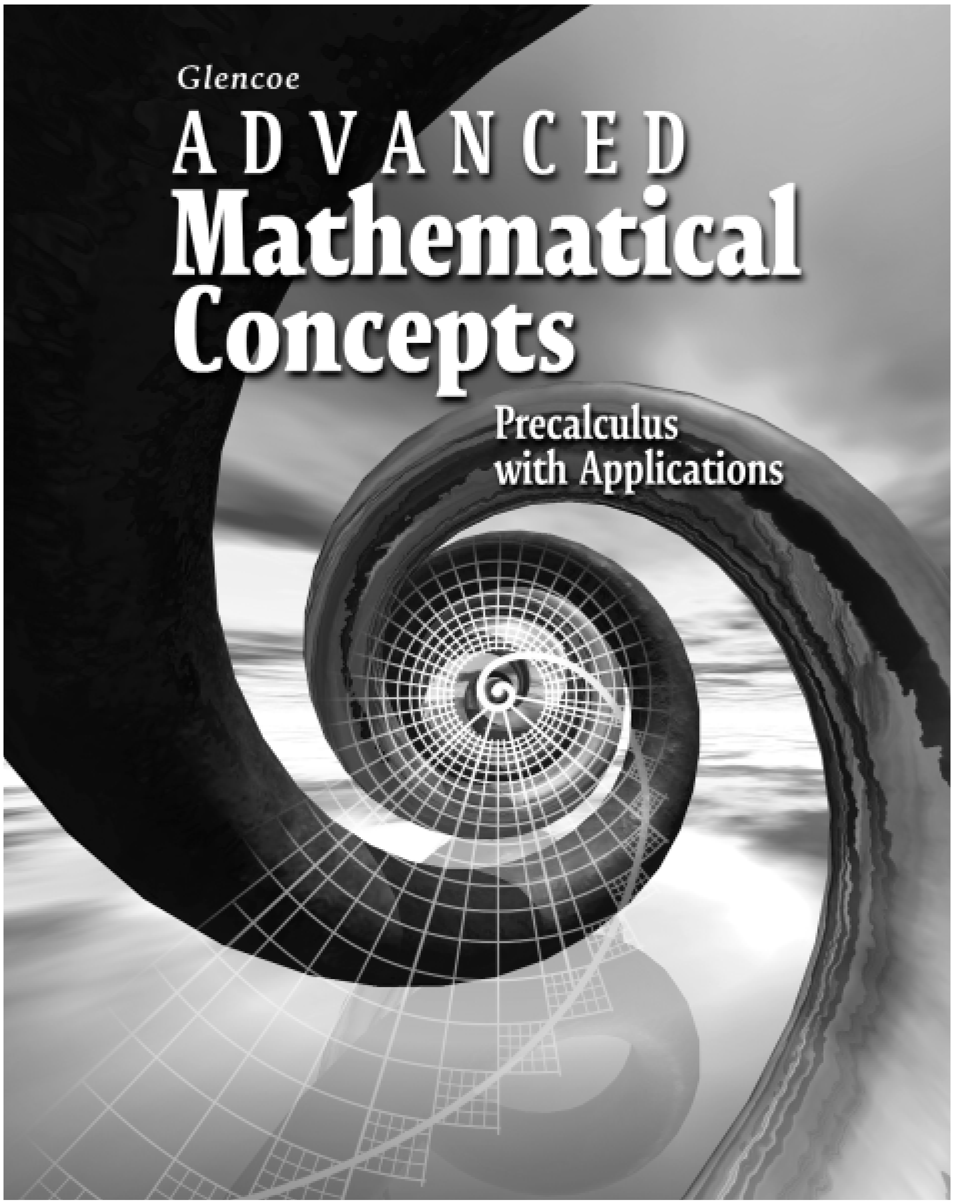
Chapter 6
Resource Masters



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StudentWorks™ This CD-ROM includes the entire Student Edition along with the Study Guide, Practice, and Enrichment masters.

TeacherWorks™ All of the materials found in this booklet are included for viewing and printing in the *Advanced Mathematical Concepts TeacherWorks* CD-ROM.

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Advanced Mathematical Concepts
Chapter 6 Resource Masters

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Contents

A Teacher's Guide to Using the Chapter 6 Resource Masters

The ***Fast File*** Chapter Resource system allows you to conveniently file the resources you use most often. The *Chapter 6 Resource Masters* include the core materials needed for Chapter 6. These materials include worksheets, extensions, and assessment options. The answers for these pages appear at the back of this booklet.

All of the materials found in this booklet are included for viewing and printing in the *Advanced Mathematical Concepts TeacherWorks* CD-ROM.

Vocabulary Builder Pages vii-viii include a student study tool that presents the key vocabulary terms from the chapter. Students are to record definitions and/or examples for each term. You may suggest that students highlight or star the terms with which they are not familiar.

When to Use Give these pages to students before beginning Lesson 6-1. Remind them to add definitions and examples as they complete each lesson.

Study Guide There is one Study Guide master for each lesson.

When to Use Use these masters as reteaching activities for students who need additional reinforcement. These pages can also be used in conjunction with the Student Edition as an instructional tool for those students who have been absent.

Practice There is one master for each lesson. These problems more closely follow the structure of the Practice section of the Student Edition exercises. These exercises are of average difficulty.

When to Use These provide additional practice options or may be used as homework for second day teaching of the lesson.

Enrichment There is one master for each lesson. These activities may extend the concepts in the lesson, offer a historical or multicultural look at the concepts, or widen students' perspectives on the mathematics they are learning. These are not written exclusively for honors students, but are accessible for use with all levels of students.

When to Use These may be used as extra credit, short-term projects, or as activities for days when class periods are shortened.

Assessment Options

The assessment section of the *Chapter 6 Resources Masters* offers a wide range of assessment tools for intermediate and final assessment. The following lists describe each assessment master and its intended use.

Chapter Assessments

Chapter Tests

- *Forms 1A, 1B, and 1C* Form 1 tests contain multiple-choice questions. Form 1A is intended for use with honors-level students, Form 1B is intended for use with average-level students, and Form 1C is intended for use with basic-level students. These tests are similar in format to offer comparable testing situations.

- *Forms 2A, 2B, and 2C* Form 2 tests are composed of free-response questions. Form 2A is intended for use with honors-level students, Form 2B is intended for use with average-level students, and Form 2C is intended for use with basic-level students. These tests are similar in format to offer comparable testing situations.

  All of the above tests include a challenging Bonus question.

- The **Extended Response Assessment** includes performance assessment tasks that are suitable for all students. A scoring rubric is included for evaluation guidelines. Sample answers are provided for assessment.

Intermediate Assessment

- A **Mid-Chapter Test** provides an option to assess the first half of the chapter. It is composed of free-response questions.

- Four free-response **quizzes** are included to offer assessment at appropriate intervals in the chapter.

Continuing Assessment

- The **SAT and ACT Practice** offers continuing review of concepts in various formats, which may appear on standardized tests that they may encounter. This practice includes multiple-choice, quantitative-comparison, and grid-in questions. Bubble-in and grid-in answer sections are provided on the master.

- The **Cumulative Review** provides students an opportunity to reinforce and retain skills as they proceed through their study of advanced mathematics. It can also be used as a test. The master includes free-response questions.

Answers

- Page A1 is an answer sheet for the SAT and ACT Practice questions that appear in the Student Edition on page 419. Page A2 is an answer sheet for the SAT and ACT Practice master. These improve students' familiarity with the answer formats they may encounter in test taking.

- The answers for the lesson-by-lesson masters are provided as reduced pages with answers appearing in red.

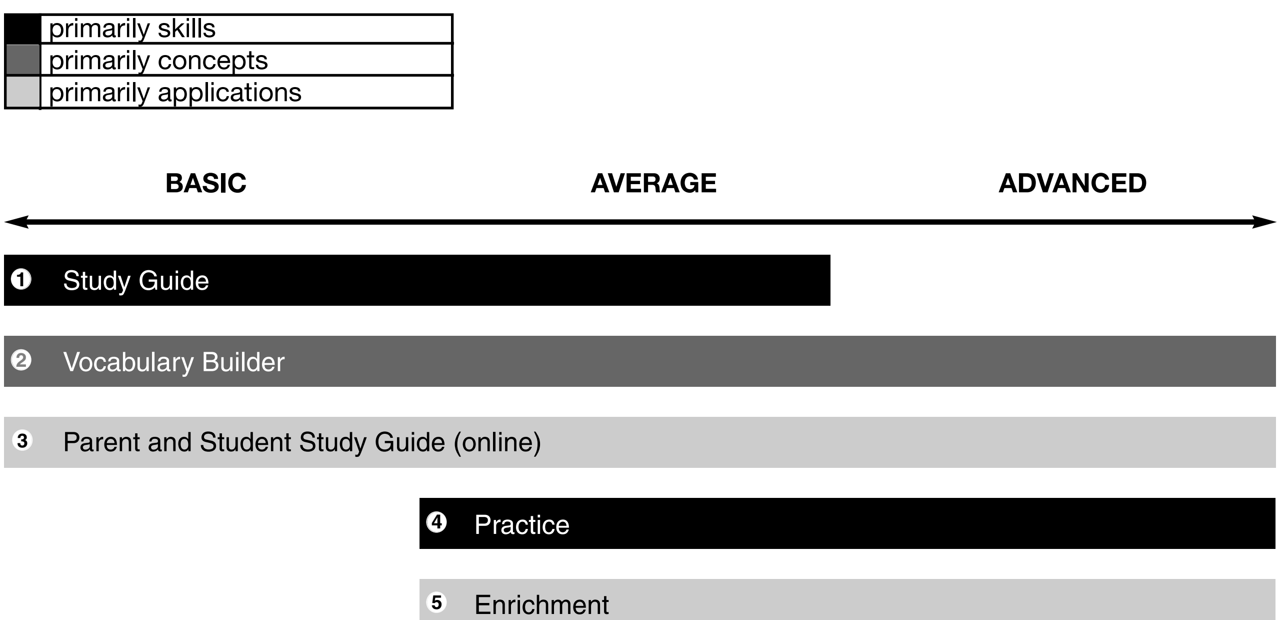
- Full-size answer keys are provided for the assessment options in this booklet.

Chapter 6 Leveled Worksheets

Glencoe's **leveled worksheets** are helpful for meeting the needs of every student in a variety of ways. These worksheets, many of which are found in the **FAST FILE Chapter Resource Masters**, are shown in the chart below.

- **Study Guide** masters provide worked-out examples as well as practice problems.
- Each chapter's **Vocabulary Builder** master provides students the opportunity to write out key concepts and definitions in their own words.
- **Practice** masters provide average-level problems for students who are moving at a regular pace.
- **Enrichment** masters offer students the opportunity to extend their learning.

Five Different Options to Meet the Needs of Every Student in a Variety of Ways



Chapter 6

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Reading to Learn Mathematics

Vocabulary Builder

This is an alphabetical list of the key vocabulary terms you will learn in Chapter 6. As you study the chapter, complete each term's definition or description. Remember to add the page number where you found the term.

Vocabulary Term	Found on Page	Definition/Description/Example
amplitude		
angular displacement		
angular velocity		
central angle		
circular arc		
compound function		
dimensional analysis		
linear velocity		
midline		
period		

(continued on the next page)

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Reading to Learn Mathematics

Vocabulary Builder *(continued)*

Vocabulary Term	Found on Page	Definition/Description/Example
periodic		
phase shift		
principal values		
radian		
sector		
sinusoidal function		

6-1

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Study Guide

Angles and Radian Measure

An angle of one complete revolution can be represented either by 360° or by 2π radians. Thus, the following formulas can be used to relate degree and **radian** measures.

Degree/Radian Conversion Formulas	1 radian $= \frac{180}{\pi}$ degrees or about 57.3° 1 degree $= \frac{\pi}{180}$ radians or about 0.017 radian

Example 1 **a. Change 36° to radian measure in terms of π.**
b. Change $-\frac{17\pi}{3}$ radians to degree measure.

a. $36° = 36° \times \frac{\pi}{180°}$
$= \frac{\pi}{5}$

b. $-\frac{17\pi}{3} = -\frac{17\pi}{3} \times \frac{180°}{\pi}$
$= -1020°$

Example 2 **Evaluate sin $\frac{3\pi}{4}$.**

The reference angle for $\frac{3\pi}{4}$ is $\frac{\pi}{4}$. Since $\frac{\pi}{4} = 45°$, the terminal side of the angle intersects the unit circle at a point with coordinates of $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$. Because the terminal side of $\frac{3\pi}{4}$ lies in Quadrant II, the x-coordinate is negative and the y-coordinate is positive. Therefore, $\sin \frac{3\pi}{4} = \frac{\sqrt{2}}{2}$.

Example 3 **Given a central angle of 147°, find the length of its intercepted arc in a circle of radius 10 centimeters. Round to the nearest tenth.**

First convert the measure of the central angle from degrees to radians.

$147° = 147° \times \frac{\pi}{180°}$ *1 degree* $= \frac{\pi}{180°}$
$= \frac{49\pi}{60}$

Then find the length of the arc.

$s = r\theta$ *Formula for the length of an arc*

$s = 10\left(\frac{49\pi}{60}\right)$ $r = 10, \theta = \frac{49\pi}{60}$

$s \approx 25.65634$

The length of the arc is about 25.7 cm.

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Practice

Angles and Radian Measure

Change each degree measure to radian measure in terms of π.

1. $-250°$

2. $6°$

3. $-145°$

4. $870°$

5. $18°$

6. $-820°$

Change each radian measure to degree measure. Round to the nearest tenth, if necessary.

7. 4π

8. $\frac{13\pi}{30}$

9. -1

10. $\frac{3\pi}{16}$

11. -2.56

12. $-\frac{7\pi}{9}$

Evaluate each expression.

13. $\tan \frac{\pi}{4}$

14. $\cos \frac{3\pi}{2}$

15. $\sin \frac{3\pi}{2}$

16. $\tan \frac{11\pi}{6}$

17. $\cos \frac{3\pi}{4}$

18. $\sin \frac{5\pi}{3}$

Given the measurement of a central angle, find the length of its intercepted arc in a circle of radius 10 centimeters. Round to the nearest tenth.

19. $\frac{\pi}{6}$

20. $\frac{3\pi}{5}$

21. $\frac{\pi}{2}$

Find the area of each sector, given its central angle θ and the radius of the circle. Round to the nearest tenth.

22. $\theta = \frac{\pi}{6}, r = 14$

23. $\theta = \frac{7\pi}{4}, r = 4$

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6-1

Enrichment

Angle Measurement: The Mil

The **mil** is an angle measurement used by the military. The military uses the mil because it is easy and accurate for measurements involving long distances. Determining the angle to use to hit a target in long-range artillery firing is one example.

In ordinary measurement, 1 mil = $\frac{1}{1000}$ inch. For angle measurement, this means that an angle measuring one mil would subtend an arc of length 1 unit, with the entire circle being 1000 mils around. So, the circumference becomes $2\pi \cdot 1000$, or about 6283.18 units. The military rounds this number to 6400 for convenience. Thus,

$$1 \text{ mil} = \frac{1}{6400} \text{ revolution around a circle}$$

So, 6400 mil = 2π radians.

Example **Change 3200 mil to radian measure.**

$$\frac{6400 \text{ mil}}{3200 \text{ mil}} = \frac{2\pi}{x}$$

$$x = \pi$$

Change each mil measurement to radian measure.

1. 1600 mil

2. 800 mil

3. 4800 mil

4. 2400 mil

Change each radian measure to mil measurement. Round your answers to the nearest tenth, where necessary.

5. $\frac{\pi}{8}$

6. $\frac{5\pi}{4}$

7. $\frac{\pi}{12}$

8. $\frac{\pi}{6}$

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6-2

Study Guide

Linear and Angular Velocity

As a circular object rotates about its center, an object at the edge moves through an angle relative to the object's starting position. That is known as the **angular displacement,** or angle of rotation. **Angular velocity** ω is given by $\omega = \frac{\theta}{t}$, where θ is the angular displacement in radians and t is time. **Linear velocity** v is given by $v = r\frac{\theta}{t}$, where $\frac{\theta}{t}$ represents the angular velocity in radians per unit of time. Since $\omega = \frac{\theta}{t}$, this formula can also be written as $v = r\omega$.

Example 1 **Determine the angular displacement in radians of 3.5 revolutions. Round to the nearest tenth.**

Each revolution equals 2π radians. For 3.5 revolutions, the number of radians is $3.5 \times 2\pi$, or 7π. 7π radians equals about 22.0 radians.

Example 2 **Determine the angular velocity if 8.2 revolutions are completed in 3 seconds. Round to the nearest tenth.**

The angular displacement is $8.2 \times 2\pi$, or 16.4π radians.

$\omega = \frac{\theta}{t}$

$\omega = \frac{16.4\pi}{3}$ *$\theta = 16.4\pi, t = 3$*

$\omega \approx 17.17403984$ *Use a calculator.*

The angular velocity is about 17.2 radians per second.

Example 3 **Determine the linear velocity of a point rotating at an angular velocity of 13π radians per second at a distance of 7 centimeters from the center of the rotating object. Round to the nearest tenth.**

$v = r\omega$

$v = 7(13\pi)$ *$r = 7, \omega = 13\pi$*

$v \approx 285.8849315$ *Use a calculator.*

The linear velocity is about 285.9 centimeters per second.

6-2

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Practice

Linear and Angular Velocity

Determine each angular displacement in radians. Round to the nearest tenth.

1. 6 revolutions

2. 4.3 revolutions

3. 85 revolutions

4. 11.5 revolutions

5. 7.7 revolutions

6. 17.8 revolutions

Determine each angular velocity. Round to the nearest tenth.

7. 2.6 revolutions in 6 seconds

8. 7.9 revolutions in 11 seconds

9. 118.3 revolutions in 19 minutes

10. 5.5 revolutions in 4 minutes

11. 22.4 revolutions in 15 seconds

12. 14 revolutions in 2 minutes

Determine the linear velocity of a point rotating at the given angular velocity at a distance r from the center of the rotating object. Round to the nearest tenth.

13. $\omega = 14.3$ radians per second, $r = 7$ centimeters

14. $\omega = 28$ radians per second, $r = 2$ feet

15. $\omega = 5.4\pi$ radians per minute, $r = 1.3$ meters

16. $\omega = 41.7\pi$ radians per second, $r = 18$ inches

17. $\omega = 234$ radians per minute, $r = 31$ inches

18. ***Clocks*** Suppose the second hand on a clock is 3 inches long. Find the linear velocity of the tip of the second hand.

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Enrichment

Angular Acceleration

An object traveling in a circular path experiences linear velocity and angular velocity. It may also experience **angular acceleration**. Angular acceleration is the rate of change in angular velocity with respect to time.

At time $t = 0$, there is an **initial angular velocity**. At the end of time t, there is a **final angular velocity**. Then the angular acceleration α of the object can be defined as

$$\alpha = \frac{\text{final angular velocity} - \text{initial angular velocity}}{\text{time}}.$$

The units for angular acceleration are usually rad/s^2 or rev/min^2.

Example **A record has a small chip on its edge. If the record begins at rest and then goes to 45 revolutions per minute in 30 seconds, what is the angular acceleration of the chip?**

The record starts at rest, so the initial angular velocity is 0. The final angular velocity is 45 revolutions/minute. Thus, the angular acceleration is

$$\alpha = \frac{45 - 0}{\frac{1}{2}}$$

$= 90$ rev/min^2.

Solve.

1. The record in the example was playing at 45 rev/min. A power surge lasting 2 seconds caused the record to speed up to 80 rev/min. What was the angular acceleration of the chip then?

2. When a car enters a curve in the road, the tires are turning at an angular velocity of 50 ft/s. At the end of the curve, the angular velocity of the tires is 60 ft/s. If the curve is an arc of a circle with radius 2000 feet and central angle $\theta = \frac{\pi}{4}$, and the car travels at a constant linear velocity of 40 mph, what is the angular acceleration?

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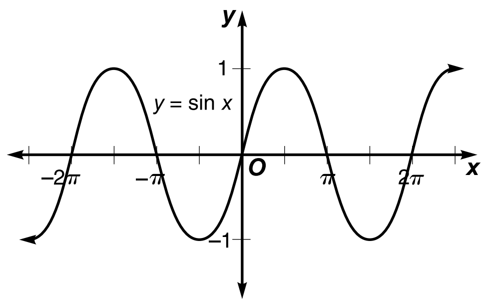
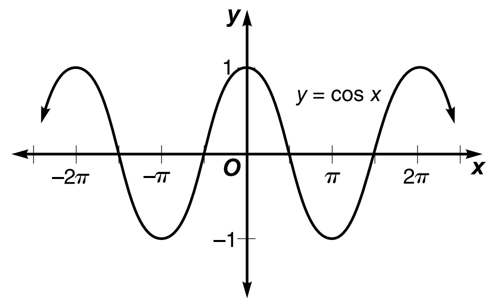
Study Guide

Graphing Sine and Cosine Functions

If the values of a function are the same for each given interval of the domain, the function is said to be **periodic.** Consider the graphs of $y = \sin x$ and $y = \cos x$ shown below. Notice that for both graphs the period is 2π and the range is from -1 to 1, inclusive.





Properties of the Graph of $y = \sin x$	**Properties of the Graph of $y = \cos x$**
The x-intercepts are located at πn, where n is an integer.	The x-intercepts are located at $\frac{\pi}{2} + \pi n$, where n is an integer.
The y-intercept is 0.	The y-intercept is 1.
The maximum values are $y = 1$ and occur when $x = \frac{\pi}{2} + 2\pi n$, when n is an integer.	The maximum values are $y = 1$ and occur when $x = \pi n$, where n is an even integer.
The minimum values are $y = -1$ and occur when $x = \frac{3\pi}{2} + 2\pi n$, where n is an integer.	The minimum values are $y = -1$ and occur when $x = \pi n$, where n is an odd integer.

Example 1 **Find $\sin \frac{7\pi}{2}$ by referring to the graph of the sine function.**

The period of the sine function is 2π. Since $\frac{7\pi}{2} > 2\pi$, rewrite $\frac{7\pi}{2}$ as a sum involving 2π.

