

GEOM - §10-3 Notes

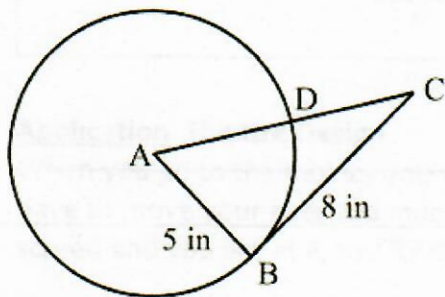
GEOMETRY NOTES

10.3 Inscribed Angles

Objectives: Use properties inscribed angles.
Use properties of inscribed polygons.

Warm-Up

1. If BC is tangent to circle A, find AC.



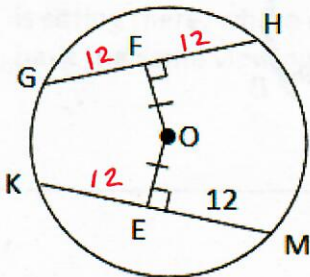
$$AC^2 = 5^2 + 8^2$$

$$AC^2 = 25 + 64$$

$$AC^2 = 89$$

$$AC = \sqrt{89} \text{ in}$$

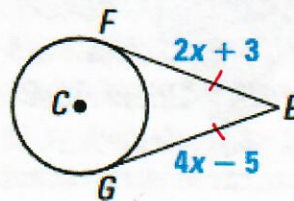
2. Find GH



$$GH = 2(12)$$

$$= 24$$

3. \overline{EF} and \overline{EG} are tangent to circle C. Find the value of x.



$$2x + 3 = 4x - 5$$

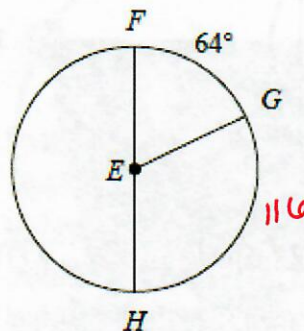
$$3 = 2x - 5$$

$$8 = 2x$$

$$4 = x$$

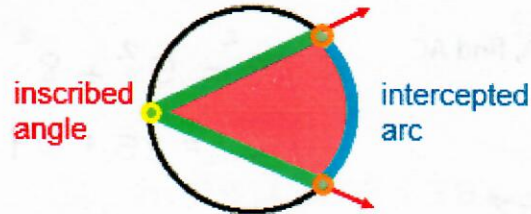
4. Consider circle E with diameter FH. Find:

- a. $m\angle FEG = 64^\circ$
 b. $m\angle GEH = 180 - 64 = 116$
 c. $m\widehat{FH} = 180^\circ$
 d. $m\widehat{GHF} = 180 + 116$
 $= 296$



Definitions Inscribed Angle

An **inscribed angle** is an angle whose vertex is on a circle and whose sides contain chords of the circle. The arcs that lie in the interior of an inscribed angle and has endpoints on the angle is called the **intercepted arc** of the angle.

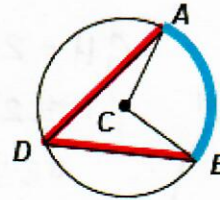


THEOREM 10.8 Measure of an Inscribed Angle

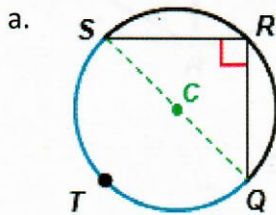
IF an angle is inscribed in a circle,
THEN its measure is half the measure of its intercepted arc.

$$m\angle ADB = \frac{1}{2}m\widehat{AB}$$

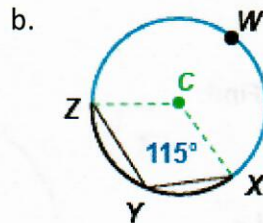
$$m\widehat{AB} = 2m\angle ADB$$



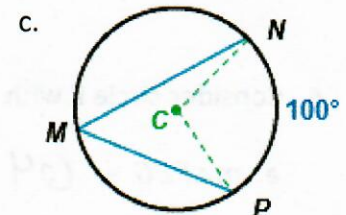
Example 1 Find the measure of the intercepted arc or inscribed angle.



