ALG I - §11-2 NOTES

Algebra I Notes

11.2 Multiplying and Dividing Rational Expressions

Objectives:	To multiply and divide rational expressions. To simplify complex fractions.

Warm-Up: Simplify each expression. State any excluded values.

1. $\frac{m+6}{m^2-m-42}$

$-7 \cdot +6 = -42$, $-7 + 6 = -1$

$= \frac{(m+6)}{(m-7)(m+6)}$

$= \frac{1}{(m-7)}$

2. $\frac{4-x^2}{x-2}$

$= \frac{-x^2+4}{x-2}$

$= \frac{-1(x^2-4)}{(x-2)}$

$= \frac{-1(x+2)(x-2)}{(x-2)}$

$= -1(x+2)$

$= -x-2$

RECALL: Multiplying fractions $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$ where b ≠ 0 and d ≠ 0.

Example 1 Multiply each rational expression. State any excluded values.

a. $\frac{6a^2}{a^2} \cdot \frac{-2}{a^3}$

$= \frac{-12a^2}{a^5}$

$= -\frac{12}{a^3}$; $a \neq 0$

b. $\frac{x+3}{x} \cdot \frac{x-5}{x+3}$

$= \frac{x-5}{x}$; $x \neq 0$

Example 2 Multiply each rational expression. State any excluded values.

a. $\frac{x+5}{7x-21} \cdot \frac{14x}{x^2+3x-10}$

$= \frac{(x+5)}{7(x-3)} \cdot \frac{2\,14x}{(x+5)(x-2)}$

$= \frac{2x}{(x-3)(x-2)}$

$x-3 \neq 0$; $x-2 \neq 0$

$x \neq 3$ $\quad x \neq 2$

b. $\frac{2m+5}{3m-6} \cdot (m^2+m-6)$

$= \frac{(2m+5)}{3(m-2)} \cdot (m+3)(m-2)$

$= \frac{(2m+5)(m+3)}{3}$

denominator is a constant
∴ no excluded values

RECALL: Dividing fractions $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$ where b ≠ 0, c ≠ 0, and d ≠ 0.

Example 3 Divide each rational expression. State any excluded values.

a. $\frac{x}{x+y} \div \frac{xy}{x+y}$

$= \frac{x}{(x+y)} \cdot \frac{(x+y)}{xy}$

$= \frac{1}{y}$; $y \neq 0$

b. $\frac{x^2-25}{4x+28} \div \frac{x-5}{x^2+9x+14}$

$= \frac{x^2-25}{4x+28} \cdot \frac{x^2+9x+14}{x-5}$

$= \frac{(x-5)(x+5)}{4(x+7)} \cdot \frac{(x+2)(x+7)}{(x-5)}$

$= \frac{(x+5)(x+2)}{4}$

PRACTICE: Multiply or divide each rational expression.

1. $\dfrac{\frac{3x^2+2x+1}{8x}}{\frac{12x^2+8x+4}{16x}}$

$= \dfrac{3x^2+2x+1}{8x} \div \dfrac{12x^2+8x+4}{16x}$

$= \dfrac{3x^2+2x+1}{8x} \cdot \dfrac{16x}{12x^2+8x+4}$

$= \dfrac{(3x^2+2x+1)}{8x} \cdot \dfrac{16x^2}{4(3x^2+2x+1)}$

$= \dfrac{2}{4}$

$= \dfrac{1}{2}$

2. $\dfrac{3x^2}{x+2} \cdot \dfrac{x^2+3x+2}{x}$

$= \dfrac{3x^{x^2}}{(x+2)} \cdot \dfrac{(x+2)(x+1)}{x}$

$= 3x(x+1)$

3. $\dfrac{x^2+2x+1}{x^2-1} \cdot (x^2+2x-3)$

$= \dfrac{(x+1)(x+1)}{(x+1)(x-1)}(x+3)(x-1)$

$= (x+1)(x+3)$

4. $\dfrac{3x^2-12x}{5x} \div (x^2-3x-4)$

$= \dfrac{3x^2-12x}{5x} \cdot \dfrac{1}{(x^2-3x-4)}$

$= \dfrac{3x(x-4)}{5x} \cdot \dfrac{1}{(x-4)(x+1)}$

$= \dfrac{3}{5(x+1)}$; $x+1 \neq 0$

$x \neq -1$