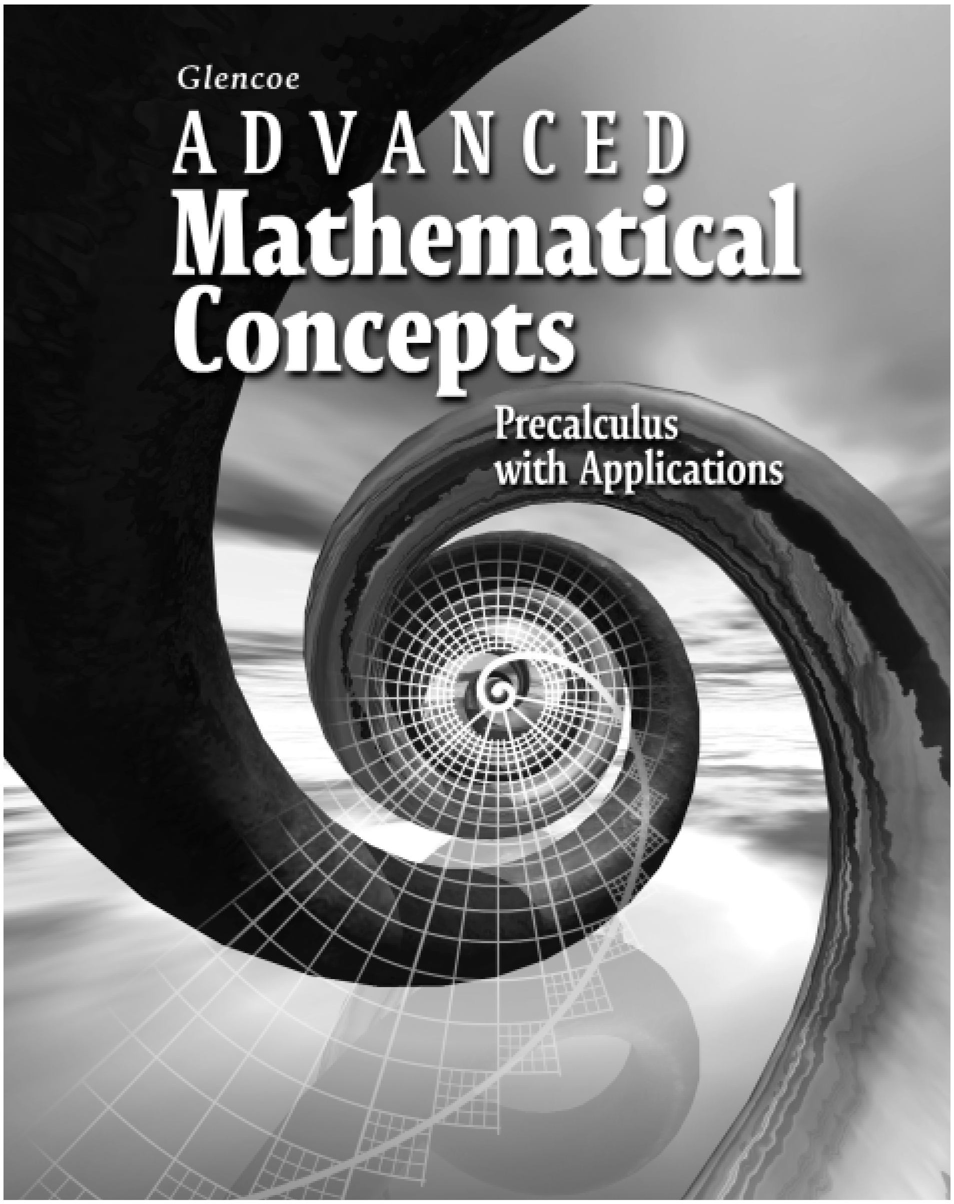
Chapter 3
Resource Masters



Glencoe

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StudentWorks™ This CD-ROM includes the entire Student Edition along with the Study Guide, Practice, and Enrichment masters.

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Advanced Mathematical Concepts
Chapter 3 Resource Masters

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Contents

A Teacher's Guide to Using the Chapter 3 Resource Masters

The ***Fast File*** Chapter Resource system allows you to conveniently file the resources you use most often. The *Chapter 3 Resource Masters* include the core materials needed for Chapter 3. These materials include worksheets, extensions, and assessment options. The answers for these pages appear at the back of this booklet.

All of the materials found in this booklet are included for viewing and printing in the *Advanced Mathematical Concepts TeacherWorks* CD-ROM.

Vocabulary Builder Pages vii-x include a student study tool that presents the key vocabulary terms from the chapter. Students are to record definitions and/or examples for each term. You may suggest that students highlight or star the terms with which they are not familiar.

When to Use Give these pages to students before beginning Lesson 3-1. Remind them to add definitions and examples as they complete each lesson.

Study Guide There is one Study Guide master for each lesson.

When to Use Use these masters as reteaching activities for students who need additional reinforcement. These pages can also be used in conjunction with the Student Edition as an instructional tool for those students who have been absent.

Practice There is one master for each lesson. These problems more closely follow the structure of the Practice section of the Student Edition exercises. These exercises are of average difficulty.

When to Use These provide additional practice options or may be used as homework for second day teaching of the lesson.

Enrichment There is one master for each lesson. These activities may extend the concepts in the lesson, offer a historical or multicultural look at the concepts, or widen students' perspectives on the mathematics they are learning. These are not written exclusively for honors students, but are accessible for use with all levels of students.

When to Use These may be used as extra credit, short-term projects, or as activities for days when class periods are shortened.

Assessment Options

The assessment section of the *Chapter 3 Resources Masters* offers a wide range of assessment tools for intermediate and final assessment. The following lists describe each assessment master and its intended use.

Chapter Assessments

Chapter Tests

- *Forms 1A, 1B, and 1C* Form 1 tests contain multiple-choice questions. Form 1A is intended for use with honors-level students, Form 1B is intended for use with average-level students, and Form 1C is intended for use with basic-level students. These tests are similar in format to offer comparable testing situations.
- *Forms 2A, 2B, and 2C* Form 2 tests are composed of free-response questions. Form 2A is intended for use with honors-level students, Form 2B is intended for use with average-level students, and Form 2C is intended for use with basic-level students. These tests are similar in format to offer comparable testing situations.

  All of the above tests include a challenging Bonus question.
- The **Extended Response Assessment** includes performance assessment tasks that are suitable for all students. A scoring rubric is included for evaluation guidelines. Sample answers are provided for assessment.

Intermediate Assessment

- A **Mid-Chapter Test** provides an option to assess the first half of the chapter. It is composed of free-response questions.
- Four free-response **quizzes** are included to offer assessment at appropriate intervals in the chapter.

Continuing Assessment

- The **SAT and ACT Practice** offers continuing review of concepts in various formats, which may appear on standardized tests that they may encounter. This practice includes multiple-choice, quantitative-comparison, and grid-in questions. Bubble-in and grid-in answer sections are provided on the master.
- The **Cumulative Review** provides students an opportunity to reinforce and retain skills as they proceed through their study of advanced mathematics. It can also be used as a test. The master includes free-response questions.

