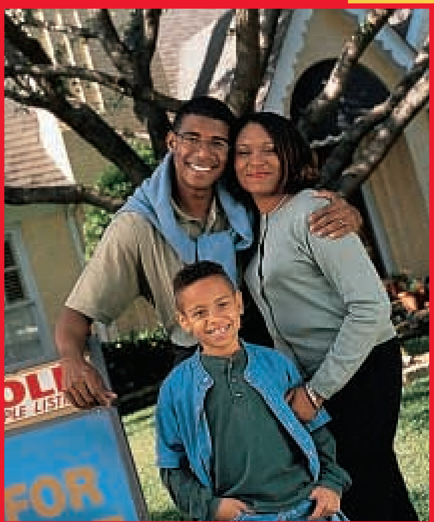
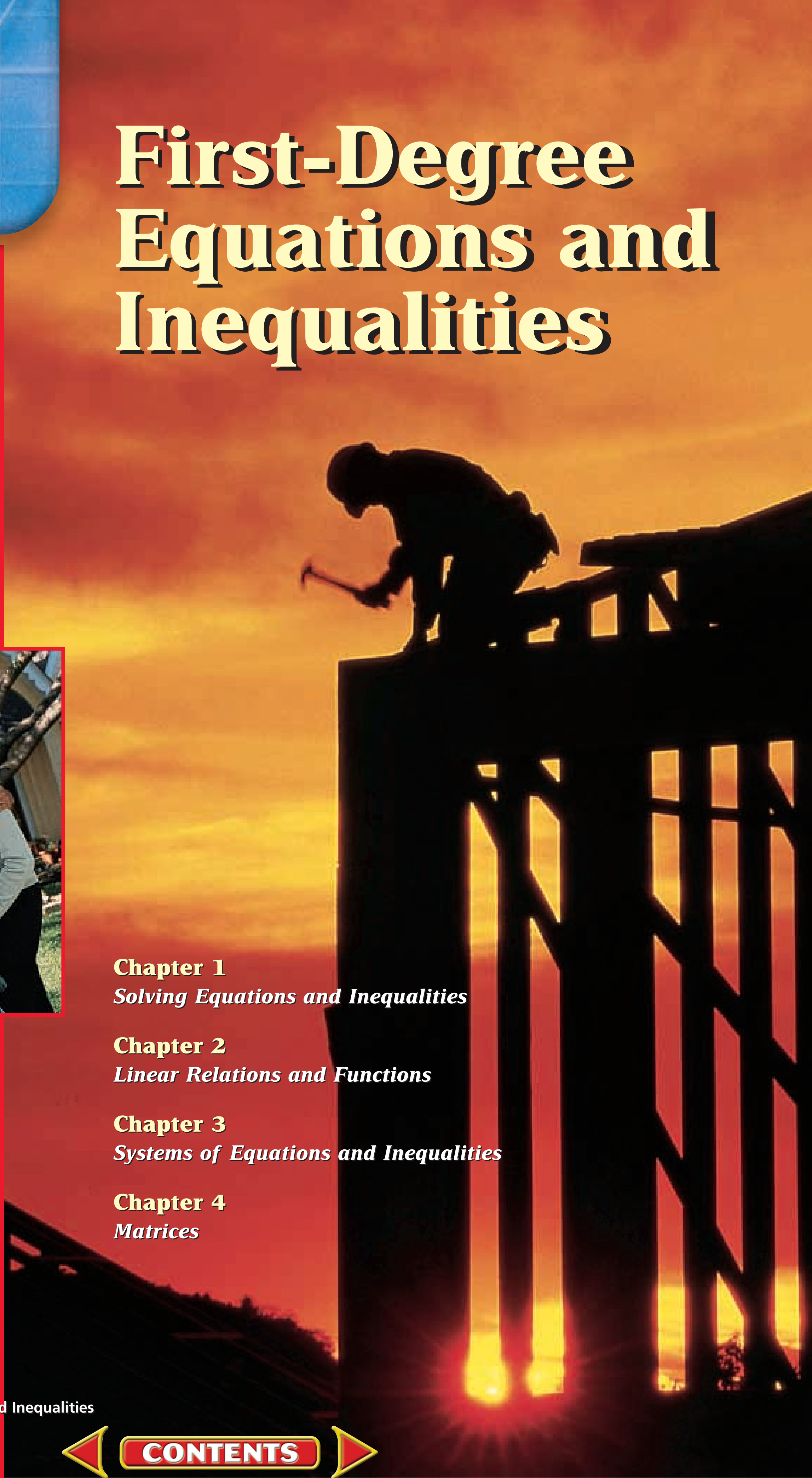
UNIT 1

First-Degree Equations and Inequalities

You can model and analyze real-world situations by using algebra. In this unit, you will solve and graph linear equations and inequalities and use matrices.





Chapter 1
Solving Equations and Inequalities

Chapter 2
Linear Relations and Functions

Chapter 3
Systems of Equations and Inequalities

Chapter 4
Matrices

CONTENTS

WebQuest Internet Project

Lessons in Home Buying, Selling

Source: *USA TODAY,* November 18, 1999

"'Buying a home,' says Housing and Urban Development Secretary Andrew Cuomo, 'is the most expensive, most complicated and most intimidating financial transaction most Americans ever make.'" In this project, you will be exploring how functions and equations relate to buying a home and your income.

Log on to www.algebra2.com/webquest. Begin your WebQuest by reading the Task.

Then continue working on your WebQuest as you study Unit 1.

Lesson	1-3	2-5	3-2	4-6
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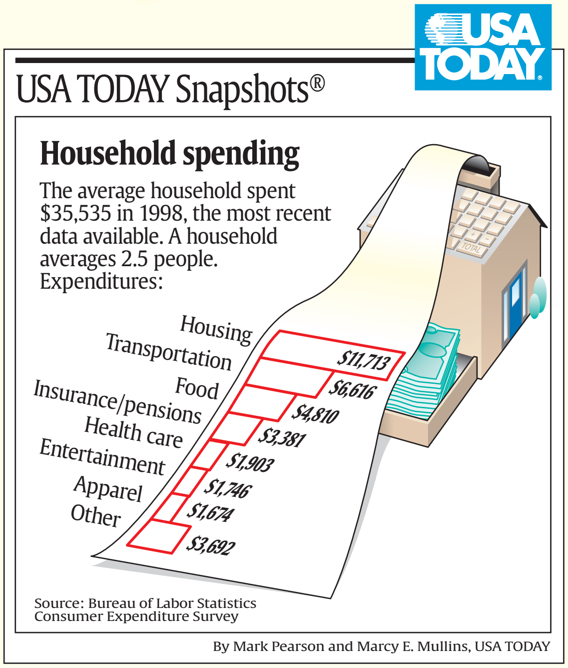


Chapter 1 Solving Equations and Inequalities

What You'll Learn

- **Lesson 1-1** Simplify and evaluate algebraic expressions.
- **Lesson 1-2** Classify and use the properties of real numbers.
- **Lesson 1-3** Solve equations.
- **Lesson 1-4** Solve absolute value equations.
- **Lessons 1-5 and 1-6** Solve and graph inequalities.

Key Vocabulary

- order of operations (p. 6)
- algebraic expression (p. 7)
- Distributive Property (p. 12)
- equation (p. 20)
- absolute value (p. 28)

Why It's Important

Algebra allows you to write expressions, equations, and inequalities that hold true for most or all values of variables. Because of this, algebra is an important tool for describing relationships among quantities in the real world. For example, the angle at which you view fireworks and the time it takes you to hear the sound are related to the width of the fireworks burst. A change in one of the quantities will cause one or both of the other quantities to change.

In Lesson 1-1, you will use the formula that relates these quantities.

Getting Started

Prerequisite Skills To be successful in this chapter, you'll need to master these skills and be able to apply them in problem-solving situations. Review these skills before beginning Chapter 1.

For Lessons 1-1 through 1-3 — Operations with Rational Numbers

Simplify.

1. $20 - 0.16$
2. $12.2 + (-8.45)$
3. $-3.01 - 14.5$
4. $-1.8 + 17$
5. $\frac{1}{4} - \frac{2}{3}$
6. $\frac{3}{5} + (-6)$
7. $-7\frac{1}{2} + 5\frac{1}{3}$
8. $-11\frac{5}{8} - \left(-4\frac{3}{7}\right)$
9. $(0.15)(3.2)$
10. $2 \div (-0.4)$
11. $(-1.21) \div (-1.1)$
12. $(-9)(0.036)$
13. $-4 \div \frac{3}{2}$
14. $\left(\frac{5}{4}\right)\left(-\frac{3}{10}\right)$
15. $\left(-2\frac{3}{4}\right)\left(-3\frac{1}{5}\right)$
16. $7\frac{1}{8} \div (-2)$

For Lesson 1-1 — Powers

Evaluate each power.

17. 2^3
18. 5^3
19. $(-7)^2$
20. $(-1)^3$
21. $(-0.8)^2$
22. $-(1.2)^2$
23. $\left(\frac{2}{3}\right)^2$
24. $\left(-\frac{4}{11}\right)^2$

For Lesson 1-5 — Compare Real Numbers

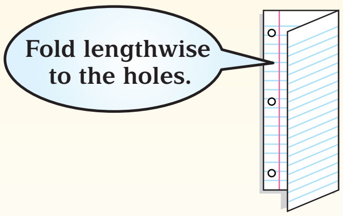
Identify each statement as *true* or *false*.

25. $-5 < -7$
26. $6 > -8$
27. $-2 \geq -2$
28. $-3 \geq -3.01$
29. $-9.02 < -9.2$
30. $\frac{1}{5} < \frac{1}{8}$
31. $\frac{2}{5} \geq \frac{16}{40}$
32. $\frac{3}{4} > 0.8$

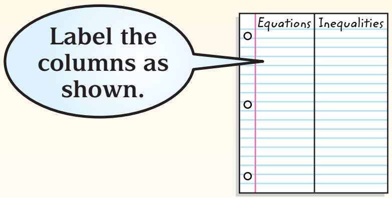
FOLDABLES™ Study Organizer

Make this Foldable to help you organize information about relations and functions. Begin with one sheet of notebook paper.

Step 1 Fold



Step 2 Cut and Label



Reading and Writing As you read and study the chapter, write notes, examples, and graphs in each column.

1-1 Expressions and Formulas

What You'll Learn

- Use the order of operations to evaluate expressions.
- Use formulas.

Vocabulary

- order of operations
- variable
- algebraic expression
- formula

How are formulas used by nurses?

Intravenous or IV fluid must be given at a specific rate, neither too fast nor too slow. A nurse setting up an IV must control the flow rate F, in drops per minute. They use the formula $F = \frac{V \times d}{t}$, where V is the volume of the solution in milliliters, d is the drop factor in drops per milliliter, and t is the time in minutes. Suppose a doctor orders 1500 milliliters of IV saline to be given over 12 hours, or 12×60 minutes. Using a drop factor of 15 drops per milliliter, the expression $\frac{1500 \times 15}{12 \times 60}$ gives the correct flow rate for this patient's IV.

ORDER OF OPERATIONS A numerical expression such as $\frac{1500 \times 15}{12 \times 60}$ must have exactly one value. In order to find that value, you must follow the **order of operations**.

Key Concept — *Order of Operations*

Step 1 Evaluate expressions inside grouping symbols, such as parentheses, ( ), brackets, [ ], braces, { }, and fraction bars, as in $\frac{5+7}{2}$.

Step 2 Evaluate all powers.

Step 3 Do all multiplications and/or divisions from left to right.

Step 4 Do all additions and/or subtractions from left to right.

Grouping symbols can be used to change or clarify the order of operations. When calculating the value of an expression, begin with the innermost set of grouping symbols.

Example 1 *Simplify an Expression*

Find the value of $[2(10 - 4)^2 + 3] \div 5$.

$[2(10 - 4)^2 + 3] \div 5 = [2(6)^2 + 3] \div 5$ First subtract 4 from 10.

$= [2(36) + 3] \div 5$ Then square 6.

$= (72 + 3) \div 5$ Multiply 36 by 2.

$= 75 \div 5$ Add 72 and 3.

$= 15$ Finally, divide 75 by 5.

The value is 15.

Scientific calculators follow the order of operations.

Graphing Calculator Investigation

Order of Operations

Think and Discuss

1. Simplify $8 - 2 \times 4 + 5$ using a graphing calculator.
2. Describe the procedure the calculator used to get the answer.
3. Where should parentheses be inserted in $8 - 2 \times 4 + 5$ so that the expression has each of the following values?
   a. -10 **b.** 29 **c.** -5
4. Evaluate $18^2 \div (2 \times 3)$ using your calculator. Explain how the answer was calculated.
5. If you remove the parentheses in Exercise 4, would the solution remain the same? Explain.

Variables are symbols, usually letters, used to represent unknown quantities. Expressions that contain at least one variable are called **algebraic expressions**. You can evaluate an algebraic expression by replacing each variable with a number and then applying the order of operations.

Study Tip

Common Misconception
A common error in this type of problem is to subtract before multiplying.

$64 - 1.5(9.5) \neq 62.5(9.5)$

Remember to follow the order of operations.

Example 2 Evaluate an Expression

Evaluate $x^2 - y(x + y)$ if $x = 8$ and $y = 1.5$.

$x^2 - y(x + y) = 8^2 - 1.5(8 + 1.5)$ Replace x with 8 and y with 1.5.

$= 64 - 1.5(8 + 1.5)$ Find 8^2.

$= 64 - 1.5(9.5)$ Add 8 and 1.5.

$= 64 - 14.25$ Multiply 1.5 and 9.5.

$= 49.75$ Subtract 14.25 from 64.

The value is 49.75.

Example 3 Expression Containing a Fraction Bar

Evaluate $\frac{a^3 + 2bc}{c^2 - 5}$ if $a = 2$, $b = -4$, and $c = -3$.

The fraction bar acts as both an operation symbol, indicating division, and as a grouping symbol. Evaluate the expressions in the numerator and denominator separately before dividing.

$\frac{a^3 + 2bc}{c^2 - 5} = \frac{2^3 + 2(-4)(-3)}{(-3)^2 - 5}$ $a = 2$, $b = -4$, and $c = -3$

$= \frac{8 + (-8)(-3)}{9 - 5}$ Evaluate the numerator and the denominator separately.

$= \frac{8 + 24}{9 - 5}$ Multiply -8 by -3.

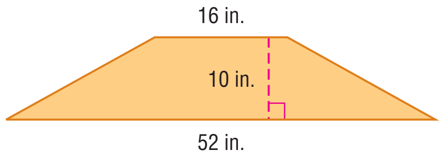
$= \frac{32}{4}$ or 8 Simplify the numerator and the denominator. Then divide.

The value is 8.

FORMULAS A **formula** is a mathematical sentence that expresses the relationship between certain quantities. If you know the value of every variable in the formula except one, you can find the value of the remaining variable.

Example 4 Use a Formula

GEOMETRY The formula for the area A of a trapezoid is $A = \frac{1}{2}h(b_1 + b_2)$, where h represents the height, and b_1 and b_2 represent the measures of the bases. Find the area of the trapezoid shown below.



Substitute each value given into the formula. Then evaluate the expression using the order of operations.

$A = \frac{1}{2}h(b_1 + b_2)$	Area of a trapezoid
$= \frac{1}{2}(10)(16 + 52)$	Replace h with 10, b_1 with 16, and b_2 with 52.
$= \frac{1}{2}(10)(68)$	Add 16 and 52.
$= 5(68)$	Divide 10 by 2.
$= 340$	Multiply 5 by 68.

The area of the trapezoid is 340 square inches.

Check for Understanding

Concept Check

1. **Describe** how you would evaluate the expression $a + b[(c + d) \div e]$ given values for a, b, c, d, and e.
2. **OPEN ENDED** Give an example of an expression where subtraction is performed before division and the symbols ( ), [ ], or { } are not used.
3. **Determine** which expression below represents the amount of change someone would receive from a \$50 bill if they purchased 2 children's tickets at \$4.25 each and 3 adult tickets at \$7 each at a movie theater. Explain.

   a. $50 - 2 \times 4.25 + 3 \times 7$
   b. $50 - (2 \times 4.25 + 3 \times 7)$
   c. $(50 - 2 \times 4.25) + 3 \times 7$
   d. $50 - (2 \times 4.25) + (3 \times 7)$

Guided Practice

Find the value of each expression.

4. $8(3 + 6)$
5. $10 - 8 \div 2$
6. $14 \cdot 2 - 5$
7. $[9 + 3(5 - 7)] \div 3$
8. $[6 - (12 - 8)^2] \div 5$
9. $\frac{17(2 + 26)}{4}$

Evaluate each expression if $x = 4$, $y = -2$, and $z = 6$.

10. $z - x + y$
11. $x + (y - 1)^3$
12. $x + [3(y + z) - y]$

Application

BANKING **For Exercises 13–15, use the following information.**
Simple interest is calculated using the formula $I = prt$, where p represents the principal in dollars, r represents the annual interest rate, and t represents the time in years. Find the simple interest I given each of the following values.

13. $p = \$1800$, $r = 6\%$, $t = 4$ years
14. $p = \$5000$, $r = 3.75\%$, $t = 10$ years
15. $p = \$31{,}000$, $r = 2\frac{1}{2}\%$, $t = 18$ months

Practice and Apply

Homework Help

For Exercises	See Examples
16–37	1, 3
38–50	2, 3
51–54	4

Extra Practice

See page 828.

Find the value of each expression.

