PRECALC – §7-4 NOTES

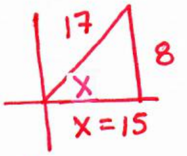
PRECALCULUS NOTES

7.4 Double-Angle and Half-Angle Identities

Objectives: Use the double-angle and half-angle identities for the sine, cosine, and tangent

Warm-Up

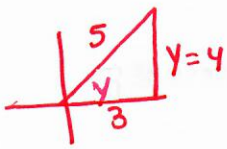
Find the exact value of $\tan(x-y)$ *if* $\sin x = \frac{8}{17}$ *and* $\cos y = \frac{3}{5}$ *if* $0 < x < \frac{\pi}{2}$ *and* $0 < y < \frac{\pi}{2}$.



$x^2 = 17^2 - 8^2$

$x^2 = 225$

$x = 15$



$$\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

$$= \frac{\frac{8}{15} - \frac{4}{3}}{1 + \left(\frac{8}{15}\right)\left(\frac{4}{3}\right)} \quad \frac{20}{15}$$

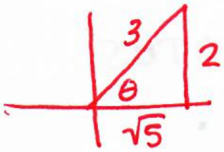
$$= \frac{-\frac{12}{15}}{1 + \frac{32}{45}}$$

$$= \frac{-\frac{12}{15}}{\frac{77}{45}}$$

$$= -\frac{12}{\cancel{15}}\left(\frac{\overset{3}{\cancel{45}}}{77}\right)$$

$$= -\frac{36}{77}$$

Double-Angle Identities:	**Half-Angle Identities:**
$\sin 2\theta = 2\, sin\theta\, cos\theta$ $\cos 2\theta = cos^2\theta - sin^2\theta$ $\cos 2\theta = 2\, cos^2\theta - 1$ $\cos 2\theta = 1 - 2\, sin^2\theta$ $\tan 2\theta = \frac{2\tan\theta}{1-tan^2\theta}$	$sin\, \frac{a}{2} = \pm\sqrt{\frac{1 - cos\, a}{2}}$ $cos\, \frac{a}{2} = \pm\sqrt{\frac{1 + cos\, a}{2}}$ $tan\, \frac{a}{2} = \pm\sqrt{\frac{1 - cos\, a}{1 + cos\, a}}$ *where* $cos\, a \neq -1$



$2^2 + x^2 = 3^2$
$4 + x^2 = 9$
$x = \sqrt{5}$

OR

$\sin^2\theta + \cos^2\theta = 1$
$\left(\frac{2}{3}\right)^2 + \cos^2\theta = 1$
$\frac{4}{9} + \cos^2\theta = 1$
$\cos^2\theta = \frac{5}{9}$
$\cos\theta = \frac{\sqrt{5}}{3}$

Example 1 If $\sin\theta = \frac{2}{3}$, and θ has a terminal side in the first quadrant, find the exact value of each.

a. sin 2θ $= 2\sin\theta\cos\theta$

$= 2\left(\frac{2}{3}\right)\left(\frac{\sqrt{5}}{3}\right)$

$= \frac{4}{3}\left(\frac{\sqrt{5}}{3}\right)$

$= \frac{4\sqrt{5}}{9}$

b. cos 2θ $= \cos^2\theta - \sin^2\theta$

$= \frac{5}{9} - \left(\frac{2}{3}\right)^2$

$= \frac{5}{9} - \frac{4}{9}$

$= \frac{1}{9}$

c. tan 2θ $= \dfrac{2\tan\theta}{1-\tan^2\theta}$

$= \dfrac{2\left(\frac{2\sqrt{5}}{5}\right)}{1-\left(\frac{2\sqrt{5}}{5}\right)^2}$

$= \dfrac{\frac{4\sqrt{5}}{5}}{1-\frac{20}{25}}$

$= \dfrac{\frac{4\sqrt{5}}{5}}{\frac{1}{5}}$

$= 4\sqrt{5}$

d. cos 4θ $= \cos 2(\overbrace{2\theta}^{\theta})$

$= \cos^2(2\theta) - \sin^2(2\theta)$

$= \underbrace{\left(\frac{1}{9}\right)^2}_{\text{part (b) ans}} - \underbrace{\left(\frac{4\sqrt{5}}{9}\right)^2}_{\text{part (a) ans}}$

$= \frac{1}{81} - \frac{16\cdot 5}{9}$

$= \frac{1}{81} - \frac{80}{81}$

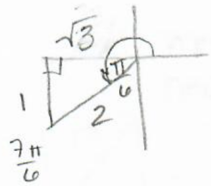
$= -\frac{79}{81}$

QII

S | A

$\frac{7\pi}{12}$

T | C



Example 2 Use half-angle identities to find the exact value of each function.

a. $\sin\frac{7\pi}{12} = \sin\frac{\frac{7\pi}{6}}{2}$

$$= \sqrt{\frac{1-\cos\frac{7\pi}{6}}{2}}$$

$$= \sqrt{\frac{1-\left(-\frac{\sqrt{3}}{2}\right)}{2}}\left(\frac{2}{2}\right)$$

$$= \frac{\sqrt{2+\sqrt{3}}}{2}$$

QI

b. $\cos 67.5° = \cos\frac{135}{2}$

$$= \sqrt{\frac{1+\cos 135}{2}}$$

$$= \sqrt{\frac{1-\frac{\sqrt{2}}{2}}{2}}\left(\frac{2}{2}\right)$$

$$= \frac{\sqrt{2-\sqrt{2}}}{2}$$

Example 3 Verify that $\frac{\cos 2\theta}{1+\sin 2\theta} = \frac{\cot\theta - 1}{\cot\theta + 1}$ is an identity.

$$\frac{\cos 2\theta}{1+\sin 2\theta} \overset{?}{=} \frac{\frac{\cos\theta}{\sin\theta} - 1\left(\frac{\sin\theta}{\sin\theta}\right)}{\frac{\cos\theta}{\sin\theta} + 1\left(\frac{\sin\theta}{\sin\theta}\right)}$$

$$\overset{?}{=} \frac{\frac{\cos\theta - \sin\theta}{\sin\theta}}{\frac{\cos\theta + \sin\theta}{\sin\theta}}$$

$$\overset{?}{=} \frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta}\left(\frac{\cos\theta + \sin\theta}{\cos\theta + \sin\theta}\right)$$

$$\overset{?}{=} \frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta + 2\cos\theta\sin\theta + \sin^2\theta}$$

$$\overset{?}{=} \frac{\cos 2\theta}{1 + 2\cos\theta\sin\theta}$$

$$= \frac{\cos 2\theta}{1+\sin 2\theta} \checkmark$$