Answers

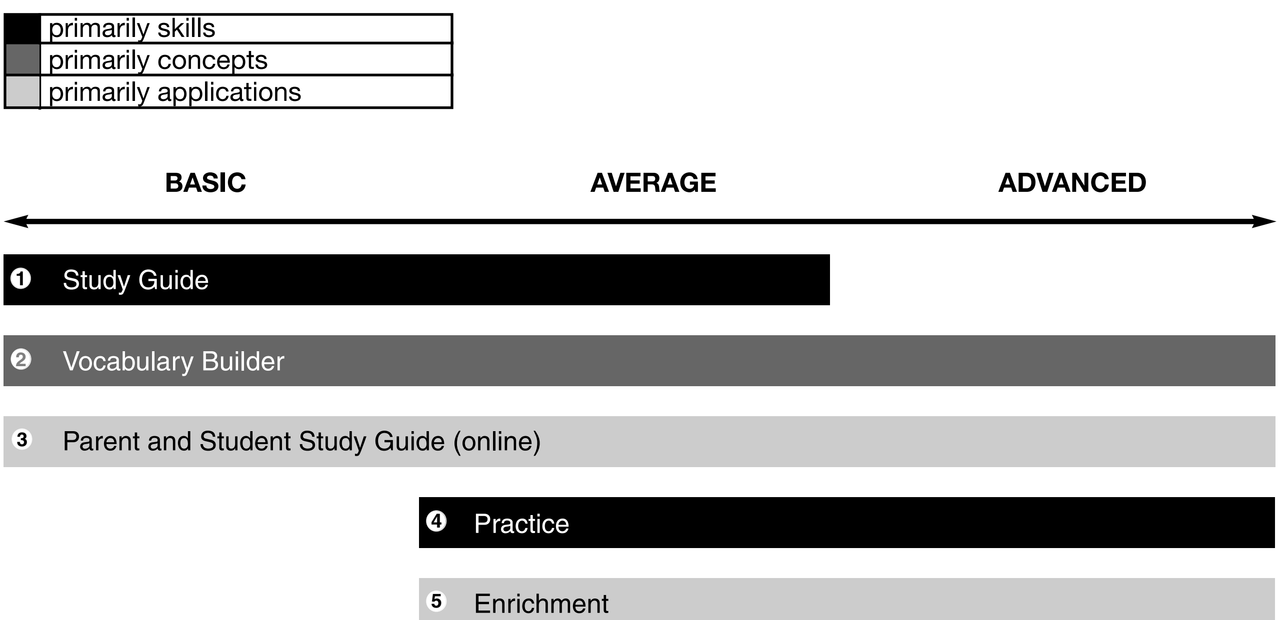
- Page A1 is an answer sheet for the SAT and ACT Practice questions that appear in the Student Edition on page 203. Page A2 is an answer sheet for the SAT and ACT Practice master. These improve students' familiarity with the answer formats they may encounter in test taking.
- The answers for the lesson-by-lesson masters are provided as reduced pages with answers appearing in red.
- Full-size answer keys are provided for the assessment options in this booklet.

Chapter 3 Leveled Worksheets

Glencoe's **leveled worksheets** are helpful for meeting the needs of every student in a variety of ways. These worksheets, many of which are found in the **FAST FILE Chapter Resource Masters**, are shown in the chart below.

- **Study Guide** masters provide worked-out examples as well as practice problems.
- Each chapter's **Vocabulary Builder** master provides students the opportunity to write out key concepts and definitions in their own words.
- **Practice** masters provide average-level problems for students who are moving at a regular pace.
- **Enrichment** masters offer students the opportunity to extend their learning.

Five Different Options to Meet the Needs of Every Student in a Variety of Ways



Chapter 3

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Reading to Learn Mathematics

Vocabulary Builder

This is an alphabetical list of the key vocabulary terms you will learn in Chapter 3. As you study the chapter, complete each term's definition or description. Remember to add the page number where you found the term.

Vocabulary Term	Found on Page	Definition/Description/Example
absolute maximum		
absolute minimum		
asymptotes		
constant function		
constant of variation		
continuous		
critical point		
decreasing function		
direct variation		
discontinuous		

(continued on the next page)

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Reading to Learn Mathematics

Vocabulary Builder *(continued)*

Vocabulary Term	Found on Page	Definition/Description/Example
end behavior		
even function		
everywhere discontinuous		
extremum		
horizontal asymptote		
horizontal line test		
image point		
increasing function		
infinite discontinuity		
inverse function		
inverse process		

(continued on the next page)

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Reading to Learn Mathematics

Vocabulary Builder *(continued)*

Vocabulary Term	Found on Page	Definition/Description/Example
inversely proportional		
inverse relations		
inverse variation		
jump discontinuity		
line symmetry		
maximum		
minimum		
monotonicity		
odd function		
parent graph		
point discontinuity		

(continued on the next page)

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Reading to Learn Mathematics

Vocabulary Builder *(continued)*

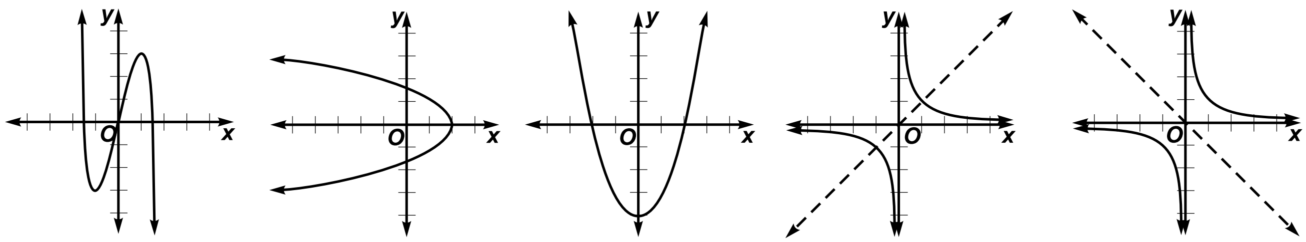
Vocabulary Term	Found on Page	Definition/Description/Example
point of inflection		
point symmetry		
rational function		
relative extremum		
relative maximum		
relative minimum		
slant asymptote		
symmetry with respect to the origin		
vertical asymptote		

Study Guide

Symmetry and Coordinate Graphs

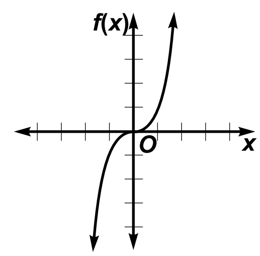
One type of symmetry a graph may have is **point symmetry.** A common point of symmetry is the origin. Another type of symmetry is **line symmetry.** Some common lines of symmetry are the x-axis, the y-axis, and the lines $y = x$ and $y = -x$.

point | **x-axis** | **y-axis** | **$y = x$** | **$y = -x$**



Example 1 **Determine whether $f(x) = x^3$ is symmetric with respect to the origin.**

If $f(-x) = -f(x)$, the graph has point symmetry.



Find $f(-x)$.	Find $-f(x)$.
$f(-x) = (-x)^3$	$-f(x) = -x^3$
$f(-x) = -x^3$	

The graph of $f(x) = x^3$ is symmetric with respect to the origin because $f(-x) = -f(x)$.

Example 2 **Determine whether the graph of $x^2 + 2 = y^2$ is symmetric with respect to the x-axis, the y-axis, the line $y = x$, the line $y = -x$, or none of these.**

Substituting (a, b) into the equation yields $a^2 + 2 = b^2$. Check to see if each test produces an equation equivalent to $a^2 + 2 = b^2$.

x-axis	$a^2 + 2 = (-b)^2$	*Substitute $(a, -b)$ into the equation.*
	$a^2 + 2 = b^2$	*Equivalent to $a^2 + 2 = b^2$*
y-axis	$(-a)^2 + 2 = b^2$	*Substitute $(-a, b)$ into the equation.*
	$a^2 + 2 = b^2$	*Equivalent to $a^2 + 2 = b^2$*
$y = x$	$(b)^2 + 2 = (a)^2$	*Substitute (b, a) into the equation.*
	$a^2 - 2 = b^2$	*Not equivalent to $a^2 + 2 = b^2$*
$y = -x$	$(-b)^2 + 2 = (-a)^2$	*Substitute $(-b, -a)$ into the equation.*
	$b^2 + 2 = a^2$	*Simplify.*
	$a^2 - 2 = b^2$	*Not equivalent to $a^2 + 2 = b^2$*

Therefore, the graph of $x^2 + 2 = y^2$ is symmetric with respect to the x-axis and the y-axis.