$$m\widehat{STQ} = 2(90) \\ = 180^\circ$$



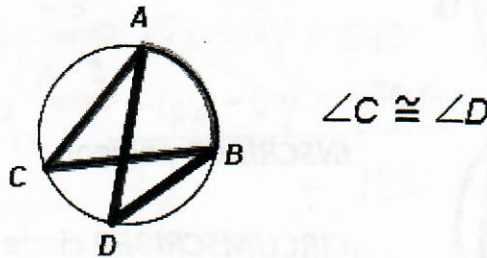
$$m\widehat{XWZ} = 2(115) \\ = 230^\circ$$



$$m\angle NMP = \frac{1}{2}(100) \\ = 50^\circ$$

THEOREM 10.9

IF two inscribed angles of a circle intercept the same arc,
THEN the angles are congruent.

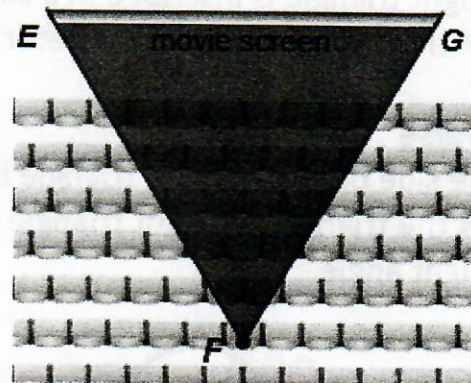


Application Theatre Design

When you go to the movies you want to be close to the movie screen, but you don't want to have to move your eyes too much to see the edges of the picture. If E and G are the ends of the screen and you are at F , $m\angle EFG$ is called your viewing angle.

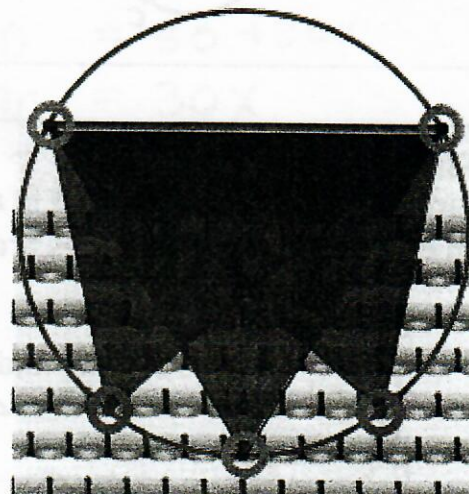
Question:

You decide that that middle of the sixth row has the best viewing angle. If someone is sitting there, where else can you sit to have the same viewing angle?



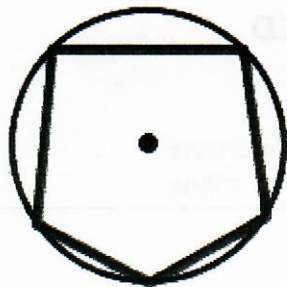
Answer:

Draw the circle that is determined by the endpoints of the screen and the sixth row center seat. Any other location on the circle will have the same viewing angle.



Definitions

IF all of the vertices of a polygon lie on a circle,
THEN the polygon is **inscribed** in the circle and the circle is **circumscribed** about the polygon.



INSCRIBED polygon

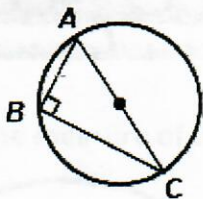
CIRCUMSCRIBED circle

Theorem 10.10

IF a right triangle is inscribed in a circle,
THEN the hypotenuse is a diameter of the circle.

Converse

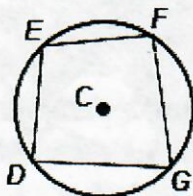
IF one side of an inscribed triangle is a diameter of a circle,
THEN the triangle is a right triangle and the angle opposite the diameter is the right angle.



$\angle B$ is a right angle if and only if \overline{AC} is a diameter of the circle.

