Geom - §10-4 Notes

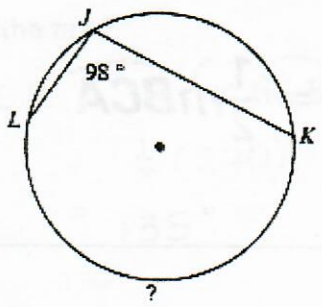
GEOMETRY NOTES

10.4 Other Angle Relationships in Circles

Objectives: Use angles formed by tangents and chords.
Use angles formed by lines that intersect a circle.

Warm-Up Find the measure of the arc or angle indicated.

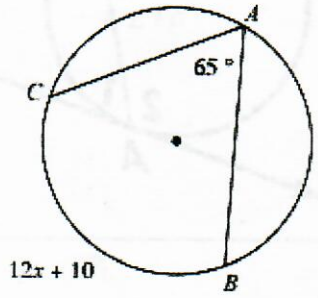
1.



$x = 2(98)$

$x = 196$

2.



$2(65) = 12x + 10$

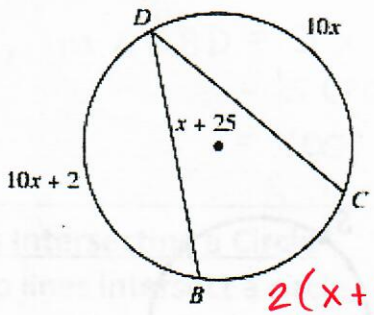
$130 = 12x + 10$

$120 = 12x$

$10 = x$

$m\overset{\frown}{BC} = 12(10) + 10 = 120 + 10 = 130$

3.



$2(x + 25)$

$(10x + 2) + 10x + (2x + 50) = 360$

$22x + 52 = 360$

$22x = 308$

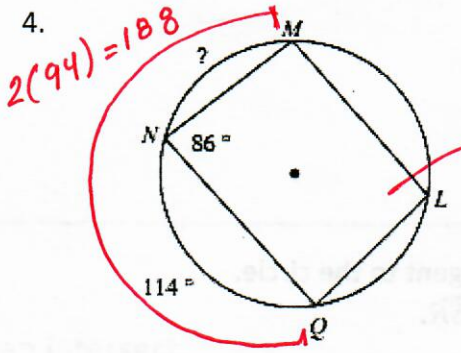
$x = 14$

$\angle BDC = x + 25$

$= 14 + 25$

$= 39$

4.



$2(94) = 188$

$180 - 86 = 94$

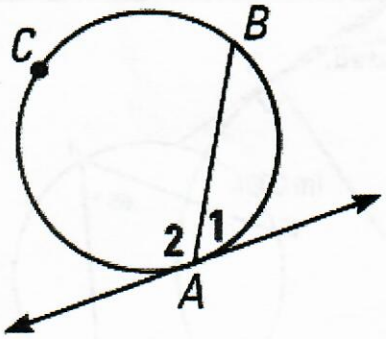
$x + 114 = 2(94)$

$x + 114 = 188$

$x = 74$

Theorem 10.12

IF a tangent and a chord intersect at a point on a circle,
THEN the measure of each angle formed is one half the measure of its intercepted arc.

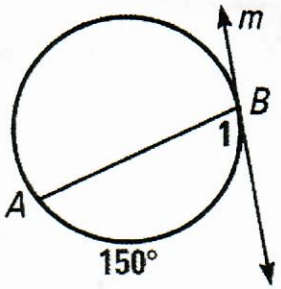


$$m\angle 1 = \frac{1}{2}m\overset{\frown}{AB}$$

$$m\angle 2 = \frac{1}{2}m\overset{\frown}{BCA}$$

Example 1

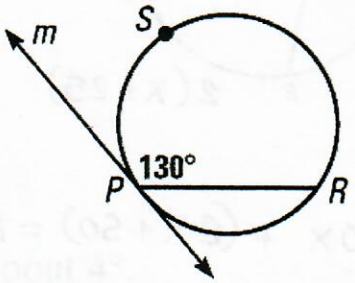
Line m is tangent to the circle.
Find the $m\angle 1$.



$m\angle 1 = \frac{1}{2}(150)$
$= 75°$

Example 2

Line m is tangent to the circle.
Find the $m\overset{\frown}{PSR}$.



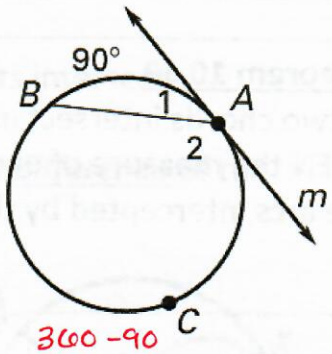
$m\overset{\frown}{PSR} = 2(130)$
$= 260°$

Example 3

Line m is tangent to the circle.

a. Find the $m\angle 1$.

b. Find the $m\overset{\frown}{ACB}$.