Practice

Symmetry and Coordinate Graphs

Determine whether the graph of each function is symmetric with respect to the origin.

1. $f(x) = \frac{-12}{x}$

2. $f(x) = x^5 - 2$

3. $f(x) = x^3 - 4x$

4. $f(x) = \frac{x^2}{3 - x}$

Determine whether the graph of each equation is symmetric with respect to the x-axis, the y-axis, the line y = x, the line y = −x, or none of these.

5. $x + y = 6$

6. $x^2 + y = 2$

7. $xy = 3$

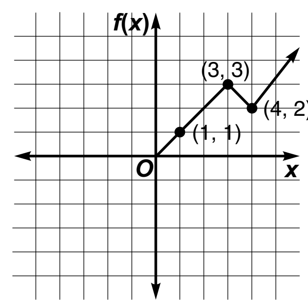
8. $x^3 + y^2 = 4$

9. $y = 4x$

10. $y = x^2 - 1$

11. Is $f(x) = |x|$ an even function, an odd function, or neither?

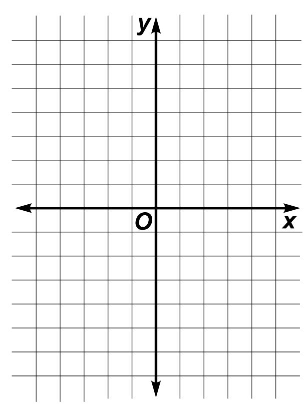
Refer to the graph at the right for Exercises 12 and 13.



12. Complete the graph so that it is the graph of an odd function.

13. Complete the graph so that it is the graph of an even function.

14. ***Geometry*** Cameron told her friend Juanita that the graph of $|y| = 6 - |3x|$ has the shape of a geometric figure. Determine whether the graph of $|y| = 6 - |3x|$ is symmetric with respect to the x-axis, the y-axis, both, or neither. Then make a sketch of the graph. Is Cameron correct?



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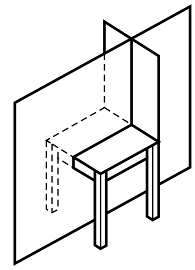
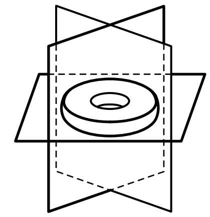
Enrichment

Symmetry in Three-Dimensional Figures

A solid figure that can be superimposed, point for point, on its mirror image has a *plane of symmetry*. A symmetrical solid object may have a finite or infinite number of planes of symmetry. The chair in the illustration at the right has just one plane of symmetry; the doughnut has infinitely many planes of symmetry, three of which are shown.





Determine the number of planes of symmetry for each object and describe the planes.

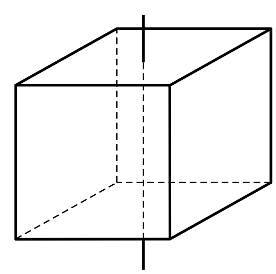
1. a brick

2. a tennis ball

3. a soup can

4. a square pyramid

5. a cube



Solid figures can also have *rotational symmetry*. For example, the axis drawn through the cube in the illustration is a fourfold axis of symmetry because the cube can be rotated about this axis into four different positions that are exactly alike.

6. How many four-fold axes of symmetry does a cube have? Use a die to help you locate them.

7. A cube has 6 two-fold axes of symmetry. In the space at the right, draw one of these axes.

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3-2

Study Guide

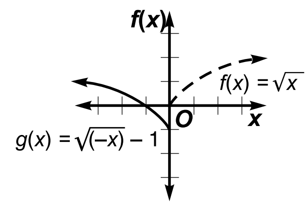
Families of Graphs

A **parent graph** is a basic graph that is transformed to create other members in a family of graphs. The transformed graph may appear in a different location, but it will resemble the parent graph.

A **reflection** flips a graph over a line called the *axis of symmetry*.
A **translation** moves a graph vertically or horizontally.
A **dilation** expands or compresses a graph vertically or horizontally.

Example 1 **Describe how the graphs of $f(x) = \sqrt{x}$ and $g(x) = \sqrt{-x} - 1$ are related.**

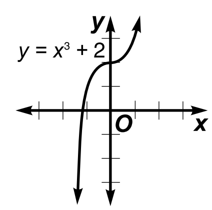
The graph of $g(x)$ is a reflection of the graph of $f(x)$ over the y-axis and then translated down 1 unit.



Example 2 **Use the graph of the given parent function to sketch the graph of each related function.**

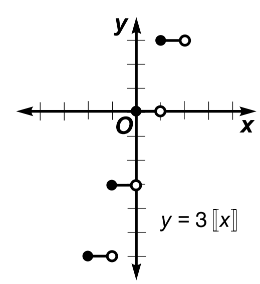
a. $f(x) = x^3$; $y = x^3 + 2$

When 2 is added to the parent function, the graph of the parent function moves up 2 units.



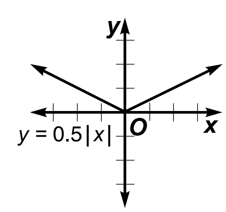
b. $f(x) = [\![x]\!]$; $y = 3[\![x]\!]$

The parent function is expanded vertically by a factor of 3, so the vertical distance between the steps is 3 units.



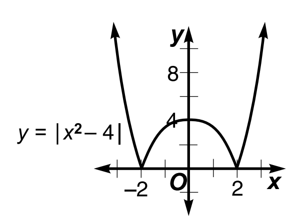
c. $f(x) = |x|$; $y = 0.5|x|$

When $|x|$ is multiplied by a constant greater than 0 but less than 1, the graph compresses vertically, in this case, by a factor of 0.5.



d. $f(x) = x^2$; $y = |x^2 - 4|$

The parent function is translated down 4 units and then any portion of the graph below the x-axis is reflected so that it is above the x-axis.



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3-2

Practice

Families of Graphs

Describe how the graphs of f(x) and g(x) are related.

1. $f(x) = x^2$ and $g(x) = (x + 3)^2 - 1$

2. $f(x) = |x|$ and $g(x) = -|2x|$

Use the graph of the given parent function to describe the graph of each related function.

3. $f(x) = x^3$

a. $y = 2x^3$

b. $y = -0.5(x - 2)^3$

c. $y = |(x + 1)^3|$

4. $f(x) = \sqrt{x}$

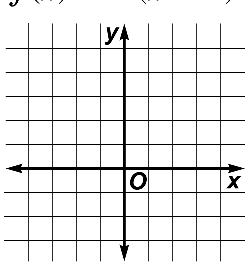
a. $y = \sqrt{x + 3} + 1$

b. $y = \sqrt{-x} - 2$

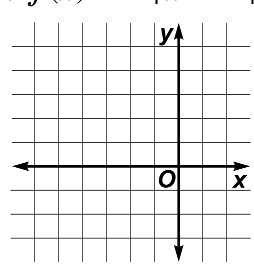
c. $y = \sqrt{0.25x} - 4$

Sketch the graph of each function.