$\frac{7\pi}{2} = 2\pi(1) + \frac{3\pi}{2}$ *This is a form of* $\frac{3\pi}{2} + 2\pi n$.

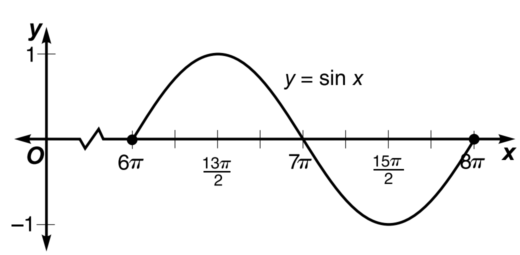
So, $\sin \frac{7\pi}{2} = \sin \frac{3\pi}{2}$ or -1.

Example 2 **Find the values of θ for which $\cos \theta = 0$ is true.**

Since $\cos \theta = 0$ indicates the x-intercepts of the cosine function, $\cos \theta = 0$ if $\theta = \frac{\pi}{2} + \pi n$, where n is an integer.

Example 3 **Graph $y = \sin x$ for $6\pi \le x \le 8\pi$.**

The graph crosses the x-axis at 6π, 7π, and 8π. Its maximum value of 1 is at $x = \frac{13\pi}{2}$, and its minimum value of -1 is at $x = \frac{15\pi}{2}$. Use this information to sketch the graph.



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6-3

Practice

Graphing Sine and Cosine Functions

Find each value by referring to the graph of the sine or the cosine function.

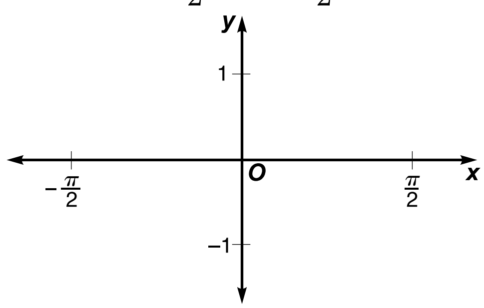
1. $\cos \pi$ **2.** $\sin \frac{3\pi}{2}$ **3.** $\sin\left(-\frac{7\pi}{2}\right)$

Find the values of θ for which each equation is true.

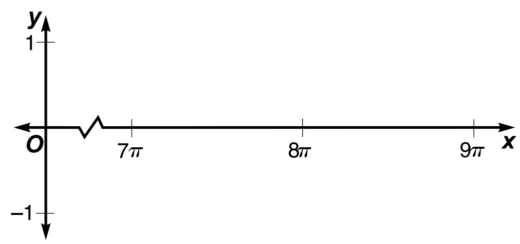
4. $\sin \theta = 0$ **5.** $\cos \theta = 1$ **6.** $\cos \theta = -1$

Graph each function for the given interval.

7. $y = \sin x$; $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$

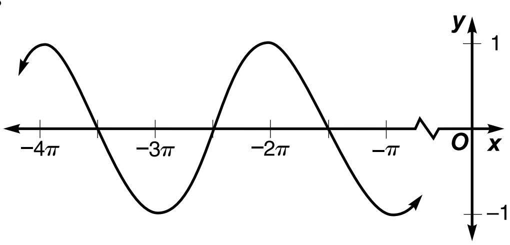


8. $y = \cos x$; $7\pi \le x \le 9\pi$

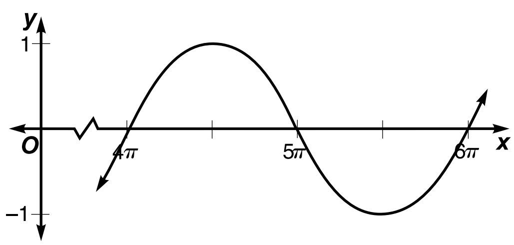


Determine whether each graph is y = sin x, y = cos x, or neither.

9.



10.



11. ***Meteorology*** The equation $y = 70.5 + 19.5 \sin\left[\frac{\pi}{6}(t - 4)\right]$ models the average monthly temperature for Phoenix, Arizona, in degrees Fahrenheit. In this equation, t denotes the number of months, with $t = 1$ representing January. What is the average monthly temperature for July?

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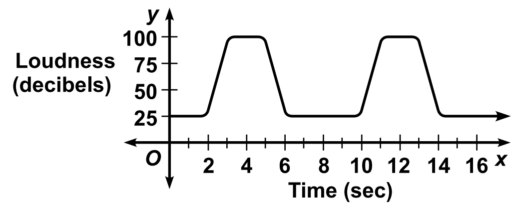
Enrichment

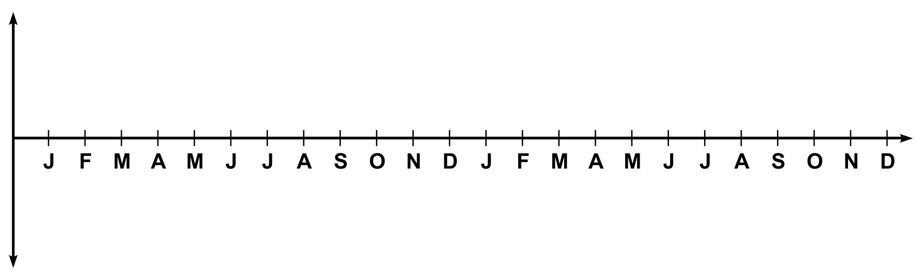
Periodic Phenomena

Periodic phenomena are common in everyday life. The first graph portrays the loudness of a foghorn as a function of time. The sound rises quickly to its loudest level, holds for about two seconds, drops off a little more quickly than it rose, then remains quiet for about four seconds before beginning a new cycle. The **period** of the cycle is eight seconds.



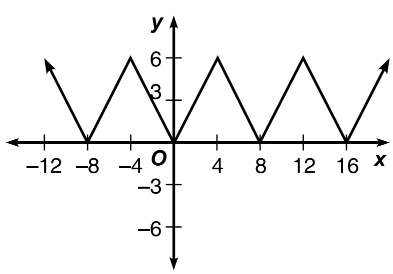
1. Give three examples of periodic phenomena, together with a typical period for each.

2. Sunrise is at 8 A.M. on December 21 in Function Junction and at 6 A.M. on June 21. Sketch a two-year graph of sunrise times in Function Junction.



State whether each function is periodic. If it is, give its period.

3.



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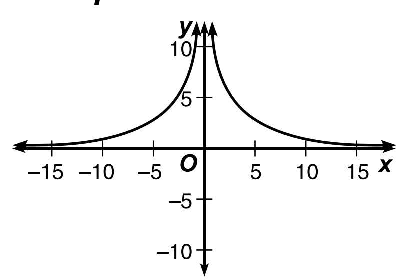
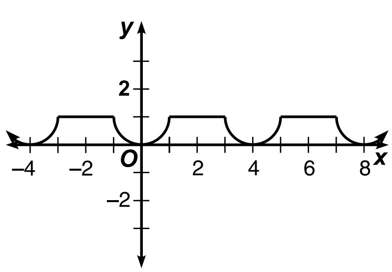
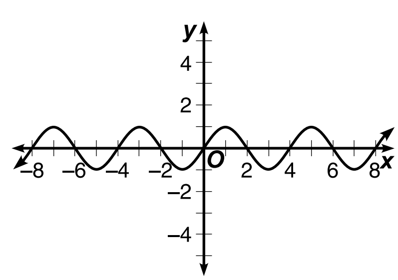


5.



6.



7. A student graphed a periodic function with a period of n. The student then translated the graph c units to the right and obtained the original graph. Describe the relationship between c and n.

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Study Guide

Amplitude and Period of Sine and Cosine Functions

The **amplitude** of the functions $y = A \sin \theta$ and $y = A \cos \theta$ is the absolute value of A, or $|A|$. The period of the functions $y = \sin k\theta$ and $y = \cos k\theta$ is $\frac{2\pi}{k}$, where $k > 0$.

Example 1 **State the amplitude and period for the function $y = -2 \cos \frac{\theta}{4}$.**

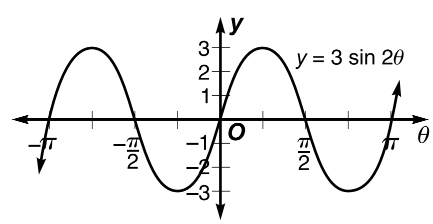
The definition of *amplitude* states that the amplitude of $y = A \cos \theta$ is $|A|$. Therefore, the amplitude of $y = -2 \cos \frac{\theta}{4}$ is $|-2|$, or 2.

The definition of *period* states that the period of $y = \cos k\theta$ is $\frac{2\pi}{k}$. Since $-2 \cos \frac{\theta}{4}$ equals $-2 \cos \left(\frac{1}{4}\theta\right)$, the period is $\frac{}{\frac{1}{4}}$ or 8π.

Example 2 **State the amplitude and period for the function $y = 3 \sin 2\theta$. Then graph the function.**

Since $A = 3$, the amplitude is $|3|$ or 3.
Since $k = 2$, the period is $\frac{2\pi}{2}$ or π.

Use the amplitude and period above and the basic shape of the sine function to graph the equation.



Example 3 **Write an equation of the sine function with amplitude 6.7 and period 3π.**

The form of the equation will be $y = A \sin k\theta$. First find the possible values of A for an amplitude of 6.7.

$|A| = 6.7$
$A = 6.7$ or -6.7

Since there are two values of A, two possible equations exist.

Now find the value of k when the period is 3π.

$\frac{2\pi}{k} = 3\pi$ *The period of the sine function is* $\frac{2\pi}{k}$.

$k = \frac{2\pi}{3\pi}$ or $\frac{2}{3}$

The possible equations are $y = 6.7 \sin \frac{2}{3}\theta$ or $y = -6.7 \sin \frac{2}{3}\theta$.

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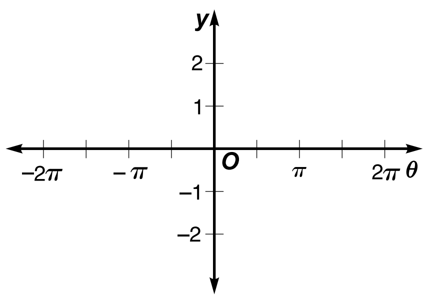
6-4

Practice

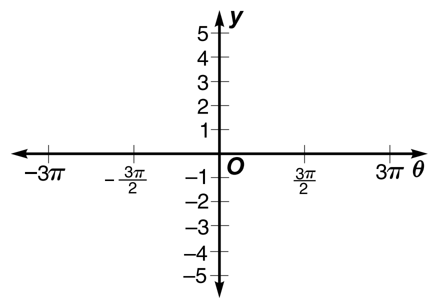
Amplitude and Period of Sine and Cosine Functions

State the amplitude and period for each function. Then graph each function.

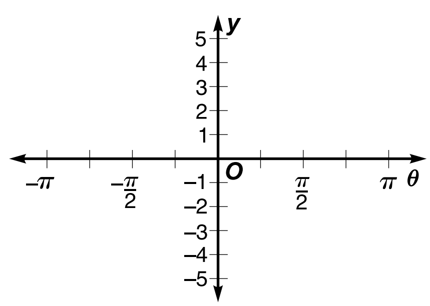
1. $y = -2 \sin \theta$



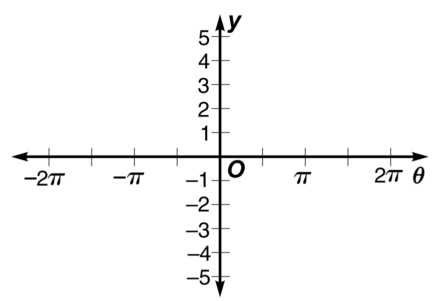
2. $y = 4 \cos \frac{\theta}{3}$



3. $y = 1.5 \cos 4\theta$



4. $y = -\frac{2}{3} \sin \frac{\theta}{2}$



Write an equation of the sine function with each amplitude and period.

5. amplitude = 3, period = 2π

6. amplitude = 8.5, period = 6π

Write an equation of the cosine function with each amplitude and period.

7. amplitude = 0.5, period = 0.2π

8. amplitude = $\frac{1}{5}$, period = $\frac{2}{5}\pi$

9. ***Music*** A piano tuner strikes a tuning fork for note A above middle C and sets in motion vibrations that can be modeled by the equation $y = 0.001 \sin 880\pi t$. Find the amplitude and period for the function.

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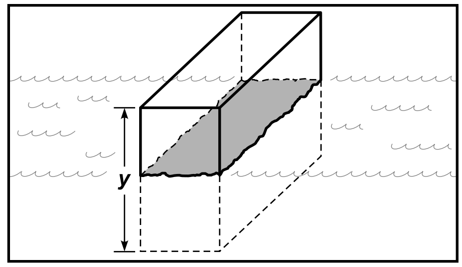
6-4

Enrichment

Mass of a Floating Object



An object bobbing up and down in the water exhibits periodic motion. The greater the mass of the object (think of an ocean liner and a buoy), the longer the period of *oscillation* (up and down motion). The greater the horizontal cross-sectional area of the object, the shorter the period. If you know the period and the cross-sectional area, you can find the mass of the object.

Imagine a point on the waterline of a stationary floating object. Let y represent the vertical position of the point above or below the waterline when the object begins to oscillate. ($y = 0$ represents the waterline.) If we neglect air and water resistance, the equation of motion of the object is

$$y = A \sin\left(\sqrt{\frac{9800C}{M}}\, t\right),$$

where A is the amplitude of the oscillation, C is the horizontal cross-sectional area of the object in square meters, M is the mass of the object in kilograms, and t is the elapsed time in seconds since the beginning of the oscillation. The argument of the sine is measured in radians and y is measured in meters.

1. A 4-kg log has a cross-sectional area of 0.2 m^2. A point on the log has a maximum displacement of 0.4 m above or below the water line. Find the vertical position of the point 5 seconds after the log begins to bob.

2. Find an expression for the period of an oscillating floating object.

3. Find the period of the log described in Exercise 1.

4. A buoy bobs up and down with a period of 0.6 seconds. The mean cross-sectional area of the buoy is 1.3 m^2. Use your equation for the period of an oscillating floating object to find the mass of the buoy.

5. Write an equation of motion of the buoy described in Exercise 4 if the amplitude is 0.45 m.

6-5

Study Guide

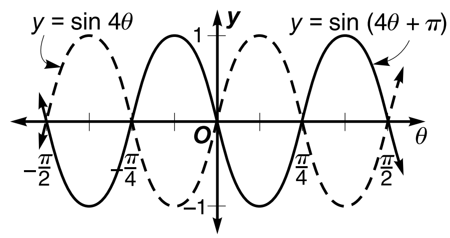
Translations of Sine and Cosine Functions

A horizontal translation of a trigonometric function is called a **phase shift**. The phase shift of the functions $y = A \sin (k\theta + c)$ and $y = A \cos (k\theta + c)$ is $-\frac{c}{k}$, where $k > 0$. If $c > 0$, the shift is to the left. If $c < 0$, the shift is to the right. The **vertical shift** of the functions $y = A \sin (k\theta + c) + h$ and $y = A \cos (k\theta + c) + h$ is h. If $h > 0$, the shift is upward. If $h < 0$, the shift is downward. The **midline** about which the graph oscillates is $y = h$.

Example 1 **State the phase shift for $y = \sin (4\theta + \pi)$. Then graph the function.**

The phase shift of the function is $-\frac{c}{k}$ or $-\frac{\pi}{4}$.

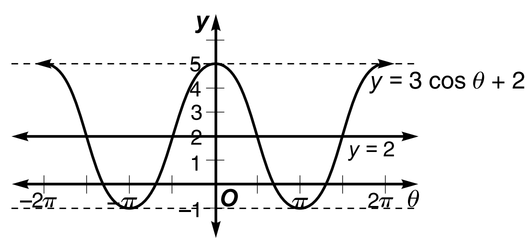
To graph $y = \sin (4\theta + \pi)$, consider the graph of $y = \sin 4\theta$. The graph of $y = \sin 4\theta$ has an amplitude of 1 and a period of $\frac{\pi}{2}$. Graph this function, then shift the graph $-\frac{\pi}{4}$.



Example 2 **State the vertical shift and the equation of the midline for $y = 3 \cos \theta + 2$. Then graph the function.**

The vertical shift is 2 units upward. The midline is the graph of $y = 2$.

To graph the function, draw the midline. Since the amplitude of the function is $|3|$, or 3, draw dashed lines parallel to the midline which are 3 units above and below $y = 2$. That is, $y = 5$ and $y = -1$. Then draw the cosine curve with a period of 2π.



Example 3 **Write an equation of the cosine function with amplitude 2.9, period $\frac{2\pi}{5}$, phase shift $-\frac{\pi}{2}$, and vertical shift −3.**

The form of the equation will be $y = A \cos (k\theta + c) + h$. Find the values of A, k, c, and h.

A: $|A| = 2.9$
$A = 2.9$ or -2.9

k: $\frac{2\pi}{k} = \frac{2\pi}{5}$ *The period is* $\frac{2\pi}{5}$.
$k = 5$

c: $-\frac{c}{k} = -\frac{\pi}{2}$ *The phase shift is* $-\frac{\pi}{2}$.
$-\frac{c}{5} = -\frac{\pi}{2}$ $k = 5$
$c = \frac{5\pi}{2}$

h: $h = -3$

The possible equations are $y = \pm 2.9 \cos \left(5\theta + \frac{5\pi}{2}\right) - 3$.

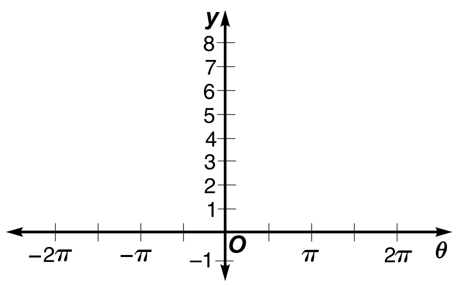
6-5

Practice

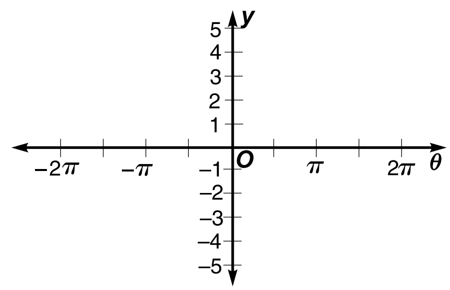
Translations of Sine and Cosine Functions

State the vertical shift and the equation of the midline for each function. Then graph each function.

1. $y = 4 \cos \theta + 4$

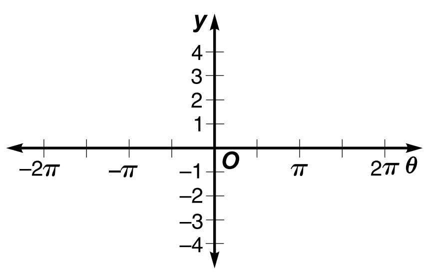


2. $y = \sin 2\theta - 2$

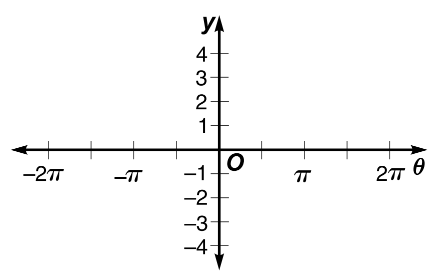


State the amplitude, period, phase shift, and vertical shift for each function. Then graph the function.

3. $y = 2 \sin\left(\theta + \frac{\pi}{2}\right) - 3$



4. $y = \frac{1}{2} \cos (2\theta - \pi) + 2$



Write an equation of the specified function with each amplitude, period, phase shift, and vertical shift.

5. sine function: amplitude = 15, period = 4π, phase shift = $\frac{\pi}{2}$, vertical shift = -10

6. cosine function: amplitude = $\frac{2}{3}$, period = $\frac{\pi}{3}$, phase shift = $-\frac{\pi}{3}$, vertical shift = 5

7. sine function: amplitude = 6, period = π, phase shift = 0, vertical shift = $-\frac{3}{2}$

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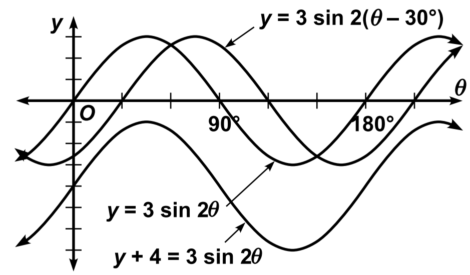
Enrichment

Translating Graphs of Trigonometric Functions

In Lesson 3-2, you learned how changes in a polynomial function affect the graph of the function. If $a > 0$, the graph of $y \pm a = f(x)$ translates the graph of $f(x)$ downward or upward a units. The graph of $y = f(x \pm a)$ translates the graph of $f(x)$ left or right a units. These results apply to trigonometric functions as well.

Example 1 **Graph $y = 3 \sin 2\theta$, $y = 3 \sin 2(\theta - 30°)$, and $y + 4 = 3 \sin 2\theta$ on the same coordinate axes.**

Obtain the graph of $y = 3 \sin 2(\theta - 30°)$ by translating the graph of $y = 3 \sin 2\theta$ 30° to the right. Obtain the graph of $y + 4 = 3 \sin 2\theta$ by translating the graph of $y = 3 \sin 2\theta$ downward 4 units.



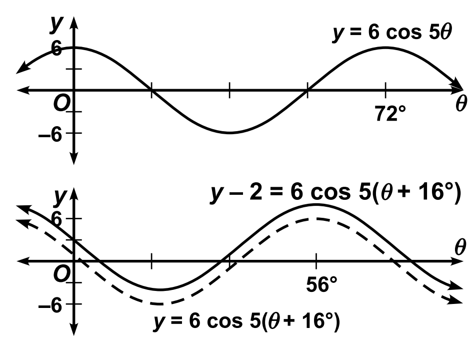
Example 2 **Graph one cycle of $y = 6 \cos (5\theta + 80°) + 2$.**

Step 1 Isolate the term involving the trigonometric function.
$y - 2 = 6 \cos (5\theta + 80°)$

Step 2 Factor out the coefficient of θ.
$y - 2 = 6 \cos 5(\theta + 16°)$

Step 3 Sketch $y = 6 \cos 5\theta$.

