PRECALC - §10-5 NOTES

PRECALCULUS NOTES
10.5 Parabolas

Objectives: **Use and determine the standard and general forms of the equation of a parabola. Graph parabolas.**

Warm-Up Consider the equation of the hyperbola: $\frac{(x+2)^2}{64} - \frac{(y-3)^2}{36} = 1$

$a^2 = 64 \quad b^2 = 36$
$a = 8 \quad b = 6$

a. State the direction and the equation of the transverse axis.

left/right

$y = 3$

$c^2 = a^2 + b^2$
$c^2 = 64 + 36$
$c^2 = 100$
$c = 10$

b. Determine the center. $(h, k) = (-2, 3)$

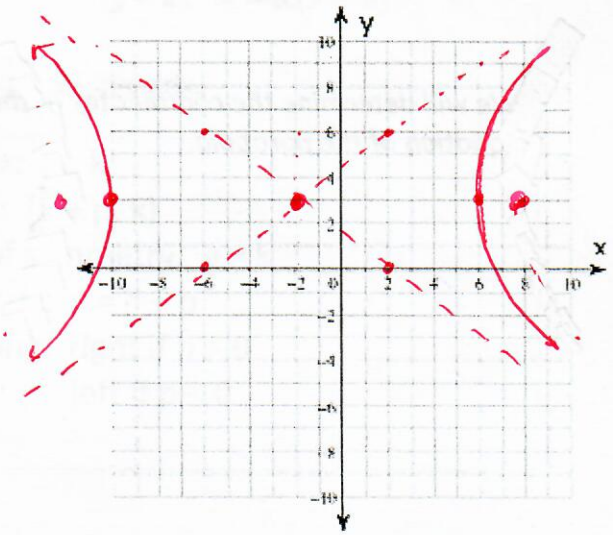
c. Determine the vertices. $(h \pm a, k) = (-2 \pm 8, 3) <$ $(6, 3)$, $(-10, 3)$

d. Determine the foci. $(h \pm c, k) = (-2 \pm 10, 3) <$ $(8, 3)$, $(-12, 3)$

e. Determine the equation of the asymptotes.

$y - k = \pm \frac{b}{a}(x - h)$
$y - 3 = \pm \frac{6}{8}(x + 2)$
$y - 3 = \pm \frac{3}{4}(x + 2)$

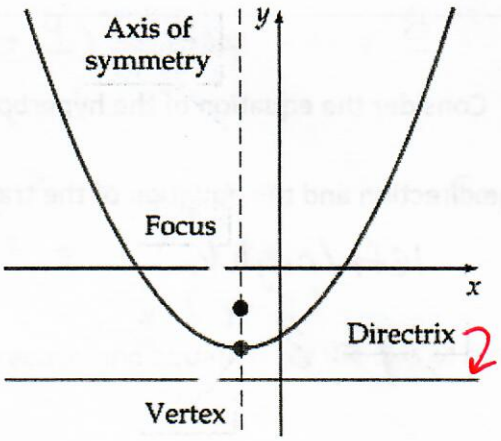
f. Sketch a graph of the hyperbola.



DEFINITION – PARABOLA

A parabola is the set of all points in a plane, that are the same distance from a given point called the focus, and a given line called the directrix.

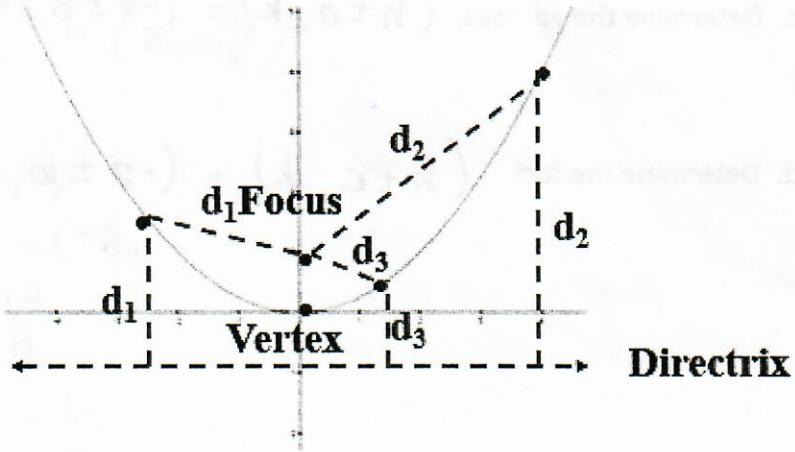
Notice that the axis of symmetry is the line through the focus, perpendicular to the directrix.