16. $18 + 6 \div 3$
17. $7 - 20 \div 5$
18. $3(8 + 3) - 4$
19. $(6 + 7)2 - 1$
20. $2(6^2 - 9)$
21. $-2(3^2 + 8)$
22. $2 + 8(5) \div 2 - 3$
23. $4 + 64 \div (8 \times 4) \div 2$
24. $[38 - (8 - 3)] \div 3$
25. $10 - [5 + 9(4)]$
26. $1 - \{30 \div [7 + 3(-4)]\}$
27. $12 + \{10 \div [11 - 3(2)]\}$
28. $\frac{1}{3}(4 - 7^2)$
29. $\frac{1}{2}[9 + 5(-3)]$
30. $\frac{16(9 - 22)}{4}$
31. $\frac{45(4 + 32)}{10}$
32. $0.3(1.5 + 24) \div 0.5$
33. $1.6(0.7 + 3.3) \div 2.5$
34. $\frac{1}{5} - \frac{20(81 \div 9)}{25}$
35. $\frac{12(52 \div 2^2)}{6} - \frac{2}{3}$
36. **BICYCLING** The amount of pollutants saved by riding a bicycle rather than driving a car is calculated by adding the organic gases, carbon monoxide, and nitrous oxides emitted. To find the pounds of pollutants created by starting a typical car 10 times and driving it for 50 miles, find the value of the expression $\frac{(52.84 \times 10) + (5.955 \times 50)}{454}$.
37. **NURSING** Determine the IV flow rate for the patient described at the beginning of the lesson by finding the value of $\frac{1500 \times 15}{12 \times 60}$.

More About. . .

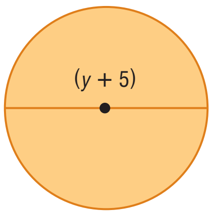
Bicycling

In order to increase awareness and acceptance of bicycling throughout the country, communities, corporations, clubs, and individuals are invited to join in sponsoring bicycling activities during the month of May, National Bike Month.

Source: League of American Bicyclists

Evaluate each expression if $w = 6$, $x = 0.4$, $y = \frac{1}{2}$, and $z = -3$.

38. $w + x + z$
39. $w + 12 \div z$
40. $w(8 - y)$
41. $z(x + 1)$
42. $w - 3x + y$
43. $5x + 2z$
44. $z^4 - w$
45. $(5 - w)^2 + x$
46. $\frac{5wx}{z}$
47. $\frac{2z - 15x}{3y}$
48. $(x - y)^2 - 2wz$
49. $\frac{1}{y} + \frac{1}{w}$
50. **GEOMETRY** The formula for the area A of a circle with diameter d is $A = \pi\left(\frac{d}{2}\right)^2$. Write an expression to represent the area of the circle.



51. Find the value of ab^n if $n = 3$, $a = 2000$, and $b = -\frac{1}{5}$.

52. **MEDICINE** Suppose a patient must take a blood pressure medication that is dispensed in 125-milligram tablets. The dosage is 15 milligrams per kilogram of body weight and is given every 8 hours. If the patient weighs 25 kilograms, how many tablets would be needed for a 30-day supply? Use the formula $n = 24d \div [8(b \times 15 \div 125)]$, where n is the number of tablets, d is the number of days the supply should last, and b is the body weight of the patient in kilograms.

53. **MONEY** In 1950, the average price of a car was about \$2000. This may sound inexpensive, but the average income in 1950 was much less than it is now. To compare dollar amounts over time, use the formula $V = \frac{A}{S}C$, where A is the old dollar amount, S is the starting year's Consumer Price Index (CPI), C is the converting year's CPI, and V is the current value of the old dollar amount. Buying a car for \$2000 in 1950 was like buying a car for how much money in 2000?

Year	Average CPI
1950	42.1
1960	29.6
1970	38.8
1980	82.4
1990	130.7
2000	174.0

Source: U.S. Department of Labor

Online Research **Data Update** What is the current Consumer Price Index? Visit www.algebra2.com/data_update to learn more.

More About. . .

Fireworks

To estimate the width w in feet of a firework burst, use the formula $w = 20At$. In this formula, A is the estimated viewing angle of the firework display and t is the time in seconds from the instant you see the light until you hear the sound.

Source: www.efg2.com

54. **FIREWORKS** Suppose you are about a mile from a fireworks display. You count 5 seconds between seeing the light and hearing the sound of the firework display. You estimate the viewing angle is about 4°. Using the information at the left, estimate the width of the firework display.

55. **CRITICAL THINKING** Write expressions having values from one to ten using exactly four 4s. You may use any combination of the operation symbols $+$, $-$, $\times$, $\div$, and/or grouping symbols, but no other numbers are allowed. An example of such an expression with a value of zero is $(4 + 4) - (4 + 4)$.

56. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How are formulas used by nurses?

Include the following in your answer:

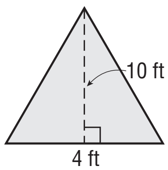
- an explanation of why a formula for the flow rate of an IV is more useful than a table of specific IV flow rates, and
- a description of the impact of using a formula, such as the one for IV flow rate, incorrectly.

Standardized Test Practice

57. Find the value of $1 + 3(5 - 17) \div 2 \times 6$.

   (A) -4 (B) 109
   (C) -107 (D) -144

58. The following are the dimensions of four rectangles. Which rectangle has the same area as the triangle at the right?



   (A) 1.6 ft by 25 ft (B) 5 ft by 16 ft
   (C) 3.5 ft by 4 ft (D) 0.4 ft by 50 ft

Maintain Your Skills

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Evaluate each expression.**

59. $\sqrt{9}$ 60. $\sqrt{16}$ 61. $\sqrt{100}$ 62. $\sqrt{169}$

63. $-\sqrt{4}$ 64. $-\sqrt{25}$ 65. $\sqrt{\frac{4}{9}}$ 66. $\sqrt{\frac{36}{49}}$

1-2 Properties of Real Numbers

What You'll Learn

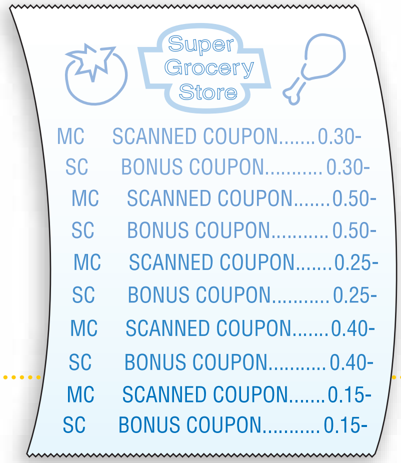
- Classify real numbers.
- Use the properties of real numbers to evaluate expressions.

Vocabulary

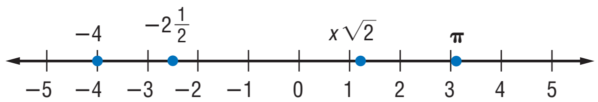
- real numbers
- rational numbers
- irrational numbers

How is the Distributive Property useful in calculating store savings?

Manufacturers often offer coupons to get consumers to try their products. Some grocery stores try to attract customers by doubling the value of manufacturers' coupons. You can use the Distributive Property to calculate these savings.



REAL NUMBERS All of the numbers that you use in everyday life are **real numbers**. Each real number corresponds to exactly one point on the number line, and every point on the number line represents exactly one real number.



Real numbers can be classified as either **rational** or **irrational**.

Study Tip

Reading Math
A *ratio* is the comparison of two numbers by division.

Key Concept — Real Numbers

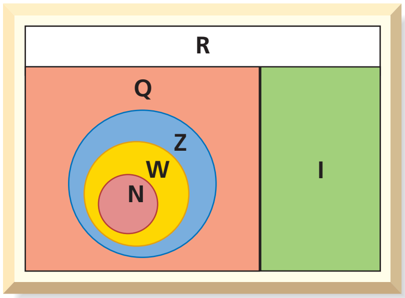
Rational Numbers

- **Words** A rational number can be expressed as a ratio $\frac{m}{n}$, where m and n are integers and n is not zero. The decimal form of a rational number is either a terminating or repeating decimal.
- **Examples** $\frac{1}{6}$, 1.9, 2.575757..., -3, $\sqrt{4}$, 0

Irrational Numbers

- **Words** A real number that is not rational is irrational. The decimal form of an irrational number neither terminates nor repeats.
- **Examples** $\sqrt{5}$, π, 0.010010001...

The sets of natural numbers, {1, 2, 3, 4, 5, ...}, whole numbers, {0, 1, 2, 3, 4, ...}, and integers, {..., −3, − 2, − 1, 0, 1, 2, ...} are all subsets of the rational numbers. The whole numbers are a subset of the rational numbers because every whole number n is equal to $\frac{n}{1}$.



The Venn diagram shows the relationships among these sets of numbers.

R = reals	Q = rationals
I = irrationals	Z = integers
W = wholes	N = naturals

The square root of any whole number is either a whole number or it is irrational. For example, $\sqrt{36}$ is a whole number, but $\sqrt{35}$, since it lies between 5 and 6, must be irrational.

Example 1 Classify Numbers

Name the sets of numbers to which each number belongs.

a. $\sqrt{16}$

$\sqrt{16} = 4$ — naturals (N), wholes (W), integers (Z), rationals (Q), reals (R)

b. -185 — integers (Z), rationals (Q), and reals (R)

c. $\sqrt{20}$ — irrationals (I) and reals (R)

$\sqrt{20}$ lies between 4 and 5 so it is not a whole number.

d. $-\frac{7}{8}$ — rationals (Q) and reals (R)

e. $0.\overline{45}$ — rationals (Q) and reals (R)

The bar over the 45 indicates that those digits repeat forever.

Study Tip

Common Misconception
Do not assume that a number is irrational because it is expressed using the square root symbol. Find its value first.

PROPERTIES OF REAL NUMBERS The real number system is an example of a mathematical structure called a *field*. Some of the properties of a field are summarized in the table below.

Key Concepts — *Real Number Properties*

For any real numbers a, b, and c:

Property	Addition	Multiplication
Commutative	$a + b = b + a$	$a \cdot b = b \cdot a$
Associative	$(a + b) + c = a + (b + c)$	$(a \cdot b) \cdot c = a \cdot (b \cdot c)$
Identity	$a + 0 = a = 0 + a$	$a \cdot 1 = a = 1 \cdot a$
Inverse	$a + (-a) = 0 = (-a) + a$	If $a \neq 0$, then $a \cdot \frac{1}{a} = 1 = \frac{1}{a} \cdot a$.
Distributive	$a(b + c) = ab + ac$ and $(b + c)a = ba + ca$	

Study Tip

Reading Math
$-a$ is read the opposite of a.

Example 2 Identify Properties of Real Numbers

Name the property illustrated by each equation.

a. $(5 + 7) + 8 = 8 + (5 + 7)$

Commutative Property of Addition

The Commutative Property says that the order in which you add does not change the sum.

b. $3(4x) = (3 \cdot 4)x$

Associative Property of Multiplication

The Associative Property says that the way you group three numbers when multiplying does not change the product.

Example 3 Additive and Multiplicative Inverses

Identify the additive inverse and multiplicative inverse for each number.

a. $-1\frac{3}{4}$

Since $-1\frac{3}{4} + \left(1\frac{3}{4}\right) = 0$, the additive inverse of $-1\frac{3}{4}$ is $1\frac{3}{4}$.

Since $-1\frac{3}{4} = -\frac{7}{4}$ and $\left(-\frac{7}{4}\right)\left(-\frac{4}{7}\right) = 1$, the multiplicative inverse of $-1\frac{3}{4}$ is $-\frac{4}{7}$.

b. 1.25

Since $1.25 + (-1.25) = 0$, the additive inverse of 1.25 is -1.25.

The multiplicative inverse of 1.25 is $\frac{1}{1.25}$ or 0.8.

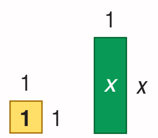
CHECK Notice that $1.25 \times 0.8 = 1$. ✓

You can model the Distributive Property using algebra tiles.

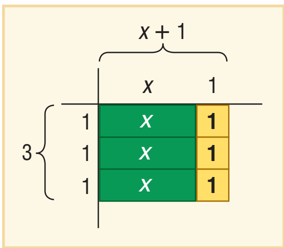
Algebra Activity

Distributive Property

- A 1 tile is a square that is 1 unit wide and 1 unit long. Its area is 1 square unit. An x tile is a rectangle that is 1 unit wide and x units long. Its area is x square units.



- To find the product $3(x + 1)$, model a rectangle with a width of 3 and a length of $x + 1$. Use your algebra tiles to mark off the dimensions on a product mat. Then make the rectangle with algebra tiles.
- The rectangle has 3 x tiles and 3 1 tiles. The area of the rectangle is $x + x + x + 1 + 1 + 1$ or $3x + 3$. Thus, $3(x + 1) = 3x + 3$.



Model and Analyze

Tell whether each statement is *true* or *false*. Justify your answer with algebra tiles and a drawing.

1. $4(x + 2) = 4x + 2$

2. $3(2x + 4) = 6x + 7$

3. $2(3x + 5) = 6x + 10$

4. $(4x + 1)5 = 4x + 5$

The Distributive Property is often used in real-world applications.

Example 4 Use the Distributive Property to Solve a Problem

FOOD SERVICE A restaurant adds a 20% tip to the bills of parties of 6 or more people. Suppose a server waits on five such tables. The bill without the tip for each party is listed in the table. How much did the server make in tips during this shift?

Party 1	Party 2	Party 3	Party 4	Party 5
$185.45	$205.20	$195.05	$245.80	$262.00

There are two ways to find the total amount of tips received.

Method 1

Multiply each dollar amount by 20% or 0.2 and then add.

$$\begin{aligned} T &= 0.2(185.45) + 0.2(205.20) + 0.2(195.05) + 0.2(245.80) + 0.2(262) \\ &= 37.09 + 41.04 + 39.01 + 49.16 + 52.40 \\ &= 218.70 \end{aligned}$$

Method 2

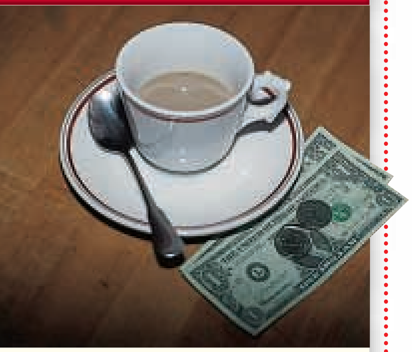
Add the bills of all the parties and then multiply the total by 0.2.

$$\begin{aligned} T &= 0.2(185.45 + 205.20 + 195.05 + 245.80 + 262) \\ &= 0.2(1093.50) \\ &= 218.70 \end{aligned}$$

The server made $218.70 during this shift.

Notice that both methods result in the same answer.

More About. . .



Food Service

Leaving a "tip" began in 18th century English coffee houses and is believed to have originally stood for "To Insure Promptness." Today, the American Automobile Association suggests leaving a 15% tip.

Source: Market Facts, Inc.

The properties of real numbers can be used to simplify algebraic expressions.

Example 5 Simplify an Expression

Simplify $2(5m + n) + 3(2m - 4n)$.

$$\begin{aligned} &2(5m + n) + 3(2m - 4n) \\ &= 2(5m) + 2(n) + 3(2m) - 3(4n) && \text{Distributive Property} \\ &= 10m + 2n + 6m - 12n && \text{Multiply.} \\ &= 10m + 6m + 2n - 12n && \text{Commutative Property (+)} \\ &= (10 + 6)m + (2 - 12)n && \text{Distributive Property} \\ &= 16m - 10n && \text{Simplify.} \end{aligned}$$

Check for Understanding

Concept Check

1. **OPEN ENDED** Give an example of each type of number.
   - **a.** natural
   - **b.** whole
   - **c.** integer
   - **d.** rational
   - **e.** irrational
   - **f.** real
2. **Explain** why $\frac{\sqrt{3}}{2}$ is *not* a rational number.
3. **Disprove** the following statement by giving a counterexample. A **counterexample** is a specific case that shows that a statement is false. Explain.

   Every real number has a multiplicative inverse.

Guided Practice

Name the sets of numbers to which each number belongs.

4. -4 5. 45 6. $6.\overline{23}$

Name the property illustrated by each equation.

7. $\frac{2}{3} \cdot \frac{3}{2} = 1$ 8. $(a + 4) + 2 = a + (4 + 2)$ 9. $4x + 0 = 4x$

Identify the additive inverse and multiplicative inverse for each number.

10. -8 11. $\frac{1}{3}$ 12. 1.5

Simplify each expression.

13. $3x + 4y - 5x$
14. $9p - 2n + 4p + 2n$
15. $3(5c + 4d) + 6(d - 2c)$
16. $\frac{1}{2}(16 - 4a) - \frac{3}{4}(12 + 20a)$

Application

BAND BOOSTERS **For Exercises 17 and 18, use the information below and in the table.**
Ashley is selling chocolate bars for $1.50 each to raise money for the band.

17. Write an expression to represent the total amount of money Ashley raised during this week.
18. Evaluate the expression from Exercise 17 by using the Distributive Property.

Ashley's Sales for One Week

Day	Bars Sold
Monday	10
Tuesday	15
Wednesday	12
Thursday	8
Friday	19
Saturday	22
Sunday	31

Practice and Apply

Homework Help

For Exercises	See Examples
19–27, 40–42, 59–62	1
28–39	2
43–48	3
63–65	4
49–58, 66–69	5

Extra Practice

See page 828.

Name the sets of numbers to which each number belongs.

19. 0 20. $-\frac{2}{9}$ 21. $\sqrt{121}$ 22. -4.55
23. $\sqrt{10}$ 24. -31 25. $\frac{12}{2}$ 26. $\frac{3\pi}{2}$
27. Name the sets of numbers to which all of the following numbers belong. Then arrange the numbers in order from least to greatest.
$2.\overline{49}, 2.4\overline{9}, 2.4, 2.49, 2.\overline{9}$

Name the property illustrated by each equation.