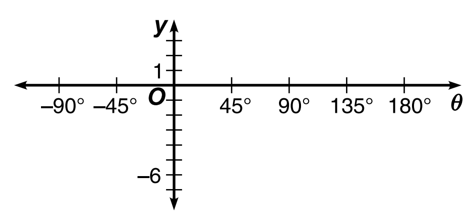
Step 4 Translate $y = 6 \cos 5\theta$ to obtain the graph of $y - 2 = 6 \cos 5(\theta + 16°)$.



Sketch these graphs on the same coordinate axes.

1. $y = 3 \sin 2(\theta + 45°)$

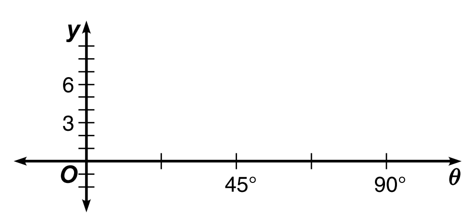
2. $y + 5 = 3 \sin 2(\theta + 90°)$



Graph one cycle of each curve on the same coordinate axes.

3. $y = 6 \cos (4\theta + 360°) + 3$

4. $y = 6 \cos 4\theta + 3$



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Study Guide

Modeling Real-World Data with Sinusoidal Functions

Example **The table shows the average monthly temperatures for Ann Arbor, Michigan. Write a sinusoidal function that models the average monthly temperatures, using $t = 1$ to represent January. Temperatures are in degrees Fahrenheit (°F).**

Jan.	30°
Feb.	34°
Mar.	45°
Apr.	59°
May	71°
June	80°
July	84°
Aug.	81°
Sept.	74°
Oct.	62°
Nov.	48°
Dec.	35°

These data can be modeled by a function of the form $y = A \sin (kt + c) + h$, where t is the time in months.

First, find A, h, and k.

A: $A = \frac{84 - 30}{2}$ or 27 — *A is half the difference between the greatest temperature and the least temperature.*

h: $h = \frac{84 + 30}{2}$ or 57 — *h is half the sum of the greatest value and the least value.*

k: $\frac{2\pi}{k} = 12$ — *The period is 12.*

$k = \frac{\pi}{6}$

Substitute these values into the general form of the function.

$y = A \sin (kt + c) + h \qquad y = 27 \sin \left(\frac{\pi}{6}t + c\right) + 57$

To compute c, substitute one of the coordinate pairs into the equation.

$y = 27 \sin \left(\frac{\pi}{6}t + c\right) + 57$	
$30 = 27 \sin \left[\frac{\pi}{6}(1) + c\right] + 57$	*Use (t, y) = (1, 30).*
$-27 = 27 \sin \left(\frac{\pi}{6} + c\right)$	*Subtract 57 from each side.*
$-\frac{27}{27} = \sin \left(\frac{\pi}{6} + c\right)$	*Divide each side by 27.*
$\sin^{-1}(-1) = \frac{\pi}{6} + c$	*Definition of inverse*
$\sin^{-1}(-1) - \frac{\pi}{6} = c$	*Subtract $\frac{\pi}{6}$ from each side.*
$-2.094395102 \approx c$	*Use a calculator.*

The function $y = 27 \sin \left(\frac{\pi}{6}t - 2.09\right) + 57$ is one model for the average monthly temperature in Ann Arbor, Michigan.

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Practice

Modeling Real-World Data with Sinusoidal Functions

1. ***Meteorology*** The average monthly temperatures in degrees Fahrenheit (°F) for Baltimore, Maryland, are given below.

Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
32°	35°	44°	53°	63°	73°	77°	76°	69°	57°	47°	37°

a. Find the amplitude of a sinusoidal function that models the monthly temperatures.

b. Find the vertical shift of a sinusoidal function that models the monthly temperatures.

c. What is the period of a sinusoidal function that models the monthly temperatures?

d. Write a sinusoidal function that models the monthly temperatures, using $t = 1$ to represent January.

e. According to your model, what is the average temperature in July? How does this compare with the actual average?

f. According to your model, what is the average temperature in December? How does this compare with the actual average?

2. ***Boating*** A buoy, bobbing up and down in the water as waves move past it, moves from its highest point to its lowest point and back to its highest point every 10 seconds. The distance between its highest and lowest points is 3 feet.

a. What is the amplitude of a sinusoidal function that models the bobbing buoy?

b. What is the period of a sinusoidal function that models the bobbing buoy?

c. Write a sinusoidal function that models the bobbing buoy, using $t = 0$ at its highest point.

d. According to your model, what is the height of the buoy at $t = 2$ seconds?

e. According to your model, what is the height of the buoy at $t = 6$ seconds?

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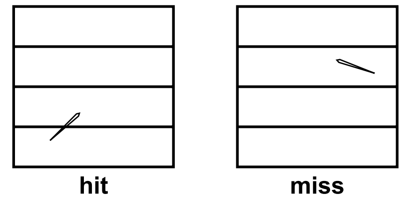
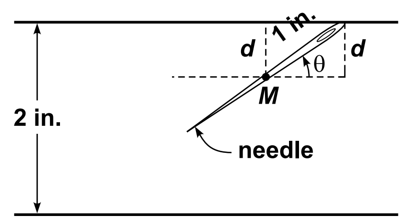
Enrichment

Approximating π

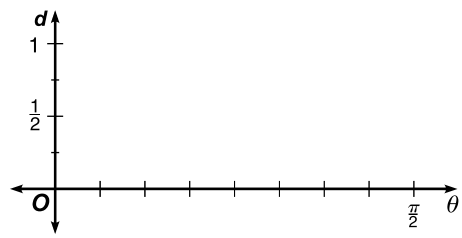
During the eighteenth century, the French scientist George de Buffon developed an experimental method for approximating π using probability. Buffon's method requires tossing a needle randomly onto an array of parallel and equidistant lines. If the needle intersects a line, it is a "hit." Otherwise, it is a "miss." The length of the needle must be less than or equal to the distance between the lines. For simplicity, we will demonstrate the method and its proof using a 2-inch needle and lines 2 inches apart.





1. Assume that the needle falls at an angle θ with the horizontal, and that the tip of the needle just touches a line. Find the distance d of the needle's midpoint M from the line.

2. Graph the function that relates θ and d for $0 \leq \theta \leq \frac{\pi}{2}$.



3. Suppose that the needle lands at an angle θ but a distance less than d. Is the toss a hit or a miss?

4. Shade the portion of the graph containing points that represent hits.

5. The area A under the curve you have drawn between $x = a$ and $x = b$ is given by $A = \cos a - \cos b$. Find the area of the shaded region of your graph.

6. Draw a rectangle around the graph in Exercise 2 for $d = 0$ to 1 and $\theta = 0$ to $\frac{\pi}{2}$. The area of the rectangle is $1 \times \frac{\pi}{2} = \frac{\pi}{2}$.

The probability P of a hit is the area of the set of all "hit" points divided by the area of the set of all possible landing points. Complete the final fraction:

$$P = \frac{\text{hit points}}{\text{all points}} = \frac{\text{shaded area}}{\text{total area}} = \frac{1}{\frac{\pi}{2}}.$$

7. Use the first and last expressions in the above equation to write π in terms of P.

8. The Italian mathematician Lazzerini made 3408 needle tosses, scoring 2169 hits. Calculate Lazzerini's experimental value of π.

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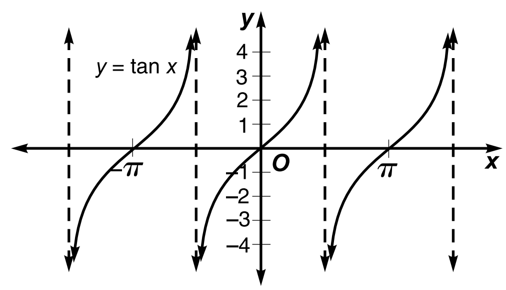
Study Guide

Graphing Other Trigonometric Functions

The period of functions $y = \csc k\theta$ and $y = \sec k\theta$ is $\frac{2\pi}{k}$, where $k > 0$. The period of functions $y = \tan k\theta$ and $y = \cot k\theta$ is $\frac{\pi}{k}$, where $k > 0$. The phase shift and vertical shift work the same way for all trigonometric functions. For example, the phase shift of the function $y = \tan(k\theta + c) + h$ is $-\frac{c}{k}$, and its vertical shift is h.

Example 1 **Graph $y = \tan x$.**

To graph $y = \tan x$, first draw the asymptotes located at $x = \frac{\pi}{2}n$, where n is an odd integer. Then plot the following coordinate pairs and draw the curves.



$\left(-\frac{5\pi}{4}, -1\right)$, $(-\pi, 0)$, $\left(-\frac{3\pi}{4}, 1\right)$, $\left(-\frac{\pi}{4}, -1\right)$, $(0, 0)$, $\left(\frac{\pi}{4}, 1\right)$, $\left(\frac{3\pi}{4}, -1\right)$, $(\pi, 0)$, $\left(\frac{5\pi}{4}, 1\right)$

Notice that the range values for the interval $-\frac{3\pi}{2} \leq x \leq -\frac{\pi}{2}$ repeat for the intervals $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$ and $\frac{\pi}{2} \leq x \leq \frac{3\pi}{2}$. So, the tangent function is a periodic function with a period of $\frac{\pi}{k}$ or π.

Example 2 **Graph $y = \sec(2\theta + \pi) + 4$.**

Since $k = 2$, the period is $\frac{2\pi}{2}$ or π. Since $c = \pi$, the phase shift is $-\frac{\pi}{2}$. The vertical shift is 4.

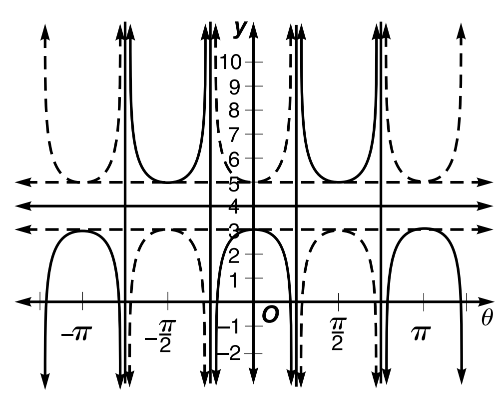
Using this information, follow the steps for graphing a secant function.

Step 1 Draw the midline, which is the graph of $y = 4$.

Step 2 Draw dashed lines parallel to the midline, which are 1 unit above and below $y = 4$.

Step 3 Draw the secant curve with a period of π.

Step 4 Shift the graph $\frac{\pi}{2}$ units to the left.



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Practice

Graphing Other Trigonometric Functions

Find each value by referring to the graphs of the trigonometric functions.

1. $\tan\left(-\frac{3\pi}{2}\right)$

2. $\cot\left(\frac{3\pi}{2}\right)$

3. $\sec 4\pi$

4. $\csc\left(-\frac{7\pi}{2}\right)$

Find the values of θ for which each equation is true.

5. $\tan\theta = 0$

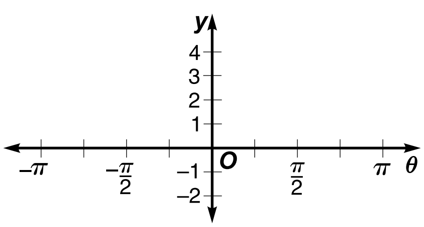
6. $\cot\theta = 0$

7. $\csc\theta = 1$

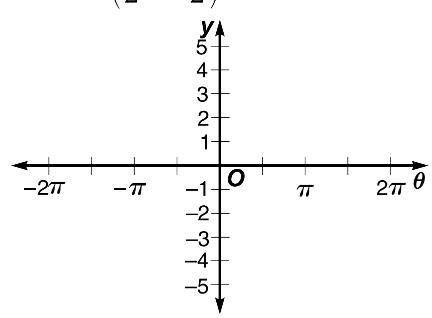
8. $\sec\theta = -1$

Graph each function.

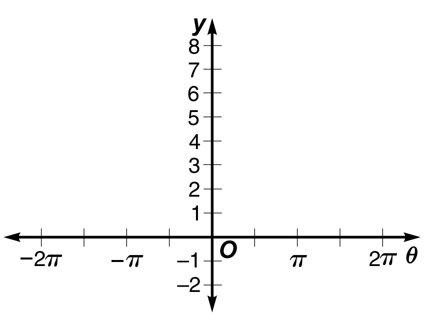
9. $y = \tan(2\theta + \pi) + 1$



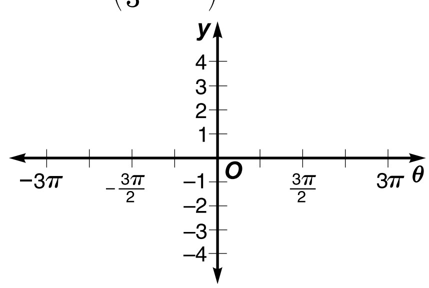
10. $y = \cot\left(\frac{\theta}{2} - \frac{\pi}{2}\right) - 2$



11. $y = \csc\theta + 3$



12. $y = \sec\left(\frac{\theta}{3} + \pi\right) - 1$



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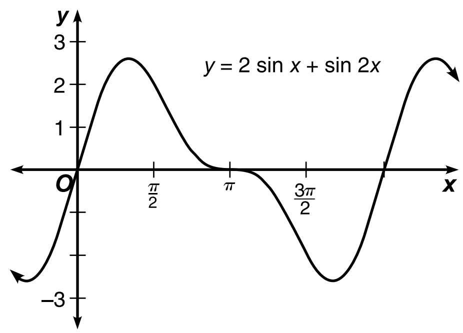
Enrichment

Reading Mathematics: Understanding Graphs

Technically, a graph is a set of points where pairs of points are connected by a set of segments and/or arcs. If the graph is the graph of an equation, the set of points consists of those points whose coordinates satisfy the equation.

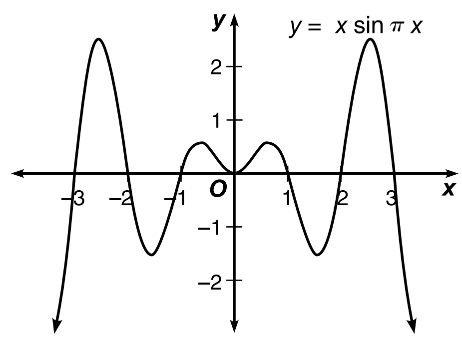
Practically speaking, to see a graph this way is as useless as seeing a word as a collection of letters. The full meaning of a graph and its value as a tool of understanding can be grasped only by viewing the graph as a whole. It is more useful to see a graph not just as a set of points, but as a picture of a function. The following suggestions, based on the idea of a graph as a picture, may help you reach a deeper understanding of the meaning of graphs.



a. ***Read the equation of the graph as a title.*** Get a sense of the behavior of the function by describing its characteristics to yourself in general terms. The graph shown depicts the function $y = 2 \sin x + \sin 2x$. In the region shown, the function increases, decreases, then increases again. It looks a bit like a sine curve but with steeper sides, sharper peaks and valleys, and a point of inflection at $x = \pi$.

b. ***Focus on the details.*** View them not as isolated or unrelated facts but as traits of the function that distinguish it from other functions. Think of the graph as a point that moves through the coordinate plane sketching a profile of the function. Use the function to guess the behavior of the graph beyond the region shown. The graph of $y = 2 \sin x + \sin 2x$ appears to exhibit point symmetry about the point of inflection $x = \pi$. It intersects the x-axis at 0, π, and 2π, and reaches a relative maximum of $y \approx 2.6$ at $x = \frac{\pi}{3}$ and a relative minimum of $y \approx -2.6$ at $x = \frac{5\pi}{3}$. Since the maximum value of $2 \sin x$ is 2 and the maximum value of $\sin 2x$ is 1, $y = 2 \sin x + \sin 2x$ will never exceed 3.

Discuss the graph at the right. Use the above discussion as a model. You should discuss the graph's shape, critical points, and symmetry.



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Study Guide

Trigonometric Inverses and Their Graphs

The inverses of the Sine, Cosine, and Tangent functions are called Arcsine, Arccosine, and Arctangent, respectively. The capital letters are used to represent the functions with restricted domains. The graphs of Arcsine, Arccosine, and Arctangent are defined as follows.

Arcsine Function	Given $y = \text{Sin } x$, the inverse Sine function is defined by the equation $y = \text{Sin}^{-1} x$ or $y = \text{Arcsin } x$.
Arccosine Function	Given $y = \text{Cos } x$, the inverse Cosine function is defined by the equation $y = \text{Cos}^{-1} x$ or $y = \text{Arccos } x$.
Arctangent Function	Given $y = \text{Tan } x$, the inverse Tangent function is defined by the equation $y = \text{Tan}^{-1} x$ or $y = \text{Arctan } x$.

Example 1 **Write the equation for the inverse of $y = \text{Arcsin } 2x$. Then graph the function and its inverse.**

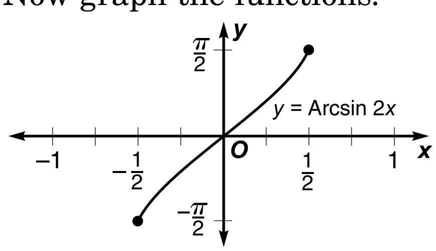
$y = \text{Arcsin } 2x$

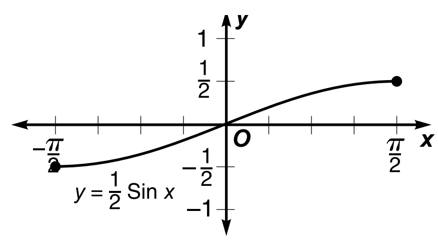
$x = \text{Arcsin } 2y$ *Exchange x and y.*

$\text{Sin } x = 2y$ *Definition of Arcsin function*

$\frac{1}{2} \text{Sin } x = y$ *Divide each side by 2.*

Now graph the functions.





Example 2 **Find each value.**

a. $\text{Arctan}\left(-\frac{\sqrt{3}}{3}\right)$

Let $\theta = \text{Arctan}\left(-\frac{\sqrt{3}}{3}\right)$.   *Arctan $\left(-\frac{\sqrt{3}}{3}\right)$ means that angle whose tan is $-\frac{\sqrt{3}}{3}$.*

$\text{Tan } \theta = -\frac{\sqrt{3}}{3}$ *Definition of Arctan function*

$\theta = -\frac{\pi}{6}$

b. $\text{Cos}^{-1}\left(\sin \frac{\pi}{2}\right)$

If $y = \sin \frac{\pi}{2}$, then $y = 1$.

$\text{Cos}^{-1}\left(\sin \frac{\pi}{2}\right) = \text{Cos}^{-1} 1$ *Replace sin $\frac{\pi}{2}$ with 1.*

$= 0$

6-8

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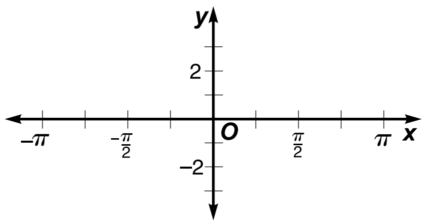
Practice

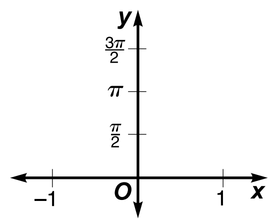
Trigonometric Inverses and Their Graphs

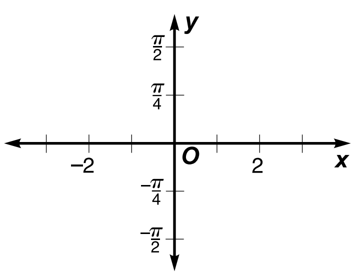
Write the equation for the inverse of each function. Then graph the function and its inverse.

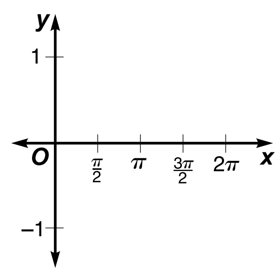
1. $y = \tan 2x$

2. $y = \frac{\pi}{2} + \text{Arccos } x$









Find each value.

3. $\text{Arccos}\,(-1)$

4. $\text{Arctan } 1$

5. $\text{Arcsin}\left(-\frac{1}{2}\right)$

6. $\text{Sin}^{-1} \frac{\sqrt{3}}{2}$

7. $\text{Cos}^{-1}\left(\sin \frac{\pi}{3}\right)$

8. $\tan\left(\text{Sin}^{-1} 1 - \text{Cos}^{-1} \frac{1}{2}\right)$

9. ***Weather*** The equation $y = 10 \sin\left(\frac{\pi}{6}t - \frac{2\pi}{3}\right) + 57$ models the average monthly temperatures for Napa, California. In this equation, t denotes the number of months with January represented by $t = 1$. During which two months is the average temperature 62°?

6-8

Enrichment

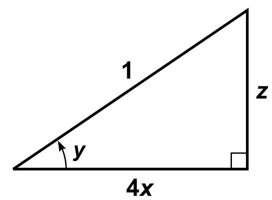
Algebraic Trigonometric Expressions

In Lesson 6-4, you learned how to use right triangles to find exact values of functions of inverse trigonometric functions. In calculus it is sometimes necessary to convert trigonometric expressions into algebraic ones. You can use the same method to do this.

Example **Write sin (arccos 4*x*) as an algebraic expression in *x*.**

Let $y = \arccos 4x$ and let z = side opposite $\angle y$.

$(4x)^2 + z^2 = 1^2$	*Pythagorean Theorem*
$16x^2 + z^2 = 1$	
$z^2 = 1 - 16x^2$	
$z = \sqrt{1 - 16x^2}$	*Take the square root of each side.*
$\sin y = \frac{z}{1}$	*Definition of sine*
$\sin y = \sqrt{1 - 16x^2}$	



Therefore, $\sin(\arccos 4x) = \sqrt{1 - 16x^2}$.

Write each of the following as an algebraic expression in x .

1. $\cot(\arccos 4x)$

2. $\sin(\arctan x)$

3. $\cos\left(\arctan \frac{x}{3}\right)$

4. $\sin[\operatorname{arcsec}(x - 2)]$

5. $\cos\left(\arcsin \frac{x - h}{r}\right)$

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Chapter 6 Test, Form 1A

Write the letter for the correct answer in the blank at the right of each problem.

