§10-3 Key

Algebra I

10.3 Operations with Radical Expressions

Objectives: To simplify sums and differences of radical expressions
To simplify products and quotients of radical expressions

Warm-Up: Simplify each expression.

1. $-3\sqrt{200x^2}$

$= -3\sqrt{100 \cdot 2 \cdot x^2}$

$= -3(10)(x)\sqrt{2}$

$= -30x\sqrt{2}$

2. $15\sqrt{12m} \cdot \frac{1}{3}\sqrt{8m^2}$

$= (15)(\frac{1}{3})\sqrt{96\,m^3}$

$= 5\sqrt{16 \cdot 6 \cdot m^2 \cdot m}$

$= 5(4)(m)\sqrt{6m}$

$= 20m\sqrt{6m}$

3. $\sqrt{\frac{5}{6a}}$

$= \frac{\sqrt{5}}{\sqrt{6a}}\left(\frac{\sqrt{6a}}{\sqrt{6a}}\right)$

$= \frac{\sqrt{30a}}{6a}$

Ex. 1 Simplify by combining like radicals.

a. $6\sqrt{11} + 9\sqrt{11}$

$= 15\sqrt{11}$

b. $1\sqrt{3} - 5\sqrt{3}$

$= -4\sqrt{3}$

c. $4\sqrt{5} - 2\sqrt{5}$

$= 2\sqrt{5}$

Ex. 2 Simplify each radical expression and combine like radicals.

a. $5\sqrt{3} - \sqrt{12}$

$= 5\sqrt{3} - \sqrt{4 \cdot 3}$

$= 5\sqrt{3} - 2\sqrt{3}$

$= 3\sqrt{3}$

b. $4\sqrt{7} + 2\sqrt{28}$

$= 4\sqrt{7} + 2\sqrt{4 \cdot 7}$

$= 4\sqrt{7} + (2)(2)\sqrt{7}$

$= 4\sqrt{7} + 4\sqrt{7}$

$= 8\sqrt{7}$

c. $-5\sqrt{32} - 4\sqrt{18}$

$= -5\sqrt{16 \cdot 2} - 4\sqrt{9 \cdot 2}$

$= -5(4)\sqrt{2} - 4(3)\sqrt{2}$

$= -20\sqrt{2} - 12\sqrt{2}$

$= -32\sqrt{2}$

Ex. 3 **Simplify each product.**

a. $\sqrt{10}\,(\sqrt{6}+3)$

$= \sqrt{60} + 3\sqrt{10}$

$= \sqrt{4 \cdot 15} + 3\sqrt{10}$

$= 2\sqrt{15} + 3\sqrt{10}$

b. $(\sqrt{6}-2\sqrt{3})(\sqrt{6}+\sqrt{3})$

$=$ F O I L

$6 + \sqrt{18} - 2\sqrt{18} - 2(3)$

$= 6 - \sqrt{18} - 6$

$= -\sqrt{9 \cdot 2}$

$= -3\sqrt{2}$

c. $(\sqrt{11}-2)^2$

$= (\sqrt{11}-2)(\sqrt{11}-2)$

$=$ F O I L

$11 - 2\sqrt{11} - 2\sqrt{11} + 4$

$= 11 - 4\sqrt{11} + 4$

$= 15 - 4\sqrt{11}$

RECALL: Simplify $\frac{2}{\sqrt{5}} \cdot \left(\frac{\sqrt{5}}{\sqrt{5}}\right)$

$= \frac{2\sqrt{5}}{\sqrt{25}}$

$= \frac{2\sqrt{5}}{5}$

Consider the binomials $\sqrt{3}+5$ *and* $\sqrt{3}-5$. These are called conjugates.

Ex. 4 **Simplify each quotient using conjugates.**

a. $\frac{3}{\sqrt{5}-2} \left(\frac{\sqrt{5}+2}{\sqrt{5}+2}\right)$

$= \frac{3\sqrt{5}+6}{5 + 2\sqrt{5} - 2\sqrt{5} - 4}$

$= \frac{3\sqrt{5}+6}{1}$

$= 3\sqrt{5} + 6$

b. $\frac{10}{\sqrt{3}+\sqrt{5}} \left(\frac{\sqrt{3}-\sqrt{5}}{\sqrt{3}-\sqrt{5}}\right)$

$= \frac{10\sqrt{3} - 10\sqrt{5}}{3 - \sqrt{15} + \sqrt{15} - 5}$

$= \frac{10\sqrt{3} - 10\sqrt{5}}{-2}$

$= -5\sqrt{3} + 5\sqrt{5}$

c. $\frac{\sqrt{2}}{3+\sqrt{7}} \left(\frac{3-\sqrt{7}}{3-\sqrt{7}}\right)$

$= \frac{3\sqrt{2} - \sqrt{14}}{9 - 3\sqrt{7} + 3\sqrt{7} - 7}$

$= \frac{3\sqrt{2} + \sqrt{14}}{2}$

d. $\frac{5}{\sqrt{2}-4} \left(\frac{\sqrt{2}+4}{\sqrt{2}+4}\right)$

$= \frac{5\sqrt{2} + 20}{2 + 4\sqrt{2} - 4\sqrt{2} - 16}$

$= \frac{5\sqrt{2} + 20}{-14}$

$= \frac{-5\sqrt{2} - 20}{14}$