Take a closer look:

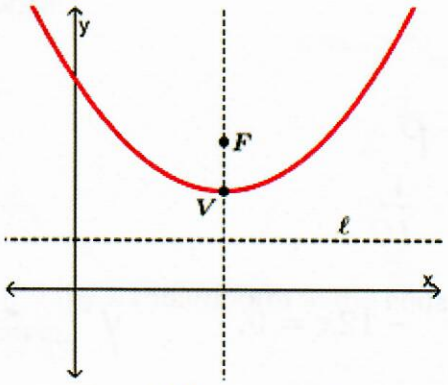
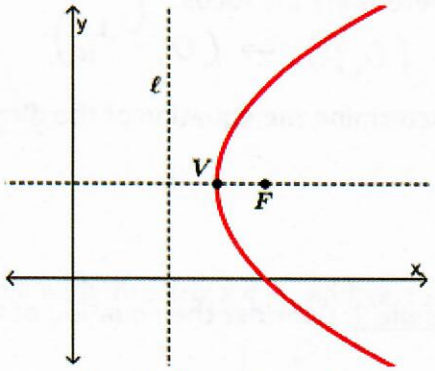
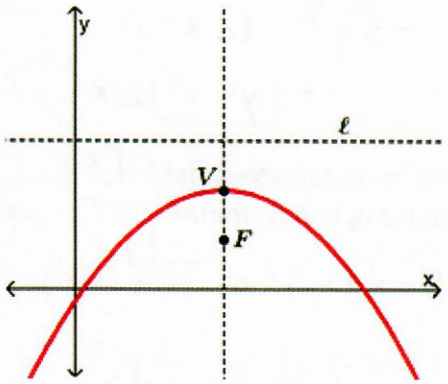
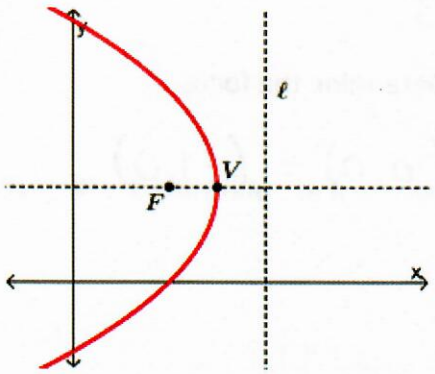
The vertex is located at the midpoint between the focus and the directrix.

The distance from the focus to any point on the parabola is equal to the distance from that point to the directrix.



We will determine the coordinates of the focus, and the equation of the directrix, given the equation of the parabola.

Standard Equations of Parabolas

Opens up/down	Opens left/right
Center at (0, 0): $x^2 = 4py$	Center at (0, 0): $y^2 = 4px$
Center at (h, k): $(x-h)^2 = 4p(y-k)$	Center at (h, k): $(y-k)^2 = 4p(x-h)$
 $(x-h)^2 = 4c(y-k)$	 $(y-k)^2 = 4c(x-h)$
 $(x-h)^2 = -4c(y-k)$	 $(y-k)^2 = -4c(x-h)$
Where l is the directrix.	Where l is the directrix.
vertex: (h, k) focus: (h, k + p) axis of symmetry: x = h directrix: y = k – p opening: up if p > 0 down if p < 0	vertex: (h, k) focus: (h + p, k) axis of symmetry: y = k directrix: x = h – p opening: right if p > 0 left if p < 0

Example 1 Consider the equation of the parabola $y = 4x^2$.

a. State the direction of opening.

up

$x^2 = 4py$

$x^2 = 4p(4x^2)$

$x^2 = 16px^2$

$1 = 16p$

$\frac{1}{16} = p$

b. Determine the focus.

$(0, p) \rightarrow (0, \frac{1}{16})$

c. Determine the equation of the directrix.

$y = -p$

$y = -\frac{1}{16}$

Example 2 Consider the equation of the parabola $-3y^2 - 12x = 0$.

y^2 ← →

a. State the direction of opening.

left or right

$y^2 = 4px$

$-3y^2 - 12x = 0$

$-3y^2 = 12x$

$y^2 = \underbrace{-4x}_{4px}$

$4px = -4x$

$4p = -4$

$p = -1$

b. Determine the focus.