1. Change 1400° to radian measure in terms of π. **1.** ________

A. $\frac{70\pi}{9}$ **B.** $\frac{35\pi}{9}$ **C.** $\frac{140\pi}{9}$ **D.** None of these

2. Change $\frac{29\pi}{37}$ radians to degree measure. **2.** ________

A. 5220° **B.** 141.1° **C.** 167.6° **D.** 66.6°

3. Determine the angular velocity if 0.75 revolutions are completed in 0.05 seconds. **3.** ________

A. 4.7 radians/s **B.** 47.1 radians/s
C. 9.4 radians/s **D.** 94 radians/s

4. Determine the linear velocity of a point rotating at 25 revolutions per minute at a distance of 2 feet from the center of the rotating object. **4.** ________

A. 2.6 ft/s **B.** 314.2 ft/s **C.** 5.2 ft/s **D.** 78.5 ft/s

5. There are 20 rollers under a conveyor belt and each roller has a radius of 15 inches. The rollers turn at a rate of 40 revolutions per minute. What is the linear velocity of the conveyor belt? **5.** ________

A. 3769.9 ft/s **B.** 104.7 ft/s **C.** 5.2 ft/s **D.** 62.8 ft/s

6. Find the degree measure of the central angle associated with an arc that is 16 inches long in a circle with a radius of 12 inches. **6.** ________

A. 76.4° **B.** 270.0° **C.** 283.6° **D.** 43.0°

7. Find the area of a sector if the central angle measures 105° and the radius of the circle is 4.2 meters. **7.** ________

A. 7.7 m^2 **B.** 16.2 m^2 **C.** 32.3 m^2 **D.** 926.1 m^2

8. Write an equation of the sine function with amplitude 3, period $\frac{3\pi}{2}$, and phase shift $\frac{\pi}{4}$. **8.** ________

A. $y = 3 \sin\left(\frac{3x}{2} - \frac{\pi}{4}\right)$ **B.** $y = -3 \sin\left(\frac{4x}{3} - \frac{\pi}{4}\right)$
C. $y = -3 \sin\left(\frac{3x}{2} - \frac{3\pi}{8}\right)$ **D.** $y = 3 \sin\left(\frac{4x}{3} - \frac{\pi}{3}\right)$

9. Write an equation of the tangent function with period $\frac{3\pi}{8}$, phase shift $-\frac{\pi}{5}$, and vertical shift -2. **9.** ________

A. $y = \tan\left(\frac{8x}{3} - \frac{8\pi}{15}\right) - 2$ **B.** $y = \tan\left(\frac{8x}{3} + \frac{8\pi}{15}\right) - 2$
C. $y = \tan\left(\frac{16x}{3} + \frac{3\pi}{80}\right) - 2$ **D.** $y = \tan\left(\frac{8x}{3} - \frac{3\pi}{40}\right) - 2$

10. State the amplitude, period, and phase shift of the function $y = -3 \cos\left(3x + \frac{3\pi}{2}\right)$. **10.** ________

A. $3; 2\pi; -\frac{\pi}{2}$ **B.** $3; \frac{2\pi}{3}; -\frac{\pi}{2}$ **C.** $-3; 2\pi; \frac{\pi}{2}$ **D.** $3; \frac{2\pi}{3}; \frac{3\pi}{2}$

11. State the period and phase shift of the function $y = -4 \tan\left(\frac{1}{2}x + \frac{3\pi}{8}\right)$. **11.** ________

A. $2\pi, -\frac{3\pi}{4}$ **B.** $\pi, \frac{3\pi}{8}$ **C.** $2\pi, \frac{3\pi}{8}$ **D.** $\pi, -\frac{3\pi}{8}$

12. What is the equation for the inverse of $y = \text{Cos } x + 3$? **12.** ________

A. $y = \text{Arccos } (x + 3)$ **B.** $y = \text{Arccos } x - 3$
C. $y = \text{Arccos } x + 3$ **D.** $y = \text{Arccos } (x - 3)$

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Chapter 6 Test, Form 1A (continued)

13. Evaluate $\tan\left(\text{Cos}^{-1}\frac{\sqrt{3}}{2} + \text{Tan}^{-1}\frac{\sqrt{3}}{3}\right)$. **13.** ________

A. $\frac{\sqrt{3}}{3}$ **B.** $\sqrt{3}$ **C.** 0 **D.** undefined

The paddle wheel of a boat measures 16 feet in diameter and is revolving at a rate of 20 rpm. The maximum depth of the paddle wheel under water is 1 foot. Suppose a point is located at the lowest point of the wheel at t = 0.

14. Write a cosine function with phase shift 0 for the height of the initial point after t seconds. **14.** ________

A. $h = 8\cos\left(\frac{2\pi}{3}t\right) + 7$ **B.** $h = -8\cos 3t + 7$

C. $h = -8\cos\left(\frac{2\pi}{3}t\right) + 7$ **D.** $h = 8\cos 3t + 7$

15. Use your function to find the height of the initial point after 55 seconds. **15.** ________

A. 7.5 ft **B.** 11 ft **C.** 10.4 ft **D.** 6.5 ft

16. Find the values of x for which the equation $\sin x = -1$ is true. **16.** ________

A. $2\pi n$ **B.** $\frac{\pi}{2} + 2\pi n$ **C.** $\pi + 2\pi n$ **D.** $\frac{3\pi}{2} + 2\pi n$

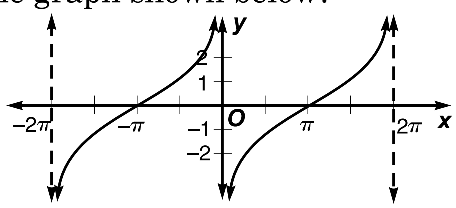
17. What is the equation of the graph shown below? **17.** ________

A. $y = \tan\frac{x}{2}$

B. $y = -\cot 2x$

C. $y = -\cot\frac{x}{2}$

D. $y = \tan 2x$



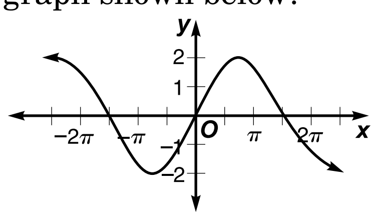
18. What is the equation of the graph shown below? **18.** ________

A. $y = -2\sin\left(\frac{2x}{3} - \pi\right)$

B. $y = 2\cos\left(\frac{x}{3} - \frac{\pi}{2}\right)$

C. $y = -2\sin\left(\frac{2x}{3} - \frac{\pi}{2}\right)$

D. $y = 2\cos\left(\frac{2x}{3} - \pi\right)$



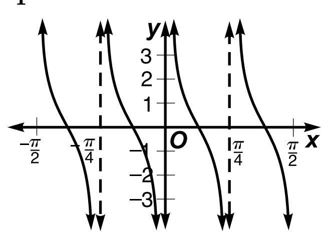
19. What is the equation of the graph shown below? **19.** ________

A. $y = \tan\left(4x - \frac{\pi}{2}\right)$

B. $y = \tan(4x - \pi)$

C. $y = \cot\left(4x - \frac{\pi}{2}\right)$

D. $y = \cot(4x - \pi)$



20. State the domain and range of the relation $y = \text{Arctan}\, x$. **20.** ________

A. D: {all real numbers}; R: $\left\{-\frac{\pi}{2} < y < \frac{\pi}{2}\right\}$

B. D: {all real numbers}; R: $\{0 \leq y \leq \pi\}$

C. D: $\left\{-\frac{\pi}{2} < y < \frac{\pi}{2}\right\}$; R: {all real numbers}

D. D: $\{0 \leq y \leq \pi\}$; R: {all real numbers}

Bonus Evaluate $\cos\left(2\pi + \text{Arctan}\frac{4}{3}\right)$. **Bonus:** ________

A. $\frac{24}{25}$ **B.** $-\frac{7}{25}$ **C.** $\frac{4}{5}$ **D.** $\frac{3}{5}$

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Chapter 6

Chapter 6 Test, Form 1B

Write the letter for the correct answer in the blank at the right of each problem.

1. Change $-435°$ to radian measure in terms of π. **1.** ________

A. $\frac{5\pi}{12}$ **B.** $-\frac{5\pi}{12}$ **C.** $\frac{29\pi}{12}$ **D.** $-\frac{29\pi}{12}$

2. Change $\frac{13\pi}{15}$ radians to degree measure. **2.** ________

A. 355° **B.** 156° **C.** 162° **D.** 207.7°

3. Determine the angular velocity if 65.7 revolutions are completed in 12 seconds. **3.** ________

A. 5.5 radians/s **B.** 34.4 radians/s
C. 17.2 radians/s **D.** 125.5 radians/s

4. Determine the linear velocity of a point rotating at an angular velocity of 62π radians per minute at a distance of 5 centimeters from the center of the rotating object. **4.** ________

A. 973.9 cm/min **B.** 310 cm/min **C.** 39.0 cm/min **D.** 1947.8 cm/min

5. There are three rollers under a conveyor belt, and each roller has a radius of 8 centimeters. The rollers turn at a rate of 2 revolutions per second. What is the linear velocity of the conveyor belt? **5.** ________

A. 0.50 m/s **B.** 50.26 m/s **C.** 100.53 m/s **D.** 1.005 m/s

6. Find the degree measure of the central angle associated with an arc that is 21 centimeters long in a circle with a radius of 4 centimeters. **6.** ________

A. 10.9° **B.** 59.2° **C.** 68.6° **D.** 300.8°

7. Find the area of a sector if the central angle measures 40° and the radius of the circle is 12.5 centimeters. **7.** ________

A. 54.5 cm² **B.** 109.1 cm² **C.** 8.7 cm² **D.** 4.4 cm²

8. Write an equation of the cosine function with amplitude 2, period π, and phase shift $\frac{\pi}{2}$. **8.** ________

A. $y = -2\cos\left(\frac{x}{2} - \frac{\pi}{4}\right)$ **B.** $y = 2\cos\left(\frac{x}{2} + \pi\right)$

C. $y = -2\cos\left(2x - \frac{\pi}{2}\right)$ **D.** $y = 2\cos\left(2x - \pi\right)$

9. Write an equation of the tangent function with period 3π, phase shift $-\frac{\pi}{4}$, and vertical shift 2. **9.** ________

A. $y = \tan\left(\frac{x}{3} - \frac{3\pi}{4}\right) + 2$ **B.** $y = \tan\left(3x + \frac{\pi}{12}\right) + 2$

C. $y = \tan\left(\frac{x}{3} + \frac{\pi}{12}\right) + 2$ **D.** $y = \tan\left(3x + \frac{3\pi}{4}\right) + 2$

10. State the amplitude, period, and phase shift of the function $y = \frac{1}{3}\sin\left(2x - \frac{\pi}{3}\right)$. **10.** ________

A. $\frac{1}{3}; \pi; \frac{\pi}{6}$ **B.** $\frac{1}{3}; 4\pi; -\frac{\pi}{6}$ **C.** $\frac{1}{3}; \pi; -\frac{\pi}{3}$ **D.** $\frac{1}{3}; 4\pi; \frac{\pi}{3}$

11. State the period and phase shift of the function $y = \frac{1}{2}\cot\left(2x - \frac{\pi}{4}\right)$. **11.** ________

A. $\frac{\pi}{2}; -\frac{\pi}{16}$ **B.** $\frac{\pi}{2}; \frac{\pi}{8}$ **C.** $\frac{\pi}{4}; -\frac{\pi}{4}$ **D.** $\frac{\pi}{2}; \frac{\pi}{4}$

Chapter 6 Test, Form 1B (continued)

12. What is the equation for the inverse of $y = \frac{1}{2}$ Sin x? **12.** ________

A. $y = \text{Arcsin } x$ **B.** $y = \text{Arcsin } \frac{1}{2}x$ **C.** $y = \text{Arcsin } 2x$ **D.** $y = 2 \text{ Arcsin } x$

13. Evaluate $\cos\left(\text{Tan}^{-1} \frac{\sqrt{3}}{3} + \text{Sin}^{-1} \frac{1}{2}\right)$. **13.** ________

A. $\frac{\sqrt{3}}{2}$ **B.** $\sqrt{3}$ **C.** 0 **D.** $\frac{1}{2}$

Kala is jumping rope, and the rope touches the ground every time she jumps. She jumps at the rate of 40 jumps per minute, and the distance from the ground to the midpoint of the rope at its highest point is 5 feet. At t = 0 the height of the midpoint is zero.

14. Write a function with phase shift 0 for the height of the midpoint of the rope above the ground after t seconds. **14.** ________

A. $h = 2.5 \cos (3\pi t) + 2.5$ **B.** $h = 2.5 \sin (3\pi t) + 2.5$

C. $h = -2.5 \cos \left(\frac{4\pi}{3}t\right) + 2.5$ **D.** $h = -2.5 \sin \left(\frac{4\pi}{3}t\right) + 2.5$

15. Use your function to find the height of the midpoint of her rope after 32 seconds. **15.** ________

A. 4.2 ft **B.** 2.5 ft **C.** 0.33 ft **D.** 3.75 ft

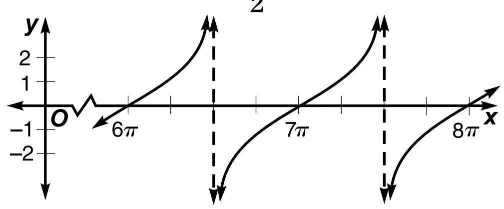
16. Find the values of x for which the equation $\cos x = -1$ is true. Let k represent an integer. **16.** ________

A. $2\pi k$ **B.** $\frac{\pi}{2} + 2\pi k$ **C.** $\pi + 2\pi k$ **D.** $\frac{3\pi}{2} + 2\pi k$

17. What is the equation of the graph shown at the right? **17.** ________

A. $y = \tan x$ **B.** $y = \cot x$

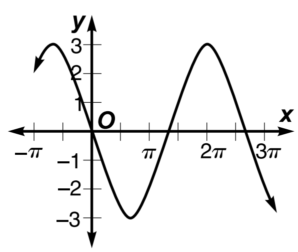
C. $y = \cot 2x$ **D.** $y = \tan 2x$



18. What is the equation of the graph shown at the right? **18.** ________

A. $y = 3 \cos \frac{2x}{3}$ **B.** $y = 3 \cos \frac{3x}{4}$

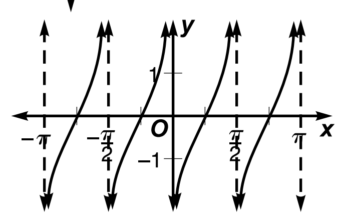
C. $y = -3 \sin \frac{3x}{4}$ **D.** $y = -3 \sin \frac{2x}{3}$



19. What is the equation of the graph shown at the right? **19.** ________

A. $y = \tan 2x$ **B.** $y = \tan\left(2x + \frac{\pi}{2}\right)$

C. $y = \cot 2x$ **D.** $y = \cot\left(2x + \frac{\pi}{2}\right)$



20. State the domain and range of the relation $y = \arcsin x$. **20.** ________

A. D:{all reals}; R: $\left\{-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}\right\}$ **B.** D: {all reals}; R: $\{0 \leq y \leq \pi\}$

C. D: $\{-1 \leq x \leq 1\}$; R: {all reals} **D.** D: $\{-1 \leq x \leq 1\}$; R: $\left\{-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}\right\}$

Bonus Evaluate $\cos\left(\text{Arctan } \frac{3}{4}\right)$. **Bonus:** ________

A. $\frac{7}{25}$ **B.** $\frac{24}{25}$ **C.** $\frac{4}{5}$ **D.** $\frac{3}{5}$

Chapter 6

NAME ______________________ DATE ______________ PERIOD ________

Chapter 6 Test, Form 1C

Write the letter for the correct answer in the blank at the right of each problem.

1. Change $-54°$ to radian measure in terms of π. **1.** ________

A. $\frac{5\pi}{12}$ **B.** $-\frac{5\pi}{12}$ **C.** $-\frac{3\pi}{10}$ **D.** $\frac{3\pi}{10}$

2. Change $\frac{6\pi}{5}$ radians to degree measure. **2.** ________

A. 190° **B.** 216° **C.** 67° **D.** 65.8°

3. Determine the angular velocity if 29 revolutions are completed in 2 seconds. **3.** ________

A. 14.5 radians/s **B.** 45.6 radians/s
C. 91.1 radians/s **D.** 143.1 radians/s

4. Determine the linear velocity of a point rotating at an angular velocity of 15 π radians per second at a distance of 12 feet from the center of the rotating object. **4.** ________

A. 565.5 ft/s **B.** 56.5 ft/s **C.** 180 ft/s **D.** 3.9 ft/s

5. Each roller under a conveyor belt has a radius of 0.5 meters. The rollers turn at a rate of 30 revolutions per minute. What is the linear velocity of the conveyor belt? **5.** ________

A. 94.25 m/s **B.** 1.57 m/s **C.** 6.28 m/s **D.** 4.71 m/s

6. Find the degree measure of the central angle associated with an arc that is 13.8 centimeters long in a circle with a radius of 6 centimeters. **6.** ________

A. 2.3° **B.** 414° **C.** 131.8° **D.** 65.9°

7. Find the area of a sector if the central angle measures 30° and the radius of the circle is 15 centimeters. **7.** ________

A. 58.9 cm^2 **B.** 117.8 cm^2 **C.** 3.9 cm^2 **D.** 7.9 cm^2

8. Write an equation of the sine function with amplitude 5, period 3π, and phase shift $-\pi$. **8.** ________

A. $y = \pm 5 \sin\left(\frac{2x}{3} - \frac{3\pi}{2}\right)$ **B.** $y = \pm 5 \sin\left(\frac{3x}{2} + \frac{2\pi}{3}\right)$

C. $y = \pm 5 \sin\left(\frac{2x}{3} - \pi\right)$ **D.** $y = \pm 5 \sin\left(\frac{2x}{3} + \frac{2\pi}{3}\right)$

9. Write an equation of the tangent function with period $\frac{\pi}{4}$, phase shift π, and vertical shift 1. **9.** ________

A. $y = \tan\left(4x - \frac{\pi}{4}\right) + 1$ **B.** $y = \tan(4x - 4\pi) + 1$

C. $y = \tan\left(\frac{x}{4} - \pi\right) + 1$ **D.** $y = \tan(4x + 4\pi) + 1$

10. State the amplitude, period, and phase shift of the function $y = -0.4 \sin\left(10x + \frac{\pi}{2}\right)$. **10.** ________

A. $0.4; \frac{\pi}{5}; -\frac{\pi}{20}$ **B.** $0.4; \frac{\pi}{5}; \frac{\pi}{20}$ **C.** $0.4; 5\pi; \frac{1}{20}$ **D.** $0.4; \frac{\pi}{5}; -\frac{1}{20}$

11. State the period and phase shift of the function $y = 3 \tan\left(4x - \frac{\pi}{3}\right)$. **11.** ________

A. $\frac{\pi}{4}; -\frac{\pi}{3}$ **B.** $4\pi; \frac{\pi}{12}$ **C.** $\frac{\pi}{4}; -\frac{\pi}{12}$ **D.** $\frac{\pi}{4}; \frac{\pi}{12}$

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Chapter 6 Test, Form 1C (continued)

12. What is the equation for the inverse of $y = \text{Cos } x + 1$? **12.** ________

A. $y = \text{Arccos } x$ **B.** $y = \text{Arccos }(x - 1)$

C. $y = 1 + \text{Arccos } x$ **D.** $y = \text{Arccos } x + 1$

13. Evaluate $\text{Cos}^{-1}\left(\tan \frac{\pi}{4}\right)$. **13.** ________

A. $\frac{\sqrt{2}}{2}$ **B.** π **C.** 0 **D.** $\frac{\pi}{4}$

A tractor tire has a diameter of 6 feet and is revolving at a rate of 45 rpm. At t = 0, a certain point on the tread of the tire is at height 0.

