MATH 1610/MATH 1552
5.1 Random Variables

DEFINITIONS

Random variable – a numerical description of the outcomes of an experiment

Discrete random variable – a random variable that may assume either a finite number of values or an infinite sequence of values such as 0, 1, 2, 3, …

Continuous random variable – a random variable that may assume any numerical value in an interval or a collection of intervals.

Examples of DISCRETE Random Variables

Experiment	Random Variable (x)	Possible Values for the Random Variable
Contact 5 customers	Number of customers who place an order	0, 1, 2, 3, 4, 5
Operate a restaurant for 1 day	Number of customers	0, 1, 2, 3, 4, …
Sell an automobile	Gender of the customer	0 if male, 1 if female

Examples of CONTINUOUS Random Variables

Experiment	Random Variable (x)	Possible Values for the Random Variable
Operate a bank	Time between customer arrivals in minutes	$x \geq 0$
Fill a soft drink cup (max of 12.1 ounces)	Number of ounces	$0 \leq x \leq 12.1$
Construct a new library	Percentage of project complete after 6 months	$0 \leq x \leq 100$

Example 1

Define a random variable, and determine if each situation is discrete or continuous.

Question	Random Variable	Type
Size of a class		
Distance from home to school		
Rolling 2 dice		

Example 2

Consider the experiment of tossing a coin three times.

a. Define a random variable that represents the number of tails occurring on the three tosses.

b. List the experimental outcomes and show what value the random variable would assume for each of the experimental outcomes.

d. Is the random variable discrete or continuous?

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5.2 Developing Discrete Probability Distributions

DISCRETE Probability Distributions for a Random Variable

- describes how probabilities are distributed over the values of the random variable.
- can be described using a table, a graph or a formula

The ***probability distribution*** is defined by a probability function, $\boldsymbol{f(x)}$, that provides the probability for each value of the random variable.

Required conditions for a ***discrete probability function***:

$$f(x) \geq 0$$
$$\sum f(x) = 1$$

Two Types of Discrete Probability Distributions

1. Uses the rules of assigning probabilities to experimental outcomes to determine probabilities for each value to the random variable.

2. Uses a special mathematical formula to compute the probabilities for each value of the random variable.

Three methods for assigning probabilities to random variables (CH 4):

1. Classical Method

2. Subjective Method

3. Relative Frequency Method – leads to an empirical discrete distribution

Example 1
Let x = the number obtained on one roll of a die, then f(x) = the probability of x.

Complete the probability distribution.

Number Rolled (x)	Probability of x: f(x)

Example 2

From the past there is an 60% chance that the Packers will beat the Lions in a game.
If they play the Lions twice in a season:

a. Find P(Packers win both games)

b. Find P(Packers win exactly one game)

c. Make a probability distribution where x is the number of wins for the Packers.

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5.3 Expected Value and Variance

Expected Value

- $E(x) = \mu = \sum x\, f(x)$
- the mean of a random variable measuring its central location.
- A ***weighted average*** of the values the random variable may assume. The weights are the probabilities.
- the value does not have to be a value the random variable can assume.

Variance (σ^2) and Standard Deviation (σ)

- The ***variance*** summarizes the variability in the values of a random variable
- $Var(x) = \sigma^2 = \sum (x-\mu)^2 f(x)$
- The ***variance*** is a ***weighted average*** of the squared deviations of a random variable from its mean. The weights are the probabilities.

- The ***standard deviation***, σ, is defined as the positive square root of the variance

Example 1

Let x = the number of automobiles sold during a day

Automobiles Sold (x)	f(x)
0	0.18
1	0.39
2	0.24
3	0.14
4	0.04
5	0.01

a. What is the probability that 4 automobiles will be sold on any given day?

b. What is the most probable number of cars sold on any given day?

c. What is the probability of selling 3 or more cars on any given day?

d. Calculate the expected value for the number of automobiles sold during a day.