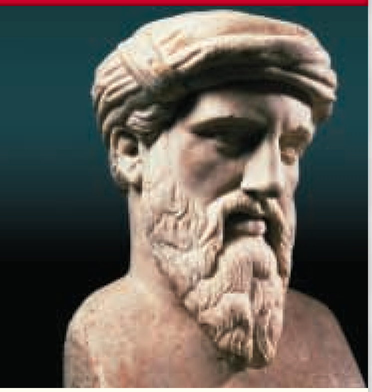
28. $5a + (-5a) = 0$
29. $(3 \cdot 4) \cdot 25 = 3 \cdot (4 \cdot 25)$
30. $-6xy + 0 = -6xy$
31. $[5 + (-2)] + (-4) = 5 + [-2 + (-4)]$
32. $(2 + 14) + 3 = 3 + (2 + 14)$
33. $\left(1\frac{2}{7}\right)\left(\frac{7}{9}\right) = 1$
34. $2\sqrt{3} + 5\sqrt{3} = (2 + 5)\sqrt{3}$
35. $ab = 1ab$

NUMBER THEORY **For Exercises 36–39, use the properties of real numbers to answer each question.**

36. If $m + n = m$, what is the value of n?
37. If $m + n = 0$, what is the value of n? What is n called with respect to m?
38. If $mn = 1$, what is the value of n? What is n called with respect to m?
39. If $mn = m$, what is the value of n?

www.algebra2.com/self_check_quiz

More About. . .



Math History

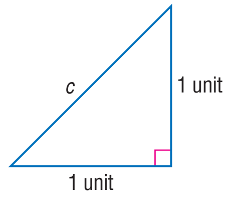
Pythagoras (572–497 B.C.), was a Greek philosopher whose followers came to be known as the Pythagoreans. It was their knowledge of what is called the Pythagorean Theorem that led to the first discovery of irrational numbers.

Source: *A History of Mathematics*

MATH HISTORY **For Exercises 40–42, use the following information.**
The Greek mathematician Pythagoras believed that all things could be described by numbers. By "number" he meant positive integers.

40. To what set of numbers was Pythagoras referring when he spoke of "numbers?"

41. Use the formula $c = \sqrt{2s^2}$ to calculate the length of the hypotenuse c, or longest side, of this right triangle using s, the length of one leg.



42. Explain why Pythagoras could not find a "number" to describe the value of c.

Name the additive inverse and multiplicative inverse for each number.

43. -10 **44.** 2.5 **45.** -0.125

46. $-\frac{5}{8}$ **47.** $\frac{4}{3}$ **48.** $-4\frac{3}{5}$

Simplify each expression.

49. $7a + 3b - 4a - 5b$

50. $3x + 5y + 7x - 3y$

51. $3(15x - 9y) + 5(4y - x)$

52. $2(10m - 7a) + 3(8a - 3m)$

53. $8(r + 7t) - 4(13t + 5r)$

54. $4(14c - 10d) - 6(d + 4c)$

55. $4(0.2m - 0.3n) - 6(0.7m - 0.5n)$

56. $7(0.2p + 0.3q) + 5(0.6p - q)$

57. $\frac{1}{4}(6 + 20y) - \frac{1}{2}(19 - 8y)$

58. $\frac{1}{6}(3x + 5y) + \frac{2}{3}\left(\frac{3}{5}x - 6y\right)$

Determine whether each statement is *true* or *false*. If false, give a counterexample.

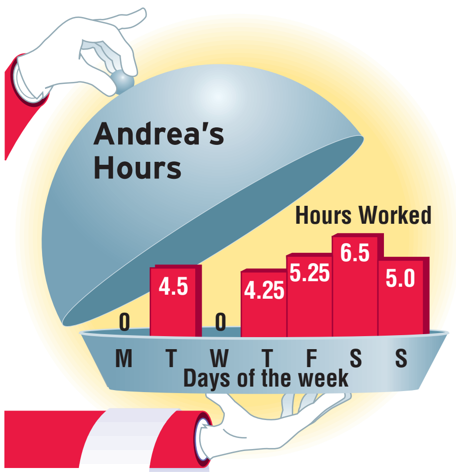
59. Every whole number is an integer.

60. Every integer is a whole number.

61. Every real number is irrational.

62. Every integer is a rational number.

WORK **For Exercises 63 and 64, use the information below and in the table.**
Andrea works as a hostess in a restaurant and is paid every two weeks.



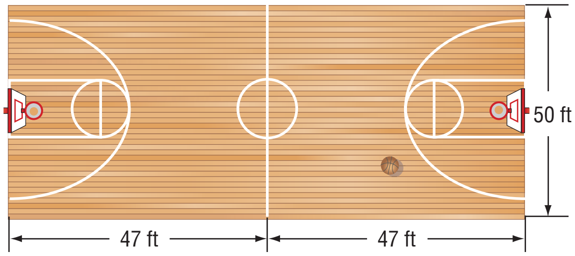
63. If Andrea earns \$6.50 an hour, illustrate the Distributive Property by writing two expressions representing Andrea's pay last week.

64. Find the mean or average number of hours Andrea worked each day, to the nearest tenth of an hour. Then use this average to predict her pay for a two-week pay period.

65. **BAKING** Mitena is making two types of cookies. The first recipe calls for $2\frac{1}{4}$ cups of flour, and the second calls for $1\frac{1}{8}$ cups of flour. If Mitena wants to make 3 batches of the first recipe and 2 batches of the second recipe, how many cups of flour will she need? Use the properties of real numbers to show how Mitena could compute this amount mentally. Justify each step.

CONTENTS

BASKETBALL For Exercises 66 and 67, use the diagram of an NCAA basketball court below.



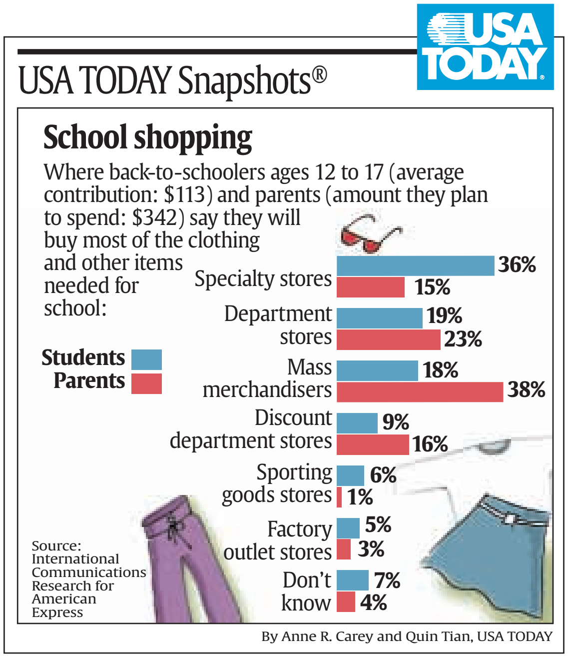
66. Illustrate the Distributive Property by writing two expressions for the area of the basketball court.

67. Evaluate the expression from Exercise 66 using the Distributive Property. What is the area of an NCAA basketball court?

SCHOOL SHOPPING For Exercises 68 and 69, use the graph at the right.

68. Illustrate the Distributive Property by writing two expressions to represent the amount that the average student spends shopping for school at specialty stores and department stores.

69. Evaluate the expression from Exercise 68 using the Distributive Property.



70. **CRITICAL THINKING** Is the Distributive Property also true for division? In other words, does $\frac{b+c}{a} = \frac{b}{a} + \frac{c}{a}$, $a \neq 0$? If so, give an example and explain why it is true. If not true, give a counterexample.

71. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How is the Distributive Property useful in calculating store savings?

Include the following in your answer:

- an explanation of how the Distributive Property could be used to calculate the coupon savings listed on a grocery receipt, and
- an example of how the Distributive Property could be used to calculate the savings from a clothing store sale where all items were discounted by the same percent.

Standardized Test Practice

A B C D

72. If a and b are natural numbers, then which of the following must also be a natural number?

I. $a - b$ **II.** ab **III.** $\frac{a}{b}$

Ⓐ I only Ⓑ II only Ⓒ III only

Ⓓ I and II only Ⓔ II and III only

73. If $x = 1.4$, find the value of $27(x + 1.2) - 26(x + 1.2)$.

Ⓐ 1 Ⓑ -0.4 Ⓒ 2.6 Ⓓ 65

CONTENTS

Extending the Lesson

For Exercises 74–77, use the following information.
The product of any two whole numbers is always a whole number. So, the set of whole numbers is said to be *closed* under multiplication. This is an example of the **Closure Property**. State whether each statement is *true* or *false*. If false, give a counterexample.

74. The set of integers is closed under multiplication.

75. The set of whole numbers is closed under subtraction.

76. The set of rational numbers is closed under addition.

77. The set of whole numbers is closed under division.

Maintain Your Skills

Mixed Review

Find the value of each expression. *(Lesson 1-1)*

78. $9(4 - 3)^5$

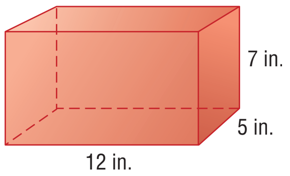
79. $5 + 9 \div 3(3) - 8$

Evaluate each expression if $a = -5$, $b = 0.25$, $c = \frac{1}{2}$, and $d = 4$. *(Lesson 1-1)*

80. $a + 2b - c$

81. $b + 3(a + d)^3$

82. GEOMETRY The formula for the surface area SA of a rectangular prism is $SA = 2\ell w + 2\ell h + 2wh$, where ℓ represents the length, w represents the width, and h represents the height. Find the surface area of the rectangular prism. *(Lesson 1-1)*



Getting Ready for the Next Lesson

PREREQUISITE SKILL **Evaluate each expression if $a = 2$, $b = -\frac{3}{4}$, and $c = 1.8$.**
*(To review **evaluating expressions**, see Lesson 1-1.)*

83. $8b - 5$

84. $\frac{2}{5}b + 1$

85. $1.5c - 7$

86. $-9(a - 6)$

Practice Quiz 1

Lessons 1-1 and 1-2

Find the value of each expression. *(Lesson 1-1)*

1. $18 - 12 \div 3$

2. $-4 + 5(7 - 2^3)$

3. $\dfrac{18 + 3 \times 4}{13 - 8}$

4. Evaluate $a^3 + b(9 - c)$ if $a = -2$, $b = \frac{1}{3}$, and $c = -12$. *(Lesson 1-1)*

5. ELECTRICITY Find the amount of current I (in amperes) produced if the electromotive force E is 2.5 volts, the circuit resistance R is 1.05 ohms, and the resistance r within a battery is 0.2 ohm. Use the formula $I = \frac{E}{R + r}$. *(Lesson 1-1)*

Name the sets of numbers to which each number belongs. *(Lesson 1-2)*

6. 3.5

7. $\sqrt{100}$

8. Name the property illustrated by $bc + (-bc) = 0$. *(Lesson 1-2)*

9. Name the additive inverse and multiplicative inverse of $\frac{6}{7}$. *(Lesson 1-2)*

10. Simplify $4(14x - 10y) - 6(x + 4y)$. *(Lesson 1-2)*

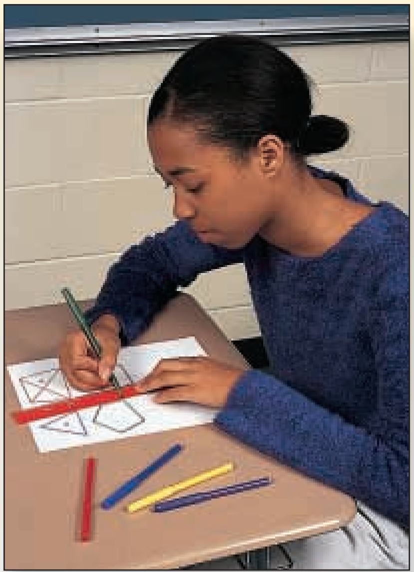
Algebra Activity

A Follow-Up of Lesson 1-2

Investigating Polygons and Patterns

Collect the Data

Use a ruler or geometry drawing software to draw six large polygons with 3, 4, 5, 6, 7, and 8 sides. The polygons do not need to be regular. Convex polygons, ones whose diagonals lie in the interior, will be best for this activity.



1. Copy the table below and complete the column labeled *Diagonals* by drawing the diagonals for all six polygons and record your results.

Figure Name	Sides (n)	Diagonals	Diagonals From One Vertex
triangle	3	0	0
quadrilateral	4	2	1
pentagon	5		
hexagon	6		
heptagon	7		
octagon	8		

Analyze the Data

2. Describe the pattern shown by the number of diagonals in the table above.

3. Complete the last column in the table above by recording the number of diagonals that can be drawn from one vertex of each polygon.

4. Write an expression in terms of n that relates the number of diagonals from one vertex to the number of sides for each polygon.

5. If a polygon has n sides, how many vertices does it have?

6. How many vertices does one diagonal connect?

Make a Conjecture

7. Write a formula in terms of n for the number of diagonals of a polygon of n sides. (*Hint:* Consider your answers to Exercises 2, 3, and 4.)

8. Draw a polygon with 10 sides. Test your formula for the decagon.

9. Explain how your formula relates to the number of vertices of the polygon and the number of diagonals that can be drawn from each vertex.

Extend the Activity

10. Draw 3 noncollinear dots on your paper. Determine the number of lines that are needed to connect each dot to every other dot. Continue by drawing 4 dots, 5 dots, and so on and finding the number of lines to connect them.

11. Copy and complete the table at the right.

12. Use any method to find a formula that relates the number of dots, x, to the number of lines, y.

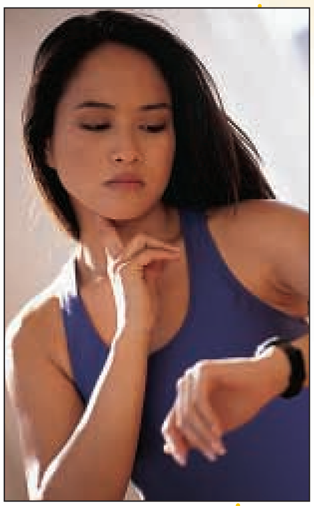
13. Explain why the formula works.

Dots (x)	Connection Lines (y)
3	3
4	
5	
6	
7	
8	

1-3 Solving Equations

What You'll Learn

- Translate verbal expressions into algebraic expressions and equations, and vice versa.
- Solve equations using the properties of equality.



Vocabulary

- open sentence
- equation
- solution

How can you find the most effective level of intensity for your workout?

When exercising, one goal is to find the best level of intensity as a percent of your maximum heart rate. To find the intensity level, multiply 6 and P, your 10-second pulse count. Then divide by the difference of 220 and your age A.

$$\overbrace{6 \times P}^{\text{Multiply 6 and your pulse rate}} \quad \overbrace{\div}^{\text{and divide by}} \quad \overbrace{(220 - A)}^{\text{the difference of 220 and your age.}}$$

VERBAL EXPRESSIONS TO ALGEBRAIC EXPRESSIONS Verbal expressions can be translated into algebraic or mathematical expressions using the language of algebra. Any letter can be used as a variable to represent a number that is not known.

Example 1 Verbal to Algebraic Expression

Write an algebraic expression to represent each verbal expression.

a. 7 less than a number	$n - 7$
b. three times the square of a number	$3x^2$
c. the cube of a number increased by 4 times the same number	$p^3 + 4p$
d. twice the sum of a number and 5	$2(y + 5)$

A mathematical sentence containing one or more variables is called an **open sentence**. A mathematical sentence stating that two mathematical expressions are equal is called an **equation**.

Example 2 Algebraic to Verbal Sentence

Write a verbal sentence to represent each equation.

a. $10 = 12 - 2$	Ten is equal to 12 minus 2.
b. $n + (-8) = -9$	The sum of a number and -8 is -9.
c. $\frac{n}{6} = n^2$	A number divided by 6 is equal to that number squared.

Open sentences are neither true nor false until the variables have been replaced by numbers. Each replacement that results in a true sentence is called a **solution** of the open sentence.

PROPERTIES OF EQUALITY To solve equations, we can use properties of equality. Some of these *equivalence relations* are listed in the table below.

Key Concept *Properties of Equality*

Property	Symbols	Examples
Reflexive	For any real number a, $a = a$.	$-7 + n = -7 + n$
Symmetric	For all real numbers a and b, if $a = b$, then $b = a$.	If $3 = 5x - 6$, then $5x - 6 = 3$.
Transitive	For all real numbers a, b, and c, if $a = b$ and $b = c$, then $a = c$.	If $2x + 1 = 7$ and $7 = 5x - 8$, then $2x + 1 = 5x - 8$.
Substitution	If $a = b$, then a may be replaced by b and b may be replaced by a.	If $(4 + 5)m = 18$, then $9m = 18$.

Study Tip

Properties of Equality
These properties are also known as *axioms of equality*.

Example 3 Identify Properties of Equality

Name the property illustrated by each statement.

a. If $3m = 5n$ and $5n = 10p$, then $3m = 10p$.

Transitive Property of Equality

b. If $-11a + 2 = -3a$, then $-3a = -11a + 2$.

Symmetric Property of Equality

Sometimes an equation can be solved by adding the same number to each side or by subtracting the same number from each side or by multiplying or dividing each side by the same number.

Key Concept *Properties of Equality*

Addition and Subtraction Properties of Equality

- **Symbols** For any real numbers a, b, and c, if $a = b$, then $a + c = b + c$ and $a - c = b - c$.
- **Examples** If $x - 4 = 5$, then $x - 4 + 4 = 5 + 4$.
  If $n + 3 = -11$, then $n + 3 - 3 = -11 - 3$.

