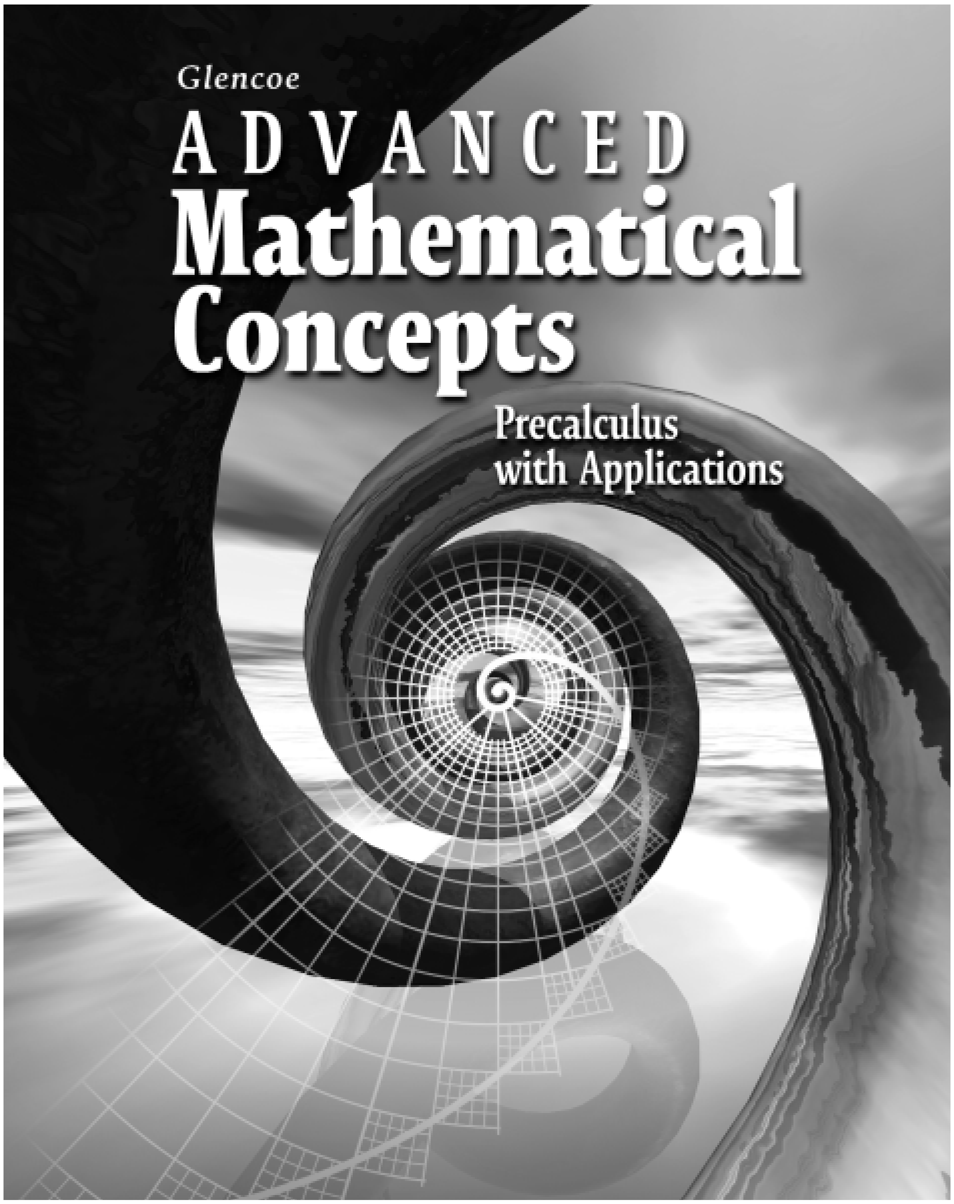
Chapter 9
Resource Masters



Glencoe

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StudentWorks™ This CD-ROM includes the entire Student Edition along with the Study Guide, Practice, and Enrichment masters.

TeacherWorks™ All of the materials found in this booklet are included for viewing and printing in the *Advanced Mathematical Concepts TeacherWorks* CD-ROM.

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Advanced Mathematical Concepts
Chapter 9 Resource Masters

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Contents

A Teacher's Guide to Using the Chapter 9 Resource Masters

The ***Fast File*** Chapter Resource system allows you to conveniently file the resources you use most often. The *Chapter 9 Resource Masters* include the core materials needed for Chapter 9. These materials include worksheets, extensions, and assessment options. The answers for these pages appear at the back of this booklet.

All of the materials found in this booklet are included for viewing and printing in the *Advanced Mathematical Concepts TeacherWorks* CD-ROM.

Vocabulary Builder Pages vii-ix include a student study tool that presents the key vocabulary terms from the chapter. Students are to record definitions and/or examples for each term. You may suggest that students highlight or star the terms with which they are not familiar.

When to Use Give these pages to students before beginning Lesson 9-1. Remind them to add definitions and examples as they complete each lesson.

Study Guide There is one Study Guide master for each lesson.

When to Use Use these masters as reteaching activities for students who need additional reinforcement. These pages can also be used in conjunction with the Student Edition as an instructional tool for those students who have been absent.

Practice There is one master for each lesson. These problems more closely follow the structure of the Practice section of the Student Edition exercises. These exercises are of average difficulty.

When to Use These provide additional practice options or may be used as homework for second day teaching of the lesson.

Enrichment There is one master for each lesson. These activities may extend the concepts in the lesson, offer a historical or multicultural look at the concepts, or widen students' perspectives on the mathematics they are learning. These are not written exclusively for honors students, but are accessible for use with all levels of students.

When to Use These may be used as extra credit, short-term projects, or as activities for days when class periods are shortened.

Assessment Options

The assessment section of the *Chapter 9 Resources Masters* offers a wide range of assessment tools for intermediate and final assessment. The following lists describe each assessment master and its intended use.

Chapter Assessments

Chapter Tests

- *Forms 1A, 1B, and 1C* Form 1 tests contain multiple-choice questions. Form 1A is intended for use with honors-level students, Form 1B is intended for use with average-level students, and Form 1C is intended for use with basic-level students. These tests are similar in format to offer comparable testing situations.
- *Forms 2A, 2B, and 2C* Form 2 tests are composed of free-response questions. Form 2A is intended for use with honors-level students, Form 2B is intended for use with average-level students, and Form 2C is intended for use with basic-level students. These tests are similar in format to offer comparable testing situations.

  All of the above tests include a challenging Bonus question.
- The **Extended Response Assessment** includes performance assessment tasks that are suitable for all students. A scoring rubric is included for evaluation guidelines. Sample answers are provided for assessment.

Intermediate Assessment

- A **Mid-Chapter Test** provides an option to assess the first half of the chapter. It is composed of free-response questions.
- Four free-response **quizzes** are included to offer assessment at appropriate intervals in the chapter.

Continuing Assessment

- The **SAT and ACT Practice** offers continuing review of concepts in various formats, which may appear on standardized tests that they may encounter. This practice includes multiple-choice, quantitative-comparison, and grid-in questions. Bubble-in and grid-in answer sections are provided on the master.
- The **Cumulative Review** provides students an opportunity to reinforce and retain skills as they proceed through their study of advanced mathematics. It can also be used as a test. The master includes free-response questions.

Answers

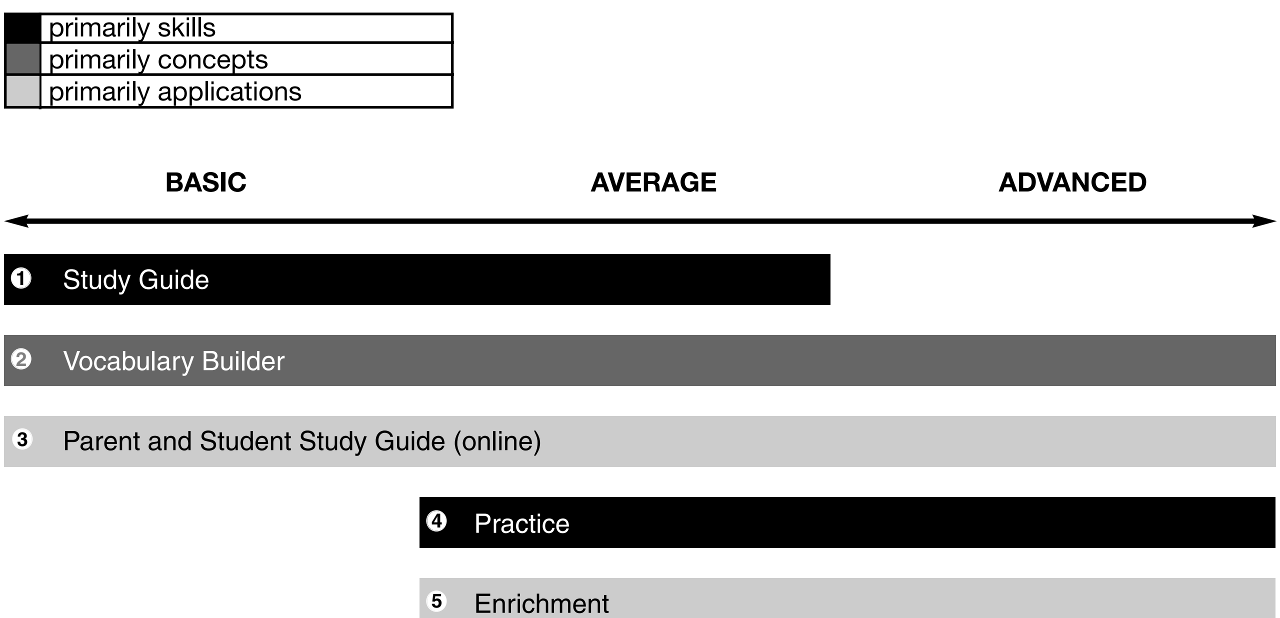
- Page A1 is an answer sheet for the SAT and ACT Practice questions that appear in the Student Edition on page 613. Page A2 is an answer sheet for the SAT and ACT Practice master. These improve students' familiarity with the answer formats they may encounter in test taking.
- The answers for the lesson-by-lesson masters are provided as reduced pages with answers appearing in red.
- Full-size answer keys are provided for the assessment options in this booklet.

Chapter 9 Leveled Worksheets

Glencoe's **leveled worksheets** are helpful for meeting the needs of every student in a variety of ways. These worksheets, many of which are found in the **FAST FILE Chapter Resource Masters**, are shown in the chart below.

- **Study Guide** masters provide worked-out examples as well as practice problems.
- Each chapter's **Vocabulary Builder** master provides students the opportunity to write out key concepts and definitions in their own words.
- **Practice** masters provide average-level problems for students who are moving at a regular pace.
- **Enrichment** masters offer students the opportunity to extend their learning.

Five Different Options to Meet the Needs of Every Student in a Variety of Ways



Chapter 9

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Reading to Learn Mathematics

Vocabulary Builder

This is an alphabetical list of the key vocabulary terms you will learn in Chapter 9. As you study the chapter, complete each term's definition or description. Remember to add the page number where you found the term.

Vocabulary Term	Found on Page	Definition/Description/Example
absolute value of a complex number		
amplitude of a complex number		
Argand plane		
argument of a complex number		
cardioid		
complex conjugates		
complex number		
complex plane		
escape set		
imaginary number		

(continued on the next page)

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Reading to Learn Mathematics

Vocabulary Builder *(continued)*

Vocabulary Term	Found on Page	Definition/Description/Example
imaginary part		
iteration		
Julia set		
lemniscate		
limaçon		
modulus		
polar axis		
polar coordinates		
polar equation		
polar form of a complex number		
polar graph		

(continued on the next page)

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Reading to Learn Mathematics

Vocabulary Builder *(continued)*

Vocabulary Term	Found on Page	Definition/Description/Example
polar plane		
pole		
prisoner set		
pure imaginary number		
real part		
rectangular form of a complex number		
rose		
spiral of Archimedes		
trigonometric form of a complex number		

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Study Guide

Polar Coordinates

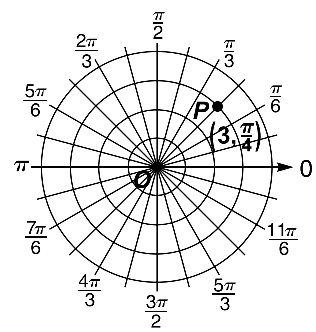
A **polar coordinate system** uses distances and angles to record the position of a point. The location of a point P can be identified by polar coordinates in the form (r, θ), where $|r|$ is the distance from the pole, or origin, to point P and θ is the measure of the angle formed by the ray from the pole to point P and the **polar axis.**

Example 1 **Graph each point.**

a. $P\left(3, \frac{\pi}{4}\right)$

Sketch the terminal side of an angle measuring $\frac{\pi}{4}$ radians in standard position.

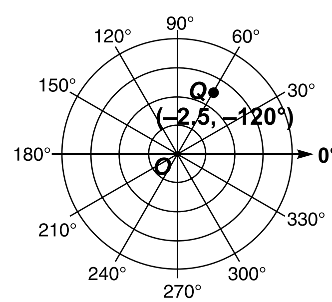
Since r is positive ($r = 3$), find the point on the terminal side of the angle that is 3 units from the pole. *Notice point P is on the third circle from the pole.*



b. **Q(−2.5, −120°)**

Negative angles are measured clockwise. Sketch the terminal side of an angle of −120° in standard position.

Since r is negative, extend the terminal side of the angle in the opposite direction. Find the point Q that is 2.5 units from the pole along this extended ray.



Example 2 **Find the distance between $P_1(3, 70°)$ and $P_2(5, 120°)$.**

$$P_1P_2 = \sqrt{r_1^2 + r_2^2 - 2r_1r_2\cos(\theta_2 - \theta_1)}$$
$$= \sqrt{3^2 + 5^2 - 2(3)(5)\cos(120° - 70°)}$$
$$\approx 3.84$$

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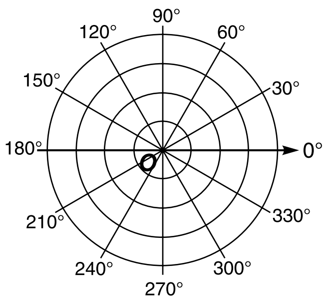
9-1

Practice

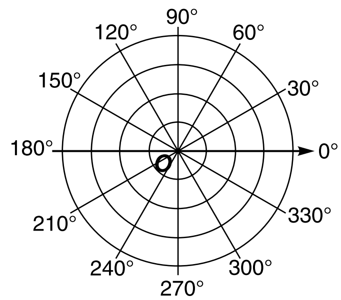
Polar Coordinates

Graph each point.

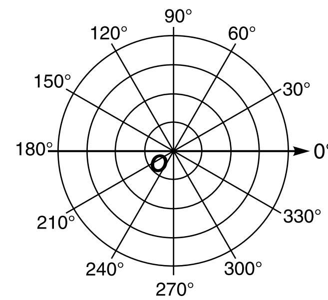
1. $(2.5, 0°)$



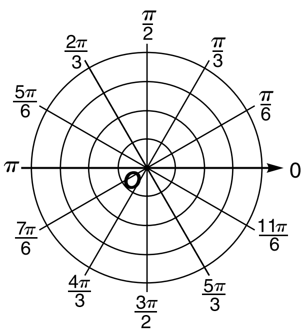
2. $(3, -135°)$



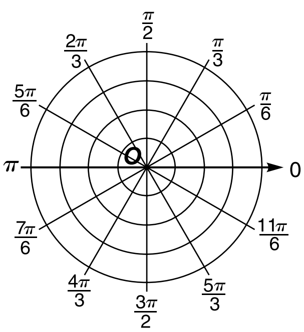
3. $(-1, -30°)$



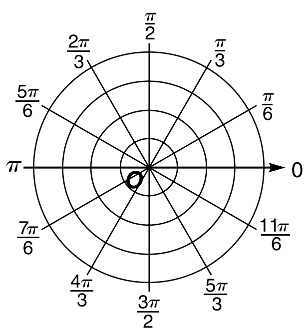
4. $\left(-2, \frac{\pi}{4}\right)$



5. $\left(1, \frac{5\pi}{4}\right)$

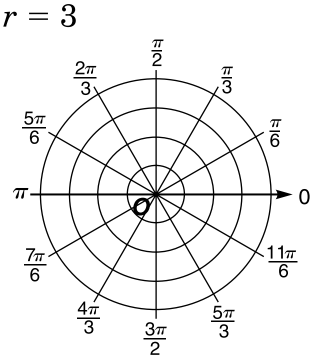


6. $\left(2, -\frac{2\pi}{3}\right)$

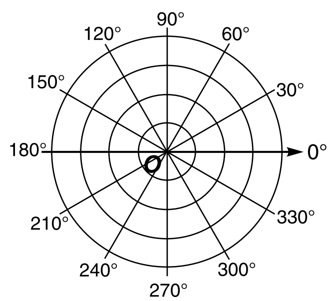


Graph each polar equation.

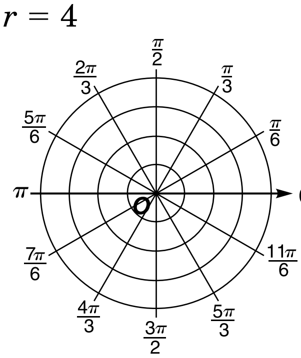
7. $r = 3$



8. $\theta = 60°$



9. $r = 4$



Find the distance between the points with the given polar coordinates.

10. $P_1(6, 90°)$ and $P_2(2, 130°)$

11. $P_1(-4, 85°)$ and $P_2(1, 105°)$

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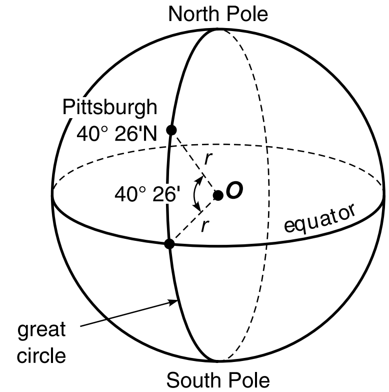
9-1

Enrichment

Distance on the Earth's Surface



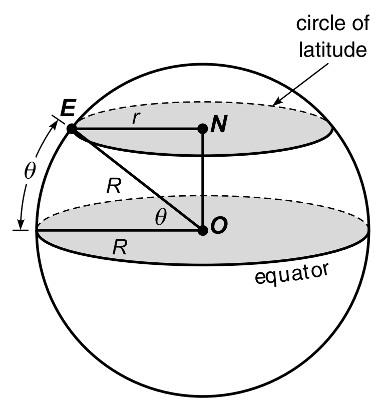
As you learned in Lesson 9-1, lines of longitude on Earth's surface intersect at the North and South Poles. A line of longitude that passes completely around Earth is called a **great circle**. All great circles have the same circumference, found by calculating the circumference of a circle with Earth's radius, 3963.2 miles. (Since Earth is slightly flattened at the poles, it is not precisely spherical. The difference is so small, however, that for most purposes it can be ignored.)

1. Find the circumference of a great circle.

On a great circle, position is measured in degrees north or south of the equator. Pittsburgh's position of 40° 26' N means that radii from Earth's center to Pittsburgh and to the point of intersection of the equator and Pittsburgh's longitude line form an angle of 40° 26'. (See the figure above.)

2. Find the length of one degree of arc on a longitude line.

3. Charleston, South Carolina (32° 46' N), and Guayaquil, Ecuador (2° 9' S), both lie on Pittsburgh's longitude line. Find the distance from Pittsburgh to each of the other cities.



Because circles of latitude are drawn parallel to the equator, their radii and circumferences grow steadily shorter as they approach the poles. The length of one degree of arc on a circle of latitude depends on how far north or south of the equator the circle is located. The figure at the right shows a circle of latitude of radius r located θ degrees north of the equator. Because the radii of the equator and the circle of latitude are parallel, $m\angle NEO = \theta$. Therefore, $\cos\theta = \frac{r}{R}$, which gives $r = R\cos\theta$, where R represents the radius of Earth.

4. Find the radius and circumference of a circle of latitude located 70° north of the equator.

5. Find the length of one degree of arc on the circle described in Exercise 4.

6. Bangor, Maine, and Salem, Oregon, are both located at latitude 44° 50' N. Their respective longitudes are 68° 46' and 123° 2' west of Greenwich. Find the distance from Bangor to Salem.

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Study Guide

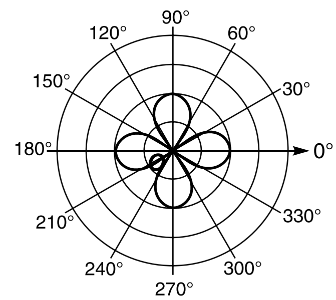
Graphs of Polar Equations

A **polar graph** is the set of all points whose coordinates (r, θ) satisfy a given polar equation. The position and shape of polar graphs can be altered by multiplying the function by a number or by adding to the function. You can also alter the graph by multiplying θ by a number or by adding to it.

Example 1 **Graph the polar equation $r = 2 \cos 2\theta$.**

Make a table of values. Graph the ordered pairs and connect them with a smooth curve.

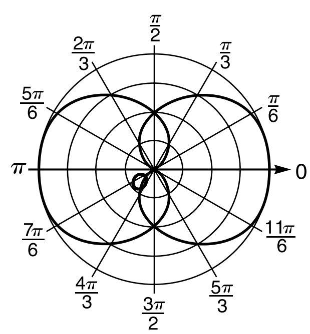
θ	$2 \cos 2\theta$	(r, θ)
0°	2	(2, 0°)
30°	1	(1, 30°)
45°	0	(0, 45°)
60°	−1	(−1, 60°)
90°	−2	(−2, 90°)
120°	−1	(−1, 120°)
135°	0	(0, 135°)
150°	1	(1, 150°)
180°	2	(2, 180°)
210°	1	(1, 210°)
225°	0	(0, 225°)
240°	−1	(−1, 240°)
270°	−2	(−2, 270°)
300°	−1	(−1, 300°)
315°	0	(0, 315°)
330°	1	(1, 330°)



This type of curve is called a *rose*. Notice that the farthest points are 2 units from the pole and the rose has 4 petals.

Example 2 **Graph the system of polar equations. Solve the system using algebra and trigonometry, and compare the solutions to those on your graph.**

$r = 2 + 2 \cos \theta$
$r = 2 - 2 \cos \theta$



To solve the system of equations, substitute $2 + 2 \cos \theta$ for r in the second equation.

$$2 + 2 \cos \theta = 2 - 2 \cos \theta$$
$$\cos \theta = 0$$
$$\theta = \frac{\pi}{2} \text{ or } \theta = \frac{3\pi}{2}$$

Substituting each angle into either of the original equations gives $r = 2$. The solutions of the system are therefore $\left(2, \frac{\pi}{2}\right)$ and $\left(2, \frac{3\pi}{2}\right)$. Tracing on the curves shows that these solutions correspond with two of the intersection points on the graph. The curves also intersect at the pole.