Automobiles Sold (x)	f(x)	$x\,f(x)$
0	0.18	
1	0.39	
2	0.24	
3	0.14	
4	0.04	
5	0.01	
Expected Value: $E(x) = \mu = \sum x\,f(x)$		

e. Interpret the ***expected value*** in terms of the given problem.

f. Assuming a 30 day month, forecast the average monthly sales.

g. Calculate the variance.

x	$x-\mu$	$(x-\mu)^2$	$f(x)$	$(x-\mu)^2 f(x)$
0				
1				
2				
3				
4				
5				
Variance	$Var(x) = \sigma^2 = \sum(x-\mu)^2 f(x)$			

h. Determine the standard deviation.

Example 2

The two largest cable providers are Comcast Cable Communications with 21. 5 million subscribers and Time Warner Cable with 11.0 million subscribers. Suppose that the management of Time Warner Cable subjectively assesses the probability distribution for the number of new subscribers next year in the state of New York as follows:

x	f(x)
100,000	0.10
200,000	0.20
300,000	0.25
400,000	0.30
500,000	0.10
600,000	0.05

Calculate the ***expected value*** and ***standard deviation***.

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1. Let X = the number of boys in a family of 5 children. The probability distribution of X is given in the table below. Find the expected value of the number of boys in a family of five.

X	f(x)
0	0.03125
1	0.15625
2	0.31250
3	0.31250
4	0.15625
5	0.03125

2. A computer monitor is comprised of multiple points of lights called pixels. It is not uncommon for a few of these pixels to be defective.

Let X = the number of defective pixels on a randomly chosen monitor.
The probability of x is as follows:

X	f(x)
0	0.2
1	0.5
2	0.2
3	0.1

a. Determine the expected value for the number of defective pixels.
b. Compute the variance and the standard deviation of the number of defective pixels.

3. A mineral economist estimated that a particular mining venture had a 40% probability of a $30 million loss, a 50% probability of a $20 million profit, and a 10% probability of a $40 million profit.

a. Create a probability distribution for the profit (X).
b. Determine the value of the expected mean.
c. Compute the variance and the standard deviation.

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5.4 Binomial Probability Distribution

Properties of a Binomial Experiment

1. The experiment consists of a sequence of ***n*** identical trials.
2. Two outcomes are possible: ***success or failure***
3. The probability of success or failure does not change from trial to trial
   success: ***p***
   failure: ***1 – p***
4. The trials are independent.

Example 1A Suppose a coin is flipped 5 times.

Random variable: let x = the number of heads

a. Determine the value of n.

b. Determine the values of x.

c. Determine the probability of success.

d. Determine the probability of failure.

Binomial Probability Formula

In ***n*** trials, ***x*** is a discrete random variable representing the number of successes where x = 0, 1, 2, 3, …, n.

$$\boldsymbol{f(x) = \binom{n}{x}\, p^x\, (1-p)^{(n-x)}}$$

$$\boldsymbol{f(x) = \left(\frac{n!}{x!\,(n-x)!}\right) p^x\, (1-p)^{(n-x)}}$$

Consider all of the ways of flipping a coin 5 times.

Coin outcome	1	2	3	4	5	
1	h	h	h	h	h	(5 h)
2	h	h	h	h	t	(4 h)
3	h	h	h	t	h	
4	h	h	t	h	h	
5	h	t	h	h	h	
6	t	h	h	h	h	
7	h	h	h	t	t	(3 h)
8	h	h	t	h	t	
9	h	t	h	h	t	
10	t	h	h	h	t	
11	h	h	t	t	h	
12	h	t	h	t	h	
13	t	h	h	t	h	
14	h	t	t	h	h	
15	t	h	t	h	h	
16	t	t	h	h	h	
17	t	t	t	h	h	(2 h)
18	t	t	h	t	h	
19	t	h	t	t	h	
20	h	t	t	t	h	
21	t	t	h	h	t	
22	t	h	t	h	t	
23	h	t	t	h	t	
24	t	h	h	t	t	
25	h	t	h	t	t	
26	h	h	t	t	t	
27	t	t	t	t	h	(1 h)
28	t	t	t	h	t	
29	t	t	h	t	t	
30	t	h	t	t	t	
31	h	t	t	t	t	
32	t	t	t	t	t	(0 h)

Example 1B Determine each probability using:

i. the list of outcomes

ii. the binomial probability function

a. Probability of 5 heads.

b. Probability of 2 heads.

c. Probability of 1 head.