Multiplication and Division Properties of Equality

- **Symbols** For any real numbers a, b, and c, if $a = b$, then $a \cdot c = b \cdot c$ and, if $c \neq 0$, $\frac{a}{c} = \frac{b}{c}$.
- **Examples** If $\frac{m}{4} = 6$, then $4 \cdot \frac{m}{4} = 4 \cdot 6$. If $-3y = 6$, then $\frac{-3y}{-3} = \frac{6}{-3}$.

Example 4 Solve One-Step Equations

Solve each equation. Check your solution.

a. $a + 4.39 = 76$

$a + 4.39 = 76$	Original equation
$a + 4.39 - 4.39 = 76 - 4.39$	Subtract 4.39 from each side.
$a = 71.61$	Simplify.

The solution is 71.61.

(continued on the next page)

CHECK $a + 4.39 = 76$ Original equation

$71.61 + 4.39 \stackrel{?}{=} 76$ Substitute 71.61 for *a*.

$76 = 76\ \checkmark$ Simplify.

Study Tip

Multiplication and Division Properties of Equality

Example 4b could also have been solved using the Division Property of Equality. Note that dividing each side of the equation by $-\frac{3}{5}$ is the same as multiplying each side by $-\frac{5}{3}$.

b. $-\frac{3}{5}d = 18$

$-\frac{3}{5}d = 18$	Original equation
$-\frac{5}{3}\left(-\frac{3}{5}\right)d = -\frac{5}{3}(18)$	Multiply each side by $-\frac{5}{3}$, the multiplicative inverse of $-\frac{3}{5}$.
$d = -30$	Simplify.

The solution is -30.

CHECK	$-\frac{3}{5}d = 18$	Original equation
	$-\frac{3}{5}(-30) \stackrel{?}{=} 18$	Substitute -30 for *d*.
	$18 = 18\ \checkmark$	Simplify.

Sometimes you must apply more than one property to solve an equation.

Example 5 Solve a Multi-Step Equation

Solve $2(2x + 3) - 3(4x - 5) = 22$.

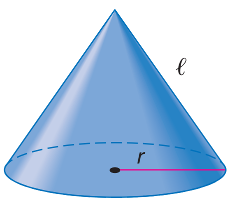
$2(2x + 3) - 3(4x - 5) = 22$	Original equation
$4x + 6 - 12x + 15 = 22$	Distributive and Substitution Properties
$-8x + 21 = 22$	Commutative, Distributive, and Substitution Properties
$-8x = 1$	Subtraction and Substitution Properties
$x = -\frac{1}{8}$	Division and Substitution Properties

The solution is $-\frac{1}{8}$.

You can use properties of equality to solve an equation or formula for a specified variable.

Example 6 Solve for a Variable

GEOMETRY **The surface area of a cone is $S = \pi r\ell + \pi r^2$, where S is the surface area, ℓ is the slant height of the cone, and r is the radius of the base. Solve the formula for ℓ.**



$S = \pi r\ell + \pi r^2$	Surface area formula
$S - \pi r^2 = \pi r\ell + \pi r^2 - \pi r^2$	Subtract πr^2 from each side.
$S - \pi r^2 = \pi r\ell$	Simplify.
$\frac{S - \pi r^2}{\pi r} = \frac{\pi r\ell}{\pi r}$	Divide each side by πr.
$\frac{S - \pi r^2}{\pi r} = \ell$	Simplify.

Many standardized test questions can be solved by using properties of equality.

Standardized Test Practice

Example 7 Apply Properties of Equality

Multiple-Choice Test Item

If $3n - 8 = \frac{9}{5}$, what is the value of $3n - 3$?

(A) $\frac{34}{5}$ (B) $\frac{49}{15}$ (C) $-\frac{16}{5}$ (D) $-\frac{27}{5}$

Read the Test Item

You are asked to find the value of the expression $3n - 3$. Your first thought might be to find the value of n and then evaluate the expression using this value. Notice, however, that you are *not* required to find the value of n. Instead, you can use the Addition Property of Equality on the given equation to find the value of $3n - 3$.

Solve the Test Item

$$3n - 8 = \frac{9}{5} \quad \text{Original equation}$$

$$3n - 8 + 5 = \frac{9}{5} + 5 \quad \text{Add 5 to each side.}$$

$$3n - 3 = \frac{34}{5} \quad \frac{9}{5} + 5 = \frac{9}{5} + \frac{25}{5} \text{ or } \frac{34}{5}$$

The answer is A.

The Princeton Review

Test-Taking Tip

If a problem seems to require lengthy calculations, look for a shortcut. There is probably a quicker way to solve it. Try using properties of equality.

To solve a word problem, it is often necessary to define a variable and write an equation. Then solve by applying the properties of equality.

Example 8 Write an Equation

HOME IMPROVEMENT Josh and Pam have bought an older home that needs some repair. After budgeting a total of \$1685 for home improvements, they started by spending \$425 on small improvements. They would like to replace six interior doors next. What is the maximum amount they can afford to spend on each door?

Explore Let c represent the cost to replace each door.

Plan Write and solve an equation to find the value of c.

The number of doors	times	the cost to replace each door	plus	previous expenses	equals	the total cost.
6	$\cdot$	c	$+$	425	$=$	1685

Solve

$$6c + 425 = 1685 \quad \text{Original equation}$$

$$6c + 425 - 425 = 1685 - 425 \quad \text{Subtract 425 from each side.}$$

$$6c = 1260 \quad \text{Simplify.}$$

$$\frac{6c}{6} = \frac{1260}{6} \quad \text{Divide each side by 6.}$$

$$c = 210 \quad \text{Simplify.}$$

They can afford to spend \$210 on each door.

Examine The total cost to replace six doors at \$210 each is 6(210) or \$1260. Add the other expenses of \$425 to that, and the total home improvement bill is 1260 + 425 or \$1685. Thus, the answer is correct.

More About. . .

Home Improvement

Previously occupied homes account for approximately 85% of all U.S. home sales. Most homeowners remodel within 18 months of purchase. The top two remodeling projects are kitchens and baths.

Source: National Association of Remodeling Industry

Check for Understanding

Concept Check

1. **OPEN ENDED** Write an equation whose solution is -7.
2. **Determine** whether the following statement is *sometimes*, *always*, or *never* true. Explain.

   Dividing each side of an equation by the same expression produces an equivalent equation.

3. **FIND THE ERROR** Crystal and Jamal are solving $C = \frac{5}{9}(F - 32)$ for F.

Crystal

$$C = \frac{5}{9}(F - 32)$$
$$C + 32 = \frac{5}{9}F$$
$$\frac{9}{5}(C + 32) = F$$

Jamal

$$C = \frac{5}{9}(F - 32)$$
$$\frac{9}{5}C = F - 32$$
$$\frac{9}{5}C + 32 = F$$

Who is correct? Explain your reasoning.

Guided Practice

Write an algebraic expression to represent each verbal expression.

4. five increased by four times a number
5. twice a number decreased by the cube of the same number

Write a verbal expression to represent each equation.

6. $9n - 3 = 6$
7. $5 + 3x^2 = 2x$

Name the property illustrated by each statement.

8. $(3x + 2) - 5 = (3x + 2) - 5$
9. If $4c = 15$, then $4c + 2 = 15 + 2$.

Solve each equation. Check your solution.

10. $y + 14 = -7$
11. $7 + 3x = 49$
12. $-4(b + 7) = -12$
13. $7q + q - 3q = -24$
14. $1.8a - 5 = -2.3$
15. $-\frac{3}{4}n + 1 = -11$

Solve each equation or formula for the specified variable.

16. $4y - 2n = 9$, for y
17. $I = prt$, for p

Standardized Test Practice

18. If $4x + 7 = 18$, what is the value of $12x + 21$?

    (A) 2.75 (B) 32 (C) 33 (D) 54

Practice and Apply

Homework Help

For Exercises	See Examples
19–28	1
29–34	2
35–40	3
41–56	4, 5
57–62	6
63–74	7

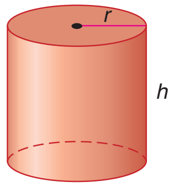
Extra Practice

See page 828.

Write an algebraic expression to represent each verbal expression.

19. the sum of 5 and three times a number
20. seven more than the product of a number and 10
21. four less than the square of a number
22. the product of the cube of a number and -6
23. five times the sum of 9 and a number
24. twice the sum of a number and 8
25. the square of the quotient of a number and 4
26. the cube of the difference of a number and 7

GEOMETRY **For Exercises 27 and 28, use the following information.**



The formula for the surface area of a cylinder with radius r and height h is π times twice the product of the radius and height plus twice the product of π and the square of the radius.

27. Translate this verbal expression of the formula into an algebraic expression.

28. Write an equivalent expression using the Distributive Property.

Write a verbal expression to represent each equation.

29. $x - 5 = 12$

30. $2n + 3 = -1$

31. $y^2 = 4y$

32. $3a^3 = a + 4$

33. $\frac{b}{4} = 2(b + 1)$

34. $7 - \frac{1}{2}x = \frac{3}{x^2}$

Name the property illustrated by each statement.

35. If $[3(-2)]z = 24$, then $-6z = 24$.

36. If $5 + b = 13$, then $b = 8$.

37. If $2x = 3d$ and $3d = -4$, then $2x = -4$.

38. If $g - t = n$, then $g = n + t$.

39. If $14 = \frac{x}{2} + 11$, then $\frac{x}{2} + 11 = 14$.

40. If $y - 2 = -8$, then $3(y - 2) = 3(-8)$.

Solve each equation. Check your solution.

41. $2p + 15 = 29$

42. $14 - 3n = -10$

43. $7a - 3a + 2a - a = 16$

44. $x + 9x - 6x + 4x = 20$

45. $\frac{1}{9} - \frac{2}{3}b = \frac{1}{18}$

46. $\frac{5}{8} + \frac{3}{4}x = \frac{1}{16}$

47. $27 = -9(y + 5)$

48. $-7(p + 8) = 21$

49. $3f - 2 = 4f + 5$

50. $3d + 7 = 6d + 5$

51. $4.3n + 1 = 7 - 1.7n$

52. $1.7x - 8 = 2.7x + 4$

53. $3(2z + 25) - 2(z - 1) = 78$

54. $4(k + 3) + 2 = 4.5(k + 1)$

55. $\frac{3}{11}a - 1 = \frac{7}{11}a + 9$

56. $\frac{2}{5}x + \frac{3}{7} = 1 - \frac{4}{7}x$

Solve each equation or formula for the specified variable.

57. $d = rt$, for r

58. $x = \frac{-b}{2a}$, for a

59. $V = \frac{1}{3}\pi r^2 h$, for h

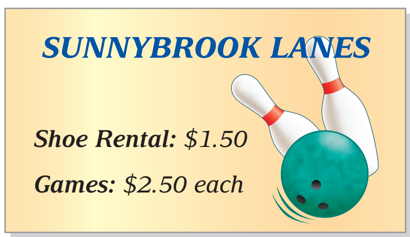
60. $A = \frac{1}{2}h(a + b)$, for b

61. $\frac{a(b - 2)}{c - 3} = x$, for b

62. $x = \frac{y}{y + 4}$, for y

Define a variable, write an equation, and solve the problem.

63. BOWLING Jon and Morgan arrive at Sunnybrook Lanes with $16.75. Find the maximum number of games they can bowl if they each rent shoes.

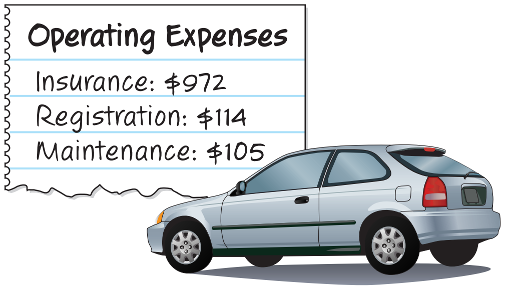


www.algebra2.com/self_check_quiz

For Exercises 64–70, define a variable, write an equation, and solve the problem.

64. **GEOMETRY** The perimeter of a regular octagon is 124 inches. Find the length of each side.

65. **CAR EXPENSES** Benito spent \$1837 to operate his car last year. Some of these expenses are listed below. Benito's only other expense was for gasoline. If he drove 7600 miles, what was the average cost of the gasoline per mile?



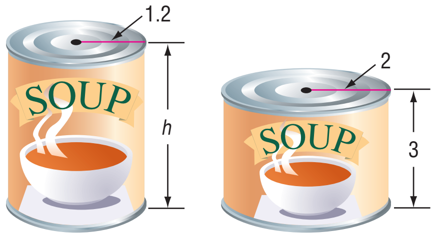
66. **SCHOOL** A school conference room can seat a maximum of 83 people. The principal and two counselors need to meet with the school's student athletes to discuss eligibility requirements. If each student must bring a parent with them, what is the maximum number of students that can attend each meeting?

67. **FAMILY** Chun-Wei's mother is 8 more than twice his age. His father is three years older than his mother is. If all three family members have lived 94 years, how old is each family member?

68. **SCHOOL TRIP** The Parent Teacher Organization has raised \$1800 to help pay for a trip to an amusement park. They ask that there be one adult for every five students attending. Adult tickets cost \$45 and student tickets cost \$30. If the group wants to take 50 students, how much will each student need to pay so that adults agreeing to chaperone pay nothing?

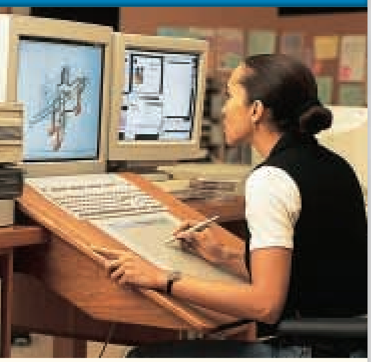
69. **BUSINESS** A trucking company is hired to deliver 125 lamps for \$12 each. The company agrees to pay \$45 for each lamp that is broken during transport. If the trucking company needs to receive a minimum payment of \$1364 for the shipment to cover their expenses, find the maximum number of lamps they can afford to break during the trip.

70. **PACKAGING** Two designs for a soup can are shown at the right. If each can holds the same amount of soup, what is the height of can A?



Can A Can B

Career Choices



Industrial Design

Industrial designers use research on product use, marketing, materials, and production methods to create functional and appealing packaging designs.

Online Research

For information about a career as an industrial designer, visit: www.algebra2.com/careers

RAILROADS **For Exercises 71–73, use the following information.**

The First Transcontinental Railroad was built by two companies. The Central Pacific began building eastward from Sacramento, California, while the Union Pacific built westward from Omaha, Nebraska. The two lines met at Promontory, Utah, in 1869, about 6 years after construction began.

71. The Central Pacific Company laid an average of 9.6 miles of track per month. Together the two companies laid a total of 1775 miles of track. Determine the average number of miles of track laid per month by the Union Pacific Company.

72. About how many miles of track did each company lay?

73. Why do you think the Union Pacific was able to lay track so much more quickly than the Central Pacific?

CONTENTS

74. **MONEY** Allison is saving money to buy a video game system. In the first week, her savings were \$8 less than $\frac{2}{5}$ the price of the system. In the second week, she saved 50 cents more than $\frac{1}{2}$ the price of the system. She was still \$37 short. Find the price of the system.

75. **CRITICAL THINKING** Write a verbal expression to represent the algebraic expression $3(x - 5) + 4x(x + 1)$.

WebQuest

You can write and solve equations to determine the monthly payment for a home. Visit www.algebra2.com/webquest to continue work on your WebQuest project.

76. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How can you find the most effective level of intensity for your workout?

Include the following in your answer:

- an explanation of how to find the age of a person who is exercising at an 80% level of intensity I with a pulse count of 27, and
- a description of when it would be desirable to solve a formula like the one given for a specified variable.

Standardized Test Practice

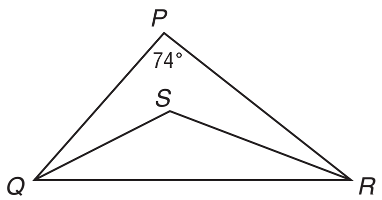
77. If $-6x + 10 = 17$, then $3x - 5 =$

(A) $-\frac{7}{6}$. (B) $-\frac{17}{2}$. (C) 2. (D) $\frac{19}{3}$. (E) $\frac{5}{3}$.

78. In triangle PQR, $\overline{QS}$ and $\overline{SR}$ are angle bisectors and angle $P = 74°$. How many degrees are there in angle QSR?

(A) 106 (B) 121 (C) 125

(D) 127 (E) 143



Maintain Your Skills

Mixed Review

Simplify each expression. *(Lesson 1-2)*

79. $2x + 9y + 4z - y - 8x$

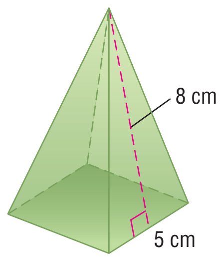
80. $4(2a + 5b) - 3(4b - a)$

Evaluate each expression if $a = 3$, $b = -2$, and $c = 1.2$. *(Lesson 1-1)*

81. $a - [b(a - c)]$

82. $c^2 - ab$

83. **GEOMETRY** The formula for the surface area S of a regular pyramid is $S = \frac{1}{2}P\ell + B$, where P is the perimeter of the base, ℓ is the slant height, and B is the area of the base. Find the surface area of the square-based pyramid shown at the right. *(Lesson 1-1)*



Getting Ready for the Next Lesson

PREREQUISITE SKILL **Identify the additive inverse for each number or expression.** *(To review **additive inverses**, see Lesson 1-2.)*

84. 5

85. -3

86. 2.5

87. $\frac{1}{4}$

88. $-3x$

89. $5 - 6y$

1-4 Solving Absolute Value Equations

What You'll Learn

- Evaluate expressions involving absolute values.
- Solve absolute value equations.