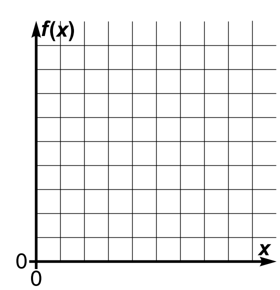
5. $f(x) = -(x - 1)^2 + 1$



6. $f(x) = 2|x + 2| - 3$



7. ***Consumer Costs*** During her free time, Jill baby-sits the neighborhood children. She charges \$4.50 for each whole hour or any fraction of an hour. Write and graph a function that shows the cost of x hours of baby-sitting.



3-2

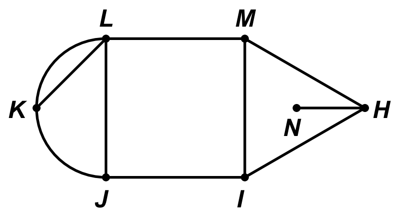
Enrichment

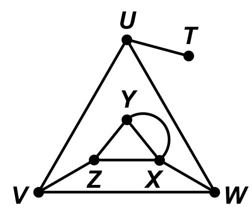
Isomorphic Graphs

A **graph** G is a collection of points in which a pair of points, called **vertices**, are connected by a set of segments or arcs, called **edges**. The **degree** of vertex C, denoted deg (C), is the number of edges connected to that vertex. We say two graphs are **isomorphic** if they have the same structure. The definition below will help you determine whether two graphs are isomorphic.

> A graph G' is isomorphic to a graph G if the following conditions hold.
> 1. G and G' have the same number of vertices and edges.
> 2. The degree of each vertex in G is the same as the degree of each corresponding vertex in G'.
> 3. If two vertices in G are joined by $k (k \geq 0)$ edges, then the two corresponding vertices in G' are also joined by k edges.

Example **In the graphs below *HIJKLMN* $\leftrightarrow$ *TUVWXYZ*. Determine whether the graphs are isomorphic.**





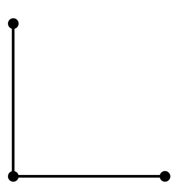
Number of vertices in G: 7
Number of edges in G: 10
deg (H): 3 deg (I): 3
deg (J): 3 deg (K): 3
deg (L): 4 deg (M): 3
deg (N): 1

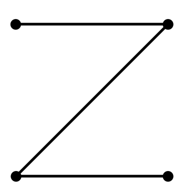
Number of vertices in G': 7
Number of edges in G': 10
deg (T): 1 deg (U): 3
deg (V): 3 deg (W): 3
deg (X): 4 deg (Y): 3
deg (Z): 3

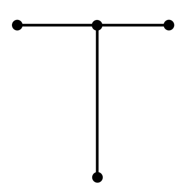
Since there are the same number of vertices and the same number of edges and there are five vertices of degree 3, one vertex of degree 4, and one vertex of degree 1 in both graphs, we can assume they are isomorphic.

Each graph in Row A is isomorphic to one graph in Row B. Match the graphs that are isomorphic.

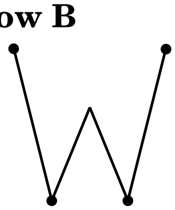
Row A

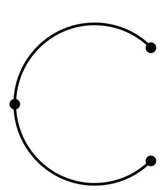
1.


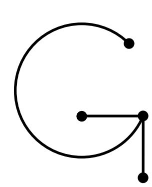
2.


3.


Row B

a.


b.


c.


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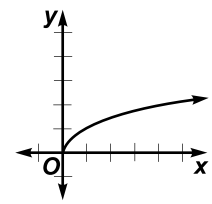
Study Guide

Graphs of Nonlinear Inequalities

Graphing an inequality in two variables identifies all ordered pairs that satisfy the inequality. The first step in graphing nonlinear inequalities is graphing the boundary.

Example 1 **Graph $y < \sqrt{x-3} + 2$.**

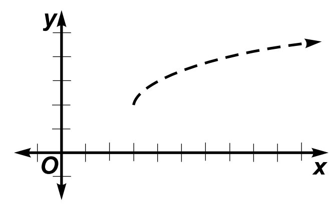


The boundary of the inequality is the graph of $y = \sqrt{x-3} + 2$. To graph the boundary curve, start with the parent graph $y = \sqrt{x}$. Analyze the boundary equation to determine how the boundary relates to the parent graph.

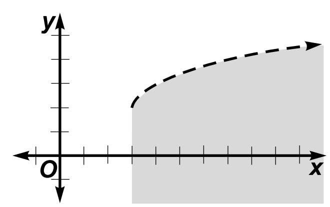
$$y = \sqrt{x - 3} + 2$$

$\uparrow$ *move 3 units right* $\quad$ $\uparrow$ *move 2 units up*

Since the boundary is not included in the inequality, the graph is drawn as a dashed curve.



The inequality states that the y-values of the solution are less than the y-values on the graph of $y = \sqrt{x-3} + 2$. Therefore, for a particular value of x, all of the points in the plane that lie below the curve have y-values less than $\sqrt{x-3} + 2$. This portion of the graph should be shaded.



To verify numerically, test a point not on the boundary.

$y < \sqrt{x-3} + 2$

$0 \overset{?}{<} \sqrt{4-3} + 2$ *Replace (x, y) with (4, 0).*

$0 < 3$ ✓ *True*

Since (4, 0) satisfies the inequality, the correct region is shaded.

Example 2 **Solve $|x - 3| - 2 > 7$.**

Two cases must be solved. In one case, $x - 3$ is negative, and in the other, $x - 3$ is positive.

Case 1 If $a < 0$, then $|a| = -a$.

$$-(x-3) - 2 > 7$$
$$-x + 3 - 2 > 7$$
$$-x > 6$$
$$x < -6$$

Case 2 If $a > 0$, then $|a| = a$.

$$x - 3 - 2 > 7$$
$$x - 5 > 7$$
$$x > 12$$

The solution set is $\{x \mid x < -6 \text{ or } x > 12\}$.

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Practice

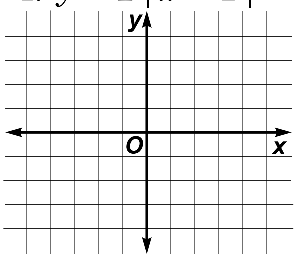
Graphs of Nonlinear Inequalities

Determine whether the ordered pair is a solution for the given inequality. Write yes or no.

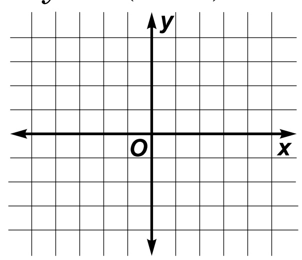
1. $y > (x + 2)^2 + 3, (-2, 6)$ **2.** $y < (x - 3)^3 + 2, (4, 5)$ **3.** $y \leq |2x - 4| - 1, (-4, 1)$

Graph each inequality.

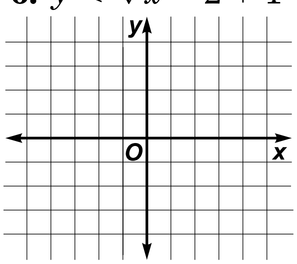
4. $y \leq 2|x - 1|$



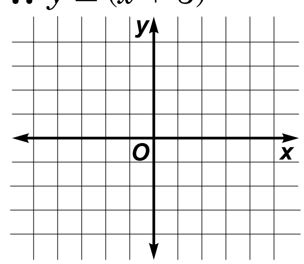
5. $y > 2(x - 1)^2$



6. $y < \sqrt{x - 2} + 1$



7. $y \geq (x + 3)^3$



Solve each inequality.

