frac{15}{8}$
2. B
3. $B = 70°$, $a \approx 3.6$, $c \approx 10.6$
4. Sample answers: 135°, −585°

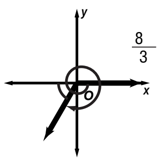


5. Sample answers: $\frac{7\pi}{3}$, $-\frac{5\pi}{3}$



Quiz (Lessons 13–3 and 13–4)
Page 831

1. $\sin \theta = \frac{\sqrt{10}}{10}$; $\cos \theta = -\frac{3\sqrt{10}}{10}$; $\tan \theta = -\frac{1}{3}$; $\csc \theta = \sqrt{10}$; $\sec \theta = -\frac{\sqrt{10}}{3}$; $\cot \theta = -3$
2. $-\frac{\sqrt{2}}{2}$
3. $\frac{\pi}{3}$



4. 89.3 in^2
5. two; $B \approx 18.9°$, $C \approx 146.1°$, $c \approx 25.8$; $B \approx 161.1°$, $C \approx 3.9°$, $c \approx 3.1$

Quiz (Lessons 13–5 and 13–6)
Page 832

1. Law of Sines; $B \approx 99.3°$, $C \approx 30.7°$, $b \approx 11.6$
2. Law of Cosines; $A \approx 46.6°$, $B \approx 104.5°$, $C \approx 28.9°$
3. Law of Cosines; $B \approx 26.2°$, $C \approx 117.8°$, $a \approx 8.0$
4. Law of Cosines; no solution
5. $-\frac{1}{2}$
6. $-\frac{\sqrt{3}}{2}$
7. −1
8. $-\frac{\sqrt{2}}{2}$
9. 5
10. π

Quiz (Lessons 13–7)
Page 832

1. $\theta = \text{Tan}^{-1} y$ or $\theta = \text{Arctan } y$
2. 60° or $\frac{\pi}{3}$
3. 0°
4. 0.80
5. 0.17

Chapter 13 Assessment Answer Key

Mid-Chapter Test
Page 833

1. B
2. B
3. A
4. C
5. B
6. B
7. $\sin\theta = \frac{\sqrt{55}}{8}$; $\cos\theta = \frac{3}{8}$; $\tan\theta = \frac{\sqrt{55}}{3}$; $\csc\theta = \frac{8\sqrt{55}}{55}$; $\sec\theta = \frac{8}{3}$; $\cot\theta = \frac{3\sqrt{55}}{55}$
8. $B = 50°$, $a \approx 8.4$, $c \approx 13.1$
9. $\sin\theta = \frac{2\sqrt{5}}{5}$; $\cos\theta = -\frac{\sqrt{5}}{5}$; $\tan\theta = -2$; $\csc\theta = \frac{\sqrt{5}}{2}$; $\sec\theta = -\sqrt{5}$; $\cot\theta = -\frac{1}{2}$
10. 1359.1 ft^2
11. one; $B \approx 19.7°$, $C \approx 108.3°$, $c \approx 8.4$
12. no solution

Cumulative Review
Page 834

1. $y = -\frac{2}{3}x + 1$
2. $x = -2$, $y = 5$
3. 100.8572
4. $52 + 23i$
5.
6. $y = 4(x - 3)^2 - 16$; $(3, -16)$; $x = 3$; up
7. $13(n^2)^2 - 52(n^2) = 0$; $-2, 0, 2$
8. $(0, \pm 4)$; $(0, \pm 2\sqrt{5})$; $y = \pm 2x$
9. 18
10. $m^5 + 15m^4 + 90m^3 + 270m^2 + 405m + 243$
11. 5040
12. 15.6, 12, no mode, 8.06
13. $A = 41°$, $b \approx 10.4$, $c \approx 13.7$
14. $A \approx 112°$, $B \approx 24°$, $C \approx 44°$
15. 315°
16. 45°

Answers

Chapter 13 Assessment Answer Key

Standardized Test Practice

Page 835

1. D
2. E
3. B
4. F
5. A
6. G
7. D
8. F
9. A
10. H

Page 836

11. 10
12. 196
13. 720
14. 136
15. C
16. B
17. A
18. C