Vocabulary

- absolute value
- empty set

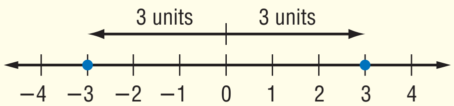
How can an absolute value equation describe the magnitude of an earthquake?

Seismologists use the Richter scale to express the magnitudes of earthquakes. This scale ranges from 1 to 10, 10 being the highest. The uncertainty in the estimate of a magnitude E is about plus or minus 0.3 unit. This means that an earthquake with a magnitude estimated at 6.1 on the Richter scale might actually have a magnitude as low as 5.8 or as high as 6.4. These extremes can be described by the absolute value equation $|E - 6.1| = 0.3$.

ABSOLUTE VALUE EXPRESSIONS The **absolute value** of a number is its distance from 0 on the number line. Since distance is nonnegative, the absolute value of a number is always nonnegative. The symbol $|x|$ is used to represent the absolute value of a number x.

Key Concept — *Absolute Value*

- **Words** For any real number a, if a is positive or zero, the absolute value of a is a. If a is negative, the absolute value of a is the opposite of a.
- **Symbols** For any real number a, $|a| = a$ if $a \geq 0$, and $|a| = -a$ if $a < 0$.
- **Model** $|-3| = 3$ and $|3| = 3$



When evaluating expressions that contain absolute values, the absolute value bars act as a grouping symbol. Perform any operations inside the absolute value bars first.

Example 1 *Evaluate an Expression with Absolute Value*

Evaluate $1.4 + |5y - 7|$ if $y = -3$.

$$
\begin{aligned}
1.4 + |5y - 7| &= 1.4 + |5(-3) - 7| && \text{Replace } y \text{ with } -3. \\
&= 1.4 + |-15 - 7| && \text{Simplify } 5(-3) \text{ first.} \\
&= 1.4 + |-22| && \text{Subtract 7 from } -15. \\
&= 1.4 + 22 && |-22| = 22 \\
&= 23.4 && \text{Add.}
\end{aligned}
$$

The value is 23.4.

ABSOLUTE VALUE EQUATIONS Some equations contain absolute value expressions. The definition of absolute value is used in solving these equations. For any real numbers a and b, where $b \geq 0$, if $|a| = b$, then $a = b$ or $-a = b$. This second case is often written as $a = -b$.

Example 2 Solve an Absolute Value Equation

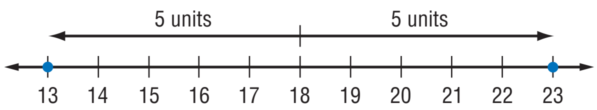
Solve $|x - 18| = 5$. Check your solutions.

Case 1		or	**Case 2**
	$a = b$		$a = -b$
	$x - 18 = 5$		$x - 18 = -5$
	$x - 18 + 18 = 5 + 18$		$x - 18 + 18 = -5 + 18$
	$x = 23$		$x = 13$

CHECK		
	$|x - 18| = 5$	$|x - 18| = 5$
	$|23 - 18| \stackrel{?}{=} 5$	$|13 - 18| \stackrel{?}{=} 5$
	$|5| \stackrel{?}{=} 5$	$|-5| \stackrel{?}{=} 5$
	$5 = 5$ ✓	$5 = 5$ ✓

The solutions are 23 or 13. Thus, the solution set is {13, 23}.

On the number line, we can see that each answer is 5 units away from 18.



Because the absolute value of a number is always positive or zero, an equation like $|x| = -5$ is never true. Thus, it has no solution. The solution set for this type of equation is the **empty set**, symbolized by { } or $\varnothing$.

Study Tip

Common Misconception
For an equation like the one in Example 3, there is no need to consider the two cases. Remember to check your solutions in the original equation to prevent this error.

Example 3 No Solution

Solve $|5x - 6| + 9 = 0$.

$|5x - 6| + 9 = 0$ Original equation

$|5x - 6| = -9$ Subtract 9 from each side.

This sentence is *never* true. So the solution set is $\varnothing$.

It is important to check your answers when solving absolute value equations. Even if the correct procedure for solving the equation is used, the answers may not be actual solutions of the original equation.

Example 4 One Solution

Solve $|x + 6| = 3x - 2$. Check your solutions.

Case 1		or	**Case 2**
	$a = b$		$a = -b$
	$x + 6 = 3x - 2$		$x + 6 = -(3x - 2)$
	$6 = 2x - 2$		$x + 6 = -3x + 2$
	$8 = 2x$		$4x + 6 = 2$
	$4 = x$		$4x = -4$
			$x = -1$

There appear to be two solutions, 4 or -1.

(continued on the next page)

CHECK

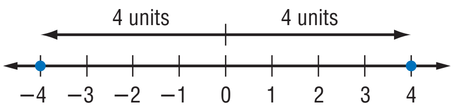
$$\begin{array}{rcl} |x + 6| &=& 3x - 2 \\ |4 + 6| &\stackrel{?}{=}& 3(4) - 2 \\ |10| &\stackrel{?}{=}& 12 - 2 \\ 10 &=& 10 \ \checkmark \end{array} \quad \text{or} \quad \begin{array}{rcl} |x + 6| &=& 3x - 2 \\ |-1 + 6| &\stackrel{?}{=}& 3(-1) - 2 \\ |5| &\stackrel{?}{=}& -3 - 2 \\ \cancel{5 = -5} \end{array}$$

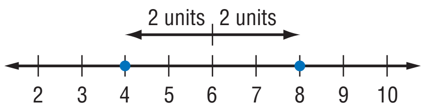
Since $5 \neq -5$, the only solution is 4. Thus, the solution set is $\{4\}$.

Check for Understanding

Concept Check

1. **Explain** why if the absolute value of a number is always nonnegative, $|a|$ can equal $-a$.
2. **Write** an absolute value equation for each solution set graphed below.

   a. 

   b. 

3. **Determine** whether the following statement is *sometimes*, *always*, or *never* true. Explain.
   For all real numbers a and b, $a \neq 0$, the equation $|ax + b| = 0$ will have one solution.
4. **OPEN ENDED** Write and evaluate an expression with absolute value.

Guided Practice

Evaluate each expression if $a = -4$ and $b = 1.5$.

5. $|a + 12|$
6. $|-6b|$
7. $-|a + 21|$

Solve each equation. Check your solutions.

8. $|x + 4| = 17$
9. $|b + 15| = 3$
10. $|a - 9| = 20$
11. $|y - 2| = 34$
12. $|2w + 3| + 6 = 2$
13. $|c - 2| = 2c - 10$

Application

FOOD **For Exercises 14–16, use the following information.**
A meat thermometer is used to assure that a safe temperature has been reached to destroy bacteria. Most meat thermometers are accurate to within plus or minus 2°F. **Source:** U.S. Department of Agriculture

14. The ham you are baking needs to reach an internal temperature of 160°F. If the thermometer reads 160°F, write an equation to determine the least and greatest temperatures of the meat.
15. Solve the equation you wrote in Exercise 14.
16. To what temperature reading should you bake a ham to ensure that the minimum internal temperature is reached? Explain.

Practice and Apply

Evaluate each expression if $a = -5$, $b = 6$, and $c = 2.8$.

17. $|-3a|$
18. $|-4b|$
19. $|a + 5|$
20. $|2 - b|$
21. $|2b - 15|$
22. $|4a + 7|$
23. $-|18 - 5c|$
24. $-|c - a|$
25. $6 - |3c + 7|$
26. $9 - |-2b + 8|$
27. $3|a - 10| + |2a|$
28. $|a - b| - |10c - a|$

Homework Help

For Exercises	See Examples
17–28	1
29–49	2–4

Extra Practice

See page 829.

Solve each equation. Check your solutions.

29. $|x - 25| = 17$

30. $|y + 9| = 21$

31. $|a + 12| = 33$

32. $2|b + 4| = 48$

33. $8|w - 7| = 72$

34. $|3x + 5| = 11$

35. $|2z - 3| = 0$

36. $|6c - 1| = -2$

37. $7|4x - 13| = 35$

38. $-3|2n + 5| = -9$

39. $-12|9x + 1| = 144$

40. $|5x + 9| + 6 = 1$

41. $|a - 3| - 14 = -6$

42. $3|p - 5| = 2p$

43. $3|2a + 7| = 3a + 12$

44. $|3x - 7| - 5 = -3$

45. $4|3t + 8| = 16t$

46. $|15 + m| = -2m + 3$

47. **COFFEE** Some say that to brew an excellent cup of coffee, you must have a brewing temperature of 200°F, plus or minus five degrees. Write and solve an equation describing the maximum and minimum brewing temperatures for an excellent cup of coffee.

48. **MANUFACTURING** A machine is used to fill each of several bags with 16 ounces of sugar. After the bags are filled, another machine weighs them. If the bag weighs 0.3 ounce more or less than the desired weight, the bag is rejected. Write an equation to find the heaviest and lightest bag the machine will approve.

49. **METEOROLOGY** The atmosphere of Earth is divided into four layers based on temperature variations. The troposphere is the layer closest to the planet. The average upper boundary of the layer is about 13 kilometers above Earth's surface. This height varies with latitude and with the seasons by as much as 5 kilometers. Write and solve an equation describing the maximum and minimum heights of the upper bound of the troposphere.

More About. . .

Meteorology

The troposphere is characterized by the density of its air and an average vertical temperature change of 6°C per kilometer. All weather phenomena occur within the troposphere.

Source: NASA

CRITICAL THINKING **For Exercises 50 and 51, determine whether each statement is *sometimes*, *always*, or *never* true. Explain your reasoning.**

50. If a and b are real numbers, then $|a + b| = |a| + |b|$.

51. If a, b, and c are real numbers, then $c|a + b| = |ca + cb|$.

52. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How can an absolute value equation describe the magnitude of an earthquake?

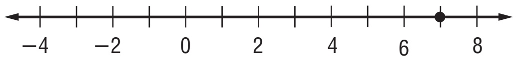
Include the following in your answer:

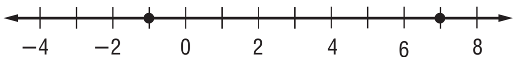
- a verbal and graphical explanation of how $|E - 6.1| = 0.3$ describes the possible extremes in the variation of the earthquake's magnitude, and
- an equation to describe the extremes for a different magnitude.

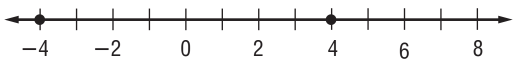
Standardized Test Practice

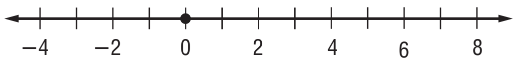
A B C D

53. Which of the graphs below represents the solution set for $|x - 3| - 4 = 0$?

(A)


(B)


(C)


(D)


54. Find the value of $-|-9| - |4| - 3|5 - 7|$.

(A) -19 (B) -11 (C) -7 (D) 11

Extending the Lesson

For Exercises 55–58, consider the equation $|x + 1| + 2 = |x + 4|$.

55. To solve this equation, we must consider the case where $x + 4 \geq 0$ and the case where $x + 4 < 0$. Write the equations for each of these cases.

56. Notice that each equation you wrote in Exercise 55 has two cases. For each equation, write two other equations taking into consideration the case where $x + 1 \geq 0$ and the case where $x + 1 < 0$.

57. Solve each equation you wrote in Exercise 56. Then, check each solution in the original equation, $|x + 1| + 2 = |x + 4|$. What are the solution(s) to this absolute value equation?

58. **MAKE A CONJECTURE** For equations with one set of absolute value symbols, two cases must be considered. For an equation with two sets of absolute value symbols, four cases must be considered. How many cases must be considered for an equation containing three sets of absolute value symbols?

Maintain Your Skills

Mixed Review

Write an algebraic expression to represent each verbal expression. *(Lesson 1-3)*

59. twice the difference of a number and 11

60. the product of the square of a number and 5

Solve each equation. Check your solution. *(Lesson 1-3)*

61. $3x + 6 = 22$

62. $7p - 4 = 3(4 + 5p)$

63. $\frac{5}{7}y - 3 = \frac{3}{7}y + 1$

Name the property illustrated by each equation. *(Lesson 1-2)*

64. $(5 + 9) + 13 = 13 + (5 + 9)$

65. $m(4 - 3) = m \cdot 4 - m \cdot 3$

66. $\left(\frac{1}{4}\right)4 = 1$

67. $5x + 0 = 5x$

Determine whether each statement is *true* or *false*. If false, give a counterexample. *(Lesson 1-2)*

68. Every real number is a rational number.

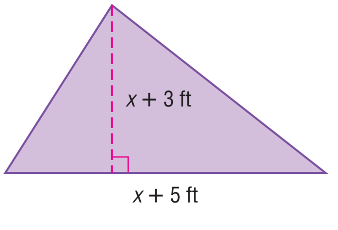
69. Every natural number is an integer.

70. Every irrational number is a real number.

71. Every rational number is an integer.

GEOMETRY **For Exercises 72 and 73, use the following information.**

The formula for the area A of a triangle is $A = \frac{1}{2}bh$, where b is the measure of the base and h is the measure of the height. *(Lesson 1-1)*



72. Write an expression to represent the area of the triangle above.

73. Evaluate the expression you wrote in Exercise 72 for $x = 23$.

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Solve each equation.** *(To review **solving equations**, see page 20.)*

74. $14y - 3 = 25$

75. $4.2x + 6.4 = 40$

76. $7w + 2 = 3w - 6$

77. $2(a - 1) = 8a - 6$

78. $48 + 5y = 96 - 3y$

79. $\frac{2x + 3}{5} = \frac{3}{10}$

1-5 Solving Inequalities

What You'll Learn

- Solve inequalities.
- Solve real-world problems involving inequalities.

Vocabulary

- set-builder notation
- interval notation

How can inequalities be used to compare phone plans?

Kuni is trying to decide between two rate plans offered by a wireless phone company.

	Plan 1	Plan 2
Monthly Access Fee	\$35.00	\$55.00
Minutes Included	150	400
Additional Minutes	40¢	35¢

To compare these two rate plans, we can use inequalities. The monthly access fee for Plan 1 is less than the fee for Plan 2, $\$35 < \$55$. However, the additional minutes fee for Plan 1 is greater than that of Plan 2, $\$0.40 > \$0.35$.

SOLVE INEQUALITIES For any two real numbers, a and b, exactly one of the following statements is true.

$$a < b \qquad a = b \qquad a > b$$

This is known as the **Trichotomy Property** or the *property of order*.

Adding the same number to, or subtracting the same number from, each side of an inequality does not change the truth of the inequality.

Study Tip

Properties of Inequality
The properties of inequality are also known as *axioms* of inequality.

Key Concept — *Properties of Inequality*

Addition Property of Inequality

- **Words** For any real numbers, a, b, and c:
  If $a > b$, then $a + c > b + c$.
  If $a < b$, then $a + c < b + c$.
- **Example**
  $3 < 5$
  $3 + (-4) < 5 + (-4)$
  $-1 < 1$

Subtraction Property of Inequality

- **Words** For any real numbers, a, b, and c:
  If $a > b$, then $a - c > b - c$.
  If $a < b$, then $a - c < b - c$.
- **Example**
  $2 > -7$
  $2 - 8 > -7 - 8$
  $-6 > -15$

These properties are also true for $\le$ and $\ge$.

These properties can be used to solve inequalities. The solution sets of inequalities in one variable can then be graphed on number lines. Use a circle with an arrow to the left for $<$ and an arrow to the right for $>$. Use and a dot with an arrow to the left for $\le$ and an arrow to the right for $\ge$.

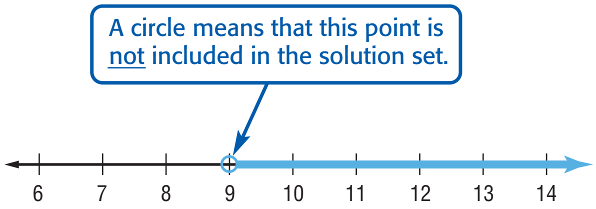
Example 1 Solve an Inequality Using Addition or Subtraction

Solve $7x - 5 > 6x + 4$. Graph the solution set on a number line.

$7x - 5 > 6x + 4$	Original inequality
$7x - 5 + (-6x) > 6x + 4 + (-6x)$	Add $-6x$ to each side.
$x - 5 > 4$	Simplify.
$x - 5 + 5 > 4 + 5$	Add 5 to each side.
$x > 9$	Simplify.

Any real number greater than 9 is a solution of this inequality.

The graph of the solution set is shown at the right.



CHECK Substitute 9 for x in $7x - 5 > 6x + 4$. The two sides should be equal. Then substitute a number greater than 9. The inequality should be true.

Multiplying or dividing each side of an inequality by a positive number does not change the truth of the inequality. However, multiplying or dividing each side of an inequality by a *negative* number requires that the order of the inequality be *reversed*. For example, to reverse $\leq$, replace it with $\geq$.

Key Concept — Properties of Inequality

Multiplication Property of Inequality

- **Words** For any real numbers, a, b, and c, where

c is positive: if $a > b$, then $ac > bc$. if $a < b$, then $ac < bc$.

c is negative: if $a > b$, then $ac < bc$. if $a < b$, then $ac > bc$.