14. Write a function with phase shift 0 for the height of the point above the ground after t seconds. **14.** ________

A. $h = 3 \cos\left(\frac{3\pi}{2}t\right) + 3$ **B.** $h = -3 \cos\left(\frac{3\pi}{2}t\right) + 3$

C. $h = 3 \sin\left(\frac{8\pi}{3}t\right) + 3$ **D.** $h = -3 \sin\left(\frac{8\pi}{3}t\right) + 3$

15. Use your function to find the height of the point after 1 minute. **15.** ________

A. 6 ft **B.** 3 ft **C.** 0 ft **D.** 1.5 ft

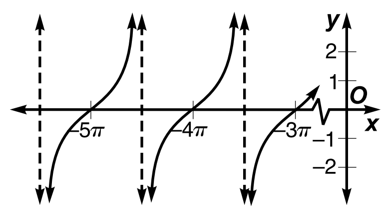
16. Find the values of x for which the equation $\cos x = 1$ is true. **16.** ________

A. $2\pi n$ **B.** $\frac{\pi}{2} + 2\pi n$ **C.** $\pi + 2\pi n$ **D.** $\frac{3\pi}{2} + 2\pi n$

17. What is the equation of the graph shown at the right? **17.** ________

A. $y = \tan x$ **B.** $y = \cot x$

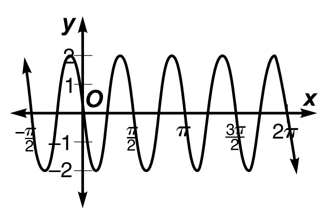
C. $y = \cot 2x$ **D.** $y = \tan 2x$



18. What is the equation of the graph shown at the right? **18.** ________

A. $y = 2 \cos \frac{x}{4}$ **B.** $y = 2 \cos 4x$

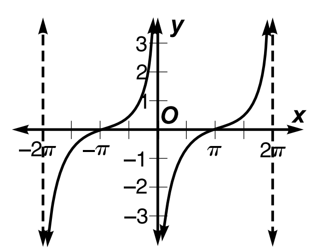
C. $y = -2 \sin \frac{x}{4}$ **D.** $y = -2 \sin 4x$



19. What is the equation of the graph shown at the right? **19.** ________

A. $y = \tan\left(\frac{x}{2} + \pi\right)$ **B.** $y = \tan\left(\frac{x}{2} + \frac{\pi}{2}\right)$

C. $y = \tan\left(\frac{x}{4} + \pi\right)$ **D.** $y = \tan\left(\frac{x}{4} + \frac{\pi}{4}\right)$



20. State the domain and range of the relation $y = \arccos x$. **20.** ________

A. D: {all real numbers}; R: $\left\{-\frac{\pi}{2} \le y \le \frac{\pi}{2}\right\}$

B. D: {all real numbers}; R: $\{0 \le y \le \pi\}$

C. D: $\{-1 \le x \le 1\}$; R: {all real numbers}

D. D: $\{-1 \le x \le 1\}$; R: $\left\{-\frac{\pi}{2} \le y \le \frac{\pi}{2}\right\}$

Bonus Evaluate $\cot\left(\text{Arctan } \frac{3}{5}\right)$. **Bonus:** ________

A. $\frac{3}{4}$ **B.** $\frac{4}{3}$ **C.** $\frac{4}{5}$ **D.** $\frac{5}{3}$

NAME ____________________ DATE ____________ PERIOD ________

Chapter 6 Test, Form 2A

1. Change $-312°$ to radian measure in terms of π. **1.** ____________

2. Change $-\frac{23\pi}{6}$ radians to degree measure. **2.** ____________

3. Determine the angular velocity if 11.3 revolutions are completed in 3.9 seconds. Round to the nearest tenth. **3.** ____________

4. Determine the linear velocity of a point rotating at 15 revolutions per minute at a distance of 3.04 meters from the center of a rotating object. Round to the nearest tenth. **4.** ____________

5. A gyroscope of radius 18 centimeters rotates 35 times per minute. Find the linear velocity of a point on the edge of the gyroscope. **5.** ____________

6. An arc is 0.04 meters long and is intercepted by a central angle of $\frac{\pi}{8}$ radians. Find the diameter of the circle. **6.** ____________

7. Find the area of sector if the central angle measures 225° and the radius of the circle is 11.04 meters. **7.** ____________

8. Write an equation of the cosine function with amplitude $\frac{2}{3}$, period 1.8, phase shift -5.2, and vertical shift 3.9. **8.** ____________

9. Write an equation of the cotangent function with period $\frac{3\pi}{2}$, phase shift $\frac{3\pi}{2}$, and vertical shift $-\frac{4}{3}$. **9.** ____________

10. State the amplitude, period, phase shift, and vertical shift for $y = 2 - \sin\left(3x - \frac{\pi}{5}\right)$. **10.** ____________

11. State the period, phase shift, and vertical shift for $y = -2 + 3\tan(4x + \pi)$. **11.** ____________

12. Write the equation for the inverse of $y = 4 \text{ Arccot}\left(\frac{3x}{4} - \frac{2}{3}\right)$. **12.** ____________

13. Evaluate $\sin\left[\text{Cos}^{-1}\left(-\frac{\sqrt{2}}{2}\right) - \frac{\pi}{4}\right]$. **13.** ____________

NAME ______________________ DATE ____________ PERIOD ________

Chapter 6 Test, Form 2A (continued)

The table shows the average monthly temperatures (°F) for Detroit, Michigan.

Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
25.3°	27.1°	35.8°	48.2°	59.5°	69.1°	73.8°	72.1°	64.6°	53.4°	41.4°	30.2°

14. Write a sinusoidal function that models the monthly temperatures in Detroit, using $t = 1$ to represent January.

14. ______________________

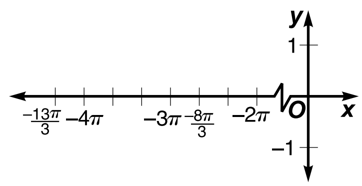
15. According to your model, what is the average temperature in October?

15. ______________________

Graph each function.

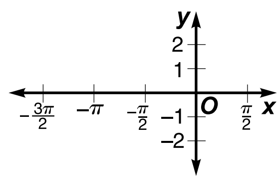
16. $y = \sin x$ for $-\frac{13\pi}{3} \le x \le -\frac{8\pi}{3}$

16.



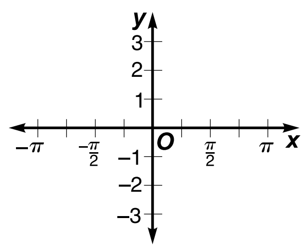
17. $y = \cot x$ for $-\frac{3\pi}{2} \le x \le \frac{\pi}{2}$

17.



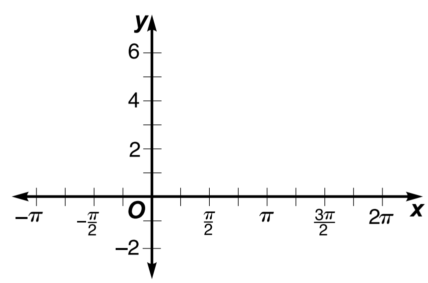
18. $y = -\frac{5}{2} \cos(3x + \pi)$

18.



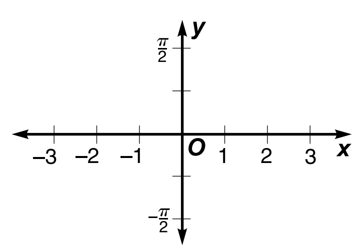
19. $y = \csc\left(2x - \frac{\pi}{2}\right) + 3$

19.



20. $y = \text{Arctan } x$

20.



Bonus Evaluate $\sin\left(2\pi + \text{Arctan } \frac{12}{5}\right)$.

Bonus: ______________________

NAME ______________________ DATE ______________ PERIOD _________

Chapter 6

Chapter 6 Test, Form 2B

1. Change 585° to radian measure in terms of π. **1.** ______________

2. Change $\frac{11\pi}{3}$ radians to degree measure. **2.** ______________

3. Determine the angular velocity if 12.5 revolutions are completed in 8 seconds. Round to the nearest tenth. **3.** ______________

4. Determine the linear velocity of a point rotating at an angular velocity of 84π radians per minute at a distance of 2 meters from the center of the rotating object. Round to the nearest tenth. **4.** ______________

5. The minute hand of a clock is 7 centimeters long. Find the linear velocity of the tip of the minute hand. **5.** ______________

6. An arc is 21.4 centimeters long and is intercepted by a central angle of $\frac{3\pi}{8}$ radians. Find the diameter of the circle. **6.** ______________

7. Find the area of a sector if the central angle measures $\frac{3\pi}{10}$ radians and the radius of the circle is 52 centimeters. **7.** ______________

8. Write an equation of the cosine function with amplitude 4, period 6, phase shift $-\pi$, and vertical shift -5. **8.** ______________

9. Write an equation of the tangent function with period 2π, phase shift $\frac{\pi}{4}$, and vertical shift -1. **9.** ______________

10. State the amplitude, period, phase shift, and vertical shift for $y = -4 \cos\left(\frac{x}{2} + \frac{\pi}{2}\right) + 1$. **10.** ______________

11. State the period, phase shift, and vertical shift for $y = \cot\left(\frac{x}{4} + \frac{\pi}{2}\right) + 3$. **11.** ______________

12. Write the equation for the inverse of $y = \text{Arctan}\,(x + 3)$. **12.** ______________

13. Evaluate $\sin\,(2\,\text{Tan}^{-1}\sqrt{3})$. **13.** ______________

NAME ______________________ DATE ____________ PERIOD ________

Chapter 6 Test, Form 2B (continued)

The average monthly temperatures in degrees Fahrenheit for the Moline, Illinois, Quad City Airport are given below.

Jan.	Feb.	March	April	May	June
21.0°	25.7°	36.9°	50.4°	61.2°	70.9°

July	Aug.	Sept.	Oct.	Nov.	Dec
74.8°	72.9°	64.6°	53.2°	38.8°	26.2°

14. Write a sinusoidal function that models the monthly temperatures in Moline, using $t = 1$ to represent January.

14. ______________

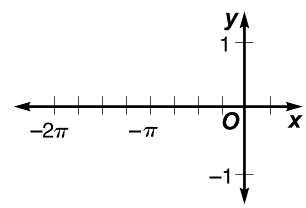
15. According to your model, what is the average temperature in May?

15. ______________

Graph each function.

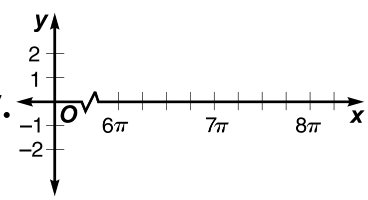
16. $y = \cos x$ for $-\frac{9\pi}{4} \le x \le \frac{\pi}{4}$

16.



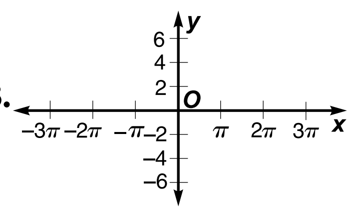
17. $y = \tan x$ for $6\pi \le x \le 8\pi$

17.



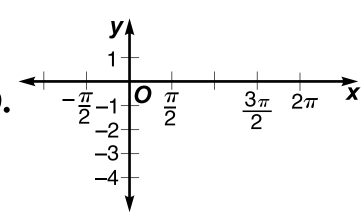
18. $y = -6 \sin\left(\frac{2x}{3}\right)$

18.



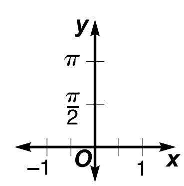
19. $y = \csc\left(x - \frac{\pi}{4}\right) - 2$

19.



20. $y = \text{Arccos}\, x$

20.



Bonus Evaluate $\sec\left(\text{Arccot}\, \frac{3}{5}\right)$.

Bonus: ______________

NAME ______________________ DATE ______________ PERIOD ________

Chapter 6 Test, Form 2C

1. Change 405° to radian measure in terms of π. **1.** ________

2. Change $\frac{5\pi}{12}$ radians to degree measure. **2.** ________

3. Determine the angular velocity if 88 revolutions are completed in 5 seconds. Round to the nearest tenth. **3.** ________

4. Determine the linear velocity of a point rotating at an angular velocity of 7π radians per second at a distance of 10 feet from the center of the rotating object. Round to the nearest tenth. **4.** ________

5. The second hand of a clock is 10 inches long. Find the linear velocity of the tip of the second hand. **5.** ________

6. Find the length of the arc intercepted by a central angle of $\frac{\pi}{8}$ radians on a circle of radius 8 inches. **6.** ________

7. Find the area of a sector if the central angle measures $\frac{\pi}{3}$ radians and the radius of the circle is 9 meters. **7.** ________

8. Write an equation of the sine function with amplitude 2, period 5π, phase shift $-\frac{\pi}{2}$, and vertical shift 3. **8.** ________

9. Write an equation of the tangent function with period 3π, phase shift $-\pi$, and vertical shift 2. **9.** ________

10. State the amplitude, period, phase shift, and vertical shift for $y = \frac{3}{2}\sin\left(2x - \frac{\pi}{4}\right)$. **10.** ________

11. State the period, phase shift, and vertical shift for $y = \tan(3x - \pi) - 2$. **11.** ________

12. Write the equation for the inverse of $y = \text{Arccos}\,(x - 5)$. **12.** ________

13. Evaluate $\tan\left(\frac{1}{2}\text{Cos}^{-1}\,0\right)$. **13.** ________

NAME ______________________ DATE ______________ PERIOD ________

Chapter 6

Chapter 6 Test, Form 2C (continued)

The average monthly temperatures in degrees Fahrenheit for the Spokane International Airport, in Washington, are given below.

Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
27.1°	33.3°	38.7°	45.9°	53.8°	61.9°	68.7°	68.4°	58.8°	47.3°	35.1°	27.9°

14. Write a sinusoidal function that models the monthly temperatures in Spokane, using $t = 1$ to represent January.

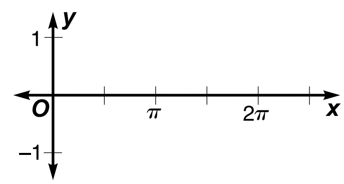
14. ____________________

15. According to your model, what is the average temperature in March?

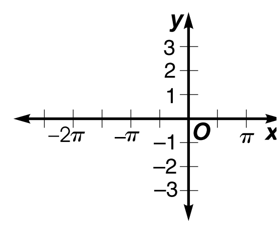
15. ____________________

Graph each function.

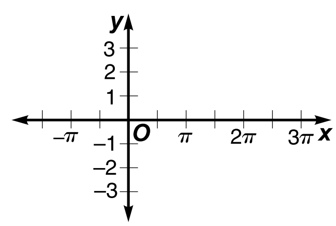
16. $y = \cos x$ for $\frac{\pi}{2} \leq x \leq \frac{5\pi}{2}$

16. 

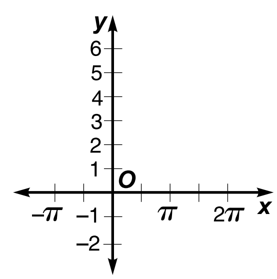
17. $y = \tan x$ for $-\frac{5\pi}{2} \leq x \leq \frac{\pi}{2}$

17. 

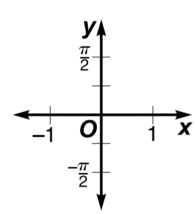
18. $y = 3 \sin \left(\frac{2x}{3}\right)$

18. 

19. $y = \sec (x - \pi) + 3$

19. 

20. $y = \text{Arcsin } x$

20. 

Bonus Evaluate $\tan \left(\text{Arccos } \frac{2}{3}\right)$.

Bonus: ____________________

NAME ______________________________ DATE ______________ PERIOD ________

Chapter 6

Chapter 6 Open-Ended Assessment

Instructions: ***Demonstrate your knowledge by giving a clear, concise solution to each problem. Be sure to include all relevant drawings and justify your answers. You may show your solution in more than one way or investigate beyond the requirements of the problem.***

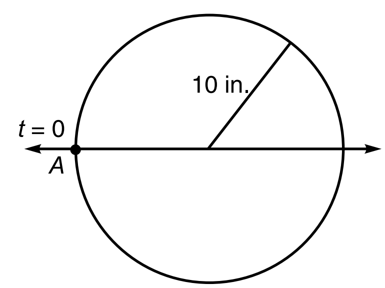
1. a. Explain what is meant by a sine function with an amplitude of 3. Draw a graph in your explanation.

b. Explain what is meant by a cosine function with a period of π. Draw a graph in your explanation.

c. Explain what is meant by a tangent function with a phase shift of $-\frac{\pi}{4}$. Draw a graph in your explanation.

d. Choose an amplitude, a period, and a phase shift for a sine function. Write the equation for these attributes and graph it.

2. a. Write an equation to describe the motion of point A as the wheel shown at the right turns counterclockwise in place. The wheel completes a revolution every 20 seconds.



b. How would the equation change if the radius of the wheel were 5 inches?

c. How would the equation change if the wheel completed a revolution every 10 seconds?

d. How would the equation change if point A were at the top of the circle at $t = 0$?

3. Give an example of a rotating object. (Engineering and the sciences are good sources of examples.) What is the angular velocity of the object? Choose a point on the object and find its linear velocity.

NAME ______________________ DATE ____________ PERIOD ________

Chapter 6 Mid-Chapter Test (Lessons 6-1 through 6-4)

1. Change $-42°$ to radian measure in terms of π. **1.** __________

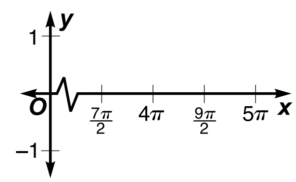
2. Change $\frac{4\pi}{15}$ radians to degree measure. **2.** __________

3. Given a central angle of 76.4°, find the length of the intercepted arc in a circle of radius 6 centimeters. Round to the nearest tenth. **3.** __________

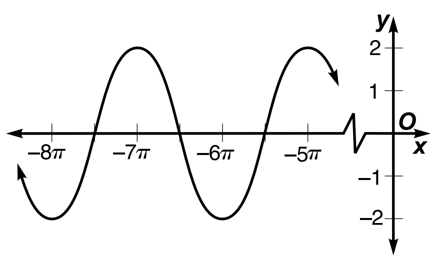
4. Find the area of a sector if the central angle measures $\frac{7\pi}{12}$ radians and the radius of the circle is 2.6 meters. Round to the nearest tenth. **4.** __________

5. A belt runs a pulley that has a diameter of 12 centimeters. If the pulley rotates at 80 revolutions per minute, what is its angular velocity in radians per second and its linear velocity in centimeters per second? **5.** __________

6. Graph $y = \cos x$ for $\frac{7\pi}{2} \le x \le 5\pi$. **6.**



7. Determine whether the graph represents $y = \sin x$, $y = \cos x$, or neither. Explain. **7.** __________



State the amplitude and period for each function.

8. $y = \frac{8}{3} \cos \frac{6\theta}{5}$ **8.** __________

9. $y = -2.3 \sin \frac{5\theta}{6}$ **9.** __________

10. Write an equation of the sine function with amplitude 5 and period $\frac{5\pi}{6}$. **10.** __________

NAME ______________________ DATE ______________ PERIOD ________

Chapter 6, Quiz A (Lessons 6-1 and 6-2)

1. Change 700° to radian measure in terms of π. **1.** ______________

2. Change $-\frac{\pi}{12}$ radians to degree measure. **2.** ______________

3. Find the area of a sector if the central angle measures 66° and the radius of the circle is 12.1 yards. Round to the nearest tenth. **3.** ______________

4. Determine the angular velocity if 57 revolutions are completed in 8 minutes. Round to the nearest tenth. **4.** ______________

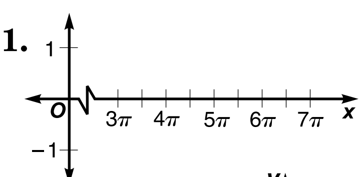
5. Determine the linear velocity of a point that rotates $\frac{5\pi}{18}$ radians in 5 seconds and is a distance of 10 centimeters from the center of the rotating object. **5.** ______________

NAME ______________________ DATE ______________ PERIOD ________

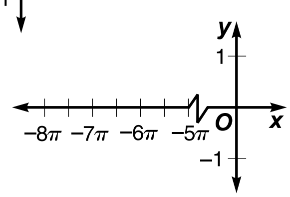
Chapter 6, Quiz B (Lessons 6-3 and 6-4)

Graph each function for the given interval.

1. $y = \cos x,\ 3\pi \le x \le \frac{13\pi}{2}$ **1.**



2. $y = \sin x,\ -8\pi \le x \le -5\pi$ **2.**



State the amplitude and period for each function.

3. $y = -\frac{3}{2}\cos 5\theta$ **3.** ______________

4. $y = 0.7 \sin \frac{3\theta}{2}$ **4.** ______________

5. Write an equation of the sine function with amplitude $\frac{8}{5}$ and period $\frac{5\pi}{3}$. **5.** ______________

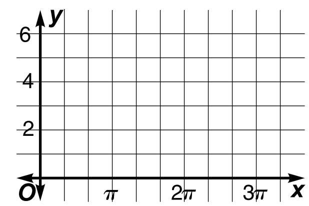
NAME ______________________ DATE ______________ PERIOD ________

Chapter 6, Quiz C (Lessons 6-5 and 6-6)

1. State the phase shift and vertical shift for $y = -3 \cos\left(\frac{\theta}{3} - 2\pi\right)$. **1.** ______________

2. Write an equation of a cosine function with amplitude 2.4, period 8.2, phase shift $\frac{\pi}{3}$, and vertical shift 0.2. **2.** ______________

3. Graph $y = \frac{1}{2}x - \sin x$. **3.**



The average monthly temperature in degrees Fahrenheit for the city of Wichita, Kansas, are given below.

Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
31.3°	35.2°	45.0°	56.1°	65.3°	75.4°	80.4°	79.3°	70.9°	59.2°	44.8°	34.5°

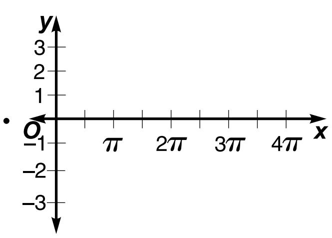
4. Write a sinusoidal function that models the monthly temperatures, using $t = 1$ to represent January. **4.** ______________

5. According to your model, what is the average monthly temperature in August? **5.** ______________

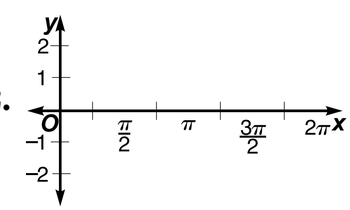
NAME ______________________ DATE ______________ PERIOD ________

Chapter 6, Quiz D (Lessons 6-7 and 6-8)

1. Graph $y = \sec x$ for $\pi \le x \le 4\pi$. **1.**



2. Graph $y = \tan\left(2x + \frac{\pi}{2}\right)$ for $0 \le x \le 2\pi$. **2.**



3. Write an equation for a cotangent function with period $\frac{\pi}{3}$, phase shift $-\frac{\pi}{12}$, and vertical shift -4. **3.** ______________

4. Write the equation for the inverse of $y = \text{Arcsin } \frac{x}{2}$. **4.** ______________

5. Evaluate $\tan\left[\text{Sin}^{-1}\left(-\frac{\sqrt{3}}{2}\right) + \text{Cos}^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right]$. **5.** ______________

NAME ______________________ DATE ______________ PERIOD ________

Chapter 6 SAT and ACT Practice

After working each problem, record the correct answer on the answer sheet provided or use your own paper.

Multiple Choice

1. Find $\cos \theta$ if $\sin \theta = -\frac{1}{3}$ and $\tan \theta > 0$.

A $\frac{\sqrt{2}}{3}$ **B** $\frac{2}{3}$

C $-\frac{1}{3}$ **D** $-\frac{\sqrt{2}}{3}$

E $-\frac{2\sqrt{2}}{3}$

2. Evaluate $(2 \sin \theta)(\cos \theta)$ if $\theta = 150°$.

A $-\sqrt{3}$ **B** $\sqrt{3}$

C $\frac{\sqrt{3}}{2}$ **D** $-\frac{\sqrt{3}}{2}$

E None of these

3. If the product of $(1 - 3)$, $(3 - 7)$, and $(7 - 13)$ is equal to two times the sum of 12 and x, then $x =$

A -36
B 48
C -60
D -108
E 12

4. From which of the following statements can you determine that $m > n$?

I. $2m + n > 12$ **II.** $m + n = 7$

A Both I alone and II alone
B Neither I nor II, nor I and II together
C I alone, but not II
D II alone, but not I
E I and II together, but neither alone

5. Solve $\sin^2 x - 1 = 0$ for $0° < x < 360°$.

A 90°
B 180°
C 270°
D All of the above
E Both A and C

6. At a point on the ground 27.6 meters from the foot of a flagpole, the angle of elevation to the top of the pole is 60°. What is the height of the flagpole?

