

PRECALC - § 7-2 NOTES

PRECALCULUS NOTES

7.2 Verifying Trigonometric Identities

Objectives: Use the basic trigonometric identities to verify other identities.
Find numerical values of trigonometric functions.

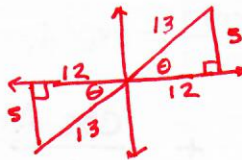
Warm-Up

1. If $\csc \theta = \frac{8}{5}$, find $\sin \theta$.

$$\frac{1}{\sin \theta} = \frac{8}{5}$$

$$\sin \theta = \frac{5}{8}$$

2. If $\cot \theta = \frac{12}{5}$, find $\sin \theta$



$$\tan \theta = \frac{5}{12}$$

$$\sin \theta = \pm \frac{5}{13}$$

3. Simplify $\cos^2 x \tan^2 x + \cos^2 x$

$$= \cos^2 x \left(\frac{\sin^2 x}{\cos^2 x} \right) + \cos^2 x$$

$$= \sin^2 x + \cos^2 x$$

$$= 1$$

If both sides are equally complicated you could transform both sides

Strategies for verifying trigonometric identities:

- Transform the more complicated side
- Substitute one or more basic trig identities to simplify expressions
- Factor or multiply to simplify expressions
- Multiply by an expression equal to one
- Express all trig functions in terms of sine or cosine
- NEVER cross over the equal sign (you are not assuming equality)

Example 1 Verify that $\sec^2 x - \tan x \cot x = \tan^2 x$ is an identity.

$$\begin{aligned}\sec^2 x - \tan x \left(\frac{1}{\tan x}\right) &\stackrel{?}{=} \tan^2 x \\ \sec^2 x - 1 &\stackrel{?}{=} \tan^2 x \\ (\tan^2 x + 1) - 1 &\stackrel{?}{=} \tan^2 x \\ \tan^2 x &= \tan^2 x\end{aligned}$$

Example 2 Verify that $\frac{\sin A}{\csc A} + \frac{\cos A}{\sec A} = \csc^2 A - \cot^2 A$ is an identity.

$$\begin{aligned}\frac{\sin A}{\frac{1}{\sin A}} + \frac{\cos A}{\frac{1}{\cos A}} &\stackrel{?}{=} \csc^2 A - \cot^2 A \\ \sin^2 A + \cos^2 A &\stackrel{?}{=} \frac{1}{\sin^2 A} - \frac{\cos^2 A}{\sin^2 A} \\ 1 &\stackrel{?}{=} \frac{1 - \cos^2 A}{\sin^2 A} \\ 1 &\stackrel{?}{=} \frac{\sin^2 A}{\sin^2 A} \\ 1 &= 1 \checkmark\end{aligned}$$

Example 3 Verify that $\frac{1 + \tan^2 x}{\csc x \sec x} = \tan x$ is an identity.

$$\begin{aligned}\frac{\sec^2 x}{\csc x \sec x} &\stackrel{?}{=} \tan x \\ \frac{\sec x}{\csc x} &\stackrel{?}{=} \tan x \\ \frac{\frac{1}{\cos x}}{\frac{1}{\sin x}} &\stackrel{?}{=} \tan x \\ \frac{\sin x}{\cos x} &\stackrel{?}{=} \tan x \\ \tan x &= \tan x \checkmark\end{aligned}$$

Example 4 Verify that $(\sec \theta - \tan \theta)(1 + \sin \theta) = \cos \theta$ is an identity.

$$\left(\frac{1}{\cos \theta} - \frac{\sin \theta}{\cos \theta}\right)(1 + \sin \theta) \stackrel{?}{=} \cos \theta$$

$$\left(\frac{1 - \sin \theta}{\cos \theta}\right)\left(\frac{1 + \sin \theta}{1}\right) \stackrel{?}{=} \cos \theta$$

$$\frac{1 - \sin^2 \theta}{\cos \theta} \stackrel{?}{=} \cos \theta$$

$$\frac{\cos^2 \theta}{\cos \theta} \stackrel{?}{=} \cos \theta$$

$$\cos \theta = \cos \theta \quad \checkmark$$

Example 5 Find a numerical value of one trigonometric function if $\frac{\cot x}{\cos x} = 2$

$$\frac{\cot x}{\cos x} = 2$$

$$\left(\frac{\cos x}{\sin x}\right)\left(\frac{1}{\cos x}\right) = 2$$

$$\frac{1}{\sin x} = 2$$

$$\csc x = 2$$

Example 6 Find a numerical value of one trigonometric function if $\frac{\cot x}{\csc x} = 0.6$

$$\frac{\cot x}{\csc x} = 0.6$$

$$\frac{\left(\frac{\cos x}{\sin x}\right)}{\frac{1}{\sin x}} = 0.6$$

$$\left(\frac{\cos x}{\sin x}\right)\left(\frac{\sin x}{1}\right) = 0.6$$

$$\cos x = 0.6$$