- **Examples**

$-2 < 3$
$4(-2) < 4(3)$
$-8 < 12$

$5 > -1$
$(-3)(5) < (-3)(1)$
$-15 < 3$

Division Property of Inequality

- **Words** For any real numbers, a, b, and c, where

c is positive: if $a > b$, then $\frac{a}{c} > \frac{b}{c}$. if $a < b$, then $\frac{a}{c} < \frac{b}{c}$.

c is negative: if $a > b$, then $\frac{a}{c} < \frac{b}{c}$. if $a < b$, then $\frac{a}{c} > \frac{b}{c}$.

- **Examples**

$-18 < -9$
$\frac{-18}{3} < \frac{-9}{3}$
$-6 < -3$

$12 > 8$
$\frac{12}{-2} < \frac{8}{-2}$
$-6 < -4$

These properties are also true for $\leq$ and $\geq$.

Study Tip

Reading Math
$\{x \mid x > 9\}$ is read *the set of all numbers x such that x is greater than 9.*

The solution set of an inequality can be expressed by using **set-builder notation**. For example, the solution set in Example 1 can be expressed as $\{x \mid x > 9\}$.

CONTENTS

Example 2 Solve an Inequality Using Multiplication or Division

Solve $-0.25y \geq 2$. Graph the solution set on a number line.

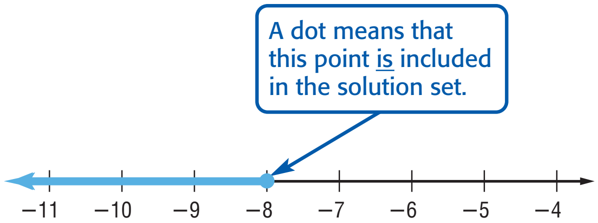
$-0.25y \geq 2$ Original inequality

$\frac{-0.25y}{-0.25} \leq \frac{2}{-0.25}$ Divide each side by -0.25, reversing the inequality symbol.

$y \leq -8$ Simplify.

The solution set is $\{y \mid y \leq -8\}$.

The graph of the solution set is shown below.



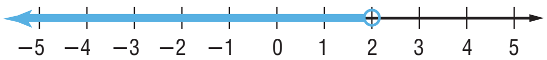
Study Tip

Reading Math

The symbol $+\infty$ is read *positive infinity*, and the symbol $-\infty$ is read *negative infinity*.

The solution set of an inequality can also be described by using **interval notation**. The infinity symbols $+\infty$ and $-\infty$ are used to indicate that a set is unbounded in the positive or negative direction, respectively. To indicate that an endpoint *is not* included in the set, a parenthesis, (or), is used.

$x < 2$ interval notation



$(-\infty, 2)$

A bracket is used to indicate that the endpoint, -2, *is* included in the solution set below. Parentheses are always used with the symbols $+\infty$ and $-\infty$, because they do not include endpoints.

$x \geq -2$ interval notation

$[-2, +\infty)$

Study Tip

Solutions to Inequalities

When solving an inequality,

- if you arrive at a false statement, such as $3 > 5$, then the solution set for that inequality is the empty set, $\varnothing$.
- if you arrive at a true statement such as $3 > -1$, then the solution set for that inequality is the set of all real numbers.

Example 3 Solve a Multi-Step Inequality

Solve $-m \leq \frac{m+4}{9}$. Graph the solution set on a number line.

$-m \leq \frac{m+4}{9}$ Original inequality

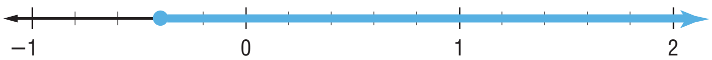
$-9m \leq m + 4$ Multiply each side by 9.

$-10m \leq 4$ Add $-m$ to each side.

$m \geq -\frac{4}{10}$ Divide each side by –10, reversing the inequality symbol.

$m \geq -\frac{2}{5}$ Simplify.

The solution set is $\left[-\frac{2}{5}, +\infty\right)$ and is graphed below.



REAL-WORLD PROBLEMS WITH INEQUALITIES

Inequalities can be used to solve many verbal and real-world problems.

Example 4 Write an Inequality

DELIVERIES Craig is delivering boxes of paper to each floor of an office building. Each box weighs 64 pounds, and Craig weighs 160 pounds. If the maximum capacity of the elevator is 2000 pounds, how many boxes can Craig safely take on each elevator trip?

Explore Let b = the number of boxes Craig can safely take on each trip. A maximum capacity of 2000 pounds means that this weight must be less than or equal to 2000.

Plan The total weight of the boxes is $64b$. Craig's weight plus the total weight of the boxes must be less than or equal to 2000. Write an inequality.

Craig's weight	plus	the weight of the boxes	is less than or equal to	2000.
160	+	$64b$	$\leq$	2000

Solve

$160 + 64b \leq 2000$ Original inequality

$160 - 160 + 64b \leq 2000 - 160$ Subtract 160 from each side.

$64b \leq 1840$ Simplify.

$\frac{64b}{64} \leq \frac{1840}{64}$ Divide each side by 64.

$b \leq 28.75$ Simplify.

Examine Since he cannot take a fraction of a box, Craig can take no more than 28 boxes per trip and still meet the safety requirements of the elevator.

Study Tip

Inequality Phrases

- $<$ is less than; is fewer than
- $>$ is greater than; is more than
- $\leq$ is at most; is no more than; is less than or equal to
- $\geq$ is at least; is no less than; is greater than or equal to

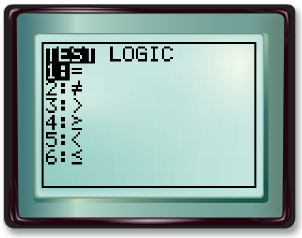
You can use a graphing calculator to find the solution set for an inequality.

Graphing Calculator Investigation

Solving Inequalities

The inequality symbols in the TEST menu on the TI-83 Plus are called *relational operators*. They compare values and return 1 if the test is true or 0 if the test is false.

You can use these relational operators to find the solution set of an inequality in one variable.



Think and Discuss

1. Clear the Y= list. Enter $11x + 3 \geq 2x - 6$ as Y1. Put your calculator in DOT mode. Then, graph in the standard viewing window. Describe the graph.
2. Using the TRACE function, investigate the graph. What values of x are on the graph? What values of y are on the graph?
3. Based on your investigation, what inequality is graphed?
4. Solve $11x + 3 \geq 2x - 6$ algebraically. How does your solution compare to the inequality you wrote in Exercise 3?

Check for Understanding

Concept Check

1. **Explain** why it is not necessary to state a division property for inequalities.
2. **Write** an inequality using the $>$ symbol whose solution set is graphed below.

3. **OPEN ENDED** Write an inequality for which the solution set is the empty set.

Guided Practice

Solve each inequality. Describe the solution set using set-builder or interval notation. Then graph the solution set on a number line.

4. $a + 2 < 3.5$
5. $5 \geq 3x$
6. $11 - c \leq 8$
7. $4y + 7 > 31$
8. $2w + 19 < 5$
9. $-0.6p < -9$
10. $\frac{n}{12} + 15 \leq 13$
11. $\frac{5z + 2}{4} < \frac{5z}{4} + 2$

Define a variable and write an inequality for each problem. Then solve.

12. The product of 12 and a number is greater than 36.
13. Three less than twice a number is at most 5.

Application

14. **SCHOOL** The final grade for a class is calculated by taking 75% of the average test score and adding 25% of the score on the final exam. If all scores are out of 100 and a student has a 76 test average, what score does the student need to make on the final exam to have a final grade of at least 80?

Practice and Apply

Homework Help

For Exercises	See Examples
15–40	1–3
41–51	4

Extra Practice

See page 829.

Solve each inequality. Describe the solution set using set-builder or interval notation. Then, graph the solution set on a number line.

15. $n + 4 \geq -7$
16. $b - 3 \leq 15$
17. $5x < 35$
18. $\frac{d}{2} > -4$
19. $\frac{g}{-3} \geq -9$
20. $-8p \geq 24$
21. $13 - 4k \leq 27$
22. $14 > 7y - 21$
23. $-27 < 8m + 5$
24. $6b + 11 \geq 15$
25. $2(4t + 9) \leq 18$
26. $90 \geq 5(2r + 6)$
27. $14 - 8n \leq 0$
28. $-4(5w - 8) < 33$
29. $0.02x + 5.58 < 0$
30. $1.5 - 0.25c < 6$
31. $6d + 3 \geq 5d - 2$
32. $9z + 2 > 4z + 15$
33. $2(g + 4) < 3g - 2(g - 5)$
34. $3(a + 4) - 2(3a + 4) \leq 4a - 1$
35. $y < \frac{-y + 2}{9}$
36. $\frac{1 - 4p}{5} < 0.2$
37. $\frac{4x + 2}{6} < \frac{2x + 1}{3}$
38. $12\left(\frac{1}{4} - \frac{n}{3}\right) \leq -6n$

39. **PART-TIME JOB** David earns \$5.60 an hour working at Box Office Videos. Each week, 25% of his total pay is deducted for taxes. If David wants his take-home pay to be at least \$105 a week, solve the inequality $5.6x - 0.25(5.6x) \geq 105$ to determine how many hours he must work.

40. **STATE FAIR** Juan's parents gave him \$35 to spend at the State Fair. He spends \$13.25 for food. If rides at the fair cost \$1.50 each, solve the inequality $1.5n + 13.25 \leq 35$ to determine how many rides he can afford.

More About. . .

Child Care

In 1995, 55% of children ages three to five were enrolled in center-based child care programs. Parents cared for 26% of children, relatives cared for 19% of children, and non-relatives cared for 17% of children.

Source: National Center for Education Statistics

Define a variable and write an inequality for each problem. Then solve.

41. The sum of a number and 8 is more than 2.

42. The product of -4 and a number is at least 35.

43. The difference of one half of a number and 7 is greater than or equal to 5.

44. One more than the product of -3 and a number is less than 16.

45. Twice the sum of a number and 5 is no more than 3 times that same number increased by 11.

46. 9 less than a number is at most that same number divided by 2.

47. **CHILD CARE** By Ohio law, when children are napping, the number of children per child care staff member may be as many as twice the maximum listed at the right. Write and solve an inequality to determine how many staff members are required to be present in a room where 17 children are napping and the youngest child is 18 months old.

Maximum Number of Children Per Child Care Staff Member
At least one child care staff member caring for:
Every 5 infants less than 12 months old (or 2 for every 12)
Every 6 infants who are at least 12 months olds, but less than 18 months old
Every 7 toddlers who are at least 18 months old, but less than 30 months old
Every 8 toddlers who are at least 30 months old, but less than 3 years old

Source: Ohio Department of Job and Family Services

CAR SALES **For Exercises 48 and 49, use the following information.**
Mrs. Lucas earns a salary of \$24,000 per year plus 1.5% commission on her sales. If the average price of a car she sells is \$30,500, about how many cars must she sell to make an annual income of at least \$40,000?

48. Write an inequality to describe this situation.

49. Solve the inequality and interpret the solution.

TEST GRADES **For Exercises 50 and 51, use the following information.**
Ahmik's scores on the first four of five 100-point history tests were 85, 91, 89, and 94.

50. If a grade of at least 90 is an A, write an inequality to find the score Ahmik must receive on the fifth test to have an A test average.

51. Solve the inequality and interpret the solution.

52. **CRITICAL THINKING** Which of the following properties hold for inequalities? Explain your reasoning or give a counterexample.

a. Reflexive **b.** Symmetric **c.** Transitive

53. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How can inequalities be used to compare phone plans?

Include the following in your answer:

- an inequality comparing the number of minutes offered by each plan, and
- an explanation of how Kuni might determine when Plan 1 might be cheaper than Plan 2 if she typically uses more than 150 but less than 400 minutes.

Standardized Test Practice

54. If $4 - 5n \geq -1$, then n could equal all of the following EXCEPT

Ⓐ $-\frac{1}{5}$.   Ⓑ $\frac{1}{5}$.   Ⓒ 1.   Ⓓ 2.

55. If $a < b$ and $c < 0$, which of the following are true?

I. $ac > bc$ **II.** $a + c < b + c$ **III.** $a - c > b - c$

Ⓐ I only   Ⓑ II only   Ⓒ III only

Ⓓ I and II only   Ⓔ I, II, and III

Graphing Calculator

Use a graphing calculator to solve each inequality.

56. $-5x - 8 < 7$ 57. $-4(6x - 3) \leq 60$ 58. $3(x + 3) \geq 2(x + 4)$

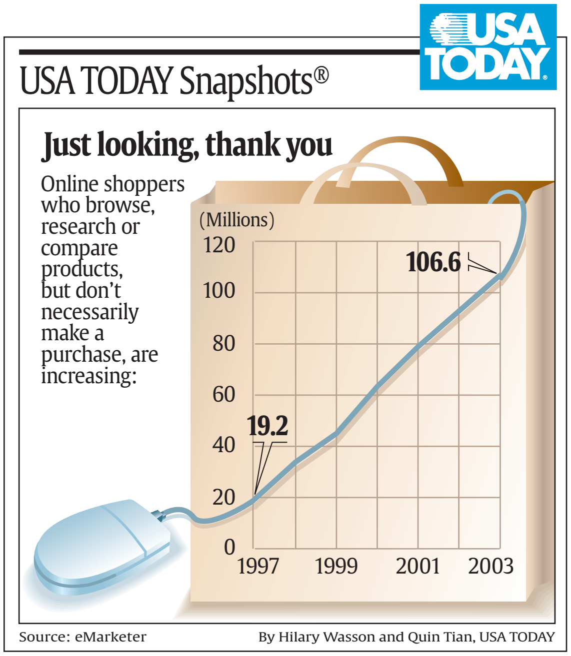
Maintain Your Skills

Mixed Review

Solve each equation. Check your solutions. *(Lesson 1-4)*

59. $|x - 3| = 17$ 60. $8|4x - 3| = 64$ 61. $|x + 1| = x$

62. **SHOPPING** On average, by how much did the number of people who just browse, but not necessarily buy, online increase each year from 1997 to 2003? Define a variable, write an equation, and solve the problem. *(Lesson 1-3)*



Name the sets of numbers to which each number belongs. *(Lesson 1-2)*

63. 31   64. $-4.\overline{2}$ 65. $\sqrt{7}$

66. **BABY-SITTING** Jenny baby-sat for $5\frac{1}{2}$ hours on Friday night and 8 hours on Saturday. She charges \$4.25 per hour. Use the Distributive Property to write two equivalent expressions that represent how much money Jenny earned. *(Lesson 1-2)*

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Solve each equation. Check your solutions.**
*(To review **solving absolute value equations**, see Lesson 1-4.)*

67. $|x| = 7$ 68. $|x + 5| = 18$ 69. $|5y - 8| = 12$

70. $|2x - 36| = 14$ 71. $2|w + 6| = 10$ 72. $|x + 4| + 3 = 17$

Practice Quiz 2

Lessons 1-3 through 1-5

1. Solve $2d + 5 = 8d + 2$. Check your solution. *(Lesson 1-3)*
2. Solve $s = \frac{1}{2}gt^2$ for g. *(Lesson 1-3)*
3. Evaluate $|x - 3y|$ if $x = -8$ and $y = 2$. *(Lesson 1-4)*
4. Solve $3|3x + 2| = 51$. Check your solutions. *(Lesson 1-4)*
5. Solve $2(m - 5) - 3(2m - 5) < 5m + 1$. Describe the solution set using set-builder or interval notation. Then graph the solution set on a number line. *(Lesson 1-5)*

1-6 Solving Compound and Absolute Value Inequalities

What You'll Learn

- Solve compound inequalities.
- Solve absolute value inequalities.

How are compound inequalities used in medicine?

One test used to determine whether a patient is diabetic and requires insulin is a glucose tolerance test. Patients start the test in a *fasting state*, meaning they have had no food or drink except water for at least 10 but no more than 16 hours. The acceptable number of hours h for fasting can be described by the following compound inequality.

$$h \geq 10 \text{ and } h \leq 16$$

Vocabulary

- compound inequality
- intersection
- union

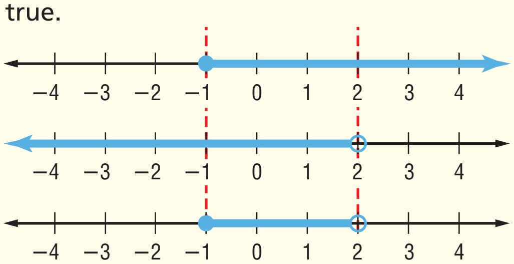
COMPOUND INEQUALITIES A **compound inequality** consists of two inequalities joined by the word *and* or the word *or*. To solve a compound inequality, you must solve each part of the inequality. The graph of a compound inequality containing *and* is the **intersection** of the solutions sets of the two inequalities.

Key Concept — "And" Compound Inequalities

- **Words** A compound inequality containing the word *and* is true if and only if *both* inequalities are true.
- **Example** $x \geq -1$

  $x < 2$

  $x \geq -1$ and $x < 2$



Another way of writing $x \geq -1$ and $x < 2$ is $-1 \leq x < 2$. Both forms are read *x is greater than or equal to −1 and less than 2.*

Study Tip

Interval Notation

The compound inequality $-1 \leq x < 2$ can be written as $[-1, 2)$, indicating that the solution set is the set of all numbers between -1 and 2, including -1, but not including 2.

Example 1 Solve an "and" Compound Inequality

Solve $13 < 2x + 7 \leq 17$. Graph the solution set on a number line.