A 15.9 m
B 47.8 m
C 23.9 m
D 13.8 m
E None of these

7. $\triangle ABC$ is an isosceles triangle, and the coordinates of two vertices are $A(-3, 2)$ and $B(1, -2)$. What are the coordinates of C?

A $(4, 3)$
B $(-1, 3)$
C $(-3, -1)$
D $(3, 4)$
E None of these

8. Determine the coordinates of Q, an endpoint of $\overline{PQ}$, given that the other endpoint is $P(-2, 4)$ and the midpoint is $M(1, 5)$.

A $(4, 14)$
B $(0, 6)$
C $(4, 6)$
D $\left(-\frac{1}{2}, \frac{9}{2}\right)$
E $(5, 6)$

9. $\frac{x^2y^7}{(x^2y^3)^2} =$

A $\frac{y^2}{x^2}$ **B** $\frac{y}{x}$

C $\frac{y}{x^2}$ **D** $\frac{1}{x^2}$

E y

10. If $kx^2 = k^2$, for what values of k will there be exactly two real values of x?

A All values of k
B All values of $k \neq 0$
C All values of $k < 0$
D All values of $k > 0$
E None of these

NAME ______________________ DATE ____________ PERIOD ________

Chapter 6 SAT and ACT Practice (continued)

11. If $\cot \theta = \frac{8}{15}$ and $\cos \theta < 0$, evaluate $\sqrt{\frac{1 - \cos \theta}{17}}$.

A $\frac{5}{17}$

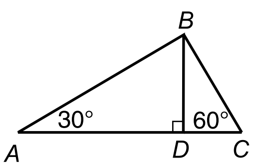
B $\frac{5\sqrt{17}}{17}$

C $\frac{85}{17}$

D 5

E $\frac{\sqrt{19}}{17}$

12. In $\triangle ABC$ shown below, $A = 30°$, $C = 60°$, and $AC = 10$. Find BD.



A $\frac{5}{2}$

B $\frac{5\sqrt{3}}{2}$

C $\frac{\sqrt{3}}{2}$

D $\frac{1}{2}$

E $\frac{\sqrt{3}}{3}$

13. For what value of k will the line $3x + ky = 8$ be perpendicular to the line $4x - 3y = 6$?

A 6
B 4
C 3
D -3
E -4

14. Which line is a perpendicular bisector to $\overline{AB}$ with endpoints $A(1, 3)$ and $B(1, -2)$?

A $x - 2y = 1$
B $2x + 2y = 1$
C $y = 1$
D $x = \frac{1}{2}$
E None of these

15. For any integer k, which of the following represents three consecutive even integers?

A $2k, 4k, 6k$
B $k, k + 1, k + 2$
C $k, k + 2, k + 4$
D $4k, 4k + 1, 4k + 2$
E $2k, 2k + 2, 2k + 4$

16. If y varies directly as the square of x, what will be the effect on y of doubling x?

A y will double
B y will be half as large
C y will be 4 times as large
D y will decrease in size
E None of these

17–18. Quantitative Comparison

A if the quantity of Column A is greater
B if the quantity in Column B is greater
C if the two quantities are equal
D if the relationship cannot be determined from the information given

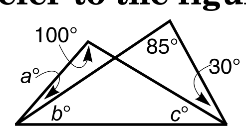
17. k is a positive integer.

Column A	Column B
$(-1)^{2k}$	$(-1)^{2k+1}$

18. $4(x + y) = 24$ and $3x - y = 6$

Column A	Column B
x	y

19–20. Refer to the figure below.



19. Grid-In Find the value of a.

20. Grid-In Find the value of $b + c$.

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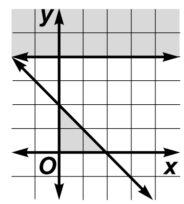
Chapter 6

Chapter 6 Cumulative Review (Chapters 1-6)

1. State the domain of $[f \circ g](x)$ for $f(x) = \frac{8}{x}$ and $g(x) = x - 8$. **1.** ______________

2. State whether the linear programming problem represented by the graph below *is infeasible, is unbounded, has an optimal solution,* or *has alternate optimal solutions* for finding a minumum. **2.** ______________



△ABC has vertices at A(-2, 3), B(1, 7), and C(4, -3). Find the coordinates of the vertices of the triangle after each of the following transformations.

3. dilation of scale factor 3 **3.** ______________

4. reflection over the y-axis **4.** ______________

5. rotation of 270° counterclockwise about the origin **5.** ______________

6. Find the value of k so that the remainder of $(3x^3 - 10x^2 + kx - 6) \div (x - 3)$ is 0. **6.** ______________

7. Solve $3|2x - 4| < 20$. **7.** ______________

8. Find the area of $\triangle ABC$ if $A = 24.4°$, $B = 56.3°$, and $c = 78.4$ centimeters. **8.** ______________

9. Find the area of $\triangle ABC$ if $a = 15$ inches, $b = 19$ inches, and $c = 24$ inches. **9.** ______________

10. An oar floating on the water bobs up and down, covering a distance of 12 feet from its lowest point to its highest point. The oar moves from its lowest point to its highest point and back to its lowest point every 15 seconds. Write a cosine function with phase shift 0 for the height of the oar after t seconds. **10.** ______________

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Trigonometry Semester Test

Write the letter for the correct answer in the blank at the right of each problem.

1. Given that x is an integer and $0 \leq x < 4$, state the relation represented by $y = x^2 - 1$ by listing a set of ordered pairs. Then state whether the relation is a function. **1.** ________

A. $\{(0, -1), (1, 0), (2, 3), (3, 8)\}$; no
B. $\{(0, -1), (1, 0), (2, 3), (3, 8)\}$; yes
C. $\{(-1, 0), (0, 1), (3, 2), (8, 3)\}$; no
D. $\{(0, 0), (1, 1), (2, 4), (3, 9)\}$; yes

2. Find the zero of the function $f(x) = 2x - 3$. **2.** ________

A. $-\frac{2}{3}$ **B.** $\frac{2}{3}$ **C.** $\frac{3}{2}$ **D.** $-\frac{3}{2}$

3. Which angle is not coterminal with $-45°$? **3.** ________

A. $-\frac{\pi}{4}$ **B.** $315°$ **C.** $\frac{7\pi}{2}$ **D.** $675°$

4. Evaluate $\cos\left(\text{Cos}^{-1} \frac{1}{2}\right)$. **4.** ________

A. $\frac{\sqrt{2}}{2}$ **B.** $\frac{\sqrt{3}}{2}$ **C.** $\frac{1}{2}$ **D.** 1

5. Which function is an even function? **5.** ________

A. $y = x^3$ **B.** $y = x^2 + x$ **C.** $y = x|x|$ **D.** $y = -x^6 - 5$

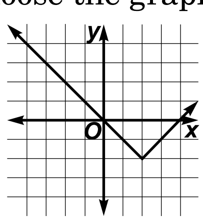
6. Find the slope of the line passing through $(-1, 1)$ and $(1, 3)$. **6.** ________

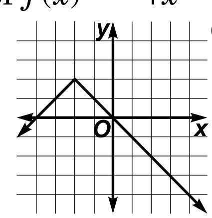
A. -1 **B.** 1 **C.** 2 **D.** undefined

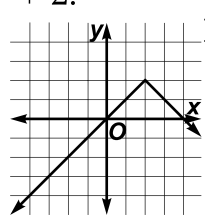
7. Write the equation of the line that passes through $(1, 2)$ and is parallel to the line $x + 3y + 1 = 0$. **7.** ________

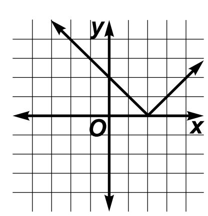
A. $3x + y - 5 = 0$ **B.** $x + 3y - 7 = 0$
C. $x - 3y + 5 = 0$ **D.** $\frac{1}{3}x + y - 3 = 0$

8. Choose the graph of $f(x) = -|x - 2| + 2$. **8.** ________

A.


B.


C.


D.


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Trigonometry Semester Test (continued)

9. Write the slope-intercept form of the equation of the line passing through $(-1, 0)$ and $(2, 3)$. **9.** ________

A. $y = x + 1$ **B.** $y = 2x + 2$ **C.** $y = -x - 1$ **D.** $y = 3x - 1$

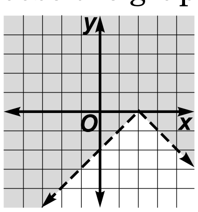
10. Write the polynomial equation of least degree with a leading coefficient of 1 and roots 0, $2\boldsymbol{i}$, and $-2\boldsymbol{i}$. **10.** ________

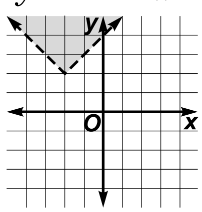
A. $x^2 + 4 = 0$ **B.** $x^3 + 4x = 0$ **C.** $4x^2 = 0$ **D.** $4x^3 = 0$

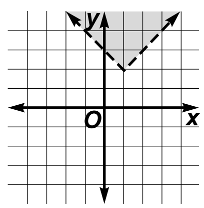
11. Find the inverse of $y = (x - 1)^3$. **11.** ________

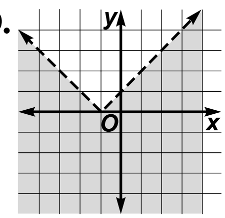
A. $y = \sqrt[3]{x} - 1$ **B.** $y = \sqrt[3]{x} + 1$ **C.** $y = \sqrt[3]{x - 1}$ **D.** $y = \sqrt[3]{x + 1}$

12. Choose the graph of $y - 2 > |x - 1|$. **12.** ________

A.


B.


C.


D.


13. Find the x-intercept(s) of the graph of the function $f(x) = (x - 2)(x^2 - 25)$. **13.** ________

A. 2, 5 **B.** 2, 25 **C.** $-2, -25$ **D.** $-5, 2, 5$

14. Find $A - B$ if $A = \begin{bmatrix} 5 & 4 \\ 3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix}$. **14.** ________

A. $\begin{bmatrix} 7 & 6 \\ 4 & 5 \end{bmatrix}$ **B.** $\begin{bmatrix} 3 & 2 \\ 2 & -1 \end{bmatrix}$ **C.** $\begin{bmatrix} 2 & -1 \\ 3 & 3 \end{bmatrix}$ **D.** $\begin{bmatrix} -2 & 9 \\ 6 & 7 \end{bmatrix}$

15. Find the discriminant of $2m^2 + 3m + 1 = 0$ and describe the nature of the roots. **15.** ________

A. -6, imaginary **B.** 3, real
C. 4, imaginary **D.** 1, real

16. Which best describes the graph of $f(x) = \frac{x^2 - 4}{x - 2}$? **16.** ________

A. The graph has infinite discontinuity.
B. The graph has jump discontinuity.
C. The graph has point discontinuity.
D. The graph is continuous.

17. Solve $\sin \theta = 1$ for all values of θ. Assume k is any integer. **17.** ________

A. $90° + 360k°$ **B.** $360k°$
C. $180° + 360k°$ **D.** $270° + 360k°$

Trigonometry Semester Test (continued)

18. Find the value of $\begin{vmatrix} 3 & 2 \\ 2 & 3 \end{vmatrix}$. **18.** ________

A. -5 **B.** 13 **C.** 5 **D.** 2

19. List all possible rational zeros of $f(x) = 2x^3 + 3x^2 + 2x + 5$. **19.** ________

A. $\pm 1, \pm 2, \pm 5$ **B.** $\pm 1, \pm 5, \pm 3, \pm \frac{1}{2}, \pm \frac{3}{2}$

C. $\pm 1, \pm 5, \pm \frac{1}{2}, \pm \frac{5}{2}$ **D.** $\pm 1, \pm 5$

20. A section of highway is 5.1 kilometers long and rises at a uniform grade, making a 2.9° angle with the horizontal. What is the change in elevation of this section of highway to the nearest thousandth? **20.** ________

A. 5.093 km **B.** 0.258 km **C.** 4.193 km **D.** 0.276 km

21. Use the Remainder Theorem to find the remainder for $(x^3 - 2x^2 + 2x + 3) \div (x - 1)$. **21.** ________

A. 4 **B.** 1 **C.** 2 **D.** 5

22. Find the multiplicative inverse of $\begin{bmatrix} 4 & 2 \\ 3 & 2 \end{bmatrix}$. **22.** ________

A. $\begin{bmatrix} 2 & -2 \\ -3 & 4 \end{bmatrix}$ **B.** $\begin{bmatrix} 3 & 2 \\ 2 & -1 \end{bmatrix}$ **C.** $\begin{bmatrix} 1 & -1 \\ -\frac{3}{2} & 2 \end{bmatrix}$ **D.** $\begin{bmatrix} 2 & 1 \\ \frac{3}{2} & 1 \end{bmatrix}$

23. The graph of $y = x^4 + 1$ is symmetric with respect to **23.** ________

A. the x-axis. **B.** the y-axis.

C. the line $y = x$. **D.** the line $y = -x$.

24. Find one positive and one negative angle that are coterminal with an angle measuring $\frac{3\pi}{4}$. **24.** ________

A. $\frac{3\pi}{4}, -\frac{11\pi}{2}$ **B.** $\frac{5\pi}{4}, -\frac{11\pi}{4}$ **C.** $\frac{11\pi}{4}, -\frac{5\pi}{4}$ **D.** $\frac{3\pi}{2}, -\frac{\pi}{4}$

25. If $\sin \theta = -\frac{\sqrt{2}}{2}$ and θ lies in Quadrant III, find $\cot \theta$. **25.** ________

A. $-\frac{\sqrt{3}}{3}$ **B.** 1 **C.** $\sqrt{3}$ **D.** -1

Trigonometry Semester Test (continued)

26. State the amplitude, period, and phase shift of the function $y = 3 \sin (2x - \pi)$. **26.** ________________

27. Find the value of $\text{Cos}^{-1}\left(\sin \frac{\pi}{6}\right)$. **27.** ________________

28. Find the values of x and y for which $\begin{bmatrix} x \\ x + 2 \\ 2y \end{bmatrix} = \begin{bmatrix} 3 + y \\ 6 \\ 2 \end{bmatrix}$. **28.** ________________

29. If α is a first quadrant angle and $\cos \alpha = \frac{1}{3}$, find $\sin \alpha$. **29.** ________________

30. Find the value of $\sec \theta$ for angle θ in standard position if a point with coordinates $(-3, 4)$ lies on its terminal side. **30.** ________________

31. Given $f(x) = x^2 + |x|$, find $f(-2)$. **31.** ________________

32. Find the slope and y-intercept of the line passing through $(-2, 4)$ and $(3, 2)$. **32.** ________________

33. Determine the slant asymptote for the graph of $f(x) = \frac{x^2 - x - 1}{x - 2}$. **33.** ________________

34. Determine whether the system is *consistent and independent, consistent and dependent,* or *inconsistent.* **34.** ________________

$$-6x + 3y = 0$$
$$-4x + 2y = 2$$

35. Solve $2 \sin x - \csc x = 0$ for $0° \leq x \leq 180°$. **35.** ________________

36. If $f(x) = x^2 + 1$ and $g(x) = x + 1$, find $[f \circ g](x)$. **36.** ________________

37. Write the slope-intercept form of the equation of the line passing through $(1, 7)$ and $(-3, -1)$. **37.** ________________

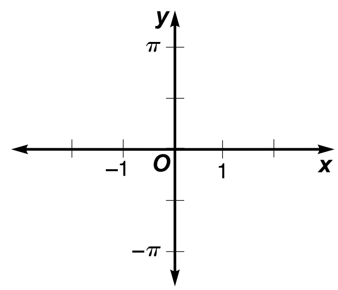
38. Solve $2x^2 + 4x + 2 = 1$ by using the Quadratic Formula. **38.** ________________

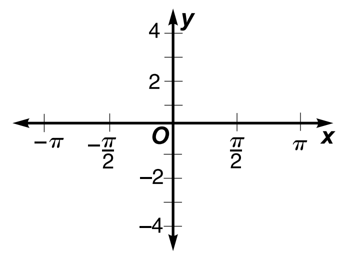
39. Describe the transformation that relates the graph of $y = \tan\left(x - \frac{\pi}{4}\right)$ to the parent graph $y = \tan x$. **39.** ________________

Trigonometry Semester Test (continued)

40. Find AB if $A = \begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -1 \\ 2 & 5 \end{bmatrix}$. **40.** ________________

41. Write the equation for the inverse of $y = \arctan x$. Then graph the function and its inverse. **41.** ________________





42. Given a central angle of 75°, find the length of the angle's intercepted arc in a circle of radius 8 centimeters. Round to the nearest thousandth. **42.** ________________

43. Determine whether the graph of $y = \frac{2}{x^3} + 3$ has *infinite discontinuity, jump discontinuity, point discontinuity,* or is *continuous.* **43.** ________________

44. Determine the rational zeros of $f(x) = 6x^3 - 11x^2 + 6x - 1$. **44.** ________________

45. State the amplitude and period of $y = -3 \cos 2x$. **45.** ________________

46. Find an equation for a sine function with amplitude 2, period π, phase shift 0, and vertical shift 1. **46.** ________________

47. Find the value of $\begin{vmatrix} 2 & 3 \\ 2 & 5 \end{vmatrix}$. **47.** ________________

48. Find the multiplicative inverse of $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$, if it exists. **48.** ________________

49. The function $f(x) = -2x^2 + 4x - 1$ has a critical point when $x = 1$. Identify the point as a *maximum,* a *minimum,* or a *point of inflection,* and state its coordinates. **49.** ________________

50. Use the Law of Cosines to solve $\triangle ABC$ with $a = 15$, $b = 20$, and $C = 95°$. Round to the nearest tenth. **50.** ________________

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NAME ______________________________ DATE ______________ PERIOD ________

SAT and ACT Practice Answer Sheet

(10 Questions)

1 (A) (B) (C) (D) (E)

2 (A) (B) (C) (D) (E)

3 (A) (B) (C) (D) (E)

4 (A) (B) (C) (D) (E)

5 (A) (B) (C) (D) (E)

6 (A) (B) (C) (D) (E)

7 (A) (B) (C) (D) (E)

8 (A) (B) (C) (D) (E)

9 (A) (B) (C) (D) (E)

10

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	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

NAME ______________________________ DATE _______________ PERIOD ________

SAT and ACT Practice Answer Sheet

(20 Questions)

1 (A) (B) (C) (D) (E)

2 (A) (B) (C) (D) (E)

3 (A) (B) (C) (D) (E)

4 (A) (B) (C) (D) (E)

5 (A) (B) (C) (D) (E)

6 (A) (B) (C) (D) (E)

7 (A) (B) (C) (D) (E)

8 (A) (B) (C) (D) (E)

9 (A) (B) (C) (D) (E)

10 (A) (B) (C) (D) (E)

11 (A) (B) (C) (D) (E)

12 (A) (B) (C) (D) (E)

13 (A) (B) (C) (D) (E)

14 (A) (B) (C) (D) (E)

15 (A) (B) (C) (D) (E)

16 (A) (B) (C) (D) (E)

17 (A) (B) (C) (D) (E)

18 (A) (B) (C) (D) (E)

19

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.	.	.	.
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

20

	/	/	
.	.	.	.
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

6-1 NAME ______________ DATE ________ PERIOD ______

Practice

Angles and Radian Measure

Change each degree measure to radian measure in terms of π.

1. $-250°$ $-\frac{25\pi}{18}$

2. $6°$ $\frac{\pi}{30}$

3. $-145°$ $-\frac{29\pi}{36}$

4. $870°$ $\frac{29\pi}{6}$

5. $18°$ $\frac{\pi}{10}$

6. $-820°$ $-\frac{41\pi}{9}$

Change each radian measure to degree measure. Round to the nearest tenth, if necessary.

7. 4π 720°

8. $\frac{13\pi}{30}$ 78°

9. -1 −57.3°

10. $\frac{3\pi}{16}$ 33.8°

11. -2.56 −146.7°

12. $-\frac{7\pi}{9}$ −140°

Evaluate each expression.

13. $\tan \frac{\pi}{4}$ 1

14. $\cos \frac{3\pi}{2}$ 0

15. $\sin \frac{3\pi}{2}$ −1

16. $\tan \frac{11\pi}{6}$ $-\frac{\sqrt{3}}{3}$

17. $\cos \frac{3\pi}{4}$ $-\frac{\sqrt{2}}{2}$

18. $\sin \frac{5\pi}{3}$ $-\frac{\sqrt{3}}{2}$

Given the measurement of a central angle, find the length of its intercepted arc in a circle of radius 10 centimeters. Round to the nearest tenth.

19. $\frac{\pi}{6}$ 5.2 cm

20. $\frac{3\pi}{5}$ 18.8 cm

21. $\frac{\pi}{2}$ 15.7 cm

Find the area of each sector, given its central angle θ and the radius of the circle. Round to the nearest tenth.

22. $\theta = \frac{\pi}{6}, r = 14$ 51.3 units^2

23. $\theta = \frac{7\pi}{4}, r = 4$ 44.0 units^2

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Enrichment

Angle Measurement: The Mil

The **mil** is an angle measurement used by the military. The military uses the mil because it is easy and accurate for measurements involving long distances. Determining the angle to use to hit a target in long-range artillery firing is one example.

In ordinary measurement, 1 mil = $\frac{1}{1000}$ inch. For angle measurement, this means that an angle measuring one mil would subtend an arc of length 1 unit, with the entire circle being 1000 mils around. So, the circumference becomes $2\pi \cdot 1000$, or about 6283.18 units. The military rounds this number to 6400 for convenience. Thus,

$$1 \text{ mil} = \frac{1}{6400} \text{ revolution around a circle}$$

So, 6400 mil = 2π radians.

Example **Change 3200 mil to radian measure.**

$$\frac{6400 \text{ mil}}{3200 \text{ mil}} = \frac{2\pi}{x}$$
$$x = \pi$$

Change each mil measurement to radian measure.

