

GEOMETRY - §10-6 NOTES

GEOMETRY NOTES

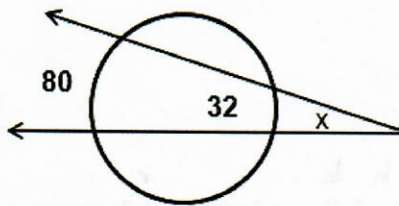
10.6 Equations of Circles

Objectives: Write the equation of a circle.

Use the equation of a circle and its graph to solve problems.

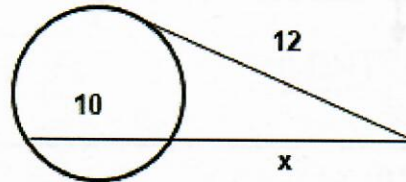
Warm-Up Write an algebraic equation to solve each.

1. Find $m\angle x$.



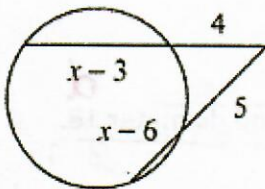
$$\begin{aligned} m\angle x &= \frac{1}{2}(80 - 32) \\ &= \frac{1}{2}(48) \\ &= 24 \end{aligned}$$

2. Find the length of x .



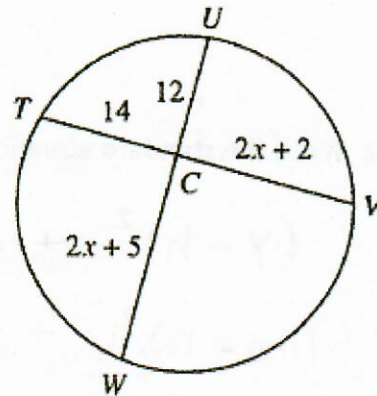
$$\begin{aligned} x(x+10) &= 12^2 \\ x^2 + 10x &= 144 \\ x^2 + 10x - 144 &= 0 \\ (x+18)(x-8) &= 0 \\ x+18=0 &\text{ or } x-8=0 \\ \cancel{x=-18} &\quad x=8 \end{aligned}$$

3. Find the value of x .



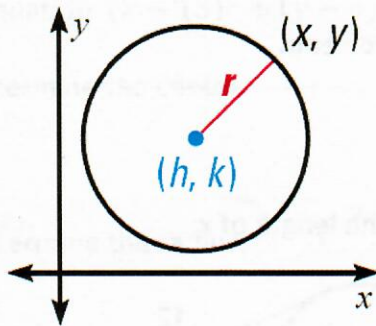
$$\begin{aligned} 4(4+x-3) &= 5(5+x-6) \\ 4(x+1) &= 5(x-1) \\ 4x+4 &= 5x-5 \\ 4 &= x-5 \\ 9 &= x \end{aligned}$$

4. Find the value of x .



$$\begin{aligned} 14(2x+2) &= 12(2x+5) \\ 28x+28 &= 24x+60 \\ 4x+28 &= 60 \\ 4x &= 32 \\ x &= 8 \end{aligned}$$

STANDARD EQUATION OF A CIRCLE



Center: (h, k)

Radius: r

EQUATION:

$$(x - h)^2 + (y - k)^2 = r^2$$

Example 1 Write the standard equation of a circle with center $(-4, 0)$ and radius 7.1 .

$$(x - h)^2 + (y - k)^2 = r^2$$

$$(x - (-4))^2 + (y - 0)^2 = (7.1)^2$$

$$(x + 4)^2 + (y - 0)^2 = 50.41$$

Example 2 Write the standard equation of a circle with center $(0, -5)$ and diameter 18.

$$(x - h)^2 + (y - k)^2 = r^2$$

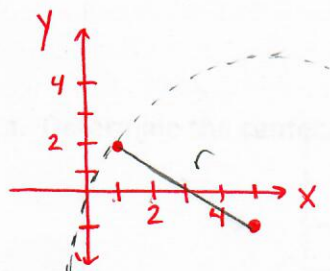
$$(x - 0)^2 + (y - (-5))^2 = (9)^2$$

$$(x - 0)^2 + (y + 5)^2 = 81$$

$$d = 18$$

$$r = 9$$

Example 3 The point $(1, 2)$ is on a circle whose center is $(5, -1)$. Write the standard equation of the circle.



$$r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$r = \sqrt{(5 - 1)^2 + (-1 - 2)^2}$$

$$r = \sqrt{4^2 + (-3)^2}$$

$$r = \sqrt{16 + 9}$$

$$r = \sqrt{25}$$

$$r = 5$$

CENTER: $(5, -1)$

RADIUS: $r = 5$

EQUATION:

$$(x - h)^2 + (y - k)^2 = r^2$$

$$(x - 5)^2 + (y - (-1))^2 = 5^2$$

$$(x - 5)^2 + (y + 1)^2 = 25$$

Example 4 Write the standard equation of the circle whose center is $(7, 0)$, and has a point on the circle located at $(27, 0)$.