Method 1

Write the compound inequality using the word *and*. Then solve each inequality.

$$\begin{array}{lcr} 13 < 2x + 7 & \text{and} & 2x + 7 \leq 17 \\ 6 < 2x & & 2x \leq 10 \\ 3 < x & & x \leq 5 \end{array}$$

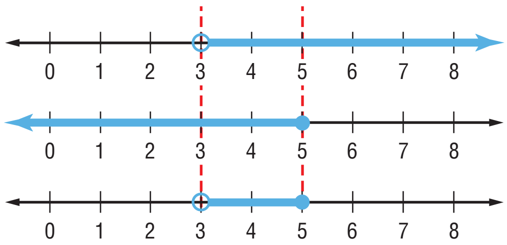
$$3 < x \leq 5$$

Method 2

Solve both parts at the same time by subtracting 7 from each part. Then divide each part by 2.

$$\begin{array}{rcl} 13 < & 2x + 7 & \leq 17 \\ 6 < & 2x & \leq 10 \\ 3 < & x & \leq 5 \end{array}$$

Graph the solution set for each inequality and find their intersection.



$x > 3$

$x \leq 5$

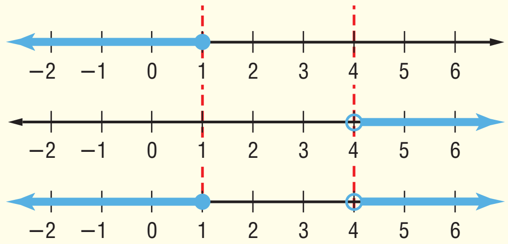
$3 < x \leq 5$

The solution set is $\{x \mid 3 < x \leq 5\}$.

The graph of a compound inequality containing *or* is the **union** of the solution sets of the two inequalities.

Key Concept — "Or" Compound Inequalities

- **Words** A compound inequality containing the word *or* is true if one or more of the inequalities is true.
- **Example** $x \leq 1$; $x > 4$; $x \leq 1$ or $x > 4$

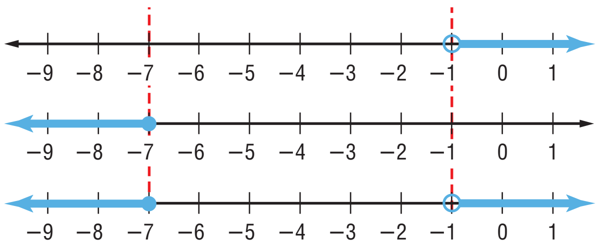


Example 2 Solve an "or" Compound Inequality

Solve $y - 2 > -3$ or $y + 4 \leq -3$. Graph the solution set on a number line.

Solve each inequality separately.

$$y - 2 > -3 \quad \text{or} \quad y + 4 \leq -3$$
$$y > -1 \qquad\qquad y \leq -7$$



$y > -1$

$y \leq -7$

$y > -1$ or $y \leq -7$

The solution set is $\{y \mid y > -1 \text{ or } y \leq -7\}$.

Study Tip

Interval Notation

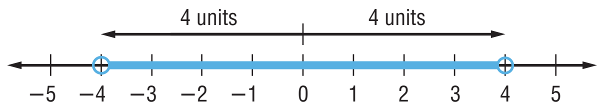
In interval notation, the symbol for the union of the two sets is $\cup$. The compound inequality $y > -1$ or $y \leq -7$ is written as $(-\infty, -7] \cup (-1, +\infty)$, indicating that all values less than and including -7 are part of the solution set. In addition, all values greater than -1, not including -1, are part of the solution set.

ABSOLUTE VALUE INEQUALITIES In Lesson 1-4, you learned that the absolute value of a number is its distance from 0 on the number line. You can use this definition to solve inequalities involving absolute value.

Example 3 Solve an Absolute Value Inequality (<)

Solve $|a| < 4$. Graph the solution set on a number line.

You can interpret $|a| < 4$ to mean that the distance between a and 0 on a number line is less than 4 units. To make $|a| < 4$ true, you must substitute numbers for a that are fewer than 4 units from 0.



Notice that the graph of $|a| < 4$ is the same as the graph of $a > -4$ and $a < 4$.

All of the numbers between -4 and 4 are less than 4 units from 0. The solution set is $\{a \mid -4 < a < 4\}$.

Study Tip

Absolute Value Inequalities

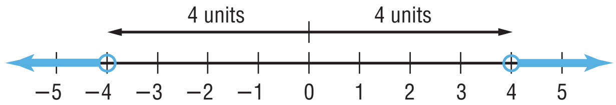
Because the absolute value of a number is never negative,

- the solution of an inequality like $|a| < -4$ is the empty set.
- the solution of an inequality like $|a| > -4$ is the set of all real numbers.

Example 4 Solve an Absolute Value Inequality (>)

Solve $|a| > 4$. Graph the solution set on a number line.

You can interpret $|a| > 4$ to mean that the distance between a and 0 is greater than 4 units. To make $|a| > 4$ true, you must substitute values for a that are greater than 4 units from 0.



Notice that the graph of $|a| > 4$ is the same as the graph of $a > 4$ or $a < -4$.

All of the numbers *not* between -4 and 4 are greater than 4 units from 0. The solution set is $\{a \mid a > 4 \text{ or } a < -4\}$.

An absolute value inequality can be solved by rewriting it as a compound inequality.

Key Concept — Absolute Value Inequalities

- **Symbols** For all real numbers a and b, $b > 0$, the following statements are true.
  1. If $|a| < b$ then $-b < a < b$.
  2. If $|a| > b$ then $a > b$ or $a < -b$.
- **Examples** If $|2x + 1| < 5$, then $-5 < 2x + 1 < 5$.
  If $|2x + 1| > 5$, then $2x + 1 > 5$ or $2x + 1 < -5$.

These statements are also true for $\leq$ and $\geq$, respectively.

Example 5 Solve a Multi-Step Absolute Value Inequality

Solve $|3x - 12| \geq 6$. Graph the solution set on a number line.

$|3x - 12| \geq 6$ is equivalent to $3x - 12 \geq 6$ or $3x - 12 \leq -6$. Solve each inequality.

$3x - 12 \geq 6$	or	$3x - 12 \leq -6$
$3x \geq 18$		$3x \leq 6$
$x \geq 6$		$x \leq 2$

The solution set is $\{x \mid x \geq 6 \text{ or } x \leq 2\}$.

CONTENTS

More About. . .

Job Hunting

When executives in a recent survey were asked to name one quality that impressed them the most about a candidate during a job interview, 32 percent said honesty and integrity.

Source: careerexplorer.net

Example 6 Write an Absolute Value Inequality

JOB HUNTING **To prepare for a job interview, Megan researches the position's requirements and pay. She discovers that the average starting salary for the position is \$38,500, but her actual starting salary could differ from the average by as much as \$2450.**

a. Write an absolute value inequality to describe this situation.

Let x = Megan's starting salary.

$$\underbrace{|38{,}500 - x|}_{\text{Her starting salary could differ from the average}} \quad \underbrace{\leq}_{\text{by as much as}} \quad \underbrace{2450}_{\$2450.}$$

b. Solve the inequality to find the range of Megan's starting salary.

Rewrite the absolute value inequality as a compound inequality. Then solve for x.

$$\begin{aligned} -2450 \leq \quad & 38{,}500 - x \quad \leq 2450 \\ -2450 - 38{,}500 \leq \quad & 38{,}500 - x - 38{,}500 \leq 2450 - 38{,}500 \\ -40{,}950 \leq \quad & -x \quad \leq -36{,}050 \\ 40{,}950 \geq \quad & x \quad \geq 36{,}050 \end{aligned}$$

The solution set is $\{x \mid 36{,}050 \leq x \leq 40{,}950\}$. Thus, Megan's starting salary will fall between \$36,050 and \$40,950, inclusive.

Check for Understanding

Concept Check

1. **Write** a compound inequality to describe the following situation. *Buy a present that costs at least \$5 and at most \$15.*

2. **OPEN ENDED** Write a compound inequality whose graph is the empty set.

3. **FIND THE ERROR** Sabrina and Isaac are solving $|3x + 7| > 2$.

Sabrina	Isaac
$\lvert 3x+7 \rvert > 2$	$\lvert 3x+7 \rvert > 2$
$3x + 7 > 2$ or $3x + 7 < -2$	$-2 < 3x + 7 < 2$
$3x > 25 \qquad 3x < -9$	$-9 < 3x < -5$
$x > -\frac{5}{3} \qquad x < -3$	$-3 < x < -\frac{5}{3}$

Who is correct? Explain your reasoning.

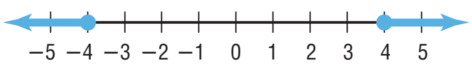
Guided Practice

Write an absolute value inequality for each of the following. Then graph the solution set on a number line.

4. all numbers between −8 and 8

5. all numbers greater than 3 and less than −3

Write an absolute value inequality for each graph.

6. 

7. 

Solve each inequality. Graph the solution set on a number line.

8. $y - 3 > 1$ or $y + 2 < 1$
9. $3 < d + 5 < 8$
10. $|a| \geq 5$
11. $|g + 4| \leq 9$
12. $|4k - 8| < 20$
13. $|w| \geq -2$

Application

14. **FLOORING** Deion estimates that he will need between 55 and 60 ceramic tiles to retile his kitchen floor. If each tile costs \$6.25, write and solve a compound inequality to determine what the cost c of the tile could be.

Practice and Apply

Homework Help

For Exercises	See Examples
15–26, 33–44	3–5
27–32, 51, 52	1, 2
45–50	6

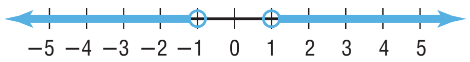
Extra Practice

See page 829.

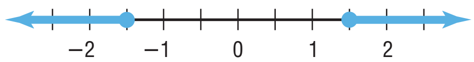
Write an absolute value inequality for each of the following. Then graph the solution set on a number line.

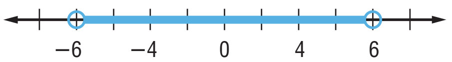
15. all numbers greater than or equal to 5 or less than or equal to -5
16. all numbers less than 7 and greater than -7
17. all numbers between -4 and 4
18. all numbers less than or equal to -6 or greater than or equal to 6
19. all numbers greater than 8 or less than -8
20. all number less than or equal to 1.2 and greater than or equal to -1.2

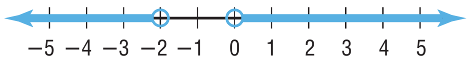
Write an absolute value inequality for each graph.

21. 

22. 

23. 

24. 

25. 

26. 

More About. . .

Betta Fish

Adult Male Size: 3 inches
Water pH: 6.8–7.4
Temperature: 75–86°F
Diet: omnivore, prefers live foods
Tank Level: top dweller
Difficulty of Care: easy to intermediate
Life Span: 2–3 years
Source: www.about.com

Solve each inequality. Graph the solution set on a number line.

27. $3p + 1 \leq 7$ or $2p - 9 \geq 7$
28. $9 < 3t + 6 < 15$
29. $-11 < -4x + 5 < 13$
30. $2c - 1 < -5$ or $3c + 2 \geq 5$
31. $-4 < 4f + 24 < 4$
32. $a + 2 > -2$ or $a - 8 < 1$
33. $|g| \leq 9$
34. $|2m| \geq 8$
35. $|3k| < 0$
36. $|-5y| < 35$
37. $|b - 4| > 6$
38. $|6r - 3| < 21$
39. $|3w + 2| \leq 5$
40. $|7x| + 4 < 0$
41. $|n| \geq n$
42. $|n| \leq n$
43. $|2n - 7| \leq 0$
44. $|n - 3| < n$

45. **BETTA FISH** A Siamese Fighting Fish, also known as a Betta fish, is one of the most recognized and colorful fish kept as a pet. Using the information at the left, write a compound inequality to describe the acceptable range of water pH levels for a male Betta.

CONTENTS

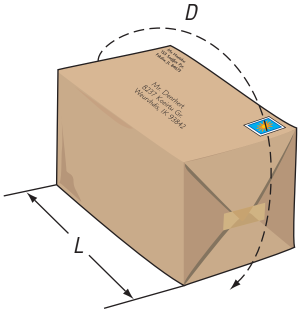
SPEED LIMITS **For Exercises 46 and 47, use the following information.**
On some interstate highways, the maximum speed a car may drive is 65 miles per hour. A tractor-trailer may not drive more than 55 miles per hour. The minimum speed for all vehicles is 45 miles per hour.

46. Write an inequality to represent the allowable speed for a car on an interstate highway.

47. Write an inequality to represent the speed at which a tractor-trailer may travel on an interstate highway.

48. **HEALTH** *Hypothermia* and *hyperthermia* are similar words but have opposite meanings. Hypothermia is defined as a lowered body temperature. Hyperthermia means an extremely high body temperature. Both conditions are potentially dangerous and occur when a person's body temperature fluctuates by more than 8° from the normal body temperature of 98.6°F. Write and solve an absolute value inequality to describe body temperatures that are considered potentially dangerous.

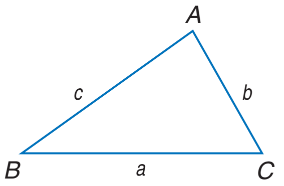
MAIL **For Exercises 49 and 50, use the following information.**
The U.S. Postal Service defines an oversized package as one for which the length L of its longest side plus the distance D around its thickest part is more than 108 inches and less than or equal to 130 inches.



49. Write a compound inequality to describe this situation.

50. If the distance around the thickest part of a package you want to mail is 24 inches, describe the range of lengths that would classify your package as oversized.

GEOMETRY **For Exercises 51 and 52, use the following information.**
The *Triangle Inequality Theorem* states that the sum of the measures of any two sides of a triangle is greater than the measure of the third side.



51. Write three inequalities to express the relationships among the sides of $\triangle ABC$.

52. Write a compound inequality to describe the range of possible measures for side c in terms of a and b. Assume that $a > b > c$. (*Hint*: Solve each inequality you wrote in Exercise 51 for c.)

53. **CRITICAL THINKING** **Graph each set on a number line.**

a. $-2 < x < 4$

b. $x < -1$ or $x > 3$

c. $(-2 < x < 4)$ and $(x < -1$ or $x > 3)$ (*Hint*: This is the intersection of the graphs in part **a** and part **b**.)

d. Solve $3 < |x + 2| \leq 8$. Explain your reasoning and graph the solution set.

54. ***WRITING IN MATH*** Answer the question that was posed at the beginning of the lesson.

How are compound inequalities used in medicine?

Include the following in your answer:

- an explanation as to when to use *and* and when to use *or* when writing a compound inequality,
- an alternative way to write $h \geq 10$ and $h \leq 16$, and
- an example of an acceptable number of hours for this fasting state and a graph to support your answer.

Standardized Test Practice

55. **SHORT RESPONSE** Solve $|2x + 11| > 1$ for x.

56. If $5 < a < 7 < b < 14$, then which of the following best defines $\frac{a}{b}$?

(A) $\frac{5}{7} < \frac{a}{b} < \frac{1}{2}$ (B) $\frac{5}{14} < \frac{a}{b} < \frac{1}{2}$

(C) $\frac{5}{7} < \frac{a}{b} < 1$ (D) $\frac{5}{14} < \frac{a}{b} < 1$

Graphing Calculator

LOGIC MENU **For Exercises 57–60, use the following information.**
You can use the operators in the LOGIC menu on the TI-83 Plus to graph compound and absolute value inequalities. To display the LOGIC menu, press [2nd] [TEST] [▶].

57. Clear the Y= list. Enter (5x + 2 > 12) and (3x − 8 < 1) as Y1. With your calculator in DOT mode and using the standard viewing window, press [GRAPH]. Make a sketch of the graph displayed.

58. Using the TRACE function, investigate the graph. Based on your investigation, what inequality is graphed?

59. Write the expression you would enter for Y1 to find the solution set of the compound inequality $5x + 2 \geq 3$ or $5x + 2 \leq -3$. Then use the graphing calculator to find the solution set.

60. A graphing calculator can also be used to solve absolute value inequalities. Write the expression you would enter for Y1 to find the solution set of the inequality $|2x - 6| > 10$. Then use the graphing calculator to find the solution set. (*Hint*: The absolute value operator is item 1 on the MATH NUM menu.)

Maintain Your Skills

Mixed Review

Solve each inequality. Describe the solution set using set builder or interval notation. Then graph the solution set on a number line. *(Lesson 1-5)*

61. $2d + 15 \geq 3$ 62. $7x + 11 > 9x + 3$ 63. $3n + 4(n + 3) < 5(n + 2)$

64. **CONTESTS** To get a chance to win a car, you must guess the number of keys in a jar to within 5 of the actual number. Those who are within this range are given a key to try in the ignition of the car. Suppose there are 587 keys in the jar. Write and solve an equation to determine the highest and lowest guesses that will give contestants a chance to win the car. *(Lesson 1-4)*

Solve each equation. Check your solutions.