Theorem 10.11

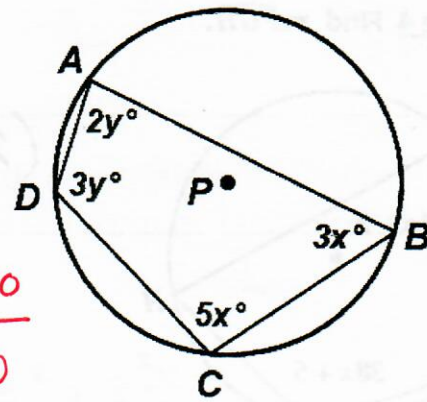
A quadrilateral can be **inscribed** in a circle
if and only if its opposite angles are supplementary.



$D, E, F,$ and G lie on some circle, $\odot C$, if and only if $m\angle D + m\angle F = 180^\circ$ and $m\angle E + m\angle G = 180^\circ$.

Example 2

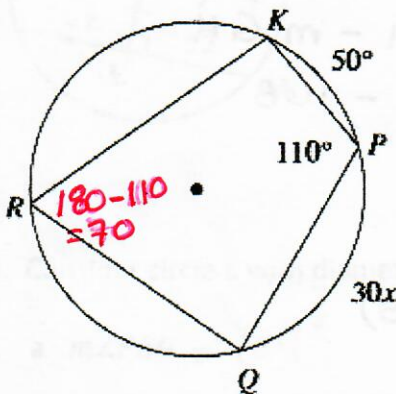
In the diagram, $ABCD$ is inscribed in $\odot P$.
Find the measure of each angle.



$$\begin{aligned} 5x + 2y &= 180 & \times 3 & \Rightarrow 15x + 6y = 540 \\ 3x + 3y &= 180 & \times -2 & \Rightarrow -6x - 6y = -360 \\ \hline 9x &= 180 \\ x &= 20 \end{aligned}$$

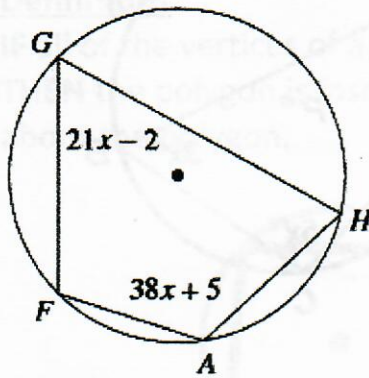
$$\begin{aligned} \text{So, } 3x + 3y &= 180 \\ 3(20) + 3y &= 180 \\ 60 + 3y &= 180 \\ 3y &= 120 \\ y &= 40 \end{aligned}$$

Example 3 Solve for x .



$$\begin{aligned} 2(m\angle KRQ) &= m\widehat{KPQ} \\ 2(70) &= 50 + 30x \\ 140 &= 50 + 30x \\ 90 &= 30x \\ 3 &= x \end{aligned}$$

Example 4 Find $m\widehat{FGH}$.



$$(21x - 2) + (38x + 5) = 180$$

$$59x + 3 = 180$$

$$59x = 177$$

$$x = 3$$

$$\text{So, } m\widehat{FGH} = 2(38x + 5)$$

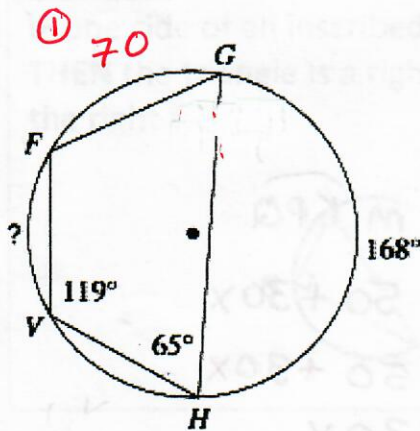
$$= 76x + 10$$

$$= 76(3) + 10$$

$$= 228 + 10$$

$$= 238^\circ$$

Example 5 Find $m\widehat{FV}$.



$$\textcircled{1} \quad m\widehat{FGH} = 2(119)$$

$$= 238$$

$$m\widehat{FG} = m\widehat{FGH} - m\widehat{GH}$$

$$= 238 - 168$$

$$= 70$$

$$\textcircled{2} \quad m\widehat{GFV} = 2(65)$$

$$= 130$$

$$m\widehat{FV} = 130 - 70$$

$$= 60$$