$m\angle 1 = \frac{1}{2}(90)$

$= 45°$

$m\overset{\frown}{ACB} = 360 - 90$

$= 270°$

$360 - 90$

$= 270$

c. Find the $m\angle 2$

$m\angle 2 = \frac{1}{2}(m\overset{\frown}{ACB})$

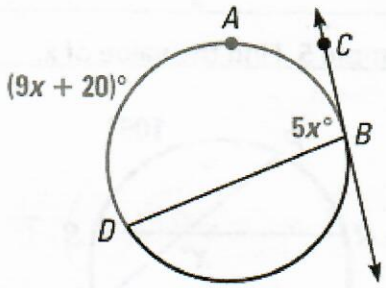
$= \frac{1}{2}(270)$

$= 135°$

Example 4

$\overleftrightarrow{BC}$ is tangent to the circle.

Find the $m\angle CBD$.



$2(5x) = 9x + 20$

$10x = 9x + 20$

$x = 20$

So, $m\angle CBD = 5x$

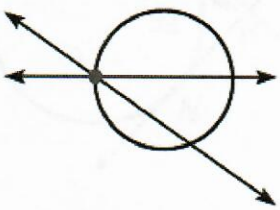
$= 5(20)$

$= 100°$

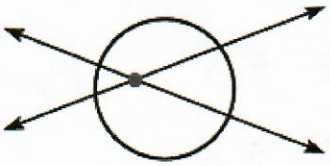
Lines Intersecting a Circle

If two lines intersect a circle,
there are three places where the lines can intersect.

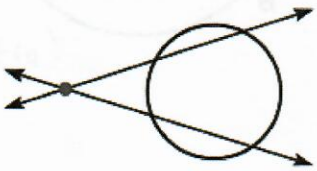
On the circle



Inside the circle



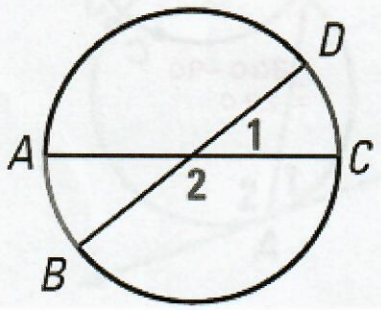
Outside the circle



Theorem 10.13

IF two chords intersect in the ***interior*** of a circle

THEN the measure of each angle is one half the ***sum*** of the measures of the arcs intercepted by the angle and its vertical angle.

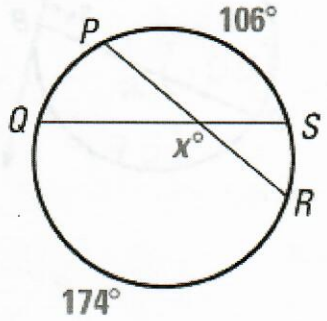


$$m\angle 1 = \frac{1}{2}(m\overset{\frown}{CD} + m\overset{\frown}{AB})$$

$$m\angle 2 = \frac{1}{2}(m\overset{\frown}{BC} + m\overset{\frown}{AD})$$

Example 5 Find the value of x.

a.



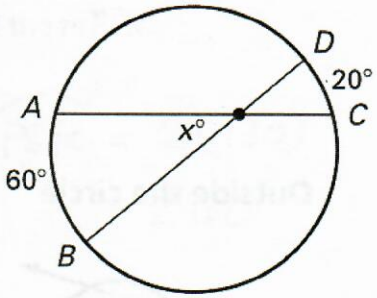
$m\angle X = \frac{1}{2}(m\overset{\frown}{PS} + m\overset{\frown}{QR})$

$m\angle X = \frac{1}{2}(106 + 174)$

$= \frac{1}{2}(280)$

$= 140^\circ$

b.



$m\angle X = \frac{1}{2}(m\overset{\frown}{CD} + m\overset{\frown}{AB})$

$m\angle X = \frac{1}{2}(20 + 60)$

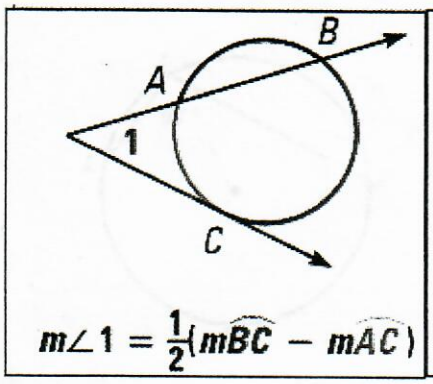
$= \frac{1}{2}(80)$

$= 40^\circ$

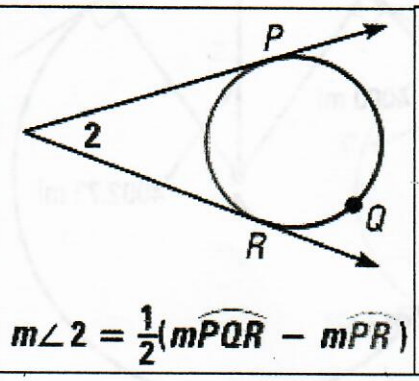
Theorem 10.14

IF a tangent and a secant, two tangents, or two secants intersect in the ***exterior*** of a circle,