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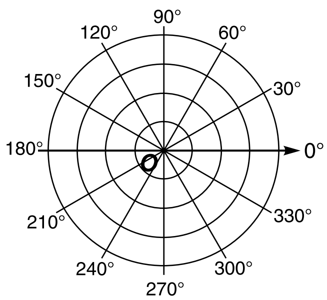
Practice

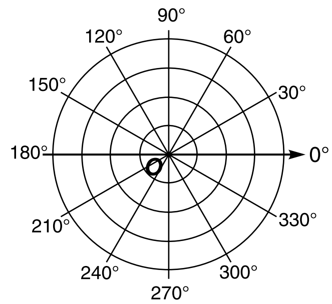
Graphs of Polar Equations

Graph each polar equation. Identify the type of curve each represents.

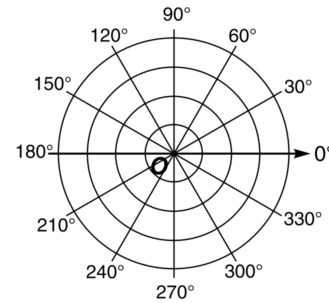
1. $r = 1 + \cos\theta$



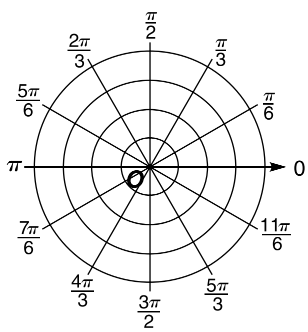
2. $r = 3\sin 3\theta$



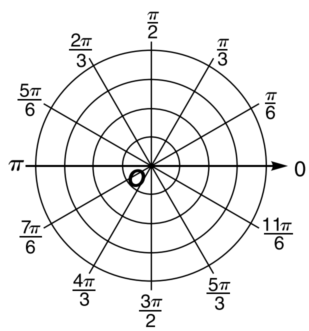
3. $r = 1 + 2\cos\theta$



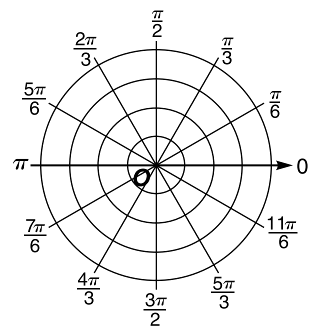
4. $r = 2 + 2\sin\theta$



5. $r = 0.5\theta$

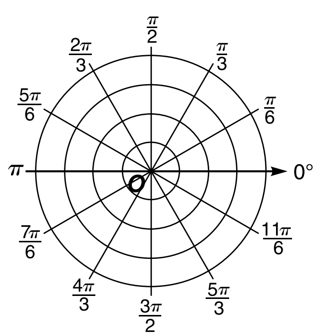


6. $r^2 = 16\cos 2\theta$

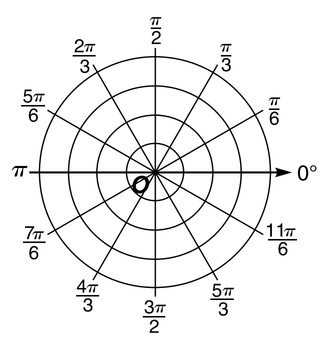


Graph each system of polar equations. Solve the system using algebra and trigonometry. Assume $0 \le \theta < 2\pi$.

7. $r = 1 + 2\sin\theta$
$r = 2 + \sin\theta$



8. $r = 1 + \cos\theta$
$r = 3\cos\theta$



9. ***Design*** Mikaela is designing a border for her stationery. Suppose she uses a rose curve. Determine an equation for designing a rose that has 8 petals with each petal 4 units long.

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Enrichment

Symmetry in Graphs of Polar Equations

It is sometimes helpful to analyze polar equations for certain properties that predict symmetry in the graph of the equation. The following rules guarantee the existence of symmetry in the graph. However, the graphs of some polar equations exhibit symmetry even though the rules do not predict it.

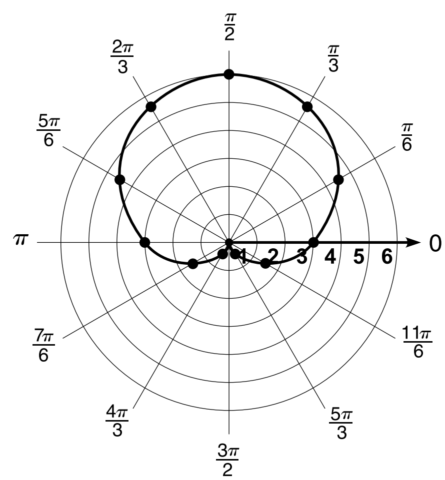
1. If replacing θ by $-\theta$ yields the same equation, then the graph of the equation is symmetric with respect to the line containing the polar axis (the x-axis in the rectangular coordinate system).

2. If replacing θ by $\pi - \theta$ yields the same equation, then the graph of the equation is symmetric with respect to the line $\theta = \frac{\pi}{2}$ (the y-axis in the rectangular coordinate system).

3. If replacing r by $-r$ yields the same equation, then the graph of the equation is symmetric with respect to the pole.

Example **Identify the symmetry of and graph $r = 3 + 3 \sin \theta$.**
Since $\sin(\pi - \theta) = \sin \theta$, by rule 2 the graph is symmetric with respect to the line $\theta = \frac{\pi}{2}$. Therefore, it is only necessary to plot points in the first and fourth quadrants.

θ	$3 + 3 \sin \theta$	(r, θ)
$-\frac{\pi}{2}$	0	$\left(0, -\frac{\pi}{2}\right)$
$-\frac{\pi}{3}$	0.4	$\left(0.4, -\frac{\pi}{3}\right)$
$-\frac{\pi}{6}$	1.5	$\left(1.5, -\frac{\pi}{6}\right)$
0	3.0	(3.0, 0)
$\frac{\pi}{6}$	4.5	$\left(4.5, \frac{\pi}{6}\right)$
$\frac{\pi}{3}$	5.6	$\left(5.6, \frac{\pi}{3}\right)$
$\frac{\pi}{2}$	6.0	$\left(6.0, \frac{\pi}{2}\right)$



The points in the second and third quadrants are found by using symmetry.

Identify the symmetry of and graph each polar equation on polar grid paper.

1. $r = 2 + 3 \cos \theta$

2. $r^2 = 4 \sin 2\theta$

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9-3

Study Guide

Polar and Rectangular Coordinates

Use the conversion formulas in the following examples to convert coordinates and equations from one coordinate system to the other.

Example 1 **Find the rectangular coordinates of each point.**

a. $P\left(3, \frac{3\pi}{4}\right)$

For $P\left(3, \frac{3\pi}{4}\right)$, $r = 3$ and $\theta = \frac{3\pi}{4}$. Use the conversion formulas $x = r\cos\theta$ and $y = r\sin\theta$.

$$x = r\cos\theta = 3\cos\frac{3\pi}{4} = 3\left(-\frac{\sqrt{2}}{2}\right) \text{ or } -\frac{3\sqrt{2}}{2}$$

$$y = r\sin\theta = 3\sin\frac{3\pi}{4} = 3\left(\frac{\sqrt{2}}{2}\right) \text{ or } \frac{3\sqrt{2}}{2}$$

The rectangular coordinates of P are $\left(-\frac{3\sqrt{2}}{2}, \frac{3\sqrt{2}}{2}\right)$, or $(-2.12, 2.12)$ to the nearest hundredth.

b. $Q(20, -60°)$

For $Q(20, -60°)$, $r = 20$ and $\theta = -60°$.

$$x = r\cos\theta = 20\cos(-60°) = 20(0.5) = 10$$

$$y = r\sin\theta = 20\sin(-60°) = 20\left(-\frac{\sqrt{3}}{2}\right) = -10\sqrt{3}$$

The rectangular coordinates of Q are $(10, -10\sqrt{3})$, or approximately $(10, -17.32)$

Example 2 **Find the polar coordinates of $R(5, -9)$.**

For $R(5, -9)$, $x = 5$ and $y = -9$.

$$r = \sqrt{x^2 + y^2} = \sqrt{5^2 + (-9)^2} = \sqrt{106} \text{ or about } 10.30$$

$$\theta = \operatorname{Arctan}\frac{y}{x} \quad x > 0$$
$$= \operatorname{Arctan}\frac{-9}{5} \approx -1.06$$

To obtain an angle between 0 and 2π you can add 2π to the θ-value. This results in $\theta = 5.22$.

The polar coordinates of R are approximately $(10.30, 5.22)$.

Example 3 **Write the polar equation $r = 5\cos\theta$ in rectangular form.**

$r = 5\cos\theta$

$r^2 = 5r\cos\theta$ *Multiply each side by r.*

$x^2 + y^2 = 5x$ *$r^2 = x^2 + y^2$ and $r\cos\theta = x$*

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Practice

Polar and Rectangular Coordinates

Find the rectangular coordinates of each point with the given polar coordinates.

1. $(6, 120°)$

2. $(-4, 45°)$

3. $\left(4, \frac{\pi}{6}\right)$

4. $\left(0, \frac{13\pi}{3}\right)$

Find the polar coordinates of each point with the given rectangular coordinates. Use $0 \leq \theta < 2\pi$ and $r \geq 0$.

5. $(2, 2)$

6. $(2, -3)$

7. $(-3, \sqrt{3})$

8. $(-5, -8)$

Write each polar equation in rectangular form.

9. $r = 4$

10. $r \cos \theta = 5$

Write each rectangular equation in polar form.

11. $x^2 + y^2 = 9$

12. $y = 3$

13. ***Surveying*** A surveyor records the polar coordinates of the location of a landmark as $(40, 62°)$. What are the rectangular coordinates?

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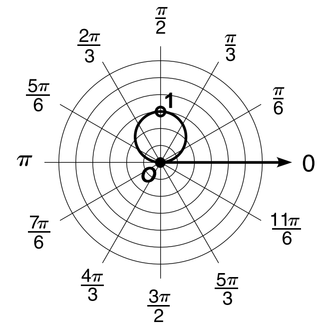
Enrichment

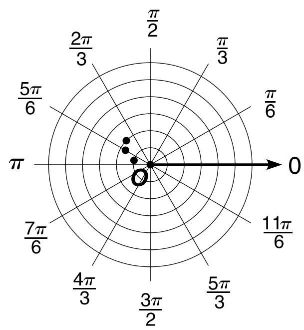
Polar Roses

The polar equation $r = a \sin n\theta$ graphs as a rose.
When $n = 1$, the rose is a circle — a flower with one leaf.

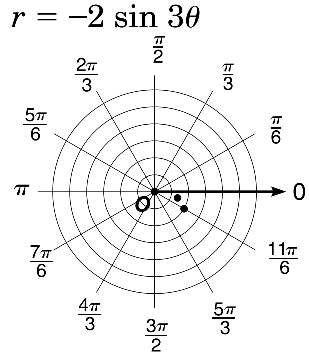


Sketch the graphs of these roses.

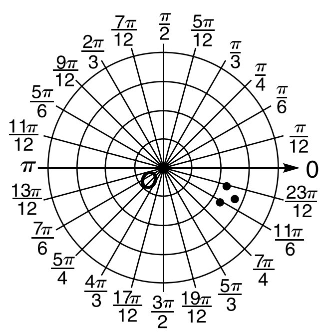
1. $r = 2 \sin 2\theta$



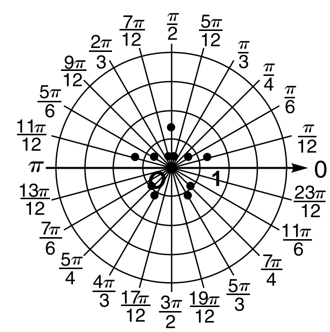
2. $r = -2 \sin 3\theta$



3. $r = -2 \sin 4\theta$



4. $r = 2 \sin 5\theta$



5. The graph of the equation $r = a \sin n\theta$ is a rose. Use your results from Exercises 1–4 to complete these conjectures.

a. The distance across a petal is __?__ units.

b. If n is an odd integer, the number of leaves is __?__.

c. If n is an even integer, the number of leaves is __?__.

6. Write $r = 2 \sin 2\theta$ in rectangular form.

7. The total area A of the three leaves in the three-leaved rose $r = a \sin 3\theta$ is given by $A = \frac{1}{4}a^2\pi$. For a four-leaved rose, the area is $A = \frac{1}{2}a^2\pi$.

a. Find the area of a four-leaved rose with $a = 6$.

b. Write the equation of a three-leaved rose with area 36π.

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Study Guide

Polar Form of a Linear Equation

Example 1 **Write the equation $x + 3y = 6$ in polar form.**

The standard form of the equation is $x + 3y - 6 = 0$. To find the values of p and ϕ, write the equation in normal form. To convert to normal form, find the value of $\pm\sqrt{A^2 + B^2}$.

$\pm\sqrt{A^2 + B^2} = \pm\sqrt{1^2 + 3^2}$ or $\pm\sqrt{10}$

Since C is negative, use $+\sqrt{10}$.

The normal form of the equation is
$\frac{1}{\sqrt{10}}x + \frac{3}{\sqrt{10}}y - \frac{6}{\sqrt{10}} = 0$ or $\frac{\sqrt{10}}{10}x + \frac{3\sqrt{10}}{10}y - \frac{3\sqrt{10}}{5} = 0$.

Using the normal form $x \cos \phi + y \sin \phi - p = 0$, we can see that $p = \frac{6}{\sqrt{10}}$ or $\frac{3\sqrt{10}}{5}$. Since $\cos \phi$ and $\sin \phi$ are both positive, the normal lies in Quadrant I.

$\tan \phi = \frac{\sin \phi}{\cos \phi}$

$\tan \phi = 3$ $\frac{3}{\sqrt{10}} \div \frac{1}{\sqrt{10}} = 3$

$\phi \approx 1.25$ *Use the Arctangent function.*

Substitute the values for p and ϕ into the polar form.

$p = r \cos(\theta - \phi)$

$\frac{3\sqrt{10}}{5} = r \cos(\theta - 1.25)$ *Polar form of $x + 3y = 6$*

Example 2 **Write $3 = r \cos(\theta - 30°)$ in rectangular form.**

$3 = r \cos(\theta - 30°)$

$3 = r(\cos \theta \cos 30° + \sin \theta \sin 30°)$ *Difference identity for cosine*

$3 = r\left(\frac{\sqrt{3}}{2} \cos \theta + \frac{1}{2} \sin \theta\right)$ *$\cos 30° = \frac{\sqrt{3}}{2}$, $\sin 30° = \frac{1}{2}$*

$3 = \frac{\sqrt{3}}{2}r \cos \theta + \frac{1}{2}r \sin \theta$ *Distributive property*

$3 = \frac{\sqrt{3}}{2}x + \frac{1}{2}y$ *$r \cos \theta = x$, $r \sin \theta = y$*

$6 = \sqrt{3}x + y$ *Multiply each side by 2.*

$0 = \sqrt{3}x + y - 6$ *Subtract 6 from each side.*

The rectangular form of $3 = r\cos(\theta - 30°)$ is $\sqrt{3}x + y - 6 = 0$.

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9-4 Practice

Polar Form of a Linear Equation

Write each equation in polar form. Round ϕ to the nearest degree.

1. $3x + 2y = 16$

2. $3x + 4y = 15$

3. $3x - 4y = 12$

4. $y = 2x - 1$

Write each equation in rectangular form.

5. $4 = r\cos\left(\theta + \frac{5\pi}{6}\right)$

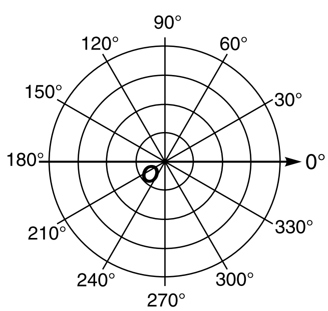
6. $2 = r\cos(\theta - 90°)$

7. $1 = r\cos\left(\theta - \frac{\pi}{4}\right)$

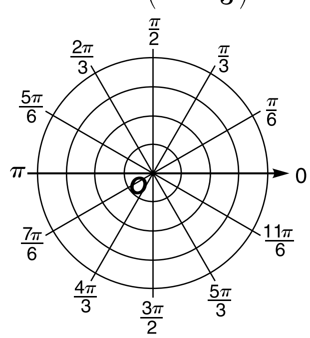
8. $3 = r\cos(\theta + 240°)$

Graph each polar equation.

9. $3 = r\cos(\theta - 60°)$



10. $1 = r\cos\left(\theta + \frac{\pi}{3}\right)$



11. ***Landscaping*** A landscaper is designing a garden with hedges through which a straight path will lead from the exterior of the garden to the interior. If the polar coordinates of the endpoints of the path are (20, 90°) and (10, 150°), where r is measured in feet, what is the equation for the path?

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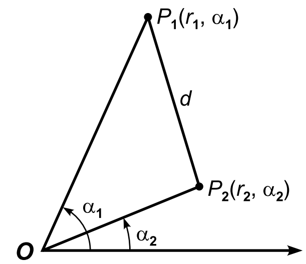
Enrichment

Distance Using Polar Coordinates



Suppose you were given the polar coordinates of two points $P_1(r_1, \alpha_1)$ and $P_2(r_2, \alpha_2)$ and were asked to find the distance d between the points. One way would be to convert to rectangular coordinates (x_1, y_1) and (x_2, y_2), and apply the distance formula

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

A more straightforward method makes use of the Law of Cosines.

1. In the above figure, the distance d between P_1 and P_2 is the length of one side of $\triangle OP_1P_2$. Find the lengths of the other two sides.

2. Determine the measure of $\angle P_1OP_2$.

3. Write an expression for d^2 using the Law of Cosines.

4. Write a formula for the distance d between the points $P_1(r_1, \alpha_1)$ and $P_2(r_2, \alpha_2)$.

5. Find the distance between the points $(3, 45°)$ and $(5, 25°)$. Round your answer to three decimal places.

6. Find the distance between the points $\left(2, \frac{\pi}{2}\right)$ and $\left(4, \frac{\pi}{8}\right)$. Round your answer to three decimal places.

7. The distance from the point $(5, 80°)$ to the point $(r, 20°)$ is $\sqrt{21}$. Find r.

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Study Guide

Simplifying Complex Numbers

Add and subtract complex numbers by performing the chosen operation on both the **real** and **imaginary parts.** Find the product of two or more complex numbers by using the same procedures used to multiply binomials. To simplify the quotient of two complex numbers, multiply the numerator and denominator by the complex conjugate of the denominator.

To find the value of i^n, let R be the remainder when n is divided by 4.	
if R = 0	$i^n = 1$
if R = 1	$i^n = i$
if R = 2	$i^n = -1$
if R = 3	$i^n = -i$

Powers of i	
$i^1 = i$	$i^2 = -1$
$i^3 = i^2 \cdot i = -i$	$i^4 = (i^2)^2 = 1$
$i^5 = i^4 \cdot i = i$	$i^6 = i^4 \cdot i^2 = -1$
$i^7 = i^4 \cdot i^3 = -i$	$i^8 = (i^2)^4 = 1$

Example 1 **Simplify each power of i.**

a. i^{30}

Method 1

$30 \div 4 = 7 \text{ R } 2$

If R = 2, $i^n = -1$.

$i^{30} = -1$

Method 2

$$\begin{aligned} i^{30} &= (i^4)^7 \cdot i^2 \\ &= (1)^7 \cdot i^2 \\ &= -1 \end{aligned}$$

b. i^{-11}

Method 1

$-11 \div 4 = -3 \text{ R } 1$

If R = 1, $i^n = i$.

$i^{-11} = i$

Method 2

$$\begin{aligned} i^{-11} &= (i^4)^{-3} \cdot i^1 \\ &= (1)^{-3} \cdot i^1 \\ &= i \end{aligned}$$

Example 2 **Simplify each expression.**

a. $(3 + 2i) + (5 - 3i)$

$$\begin{aligned} &(3 + 2i) + (5 - 3i) \\ &= (3 + 5) + (2i - 3i) \\ &= 8 - i \end{aligned}$$

b. $(8 - 4i) - (9 - 7i)$

$$\begin{aligned} &(8 - 4i) - (9 - 7i) \\ &= 8 - 4i - 9 + 7i \\ &= -1 + 3i \end{aligned}$$

Example 3 **Simplify $(4 - 2i)(5 - 3i)$.**

$$\begin{aligned} (4 - 2i)(5 - 3i) &= 5(4 - 2i) - 3i(4 - 2i) && \textit{Distributive property} \\ &= 20 - 10i - 12i + 6i^2 && \textit{Distributive property} \\ &= 20 - 10i - 12i + 6(-1) && i^2 = -1 \\ &= 14 - 22i \end{aligned}$$

Example 4 **Simplify $(4 - 5i) \div (2 + i)$.**

$$\begin{aligned} (4 - 5i) \div (2 + i) &= \frac{4 - 5i}{2 + i} \\ &= \frac{4 - 5i}{2 + i} \cdot \frac{2 - i}{2 - i} && 2 - i \textit{ is the conjugate of } 2 + i. \\ &= \frac{8 - 10i - 4i + 5i^2}{4 - i^2} \\ &= \frac{8 - 14i + 5(-1)}{4 - (-1)} && i^2 = -1 \\ &= \frac{3 - 14i}{5} \\ &= \frac{3}{5} - \frac{14}{5}i && \textit{Write the answer in the form } a + bi. \end{aligned}$$

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9-5

Practice

Simplifying Complex Numbers

Simplify.

1. $\boldsymbol{i}^{38}$

2. $\boldsymbol{i}^{-17}$

3. $(3 + 2\boldsymbol{i}) + (4 + 5\boldsymbol{i})$

4. $(-6 - 2\boldsymbol{i}) - (-8 - 3\boldsymbol{i})$

5. $(8 - \boldsymbol{i}) - (4 - \boldsymbol{i})$

6. $(1 + \boldsymbol{i})(3 - 2\boldsymbol{i})$

7. $(2 - 3\boldsymbol{i})(5 + \boldsymbol{i})$

8. $(4 + 5\boldsymbol{i})(4 - 5\boldsymbol{i})$

9. $(3 + 4\boldsymbol{i})^2$

10. $(4 + 3\boldsymbol{i}) \div (1 - 2\boldsymbol{i})$

11. $(2 + \boldsymbol{i}) \div (2 - \boldsymbol{i})$

12. $\frac{8 - 7\boldsymbol{i}}{1 - 2\boldsymbol{i}}$

13. ***Physics*** A fence post wrapped in two wires has two forces acting on it. Once force exerts 5.3 newtons due north and 4.1 newtons due east. The second force exerts 6.2 newtons due north and 2.8 newtons due east. Find the resultant force on the fence post. Write your answer as a complex number. (*Hint:* A vector with a horizontal component of magnitude a and a vertical component of magnitude b can be represented by the complex number $a + b\boldsymbol{i}$.)

