ALG I – §11-1 NOTES

Algebra I

11.1 Simplifying Rational Expressions

Objective: To simplify rational expressions.

Warm-Up: Simplify each radical expression.

1. $2\sqrt{24x^2}$

$= 2\sqrt{4 \cdot 6 \cdot x^2}$

$= 2(2)(x)\sqrt{6}$

$= 4x\sqrt{6}$

2. $3\sqrt{6y^3} \cdot 2\sqrt{2y}$

$= 6\sqrt{12y^4}$

$= 6\sqrt{4 \cdot 3 \cdot y^4}$

$= 6(2)(y^2)\sqrt{3}$

$= 12y^2\sqrt{3}$

3. $\sqrt{\frac{3}{5m}}$

$= \frac{\sqrt{3}}{\sqrt{5m}}\left(\frac{\sqrt{5m}}{\sqrt{5m}}\right)$

$= \frac{\sqrt{15m}}{5m}$

DEFINITION

An expression in the form $\frac{polynomial}{polynomial}$ is a rational expression.

Example 1 Simplify each rational expression. State any excluded values.

a. $\frac{21a^2}{7a^3}$ $\div 7a^2$ $\div 7a^2$

$= \frac{3}{a}$; $a \neq 0$

b. $\frac{18d^2}{4d+8}$

$= \frac{18d^2}{4(d+2)}$

$= \frac{9d^2}{2(d+2)}$

$d \neq -2$

c. $\frac{2n-3}{6n-9}$

$= \frac{2n-3}{3(2n-3)}$

$= \frac{1}{3}$

Example 2 Simplify $\frac{40\,(x+2)\,(x+3)}{5(x+2)\,(x+5)}$

$= \frac{8(x+3)}{(x+5)}$; $x \neq -5$

Example 3 Simplify each rational expression. State any excluded values.

a. $\frac{3x-6}{x^2+x-6}$

$= \frac{3(x-2)}{(x+3)(x-2)}$

$= \frac{3}{(x+3)}$; $x \neq -3$

b. $\frac{6c+12}{c^2+5c+6}$

$= \frac{6(c+2)}{(c+3)(c+2)}$

$= \frac{6}{(c+3)}$; $c \neq -3$

c. $\frac{a^2-3a+2}{3-3a^2}$

$= \frac{a^2-3a+2}{-3a^2+3}$

$= \frac{(a-2)(a-1)}{-3(a^2-1)}$

$= \frac{(a-2)(a-1)}{-3(a-1)(a+1)}$

$= \frac{(a-2)}{-3(a+1)}$; $a \neq -1$

d. $\frac{16+16m+3m^2}{m^2-3m-28}$

$= \frac{3m^2+16m+16}{m^2-3m-28}$

$= \frac{(3m+4)(m+4)}{(m-7)(m+4)}$

$= \frac{3m+4}{m-7}$; $m = 7$

Example 4 Simplify each rational expression. State any excluded values.

a. $\dfrac{2x^3-2x^2+5x-5}{2x^3+4x^2+5x+10}$

$= \dfrac{2x^2(x-1)+5(x-1)}{2x^2(x+2)+5(x+2)}$

$= \dfrac{(2x^2+5)(x-1)}{(2x^2+5)(x+2)}$

$= \dfrac{(x-1)}{(x+2)}$

Excluded values:

$x+2 \neq 0$

$x \neq -2$

b. $\dfrac{6x^3+3x^2-4x-2}{3x^3-9x^2-2x+6}$

$= \dfrac{3x^2(2x+1)-2(2x+1)}{3x^2(x-3)-2(x-3)}$

$= \dfrac{(3x^2-2)\cdot(2x+1)}{(3x^2-2)(x-3)}$

$= \dfrac{(2x+1)}{(x-3)}$

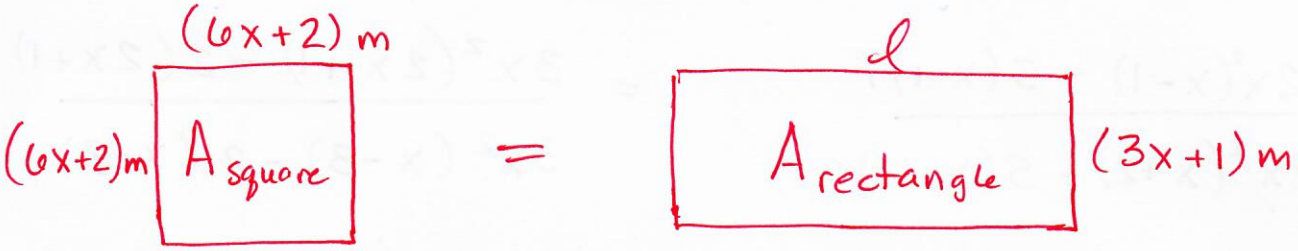
Excluded values:

$x-3 \neq 0$

$x \neq 3$

Example 5 **Application**

A square has side length (6x + 2) m. A rectangle with width (3x + 1) m has the same area as the square. What is the length of the rectangle?



Area of Square = Area of rectangle

$lw = lw$

$(6x+2)(6x+2) = l(3x+1)$

$2(3x+1)\cdot 2(3x+1) = l(3x+1)$

$\frac{4(3x+1)(3x+1)}{(3x+1)} = \frac{l(3x+1)}{(3x+1)}$

$4(3x+1)\ m = l$

OR

$(12x+4)\ m = l$