$(p, 0) = (-1, 0)$

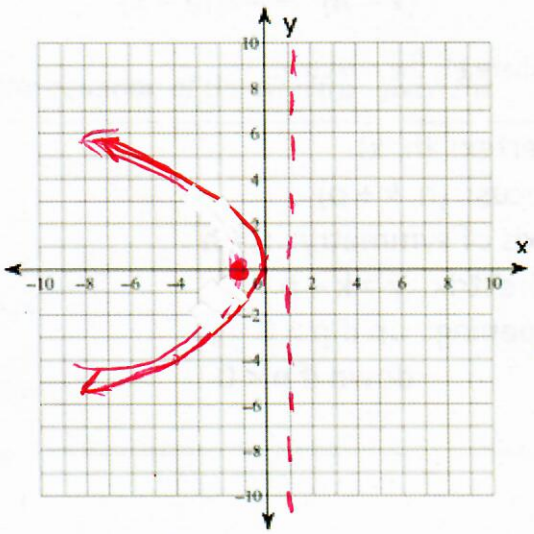
c. Determine the equation of the directrix.

$x = h - p$

$x = -p$

$x = 1$

d. Sketch a graph of the parabola.



General Form of the Equation of a Hyperbola: (standard form expanded):

$y^2 + Dx + Ey + F = 0$, when the directrix is parallel to the y-axis

or

$x^2 + Dx + Ey + F = 0$, when the directrix is parallel to the x-axis

Example 3

a. Write the standard form of the equation of the parabola with focus at (0, 3) and vertex at the origin.

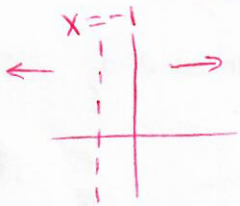
(0, p)

$x^2 = 4py$

$x^2 = 4(3)y$

$x^2 = 12y$

b. Write the standard form of the equation of the parabola with directrix x = -1, and vertex at the origin.



$y^2 = 4px$

$y^2 = 4(1)x$

$y^2 = 4x$

$x = h - p$

$x = 0 - 1$; so $p = 1$

Example 4 Consider the equation of the parabola, $y^2 = 8x + 48$. Determine the focus, vertex, directrix, axis of symmetry, and graph the parabola.

y^2

$(y-k)^2 = 4p(x-h)$

$y^2 = 8(x+6)$

$(y-0)^2 = 4(2)(x-(-6))$

vertex $(h,k) = (-6, 0)$

directrix $x = h - p$

$x = -6 - 2$

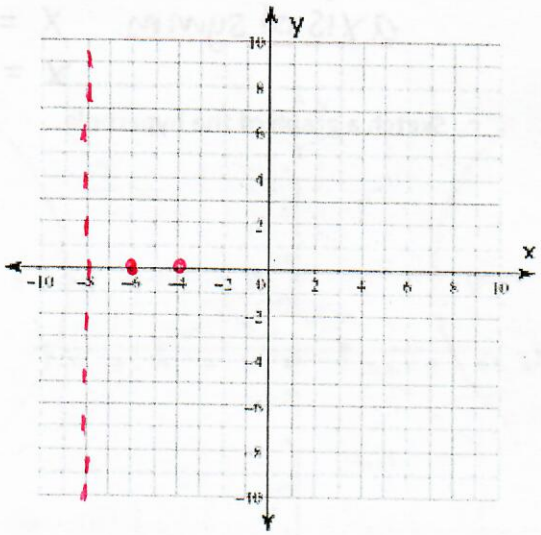
$x = -8$

focus $(h+p, k) = (-6+2, 0)$

$= (-4, 0)$

axis of symm $y = k$

$y = 0$



x^2 ↑ ↓

Example 5 Consider the equation of the parabola $2x^2 - 8x + y + 6 = 0$.

a. Write the equation in standard form. $(x-h)^2 = 4p(y-k)$

$2x^2 - 8x + \_ = -y - 6 + \_$

$2(x^2 - 4x + \underline{4}) = -y - 6 + \underline{8}$

$\frac{1}{2}(-4) \rightarrow (-2)^2$

$2(x-2)^2 = -y + 2$

$2(x-2)^2 = -(y-2)$

$(x-2)^2 = \underbrace{-\frac{1}{2}}_{4p}(y-2)$

So, $4p = -\frac{1}{2}$

$p = -\frac{1}{8}$

b. Determine the focus, vertex, equation of directrix, and equation for the axis of symmetry.

vertex $(h, k) = (2, 2)$

focus $(h, k+p) = (2, 2 - \frac{1}{8})$

$= (2, \frac{15}{8})$

directrix $y = k - p$

$y = 2 - (-\frac{1}{8})$

$y = \frac{17}{8}$

axis of symm $x = h$

$x = 2$

c. Sketch a graph of the hyperbola.