8. $|4x - 10| \leq 6$ **9.** $|x + 5| + 2 > 6$ **10.** $|2x - 2| - 1 < 7$

11. ***Measurement*** Instructions for building a birdhouse warn that the platform, which ideally measures 14.75 cm^2, should not vary in size by more than 0.30 cm^2. If it does, the preconstructed roof for the birdhouse will not fit properly.

a. Write an absolute value inequality that represents the range of possible sizes for the platform. Then solve for x to find the range.

b. Dena cut a board 14.42 cm^2. Does the platform that Dena cut fit within the acceptable range?

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Enrichment

Some Parametric Graphs

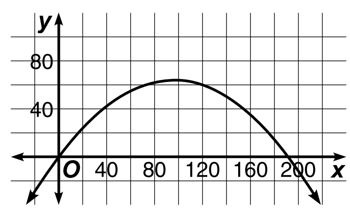
For some curves, the coordinates x and y can be written as functions of a third variable. The conditions determining the curve are given by two equations, rather than by a single equation in x and y. The third variable is called a *parameter*, and the two equations are called *parametric equations* of the curve.

For the curves you will graph on this page, the parameter is t and the parametric equations of each curve are in the form $x = f(t)$ and $y = g(t)$.

Example **Graph the curve associated with the parametric equations $x = 48t$ and $y = 64t - 16t^2$.**

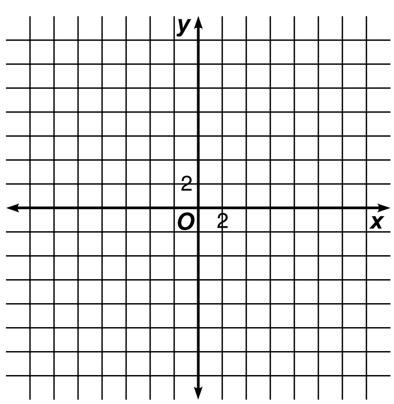
Choose values for t and make a table showing the values of all three variables. Then graph the x- and y-values.

t	x	y
–1	–48	–80
0	0	0
0.5	24	28
1	48	48
2	96	64
3	144	48
4	192	0

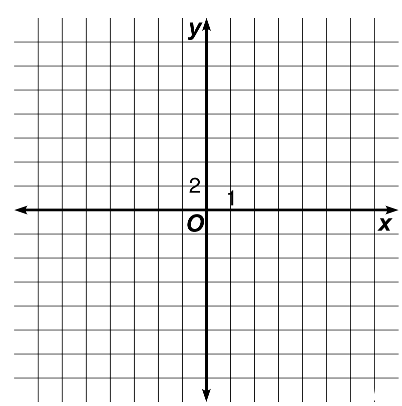


Graph each curve.

1. $x = 3t, \quad y = \dfrac{12}{t}$



2. $x = t^2 + 1, \quad y = t^3 - 1$



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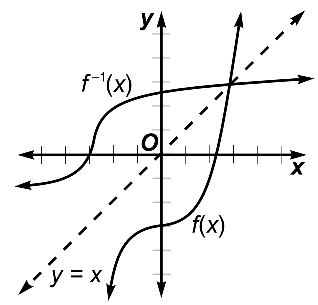
Study Guide

Inverse Functions and Relations

Two relations are inverse relations if and only if one relation contains the element (b, a) whenever the other relation contains the element (a, b). If the inverse of the function $f(x)$ is also a function, then the inverse is denoted by $f^{-1}(x)$.

Example 1 **Graph $f(x) = \frac{1}{4}x^3 - 3$ and its inverse.**

To graph the function, let $y = f(x)$. To graph $f^{-1}(x)$, interchange the x- and y-coordinates of the ordered pairs of the function.



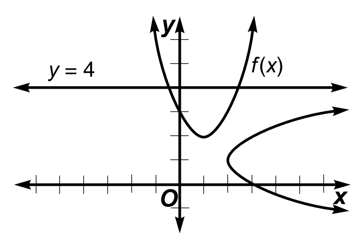
$f(x) = \frac{1}{4}x^3 - 3$	
x	y
−3	−9.75
−2	−5
−1	−3.25
0	−3
1	−2.75
2	−1
3	3.75

$f^{-1}(x)$	
x	y
−9.75	−3
−5	−2
−3.25	−1
−3	0
−2.75	1
−1	2
3.75	3

You can use the **horizontal line test** to determine if the inverse of a relation will be a function. If every horizontal line intersects the graph of the relation in at most one point, then the inverse of the relation is a function.

You can find the inverse of a relation algebraically. First, let $y = f(x)$. Then interchange x and y. Finally, solve the resulting equation for y.

Example 2 **Determine if the inverse of $f(x) = (x - 1)^2 + 2$ is a function. Then find the inverse.**



Since the line $y = 4$ intersects the graph of $f(x)$ at more than one point, the function fails the horizontal line test. Thus, the inverse of $f(x)$ is not a function.

$y = (x - 1)^2 + 2$	*Let $y = f(x)$.*
$x = (y - 1)^2 + 2$	*Interchange x and y.*
$x - 2 = (y - 1)^2$	*Isolate the expression containing y.*
$\pm\sqrt{x - 2} = y - 1$	*Take the square root of each side.*
$y = 1 \pm \sqrt{x - 2}$	*Solve for y.*

3-4

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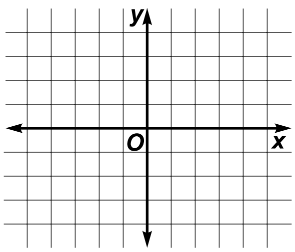
Practice

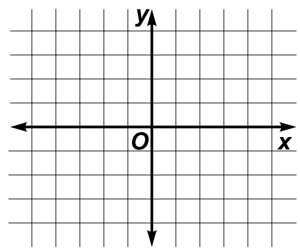
Inverse Functions and Relations

Graph each function and its inverse.

1. $f(x) = (x - 1)^3 + 1$



2. $f(x) = 3|x| + 2$



Find the inverse of f(x). Then state whether the inverse is also a function.

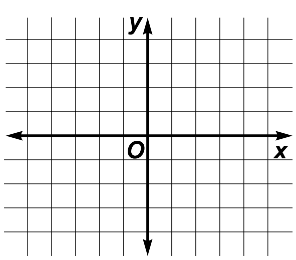
3. $f(x) = -4x^2 + 1$

4. $f(x) = \sqrt[3]{x - 1}$

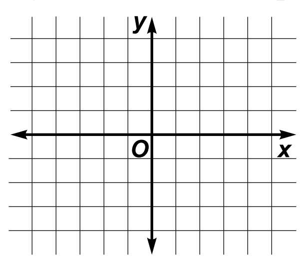
5. $f(x) = \dfrac{4}{(x - 3)^2}$

Graph each equation using the graph of the given parent function.

6. $y = -\sqrt{x + 3} - 1$, $p(x) = x^2$



7. $y = 2 + \sqrt[5]{x + 2}$, $p(x) = x^5$



8. ***Fire Fighting*** Airplanes are often used to drop water on forest fires in an effort to stop the spread of the fire. The time t it takes the water to travel from height h to the ground can be derived from the equation $h = \frac{1}{2}gt^2$ where g is the acceleration due to gravity (32 feet/second^2).

a. Write an equation that will give time as a function of height.

b. Suppose a plane drops water from a height of 1024 feet. How many seconds will it take for the water to hit the ground?