1. 1600 mil $\frac{\pi}{2}$

2. 800 mil $\frac{\pi}{4}$

3. 4800 mil $\frac{3\pi}{2}$

4. 2400 mil $\frac{3\pi}{4}$

Change each radian measure to mil measurement. Round your answers to the nearest tenth, where necessary.

5. $\frac{\pi}{8}$ 400 mil

6. $\frac{5\pi}{4}$ 4000 mil

7. $\frac{\pi}{12}$ 266.7 mil

8. $\frac{\pi}{6}$ 533.3 mil

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6-2 Practice

Linear and Angular Velocity

Determine each angular displacement in radians. Round to the nearest tenth.

1. 6 revolutions
37.7 radians

2. 4.3 revolutions
27.0 radians

3. 85 revolutions
534.1 radians

4. 11.5 revolutions
72.3 radians

5. 7.7 revolutions
48.4 radians

6. 17.8 revolutions
111.8 radians

Determine each angular velocity. Round to the nearest tenth.

7. 2.6 revolutions in 6 seconds
2.7 radians/s

8. 7.9 revolutions in 11 seconds
4.5 radians/s

9. 118.3 revolutions in 19 minutes
39.1 radians/min

10. 5.5 revolutions in 4 minutes
8.6 radians/min

11. 22.4 revolutions in 15 seconds
9.4 radians/s

12. 14 revolutions in 2 minutes
44.0 radians/min

Determine the linear velocity of a point rotating at the given angular velocity at a distance r from the center of the rotating object. Round to the nearest tenth.

13. $\omega = 14.3$ radians per second, $r = 7$ centimeters
100.1 cm/s

14. $\omega = 28$ radians per second, $r = 2$ feet
56.0 ft/s

15. $\omega = 5.4\pi$ radians per minute, $r = 1.3$ meters
22.1 m/min

16. $\omega = 41.7\pi$ radians per second, $r = 18$ inches
2358.1 in./s

17. $\omega = 234$ radians per minute, $r = 31$ inches
7254.0 in./min

18. ***Clocks*** Suppose the second hand on a clock is 3 inches long. Find the linear velocity of the tip of the second hand.
0.3 in./s

NAME ______________ DATE ________ PERIOD _____

6-2 Enrichment

Angular Acceleration

An object traveling in a circular path experiences linear velocity and angular velocity. It may also experience **angular acceleration**. Angular acceleration is the rate of change in angular velocity with respect to time.

At time $t = 0$, there is an **initial angular velocity**. At the end of time t, there is a **final angular velocity**. Then the angular acceleration α of the object can be defined as

$$\alpha = \frac{\text{final angular velocity} - \text{initial angular velocity}}{\text{time}}.$$

The units for angular acceleration are usually rad/s^2 or rev/min^2.

Example **A record has a small chip on its edge. If the record begins at rest and then goes to 45 revolutions per minute in 30 seconds, what is the angular acceleration of the chip?**

The record starts at rest, so the initial angular velocity is 0. The final angular velocity is 45 revolutions/minute. Thus, the angular acceleration is

$$\alpha = \frac{45 - 0}{\frac{1}{2}}$$

$$= 90 \text{ rev/min}^2.$$

Solve.

1. The record in the example was playing at 45 rev/min. A power surge lasting 2 seconds caused the record to speed up to 80 rev/min. What was the angular acceleration of the chip then?
1050 rev/min²

2. When a car enters a curve in the road, the tires are turning at an angular velocity of 50 ft/s. At the end of the curve, the angular velocity of the tires is 60 ft/s. If the curve is an arc of a circle with radius 2000 feet and central angle $\theta = \frac{\pi}{4}$, and the car travels at a constant linear velocity of 40 mph, what is the angular acceleration?
0.37 ft/s²

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6-3

Practice

Graphing Sine and Cosine Functions

Find each value by referring to the graph of the sine or the cosine function.

1. $\cos \pi$ **−1**

2. $\sin \frac{3\pi}{2}$ **−1**

3. $\sin\left(-\frac{7\pi}{2}\right)$ **1**

Find the values of θ for which each equation is true.

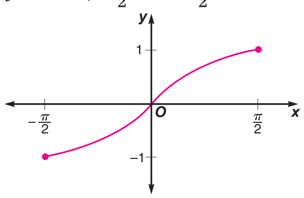
4. $\sin \theta = 0$ **πn, where n is any integer**

5. $\cos \theta = 1$ **πn, where n is an even integer**

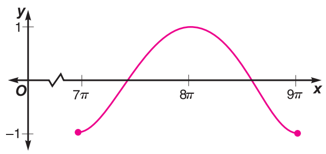
6. $\cos \theta = -1$ **πn, where n is an odd integer**

Graph each function for the given interval.

7. $y = \sin x;\ -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$

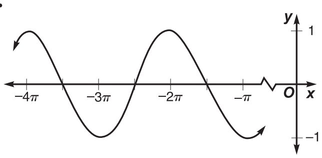


8. $y = \cos x;\ 7\pi \leq x \leq 9\pi$



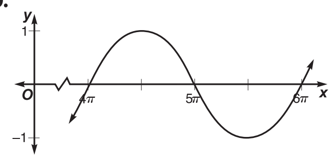
Determine whether each graph is y = sin x, y = cos x, or neither.

9.



$y = \cos x$

10.



$y = \sin x$

11. *Meteorology* The equation $y = 70.5 + 19.5 \sin\left[\frac{\pi}{6}(t - 4)\right]$ models the average monthly temperature for Phoenix, Arizona, in degrees Fahrenheit. In this equation, t denotes the number of months, with $t = 1$ representing January. What is the average monthly temperature for July? **90°F**

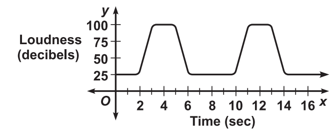
NAME ______________ DATE ______________ PERIOD ______

6-3

Enrichment

Periodic Phenomena

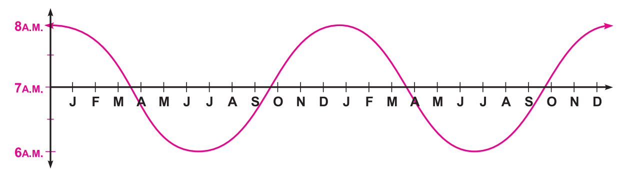
Periodic phenomena are common in everyday life. The first graph portrays the loudness of a foghorn as a function of time. The sound rises quickly to its loudest level, holds for about two seconds, drops off a little more quickly than it rose, then remains quiet for about four seconds before beginning a new cycle. The **period** of the cycle is eight seconds.



1. Give three examples of periodic phenomena, together with a typical period for each.

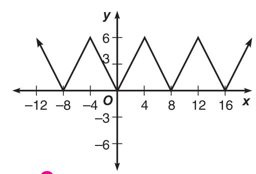
sample answers: the cycle of the moon (28 days), the swinging of a pendulum (one second), the cycle of the seasons (one year)

2. Sunrise is at 8 A.M. on December 21 in Function Junction and at 6 A.M. on June 21. Sketch a two-year graph of sunrise times in Function Junction.



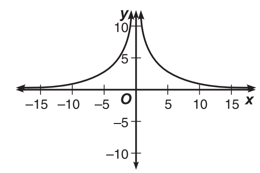
State whether each function is periodic. If it is, give its period.

3.



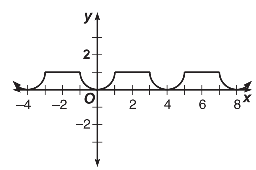
yes; 8

4.



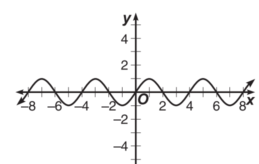
no

5.



yes; 4

6.



yes; 4

7. A student graphed a periodic function with a period of n. The student then translated the graph c units to the right and obtained the original graph. Describe the relationship between c and n. **c is a positive multiple of n.**

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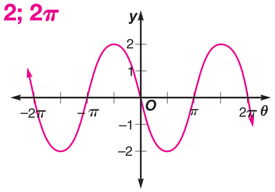
Practice

Amplitude and Period of Sine and Cosine Functions

State the amplitude and period for each function. Then graph each function.

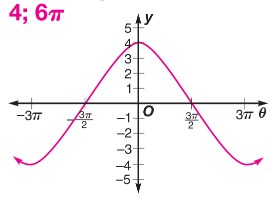
1. $y = -2 \sin \theta$

2; 2π



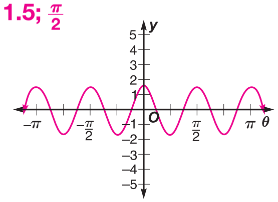
2. $y = 4 \cos \frac{\theta}{3}$

4; 6π



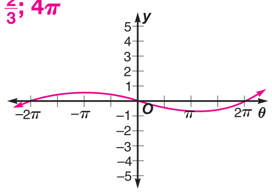
3. $y = 1.5 \cos 4\theta$

1.5; $\frac{\pi}{2}$



4. $y = -\frac{2}{3} \sin \frac{\theta}{2}$

$\frac{2}{3}$; 4π



Write an equation of the sine function with each amplitude and period.

5. amplitude = 3, period = 2π

$y = \pm 3 \sin \theta$

6. amplitude = 8.5, period = 6π

$y = \pm 8.5 \sin \frac{\theta}{3}$

Write an equation of the cosine function with each amplitude and period.

7. amplitude = 0.5, period = 0.2π

$y = \pm 0.5 \cos 10\theta$

8. amplitude = $\frac{1}{5}$, period = $\frac{2}{5}\pi$

$y = \pm \frac{1}{5} \cos 5\theta$

9. ***Music*** A piano tuner strikes a tuning fork for note A above middle C and sets in motion vibrations that can be modeled by the equation $y = 0.001 \sin 880\pi t$. Find the amplitude and period for the function.

0.001; $\frac{1}{440}$

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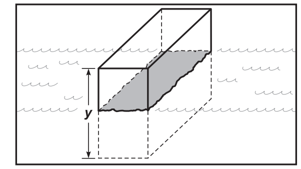
Enrichment

Mass of a Floating Object



An object bobbing up and down in the water exhibits periodic motion. The greater the mass of the object (think of an ocean liner and a buoy), the longer the period of *oscillation* (up and down motion). The greater the horizontal cross-sectional area of the object, the shorter the period. If you know the period and the cross-sectional area, you can find the mass of the object.

Imagine a point on the waterline of a stationary floating object. Let y represent the vertical position of the point above or below the waterline when the object begins to oscillate. ($y = 0$ represents the waterline.) If we neglect air and water resistance, the equation of motion of the object is

$$y = A \sin\left(\sqrt{\frac{9800C}{M}}\, t\right),$$

where A is the amplitude of the oscillation, C is the horizontal cross-sectional area of the object in square meters, M is the mass of the object in kilograms, and t is the elapsed time in seconds since the beginning of the oscillation. The argument of the sine is measured in radians and y is measured in meters.

1. A 4-kg log has a cross-sectional area of 0.2 m^2. A point on the log has a maximum displacement of 0.4 m above or below the water line. Find the vertical position of the point 5 seconds after the log begins to bob.

≈ 0.26 below the water line

2. Find an expression for the period of an oscillating floating object.

$p = 2\pi\sqrt{\frac{M}{9800C}}$

3. Find the period of the log described in Exercise 1.

≈ 0.28 seconds

4. A buoy bobs up and down with a period of 0.6 seconds. The mean cross-sectional area of the buoy is 1.3 m^2. Use your equation for the period of an oscillating floating object to find the mass of the buoy.

≈ 116.2 kg

5. Write an equation of motion of the buoy described in Exercise 4 if the amplitude is 0.45 m. **$y \approx 0.45 \sin 10.47t$**

6-5

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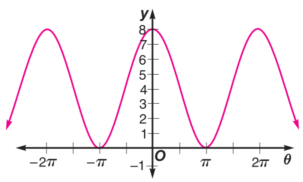
Practice

Translations of Sine and Cosine Functions

State the vertical shift and the equation of the midline for each function. Then graph each function.

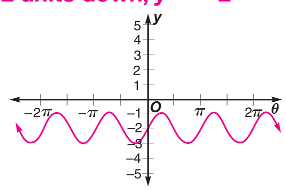
1. $y = 4\cos\theta + 4$

4 units up; $y = 4$



2. $y = \sin 2\theta - 2$

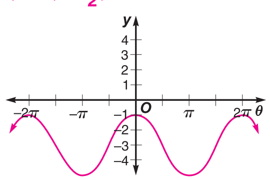
2 units down; $y = -2$



State the amplitude, period, phase shift, and vertical shift for each function. Then graph the function.

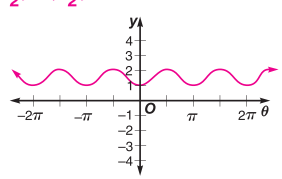
3. $y = 2\sin\left(\theta + \frac{\pi}{2}\right) - 3$

2; 2π; $-\frac{\pi}{2}$; -3



4. $y = \frac{1}{2}\cos(2\theta - \pi) + 2$

$\frac{1}{2}$; π; $\frac{\pi}{2}$; 2



Write an equation of the specified function with each amplitude, period, phase shift, and vertical shift.

5. sine function: amplitude = 15, period = 4π, phase shift = $\frac{\pi}{2}$, vertical shift = -10

$y = \pm 15\sin\left(\frac{\theta}{2} - \frac{\pi}{4}\right) - 10$

6. cosine function: amplitude = $\frac{2}{3}$, period = $\frac{\pi}{3}$, phase shift = $-\frac{\pi}{3}$, vertical shift = 5

$y = \pm\frac{2}{3}\cos(6\theta + 2\pi) + 5$

7. sine function: amplitude = 6, period = π, phase shift = 0, vertical shift = $-\frac{3}{2}$

$y = \pm 6\sin 2\theta - \frac{3}{2}$

6-5

NAME ______________ DATE ______________ PERIOD ______

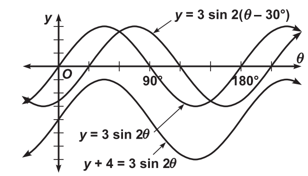
Enrichment

Translating Graphs of Trigonometric Functions

In Lesson 3-2, you learned how changes in a polynomial function affect the graph of the function. If $a > 0$, the graph of $y \pm a = f(x)$ translates the graph of $f(x)$ downward or upward a units. The graph of $y = f(x \pm a)$ translates the graph of $f(x)$ left or right a units. These results apply to trigonometric functions as well.

Example 1 **Graph $y = 3\sin 2\theta$, $y = 3\sin 2(\theta - 30°)$, and $y + 4 = 3\sin 2\theta$ on the same coordinate axes.**

Obtain the graph of $y = 3\sin 2(\theta - 30°)$ by translating the graph of $y = 3\sin 2\theta$ 30° to the right. Obtain the graph of $y + 4 = 3\sin 2\theta$ by translating the graph of $y = 3\sin 2\theta$ downward 4 units.



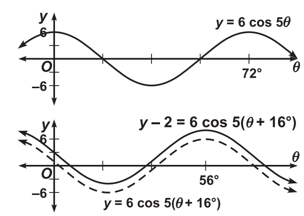
Example 2 **Graph one cycle of $y = 6\cos(5\theta + 80°) + 2$.**

Step 1 Isolate the term involving the trigonometric function.
$y - 2 = 6\cos(5\theta + 80°)$

Step 2 Factor out the coefficient of θ.
$y - 2 = 6\cos 5(\theta + 16°)$

Step 3 Sketch $y = 6\cos 5\theta$.

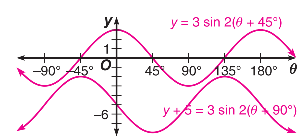
Step 4 Translate $y = 6\cos 5\theta$ to obtain the graph of $y - 2 = 6\cos 5(\theta + 16°)$.



Sketch these graphs on the same coordinate axes.

1. $y = 3\sin 2(\theta + 45°)$

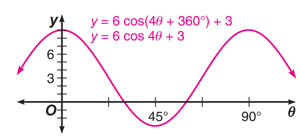
2. $y + 5 = 3\sin 2(\theta + 90°)$



Graph one cycle of each curve on the same coordinate axes.

3. $y = 6\cos(4\theta + 360°) + 3$

4. $y = 6\cos 4\theta + 3$



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6-6 Practice

Modeling Real-World Data with Sinusoidal Functions

1. ***Meteorology*** The average monthly temperatures in degrees Fahrenheit (°F) for Baltimore, Maryland, are given below.

Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
32°	35°	44°	53°	63°	73°	77°	76°	69°	57°	47°	37°

a. Find the amplitude of a sinusoidal function that models the monthly temperatures.
22.5°

b. Find the vertical shift of a sinusoidal function that models the monthly temperatures.
54.5°

c. What is the period of a sinusoidal function that models the monthly temperatures?
12 months

d. Write a sinusoidal function that models the monthly temperatures, using $t = 1$ to represent January.
Sample answer: $y = 22.5 \cos\left(\frac{\pi}{6}t + 2.62\right) + 54.5$

e. According to your model, what is the average temperature in July? How does this compare with the actual average?
Sample answer: 77°; the average temperature and the model are the same.

f. According to your model, what is the average temperature in December? How does this compare with the actual average?
Sample answer: 35°; the average temperature is 37°; the model is 2° less.

2. ***Boating*** A buoy, bobbing up and down in the water as waves move past it, moves from its highest point to its lowest point and back to its highest point every 10 seconds. The distance between its highest and lowest points is 3 feet.

a. What is the amplitude of a sinusoidal function that models the bobbing buoy? **1.5**

b. What is the period of a sinusoidal function that models the bobbing buoy? **10 s**

c. Write a sinusoidal function that models the bobbing buoy, using $t = 0$ at its highest point. **Sample answer:** $1.5 \cos\left(\frac{\pi}{5}t\right)$

d. According to your model, what is the height of the buoy at $t = 2$ seconds? **about 0.46 ft**

e. According to your model, what is the height of the buoy at $t = 6$ seconds? **about −1.21 ft**

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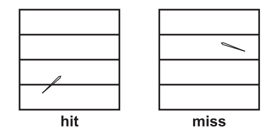
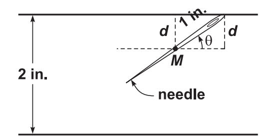
6-6 Enrichment

Approximating π

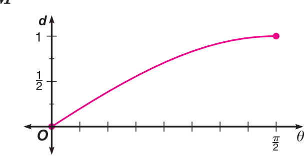
During the eighteenth century, the French scientist George de Buffon developed an experimental method for approximating π using probability. Buffon's method requires tossing a needle randomly onto an array of parallel and equidistant lines. If the needle intersects a line, it is a "hit." Otherwise, it is a "miss." The length of the needle must be less than or equal to the distance between the lines. For simplicity, we will demonstrate the method and its proof using a 2-inch needle and lines 2 inches apart.





1. Assume that the needle falls at an angle θ with the horizontal, and that the tip of the needle just touches a line. Find the distance d of the needle's midpoint M from the line.
sin θ

2. Graph the function that relates θ and d for $0 \le \theta \le \frac{\pi}{2}$.



3. Suppose that the needle lands at an angle θ but a distance less than d. Is the toss a hit or a miss? **a hit**

4. Shade the portion of the graph containing points that represent hits. **Students should shade the lower portion of the graph.**

5. The area A under the curve you have drawn between $x = a$ and $x = b$ is given by $A = \cos a - \cos b$. Find the area of the shaded region of your graph. **1**

6. Draw a rectangle around the graph in Exercise 2 for $d = 0$ to 1 and $\theta = 0$ to $\frac{\pi}{2}$. The area of the rectangle is $1 \times \frac{\pi}{2} = \frac{\pi}{2}$.

The probability P of a hit is the area of the set of all "hit" points divided by the area of the set of all possible landing points. Complete the final fraction:

$P = \frac{\text{hit points}}{\text{all points}} = \frac{\text{shaded area}}{\text{total area}} = \frac{1}{\frac{\pi}{2}}$. $\boldsymbol{P = \frac{2}{\pi}}$

7. Use the first and last expressions in the above equation to write π in terms of P. $\pi = \frac{2}{P}$

8. The Italian mathematician Lazzerini made 3408 needle tosses, scoring 2169 hits. Calculate Lazzerini's experimental value of π.
3.1424619

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6-7 Practice

Graphing Other Trigonometric Functions

Find each value by referring to the graphs of the trigonometric functions.

1. $\tan\left(-\frac{3\pi}{2}\right)$ **undefined**

2. $\cot\left(\frac{3\pi}{2}\right)$ **0**

3. $\sec 4\pi$ **1**

4. $\csc\left(-\frac{7\pi}{2}\right)$ **1**

Find the values of θ for which each equation is true.

5. $\tan\theta = 0$ **πn, where n is an integer**

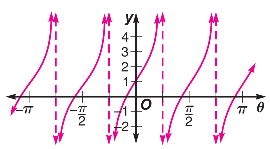
6. $\cot\theta = 0$ **$\frac{\pi}{2}n$, where n is an odd integer**

7. $\csc\theta = 1$ **$\frac{\pi}{2} + 2\pi n$, where n is an integer**

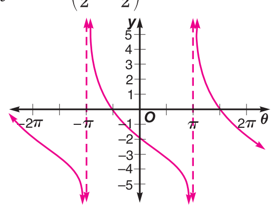
8. $\sec\theta = -1$ **πn, where n is an odd integer**

Graph each function.

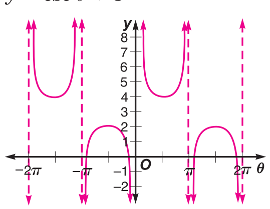
9. $y = \tan(2\theta + \pi) + 1$



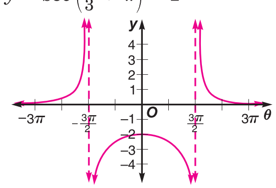
10. $y = \cot\left(\frac{\theta}{2} - \frac{\pi}{2}\right) - 2$



11. $y = \csc\theta + 3$



12. $y = \sec\left(\frac{\theta}{3} + \pi\right) - 1$



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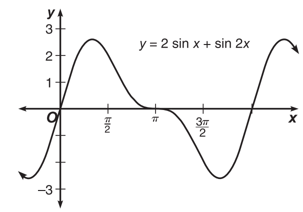
6-7 Enrichment

Reading Mathematics: Understanding Graphs

Technically, a graph is a set of points where pairs of points are connected by a set of segments and/or arcs. If the graph is the graph of an equation, the set of points consists of those points whose coordinates satisfy the equation.