Tables of binomial probabilities provide the probability of ***x*** successes in ***n*** trials for a binomial experiment.

Tables can be quicker and easier to use than the formula.

Table 1 Binomial distribution — probability function

	x	0.01	0.05	0.10	0.15	0.20	*p* 0.25	0.30	0.35	0.40	0.45	0.50	
n=1	0	.9900	.9500	.9000	.8500	.8000	.7500	.7000	.6500	.6000	.5500	.5000	1
	1	.0100	.0500	.1000	.1500	.2000	.2500	.3000	.3500	.4000	.4500	.5000	0
n=2	0	.9801	.9025	.8100	.7225	.6400	.5625	.4900	.4225	.3600	.3025	.2500	2
	1	.0198	.0950	.1800	.2550	.3200	.3750	.4200	.4550	.4800	.4950	.5000	1
	2	.0001	.0025	.0100	.0225	.0400	.0625	.0900	.1225	.1600	.2025	.2500	0
n=3	0	.9703	.8574	.7290	.6141	.5120	.4219	.3430	.2746	.2160	.1664	.1250	3
	1	.0294	.1354	.2430	.3251	.3840	.4219	.4410	.4436	.4320	.4084	.3750	2
	2	.0003	.0071	.0270	.0574	.0960	.1406	.1890	.2389	.2880	.3341	.3750	1
	3		.0001	.0010	.0034	.0080	.0156	.0270	.0429	.0640	.0911	.1250	0
n=4	0	.9606	.8145	.6561	.5220	.4096	.3164	.2401	.1785	.1296	.0915	.0625	4
	1	.0388	.1715	.2916	.3685	.4096	.4219	.4116	.3845	.3456	.2995	.2500	3
	2	.0006	.0135	.0486	.0975	.1536	.2109	.2646	.3105	.3456	.3675	.3750	2
	3		.0005	.0036	.0115	.0256	.0469	.0756	.1115	.1536	.2005	.2500	1
	4			.0001	.0005	.0016	.0039	.0081	.0150	.0256	.0410	.0625	0
n=5	0	.9510	.7738	.5905	.4437	.3277	.2373	.1681	.1160	.0778	.0503	.0313	5
	1	.0480	.2036	.3281	.3915	.4096	.3955	.3602	.3124	.2592	.2059	.1563	4
	2	.0010	.0214	.0729	.1382	.2048	.2637	.3087	.3364	.3456	.3369	.3125	3
	3		.0011	.0081	.0244	.0512	.0879	.1323	.1811	.2304	.2757	.3125	2
	4			.0005	.0022	.0064	.0146	.0284	.0488	.0768	.1128	.1563	1
	5				.0001	.0003	.0010	.0024	.0053	.0102	.0185	.0313	0

<u>Example 1C</u> Consider flipping a coin 5 times. Determine each probability using the given table.

Recall each value: n = p = q =

a. Probability of 5 heads.

b. Probability of 2 heads.

c. Probability of 1 head.

<u>Example 2</u> At a department store, the store manager estimates that there is a 30% probability that any one of the next three customers will make a purchase.
What is the probability that two of the next three customers will make a purchase?

a. Make a tree diagram showing the possible purchasing outcomes of the 3 customers.

b. Use the tree diagram to determine the probability that 2 of the 3 customers make a purchase.

c. Use the binomial probability formula to determine the probability that two of the three customers will make a purchase.

d. Use the table on the previous page to determine the probability that two of the three customers will make a purchase.

