ALG I – §9-6 NOTES

Algebra I

9.6 The Quadratic Formula and the Discriminant

Objective: To solve quadratic equations using the quadratic formula.
To find the number of solutions of a quadratic equation.

Warm-Up **Solve by factoring.**

1) $(4x+5)(x-3)=0$

$4x+5=0$ or $x-3=0$

$4x=-5$ $x=3$

$x=-\frac{5}{4}$

2) $3m^2+m=14$

$3m^2+m-14=0$

sum 1; 7, −6; −42 prod

$\underbrace{3m^2-6m}+\underbrace{7m-14}=0$

$3m(m-2)+7(m-2)=0$

$(m-2)(3m+7)=0$

$m-2=0$ or $3m+7=0$

$m=2$ $3m=-7$

$m=-\frac{7}{3}$

take note

Key Concept Quadratic Formula

Algebra

If $ax^2+bx+c=0$, and $a \neq 0$, then

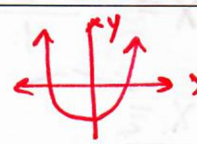
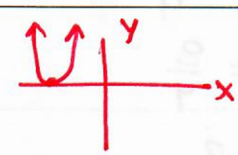
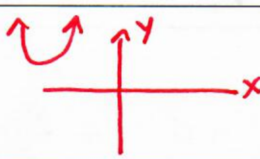
$$x=\frac{-b \pm \sqrt{b^2-4ac}}{2a} \longrightarrow b^2-4ac$$

Discriminant: b^2-4ac

Example

Suppose $2x^2+3x-5=0$. Then $a=2$, $b=3$, and $c=-5$. Therefore

$$x=\frac{-(3) \pm \sqrt{(3)^2-4(2)(-5)}}{2(2)}$$

Discriminant's Value	# and Nature of Solutions	Possible Graph
$b^2-4ac>0$	2 real solns	
$b^2-4ac=0$	1 real soln	
$b^2-4ac<0$	0 real solns	

Solve using the Quadratic Formula.
Find the discriminant, determine the nature of the roots, solve.

1) $2x^2 - 6x + 4 = 0$	2) $6m^2 + m + 12 = 0$
a = 2, b = -6, c = 4	a = 6, b = 1, c = 12
Discriminant's Value: $b^2 - 4ac = (-6)^2 - 4(2)(4) = 36 - 32 = 4$	Discriminant's Value: $b^2 - 4ac = (1)^2 - 4(6)(12) = 1 - 288 = -287$
#/Nature of Roots: 2 real solns	#/Nature of Roots: Ø real (imaginary)
Solution: $x = \frac{6 \pm \sqrt{4}}{2(2)}$, $x = \frac{6 \pm 2}{4}$, $x = \frac{6+2}{4}$, $x = \frac{8}{4}$, $x = 2$; $x = \frac{6-2}{4}$, $x = \frac{4}{4}$, $x = 1$	Solution: No solution

Solve using the Quadratic Formula.
Find the discriminant, determine the nature of the roots, solve.

3) $3y^2 + 10y = 5$	4) $2p^2 = 28p - 98$
$3y^2 + 10y - 5 = 0$	$2p^2 - 28p + 98 = 0$
a = 3, b = 10, c = -5	a = 2, b = -28, c = 98
Discriminant's Value: $b^2 - 4ac = (10)^2 - 4(3)(-5) = 100 + 60 = 160$	Discriminant's Value: $b^2 - 4ac = (-28)^2 - 4(2)(98) = 784 - 784 = 0$
#/Nature of Roots: 2 real	#/Nature of Roots: 1 real
Solution: $x = \frac{-10 \pm \sqrt{160}}{2(3)}$, $x = \frac{-10 \pm \sqrt{16 \cdot 10}}{6}$, $x = \frac{-10 \pm 4\sqrt{10}}{6}$, $x = \frac{-5 \pm 2\sqrt{10}}{3}$	Solution: $x = \frac{28 \pm \sqrt{0}}{2(2)}$, $x = \frac{28}{4}$, $x = 7$

Application Problem

Example 5 A ball is thrown up into the air and its height is represented by the equation $h = -d^2 + 10d + 5$. How far away does the ball land on the ground?

$a = -1 \quad b = 10 \quad c = 5$

$b^2 - 4ac = (10)^2 - 4(-1)(5)$

$= 100 + 20$

$= 120$

2 solutions

$d = \frac{-10 \pm \sqrt{120}}{2(-1)}$

$d = \frac{-10 \pm \sqrt{4 \cdot 30}}{-2}$

$d = \frac{-10 \pm 2\sqrt{30}}{-2}$

$d = 5 \pm \sqrt{30}$

120

12 10

6 2 2 5

Choosing the Best Method for Solving a Quadratic Equation:

Method	When to Use
Graphing	Use if you have a graphing calculator handy.
Square roots	Use if the equation has no x-term.
Factoring	Use if you can factor the equation easily.
Completing the square	Use if the coefficient of x^2 is 1, but you cannot easily factor the equation.
Quadratic formula	Use if the equation cannot be factored easily or at all.

Example 6 Which method(s) would you choose to solve each equation? Justify your reasoning.

a) $h^2 + 4h + 7 = 0$

Quadratic Formula

b) $a^2 - 4a - 12 = 0$

Factor

c) $a^2 - 144 = 0$

Square Roots

d) $24m^2 - 11m - 14 = 0$

Quadratic Formula