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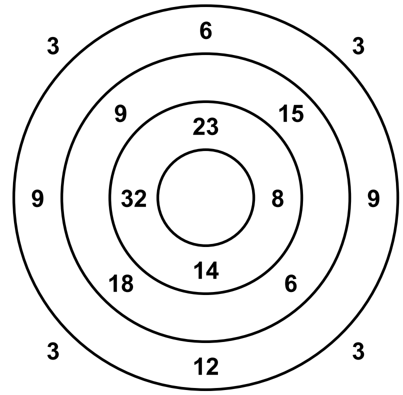
Enrichment

Cycle Quadruples



Four nonnegative integers are arranged in cyclic order to make a "cyclic quadruple." In the example, this quadruple is 23, 8, 14, and 32.

The next cyclic quadruple is formed from the absolute values of the four differences of adjacent integers:

$|23 - 8| = 15$ $|8 - 14| = 6$ $|14 - 32| = 18$ $|32 - 23| = 9$

By continuing in this manner, you will eventually get four equal integers. In the example, the equal integers appear in three steps.

Solve each problem.

1. Start with the quadruple 25, 17, 55, 47. In how many steps do the equal integers appear?

2. Some interesting things happen when one or more of the original numbers is 0. Draw a diagram showing a beginning quadruple of three zeros and one nonnegative integer. Predict how many steps it will take to reach 4 equal integers. Also, predict what that integer will be. Complete the diagram to check your predictions.

3. Start with four integers, two of them zero. If the zeros are opposite one another, how many steps does it take for the zeros to disappear?

4. Start with two equal integers and two zeros. The zeros are next to one another. How many steps does it take for the zeros to disappear?

5. Start with two nonequal integers and two zeros. The zeros are next to one another. How many steps does it take for the zeros to disappear?

6. Start with three equal integers and one zero. How many steps does it take for the zero to disappear?

7. Describe the remaining cases with one zero and tell how many steps it takes for the zero to disappear.

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Study Guide

The Complex Plane and Polar Form of Complex Numbers

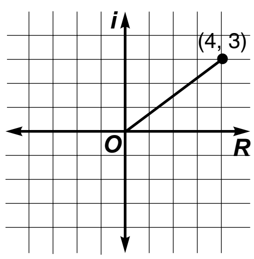
In the **complex plane,** the real axis is horizontal and the imaginary axis is vertical. The **absolute value** of a complex number is its distance from zero in the complex plane.

The polar form of the complex number $a + b\boldsymbol{i}$ is $r(\cos\theta + \boldsymbol{i}\sin\theta)$, which is often abbreviated as $r \text{ cis } \theta$. In polar form, r represents the absolute value, or **modulus,** of the complex number. The angle θ is called the **amplitude** or **argument** of the complex number.

Example 1 **Graph each number in the complex plane and find its absolute value.**

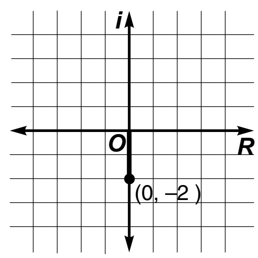
a. $\boldsymbol{z = 4 + 3i}$

$|z| = \sqrt{4^2 + 3^2}$
$= 5$



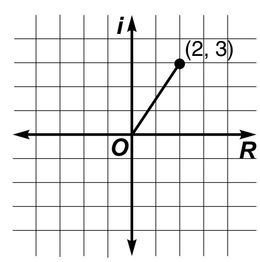
b. $\boldsymbol{z = -2i}$

$z = 0 - 2\boldsymbol{i}$
$|z| = \sqrt{0^2 + (-2)^2}$
$= 2$



Example 2 **Express the complex number 2 + 3*i* in polar form.**

First plot the number in the complex plane.



Then find the modulus.

$r = \sqrt{2^2 + 3^2}$ or $\sqrt{13}$

Now find the amplitude. Notice that θ is in Quandrant I.

$\theta = \text{Arctan } \frac{3}{2}$ $\qquad$ *$\theta = \text{Arctan } \frac{b}{a}$ if $a > 0$*

≈ 0.98

Therefore, $2 + 3\boldsymbol{i} \approx \sqrt{13}(\cos 0.98 + \boldsymbol{i}\sin 0.98)$ or $\sqrt{13} \text{ cis } 0.98$.

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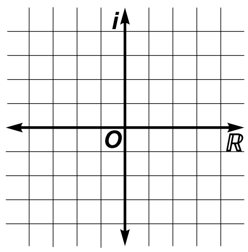
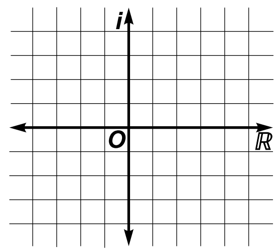
Practice

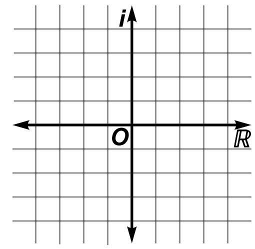
The Complex Plane and Polar Form of Complex Numbers

Graph each number in the complex plan and find its absolute value.

1. $z = 3\boldsymbol{i}$ **2.** $z = 5 + \boldsymbol{i}$ **3.** $z = -4 - 4\boldsymbol{i}$







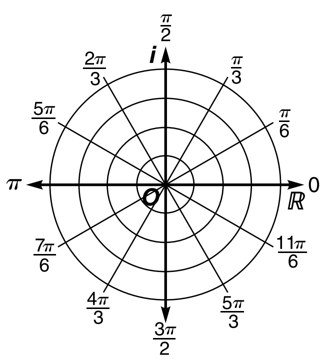
Express each complex number in polar form.

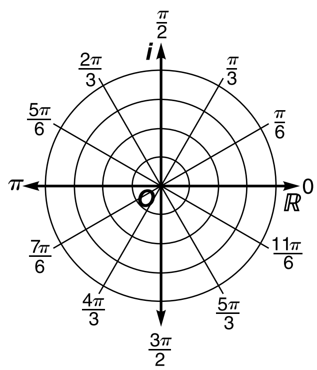
4. $3 + 4\boldsymbol{i}$ **5.** $-4 + 3\boldsymbol{i}$

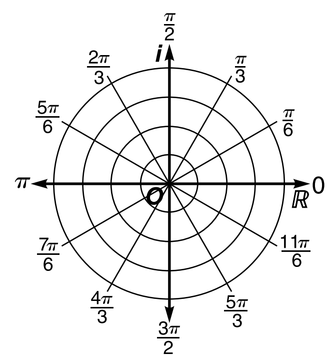
6. $-1 + \boldsymbol{i}$ **7.** $1 - \boldsymbol{i}$

Graph each complex number. Then express it in rectangular form.

8. $2\left(\cos \frac{3\pi}{4} + \boldsymbol{i} \sin \frac{3\pi}{4}\right)$ **9.** $4\left(\cos \frac{5\pi}{6} + \boldsymbol{i} \sin \frac{5\pi}{6}\right)$ **10.** $3\left(\cos \frac{4\pi}{3} + \boldsymbol{i} \sin \frac{4\pi}{3}\right)$







11. ***Vectors*** The force on an object is represented by the complex number $8 + 21\boldsymbol{i}$, where the components are measured in pounds. Find the magnitude and direction of the force.

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Enrichment

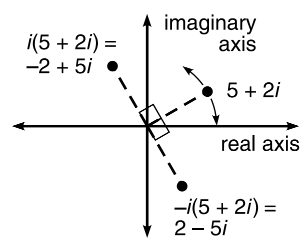
A Complex Treasure Hunt

A prospector buried a sack of gold dust. He then wrote instructions telling where the gold dust could be found:

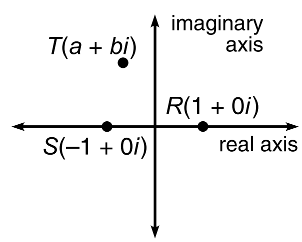
1. Start at the oak tree. Walk to the mineral spring counting the number of paces.
2. Turn 90° to the right and walk an equal number of paces. Place a stake in the ground.
3. Go back to the oak tree. Walk to the red rock counting the number of paces.
4. Turn 90° to the left and walk an equal number of paces. Place a stake in the ground.
5. Find the spot halfway between the stakes. There you will find the gold.

Years later, an expert in complex numbers found the instructions in a rusty tin can. Some additional instructions told how to get to the general area where the oak tree, the mineral spring, and the red rock could be found. The expert hurried to the area and readily located the spring and the rock. Unfortunately, hundreds of oak trees had sprung up since the prospector's day, and it was impossible to know which one was referred to in the instructions. Nevertheless, through prudent application of complex numbers, the expert found the gold. Especially helpful in the quest were the following facts.

- The distance between the graphs of two complex numbers can be represented by the absolute value of the difference between the numbers.
- Multiplication by $\boldsymbol{i}$ rotates the graph of a complex number 90° counterclockwise. Multiplication by $-\boldsymbol{i}$ rotates it 90° clockwise.



The expert drew a map on the complex plane, letting $S(-1 + 0\boldsymbol{i})$ be the spring and $R(1 + 0\boldsymbol{i})$ be the rock. Since the location of the oak tree was unknown, the expert represented it by $T(a + b\boldsymbol{i})$.



1. Find the distance from the oak tree to the spring. Express the distance as a complex number.

2. Write the complex number whose graph would be a 90° counterclockwise rotation of your answer to Exercise 1. This is where the first stake should be placed.

3. Repeat Exercises 1 and 2 for the distance from the tree to the rock. Where should the second stake be placed?

4. The gold is halfway between the stakes. Find the coordinates of the location.

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Study Guide

Products and Quotients of Complex Numbers in Polar Form

Example 1 **Find the product $2\left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}\right) \cdot 4\left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}\right)$. Then express the product in rectangular form.**

Find the modulus and amplitude of the product.

$$r = r_1 r_2 \qquad \theta = \theta_1 + \theta_2$$
$$= 2(4) \qquad = \frac{\pi}{2} + \frac{\pi}{3}$$
$$= 8 \qquad = \frac{5\pi}{6}$$

The product is $8\left(\cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6}\right)$.

Now find the rectangular form of the product.

$$8\left(\cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6}\right) = 8\left(-\frac{\sqrt{3}}{2} + \frac{1}{2}i\right) \qquad \cos \frac{5\pi}{6} = -\frac{\sqrt{3}}{2},\ \sin \frac{5\pi}{6} = \frac{1}{2}$$
$$= -4\sqrt{3} + 4i$$

The rectangular form of the product is $-4\sqrt{3} + 4i$.

Example 2 **Find the quotient $21\left(\cos \frac{7\pi}{6} + i \sin \frac{7\pi}{6}\right) \div 7\left(\cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3}\right)$. Then express the quotient in rectangular form.**

Find the modulus and amplitude of the quotient.

$$r = \frac{r_1}{r_2} \qquad \theta = \theta_1 - \theta_2$$
$$= \frac{21}{7} \qquad = \frac{7\pi}{6} - \frac{4\pi}{3}$$
$$= 3 \qquad = -\frac{\pi}{6}$$

The quotient is $3\left[\cos\left(-\frac{\pi}{6}\right) + i \sin\left(-\frac{\pi}{6}\right)\right]$.

Now find the rectangular form of the quotient.

$$3\left[\cos\left(-\frac{\pi}{6}\right) + i \sin\left(-\frac{\pi}{6}\right)\right] = 3\left[\frac{\sqrt{3}}{2} + \left(-\frac{1}{2}\right)i\right] \qquad \cos\left(-\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2},\ \sin\left(-\frac{\pi}{6}\right) = -\frac{1}{2}$$
$$= \frac{3\sqrt{3}}{2} - \frac{3}{2}i$$

The rectangular form of the quotient is $\frac{3\sqrt{3}}{2} - \frac{3}{2}i$.

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Practice

Products and Quotients of Complex Numbers in Polar Form

Find each product or quotient. Express the result in rectangular form.

1. $3\left(\cos \frac{\pi}{3} + \boldsymbol{i} \sin \frac{\pi}{3}\right) \cdot 3\left(\cos \frac{5\pi}{3} + \boldsymbol{i} \sin \frac{5\pi}{3}\right)$

2. $6\left(\cos \frac{\pi}{2} + \boldsymbol{i} \sin \frac{\pi}{2}\right) \div 2\left(\cos \frac{\pi}{3} + \boldsymbol{i} \sin \frac{\pi}{3}\right)$

3. $14\left(\cos \frac{5\pi}{4} + \boldsymbol{i} \sin \frac{5\pi}{4}\right) \div 2\left(\cos \frac{\pi}{2} + \boldsymbol{i} \sin \frac{\pi}{2}\right)$

4. $3\left(\cos \frac{5\pi}{6} + \boldsymbol{i} \sin \frac{5\pi}{6}\right) \cdot 6\left(\cos \frac{\pi}{3} + \boldsymbol{i} \sin \frac{\pi}{3}\right)$

5. $2\left(\cos \frac{\pi}{2} + \boldsymbol{i} \sin \frac{\pi}{2}\right) \cdot 2\left(\cos \frac{4\pi}{3} + \boldsymbol{i} \sin \frac{4\pi}{3}\right)$

6. $15(\cos \pi + \boldsymbol{i} \sin \pi) \div 3\left(\cos \frac{\pi}{2} + \boldsymbol{i} \sin \frac{\pi}{2}\right)$

7. ***Electricity*** Find the current in a circuit with a voltage of 12 volts and an impedance of $2 - 4\boldsymbol{j}$ ohms. Use the formula, $E = I \cdot Z$, where E is the voltage measured in volts, I is the current measured in amperes, and Z is the impedance measured in ohms.
(*Hint:* Electrical engineers use $\boldsymbol{j}$ as the imaginary unit, so they write complex numbers in the form $a + b\boldsymbol{j}$. Express each number in polar form, substitute values into the formula, and then express the current in rectangular form.)

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Enrichment

Complex Conjugates

In Lesson 9-5, you learned that complex numbers in the form $a + b\boldsymbol{i}$ and $a - b\boldsymbol{i}$ are called conjugates. You can show that two numbers are conjugates by finding the appropriate values of a and b.

1. Show that the solutions of $x^2 + 2x + 3 = 0$ are conjugates.

2. Show that the solutions of $Ax^2 + Bx + C = 0$ are conjugates when $B^2 - 4AC < 0$.

The conjugate of the complex number z is represented by $\overline{z}$.

3. $z = a + b\boldsymbol{i}$. Use $\overline{z}$ to find the reciprocal of z.

4. $z = r(\cos\theta + \boldsymbol{i}\sin\theta)$. Find $\overline{z}$. Express your answer in polar form.

Use your answer to Exercise 4 to solve Exercises 5 and 6.

5. Find $z \cdot \overline{z}$.

6. Find $z \div \overline{z}$. $(z \neq 0)$

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9-8 Practice

Powers and Roots of Complex Numbers

You can use De Moivre's Theorem, $[r(\cos\theta + \boldsymbol{i}\sin\theta)]^n = r^n(\cos n\theta + \boldsymbol{i}\sin n\theta)$, to find the powers and roots of complex numbers in polar form.

Example 1 **Find $(-1 + \sqrt{3}\boldsymbol{i})^3$.**

First, write $-1 + \sqrt{3}\boldsymbol{i}$ in polar form. Note that its graph is in Quadrant II of the complex plane.

$$r = \sqrt{(-1)^2 + (\sqrt{3})^2} = \sqrt{1 + 3} \text{ or } 2 \qquad \theta = \text{Arctan}\,\frac{\sqrt{3}}{-1} + \pi = -\frac{\pi}{3} + \pi \text{ or } \frac{2\pi}{3}$$

The polar form of $-1 + \sqrt{3}\boldsymbol{i}$ is $2\left(\cos\frac{2\pi}{3} + \boldsymbol{i}\sin\frac{2\pi}{3}\right)$.

Now use De Moivre's Theorem to find the third power.

$$(-1 + \sqrt{3}\boldsymbol{i})^3 = \left[2\left(\cos\frac{2\pi}{3} + \boldsymbol{i}\sin\frac{2\pi}{3}\right)\right]^3$$

$$= 2^3\left[\cos 3\left(\frac{2\pi}{3}\right) + \boldsymbol{i}\sin 3\left(\frac{2\pi}{3}\right)\right] \qquad \textit{De Moivre's Theorem}$$

$$= 8(\cos 2\pi + \boldsymbol{i}\sin 2\pi)$$

$$= 8(1 + 0\boldsymbol{i}) \qquad \textit{Write the result in rectangular form.}$$

$$= 8$$

Therefore, $(-1 + \sqrt{3}\boldsymbol{i})^3 = 8$.

Example 2 **Find $\sqrt[3]{64\boldsymbol{i}}$.**

$$\sqrt[3]{64\boldsymbol{i}} = (0 + 64\boldsymbol{i})^{\frac{1}{3}} \qquad \textit{a = 0, b = 64}$$

$$= \left[64\left(\cos\frac{\pi}{2} + \boldsymbol{i}\sin\frac{\pi}{2}\right)\right]^{\frac{1}{3}} \qquad \textit{Polar form: } r = \sqrt{0^2 + 64^2} \textit{ or } 64;\ \theta = \frac{\pi}{2} \textit{ since } a = 0.$$

$$= 64^{\frac{1}{3}}\left[\cos\left(\frac{1}{3}\right)\left(\frac{\pi}{2}\right) + \boldsymbol{i}\sin\left(\frac{1}{3}\right)\left(\frac{\pi}{2}\right)\right] \qquad \textit{De Moivre's Theorem}$$

$$= 4\left(\cos\frac{\pi}{6} + \boldsymbol{i}\sin\frac{\pi}{6}\right)$$

$$= 4\left(\frac{\sqrt{3}}{2} + \frac{1}{2}\boldsymbol{i}\right)$$

$$= 2\sqrt{3} + 2\boldsymbol{i}$$

Therefore, $2\sqrt{3} + 2\boldsymbol{i}$ is the principal cube root of $64\boldsymbol{i}$.

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9-8

Practice

Powers and Roots of Complex Numbers

Find each power. Express the result in rectangular form.

1. $(-2 - 2\sqrt{3}\boldsymbol{i})^3$

2. $(1 - \boldsymbol{i})^5$

3. $(-1 + \sqrt{3}\boldsymbol{i})^{12}$

4. $\left[1\left(\cos \frac{\pi}{4} + \boldsymbol{i} \sin \frac{\pi}{4}\right)\right]^{-3}$

5. $(2 + 3\boldsymbol{i})^6$

6. $(1 + \boldsymbol{i})^8$

Find each principal root. Express the result in the form a + bi with a and b rounded to the nearest hundredth.

7. $(-27\boldsymbol{i})^{\frac{1}{3}}$

8. $(8 - 8\boldsymbol{i})^{\frac{1}{3}}$

9. $\sqrt[5]{-243\boldsymbol{i}}$

10. $(-\boldsymbol{i})^{\frac{1}{3}}$

11. $\sqrt[8]{-8\boldsymbol{i}}$

12. $\sqrt[4]{-2 - 2\sqrt{3}\boldsymbol{i}}$

9-8 Enrichment

Algebraic Numbers

A complex number is said to be **algebraic** if it is a zero of a polynomial with integer coefficients. For example, if p and q are integers with no common factors and $q \neq 0$, then $\frac{p}{q}$ is a zero of $qx - p$. This shows that every rational number is algebraic. Some irrational numbers can be shown to be algebraic.

Example **Show that $1 + \sqrt{3}$ is algebraic.**

Let $x = 1 + \sqrt{3}$. Then

$$x - 1 = \sqrt{3}$$
$$(x - 1)^2 = (\sqrt{3})^2$$
$$x^2 - 2x + 1 = 3$$
$$x^2 - 2x - 2 = 0$$

Thus, $1 + \sqrt{3}$ is a zero of $x^2 - 2x - 2$, so $1 + \sqrt{3}$ is an algebraic number.

If a complex number is not algebraic, it is said to be **trancendental**. The best-known transcendental numbers are π and e. Proving that these numbers are not algebraic was a difficult task. It was not until 1873 that the French mathematician Charles Hermite was able to show that e is transcendental. It wasn't until 1882 that C. L. F. Lindemann of Munich showed that π is also transcendental.

Show that each complex number is algebraic by finding a polynomial with integer coefficients of which the given number is a zero.

1. $\sqrt{2}$

2. $\boldsymbol{i}$

3. $2 - \boldsymbol{i}$

4. $\sqrt[3]{3}$

5. $4 - \sqrt[4]{2}\boldsymbol{i}$

6. $\sqrt{3} + \boldsymbol{i}$

7. $\sqrt{1 + \sqrt[3]{5}}$

8. $\sqrt[3]{2 - \sqrt{3}}$

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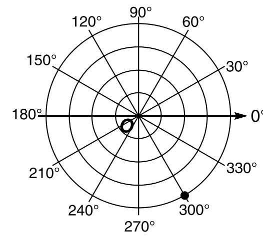
Chapter 9

Chapter 9 Test, Form 1A

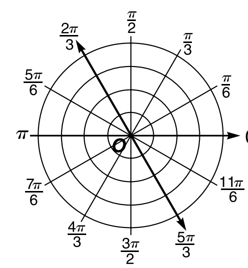
Write the letter for the correct answer in the blank at the right or each problem.

1. Find the polar coordinates that do *not* describe the point in the given graph. **1.** ________



A. $(-4, 120°)$
B. $(4, 300°)$
C. $(4, -240°)$
D. $(4, -60°)$

2. Find the equation represented in the given graph. **2.** ________

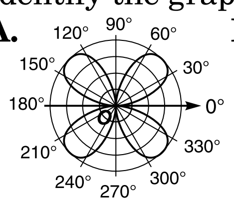


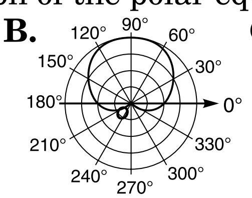
A. $\theta = -\frac{\pi}{3}$
B. $r = \frac{\pi}{3}$
C. $\theta = 2$
D. $r = \frac{2\pi}{3}$

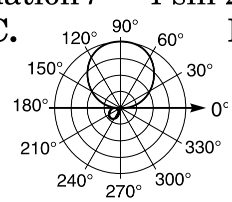
3. Find the distance between the points with polar coordinates $\left(-2.5, \frac{\pi}{6}\right)$ and $\left(-1.9, -\frac{\pi}{3}\right)$. **3.** ________

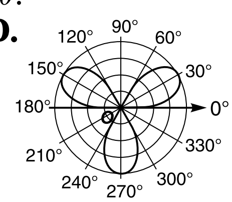
A. 3.14 **B.** 2.91 **C.** 3.49 **D.** 1.65

4. Identify the graph of the polar equation $r = 4 \sin 2\theta$. **4.** ________

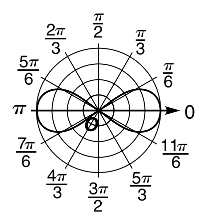
A. 