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Enrichment

An Inverse Acrostic

The puzzle on this page is called an acrostic. To solve the puzzle, work back and forth between the clues and the puzzle box. You may need a math dictionary to help with some of the clues.

1. If a relation contains the element (e, v), then the inverse of the relation must contain the element (__, __). — 17 28
2. The inverse of the function $2x$ is found by computing __ of x. — 2 29 6 27
3. The first letter and the last two letters of the meaning of the symbol f^{-1} are __ . — 31 33 14
4. This is the product of a number and its multiplicative inverse. — 20 11 34
5. If the second coordinate of the inverse of $(x, f(x))$ is y, then the first coordinate is read "__ of __". — 36 7
6. The inverse ratio of two numbers is the __ of the reciprocals of the numbers. — 24 16 19 10 4
7. If $\cdot$ is a binary operation on set S and $x \cdot e = e \cdot x = x$ for all x in S, then an identity element for the operation is __. — 18
8. To solve a matrix equation, multiply each side of the matrix equation on the __ by the inverse matrix. — 35 3 21 8
9. Two variables are inversely proportional __ their product is constant. — 13 9 22 5
10. The graph of the inverse of a linear function is a __ line. — 26 32 30 23 25 12 15 1

From President Franklin D. Roosevelt's inaugural address during the Great Depression; delivered March 4, 1933.

■	1	2	3	■	4	5	6	7	■	8	9	10	11	12	■
13	14	■	15	16	17	18	■	19	20	■	21	22	23	24	■
■	25	26	■	27	28	29	30	■	31	32	33	34	35	36	■

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Study Guide

Continuity and End Behavior

A function is **continuous** at $x = c$ if it satisfies the following three conditions.
(1) the function is defined at c; in other words, $f(c)$ exists;
(2) the function approaches the same y-value to the left and right of $x = c$; and
(3) the y-value that the function approaches from each side is $f(c)$.

Functions can be continuous or **discontinuous.** Graphs that are discontinuous can exhibit **infinite discontinuity**, **jump discontinuity**, or **point discontinuity.**

Example 1 **Determine whether each function is continuous at the given x-value. Justify your answer using the continuity test.**

a. $f(x) = 2|x| + 3; x = 2$

(1) The function is defined at $x = 2$; $f(2) = 7$.

(2) The tables below show that y approaches 7 as x approaches 2 from the left and that y approaches 7 as x approaches 2 from the right.

x	$y = f(x)$
1.9	6.8
1.99	6.98
1.999	6.998

x	$y = f(x)$
2.1	7.2
2.01	7.02
2.001	7.002

(3) Since the y-values approach 7 as x approaches 2 from both sides and $f(2) = 7$, the function is continuous at $x = 2$.

b. $f(x) = \frac{2x}{x^2 - 1}; x = 1$

Start with the first condition in the continuity test. The function is not defined at $x = 1$ because substituting 1 for x results in a denominator of zero. So the function is discontinuous at $x = 1$.

c. $f(x) = \begin{cases} 2x + 1 & \text{if } x > 2 \\ x - 1 & \text{if } x = 2 \end{cases}$

This function fails the second part of the continuity test because the values of $f(x)$ approach 1 as x approaches 2 from the left, but the values of $f(x)$ approach 5 as x approaches 2 from the right.

The **end behavior** of a function describes what the $y-$values do as $|x|$ becomes greater and greater. In general, the end behavior of any polynomial function can be modeled by the function made up solely of the term with the highest power of x and its coefficient.

Example 2 **Describe the end behavior of $p(x) = -x^5 + 2x^3 - 4$.**

Determine $f(x) = a_n x^n$ where x^n is the term in $p(x)$ with the highest power of x and a_n is its coefficient.

$f(x) = -x^5 \quad x^n = x^5 \quad a_n = -1$

Thus, by using the table on page 163 of your text, you can see that when a^n is negative and n is odd, the end behavior can be stated as $p(x) \to -\infty$ as $x \to \infty$ and $p(x) \to \infty$ as $x \to -\infty$.

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Practice

Continuity and End Behavior

Determine whether each function is continuous at the given x-value. Justify your answer using the continuity test.

1. $y = \frac{2}{3x^2}; x = -1$

2. $y = \frac{x^2 + x + 4}{2}; x = 1$

3. $y = x^3 - 2x + 2; x = 1$

4. $y = \frac{x - 2}{x + 4}; x = -4$

Describe the end behavior of each function.

5. $y = 2x^5 - 4x$

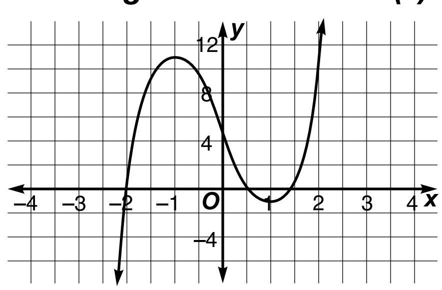
6. $y = -2x^6 + 4x^4 - 2x + 1$

7. $y = x^4 - 2x^3 + x$

8. $y = -4x^3 + 5$

Given the graph of the function, determine the interval(s) for which the function is increasing and the interval(s) for which the function is decreasing.

9.



10. ***Electronics*** Ohm's Law gives the relationship between resistance R, voltage E, and current I in a circuit as $R = \frac{E}{I}$. If the voltage remains constant but the current keeps increasing in the circuit, what happens to the resistance?

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3-5

Enrichment

Reading Mathematics

The following selection gives a definition of a continuous function as it might be defined in a college-level mathematics textbook. Notice that the writer begins by explaining the notation to be used for various types of intervals. It is a common practice for college authors to explain their notation, since, although a great deal of the notation is standard, each author usually chooses the notation he or she wishes to use.

> Throughout this book, the set S, called the domain of definition of a function, will usually be an interval. An interval is a set of numbers satisfying one of the four inequalities $a < x < b$, $a \leq x < b$, $a < x \leq b$, or $a \leq x \leq b$. In these inequalities, $a \leq b$. The usual notations for the intervals corresponding to the four inequalities are, respectively, (a, b), $[a, b)$, $(a, b]$, and $[a, b]$.
>
> An interval of the form (a, b) is called *open*, an interval of the form $[a, b)$ or $(a, b]$ is called *half-open* or *half-closed*, and an interval of the form $[a, b]$ is called *closed*.
>
> Suppose I is an interval that is either open, closed, or half-open. Suppose $f(x)$ is a function defined on I and x_0 is a point in I. We say that the function $f(x)$ is continuous at the point x_0 if the quantity $|f(x) - f(x_0)|$ becomes small as $x \in I$ approaches x_0.

Use the selection above to answer these questions.

1. What happens to the four inequalities in the first paragraph when $a = b$?

2. What happens to the four intervals in the first paragraph when $a = b$?

3. What mathematical term makes sense in this sentence?
If $f(x)$ is not __?__ at x_0, it is said to be discontinuous at x_0.