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5.4 Worksheet

<u>Example 1</u> At a department store, the store manager estimates that there is a 40% probability that any one of the next nine customers will make a purchase.
What is the probability that four of the next nine customers will make a purchase?

a. Determine the values of n, x, p, and 1-p. Use the binomial probability formula to determine the probability that four of the nine customers will make a purchase.

b. Use the table to verify the probability that four of the nine customers will make a purchase.

	x	0.01	0.05	0.10	0.15	0.20	*p* 0.25	0.30	0.35	0.40	0.45	0.50	
n=9	0	0.9135	0.6302	0.3874	0.2316	0.1342	0.0751	0.0404	0.0207	0.0101	0.0046	0.0020	9
	1	0.0830	0.2985	0.3874	0.3679	0.3020	0.2253	0.1556	0.1004	0.0605	0.0339	0.0176	8
	2	0.0034	0.0629	0.1722	0.2597	0.3020	0.3003	0.2668	0.2162	0.1612	0.1110	0.0703	7
	3	0.0001	0.0077	0.0446	0.1069	0.1762	0.2336	0.2668	0.2716	0.2508	0.2119	0.1641	6
	4		0.0006	0.0074	0.0283	0.0661	0.1168	0.1715	0.2194	0.2508	0.2600	0.2461	5
	5			0.0008	0.0050	0.0165	0.0389	0.0735	0.1181	0.1672	0.2128	0.2461	4
	6			0.0001	0.0006	0.0028	0.0087	0.0210	0.0424	0.0743	0.1160	0.1641	3
	7					0.0003	0.0012	0.0039	0.0098	0.0212	0.0407	0.0703	2
	8						0.0001	0.0004	0.0013	0.0035	0.0083	0.0176	1
	9								0.0001	0.0003	0.0008	0.0020	0

c. Use the binomial table to determine the probability that **<u>at least seven</u>** of the nine customers will make a purchase.

Example 2 Suppose a salesperson visits 10 randomly selected families. From prior experience, the salesperson knows that there is a 25% chance that a randomly selected family will purchase an insurance policy. What is the probability that six families purchase an insurance policy?

Random Variable: let x = the family purchases an insurance policy

a. Determine the values of n, x, p, and 1-p. Use the binomial probability formula to determine the probability that six of the ten families will make a purchase.

b. Use the table to verify the probability that six of the ten customers will make a purchase.

							p						
	x	0.01	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	
n=10	0	0.9044	0.5987	0.3487	0.1969	0.1074	0.0563	0.0282	0.0135	0.0060	0.0025	0.0010	10
	1	0.0914	0.3151	0.3874	0.3474	0.2684	0.1877	0.1211	0.0725	0.0403	0.0207	0.0098	9
	2	0.0042	0.0746	0.1937	0.2759	0.3020	0.2816	0.2335	0.1757	0.1209	0.0763	0.0439	8
	3	0.0001	0.0105	0.0574	0.1298	0.2013	0.2503	0.2668	0.2522	0.2150	0.1665	0.1172	7
	4		0.0010	0.0112	0.0401	0.0881	0.1460	0.2001	0.2377	0.2508	0.2384	0.2051	6
	5		0.0001	0.0015	0.0085	0.0264	0.0584	0.1029	0.1536	0.2007	0.2340	0.2461	5
	6			0.0001	0.0012	0.0055	0.0162	0.0368	0.0689	0.1115	0.1596	0.2051	4
	7				0.0001	0.0008	0.0031	0.0090	0.0212	0.0425	0.0746	0.1172	3
	8					0.0001	0.0004	0.0014	0.0043	0.0106	0.0229	0.0439	2
	9							0.0001	0.0005	0.0016	0.0042	0.0098	1
	10									0.0001	0.0003	0.0010	0

c. Use the binomial table to determine the probability that two or four or eight families will purchase an insurance policy.