B. 

C. 

D. 

5. Find the equation whose graph is given. **5.** ________



A. $r = 4 \cos 2\theta$
B. $r = 2 + 2 \cos \theta$
C. $r = 4 \cos \theta$
D. $r^2 = 16 \cos 2\theta$

6. Find the polar coordinates of the point with rectangular coordinates $(-2, 2\sqrt{3})$. **6.** ________

A. $\left(4, \frac{\pi}{3}\right)$ **B.** $\left(4, \frac{2\pi}{3}\right)$ **C.** $\left(4, \frac{5\pi}{6}\right)$ **D.** $\left(2, \frac{2\pi}{3}\right)$

7. Find the rectangular coordinates of the point with polar coordinates $\left(4, \frac{5\pi}{4}\right)$. **7.** ________

A. $(-2\sqrt{2}, -2\sqrt{2})$ **B.** $(2, 2\sqrt{3})$
C. $(2\sqrt{2}, 2\sqrt{2})$ **D.** $(-2\sqrt{3}, -2)$

8. Write the rectangular equation $x^2 + y^2 - 2x = 0$ in polar form. **8.** ________

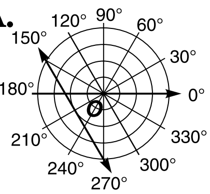
A. $r = 2 \sin \theta$ **B.** $r^2 - 2r \sin \theta = 0$
C. $r = \cos 2\theta$ **D.** $r = 2 \cos \theta$

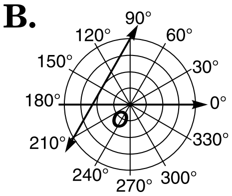
9. Write the polar equation $r^2 - 2r \sin \theta = 0$ in rectangular form. **9.** ________

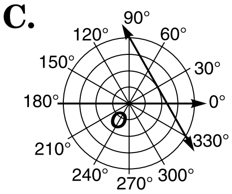
A. $x + y - 2 = 0$ **B.** $x^2 + y^2 - 2x = 0$
C. $x^2 + y^2 - 2y = 0$ **D.** $x = 2y$

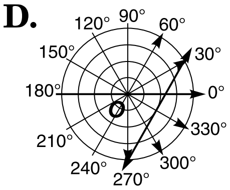
Chapter 9 Test, Form 1A (continued)

10. Identify the graph of the polar equation $r = 2 \csc(\theta + 60°)$. **10.** ________

A.


B.


C.


D.


11. Write $2x + y = 5$ in polar form. **11.** ________

A. $-\sqrt{5} = r\cos(\theta - 27°)$ **B.** $\sqrt{5} = r\cos(\theta - 27°)$

C. $-\sqrt{5} = r\cos(\theta + 27°)$ **D.** $\sqrt{5} = r\cos(\theta + 27°)$

12. Simplify $2(3 - \boldsymbol{i}^{14}) - (5 - \boldsymbol{i}^{23})$. **12.** ________

A. $1 - 3\boldsymbol{i}$ **B.** $3 - \boldsymbol{i}$ **C.** $1 - 2\boldsymbol{i}$ **D.** $1 + \boldsymbol{i}$

13. Simplify $(5 - 3\boldsymbol{i})^2$. **13.** ________

A. $16 + 30\boldsymbol{i}$ **B.** $34 - 30\boldsymbol{i}$ **C.** $16 - 30\boldsymbol{i}$ **D.** $34 + 30\boldsymbol{i}$

14. Simplify $\frac{3 + 2\boldsymbol{i}}{4 - 5\boldsymbol{i}}$. **14.** ________

A. $\frac{2}{41} - \frac{23}{41}\boldsymbol{i}$ **B.** $\frac{22}{41} + \frac{23}{41}\boldsymbol{i}$ **C.** $-\frac{2}{9} - \frac{23}{9}\boldsymbol{i}$ **D.** $\frac{2}{41} + \frac{23}{41}\boldsymbol{i}$

15. Express $5\sqrt{3} - 5\boldsymbol{i}$ in polar form. **15.** ________

A. $10\left(\cos\frac{11\pi}{6} + \boldsymbol{i}\sin\frac{11\pi}{6}\right)$ **B.** $10\left(\cos\frac{11\pi}{6} - \boldsymbol{i}\sin\frac{11\pi}{6}\right)$

C. $5\left(\cos\frac{11\pi}{6} + \boldsymbol{i}\sin\frac{11\pi}{6}\right)$ **D.** $10\left(\cos\frac{5\pi}{3} + \boldsymbol{i}\sin\frac{5\pi}{3}\right)$

16. Express $4\left(\cos\frac{3\pi}{4} + \boldsymbol{i}\sin\frac{3\pi}{4}\right)$ in rectangular form. **16.** ________

A. $-\sqrt{2} + \sqrt{2}\boldsymbol{i}$ **B.** $2\sqrt{2} - 2\sqrt{2}\boldsymbol{i}$

C. $-2\sqrt{2} - 2\sqrt{2}\boldsymbol{i}$ **D.** $-2\sqrt{2} + 2\sqrt{2}\boldsymbol{i}$

For Exercises 17 and 18, let $z_1 = 8\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$ and $z_2 = 0.5\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$.

17. Write the rectangular form of z_1z_2. **17.** ________

A. $-4\boldsymbol{i}$ **B.** 4 **C.** $4 + 4\boldsymbol{i}$ **D.** -4

18. Write the rectangular form of $\frac{z_1}{z_2}$. **18.** ________

A. $8 + 8\sqrt{3}\boldsymbol{i}$ **B.** $-8 + 8\sqrt{3}\boldsymbol{i}$ **C.** $16 + 16\sqrt{3}\boldsymbol{i}$ **D.** $8 - 8\sqrt{3}\boldsymbol{i}$

19. Simplify $(3\sqrt{3} + 3\boldsymbol{i})^{-3}$ and express the result in rectangular form. **19.** ________

A. $-216\boldsymbol{i}$ **B.** $-\frac{1}{216}\boldsymbol{i}$ **C.** $\frac{1}{216}\boldsymbol{i}$ **D.** $216\boldsymbol{i}$

20. Which of the following is *not* a root of $z^3 = 1 - \sqrt{3}\boldsymbol{i}$ to the nearest hundredth? **20.** ________

A. $-0.22 + 1.24\boldsymbol{i}$ **B.** $-0.97 - 0.81\boldsymbol{i}$

C. $1.02 - 0.65\boldsymbol{i}$ **D.** $1.18 - 0.43\boldsymbol{i}$

Bonus Find $(\cos\theta - \boldsymbol{i}\sin\theta)^2$. **Bonus:** ________

A. $\cos 2\theta + \boldsymbol{i}\sin 2\theta$ **B.** $\cos^2\theta + \boldsymbol{i}\sin^2\theta$

C. $\cos^2\theta - \boldsymbol{i}\sin^2\theta$ **D.** $\cos 2\theta - \boldsymbol{i}\sin 2\theta$

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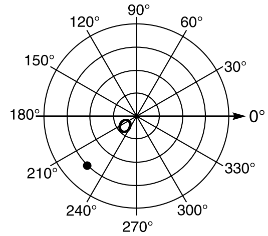
Chapter 9

Chapter 9 Test, Form 1B

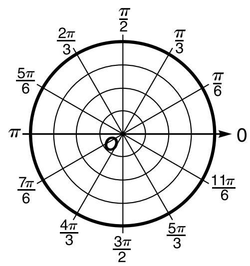
Write the letter for the correct answer in the blank at the right of each problem.

1. Find the polar coordinates that do not describe the point in the given graph. **1.** ________



A. $(-3, 45°)$
B. $(-3, -135°)$
C. $(3, 225°)$
D. $(-3, -315°)$

2. Find the equation represented in the given graph. **2.** ________

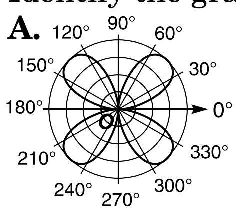


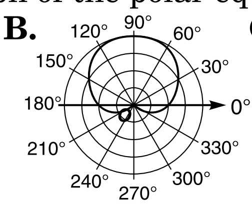
A. $r = 2$
B. $\theta = 2\pi$
C. $\theta = 4$
D. $r = 4$

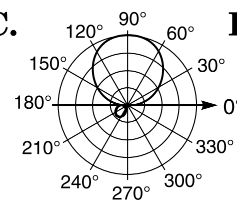
3. Find the distance between the points with polar coordinates $(3, 120°)$ and $(0.5, 49°)$. **3.** ________

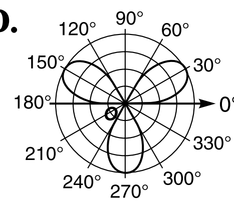
A. 2.88 **B.** 3.19 **C.** 3.49 **D.** 1.59

4. Identify the graph of the polar equation $r = 2 + 2 \sin \theta$. **4.** ________

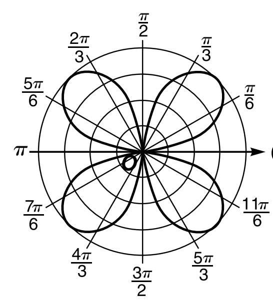
A.


B.


C.


D.


5. Find the equation whose graph is given. **5.** ________



A. $r = 4 \sin \theta$
B. $r = 2 + 2 \sin \theta$
C. $r = 4 \sin 2\theta$
D. $r^2 = 16 \sin 2\theta$

6. Find the polar coordinates of the point with rectangular coordinates $(2, -2)$. **6.** ________

A. $\left(2, \frac{\pi}{4}\right)$ **B.** $\left(\sqrt{2}, \frac{7\pi}{4}\right)$ **C.** $\left(2\sqrt{2}, \frac{\pi}{4}\right)$ **D.** $\left(2\sqrt{2}, \frac{7\pi}{4}\right)$

7. Find the rectangular coordinates of the point with polar coordinates $\left(-2, \frac{5\pi}{6}\right)$. **7.** ________

A. $(-\sqrt{3}, -1)$ **B.** $(-2\sqrt{3}, 2)$ **C.** $(\sqrt{3}, -1)$ **D.** $(2\sqrt{3}, -2)$.

8. Write the rectangular equation $y = x$ in polar form. **8.** ________

A. $\theta = 45°$ **B.** $r = \tan \theta$ **C.** $r = \cos \theta$ **D.** $\theta = 1$

9. Write the polar equation $r = 3 \sin \theta$ in rectangular form. **9.** ________

A. $y = 3x$
B. $x^2 + y^2 - 3y = 0$
C. $x^2 + y^2 - 3x = 0$
D. $x = 3y$

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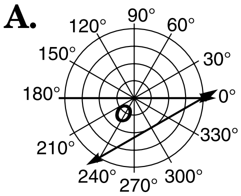
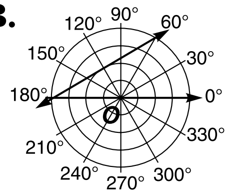
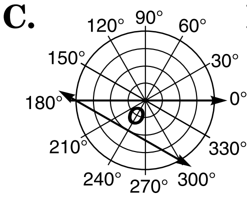
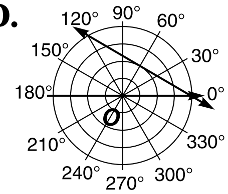
Chapter 9 Test, Form 1B (continued)

10. Identify the graph of the polar equation $r = 2 \sec(\theta + 120°)$. **10.** ________

A.


B.


C.


D.


11. Write $3x + 2y - 13 = 0$ in polar form. **11.** ________

A. $-\sqrt{13} = r\cos(\theta - 34°)$ **B.** $\sqrt{13} = r\cos(\theta - 34°)$
C. $-\sqrt{13} = r\cos(\theta + 34°)$ **D.** $\sqrt{13} = r\cos(\theta + 34°)$

12. Simplify $(3 - \boldsymbol{i}^7) - 2(\boldsymbol{i}^6 - 5\boldsymbol{i})$. **12.** ________

A. $5 + 11\boldsymbol{i}$ **B.** $5 + 9\boldsymbol{i}$ **C.** $1 + 11\boldsymbol{i}$ **D.** $5 - 9\boldsymbol{i}$

13. Simplify $(5 - 3\boldsymbol{i})(2 + 4\boldsymbol{i})$. **13.** ________

A. $-2 + 14\boldsymbol{i}$ **B.** $22 + 14\boldsymbol{i}$ **C.** $22 - 14\boldsymbol{i}$ **D.** $-2 - 14\boldsymbol{i}$

14. Simplify $\frac{5 + 2\boldsymbol{i}}{3 - 4\boldsymbol{i}}$. **14.** ________

A. $\frac{7}{25} - \frac{26}{25}\boldsymbol{i}$ **B.** $\frac{23}{25} + \frac{26}{25}\boldsymbol{i}$ **C.** $-1 - \frac{26}{7}\boldsymbol{i}$ **D.** $\frac{7}{25} + \frac{26}{25}\boldsymbol{i}$

15. Express $-2\sqrt{2} + 2\sqrt{2}\boldsymbol{i}$ in polar form. **15.** ________

A. $4\left(\cos\frac{3\pi}{4} - \boldsymbol{i}\sin\frac{3\pi}{4}\right)$ **B.** $2\left(\cos\frac{3\pi}{4} + \boldsymbol{i}\sin\frac{3\pi}{4}\right)$
C. $4\left(\cos\frac{3\pi}{4} + \boldsymbol{i}\sin\frac{3\pi}{4}\right)$ **D.** $4\left(\cos\frac{7\pi}{4} + \boldsymbol{i}\sin\frac{7\pi}{4}\right)$

16. Express $10\left(\cos\frac{7\pi}{6} + \boldsymbol{i}\sin\frac{7\pi}{6}\right)$ in rectangular form. **16.** ________

A. $-5\sqrt{3} + 5\boldsymbol{i}$ **B.** $-5 + 5\sqrt{3}\boldsymbol{i}$ **C.** $5\sqrt{3} + 5\boldsymbol{i}$ **D.** $-5\sqrt{3} - 5\boldsymbol{i}$

For Exercises 17 and 18, let $z_1 = 12\left(\cos\frac{7\pi}{6} + i\sin\frac{7\pi}{6}\right)$ ***and*** $z_2 = 3\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$.

17. Write the rectangular form of $z_1 z_2$. **17.** ________

A. $-18 + 18\sqrt{3}\boldsymbol{i}$ **B.** $-18 - 18\sqrt{3}\boldsymbol{i}$ **C.** $18 + 18\sqrt{3}\boldsymbol{i}$ **D.** $18 - 18\sqrt{3}\boldsymbol{i}$

18. Write the rectangular form of $\frac{z_1}{z_2}$. **18.** ________

A. 4 **B.** $-4\boldsymbol{i}$ **C.** -4 **D.** $4 - 4\boldsymbol{i}$

19. Simplify $(1 - \sqrt{3}\boldsymbol{i})^5$ and express the result in rectangular form. **19.** ________

A. $16 + 16\sqrt{3}\boldsymbol{i}$ **B.** $16\sqrt{3} + 16\boldsymbol{i}$ **C.** $16 - 16\sqrt{3}\boldsymbol{i}$ **D.** $-16 + 16\sqrt{3}\boldsymbol{i}$

20. Find $\sqrt[3]{27\boldsymbol{i}}$. **20.** ________

A. $\frac{3\sqrt{3}}{2} - \frac{3}{2}\boldsymbol{i}$ **B.** $\frac{3}{2} - \frac{3\sqrt{3}}{2}\boldsymbol{i}$ **C.** $-\frac{3\sqrt{3}}{2} + \frac{3}{2}\boldsymbol{i}$ **D.** $\frac{3\sqrt{3}}{2} + \frac{3}{2}\boldsymbol{i}$

Bonus Find $(\cos\theta + \boldsymbol{i}\sin\theta)^2$. **Bonus:** ________

A. $\cos 2\theta + \boldsymbol{i}\sin 2\theta$ **B.** $\cos^2\theta + \boldsymbol{i}\sin^2\theta$
C. $\cos^2\theta - \boldsymbol{i}\sin^2\theta$ **D.** $\cos 2\theta - \sin 2\theta$

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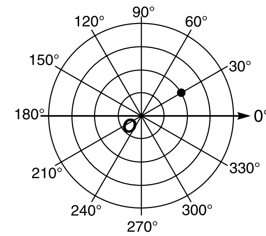
Chapter 9

Chapter 9 Test, Form 1C

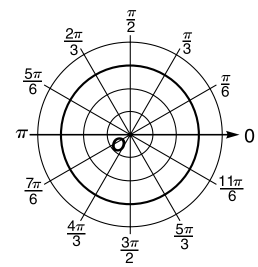
Write the letter for the correct answer in the blank at the right of each problem.

1. Find the polar coordinates that do *not* describe the point in the given graph. **1.** ________



A. $(-2, 30°)$
B. $(-2, 210°)$
C. $(2, 30°)$
D. $(-2, -150°)$

2. Find the equation represented in the given graph. **2.** ________



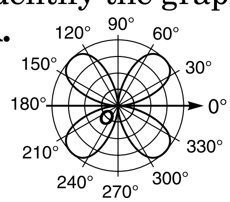
A. $\theta = 3$
B. $r = 3$
C. $\theta = 2\pi$
D. $r = 2$

3. Find the distance between the points with polar coordinates $(2, 120°)$ and $(1, 45°)$. **3.** ________

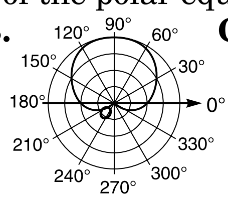
A. 1.40 **B.** 2.98 **C.** 2.46 **D.** 1.99

4. Identify the graph of the polar equation $r = 4 \sin \theta$. **4.** ________

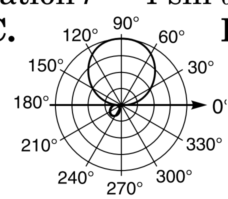
A.



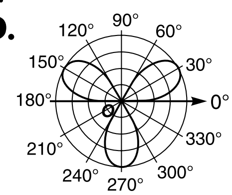
B.



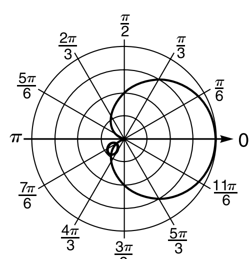
C.



D.



5. Find the equation whose graph is given. **5.** ________



A. $r = 4 \cos \theta$
B. $r = 2 - 2 \cos \theta$
C. $r = 2 + 2 \cos \theta$
D. $r = 2 + 2 \sin \theta$

6. Find the polar coordinates of the point with rectangular coordinates $(\sqrt{3}, 1)$. **6.** ________

A. $\left(2, \frac{\pi}{3}\right)$ **B.** $\left(2, \frac{\pi}{6}\right)$ **C.** $\left(2, \frac{\pi}{4}\right)$ **D.** $\left(1, \frac{\pi}{6}\right)$

7. Find the rectangular coordinates of the point with polar coordinates $(3, 180°)$. **7.** ________

A. $(-3, 0)$ **B.** $(0, 3)$ **C.** $(3, 0)$ **D.** $(0, -3)$

8. Write the rectangular equation $x = 3$ in polar form. **8.** ________

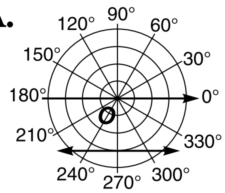
A. $r \sin \theta = 3$ **B.** $r = 3$
C. $\theta = 3$ **D.** $r \cos \theta = 3$

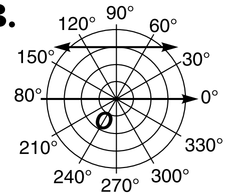
9. Write the polar equation $r = 3$ in rectangular form. **9.** ________

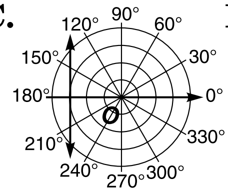
A. $x^2 - 9 = 0$ **B.** $x^2 + y^2 - 9y = 0$
C. $x^2 + y^2 = 9$ **D.** $xy = 9$

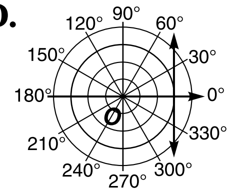
Chapter 9 Test, Form 1C (continued)

10. Identify the graph of the polar equation $r = 3 \sec (\theta + 90°)$. **10.** ________

A. 

B. 

C. 

D. 

11. Write $3x + 4y - 5 = 0$ in polar form. **11.** ________

A. $-1 = r \cos (\theta - 53°)$ **B.** $1 = r \cos (\theta - 53°)$
C. $-1 = r \cos (\theta + 53°)$ **D.** $1 = r \cos (\theta + 53°)$

12. Simplify $2(3 - 4\boldsymbol{i}) + (5 - \boldsymbol{i}^{15})$ **12.** ________

A. $10 - 7\boldsymbol{i}$ **B.** $11 - 9\boldsymbol{i}$ **C.** $12 - 8\boldsymbol{i}$ **D.** $11 - 7\boldsymbol{i}$

13. Simplify $(3 + \boldsymbol{i})(1 - \boldsymbol{i})$ **13.** ________

A. $2 - 2\boldsymbol{i}$ **B.** $2 + 2\boldsymbol{i}$ **C.** $4 - 2\boldsymbol{i}$ **D.** $4 + 2\boldsymbol{i}$

14. Simplify $\frac{2 + \boldsymbol{i}}{1 - \boldsymbol{i}}$. **14.** ________

A. $\frac{1}{2} + \frac{3}{2}\boldsymbol{i}$ **B.** $\frac{1}{2} - \frac{3}{2}\boldsymbol{i}$ **C.** $\frac{3}{2} - \frac{3}{2}\boldsymbol{i}$ **D.** $1 + 2\boldsymbol{i}$

15. Express $3\sqrt{3} + 3\boldsymbol{i}$ in polar form. **15.** ________

A. $3\left(\cos \frac{\pi}{6} + \boldsymbol{i} \sin \frac{\pi}{6}\right)$ **B.** $6\left(\cos \frac{\pi}{6} - \boldsymbol{i} \sin \frac{\pi}{6}\right)$
C. $6\left(\cos \frac{\pi}{3} + \boldsymbol{i} \sin \frac{\pi}{3}\right)$ **D.** $6\left(\cos \frac{\pi}{6} + \boldsymbol{i} \sin \frac{\pi}{6}\right)$

16. Express $2\left(\cos \frac{\pi}{3} + \boldsymbol{i} \sin \frac{\pi}{3}\right)$ in rectangular form. **16.** ________

A. $-1 + \sqrt{3}\boldsymbol{i}$ **B.** $1 + \sqrt{3}\boldsymbol{i}$ **C.** $1 - \sqrt{3}\boldsymbol{i}$ **D.** $\sqrt{3} + \boldsymbol{i}$

For Exercises 17 and 18, let $z_1 = 4(\cos 135° + i \sin 135°)$ and $z_2 = 2(\cos 45° + i \sin 45°)$.

