

PRECALCULUS NOTES

10.6 Rectangular and Parametric Forms of Conic Sections

Objectives: Recognize conic sections in their rectangular form by their equations.
Find a rectangular equation for a curve defined parametrically and vice versa.

General Equations for Conic Sections: The equation of a conic section can be written in the form:

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0, \text{ where } A, B \text{ and } C \text{ are not all } 0$$

conic	$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$
circle	$A = C$
parabola	Either A or C is zero.
ellipse	A and C have the same sign and $A \neq C$.
hyperbola	A and C have opposite signs.

Example 1 Identify each conic section.

- a. $6y^2 + 3x - 4y - 12 = 0$ $A=0; C=6$ PARABOLA
- b. $3y^2 - 2x^2 + 5y - x - 15 = 0$ $A=-2, C=3$ HYPERBOLA
- c. $9x^2 + 27y^2 - 6x - 108y + 82 = 0$ $A=9, C=27$ ELLIPSE
- d. $4x^2 + 4y^2 + 5x + 2y - 150 = 0$ $A=4, C=4$ CIRCLE

Example 2 Identify each conic section.

- a. $3x^2 - 2y^2 - 4 + 5y = -3$ $A=3, C=-3$ HYPERBOLA
- b. $8x^2 + y^2 + 3x - 7y = 0$ $A=8, C=1$ ELLIPSE
- c. $-4x^2 - 4y^2 + 6x + y = -5$ $A=-4, C=-4$ CIRCLE
- d. $y^2 - x + 3y - 9 = 0$ $A=0, C=1$ PARABOLA