$$r = \sqrt{(27 - 7)^2 + (0 - 0)^2}$$

$$r = \sqrt{20^2 + 0^2}$$

$$r = \sqrt{400}$$

$$r = 20$$

Center $(7, 0)$

Radius $r = 20$

Equation $(x - h)^2 + (y - k)^2 = r^2$

$$(x - 7)^2 + (y - 0)^2 = 20^2$$

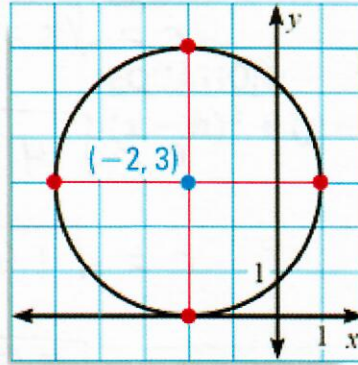
$$(x - 7)^2 + (y - 0)^2 = 400$$

GRAPHING A CIRCLE

Consider the equation of the circle: $(x + 2)^2 + (y - 3)^2 = 9$

Center: $(-2, 3)$

Radius: $r^2 = 9$
 $r = 3$



Example 5 Consider the equation $(x - 3)^2 + y^2 = 16$.

$$(x-3)^2 + (y-0)^2 = 4^2$$

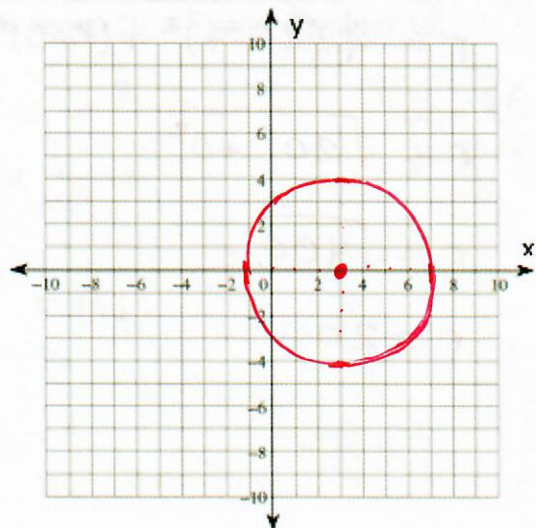
a. Determine the center.

$$\left(\begin{matrix} h \\ 3, \end{matrix} \begin{matrix} k \\ 0 \end{matrix} \right)$$

b. Determine the radius

$$r^2 = 16$$
$$r = 4$$

c. Graph the circle.



Example 6 Consider the equation $x^2 + (y - 5)^2 = 25$

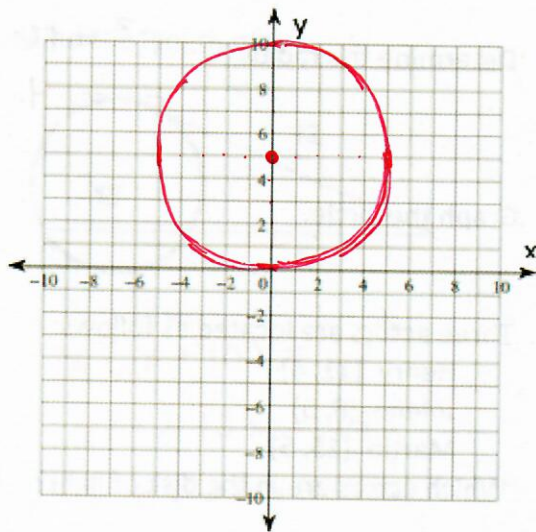
$$(x-0)^2 + (y-5)^2 = 5^2$$

a. Determine the center. $(h, k) = (0, 5)$

b. Determine the radius. $r^2 = 25$

$$r = 5$$

c. Graph the circle.



Example 7 Consider the equation $(x + 4)^2 + (y + 3)^2 = 9$.

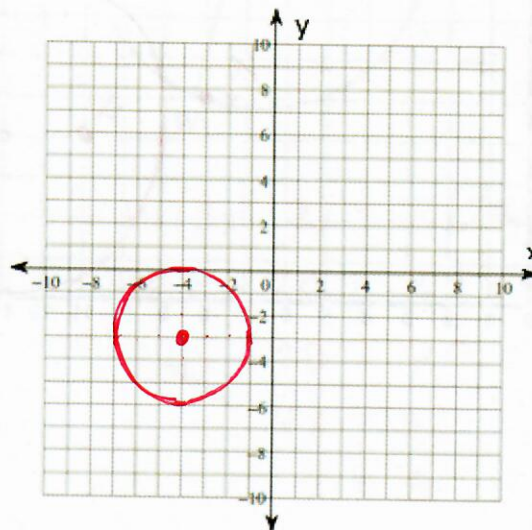
$$(x - (-4))^2 + (y - (-3))^2 = 3^2$$

a. Determine the center. $(h, k) = (-4, -3)$

b. Determine the radius. $r^2 = 9$

$$r = 3$$

c. Graph the circle.



APPLICATION Theatre Lighting

A bank of lights is arranged over a stage. Each light illuminates a circular region on the stage. A coordinate plane is used to arrange the lights, using the corner of the stage as the origin. The equation $(x - 13)^2 + (y - 4)^2 = 16$ represents one of the disks of light.

a. Determine the center. $(h, k) = (13, 4)$

b. Determine the radius. $r^2 = 16$
 $r = 4$

c. Graph the circle.

d. Three actors are located as follows:

Henry (11, 4)

Jolene (8, 5)

Martin (15, 5)

Which actors are in the disk of light? Henry and Martin are in the disk of light.