17. Write the rectangular form of $z_1 z_2$. **17.** ________

A. $-8\boldsymbol{i}$ **B.** -8 **C.** $8 + 8\boldsymbol{i}$ **D.** 8

18. Write the rectangular form of $\frac{z_1}{z_2}$. **18.** ________

A. $2\boldsymbol{i}$ **B.** -2 **C.** $-2\boldsymbol{i}$ **D.** $2 + 2\boldsymbol{i}$

19. Simplify $(\sqrt{3} + \boldsymbol{i})^4$ and express the result in rectangular form. **19.** ________

A. $8 + 8\sqrt{3}\boldsymbol{i}$ **B.** $8 - 8\sqrt{3}\boldsymbol{i}$ **C.** $16 + 16\sqrt{3}\boldsymbol{i}$ **D.** $-8 + 8\sqrt{3}\boldsymbol{i}$

20. Find $\sqrt[3]{\boldsymbol{i}}$. **20.** ________

A. $\frac{\sqrt{3}}{2} - \frac{1}{2}\boldsymbol{i}$ **B.** $-\frac{\sqrt{3}}{2} + \frac{1}{2}\boldsymbol{i}$ **C.** $\frac{\sqrt{3}}{2} + \frac{1}{2}\boldsymbol{i}$ **D.** $\frac{1}{2} + \frac{\sqrt{3}}{2}\boldsymbol{i}$

Bonus If $2 + 2\boldsymbol{i} = 2\sqrt{2}(\cos 45° + \boldsymbol{i} \sin 45°)$, find $2 - 2\boldsymbol{i}$. **Bonus:** ________

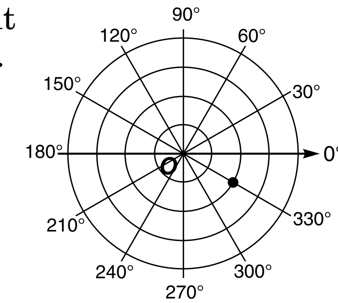
A. $2\sqrt{2}(\cos 45° + \boldsymbol{i} \sin 45°)$ **B.** $2\sqrt{2}(\cos 135° + \boldsymbol{i} \sin 135°)$
C. $2\sqrt{2}(\cos 225° + \boldsymbol{i} \sin 225°)$ **D.** $2\sqrt{2}(\cos 315° + \boldsymbol{i} \sin 315°)$

NAME ____________ DATE ____________ PERIOD ____________

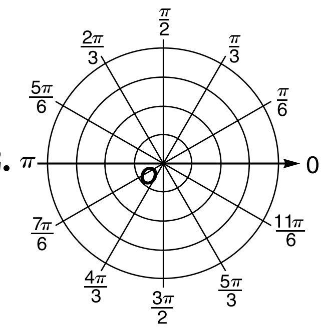
Chapter 9

Chapter 9 Test, Form 2A

1. Write the polar coordinates of the point in the graph if $r < 0$ and $0° < \theta < 180°$. **1.** ____________

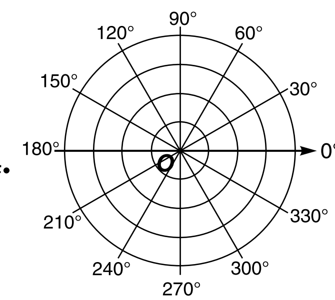


2. Graph the polar equation $r = -3$. **2.**



3. Find the distance between the points with polar coordinates $\left(-1.5, \frac{3\pi}{4}\right)$ and $\left(-2, \frac{\pi}{6}\right)$. **3.** ____________

4. Graph the polar equation $r = 4 \sin 3\theta$. **4.**



5. Identify the classical curve represented by the equation $r^2 = 16 \sin 2\theta$. **5.** ____________

6. Find the polar coordinates of the point with rectangular coordinates $(-3, -3)$. Use $0 \leq \theta < 2\pi$ and $r \geq 0$. **6.** ____________

7. Find the rectangular coordinates of the point with polar coordinates $\left(6, \frac{7\pi}{4}\right)$. **7.** ____________

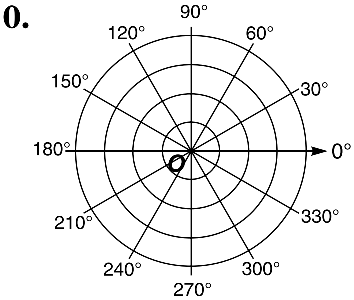
8. Write the rectangular equation $x - 2y + 5 = 0$ in polar form. Round ϕ to the nearest degree. **8.** ____________

9. Write the polar equation $r^2 \sin 2\theta = 8$ in rectangular form. **9.** ____________

Chapter 9

Chapter 9 Test, Form 2A (continued)

10. Graph the polar equation $1 = r\cos(\theta - 15°)$. **10.**



11. Write $3x + y = 10$ in polar form. **11.** ______________

12. Simplify $3(2\boldsymbol{i} + \boldsymbol{i}^{10}) - 4(8 - \boldsymbol{i}^{49})$. **12.** ______________

13. Simplify $(3 - 4\boldsymbol{i})(2 + 5\boldsymbol{i})$. **13.** ______________

14. Simplify $\frac{3 - 4\boldsymbol{i}}{2 + 5\boldsymbol{i}}$. **14.** ______________

15. Express $2 - 2\sqrt{3}\boldsymbol{i}$ in polar form. **15.** ______________

16. Express $8\left(\cos \frac{5\pi}{4} + \boldsymbol{i} \sin \frac{5\pi}{4}\right)$ in rectangular form. **16.** ______________

For Exercises 17 and 18, let $z_1 = 12\left(\cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3}\right)$ ***and*** $z_2 = 2\left(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6}\right)$.

17. Write the rectangular form of $z_1 z_2$. **17.** ______________

18. Write the rectangular form of $\frac{z_1}{z_2}$. **18.** ______________

19. Simplify $(4 - 4\boldsymbol{i})^{-2}$ and express the result in rectangular form. **19.** ______________

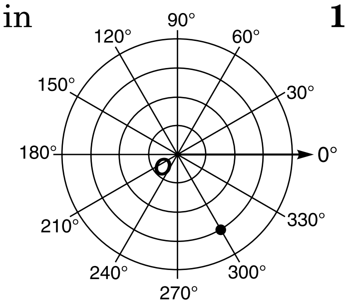
20. Solve the equation $z^3 = -2 + 2\sqrt{3}\boldsymbol{i}$. **20.** ______________

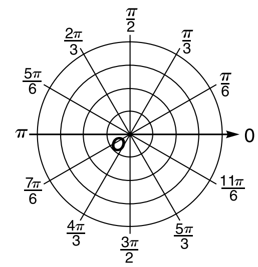
Bonus If $3 + \sqrt{3}\boldsymbol{i} = 2\sqrt{3}(\cos 30° + \boldsymbol{i} \sin 30°)$, find $3 - \sqrt{3}\boldsymbol{i}$. **Bonus:** ______________

NAME ______________________ DATE ______________ PERIOD ________

Chapter 9 Test, Form 2B

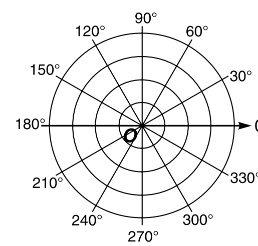
1. Write the polar coordinates of the point in the graph if $-90° < \theta < 0°$. **1.** ________________





2. Graph the polar equation $\theta = \frac{5\pi}{6}$. **2.**

3. Find the distance between the points with polar coordinates (2.5, 150°) and (1, 70°). **3.** ________________



4. Graph the polar equation $r = 2 - 2 \sin \theta$. **4.**

5. Identify the classical curve represented by the equation $r = 2 + 5 \sin \theta$. **5.** ________________

6. Find the polar coordinates of the point with rectangular coordinates $\left(1, -\sqrt{3}\right)$. Use $0 \le \theta < 2\pi$ and $r \ge 0$. **6.** ________________

7. Find the rectangular coordinates of the point with polar coordinates $\left(2, \frac{2\pi}{3}\right)$. **7.** ________________

8. Write the rectangular equation $x^2 + y^2 = 4$ in polar form. **8.** ________________

9. Write the polar equation $r^2 = 8$ in rectangular form. **9.** ________________

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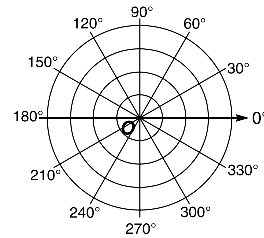
Chapter 9 Test, Form 2B (continued)

10. Graph the polar equation $r = 2 \csc (\theta - 60°)$. **10.**



11. Write $x + 2y - 5 = 0$ in polar form. **11.** ________________

12. Simplify $2(\boldsymbol{i}^{21} + 7) - (5 - \boldsymbol{i}^3)$. **12.** ________________

13. Simplify $(3 + 2\boldsymbol{i})^2$. **13.** ________________

14. Simplify $\frac{4 - 2\boldsymbol{i}}{3 + 5\boldsymbol{i}}$. **14.** ________________

15. Express $-6 + 6\boldsymbol{i}$ in polar form. **15.** ________________

16. Express $4\left(\cos \frac{\pi}{6} + \boldsymbol{i} \sin \frac{\pi}{6}\right)$ in rectangular form. **16.** ________________

For Exercises 17 and 18, let $z_1 = 8\left(\cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6}\right)$ and $z_2 = 4\left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}\right)$.

17. Write the rectangular form of $z_1 z_2$. **17.** ________________

18. Write the rectangular form of $\frac{z_1}{z_2}$. **18.** ________________

19. Simplify $\left(2\sqrt{3} - 2\boldsymbol{i}\right)^3$ and express the result in rectangular form. **19.** ________________

20. Find $\sqrt[3]{-64\boldsymbol{i}}$. **20.** ________________

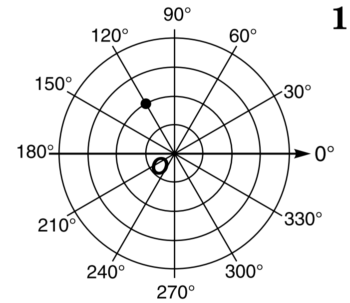
Bonus Find $\left(\cos \frac{\pi}{4} + \boldsymbol{i} \sin \frac{\pi}{4}\right)^3$. Express the result in rectangular form. **Bonus:** ________________

Chapter 9

NAME ______________________ DATE ______________ PERIOD ________

Chapter 9 Test, Form 2C

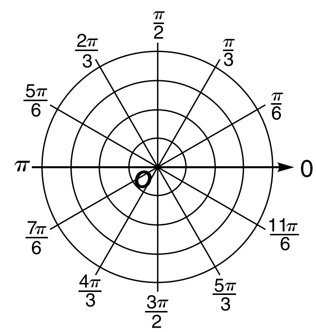
1. Write the polar coordinates of the point in the graph if $0° < \theta < 180°$.



1. ______________

2. Graph the polar equation $\theta = \frac{\pi}{3}$.

2.

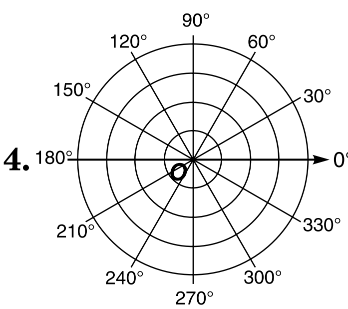


3. Find the distance between the points with polar coordinates (3.2, 120°) and (2, 45°).

3. ______________

4. Graph the polar equation $r = 4 \cos \theta$.

4.



5. Identify the classical curve represented by the equation $r = 4 \sin 2\theta$.

5. ______________

6. Find the polar coordinates of the point with rectangular coordinates (0, 1).

6. ______________

7. Find the rectangular coordinates of the point with polar coordinates $\left(2, \frac{\pi}{4}\right)$.

7. ______________

8. Write the rectangular equation $y = 2$ in polar form.

8. ______________

9. Write the polar equation $r = 3$ in rectangular form.

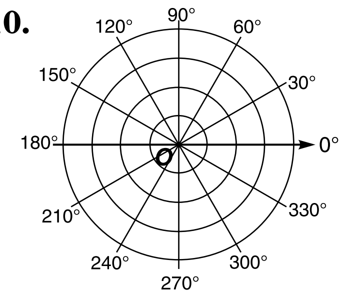
9. ______________

NAME ______________________ DATE ______________ PERIOD ________

Chapter 9 Test, Form 2C (continued)

10. Graph the polar equation $r = 2 \sec (\theta - 60°)$.

10.



11. Write $x + y - 2 = 0$ in polar form. **11.** ________________

12. Simplify $(3 - \boldsymbol{i}^{17}) + (2 + 3\boldsymbol{i})$. **12.** ________________

13. Simplify $(2 + 4\boldsymbol{i})(2 - 4\boldsymbol{i})$. **13.** ________________

14. Simplify $\frac{3 - \boldsymbol{i}}{2 + \boldsymbol{i}}$. **14.** ________________

15. Express $2\sqrt{3} + 2\boldsymbol{i}$ in polar form. **15.** ________________

16. Express $6\sqrt{2}\left(\cos \frac{3\pi}{4} + \boldsymbol{i} \sin \frac{3\pi}{4}\right)$ in rectangular form. **16.** ________________

For Exercises 17 and 18, let $z_1 = 12(\cos 240° + i \sin 240°)$ and $z_2 = 0.5(\cos 30° + i \sin 30°)$.

17. Write the rectangular form of $z_1 z_2$. **17.** ________________

18. Write the rectangular form of $\frac{z_1}{z_2}$. **18.** ________________

19. Simplify $(2 + 2\boldsymbol{i})^4$ and express the result in rectangular form. **19.** ________________

20. Find $\sqrt[3]{-8\boldsymbol{i}}$. **20.** ________________

Bonus Find $\left(\cos \frac{\pi}{4} + \boldsymbol{i} \sin \frac{\pi}{4}\right)^2$. Express the result in rectangular form. **Bonus:** ________________

Chapter 9

NAME ______________________________ DATE ________________ PERIOD ________

Chapter 9 Open-Ended Assessment

Instructions: ***Demonstrate your knowledge by giving a clear, concise solution to each problem. Be sure to include all relevant drawings and justify your answers. You may show your solution in more than one way or investigate beyond the requirements of the problem.***

1. a. Write the rectangular coordinates for a point in a plane.

b. Graph the point described in part **a**.

c. Find the polar coordinates for the point described in part **a**. Graph the point in the polar coordinate system.

d. Explain how the two graphs are related.

2. a. Write the polar coordinates for a point in a plane.

b. Graph the point described in part **a**.

c. Find the rectangular coordinates for the point described in part **a**. Graph the point in the rectangular coordinate system.

d. Explain how the two graphs are related.

3. a. Draw the graph of $r = \cos\theta$.

b. Tell how the graph of $r = 2\cos\theta$ differs from the graph in part **a**.

c. What type of classical curve is represented by $r = \cos 4\theta$?

d. What type of classical curve is represented by $r = 1 + \cos\theta$?

e. Write a polar equation for a classical curve. Graph the equation and name the type of curve.

4. a. Find two complex numbers a and b whose sum is $3 - 3\boldsymbol{i}$.

b. Express the complex numbers a and b in part **a** in polar form. Explain each step.

c. Find the product of a and b.

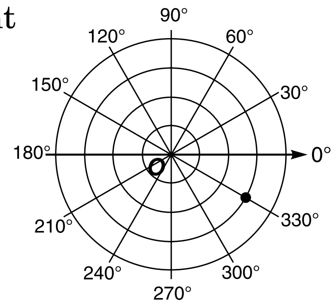
d. Show two ways to find $(3 - 3\boldsymbol{i})^4$. Then find $(3 - 3\boldsymbol{i})^4$.

e. Explain how to find $(3 - 3\boldsymbol{i})^{\frac{1}{3}}$. Then find $(3 - 3\boldsymbol{i})^{\frac{1}{3}}$.

NAME ______________________ DATE ____________ PERIOD ________

Chapter 9 Mid-Chapter Test (Lessons 9-1 through 9-4)

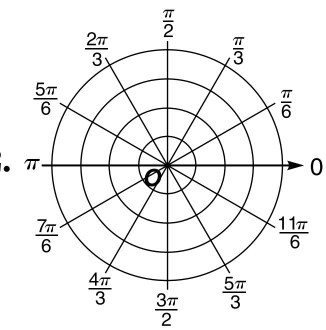
1. Write the polar coordinates of the point in the given graph if $0° < \theta < 180°$.



1. ________________

2. Graph the polar equation $r = 3$.

2.

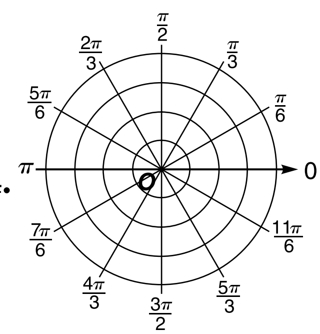


3. Find the distance between the points with polar coordinates (3, 150°) and (−2, 45°).

3. ________________

4. Graph the polar equation $r = 2 - 2 \cos \theta$.

4.



5. Identify the type of classical curve represented by the graph of $r = 3 \cos 2\theta$.

5. ________________

6. Find the polar coordinates of the point with rectangular coordinates (−3, −3). Use $0 \le \theta < 2\pi$ and $r \ge 0$.

6. ________________

7. Find the rectangular coordinates of the point with polar coordinates (4, 150°).

7. ________________

8. Write the rectangular equation $x^2 + y^2 = 5y$ in polar form.

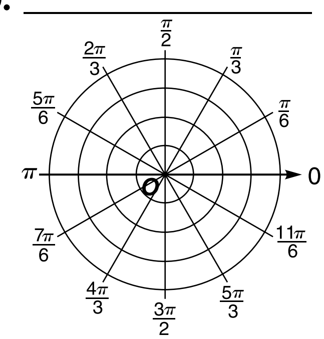
8. ________________

9. Write the polar equation $\theta = \frac{\pi}{3}$ in rectangular form.

9. ________________

10. Graph the polar equation $r = 2 \sec (\theta + \pi)$ and state the rectangular form of the linear equation.

10. ________________



Chapter 9

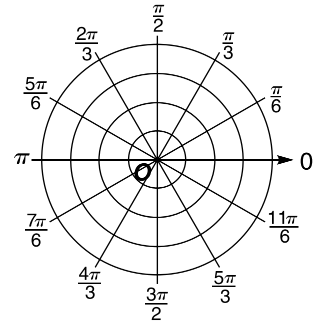
NAME ________________ DATE ________ PERIOD ______

Chapter 9, Quiz A (Lessons 9-1 and 9-2)

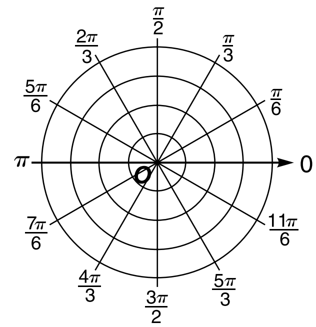
1. Write the polar coordinates of the point at (3, 30°) if $-180° < \theta < 0°$. **1.** ________________

Graph each polar equation.

2. $\theta = \frac{5\pi}{6}$ **2.**



3. $r^2 = 16 \sin 2\theta$ **3.**



4. Find the distance between the points with polar coordinates (−2, 210°) and (4, 60°). **4.** ________________

Chapter 9

NAME ________________ DATE ________ PERIOD ______

Chapter 9, Quiz B (Lessons 9-3 and 9-4)

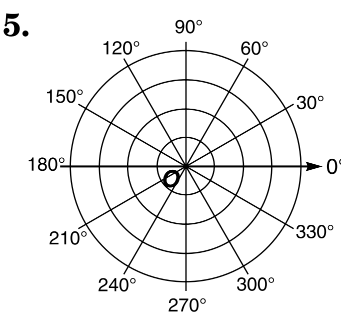
1. Find the polar coordinates of the point with rectangular coordinates $(4, -4\sqrt{3})$. Use $0 \le \theta < 2\pi$ and $r \ge 0$. **1.** ________________

2. Find the rectangular coordinates of the point with polar coordinates (6, 315°). **2.** ________________

3. Write the rectangular equation $x + 3y - 5 = 0$ in polar form. Round ϕ to the nearest degree. **3.** ________________

4. Write the polar equation $r = 5$ in rectangular form. **4.** ________________

5. Graph the polar equation $r = 3 \sec (\theta + 30°)$. **5.**



Chapter 9

NAME ________________ DATE ________ PERIOD ______

Chapter 9, Quiz C (Lessons 9-5 and 9-6)

Simplify.

1. $2(3 - \boldsymbol{i}^{11}) - (4 - \boldsymbol{i})$ **1.** ________

2. $(2 - 4\boldsymbol{i})(3 + 5\boldsymbol{i})$ **2.** ________

3. $\frac{4 - 3\boldsymbol{i}}{5 + 2\boldsymbol{i}}$ **3.** ________

4. Express $2\sqrt{3} - 2\boldsymbol{i}$ in polar form. **4.** ________

5. Express $8\left(\cos \frac{3\pi}{4} + \boldsymbol{i} \sin \frac{3\pi}{4}\right)$ in rectangular form. **5.** ________

Chapter 9

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Chapter 9, Quiz D (Lessons 9-7 and 9-8)

Find each product, quotient, or power and express the result in rectangular form. Let $z_1 = 4(\cos 120° + i \sin 120°)$ and $z_2 = 0.5(\cos 30° + i \sin 30°)$.

1. $z_1 z_2$ **1.** ________

2. $\frac{z_1}{z_2}$ **2.** ________

3. $z_1^{\,2}$ **3.** ________

Find each power or root. Express the result in rectangular form.

4. $(\sqrt{2} + \sqrt{2}\boldsymbol{i})^4$ **4.** ________

5. $\sqrt[5]{-32\boldsymbol{i}}$ **5.** ________

Chapter 9

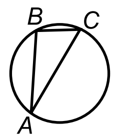
NAME ______________________ DATE ______________ PERIOD ________

Chapter 9 SAT and ACT Practice

After working each problem, record the correct answer on the answer sheet provided or use your own paper.

Multiple Choice

1. $\overline{AC}$ is a diameter of the circle at the right. Point B is on the circle such that $m\angle BAC = 2x°$. Find $m\angle BCA$.