4. What notation is used in the selection to express the fact that a number x is contained in the interval I?

5. In the space at the right, sketch the graph of the function $f(x)$ defined as follows:

$$f(x) = \begin{cases} \frac{1}{2} \text{ if } x \in \left[0, \frac{1}{2}\right) \\ 1, \text{ if } x \in \left[\frac{1}{2}, 1\right] \end{cases}$$

6. Is the function given in Exercise 5 continuous on the interval $[0, 1]$? If not, where is the function discontinuous?

3-6

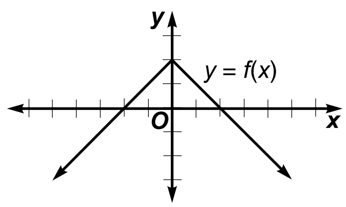
Study Guide

Critical Points and Extrema

Critical points are points on a graph at which a line drawn tangent to the curve is horizontal or vertical. A critical point may be a **maximum**, a **minimum**, or a **point of inflection.** A point of inflection is a point where the graph changes its curvature. Graphs can have an **absolute maximum**, an **absolute minimum**, a **relative maximum**, or a **relative minimum.** The general term for maximum or minimum is **extremum** (plural, *extrema*).

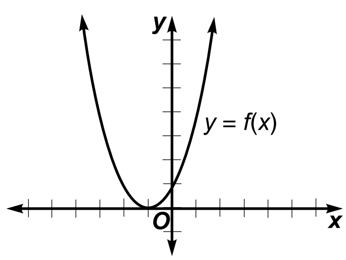
Example 1 **Locate the extrema for the graph of $y = f(x)$. Name and classify the extrema of the function.**

a.



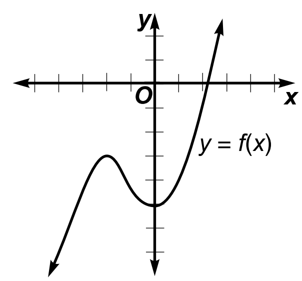
The function has an absolute maximum at (0, 2). The absolute maximum is the greatest value that a function assumes over its domain.

b.



The function has an absolute minimum at $(-1, 0)$. The absolute minimum is the least value that a function assumes over its domain.

c.



The relative maximum and minimum may not be the greatest and the least y-value for the domain, respectively, but they are the greatest and least y-value on some interval of the domain. The function has a relative maximum at $(-2, -3)$ and a relative minimum at $(0, -5)$. Because the graph indicates that the function increases or decreases without bound as x increases or decreases, there is neither an absolute maximum nor an absolute minimum.

By testing points on both sides of a critical point, you can determine whether the critical point is a relative maximum, a relative minimum, or a point of inflection.

Example 2 **The function $f(x) = 2x^6 + 2x^4 - 9x^2$ has a critical point at $x = 0$. Determine whether the critical point is the location of a maximum, a minimum, or a point of inflection.**

x	$x - 0.1$	$x + 0.1$	$f(x - 0.1)$	$f(x)$	$f(x + 0.1)$	Type of Critical Point
0	-0.1	0.1	-0.0899	0	-0.0899	maximum

Because 0 is greater than both $f(x - 0.1)$ and $f(x + 0.1)$, $x = 0$ is the location of a relative maximum.

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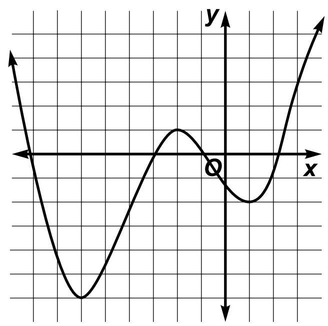
Practice

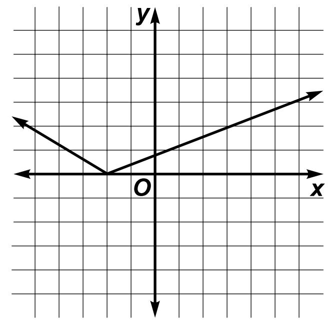
Critical Points and Extrema

Locate the extrema for the graph of $y = f(x)$. Name and classify the extrema of the function.

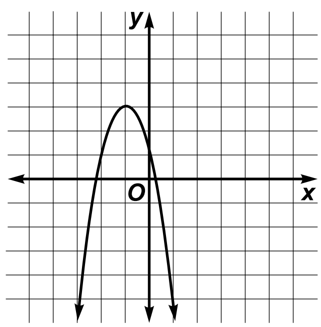
1.



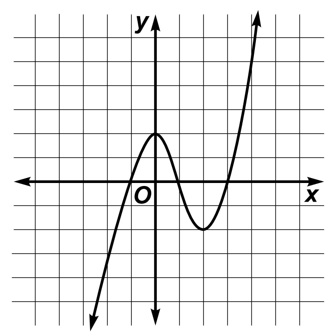
2.



3.



4.

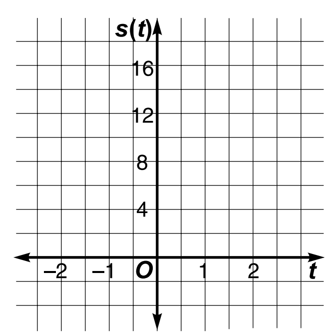


Determine whether the given critical point is the location of a maximum, a minimum, or a point of inflection.

5. $y = x^2 - 6x + 1, x = 3$ **6.** $y = x^2 - 2x - 6, x = 1$ **7.** $y = x^4 + 3x^2 - 5, x = 0$

8. $y = x^5 - 2x^3 - 2x^2, x = 0$ **9.** $y = x^3 + x^2 - x, x = -1$ **10.** $y = 2x^3 + 4, x = 0$

11. ***Physics*** Suppose that during an experiment you launch a toy rocket straight upward from a height of 6 inches with an initial velocity of 32 feet per second. The height at any time t can be modeled by the function $s(t) = -16t^2 + 32t + 0.5$ where $s(t)$ is measured in feet and t is measured in seconds. Graph the function to find the maximum height obtained by the rocket before it begins to fall.



3-6

Enrichment

"Unreal" Equations

There are some equations that cannot be graphed on the real-number coordinate system. One example is the equation $x^2 - 2x + 2y^2 + 8y + 14 = 0$. Completing the squares in x and y gives the equation $(x - 1)^2 + 2(y + 2)^2 = -5$.

For any real numbers, x and y, the values of $(x - 1)^2$ and $2(y + 2)^2$ are nonnegative. So, their sum cannot be -5. Thus, no real values of x and y satisfy the equation; only imaginary values can be solutions.

Determine whether each equation can be graphed on the real-number plane. Write yes or no.

1. $(x + 3)^2 + (y - 2)^2 = -4$

2. $x^2 - 3x + y^2 + 4y = -7$

3. $(x + 2)^2 + y^2 - 6y + 8 = 0$

4. $x^2 + 16 = 0$

5. $x^4 + 4y^2 + 4 = 0$

6. $x^2 + 4y^2 + 4xy + 16 = 0$

In Exercises 7 and 8, for what values of k :

a. will the solutions of the equation be imaginary?

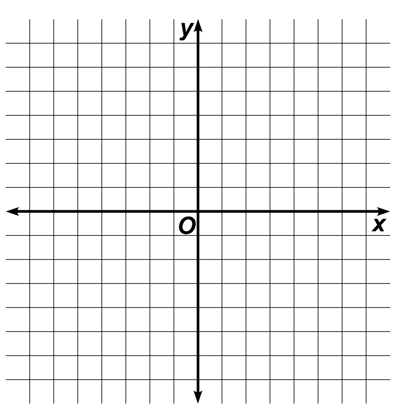
b. will the graph be a point?