65. $5|x - 3| = 65$ 66. $|2x + 7| = 15$ 67. $|8c + 7| = -4$

Name the property illustrated by each statement. *(Lesson 1-3)*

68. If $3x = 10$, then $3x + 7 = 10 + 7$.

69. If $-5 = 4y - 8$, then $4y - 8 = -5$.

70. If $-2x - 5 = 9$ and $9 = 6x + 1$, then $-2x - 5 = 6x + 1$.

Simplify each expression. *(Lesson 1-2)*

71. $6a - 2b - 3a + 9b$ 72. $-2(m - 4n) - 3(5n + 6)$

Find the value of each expression. *(Lesson 1-1)*

73. $6(5 - 8) \div 9 + 4$ 74. $(3 + 7)^2 - 16 \div 2$ 75. $\frac{7(1 - 4)}{8 - 5}$

Chapter 1

Study Guide and Review

Vocabulary and Concept Check

absolute value (p. 28)
Addition Property
  of Equality (p. 21)
  of Inequality (p. 33)
algebraic expression (p. 7)
Associative Property (p. 12)
Commutative Property (p. 12)
compound inequality (p. 40)
counterexample (p. 14)
Distributive Property (p. 12)
Division Property
  of Equality (p. 21)
  of Inequality (p. 34)
empty set (p. 29)
equation (p. 20)
formula (p. 8)
Identity Property (p. 12)
intersection (p. 40)
interval notation (p. 35)
Inverse Property (p. 12)
irrational numbers (p. 11)
Multiplication Property
  of Equality (p. 21)
  of Inequality (p. 34)
open sentence (p. 20)
order of operations (p. 6)
rational numbers (p. 11)
real numbers (p. 11)
Reflexive Property (p. 21)
set-builder notation (p. 34)
solution (p. 20)
Substitution Property (p. 21)
Subtraction Property
  of Equality (p. 21)
  of Inequality (p. 33)
Symmetric Property (p. 21)
Transitive Property (p. 21)
Trichotomy Property (p. 33)
union (p. 41)
variable (p. 7)

Choose the term from the list above that best matches each example.

1. $y > 3$ or $y < -2$
2. $0 + (-4b) = -4b$
3. $(m - 1)(-2) = -2(m - 1)$
4. $35x + 56 = 7(5x + 8)$
5. $ab + 1 = ab + 1$
6. If $2x = 3y - 4$, $3y - 4 = 7$, then $2x = 7$.
7. $4(0.25) = 1$
8. $2p + (4 + 9r) = (2p + 4) + 9r$
9. $|5n|$
10. $6y + 5z - 2(x + y)$

Lesson-by-Lesson Review

1-1 Expressions and Formulas

See pages 6–10.

Concept Summary

- Order of Operations

Step 1 Simplify the expressions inside grouping symbols, such as parentheses, ( ), brackets, [ ], braces, { }, and fraction bars.

Step 2 Evaluate all powers.

Step 3 Do all multiplications and/or divisions from left to right.

Step 4 Do all additions and/or subtractions from left to right.

Example **Evaluate $\frac{y^3}{3ab + 2}$ if $y = 4$, $a = -2$, and $b = -5$.**

$$\frac{y^3}{3ab + 2} = \frac{4^3}{3(-2)(-5) + 2} \quad y = 4,\ a = -2, \text{ and } b = -5$$

$$= \frac{64}{3(10) + 2} \quad \text{Evaluate the numerator and denominator separately.}$$

$$= \frac{64}{32} \text{ or } 2$$

Exercises Find the value of each expression. *See Example 1 on page 6.*

11. $10 + 16 \div 4 + 8$ **12.** $[21 - (9 - 2)] \div 2$ **13.** $\frac{14(8 - 15)}{2}$

Evaluate each expression if $a = 12$, $b = 0.5$, $c = -3$, and $d = \frac{1}{3}$.
See Examples 2 and 3 on page 7.

14. $6b - 5c$ **15.** $c^3 + ad$ **16.** $\frac{9c + ab}{c}$ **17.** $a[b^2(b + a)]$

1-2 Properties of Real Numbers

See pages 11–18.

Concept Summary

- Real numbers (R) can be classified as rational (Q) or irrational (I).
- Rational numbers can be classified as natural numbers (N), whole numbers (W), and/or integers (Z).
- Use the properties of real numbers to simplify algebraic expressions.

Example Simplify $4(2b + 6c) + 3b - c$.

$$\begin{aligned} 4(2b + 6c) + 3b - c &= 4(2b) + 4(6c) + 3b - c && \text{Distributive Property} \\ &= 8b + 24c + 3b - c && \text{Multiply.} \\ &= 8b + 3b + 24c - c && \text{Commutative Property } (+) \\ &= (8 + 3)b + (24 - 1)c && \text{Distributive Property} \\ &= 11b + 23c && \text{Add 3 to 8 and subtract 1 from 24.} \end{aligned}$$

Exercises Name the sets of numbers to which each value belongs.
See Example 1 on page 12.

18. $-\sqrt{9}$ **19.** $1.\overline{6}$ **20.** $\frac{35}{7}$ **21.** $\sqrt{18}$

Simplify each expression. *See Example 5 on page 14.*

22. $2m + 7n - 6m - 5n$ **23.** $-5(a - 4b) + 4b$ **24.** $2(5x + 4y) - 3(x + 8y)$

1-3 Solving Equations

See pages 20–27.

Concept Summary

- Verbal expressions can be translated into algebraic expressions using the language of algebra, using variables to represent the unknown quantities.
- Use the properties of equality to solve equations.

Example Solve $4(a + 5) - 2(a + 6) = 3$.

$$\begin{aligned} 4(a + 5) - 2(a + 6) &= 3 && \text{Original equation} \\ 4a + 20 - 2a - 12 &= 3 && \text{Distributive Property} \\ 2a + 8 &= 3 && \text{Commutative, Distributive, and Substitution Properties} \\ 2a &= -5 && \text{Subtraction Property } (=) \\ a &= -2.5 && \text{Division Property } (=) \end{aligned}$$

Exercises **Solve each equation. Check your solution.**
See Examples 3 and 4 on pages 21 and 22.

25. $x - 6 = -20$ **26.** $-\frac{2}{3}a = 14$ **27.** $7 + 5n = -58$

28. $3w + 14 = 7w + 2$ **29.** $5y + 4 = 2(y - 4)$ **30.** $\frac{n}{4} + \frac{n}{3} = \frac{1}{2}$

Solve each equation or formula for the specified variable. *See Example 5 on page 22.*

31. $Ax + By = C$ for x **32.** $\frac{a - 4b^2}{2c} = d$ for a **33.** $A = p + prt$ for p

1-4 Solving Absolute Value Equations

See pages 28–32.

Concept Summary

- For any real numbers a and b, where $b \geq 0$, if $|a| = b$, then $a = b$ or $a = -b$.

Example

Solve $|2x + 9| = 11$.

Case 1 $a = b$ **or** **Case 2** $a = -b$

$$2x + 9 = 11 \qquad 2x + 9 = -11$$
$$2x = 2 \qquad 2x = -20$$
$$x = 1 \qquad x = -10$$

The solution set is $\{1, -10\}$. Check these solutions in the original equation.

Exercises **Solve each equation. Check your solutions.**
See Examples 1–4 on pages 28–30.

34. $|x + 11| = 42$ **35.** $3|x + 6| = 36$ **36.** $|4x - 5| = -25$

37. $|x + 7| = 3x - 5$ **38.** $|y - 5| - 2 = 10$ **39.** $4|3x + 4| = 4x + 8$

1-5 Solving Inequalities

See pages 33–39.

Concept Summary

- Adding the same number to, or subtracting the same number from, each side of an inequality does not change the truth of the inequality.
- When you multiply or divide each side of an inequality by a negative number, the direction of the inequality symbol must be *reversed*.

Example

Solve $5 - 4a > 8$. Graph the solution set on a number line.

$5 - 4a > 8$ Original inequality

$-4a > 3$ Subtract 5 from each side.

$a < -\frac{3}{4}$ Divide each side by -4, reversing the inequality symbol.

The solution set is $\left\{a \,\middle|\, a < -\frac{3}{4}\right\}$.

The graph of the solution set is shown at the right.

Chapter 1 For More ...
• Extra Practice, see pages 828–829.
• Mixed Problem Solving, see page 862.

Exercises **Solve each inequality. Describe the solution set using set builder or interval notation. Then graph the solution set on a number line.**

See Examples 1–3 on pages 34–35.

40. $-7w > 28$ **41.** $3x + 4 \geq 19$ **42.** $\frac{n}{12} + 5 \leq 7$

43. $3(6 - 5a) < 12a - 36$ **44.** $2 - 3z \geq 7(8 - 2z) + 12$ **45.** $8(2x - 1) > 11x - 17$

1-6 Solving Compound and Absolute Value Inequalities

See pages 40–46.

Concept Summary

- The graph of an *and* compound inequality is the intersection of the solution sets of the two inequalities.
- The graph of an *or* compound inequality is the union of the solution sets of the two inequalities.
- For all real numbers a and b, $b > 0$, the following statements are true.
  1. If $|a| < b$ then $-b < a < b$.
  2. If $|a| > b$ then $a > b$ or $a < -b$.

Examples **Solve each inequality. Graph the solution set on a number line.**

1 $\mathbf{-19 < 4d - 7 \leq 13}$

$-19 < 4d - 7 \leq 13$ Original inequality

$-12 < 4d \leq 20$ Add 7 to each part.

$-3 < d \leq 5$ Divide each part by 4.

The solution set is $\{x \mid -3 < d \leq 5\}$.

2 $\mathbf{|2x + 4| \geq 12}$

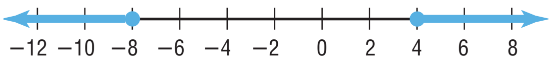
$|2x + 4| \geq 12$ is equivalent to $2x + 4 \geq 12$ or $2x + 4 \leq -12$.

$2x + 4 \geq 12$ or $2x + 4 \leq -12$ Original inequality

$2x \geq 8$ $2x \leq -16$ Subtract 4 from each side.

$x \geq 4$ $x \leq -8$ Divide each side by 2.

The solution set is $\{x \mid x \geq 4 \text{ or } x \leq -8\}$.



Exercises **Solve each inequality. Graph the solution set on a number line.**

See Examples 1–5 on pages 40–42.

46. $-1 < 3a + 2 < 14$ **47.** $-1 < 3(y - 2) \leq 9$ **48.** $|x| + 1 > 12$

49. $|2y - 9| \leq 27$ **50.** $|5n - 8| > -4$ **51.** $|3b + 11| > 1$

Chapter 1 Practice Test

Vocabulary and Concepts

Choose the term that best completes each sentence.

1. An algebraic *(equation, expression)* contains an equals sign.
2. *(Whole numbers, Rationals)* are a subset of the set of integers.
3. If $x + 3 = y$, then $y = x + 3$ is an example of the *(Transitive, Symmetric)* Property of Equality.

Skills and Applications

Find the value of each expression.

4. $[(3 + 6)^2 \div 3] \times 4$
5. $\dfrac{20 + 4 \times 3}{11 - 3}$
6. $0.5(2.3 + 25) \div 1.5$

Evaluate each expression if $a = -9$, $b = \frac{2}{3}$, $c = 8$, and $d = -6$.

7. $\dfrac{db + 4c}{a}$
8. $\dfrac{a}{b^2} + c$
9. $2b(4a + a^2)$

Name the sets of numbers to which each number belongs.

10. $\sqrt{17}$
11. 0.86
12. $\sqrt{64}$

Name the property illustrated by each equation or statement.

13. $(7 \cdot s) \cdot t = 7 \cdot (s \cdot t)$
14. If $(r + s)t = rt + st$, then $rt + st = (r + s)t$.
15. $\left(3 \cdot \frac{1}{3}\right) \cdot 7 = \left(3 \cdot \frac{1}{3}\right) \cdot 7$
16. $(6 - 2)a - 3b = 4a - 3b$
17. $(4 + x) + y = y + (4 + x)$
18. If $5(3) + 7 = 15 + 7$ and $15 + 7 = 22$, then $5(3) + 7 = 22$.

Solve each equation. Check your solution(s).

19. $5t - 3 = -2t + 10$
20. $2x - 7 - (x - 5) = 0$
21. $5m - (5 + 4m) = (3 + m) - 8$
22. $|8w + 2| + 2 = 0$
23. $12\left|\frac{1}{2}y + 3\right| = 6$
24. $2|2y - 6| + 4 = 8$

Solve each inequality. Describe the solution set using set builder or interval notation. Then graph the solution set on a number line.

25. $4 > b + 1$
26. $3q + 7 \geq 13$
27. $5(3x - 5) + x < 2(4x - 1) + 1$
28. $|5 + k| \leq 8$
29. $-12 < 7d - 5 \leq 9$
30. $|3y - 1| > 5$

For Exercises 31 and 32, define a variable, write an equation or inequality, and solve the problem.

31. **CAR RENTAL** Mrs. Denney is renting a car that gets 35 miles per gallon. The rental charge is \$19.50 a day plus 18¢ per mile. Her company will reimburse her for \$33 of this portion of her travel expenses. If Mrs. Denney rents the car for 1 day, find the maximum number of miles that will be paid for by her company.
32. **SCHOOL** To receive a B in his English class, Nick must have an average score of at least 80 on five tests. He scored 87, 89, 76, and 77 on his first four tests. What must he score on the last test to receive a B in the class?

33. **STANDARDIZED TEST PRACTICE** If $\frac{a}{b} = 8$ and $ac - 5 = 11$, then $bc =$

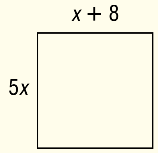
(A) 93. (B) 2. (C) $\frac{5}{8}$. (D) cannot be determined

Chapter 1 Standardized Test Practice

Part 1 Multiple Choice

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

1. In the square at the right, what is the value of x?



(A) 1 (B) 2

(C) 3 (D) 4

2. On a college math test, 18 students earned an A. This number is exactly 30% of the total number of students in the class. How many students are in the class?

(A) 5 (B) 23

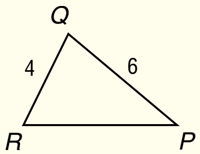
(C) 48 (D) 60

3. A student computed the average of her 7 test scores by adding the scores together and dividing this total by the number of tests. The average was 87. On her next test, she scored a 79. What is her new test average?

(A) 83 (B) 84

(C) 85 (D) 86

4. If the perimeter of $\triangle PQR$ is 3 times the length of PQ, then $PR =$ ____.



Note: Figure not drawn to scale.

(A) 4 (B) 6

(C) 7 (D) 8

5. If a different number is selected from each of the three sets shown below, what is the greatest sum these 3 numbers could have?

R = {3, 6, 7}; S = {2, 4, 7}; T = {1, 3, 7}

(A) 13 (B) 14

(C) 17 (D) 21

6. A pitcher contains a ounces of orange juice. If b ounces of juice are poured from the pitcher into each of c glasses, which expression represents the amount of juice remaining in the pitcher?

(A) $\frac{a}{b} + c$ (B) $ab - c$

(C) $a - bc$ (D) $\frac{a}{bc}$

7. The sum of three consecutive integers is 135. What is the greatest of the three integers?

(A) 43 (B) 44

(C) 45 (D) 46

8. The ratio of girls to boys in a class is 5 to 4. If there are a total of 27 students in the class, how many are girls?

(A) 15 (B) 12

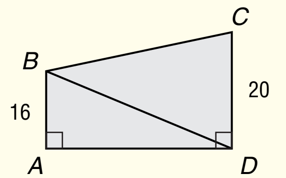
(C) 9 (D) 5

9. For which of the following ordered pairs (x, y) is $x + y > 3$ and $x - y < -2$?

(A) (0, 3) (B) (3, 4)

(C) (5, 3) (D) (2, 5)

10. If the area of $\triangle ABD$ is 280, what is the area of the polygon $ABCD$?



Note: Figure not drawn to scale.

(A) 560 (B) 630

(C) 700 (D) 840

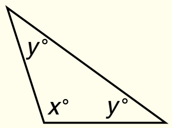
The Princeton Review Test-Taking Tip

Question 9 To solve equations or inequalities, you can replace the variables in the question with the values given in each answer choice. The answer choice that results in true statements is the correct answer choice.

Part 2 Short Response/Grid In

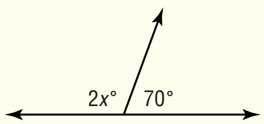
Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

11. In the triangle below, x and y are integers. If $25 < y < 30$, what is one possible value of x?



12. If n and p are each different positive integers and $n + p = 4$, what is one possible value of $3n + 4p$?

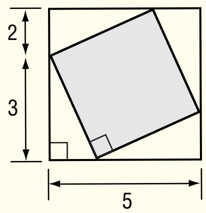
13. In the figure at the right, what is the value of x?



14. One half quart of lemonade concentrate is mixed with $1\frac{1}{2}$ quarts of water to make lemonade for 6 people. If you use the same proportions of concentrate and water, how many quarts of lemonade concentrate are needed to make lemonade for 21 people?

15. If 25 percent of 300 is equal to 500 percent of t, then t is equal to what number?

16. In the figure below, what is the area of the shaded square in square units?



17. There are 140 students in the school band. One of these students will be selected at random to be the student representative. If the probability that a brass player is selected is $\frac{2}{5}$, how many brass players are in the band?

18. A shelf holds fewer than 50 cans. If all of the cans on this shelf were put into stacks of five cans each, no cans would remain. If the same cans were put into stacks of three cans each, one can would remain. What is the greatest number of cans that could be on the shelf?

Part 3 Quantitative Comparison

Compare the quantity in Column A and the quantity in Column B. Then determine whether:

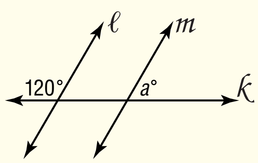
- Ⓐ **the quantity in Column A is greater;**
- Ⓑ **the quantity in Column B is greater;**
- Ⓒ **the two quantities are equal;**
- Ⓓ **the relationship cannot be determined from the information given.**

	Column A	Column B
19.	$\dfrac{\frac{3}{4}}{\left(\frac{3}{4}\right)^2}$	$\frac{4}{3}$
20.	$x + 13$	$x + 14$

21. $0 < s < \frac{3}{4}$

Column A	Column B
1	$3s$

22.



$\ell \parallel m$

Column A	Column B
120	$2a$

23. The average (arithmetic mean) of s and t is greater than the average of s and w.

Column A	Column B
w	t

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