A $6x°$
B $\left[\frac{1}{2}(180 - 2x)\right]°$
C $(2x - 90)°$
D $[2(45 - x)]°$
E It cannot be determined from the information given.

2. A chord with a length of 8 is 2 units from the center of a circle. Find the diameter.

A $\sqrt{5}$
B $2\sqrt{5}$
C $4\sqrt{5}$
D $2\sqrt{3}$
E $4\sqrt{3}$

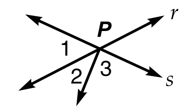
3. $2 \cos \frac{\pi}{4} =$

A 0
B $\frac{1}{2}$
C 1
D $\sqrt{2}$
E 2

4. $\left(\sin \frac{\pi}{3}\right)\left(\cos \frac{\pi}{6}\right) - \left(\cos \frac{\pi}{3}\right)\left(\sin \frac{\pi}{6}\right) =$

A $-\frac{1}{2}$
B $\frac{1}{2}$
C $\frac{3}{4}$
D 1
E $\frac{5}{4}$

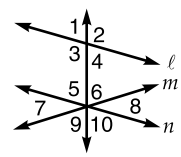
5. Given that lines r and s intersect at P, $m\angle 1 = 3x°$, and $m\angle 3 = m\angle 1$, find $m\angle 2$.



A $x°$
B $(180 - 3x)°$
C $6x°$
D $(180 - 6x)°$
E $3x°$

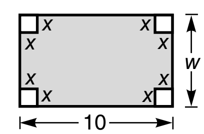
6. If $\ell \parallel n$, then

I. Angles 3 and 5 are supplementary
II. $m\angle 7 = m\angle 8$
III. $m\angle 3 = m\angle 7 + m\angle 6$



A I only
B II only
C III only
D I and II only
E I, II, and III

7. In the rectangle below, what is the area of the shaded region?



A $10w$
B $4x^2$
C $10w - 4x$
D $10w - x^2$
E $10w - 4x^2$

8. On a map, 1 inch represents 2 miles. A circle on the map has a circumference of 5π inches. What area does the circular region on the map represent?

A 10π mi^2
B 25π mi^2
C 5π mi^2
D 100π mi^2
E 50π mi^2

9. $\frac{1}{10^{15}} - \frac{1}{10^{16}} =$

A $-\frac{9}{10^{16}}$
B $\frac{9}{10^{16}}$
C $\frac{1}{10}$
D $-\frac{1}{10}$
E $\frac{1}{10^{16}}$

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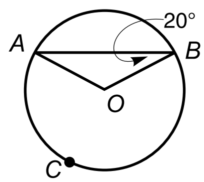
Chapter 9 SAT and ACT Practice (continued)

10. Choose the expression that is *not* equivalent to the other three.
A $4 + 2\sqrt{5}$
B $\frac{1}{2}(8 + \sqrt{80})$
C $6 + \sqrt{80} - 2 - \sqrt{20}$
D $\sqrt{2}(\sqrt{8} + \sqrt{10})$
E They are all equivalent.

11. In the circle O below, if $m\angle B = 20°$, find $m\widehat{ACB}$.
A 40°
B 140°
C 220°
D 320°
E None of these



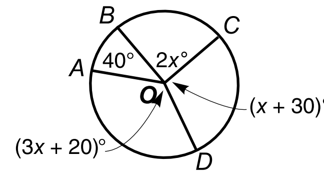
12. Given that $\overline{AB}$ is tangent to circle O at point A, $\overline{OA}$ is a radius, $OA = 6$, and $OB = 8$, find AB.
A $\sqrt{7}$ **B** $2\sqrt{7}$
C $4\sqrt{7}$ **D** 5
E 10

13. If $\frac{(x - 3)(y + 3)z}{3w} = 60$, which of the variables cannot be 3?
A x **B** y
C z **D** w
E None of these

14. A function f is described by $f(x) = 3x - 6$ and a function g is described by $g(x) = 12 - 6x$. Which of the following statements is true?
A $g(f(x)) = f(g(x))$
B $g(f(x)) = 2f(g(x))$
C $g(f(x)) = -2f(g(x))$
D $g(f(x)) = f(g(x)) + 18$
E None of these

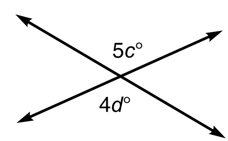
15. $\triangle ABC$ is inscribed in a circle. $m\angle A = 40°$, and $m\angle C = 80°$. Which is the shortest chord?
A $\overline{AB}$ **B** $\overline{BC}$
C $\overline{CA}$ **D** $AC = BC$
E It cannot be determined from the information given.

16. In circle O below, find $m\widehat{ADC}$.
A 45°
B 75°
C 155°
D 230°
E 245°



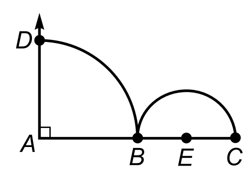
17–18. Quantitative Comparison
A if the quantity in Column A is greater
B if the quantity in Column B is greater
C if the two quantities are equal
D if the relationship cannot be determined from the information given

17.



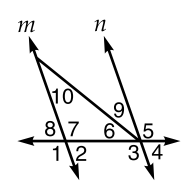
Column A	Column B
c	d

18.



Column A	Column B
Length of $\widehat{BC}$	Length of $\widehat{BD}$

19–20. Use the diagram for Exercises 19 and 20. In the diagram, $m \parallel n$.



19. Grid-In $m\angle 2$ is 60° less than twice $m\angle 3$. Find $m\angle 1$.

20. Grid-In $m\angle 10 = 3x + 30$ and $m\angle 9 = x + 40$. Find $m\angle 9$.

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Chapter 9

Chapter 9 Cumulative Review (Chapters 1-9)

1. Find $[f \circ g](x)$ for $f(x) = \frac{1}{2 + x}$ and $g(x) = 3x - 2$. **1.** ______________

2. Determine whether the graphs of $3x + 2y - 5 = 0$ and $y = \frac{2}{3}x + 4$ are parallel, coinciding, perpendicular, or none of these. **2.** ______________

3. Solve the system of equations. $5x - 3y = 11$, $x + 2y = -16$ **3.** ______________

4. Find the inverse of $\begin{bmatrix} 3 & -1 \\ 4 & 2 \end{bmatrix}$, if it exists. **4.** ______________

5. Determine whether the function $f(x) = |x^3|$ is odd, even, or neither. **5.** ______________

6. Solve the equation $2x^2 - 10x + 12 = 0$. **6.** ______________

7. List the possible rational roots of $3x^3 - 5x^2 + 6x - 2 = 0$. **7.** ______________

8. Find the measure of the reference angle for 220°. **8.** ______________

9. State the amplitude, period, phase shift, and vertical shift for $y = 5 - 3\sin(2\theta - \pi)$. **9.** ______________

10. Find the value of $\text{Cos}^{-1}\left(\tan \frac{3\pi}{4}\right)$. **10.** ______________

11. Simplify $\frac{1 - \cos^2\theta}{1 - \sin^2\theta}$. **11.** ______________

12. Write the ordered pair that represents the vector from $M(-7, 4)$ to $N(3, -1)$. **12.** ______________

13. Find the cross product $\langle -6, 3, 2\rangle \times \langle 3, 4, -1\rangle$. **13.** ______________

14. Find the distance between the points with polar coordinates (3, 150°) and (4, 70°). **14.** ______________

15. Express $2\left(\cos \frac{\pi}{6} + \boldsymbol{i}\sin \frac{\pi}{6}\right)$ in rectangular form. **15.** ______________

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Precalculus Semester Test

Write the letter for the correct answer in the blank at the right of each problem.

1. Which angle is not coterminal with $-30°$? **1.** ________

A. $-\frac{\pi}{6}$ **B.** $-750°$ **C.** $\frac{35\pi}{6}$ **D.** $750°$

2. Which ordered triple represents $\overrightarrow{CD}$ for $C(5, 0, -1)$ and $D(3, -2, 6)$? **2.** ________

A. $\langle 8, -2, 5\rangle$ **B.** $\langle -2, -2, 7\rangle$
C. $\langle -2, 2, -7\rangle$ **D.** $\langle 2, 2, -5\rangle$

3. Evaluate $\cos\left(\text{Sin}^{-1}\frac{\sqrt{3}}{2}\right)$. **3.** ________

A. $\frac{\sqrt{3}}{2}$ **B.** $\frac{1}{2}$ **C.** $\sqrt{3}$ **D.** $\frac{\sqrt{2}}{2}$

4. Write the polynomial equation of least degree with roots $7\boldsymbol{i}$ and $-7\boldsymbol{i}$. **4.** ________

A. $x^2 + 49 = 0$ **B.** $x^2 - 49x = 0$
C. $x^2 - 7 = 0$ **D.** $x^2 + 7 = 0$

5. Find the angle to the nearest degree that the normal to the line with equation $3x - y + 4 = 0$ makes with the positive x-axis. **5.** ________

A. $-18°$ **B.** $18°$ **C.** $162°$ **D.** $108°$

6. Find the x-intercepts of the graph of the function $f(x) = (x - 3)(x^2 + 4x + 3)$. **6.** ________

A. 3, 1 **B.** -9 **C.** $-3, -1, 3$ **D.** $9, -9$

7. Find the discriminant of $4m^2 + 2m + 1 = 0$ and describe the nature of the roots of the equation. **7.** ________

A. -12, imaginary **B.** 12, real
C. 4, imaginary **D.** 2, real

8. Solve $\sin\theta = -1$ for all values of θ. Assume k is any integer. **8.** ________

A. $90° + 360k°$ **B.** $180° + 360k°$ **C.** $360k°$ **D.** $270° + 360k°$

9. List all possible rational roots of $f(x) = 2x^3 + 5x^2 + 4x + 3$. **9.** ________

A. $\pm 1, \pm 2$ **B.** $\pm 1, \pm 3, \pm\frac{1}{2}, \pm\frac{3}{2}$
C. $\pm 1, \pm 2, \pm 3, \pm\frac{2}{3}$ **D.** $\pm 1, \pm 3$

10. Find the rectangular coordinates of the point with polar coordinates $\left(1, \frac{\pi}{4}\right)$. **10.** ________

A. $\left(1, \frac{1}{2}\right)$ **B.** $\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ **C.** $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ **D.** $\left(-\frac{\sqrt{3}}{2}, \frac{\sqrt{2}}{2}\right)$

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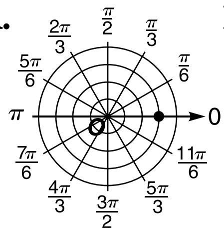
Precalculus Semester Test (continued)

11. A section of highway is 4.2 kilometers long and rises at a uniform grade making a 3.2° angle with the horizontal. What is the change in elevation of this section of highway to the nearest thousandth? **11.** ________

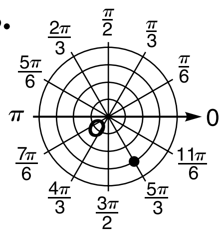
A. 0.235 km **B.** 0.013 km **C.** 4.193 km **D.** 0.234 km

12. Choose the graph of the point with polar coordinates $\left(3, \frac{\pi}{4}\right)$. **12.** ________

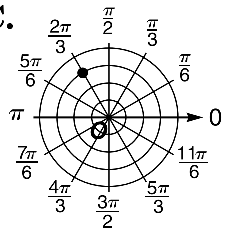
A.



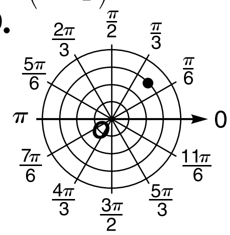
B.



C.



D.



13. Use the Remainder Theorem to find the remainder for $(2x^3 - 5x^2 + 3x + 4) \div (x - 2)$. **13.** ________

A. -6 **B.** 6 **C.** 2 **D.** 0

14. Find the polar coordinates of the point with rectangular coordinates (2, 2). **14.** ________

A. $\left(3\sqrt{2}, \frac{\pi}{3}\right)$ **B.** $\left(2\sqrt{2}, \frac{\pi}{4}\right)$ **C.** $(\sqrt{2}, \pi)$ **D.** $\left(\sqrt{2}, \frac{3\pi}{2}\right)$

15. If $\vec{\mathbf{v}}$ has magnitude 6 kilometers, $\vec{\mathbf{w}}$ has magnitude 18 kilometers, and both vectors have the same direction, which of the following is true? **15.** ________

A. $\vec{\mathbf{v}} = 3\vec{\mathbf{w}}$ **B.** $3\vec{\mathbf{v}} = \vec{\mathbf{w}}$ **C.** $\vec{\mathbf{v}} = \vec{\mathbf{w}}$ **D.** $3\vec{\mathbf{v}} = 18\vec{\mathbf{w}}$

16. Find the magnitude of $\overrightarrow{AB}$ for $A(8, 8)$ and $B(-7, 3)$. **16.** ________

A. $5\sqrt{10}$ **B.** $\sqrt{26}$ **C.** $10\sqrt{2}$ **D.** $\sqrt{123}$

17. Change 54° to radian measure in terms of π. **17.** ________

A. $\frac{5\pi}{4}$ **B.** $\frac{3\pi}{10}$ **C.** $\frac{\pi}{4}$ **D.** $\frac{4\pi}{9}$

18. Find one positive and one negative angle that are coterminal with an angle measuring $\frac{\pi}{6}$. **18.** ________

A. $\frac{\pi}{4}, -\frac{3\pi}{2}$ **B.** $\frac{13\pi}{6}, -\frac{11\pi}{6}$ **C.** $\frac{7\pi}{6}, -\frac{5\pi}{6}$ **D.** $\frac{2\pi}{3}, -\frac{2\pi}{3}$

19. Simplify $\sec\theta - \tan\theta \sin\theta$. **19.** ________

A. $\cos\theta$ **B.** $\sin\theta$ **C.** $\sec\theta$ **D.** $\csc\theta$

Precalculus Semester Test (continued)

20. Express $3\left(\cos \frac{3\pi}{2} + \boldsymbol{i} \sin \frac{3\pi}{2}\right)$ in rectangular form. **20.** ________

A. $-3\boldsymbol{i}$ **B.** $3\boldsymbol{i}$ **C.** $\frac{\sqrt{3}}{2}\boldsymbol{i}$ **D.** $\boldsymbol{i}$

21. If $\sin \theta = -\frac{1}{2}$ and θ lies in Quadrant III, find $\cot \theta$. **21.** ________

A. $-\frac{\sqrt{3}}{3}$ **B.** $\frac{\sqrt{3}}{3}$ **C.** $\sqrt{3}$ **D.** $-\sqrt{3}$

22. State the amplitude, period, and phase shift of the function $y = 2 \sin \left(3x - \frac{\pi}{3}\right)$. **22.** ________

A. $2, 3, \frac{\pi}{2}$ **B.** $3, 3, \pi$ **C.** $2, \frac{2\pi}{3}, \frac{\pi}{9}$ **D.** $2, \frac{3}{2\pi}, \frac{\pi}{9}$

23. Find the value of $\mathrm{Cos}^{-1}\left(\sin \frac{\pi}{2}\right)$. **23.** ________

A. 0 **B.** $\frac{\pi}{2}$ **C.** π **D.** $\frac{3\pi}{2}$

24. Which equation is a trigonometric identity? **24.** ________

A. $\cos 2\theta = \cos^2 \theta - \sin^2 \theta$ **B.** $\cos^2 \theta - \sin^2 \theta = 1$
C. $\sin 2\theta = \sin \theta \cos \theta$ **D.** $\cos(-\theta) = -\cos \theta$

25. If α is a first quadrant angle and $\cos \alpha = \frac{\sqrt{10}}{10}$, find $\sin 2\alpha$. **25.** ________

A. $\frac{3\sqrt{10}}{5}$ **B.** $\frac{3}{5}$ **C.** $-\frac{4}{5}$ **D.** $-\frac{3}{4}$

26. Which expression is equivalent to $\sin(90° - \theta)$? **26.** ________

A. $-\sin \theta$ **B.** $\tan \theta$ **C.** $\cos \theta$ **D.** $-\cos \theta$

27. Simplify $\boldsymbol{i}^{17}$. **27.** ________

A. $-\boldsymbol{i}$ **B.** $\boldsymbol{i}$ **C.** 1 **D.** -1

28. Write the rectangular equation $y = 1$ in polar form. **28.** ________

A. $r \cos \theta = 1$ **B.** $r = \sin \theta$ **C.** $r \sin \theta = 1$ **D.** $2r \sin \theta = 1$

29. Simplify $\boldsymbol{i}^5 + \boldsymbol{i}^3$. **29.** ________

A. 0 **B.** $2\boldsymbol{i}$ **C.** $\boldsymbol{i}$ **D.** $-2\boldsymbol{i}$

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Precalculus Semester Test (continued)

30. Find the distance from $P(1, 3)$ to the line with equation $3x + 2y = 4$. **30.** ________________

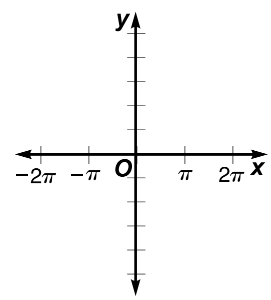
31. Solve $2 \cos x - \sec x = 1$ for $0° \leq x \leq 180°$. **31.** ________________

32. If $\vec{\mathbf{v}}$ has a magnitude of 20 and a direction of 140°, find the magnitude of its vertical and horizontal components. **32.** ________________

33. Solve $5x^2 + 10x + 6 = 3$ by using the Quadratic Formula. **33.** ________________

34. Describe the transformation that relates the graph of $y = \sin\left(x - \frac{\pi}{2}\right)$ to the parent graph $y = \sin x$. **34.** ________________

35. Graph $y = \tan\left(\frac{\theta}{2} + \frac{\pi}{4}\right) - 1$. **35.**



36. Given a central angle of 56°, find the length of its intercepted arc in a circle of radius 6 centimeters. Round your answer to the nearest thousandth. **36.** ________________

37. If $\vec{\mathbf{v}} = \langle -5, 1 \rangle$ and $\vec{\mathbf{w}} = \langle 4, -6 \rangle$, find $\vec{\mathbf{v}} - 2\vec{\mathbf{w}}$. **37.** ________________

38. Write an equation in slope-intercept form of the line with parametric equations $x = -3t - 2$ and $y = 4t + 5$. **38.** ________________

39. Find the distance between the lines with equations $6y - 8x = 18$ and $4x - 3y = 7$. **39.** ________________

Precalculus Semester Test (continued)

40. Determine the rational zeros of $f(x) = 2x^3 - 3x^2 - 18x - 8$. **40.** ____________

41. State the amplitude and period for $y = -4 \cos x$. **41.** ____________

42. Write an equation of a cosine function with amplitude 5 and period 6. **42.** ____________

43. If $\sin \alpha = \frac{1}{3}$ and $\cos \beta = \frac{3}{4}$, find $\cos (\alpha - \beta)$ if α is a first quadrant angle and β is a fourth quadrant angle. **43.** ____________

44. Approximate the positive real zeros of the function $f(x) = x^3 + 3x - 8$ to the nearest tenth. **44.** ____________

45. Evaluate $\langle 1, 5, -3 \rangle \times \langle 2, 1, 1 \rangle$. **45.** ____________

46. Use the Law of Cosines to solve $\triangle ABC$ if $a = 10$, $b = 40$, and $C = 120°$. Round answers to the nearest tenth. **46.** ____________

47. Simplify $(3 - \boldsymbol{i})(4 + 2\boldsymbol{i})$. **47.** ____________

48. Simplify $(-1 + 5\boldsymbol{i}) \div (2 + 3\boldsymbol{i})$. **48.** ____________

49. Write the rectangular form of the polar equation $r = 3$. **49.** ____________

50. Express $\sqrt{3} + \boldsymbol{i}$ in polar form. **50.** ____________

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SAT and ACT Practice Answer Sheet

(10 Questions)

1 (A) (B) (C) (D) (E)

2 (A) (B) (C) (D) (E)

3 (A) (B) (C) (D) (E)

4 (A) (B) (C) (D) (E)

5 (A) (B) (C) (D) (E)

6 (A) (B) (C) (D) (E)

7 (A) (B) (C) (D) (E)

8 (A) (B) (C) (D) (E)

9 (A) (B) (C) (D) (E)

10

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.	.	.	.
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

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SAT and ACT Practice Answer Sheet

(20 Questions)

1 (A) (B) (C) (D) (E)

2 (A) (B) (C) (D) (E)

3 (A) (B) (C) (D) (E)

4 (A) (B) (C) (D) (E)

5 (A) (B) (C) (D) (E)

6 (A) (B) (C) (D) (E)

7 (A) (B) (C) (D) (E)

8 (A) (B) (C) (D) (E)

9 (A) (B) (C) (D) (E)

10 (A) (B) (C) (D) (E)

11 (A) (B) (C) (D) (E)

12 (A) (B) (C) (D) (E)

13 (A) (B) (C) (D) (E)

14 (A) (B) (C) (D) (E)

15 (A) (B) (C) (D) (E)

16 (A) (B) (C) (D) (E)

17 (A) (B) (C) (D) (E)

18 (A) (B) (C) (D) (E)

19

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.	.	.	.
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

20

	/	/	
.	.	.	.
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

9-1

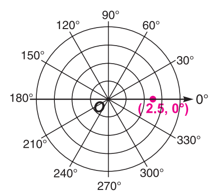
NAME ______________ DATE ______________ PERIOD ________

Practice

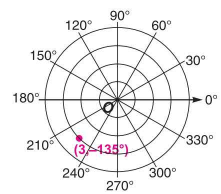
Polar Coordinates

Graph each point.

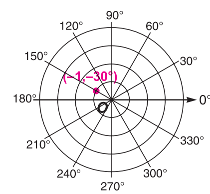
1. (2.5, 0°)



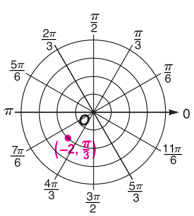
2. (3, −135°)



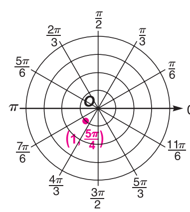
3. (−1, −30°)



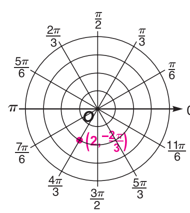
4. $\left(-2, \frac{\pi}{4}\right)$



5. $\left(1, \frac{5\pi}{4}\right)$

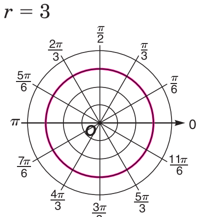


6. $\left(2, -\frac{2\pi}{3}\right)$

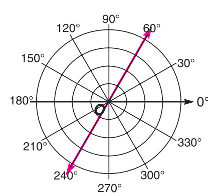


Graph each polar equation.

