

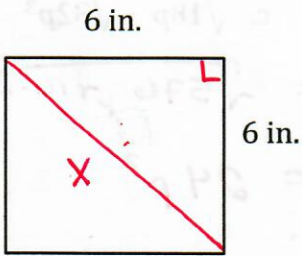
Algebra I

10.2 Simplifying Radicals

Objectives: To simplify radicals involving products and quotients.

Warm-Up:

Find the length diagonal of the below square. State the exact answer in reduced form.



$$\begin{aligned} \text{leg}^2 + \text{leg}^2 &= \text{hyp}^2 \\ 6^2 + 6^2 &= x^2 \\ 36 + 36 &= x^2 \\ \sqrt{72} &= \sqrt{x^2} \\ \sqrt{36 \cdot 2} &= x \\ 6\sqrt{2} &= x \end{aligned}$$

List the perfect squares from 1^2 to 12^2 .

1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144

Ex. 1 Simplify each radical.

a. $\sqrt{50} = \sqrt{25 \cdot 2}$
 $= \sqrt{25} \cdot \sqrt{2}$
 $= 5\sqrt{2}$

b. $\sqrt{\frac{49}{64}} = \frac{\sqrt{49}}{\sqrt{64}}$
 $= \frac{7}{8}$

c. $\sqrt{x^2y} = \sqrt{x^2} \cdot \sqrt{y}$
 $= x\sqrt{y}$

take note

Property Multiplication Property of Square Roots

Algebra

For $a \geq 0$ and $b \geq 0$, $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$.

Example

$$\sqrt{48} = \sqrt{16} \cdot \sqrt{3} = 4\sqrt{3}$$

Ex. 2 Simplify each radical expression.

$$\begin{aligned} \text{a. } 2\sqrt{24} &= 2\sqrt{4 \cdot 6} \\ &= 2\sqrt{4} \cdot \sqrt{6} \\ &= 2(2)\sqrt{6} \\ &= 4\sqrt{6} \end{aligned}$$

$$\begin{aligned} \text{b. } 3\sqrt{48x^3} &= 3\sqrt{16 \cdot 3 \cdot x^2 \cdot x} \\ &= 3(4)(x)\sqrt{3x} \\ &= 12x\sqrt{3x} \end{aligned}$$

$$\begin{aligned} \text{c. } -5m\sqrt{20m^2n} &= -5m\sqrt{4 \cdot 5m^2n} \\ &= -5m(2m)\sqrt{5n} \\ &= -10m^2\sqrt{5n} \end{aligned}$$

Ex. 3 Simplify each product.

$$\begin{aligned} \text{a. } \sqrt{6} \cdot \sqrt{8} &= \sqrt{48} \\ &= \sqrt{16 \cdot 3} \\ &= 4\sqrt{3} \end{aligned}$$

$$\begin{aligned} \text{b. } \frac{1}{2}\sqrt{12} \cdot \sqrt{15} &= \frac{1}{2}\sqrt{180} \\ &= \frac{1}{2}\sqrt{36 \cdot 5} \\ &= \frac{1}{2}(6)\sqrt{5} \\ &= 3\sqrt{5} \end{aligned}$$

$$\begin{aligned} \text{c. } \sqrt{18p} \cdot \sqrt{32p^3} &= \sqrt{576p^4} \\ &= 24p^2 \end{aligned}$$

Ex. 4 Simplify each product.

$$\begin{aligned} \text{a. } -3\sqrt{5m} \cdot 2\sqrt{8m^5} &= -6\sqrt{40m^6} \\ &= -6\sqrt{4 \cdot 10m^6} \\ &= -6(2)(m^3)\sqrt{10} \\ &= -12m^3\sqrt{10} \end{aligned}$$

$$\begin{aligned} \text{b. } 12\sqrt{14a^3} \cdot \frac{1}{4}\sqrt{21a^4} &= (12)\left(\frac{1}{4}\right)\sqrt{294a^7} \\ &= 3\sqrt{49 \cdot 6 \cdot a^6 \cdot a} \\ &= 3(7)(a^3)\sqrt{6a} \\ &= 21a^3\sqrt{6a} \end{aligned}$$

Ex. 5 Simplify each radical expression.

$$\begin{aligned} \text{a. } \sqrt{\frac{2}{3}} &= \frac{\sqrt{2}}{\sqrt{3}} \left(\frac{\sqrt{3}}{\sqrt{3}} \right) \\ &= \frac{\sqrt{6}}{3} \end{aligned}$$

$$\begin{aligned} \text{b. } \sqrt{\frac{4}{5n}} &= \frac{\sqrt{4}}{\sqrt{5n}} \\ &= \frac{2}{\sqrt{5n}} \left(\frac{\sqrt{5n}}{\sqrt{5n}} \right) \\ &= \frac{2\sqrt{5n}}{5n} \end{aligned}$$

$$\begin{aligned} \text{c. } 2\sqrt{\frac{1}{7a^3}} &= \frac{2\sqrt{1}}{\sqrt{7a^3}} = \frac{2\sqrt{7a}}{7a^2} \\ &= \frac{2}{\sqrt{7a^3}} \left(\frac{\sqrt{7a}}{\sqrt{7a}} \right) \\ &= \frac{2\sqrt{7a}}{7a^2} \end{aligned}$$

$$\begin{aligned} \text{d. } \frac{\sqrt{3}}{\sqrt{6m}} \left(\frac{\sqrt{6m}}{\sqrt{6m}} \right) &= \frac{\sqrt{18m}}{6m} \\ &= \frac{\sqrt{9 \cdot 2m}}{6m} \\ &= \frac{3\sqrt{2m}}{6m} = \frac{\sqrt{2m}}{2m} \end{aligned}$$

$$\begin{aligned} \text{e. } \frac{-8}{\sqrt{x^5}} \left(\frac{\sqrt{x}}{\sqrt{x}} \right) &= -\frac{8\sqrt{x}}{\sqrt{x^6}} \\ &= -\frac{8\sqrt{x}}{x^3} \end{aligned}$$

$$\begin{aligned} \text{f. } 3\sqrt{\frac{5}{32x}} &= \frac{3\sqrt{5}}{\sqrt{32x}} \\ &= \frac{3\sqrt{5}}{\sqrt{16 \cdot 2x}} \\ &= \frac{3\sqrt{5}}{4\sqrt{2x}} \left(\frac{\sqrt{2x}}{\sqrt{2x}} \right) \\ &= \frac{3\sqrt{10x}}{8x} \end{aligned}$$