GEOMETRY - §10-5 NOTES

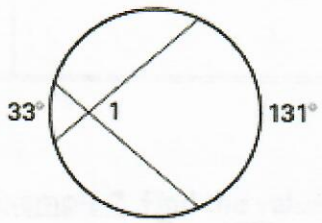
GEOMETRY NOTES

10.5 Segment Lengths in Circles

Objectives: **Find the lengths of segments of chords.**
Find the lengths of segments of tangents and secants.

Warm-Up **Write an agebraic equation to solve each.**

1. Find $m\angle 1$.

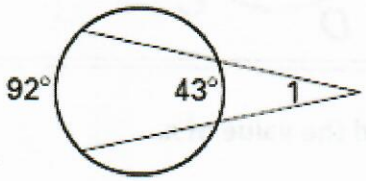


$m\angle 1 = \frac{1}{2}(33+131)$

$= \frac{1}{2}(164)$

$= 82°$

2. Find $m\angle 1$.

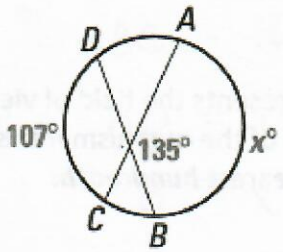


$m\angle 1 = \frac{1}{2}(92-43)$

$= \frac{1}{2}(49)$

$= 24.5°$

3. Find the value of x.

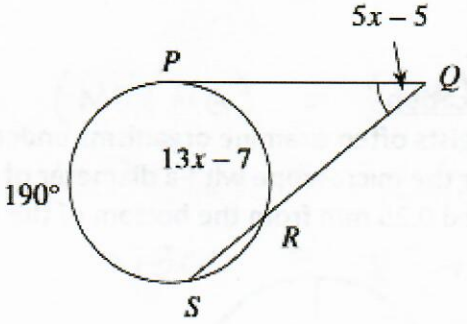


$2(135) = 2\left(\frac{1}{2}(107+x)\right)$

$270 = 107 + x$

$163 = x$

4. Find the value of x.



$5x-5 = \frac{1}{2}(190-(13x-7))$

$5x-5 = \frac{1}{2}(190-13x+7)$

$2(5x-5) = 2\left(\frac{1}{2}(197-13x)\right)$

$10x-10 = 197-13x$

$23x-10 = 197$

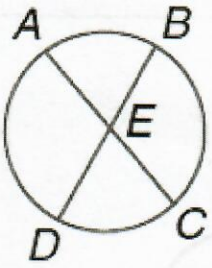
$23x = 207$

$x = 9$

Theorem 10.15

IF two chords intersect ***IN*** a circle,

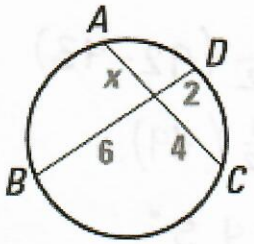
THEN the products of the measure of the segments of the chords are equal.



Example: $AE \cdot EC = BE \cdot ED$

Example 1 **Find the value of x.**

a.

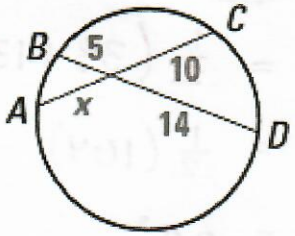


$x(4) = (6)(2)$

$4x = 12$

$x = 3$

b.



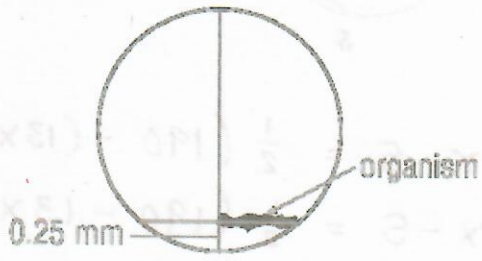
$(x)(10) = (5)(14)$

$10x = 70$

$x = 7$

Application

Biologists often examine organisms under microscopes. The circle represents the field of view under the microscope with a diameter of 2 mm. Determine the length of the organism if it is located 0.25 mm from the bottom of the field of view. Round to the nearest ***hundredth***.



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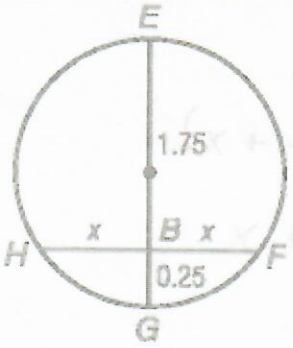


$HB \cdot BF = EB \cdot BG$

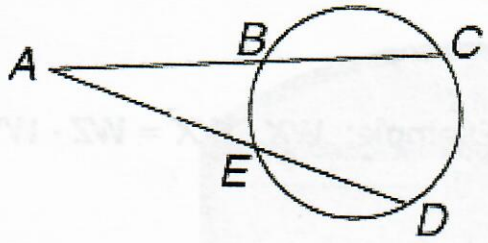
$(x)(x) = (1.75)(0.25)$

$\sqrt{x^2} = \sqrt{0.4375}$

$x \approx 0.66$ mm

Theorem 10.16

IF two secant segments are drawn to a circle from an exterior point, **THEN** the product of the measures of one secant segment and its external secant segment is equal to the product of the measures of the other secant segment and its external secant segment.