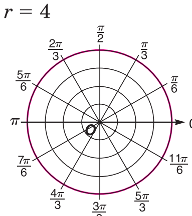
7. $r = 3$



8. $\theta = 60°$



9. $r = 4$



Find the distance between the points with the given polar coordinates.

10. $P_1(6, 90°)$ and $P_2(2, 130°)$ **4.65**

11. $P_1(-4, 85°)$ and $P_2(1, 105°)$ **4.95**

9-1

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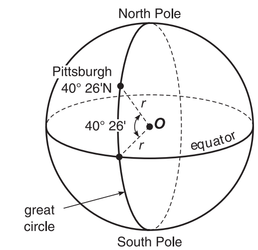
Enrichment

Distance on the Earth's Surface



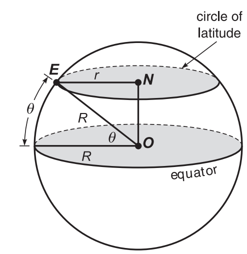
As you learned in Lesson 9-1, lines of longitude on Earth's surface intersect at the North and South Poles. A line of longitude that passes completely around Earth is called a **great circle**. All great circles have the same circumference, found by calculating the circumference of a circle with Earth's radius, 3963.2 miles. (Since Earth is slightly flattened at the poles, it is not precisely spherical. The difference is so small, however, that for most purposes it can be ignored.)

1. Find the circumference of a great circle. **24,901.5 miles**

On a great circle, position is measured in degrees north or south of the equator. Pittsburgh's position of 40° 26' N means that radii from Earth's center to Pittsburgh and to the point of intersection of the equator and Pittsburgh's longitude line form an angle of 40° 26'. (See the figure above.)

2. Find the length of one degree of arc on a longitude line. **69.2 miles**

3. Charleston, South Carolina (32° 46' N), and Guayaquil, Ecuador (2° 9' S), both lie on Pittsburgh's longitude line. Find the distance from Pittsburgh to each of the other cities. **530.3 miles; 2945.5 miles**



Because circles of latitude are drawn parallel to the equator, their radii and circumferences grow steadily shorter as they approach the poles. The length of one degree of arc on a circle of latitude depends on how far north or south of the equator the circle is located. The figure at the right shows a circle of latitude of radius r located θ degrees north of the equator. Because the radii of the equator and the circle of latitude are parallel, $m\angle NEO = \theta$. Therefore, $\cos\theta = \frac{r}{R}$, which gives $r = R\cos\theta$, where R represents the radius of Earth.

4. Find the radius and circumference of a circle of latitude located 70° north of the equator. **1355.5 miles; 8516.8 miles**

5. Find the length of one degree of arc on the circle described in Exercise 4. **23.7 miles**

6. Bangor, Maine, and Salem, Oregon, are both located at latitude 44° 50' N. Their respective longitudes are 68° 46' and 123° 2' west of Greenwich. Find the distance from Bangor to Salem. **2662.0 miles**

9-2

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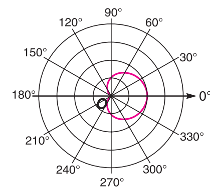
Practice

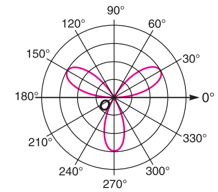
Graphs of Polar Equations

Graph each polar equation. Identify the type of curve each represents.

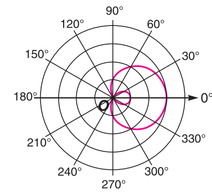
1. $r = 1 + \cos\theta$
cardioid



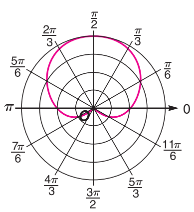
2. $r = 3\sin 3\theta$
rose



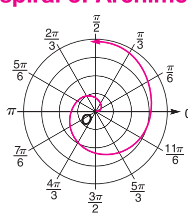
3. $r = 1 + 2\cos\theta$
limaçon



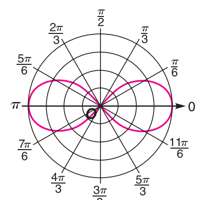
4. $r = 2 + 2\sin\theta$
cardioid



5. $r = 0.5\theta$
spiral of Archimedes

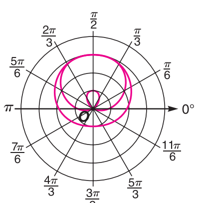


6. $r^2 = 16\cos 2\theta$
lemniscate

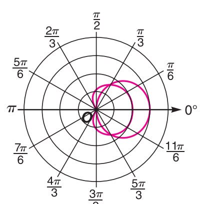


Graph each system of polar equations. Solve the system using algebra and trigonometry. Assume $0 \le \theta < 2\pi$.

7. $r = 1 + 2\sin\theta$
$r = 2 + \sin\theta$
$\left(3, \frac{\pi}{2}\right)$



8. $r = 1 + \cos\theta$
$r = 3\cos\theta$
$\left(1.5, \frac{\pi}{3}\right); \left(1.5, \frac{5\pi}{3}\right)$



9. *Design* Mikaela is designing a border for her stationery. Suppose she uses a rose curve. Determine an equation for designing a rose that has 8 petals with each petal 4 units long.
Sample answer: $r = 4\sin 4\theta$

9-2

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Enrichment

Symmetry in Graphs of Polar Equations

It is sometimes helpful to analyze polar equations for certain properties that predict symmetry in the graph of the equation. The following rules guarantee the existence of symmetry in the graph. However, the graphs of some polar equations exhibit symmetry even though the rules do not predict it.

1. If replacing θ by $-\theta$ yields the same equation, then the graph of the equation is symmetric with respect to the line containing the polar axis (the x-axis in the rectangular coordinate system).

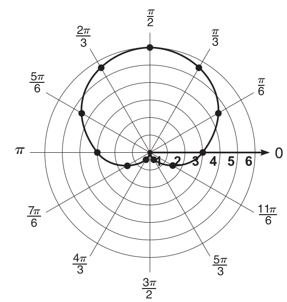
2. If replacing θ by $\pi - \theta$ yields the same equation, then the graph of the equation is symmetric with respect to the line

$\theta = \frac{\pi}{2}$ (the y-axis in the rectangular coordinate system).

3. If replacing r by $-r$ yields the same equation, then the graph of the equation is symmetric with respect to the pole.

Example **Identify the symmetry of and graph $r = 3 + 3\sin\theta$.**
Since $\sin(\pi - \theta) = \sin\theta$, by rule 2 the graph is symmetric with respect to the line $\theta = \frac{\pi}{2}$. Therefore, it is only necessary to plot points in the first and fourth quadrants.

θ	$3 + 3\sin\theta$	(r, θ)
$-\frac{\pi}{2}$	0	$\left(0, -\frac{\pi}{2}\right)$
$-\frac{\pi}{3}$	0.4	$\left(0.4, -\frac{\pi}{3}\right)$
$-\frac{\pi}{6}$	1.5	$\left(1.5, -\frac{\pi}{6}\right)$
0	3.0	(3.0, 0)
$\frac{\pi}{6}$	4.5	$\left(4.5, \frac{\pi}{6}\right)$
$\frac{\pi}{3}$	5.6	$\left(5.6, \frac{\pi}{3}\right)$
$\frac{\pi}{2}$	6.0	$\left(6.0, \frac{\pi}{2}\right)$



The points in the second and third quadrants are found by using symmetry.

See students' graphs.

Identify the symmetry of and graph each polar equation on polar grid paper.

1. $r = 2 + 3\cos\theta$
Symmetric with respect to polar axis.

2. $r^2 = 4\sin 2\theta$
Symmetric with respect to the pole.

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9-3 Practice

Polar and Rectangular Coordinates

Find the rectangular coordinates of each point with the given polar coordinates.

1. $(6, 120°)$
   $(-3, 3\sqrt{3})$
2. $(-4, 45°)$
   $(-2\sqrt{2}, -2\sqrt{2})$
3. $\left(4, \frac{\pi}{6}\right)$
   $(2\sqrt{3}, 2)$
4. $\left(0, \frac{13\pi}{3}\right)$
   $(0, 0)$

Find the polar coordinates of each point with the given rectangular coordinates. Use 0 ≤ θ < 2π and r ≥ 0.

5. $(2, 2)$
   $\left(2\sqrt{2}, \frac{\pi}{4}\right)$
6. $(2, -3)$
   $(3.61, 5.30)$
7. $(-3, \sqrt{3})$
   $\left(2\sqrt{3}, \frac{5\pi}{6}\right)$
8. $(-5, -8)$
   $(9.43, 4.15)$

Write each polar equation in rectangular form.

9. $r = 4$
   $x^2 + y^2 = 16$
10. $r \cos \theta = 5$
   $x = 5$

Write each rectangular equation in polar form.

11. $x^2 + y^2 = 9$
   $r = \pm 3$
12. $y = 3$
   $r \sin \theta = 3$ or
   $r = 3 \csc \theta$

13. ***Surveying*** A surveyor records the polar coordinates of the location of a landmark as (40, 62°). What are the rectangular coordinates?
   $(18.78, 35.32)$

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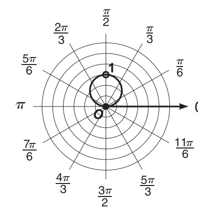
9-3 Enrichment

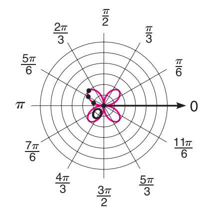
Polar Roses

The polar equation $r = a \sin n\theta$ graphs as a rose. When $n = 1$, the rose is a circle — a flower with one leaf.

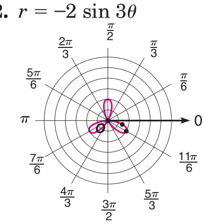


Sketch the graphs of these roses.

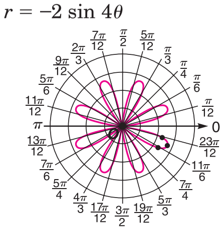
1. $r = 2 \sin 2\theta$



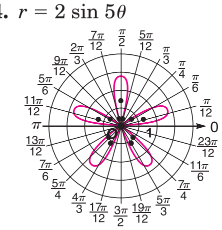
2. $r = -2 \sin 3\theta$



3. $r = -2 \sin 4\theta$



4. $r = 2 \sin 5\theta$



5. The graph of the equation $r = a \sin n\theta$ is a rose. Use your results from Exercises 1–4 to complete these conjectures.
   a. The distance across a petal is __?__ units. $|a|$
   b. If n is an odd integer, the number of leaves is __?__. n
   c. If n is an even integer, the number of leaves is __?__. $2n$
6. Write $r = 2 \sin 2\theta$ in rectangular form. $(x^2 + y^2)^2 = 16x^2y^2$
7. The total area A of the three leaves in the three-leaved rose $r = a \sin 3\theta$ is given by $A = \frac{1}{4}a^2\pi$. For a four-leaved rose, the area is $A = \frac{1}{2}a^2\pi$.
   a. Find the area of a four-leaved rose with $a = 6$. 18π
   b. Write the equation of a three-leaved rose with area 36π.
   Sample answer: $r = 12 \sin 3\theta$

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9-4 Practice

Polar Form of a Linear Equation

Write each equation in polar form. Round ϕ to the nearest degree.

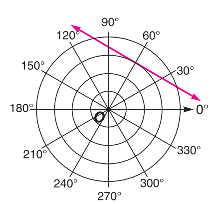
1. $3x + 2y = 16$
   $\frac{16\sqrt{13}}{13} = r\cos(\theta - 34°)$
2. $3x + 4y = 15$
   $3 = r\cos(\theta - 53°)$
3. $3x - 4y = 12$
   $-\frac{12}{5} = r\cos(\theta - 127°)$
4. $y = 2x - 1$
   $-\frac{\sqrt{5}}{5} = r\cos(\theta - 153°)$

Write each equation in rectangular form.

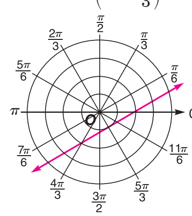
5. $4 = r\cos\left(\theta + \frac{5\pi}{6}\right)$
   $\sqrt{3}x + y + 8 = 0$
6. $2 = r\cos(\theta - 90°)$
   $y = 2$
7. $1 = r\cos\left(\theta - \frac{\pi}{4}\right)$
   $\sqrt{2}x + \sqrt{2}y - 2 = 0$
8. $3 = r\cos(\theta + 240°)$
   $x - \sqrt{3}y + 6 = 0$

Graph each polar equation.

9. $3 = r\cos(\theta - 60°)$



10. $1 = r\cos\left(\theta + \frac{\pi}{3}\right)$



11. ***Landscaping*** A landscaper is designing a garden with hedges through which a straight path will lead from the exterior of the garden to the interior. If the polar coordinates of the endpoints of the path are (20, 90°) and (10, 150°), where r is measured in feet, what is the equation for the path?
    $10 = r\cos(\theta - 150°)$

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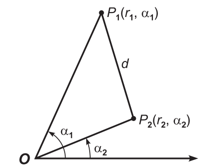
9-4 Enrichment

Distance Using Polar Coordinates



Suppose you were given the polar coordinates of two points $P_1(r_1, \alpha_1)$ and $P_2(r_2, \alpha_2)$ and were asked to find the distance d between the points. One way would be to convert to rectangular coordinates (x_1, y_1) and (x_2, y_2), and apply the distance formula

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

A more straightforward method makes use of the Law of Cosines.

1. In the above figure, the distance d between P_1 and P_2 is the length of one side of $\triangle OP_1P_2$. Find the lengths of the other two sides.
   r_1 and r_2
2. Determine the measure of $\angle P_1OP_2$.
   $|\alpha_1 - \alpha_2|$
3. Write an expression for d^2 using the Law of Cosines.
   $d^2 = r_1^2 + r_2^2 - 2r_1r_2\cos|\alpha_1 - \alpha_2|$
4. Write a formula for the distance d between the points $P_1(r_1, \alpha_1)$ and $P_2(r_2, \alpha_2)$.
   $d = \sqrt{r_1^2 + r_2^2 - 2r_1r_2\cos|\alpha_1 - \alpha_2|}$
5. Find the distance between the points (3, 45°) and (5, 25°). Round your answer to three decimal places.
   2.410
6. Find the distance between the points $\left(2, \frac{\pi}{2}\right)$ and $\left(4, \frac{\pi}{8}\right)$. Round your answer to three decimal places.
   3.725
7. The distance from the point (5, 80°) to the point (r, 20°) is $\sqrt{21}$. Find r.
   1 or 4

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9-5

Practice

Simplifying Complex Numbers

Simplify.

1. i^{38}
−1

2. i^{-17}
$-i$

3. $(3 + 2i) + (4 + 5i)$
$7 + 7i$

4. $(-6 - 2i) - (-8 - 3i)$
$2 + i$

5. $(8 - i) - (4 - i)$
4

6. $(1 + i)(3 - 2i)$
$5 + i$

7. $(2 - 3i)(5 + i)$
$13 - 13i$

8. $(4 + 5i)(4 - 5i)$
41

9. $(3 + 4i)^2$
$-7 + 24i$

10. $(4 + 3i) \div (1 - 2i)$
$-\frac{2}{5} + \frac{11}{5}i$

11. $(2 + i) \div (2 - i)$
$\frac{3}{5} + \frac{4}{5}i$

12. $\frac{8 - 7i}{1 - 2i}$
$\frac{22}{5} + \frac{9}{5}i$

13. ***Physics*** A fence post wrapped in two wires has two forces acting on it. Once force exerts 5.3 newtons due north and 4.1 newtons due east. The second force exerts 6.2 newtons due north and 2.8 newtons due east. Find the resultant force on the fence post. Write your answer as a complex number. (*Hint:* A vector with a horizontal component of magnitude a and a vertical component of magnitude b can be represented by the complex number $a + bi$.)
$(4.1 + 5.3i) + (2.8 + 6.2i) = 6.9 + 11.5i$ N

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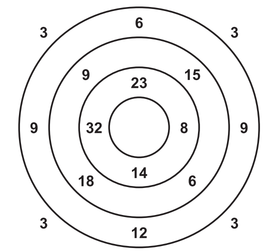
9-5

Enrichment

Cycle Quadruples

Four nonnegative integers are arranged in cyclic order to make a "cyclic quadruple." In the example, this quadruple is 23, 8, 14, and 32.



The next cyclic quadruple is formed from the absolute values of the four differences of adjacent integers:

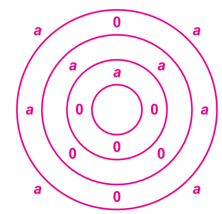
$|23 - 8| = 15 \quad |8 - 14| = 6 \quad |14 - 32| = 18 \quad |32 - 23| = 9$

By continuing in this manner, you will eventually get four equal integers. In the example, the equal integers appear in three steps.

Solve each problem.

1. Start with the quadruple 25, 17, 55, 47. In how many steps do the equal integers appear? **4 steps**

2. Some interesting things happen when one or more of the original numbers is 0. Draw a diagram showing a beginning quadruple of three zeros and one nonnegative integer. Predict how many steps it will take to reach 4 equal integers. Also, predict what that integer will be. Complete the diagram to check your predictions.
3 steps; *a*



3. Start with four integers, two of them zero. If the zeros are opposite one another, how many steps does it take for the zeros to disappear? **1 step**

4. Start with two equal integers and two zeros. The zeros are next to one another. How many steps does it take for the zeros to disappear? **2 steps**

5. Start with two nonequal integers and two zeros. The zeros are next to one another. How many steps does it take for the zeros to disappear? **4 steps**

6. Start with three equal integers and one zero. How many steps does it take for the zero to disappear? **3 steps**

7. Describe the remaining cases with one zero and tell how many steps it takes for the zero to disappear.
(1) all integers different; 1 step
(2) opposite nonzero integers equal, but different from third integer; 1 step
(3) two adjacent integers equal, but different from third integer; 2 steps

9-6

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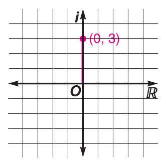
Practice

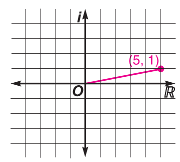
The Complex Plane and Polar Form of Complex Numbers

Graph each number in the complex plan and find its absolute value.

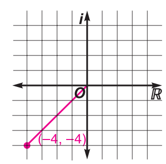
1. $z = 3i$
$|z| = 3$



2. $z = 5 + i$
$|z| = \sqrt{26}$



3. $z = -4 - 4i$
$|z| = 4\sqrt{2}$



Express each complex number in polar form.

4. $3 + 4i$
$5(\cos 0.93 + i \sin 0.93)$

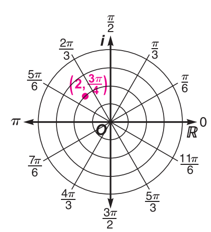
5. $-4 + 3i$
$5(\cos 2.5 + i \sin 2.5)$

6. $-1 + i$
$\sqrt{2}\left(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4}\right)$

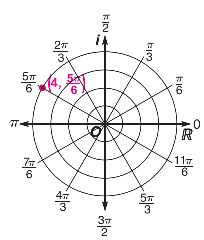
7. $1 - i$
$\sqrt{2}\left(\cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4}\right)$

Graph each complex number. Then express it in rectangular form.

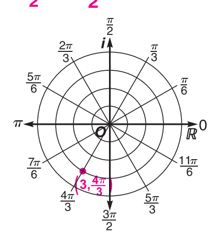
8. $2\left(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4}\right)$
$-\sqrt{2} + \sqrt{2}i$



9. $4\left(\cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6}\right)$
$-2\sqrt{3} + 2i$



10. $3\left(\cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3}\right)$
$-\frac{3}{2} - \frac{3\sqrt{3}}{2}i$



11. *Vectors* The force on an object is represented by the complex number $8 + 21i$, where the components are measured in pounds. Find the magnitude and direction of the force.
22.47 lb; 69.15°

9-6

NAME ______________ DATE ______________ PERIOD ________

Enrichment

A Complex Treasure Hunt

A prospector buried a sack of gold dust. He then wrote instructions telling where the gold dust could be found:

1. Start at the oak tree. Walk to the mineral spring counting the number of paces.

2. Turn 90° to the right and walk an equal number of paces. Place a stake in the ground.

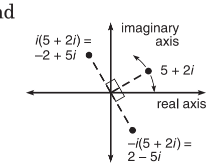
3. Go back to the oak tree. Walk to the red rock counting the number of paces.

4. Turn 90° to the left and walk an equal number of paces. Place a stake in the ground.

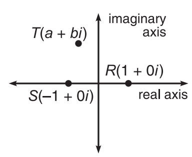
5. Find the spot halfway between the stakes. There you will find the gold.

Years later, an expert in complex numbers found the instructions in a rusty tin can. Some additional instructions told how to get to the general area where the oak tree, the mineral spring, and the red rock could be found. The expert hurried to the area and readily located the spring and the rock. Unfortunately, hundreds of oak trees had sprung up since the prospector's day, and it was impossible to know which one was referred to in the instructions. Nevertheless, through prudent application of complex numbers, the expert found the gold. Especially helpful in the quest were the following facts.

- The distance between the graphs of two complex numbers can be represented by the absolute value of the difference between the numbers.
- Multiplication by i rotates the graph of a complex number 90° counterclockwise. Multiplication by $-i$ rotates it 90° clockwise.



The expert drew a map on the complex plane, letting $S(-1 + 0i)$ be the spring and $R(1 + 0i)$ be the rock. Since the location of the oak tree was unknown, the expert represented it by $T(a + bi)$.



1. Find the distance from the oak tree to the spring. Express the distance as a complex number.
$|(a + 1) + bi|$

2. Write the complex number whose graph would be a 90° counterclockwise rotation of your answer to Exercise 1. This is where the first stake should be placed. $-b + (a + 1)i$

3. Repeat Exercises 1 and 2 for the distance from the tree to the rock. Where should the second stake be placed? $b - (a - 1)i$

4. The gold is halfway between the stakes. Find the coordinates of the location. (0 + i), the point on the imaginary axis 1 unit from the origin

NAME ______________ DATE ________ PERIOD ______

9-7 Practice

Products and Quotients of Complex Numbers in Polar Form

Find each product or quotient. Express the result in rectangular form.

