ALG I - §3-7 NOTES

Algebra I

3.7 Absolute Value Equations/ Inequalities

Objectives:	To solve absolute value equations To solve and graph absolute value inequalities

Warm Up Evaluate.

1. $|15|$ = 15
2. $|-3|$ = 3
3. $|12-18|$ = $|-6|$ = 6
4. $-|-7|$ = -7

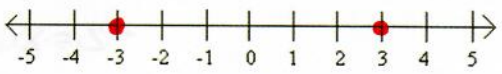
Complete each statement with <, =, or >. Use ORDER OF OPERATIONS.

5. $|3-7|$ = 4 → $|-4| = 4$, $4 = 4$
6. $|-5|+2$ > 6 → $5+2$ > 6, 7 > 6
7. $|7|-1$ < 8 → $7-1$ < 8, 6 < 8

Absolute Value: the distance a number is from 0.

An absolute value equation has a variable within the absolute value sign.
For example, $|x| = 3$. The answer for x can be 3 or -3.

$x = -3$ or $x = 3$



Example 1 Solve.

a. $|x| = 5$ → $x = -5$ or $x = 5$

b. $|x| = 12$ → $x = -12$ or $x = 12$

c. $|x| = -4$ → No soln.

PART 1: Steps for Solving Absolute Value *EQUATIONS*:

1. **ISOLATE** - isolate the absolute value
2. **SEPARATE** - separate into *TWO* equations
3. **SOLVE** - solve each equation

Example 2 Solve the absolute value equation, and check your answers.

$|x| + 5 = 11$

$-5 \quad -5$

$|x| = 6$ ISOLATE

$x = -6$ or $x = 6$ SEPARATE

SOLVE

Check:

$x = -6$

$|-6| + 5 = 11$

$6 + 5 = 11$

$11 = 11$ ✓

$x = 6$

$|6| + 5 = 11$

$6 + 5 = 11$

$11 = 11$ ✓

Example 3 Solve each absolute value equation, and check your answers mentally.

a. $3|x| = 15$

$\frac{3}{3} \quad \frac{3}{3}$

$|x| = 5$ ISOLATE

$x = -5$ or $x = 5$ SEPARATE

SOLVE

b. $4 = 3|x| - 2$

$+2 \quad +2$

$\frac{6}{3} = \frac{3|x|}{3}$ ISOLATE

$2 = |x|$ SEPARATE

$-2 = x$ or $2 = x$ SOLVE

Example 4 Solve the absolute value equation, and check your answers mentally.

$3|x + 2| - 1 = 8$

$+1 \quad +1$

$\frac{3|x+2|}{3} = \frac{9}{3}$ ISOLATE

$|x+2| = 3$ SEPARATE

$x + 2 = -3$ or $x + 2 = 3$

$-2 \quad -2 \qquad -2 \quad -2$

$x = -5 \qquad x = 1$ SOLVE

Example 5 Solve the absolute value *equation*, and check your answers mentally.

$3|4x - 1| - 5 = 10$

$+5 \quad +5$

$\frac{3|4x-1|}{3} = \frac{15}{3}$

$|4x - 1| = 5$

$4x - 1 = -5$ or $4x - 1 = 5$

$+1 \quad +1 \qquad +1 \quad +1$

$4x = -4 \qquad 4x = 6$

$x = -1 \qquad x = \frac{6}{4} = \frac{3}{2}$

ISOLATE

↓

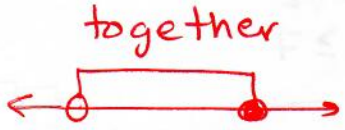
SEPARATE

↓

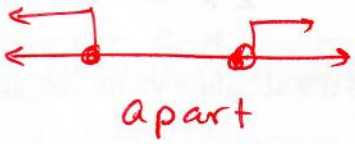
SOLVE

Part 2: Solving Absolute Value *INEQUALITIES*:

If $|x| < \#$ / $|x| \leq \#$ *then* → "and"



If $|x| > \#$ / $|x| \geq \#$ *then* → "or"



Basic Rules of Rewriting Absolute Value *INEQUALITIES*:

To solve an inequality in the form $|x| < c$

$x < c$ and $x > -c$

↑ FLIP & make opposite

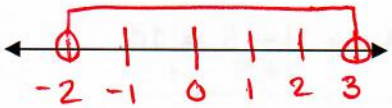
To solve an inequality in the form $|x| > c$

$x > c$ or $x < -c$

↑ FLIP & make opposite

Example 6 Solve the absolute value *inequality*, then graph your solution.

$|2x - 1| < 5$ "and" → together



$(-2, 3)$

$2x - 1 < 5$ and $2x - 1 > -5$

$2x < 6$ $\quad 2x > -4$

$x < 3$ $\quad x > -2$

Example 8 Solve the absolute value *inequality*, then graph your solution.

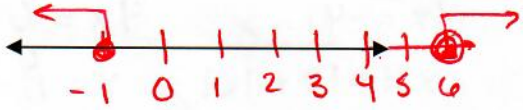
$2|2x - 5| + 1 \geq 15$ "or" → apart

$-1 \quad -1$

$2|2x-5| \geq 14$

$\frac{2}{2} \quad \frac{14}{2}$

$|2x-5| \geq 7$



$(-\infty, -1]$ or $[6, \infty)$

$2x - 5 \geq 7$ or $2x - 5 \leq -7$

$2x \geq 12$ $\quad 2x \leq -2$

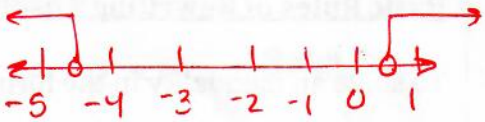
$x \geq 6$ $\quad x \leq -1$

Example 9 Solve the absolute value *inequality*, then graph your solution.

a. $6|2x + 4| > 30$ "or" → apart

$\frac{6}{6} \quad \frac{30}{6}$

$|2x+4| > 5$



$2x + 4 > 5$ or $2x + 4 < -5$

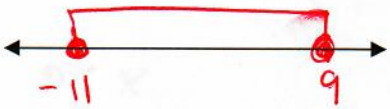
$2x > 1$ $\quad 2x < -9$

$x > \frac{1}{2}$ $\quad x < -\frac{9}{2}$

b. $|2x + 2| - 5 \leq 15$ "and" → together

$+5 \quad +5$

$|2x+2| \leq 20$



$2x + 2 \leq 20$ and $2x + 2 \geq -20$

$2x \leq 18$ $\quad 2x \geq -22$

$x \leq 9$ $\quad x \geq -11$