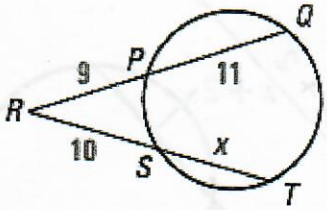


Example: $AB \cdot AC = AE \cdot AD$

(outer piece)(whole piece) = (outer piece)(whole piece)

Example 2 Find the value of x.

a.



$(RP)(RQ) = (RS)(RT)$

$(9)(20) = (10)(10+x)$

$180 = 100 + 10x$

$80 = 10x$

$8 = x$

b.

$(MO)(MG) = (MK)(MI)$

$(25)(52) = x(x+24)$

$1300 = x^2 + 24x$

$0 = x^2 + 24x - 1300$

$0 = (x-26)(x+50)$

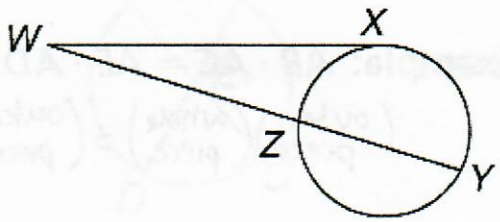
$x - 26 = 0$ or $x + 50 = 0$

$x = 26$ $x = -50$

Theorem 10.17

IF a tangent segment and a secant segment are drawn to a circle from an exterior point,

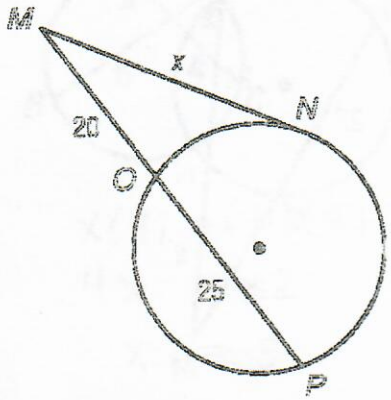
THEN the square of the measure of the tangent segment is equal to the product of the measures of the secant segment and its external secant segment.



Example: $WX \cdot WX = WZ \cdot WY$

Example 3 **Find the value of x.**

a.



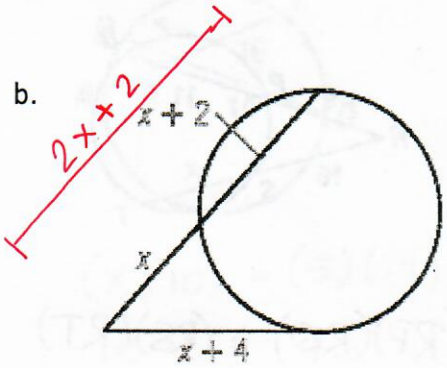
$(x)(x) = (20)(20+25)$

$(x^2) = (20)(45)$

$x^2 = 900$

$x = 30$

b.



$x(2x+2) = (x+4)^2$

$2x^2 + 2x = (x+4)(x+4)$

$2x^2 + 2x = x^2 + 4x + 4x + 16$

$2x^2 + 2x = x^2 + 8x + 16$

$x^2 - 6x - 16 = 0$

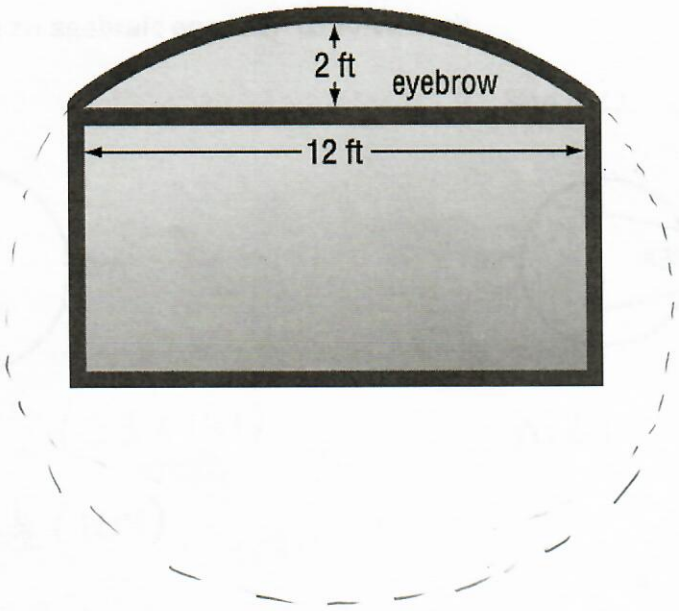
$(x-8)(x+2) = 0$

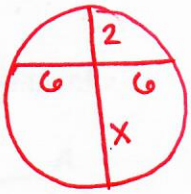
$x - 8 = 0$ or $x + 2 = 0$

$x = 8$ $x = -2$

Application

Phil is installing a new window in an addition for a client's home. The window is a rectangle with an arched top called an eyebrow. The diagram below shows the dimensions of the window. What is the radius of the circle containing the arc if the eyebrow portion of the window is not a semicircle?





$(6)(6) = 2x$

$36 = 2x$

$18 = x$

Diameter $= 2 + x$

$= 2 + 18$

$= 20$ ft

$\therefore$ radius $= \frac{1}{2}(20)$

$= 10$ ft