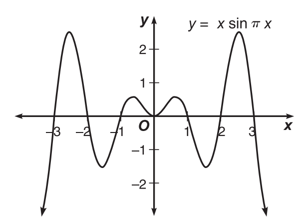
Practically speaking, to see a graph this way is as useless as seeing a word as a collection of letters. The full meaning of a graph and its value as a tool of understanding can be grasped only by viewing the graph as a whole. It is more useful to see a graph not just as a set of points, but as a picture of a function. The following suggestions, based on the idea of a graph as a picture, may help you reach a deeper understanding of the meaning of graphs.



a. ***Read the equation of the graph as a title.*** Get a sense of the behavior of the function by describing its characteristics to yourself in general terms. The graph shown depicts the function $y = 2\sin x + \sin 2x$. In the region shown, the function increases, decreases, then increases again. It looks a bit like a sine curve but with steeper sides, sharper peaks and valleys, and a point of inflection at $x = \pi$.

b. ***Focus on the details.*** View them not as isolated or unrelated facts but as traits of the function that distinguish it from other functions. Think of the graph as a point that moves through the coordinate plane sketching a profile of the function. Use the function to guess the behavior of the graph beyond the region shown. The graph of $y = 2\sin x + \sin 2x$ appears to exhibit point symmetry about the point of inflection $x = \pi$. It intersects the x-axis at 0, π, and 2π, and reaches a relative maximum of $y \approx 2.6$ at $x = \frac{\pi}{3}$ and a relative minimum of $y \approx -2.6$ at $x = \frac{5\pi}{3}$. Since the maximum value of $2\sin x$ is 2 and the maximum value of $\sin 2x$ is 1, $y = 2\sin x + \sin 2x$ will never exceed 3.

Discuss the graph at the right. Use the above discussion as a model. You should discuss the graph's shape, critical points, and symmetry.



Sample answer: The graph depicts $y = x\sin\pi x$. The region shown has the shape of a W with a dip in the center peak. It reaches relative minima of $y \approx -1.5$ at about $x = \pm1.5$ and of $y = 0$ at the origin. It reaches relative maxima of about 0.5 at about $x = \pm0.5$. The graph also shows relative maxima of about 2.5 at about $x = \pm2.5$. The graph appears to be symmetric about the y-axis.

6-8

NAME ______________ DATE ________ PERIOD _____

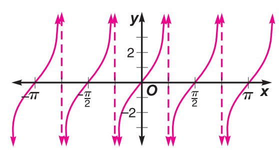
Practice

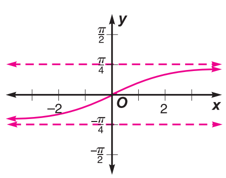
Trigonometric Inverses and Their Graphs

Write the equation for the inverse of each function. Then graph the function and its inverse.

1. $y = \tan 2x$

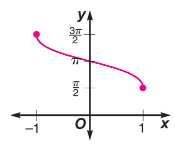
$y = \frac{1}{2}\tan^{-1} x$

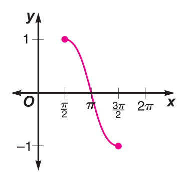




2. $y = \frac{\pi}{2} + \text{Arccos } x$

$y = \text{Cos}\left(x - \frac{\pi}{2}\right)$





Find each value.

3. Arccos (-1)
π

4. Arctan 1
$\frac{\pi}{4}$

5. Arcsin $\left(-\frac{1}{2}\right)$
$-\frac{\pi}{6}$

6. $\text{Sin}^{-1} \frac{\sqrt{3}}{2}$
$\frac{\pi}{3}$

7. $\text{Cos}^{-1}\left(\sin \frac{\pi}{3}\right)$
$\frac{\pi}{6}$

8. $\tan\left(\text{Sin}^{-1} 1 - \text{Cos}^{-1} \frac{1}{2}\right)$
$\frac{\sqrt{3}}{3}$

9. *Weather* The equation $y = 10 \sin\left(\frac{\pi}{6}t - \frac{2\pi}{3}\right) + 57$ models the average monthly temperatures for Napa, California. In this equation, t denotes the number of months with January represented by $t = 1$. During which two months is the average temperature 62°? **May and September**

6-8

NAME ______________ DATE ________ PERIOD _____

Enrichment

Algebraic Trigonometric Expressions

In Lesson 6-4, you learned how to use right triangles to find exact values of functions of inverse trigonometric functions. In calculus it is sometimes necessary to convert trigonometric expressions into algebraic ones. You can use the same method to do this.

Example **Write sin (arccos 4*x*) as an algebraic expression in *x*.**

Let $y = \arccos 4x$ and let z = side opposite $\angle y$.

$(4x)^2 + z^2 = 1^2$ *Pythagorean Theorem*

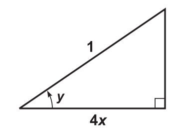
$16x^2 + z^2 = 1$

$z^2 = 1 - 16x^2$

$z = \sqrt{1 - 16x^2}$ *Take the square root of each side.*

$\sin y = \frac{z}{1}$ *Definition of sine*

$\sin y = \sqrt{1 - 16x^2}$



Therefore, $\sin(\arccos 4x) = \sqrt{1 - 16x^2}$.

Write each of the following as an algebraic expression in x .

1. $\cot(\arccos 4x)$

$\frac{4x}{\sqrt{1 - 16x^2}}$

2. $\sin(\arctan x)$

$\frac{x}{\sqrt{x^2 + 1}}$

3. $\cos\left(\arctan \frac{x}{3}\right)$

$\frac{3}{\sqrt{x^2 + 9}}$

4. $\sin[\text{arcsec}(x - 2)]$

$\frac{\sqrt{x^2 - 4x + 3}}{x - 2}$

5. $\cos\left(\arcsin \frac{x - h}{r}\right)$

$\frac{\sqrt{r^2 - x^2 + 2xh - h^2}}{r}$

Chapter 6 Answer Key

Form 1A

Page 249

1. A
2. B
3. D
4. C
5. C
6. A
7. B
8. D
9. B
10. B
11. A
12. D

Page 250

13. B
14. C
15. B
16. D
17. C
18. A
19. D
20. A

Bonus: D

Form 1B

Page 251

1. D
2. B
3. B
4. A
5. D
6. D
7. A
8. D
9. C
10. A
11. B

Page 252

12. C
13. D
14. C
15. D
16. C
17. A
18. C
19. B
20. C

Bonus: C

Chapter 6 Answer Key

Form 1C

Page 253

1. C
2. B
3. C
4. A
5. B
6. C
7. A
8. D
9. B
10. A
11. D

Page 254

12. B
13. C
14. B
15. C
16. A
17. A
18. D
19. B
20. C

Bonus: D

Form 2A

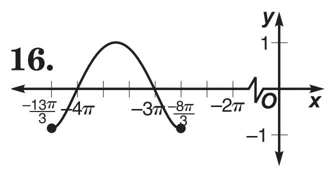
Page 255

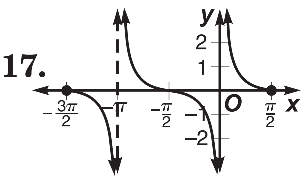
1. $-\frac{26\pi}{15}$
2. $-690°$
3. 18.2 radians/s
4. 286.5 m/min
5. Sample answer: 3958.4 cm/min
6. 0.2 m
7. 239.3 m^2
8. $y = \pm\frac{2}{3}\cos\left(\frac{\pi x}{0.9} + 5.78\pi\right) + 3.9$
9. $y = \cot\left(\frac{2x}{3} - \pi\right) - \frac{4}{3}$
10. $1; \frac{2\pi}{3}; \frac{\pi}{15}; 2$
11. $\frac{\pi}{4}; -\frac{\pi}{4}; -2$
12. $y = \frac{4}{3}\left(\frac{2}{3} + \text{Cot } \frac{x}{4}\right)$
13. 1

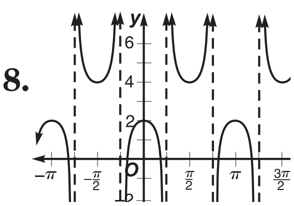
Page 256

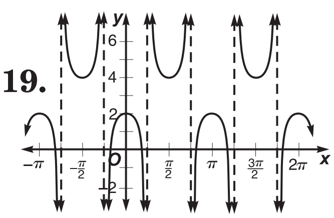
14. Sample answer: $y = 24.25 \sin\left(\frac{\pi}{6}t - \frac{2\pi}{3}\right) + 49.55$

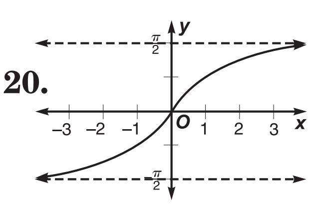
15. Sample answer: 49.55°

16. 

17. 

18. 

19. 

20. 

Bonus: $\frac{12}{13}$

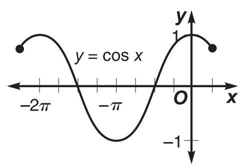
Chapter 6 Answer Key

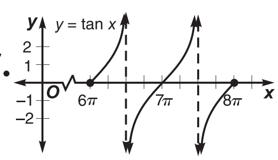
Form 2B

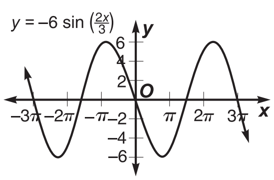
Page 257

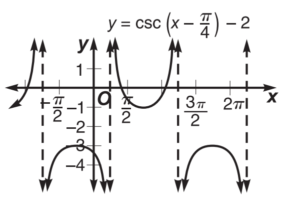
1. $\frac{13\pi}{4}$
2. 660°
3. 9.8 radians/s
4. 527.8 m/min
5. 0.7 cm/min
6. about 36.3 cm
7. about 1274 cm
8. $y = \pm 4\cos\left(\frac{\pi x}{3} + \frac{\pi^2}{3}\right) - 5$
9. $y = \tan\left(\frac{x}{2} - \frac{\pi}{8}\right) - 1$
10. $4;\ 4\pi;\ -\pi;\ 1$
11. $4\pi;\ -2\pi;\ 3$
12. $y = \text{Tan}\ x - 3$
13. $\frac{\sqrt{3}}{2}$

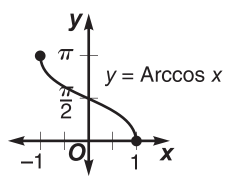
Page 258

14. Sample answer: $y = 26.9\sin\left(\frac{\pi}{6}t - \frac{2\pi}{3}\right) + 47.9$
15. Sample answer: 61.4°
16. 

17. 

18. 

19. 

20. 

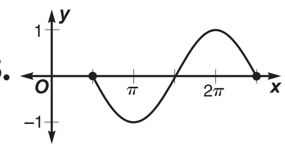
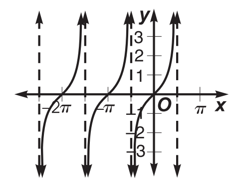
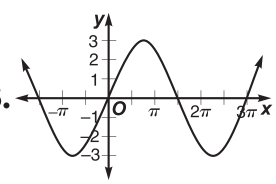
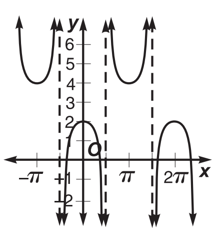
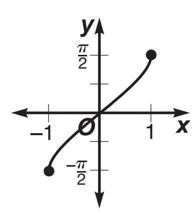
Bonus: $\frac{\sqrt{34}}{3}$

Form 2C

Page 259

1. $\frac{9\pi}{4}$
2. 75°
3. 110.6 radians/s
4. 219.9 ft/s
5. 1.05 in./s or 62.8 in./min
6. about 3.1 in.
7. about 42.4 m^2
8. $y = \pm 2\sin\left(\frac{2x}{5} + \frac{\pi}{5}\right) + 3$
9. $y = \tan\left(\frac{x}{3} + \frac{\pi}{3}\right) + 2$
10. $\frac{3}{2};\ \pi;\ \frac{\pi}{8};\ 0$
11. $\frac{\pi}{3};\ \frac{\pi}{3};\ -2$
12. $y = \text{Cos}\ x + 5$
13. 1

Page 260

14. Sample answer: $y = 20.8\sin\left(\frac{\pi}{6}t - \frac{2\pi}{3}\right) + 47.9$
15. Sample answer: 37.5°
16. 
17. 
18. 
19. 
20. 

Bonus: $\frac{\sqrt{5}}{2}$

Chapter 6 Answer Key

CHAPTER 6 SCORING RUBRIC

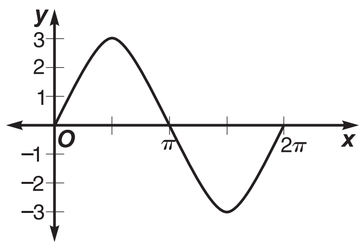
Level	Specific Criteria
3 Superior	• Shows thorough understanding of the concepts *amplitude, period,* and *phase shift of a graph.* • Uses appropriate strategies to model motion of point on wheel. • Computations are correct. • Written explanations are exemplary. • Example and analysis of rotating object are appropriate and make sense. • Graphs are accurate and appropriate. • Goes beyond requirements of some or all problems.
2 Satisfactory, with Minor Flaws	• Shows understanding of the concepts *amplitude, period,* and *phase shift of a graph.* • Uses appropriate strategies to model motion of point on wheel. • Computations are mostly correct. • Written explanations are effective. • Example and analysis of rotating object are appropriate and make sense. • Graphs are mostly accurate and appropriate. • Satisfies all requirements of problems.
1 Nearly Satisfactory, with Serious Flaws	• Shows understanding of most of the concepts *amplitude, period,* and *phase shift of a graph.* • May not use appropriate strategies to model motion of point on wheel. • Computations are mostly correct. • Written explanations are satisfactory. • Example and analysis of rotating object are mostly appropriate and sensible. • Graphs are mostly accurate and appropriate. • Satisfies most requirements of problems.
0 Unsatisfactory	• Shows little or no understanding of the concepts *amplitude, period,* and *phase shift of a graph.* • May not use appropriate strategies to model motion of point on wheel. • Computations are incorrect. • Written explanations are not satisfactory. • Example and analysis of rotating object are not appropriate and sensible. • Graphs are not accurate and appropriate. • Does not satisfy requirements of problems.

Chapter 6 Answer Key

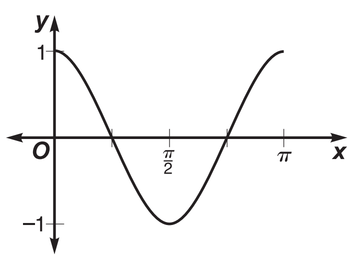
Open-Ended Assessment

Page 261

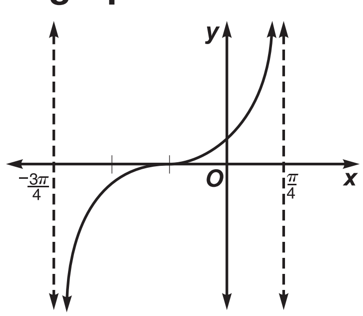
1a. **An amplitude of 3 for a sine function means that the *y* values of the function vary between +3 and −3, as shown in the graph.**



1b. **A period for π for a cosine function means it takes π units along the *x*-axis for the function to complete one cycle, as shown in the graph.**

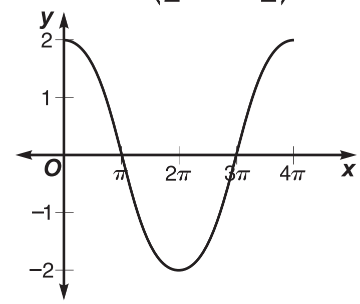


1c. **A phase shift of $-\frac{\pi}{4}$ for a tangent function means the graph has shifted $\frac{\pi}{4}$ units to the left, as shown in the graph.**



1d. **Sample answer: amplitude 2, period 4π, and phase shift $-\pi$**

$$y = 2 \sin\left(\frac{1}{2}x + \frac{\pi}{2}\right)$$



2a. $y = -10 \sin \frac{\pi}{10}t$

$\frac{x}{2\pi} = \frac{1}{20}, x = \frac{\pi}{10}$ **rev/s**

2b. **The amplitude would change from 10 to 5.** $y = -5 \sin \frac{\pi}{10}t$

2c. **The period would change from $\frac{\pi}{10}$ to $\frac{\pi}{5}$.**

$y = -10 \sin \frac{\pi}{5}t$

$\frac{x}{2\pi} = \frac{1}{10}, x = \frac{\pi}{5}$ **rev/s**

2d. **Sample answer: The function would change from sine to cosine.**

$y = 10 \cos \frac{\pi}{10}t$

3. **Earth is a rotating object. It rotates once every 24 hours. Since 24 hours = 86,400 seconds, the angular velocity is**

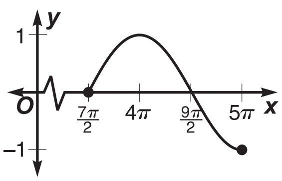
$\omega = \frac{\theta}{t} = \frac{2\pi}{86400} = 0.000277$ **radian/s.**

The radius of Earth is about 6400 kilometers. The linear velocity of a point at the equator is

$v = \frac{r\theta}{t} = \frac{6400(2\pi)}{86,400} \approx 0.465$ **km/s.**

Chapter 6 Answer Key

Mid-Chapter Test
Page 262

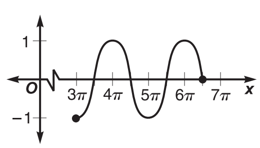
1. $-\frac{7\pi}{30}$
2. 48°
3. 8.0 cm
4. 6.2 m^2
5. $\frac{8\pi}{3}$ radians/s; 50.3 cm/s
6. 

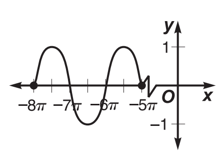
7. neither; the maximum value is 2, not 1
8. $\frac{8}{3}$; $\frac{5\pi}{3}$
9. 2.3; $\frac{12\pi}{5}$
10. $y = \pm 5 \sin \frac{12\theta}{5}$

Quiz A
Page 263

1. $\frac{35\pi}{9}$
2. −15°
3. 84.3 yd^2
4. 44.8 radians/min
5. 1.75 cm/s

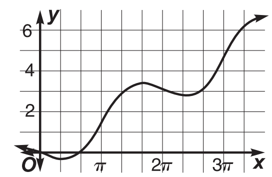
Quiz B
Page 263

1. 

2. 

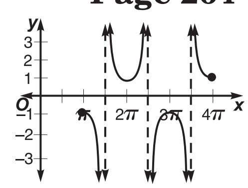
3. $\frac{3}{2}$; $\frac{2\pi}{5}$
4. 0.7; $\frac{4\pi}{3}$
5. $y = \pm \frac{8}{5} \sin \frac{6\theta}{5}$

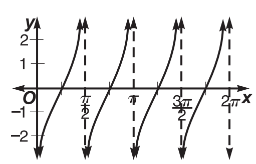
Quiz C
Page 264

1. 6π; 0
2. $y = \pm 2.4 \cos\left(\frac{\pi\theta}{4.1} - \frac{\pi^2}{12.3}\right) + 0.2$
3. 

4. Sample answer: $y = 24.5 \sin\left(\frac{\pi}{6}t - \frac{2\pi}{3}\right) + 55.85$
5. Sample answer: 77.1°

Quiz D
Page 264

1. 

2. 

3. $y = \cot\left(3x + \frac{\pi}{4}\right) - 4$
4. $y = 2 \text{ Sin } x$
5. undefined

Chapter 6 Answer Key

SAT/ACT Practice

Page 265

1. E
2. D
3. A
4. E
5. E
6. B
7. D
8. C
9. C
10. D

Page 266

11. A
12. B
13. B
14. E
15. E
16. C
17. A
18. C
19. 15
20. 65

Cumulative Review

Page 267

1. $x \neq 8$
2. infeasible
3. (−6, 9), (3, 21), (12, −9)
4. (2, 3), (−1, 7), (−4, −3)
5. (3, 2), (7, −1), (−3, −4)
6. 5
7. $-\frac{4}{3} < x < \frac{16}{3}$
8. 1070 cm^2
9. 142.5 in^2
10. $h = -6 \cos \frac{2\pi}{15} t$

Answer Key

Semester Test
Chapters 1–6

Page 269

1. B
2. C
3. C
4. C
5. D
6. B
7. B
8. C

Page 270

9. A
10. B
11. B
12. C
13. D
14. B
15. D
16. C
17. A

Page 271

18. C
19. C
20. B
21. A
22. C
23. B
24. C
25. B

Answer Key

Page 272

26. $3, \pi, \frac{\pi}{2}$

27. $\frac{\pi}{3}$

28. (4, 1)

29. $\frac{2\sqrt{2}}{3}$

30. $-\frac{5}{3}$

31. 6

32. $-\frac{2}{5}, \frac{16}{5}$

33. $f(x) = x + 1$

34. inconsistent

35. 45°, 135°

36. $x^2 + 2x + 2$

37. $y = 2x + 5$

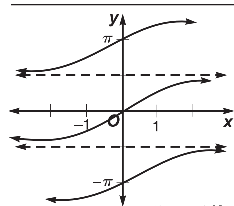
38. $\frac{-2 \pm \sqrt{2}}{2}$

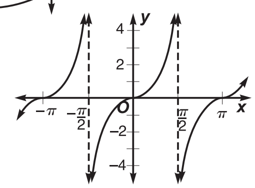
39. translated $\frac{\pi}{4}$ unit to the right

Page 273

40. $\begin{bmatrix} 7 & 9 \\ 6 & -19 \end{bmatrix}$

41. $y = \tan x$





42. 10.472 cm

43. infinite discontinuity

44. $1, \frac{1}{2}, \frac{1}{3}$

45. $3, \pi$

46. $y = \pm 2 \sin 2\theta + 1$

47. 4

48. $\begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}$

49. maximum, (1, 1)

50. c = 26.0, A = 35.0°, B = 50.0°

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