1. $3\left(\cos\frac{\pi}{3} + \boldsymbol{i}\sin\frac{\pi}{3}\right) \cdot 3\left(\cos\frac{5\pi}{3} + \boldsymbol{i}\sin\frac{5\pi}{3}\right)$
9

2. $6\left(\cos\frac{\pi}{2} + \boldsymbol{i}\sin\frac{\pi}{2}\right) \div 2\left(\cos\frac{\pi}{3} + \boldsymbol{i}\sin\frac{\pi}{3}\right)$
$\frac{3\sqrt{3}}{2} + \frac{3}{2}i$

3. $14\left(\cos\frac{5\pi}{4} + \boldsymbol{i}\sin\frac{5\pi}{4}\right) \div 2\left(\cos\frac{\pi}{2} + \boldsymbol{i}\sin\frac{\pi}{2}\right)$
$-\frac{7\sqrt{2}}{2} + \frac{7\sqrt{2}}{2}i$

4. $3\left(\cos\frac{5\pi}{6} + \boldsymbol{i}\sin\frac{5\pi}{6}\right) \cdot 6\left(\cos\frac{\pi}{3} + \boldsymbol{i}\sin\frac{\pi}{3}\right)$
$-9\sqrt{3} - 9i$

5. $2\left(\cos\frac{\pi}{2} + \boldsymbol{i}\sin\frac{\pi}{2}\right) \cdot 2\left(\cos\frac{4\pi}{3} + \boldsymbol{i}\sin\frac{4\pi}{3}\right)$
$2\sqrt{3} - 2i$

6. $15(\cos\pi + \boldsymbol{i}\sin\pi) \div 3\left(\cos\frac{\pi}{2} + \boldsymbol{i}\sin\frac{\pi}{2}\right)$
$5i$

7. ***Electricity*** Find the current in a circuit with a voltage of 12 volts and an impedance of $2 - 4\boldsymbol{j}$ ohms. Use the formula, $E = I \cdot Z$, where E is the voltage measured in volts, I is the current measured in amperes, and Z is the impedance measured in ohms.
(*Hint:* Electrical engineers use $\boldsymbol{j}$ as the imaginary unit, so they write complex numbers in the form $a + b\boldsymbol{j}$. Express each number in polar form, substitute values into the formula, and then express the current in rectangular form.)
$1.2 + 2.4j$ amps

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9-7 Enrichment

Complex Conjugates

In Lesson 9-5, you learned that complex numbers in the form $a + b\boldsymbol{i}$ and $a - b\boldsymbol{i}$ are called conjugates. You can show that two numbers are conjugates by finding the appropriate values of a and b.

1. Show that the solutions of $x^2 + 2x + 3 = 0$ are conjugates.
The solutions are $-1 + i\sqrt{2}$ and $-1 - i\sqrt{2}$, so $a = -1$ and $b = \sqrt{2}$.

2. Show that the solutions of $Ax^2 + Bx + C = 0$ are conjugates when $B^2 - 4AC < 0$.
By the quadratic formula, the solutions are
$-\frac{B}{2A} + i\frac{\sqrt{B^2 - 4AC}}{2A}$ **and** $-\frac{B}{2A} - i\frac{\sqrt{B^2 - 4AC}}{2A}$,
so $a = -\frac{B}{2A}$ **and** $b = \frac{\sqrt{B^2 - 4AC}}{2A}$.

The conjugate of the complex number z is represented by $\bar{z}$.

3. $z = a + b\boldsymbol{i}$. Use $\bar{z}$ to find the reciprocal of z.
$\frac{a - bi}{a^2 + b^2}$

4. $z = r(\cos\theta + \boldsymbol{i}\sin\theta)$. Find $\bar{z}$. Express your answer in polar form.
$r[\cos(-\theta) + i\sin(-\theta)]$

Use your answer to Exercise 4 to solve Exercises 5 and 6.

5. Find $z \cdot \bar{z}$.
$r^2 = |z|^2$

6. Find $z \div \bar{z}$. $(z \neq 0)$
$\cos 2\theta + i\sin 2\theta = \frac{z^2}{|z|^2}$

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9-8

Practice

Powers and Roots of Complex Numbers

Find each power. Express the result in rectangular form.

1. $(-2 - 2\sqrt{3}\boldsymbol{i})^3$
64

2. $(1 - \boldsymbol{i})^5$
$-4 + 4i$

3. $(-1 + \sqrt{3}\boldsymbol{i})^{12}$
4096

4. $\left[1\left(\cos \frac{\pi}{4} + \boldsymbol{i} \sin \frac{\pi}{4}\right)\right]^{-3}$
$-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i$

5. $(2 + 3\boldsymbol{i})^6$
$2035 - 828i$

6. $(1 + \boldsymbol{i})^8$
16

Find each principal root. Express the result in the form a + bi with a and b rounded to the nearest hundredth.

7. $(-27\boldsymbol{i})^{\frac{1}{3}}$
$2.60 - 1.5i$

8. $(8 - 8\boldsymbol{i})^{\frac{1}{3}}$
$2.17 - 0.58i$

9. $\sqrt[5]{-243\boldsymbol{i}}$
$2.85 - 0.93i$

10. $(-\boldsymbol{i})^{\frac{1}{3}}$
$0.87 + 0.5i$

11. $\sqrt[8]{-8\boldsymbol{i}}$
$1.27 - 0.25i$

12. $\sqrt[4]{-2 - 2\sqrt{3}\boldsymbol{i}}$
$1.22 - 0.71i$

NAME ______________ DATE __________ PERIOD ______

9-8

Enrichment

Algebraic Numbers

A complex number is said to be **algebraic** if it is a zero of a polynomial with integer coefficients. For example, if p and q are integers with no common factors and $q \neq 0$, then $\frac{p}{q}$ is a zero of $qx - p$. This shows that every rational number is algebraic. Some irrational numbers can be shown to be algebraic.

Example **Show that $1 + \sqrt{3}$ is algebraic.**

Let $x = 1 + \sqrt{3}$. Then

$$x - 1 = \sqrt{3}$$
$$(x - 1)^2 = (\sqrt{3})^2$$
$$x^2 - 2x + 1 = 3$$
$$x^2 - 2x - 2 = 0$$

Thus, $1 + \sqrt{3}$ is a zero of $x^2 - 2x - 2$, so $1 + \sqrt{3}$ is an algebraic number.

If a complex number is not algebraic, it is said to be **trancendental**. The best-known transcendental numbers are π and e. Proving that these numbers are not algebraic was a difficult task. It was not until 1873 that the French mathematician Charles Hermite was able to show that e is transcendental. It wasn't until 1882 that C. L. F. Lindemann of Munich showed that π is also transcendental.

Show that each complex number is algebraic by finding a polynomial with integer coefficients of which the given number is a zero.

1. $\sqrt{2}$
$x^2 - 2$

2. $\boldsymbol{i}$
$x^2 + 1$

3. $2 - \boldsymbol{i}$
$x^2 - 4x + 5$

4. $\sqrt[3]{3}$
$x^3 - 3$

5. $4 - \sqrt[4]{2}\boldsymbol{i}$
$x^4 - 16x^3 + 96x^2 - 256x - 254$

6. $\sqrt{3} + \boldsymbol{i}$
$x^4 - 4x^2 + 16$

7. $\sqrt{1 + \sqrt[3]{5}}$
$x^6 - 3x^4 + 3x^2 - 6$

8. $\sqrt[3]{2 - \sqrt{3}}$
$x^6 - 4x^3 + 1$

Chapter 9 Answer Key

Form 1A

Page 391		Page 392	
1.	C	10.	C
2.	A	11.	B
3.	A	12.	B
4.	A	13.	C
5.	D	14.	D
6.	B	15.	A
7.	A	16.	D
8.	D	17.	D
9.	C	18.	A
		19.	B
		20.	C
		Bonus:	D

Form 1B

Page 393		Page 394	
1.	B	10.	C
2.	D	11.	B
3.	A	12.	A
4.	B	13.	B
5.	C	14.	D
6.	D	15.	C
7.	C	16.	D
8.	A	17.	B
9.	B	18.	C
		19.	A
		20.	D
		Bonus:	A

Chapter 9 Answer Key

Form 1C

Page 395

1. A
2. B
3. D
4. C
5. C
6. B
7. A
8. D
9. C

Page 396

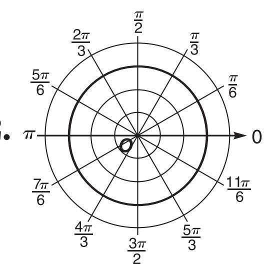
10. A
11. B
12. D
13. C
14. A
15. D
16. B
17. B
18. A
19. D
20. C

Bonus: D

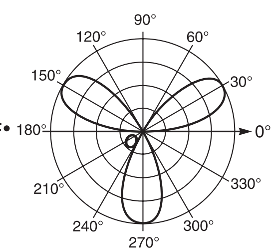
Form 2A

Page 397

1. $(-2, 150°)$

2. 

3. 2.79

4. 

5. lemniscate

6. $\left(3\sqrt{2}, \frac{5\pi}{4}\right)$

7. $(3\sqrt{2}, -3\sqrt{2})$

8. $\sqrt{5} = r \cos(\theta - 117°)$

9. $xy = 4$

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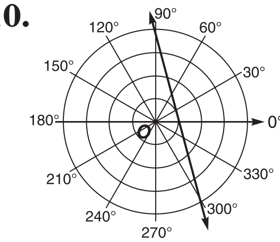
10. 

11. $\sqrt{10} = r \cos(\theta - 18°)$

12. $-35 + 10i$

13. $26 + 7i$

14. $-\frac{14}{29} - \frac{23}{29}i$

15. $4\left(\cos \frac{5\pi}{3} + i \sin \frac{5\pi}{3}\right)$

16. $-4\sqrt{2} - 4\sqrt{2}\,i$

17. $-24i$

18. $-3\sqrt{3} - 3i$

19. $\frac{1}{32}i$

20. $1.22 + 1.02i$; $-1.49 + 0.54i$; $0.28 - 1.56i$

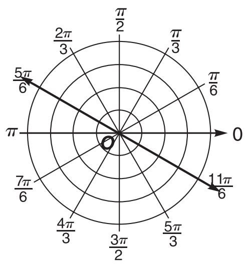
Bonus: $2\sqrt{3}(\cos 330° + i \sin 330°)$

Chapter 9 Answer Key

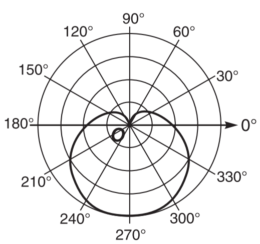
Form 2B

Page 399

1. $(3, -60°)$

2. 

3. 2.53

4. 

5. limaçon

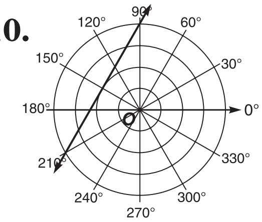
6. $\left(2, \frac{5\pi}{3}\right)$

7. $\left(-1, \sqrt{3}\right)$

8. $r = \pm 2$

9. $x^2 + y^2 = 8$

Page 400

10. 

11. $\sqrt{5} = r \cos(\theta - 63°)$

12. $9 + i$

13. $5 + 12i$

14. $\frac{1}{17} - \frac{13}{17}i$

15. $6\sqrt{2}\left(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4}\right)$

16. $2\sqrt{3} + 2i$

17. $-16\sqrt{3} - 16i$

18. $2i$

19. $-64i$

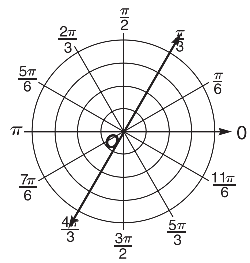
20. $2\sqrt{3} - 2i$

Bonus: $-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$

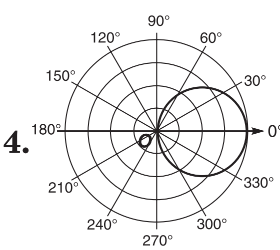
Form 2C

Page 401

1. $(2, 120°)$

2. 

3. 3.31

4. 

5. rose

6. $\left(1, \frac{\pi}{2}\right)$

7. $\left(\sqrt{2}, \sqrt{2}\right)$

8. $r \sin \theta = 2$ or $r = 2 \csc \theta$

9. $x^2 + y^2 = 9$

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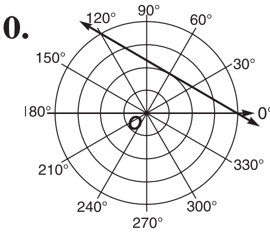
10. 

11. $\sqrt{2} = r \cos(\theta - 45°)$

12. $5 + 2i$

13. 20

14. $1 - i$

15. $4\left(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6}\right)$

16. $-6 + 6i$

17. $-6i$

18. $-12\sqrt{3} - 12i$

19. -64

20. $\sqrt{3} - i$

Bonus: i

Chapter 9 Answer Key

CHAPTER 9 SCORING RUBRIC

Level	Specific Criteria
3 Superior	• Shows thorough understanding of the concepts *polar and rectangular coordinates, polar equations,* and *sum, product, and powers of complex numbers.* • Uses appropriate strategies to find complex numbers with known sum. • Computations are correct. • Written explanations are exemplary. • Graphs are accurate and appropriate. • Goes beyond requirements of some or all problems.
2 Satisfactory, with Minor Flaws	• Shows understanding of the concepts *polar and rectangular coordinates, polar equations,* and *sum, product, and powers of complex numbers.* • Uses appropriate strategies to find complex numbers with known sum. • Computations are mostly correct. • Written explanations are effective. • Graphs are mostly accurate and appropriate. • Satisfies all requirements of problems.
1 Nearly Satisfactory, with Serious Flaws	• Shows understanding of most of the concepts *polar and rectangular coordinates, polar equations,* and *sum, product, and powers of complex numbers.* • May not use appropriate strategies to solve problems. • Computations are mostly correct. • Written explanations are satisfactory. • Diagrams and graphs are mostly accurate and appropriate. • Satisfies most requirements of problems. • Written explanations are satisfactory. • Satisfies most requirements of problems.
0 Unsatisfactory	• Shows little or no understanding of the concepts *polar and rectangular coordinates, polar equations,* and *sum, product, and powers of complex numbers.* • May not use appropriate strategies to find complex numbers with known sum. • Computations are incorrect. • Written explanations are not satisfactory. • Diagrams and graphs are not accurate or appropriate. • Does not satisfy requirements of problems.

Chapter 9 Answer Key

Open-Ended Assessment

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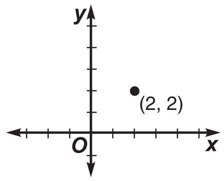
1–2. Sample answers are given

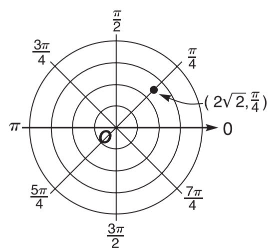
1a. (2, 2)

1b.



1c. $r = 2\sqrt{2}$

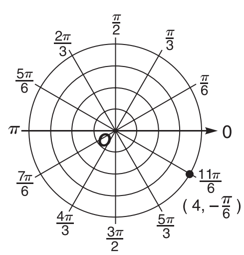
$\theta = \frac{\pi}{4}$



1d. The two graphs locate the same point in different coordinate systems. The graphs are related by the relationships $x = r\cos\theta$ and $y = r\sin\theta$.

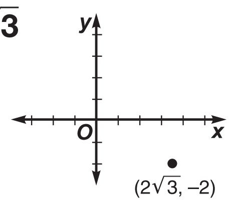
2a. $\left(4, -\frac{\pi}{6}\right)$

2b.



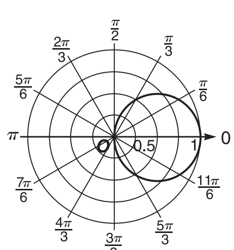
2c. $x = 4\cos\left(-\frac{\pi}{6}\right)$, or $2\sqrt{3}$

$y = 4\sin\left(-\frac{\pi}{6}\right)$, or -2



2d. The two graphs locate the same point in different coordinate systems. The graphs are related by the relationships $x = r\cos\theta$ and $y = r\sin\theta$.

3a.

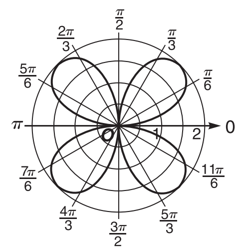


3b. The graph of $r = 2\cos\theta$ is a circle of radius 1 centered at (1, 0). Students can use the graph from part a in their description in part b.

3c. The graph is an 8-petal rose.

3d. The graph is a cardioid passing through (2, 0) and (0, π) and symmetric about $\theta = 0$.

3e. Sample answer: $r = 2\sin 2\theta$; rose



4a–4c. Sample answers are given.

4a. $(1 - i) + (2 - 2i) = 3 - 3i$

4b. $r = \sqrt{1^2 + (-1)^2} = \sqrt{2}$, $\theta =$ Arctan $\frac{-1}{1}$, or $-\frac{\pi}{4}$

The polar form of $1 - i$ is

$\sqrt{2}\left[\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\right]$.

$r = \sqrt{2^2 + (-2)^2} = 2\sqrt{2}$, $\theta =$ Arctan $\frac{-2}{2}$, or $-\frac{\pi}{4}$.

The polar form of $2 - 2i$ is

$2\sqrt{2}\left[\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\right]$.

4c. $$(1 - i)(2 - 2i) = \sqrt{2}\left[\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\right] \cdot 2\sqrt{2}\left[\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\right]$$
$$= 4\left[\cos\left(-\frac{\pi}{2}\right) + i\sin\left(-\frac{\pi}{2}\right)\right]$$
$$= -4i$$

4d. $(3 - 3i)^4 = (3 - 3i)(3 - 3i)(3 - 3i)(3 - 3i)$ or -324

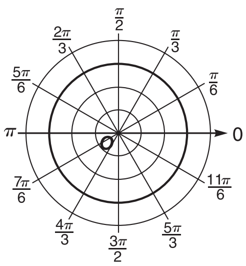
$$(3 - 3i)^4 = \left[3\sqrt{2}\left(\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\right)\right]^4$$
$$= 324[\cos(-\pi) + i\sin(-\pi)]$$
$$= -324$$

4e. $$(3 - 3i)^{\frac{1}{3}} = \left[3\sqrt{2}\left(\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\right)\right]^{\frac{1}{3}}$$
$$= \sqrt[6]{18}\left[\cos\left(-\frac{\pi}{12}\right) + i\sin\left(-\frac{\pi}{12}\right)\right]$$
$$\approx 1.56 - 0.42i$$

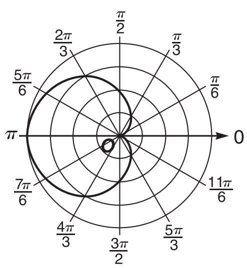
Chapter 9 Answer Key

Mid-Chapter Test
Page 404

1. $(-3, 150°)$

2. 

3. 3.15

4. 

5. rose

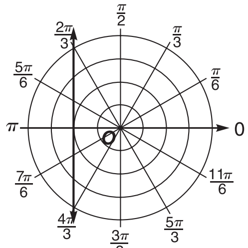
6. $\left(3\sqrt{2}, \frac{5\pi}{4}\right)$

7. $(-2\sqrt{3}, 2)$

8. $r = 5 \sin \theta$

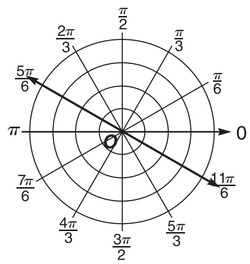
9. $\sqrt{3}x - y = 0$

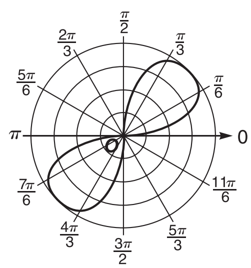
10. $x = -2$



Quiz A
Page 405

1. $(-3, -150°)$

2. 

3. 

4. 2.48

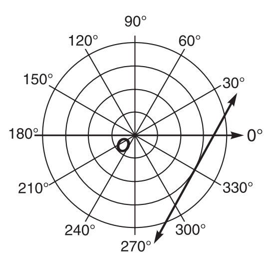
Quiz B
Page 405

1. $\left(8, \frac{5\pi}{3}\right)$

2. $(3\sqrt{2}, -3\sqrt{2})$

3. $\frac{\sqrt{10}}{2} = r \cos (\theta - 72°)$

4. $x^2 + y^2 = 25$

5. 

Quiz C
Page 406

1. $2 + 3i$

2. $26 - 2i$

3. $\frac{14}{29} - \frac{23}{29}i$

4. $4\left(\cos \frac{11\pi}{6} + i \sin \frac{11\pi}{6}\right)$

5. $-4\sqrt{2} + 4\sqrt{2}i$

Quiz D
Page 406

1. $-\sqrt{3} + i$

2. $8i$

3. $-8 - 8\sqrt{3}i$

4. -16

5. $1.90 - 0.62i$

Chapter 9 Answer Key

SAT/ACT Practice

Page 407

1. D
2. C
3. D
4. B
5. D
6. E
7. E
8. B
9. B

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10. E
11. C
12. B
13. A
14. D
15. B
16. D
17. B
18. D
19. 80
20. 45

Cumulative Review

Page 409

1. $\frac{1}{3x}$
2. perpendicular
3. (-2, −7)
4. $\frac{1}{10}\begin{bmatrix} 2 & 1 \\ -4 & 3 \end{bmatrix}$
5. even
6. 2, 3
7. $\pm1, \pm2, \pm\frac{1}{3}, \pm\frac{2}{3}$
8. 40°
9. $3; \pi; \frac{\pi}{2}; 5$
10. π
11. $\tan^2 \theta$
12. $\langle 10, -5 \rangle$
13. $\langle -11, 0, -33 \rangle$
14. 4.56
15. $\sqrt{3} + i$

Answer Key

Precalculus Semester Test

Page 411

1. D
2. B
3. B
4. A
5. C
6. C
7. A
8. D
9. B
10. C

Page 412

11. D
12. D
13. B
14. B
15. B
16. A
17. B
18. B
19. A

Page 413

20. A
21. C
22. C
23. A
24. A
25. B
26. C
27. B
28. C
29. A

Answer Key

Page 414

30. $\frac{5\sqrt{13}}{13}$

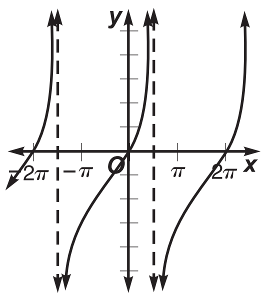
31. 0°, 120°

32. 12.86, −15.32

33. $\frac{-5 \pm \sqrt{10}}{5}$

34. translated $\frac{\pi}{2}$ units to the right

35.



36. 5.864 cm

37. $\langle -13, 13 \rangle$

38. $y = -\frac{4}{3}x + \frac{7}{3}$

39. 3.2 units

Page 415

40. −2, −0.5, 4

41. 4, 2π

42. $y = \pm 5 \cos \frac{\pi\theta}{3}$

43. $\frac{6\sqrt{2} - \sqrt{7}}{12}$

44. 1.5

45. $\langle 8, -7, -9 \rangle$

46. $A = 10.9°$, $B = 49.1°$, $c = 45.8$

47. $14 + 2i$

48. $1 + i$

49. $x^2 + y^2 = 9$

50. $2\left(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6}\right)$

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