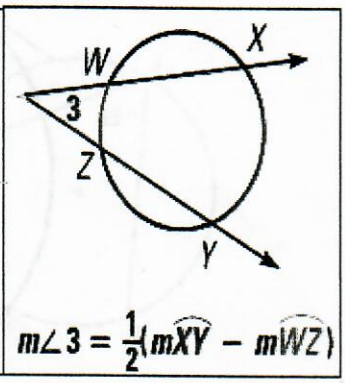
THEN the measure of the angle formed is one half the ***difference*** of the measures of the intercepted arcs.



$m\angle 1 = \frac{1}{2}(m\overset{\frown}{BC} - m\overset{\frown}{AC})$

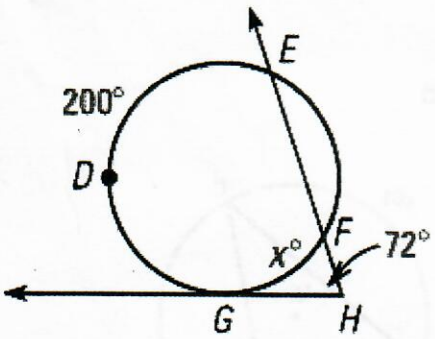


$m\angle 2 = \frac{1}{2}(m\overset{\frown}{PQR} - m\overset{\frown}{PR})$



$m\angle 3 = \frac{1}{2}(m\overset{\frown}{XY} - m\overset{\frown}{WZ})$

Example 6 Find the value of x.



$m\angle EHG = \frac{1}{2}(m\overset{\frown}{EDG} - m\overset{\frown}{FG})$

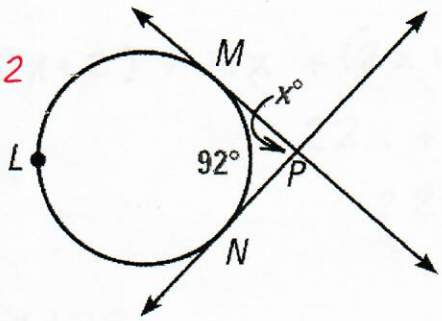
$72 = \frac{1}{2}(200 - x)$

$144 = 200 - x$

$-56 = -x$

$56^\circ = x$

Example 7 Find the value of x.



$360 - 92 = 268$

$x = \frac{1}{2}(m\overset{\frown}{MLN} - m\overset{\frown}{MN})$

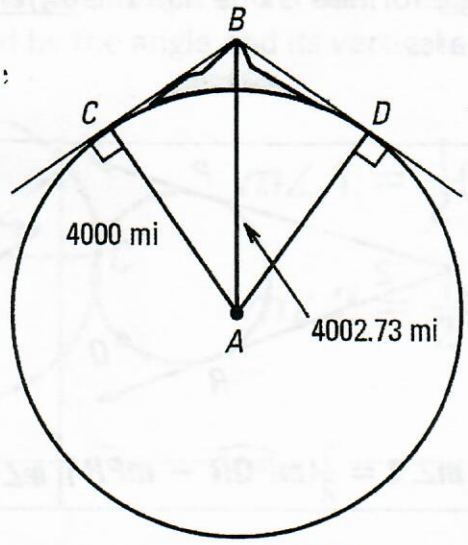
$x = \frac{1}{2}(268 - 92)$

$x = \frac{1}{2}(176)$

$x = 88^\circ$

APPLICATION **Describing the View from Mount Rainier**

You are on top of Mount Rainier on a clear day. You are about 2.73 miles above sea level. Find the measure of the arc $\overset{\frown}{CD}$ that represents the part of the earth that you can see.



Not drawn to scale

SOLUTION

$\overrightarrow{BC}$ and $\overrightarrow{BD}$ are tangent to Earth. You can solve right $\triangle BCA$ to see that $m\angle CBA \approx 87.9°$. So, $m\angle CBD \approx 175.8°$. Let $m\overset{\frown}{CD} = x°$.

$$175.8 \approx \frac{1}{2}[(360 - x) - x]$$

$$175.8 \approx \frac{1}{2}(360 - 2x)$$

$$175.8 \approx 180 - x$$

$$x \approx 4.2$$

From the peak, you can see an arc of about 4°.