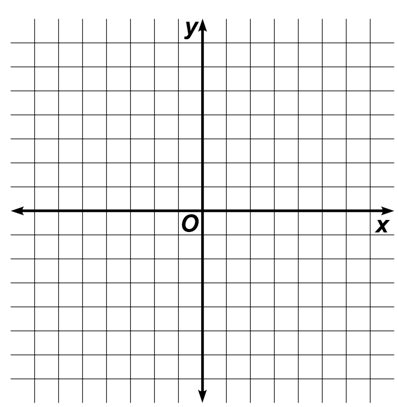
c. will the graph be a curve?

d. Choose a value of k for which the graph is a curve and sketch the curve on the axes provided.

7. $x^2 - 4x + y^2 + 8y + k = 0$

8. $x^2 + 4x + y^2 - 6y - k = 0$





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Study Guide

Graphs of Rational Functions

A **rational function** is a quotient of two polynomial functions.

The line $x = a$ is a **vertical asymptote** for a function $f(x)$ if $f(x) \to \infty$ or $f(x) \to -\infty$ as $x \to a$ from either the left or the right.

The line $y = b$ is a **horizontal asymptote** for a function $f(x)$ if $f(x) \to b$ as $x \to \infty$ or as $x \to -\infty$.

A **slant asymptote** occurs when the degree of the numerator of a rational function is exactly one greater than that of the denominator.

Example 1 **Determine the asymptotes for the graph of $f(x) = \frac{2x - 1}{x + 3}$.**

Since $f(-3)$ is undefined, there may be a vertical asymptote at $x = -3$. To verify that $x = -3$ is a vertical asymptote, check to see that $f(x) \to \infty$ or $f(x) \to -\infty$ as $x \to -3$ from either the left or the right.

x	$f(x)$
-2.9	-68
-2.99	-698
-2.999	-6998
-2.9999	-69998

The values in the table confirm that $f(x) \to -\infty$ as $x \to -3$ from the right, so there is a vertical asymptote at $x = -3$.

One way to find the horizontal asymptote is to let $f(x) = y$ and solve for x in terms of y. Then find where the function is undefined for values of y.

$$
\begin{aligned}
y &= \frac{2x - 1}{x + 3} \\
y(x + 3) &= 2x - 1 \\
xy + 3y &= 2x - 1 \\
xy - 2x &= -3y - 1 \\
x(y - 2) &= -3y - 1 \\
x &= \frac{-3y - 1}{y - 2}
\end{aligned}
$$

The rational expression $\frac{-3y - 1}{y - 2}$ is undefined for $y = 2$. Thus, the horizontal asymptote is the line $y = 2$.

Example 2 **Determine the slant asymptote for $f(x) = \frac{3x^2 - 2x + 2}{x - 1}$.**

First use division to rewrite the function.

$$
\begin{array}{r}
3x + 1 \\
x - 1\overline{)3x^2 - 2x + 2} \\
\underline{3x^2 - 3x} \\
x + 2 \\
\underline{x - 1} \\
3
\end{array}
\quad \rightarrow \quad f(x) = 3x + 1 + \frac{3}{x - 1}
$$

As $x \to \infty$, $\frac{3}{x - 1} \to 0$. Therefore, the graph of $f(x)$ will approach that of $y = 3x + 1$. This means that the line $y = 3x + 1$ is a slant asymptote for the graph of $f(x)$.

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Practice

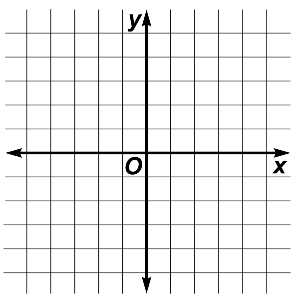
Graphs of Rational Functions

Determine the equations of the vertical and horizontal asymptotes, if any, of each function.

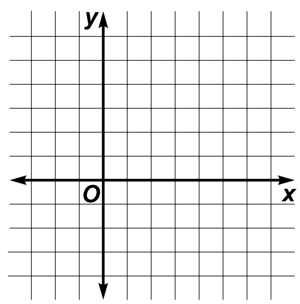
1. $f(x) = \dfrac{4}{x^2 + 1}$ **2.** $f(x) = \dfrac{2x + 1}{x + 1}$ **3.** $g(x) = \dfrac{x + 3}{(x + 1)(x - 2)}$

Use the parent graph $f(x) = \frac{1}{x}$ to graph each equation. Describe the transformation(s) that have taken place. Identify the new locations of the asymptotes.

4. $y = \dfrac{3}{x + 1} - 2$



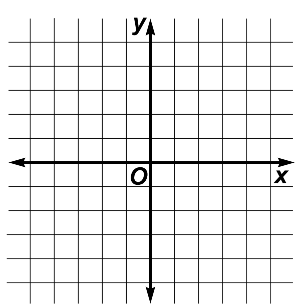
5. $y = -\dfrac{4}{x - 3} + 3$



Determine the slant asymptotes of each equation.

6. $y = \dfrac{5x^2 - 10x + 1}{x - 2}$ **7.** $y = \dfrac{x^2 - x}{x + 1}$

8. Graph the function $y = \dfrac{x^2 + x - 6}{x + 1}$.



9. ***Physics*** The illumination I from a light source is given by the formula $I = \frac{k}{d^2}$, where k is a constant and d is distance. As the distance from the light source doubles, how does the illumination change?

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Enrichment

Slant Asymptotes

The graph of $y = ax + b$, where $a \neq 0$, is called a slant asymptote of $y = f(x)$ if the graph of $f(x)$ comes closer and closer to the line as $x \to \infty$ or $x \to -\infty$.

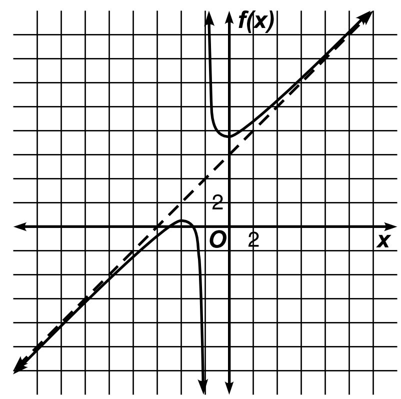
For $f(x) = 3x + 4 + \frac{2}{x}$, $y = 3x + 4$ is a slant asymptote because

$f(x) - (3x + 4) = \frac{2}{x}$, and $\frac{2}{x} \to 0$ as $x \to \infty$ or $x \to -\infty$.

Example **Find the slant asymptote of** $\boldsymbol{f(x) = \frac{x^2 + 8x + 15}{x + 2}}$.

$$\begin{array}{r|rrr} -2 & 1 & 8 & 15 \\ & & -2 & -12 \\ \hline & 1 & 6 & |\,3 \end{array}$$

Use synthetic division.



$$y = \frac{x^2 + 8x + 15}{x + 2} = x + 6 + \frac{3}{x + 2}$$

Since $\frac{3}{x + 2} \to 0$ as $x \to \infty$ or $x \to -\infty$,

$y = x + 6$ is a slant asymptote.

Use synthetic division to find the slant asymptote for each of the following.

1. $y = \frac{8x^2 - 4x + 11}{x + 5}$

2. $y = \frac{x^2 + 3x - 15}{x - 2}$

3. $y = \frac{x^2 - 2x - 18}{x - 3}$

4. $y = \frac{ax^2 + bx + c}{x - d}$

5. $y = \frac{ax^2 + bx + c}{x + d}$

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Study Guide

Direct, Inverse, and Joint Variation

A **direct variation** can be described by the equation $y = kx^n$. The k in this equation is called the **constant of variation.** To express a direct variation, we say that y varies directly as x^n. An inverse variation can be described by the equation $y = \frac{k}{x^n}$ or $x^n y = k$. When quantities are **inversely proportional,** we say they *vary inversely* with each other. **Joint variation** occurs when one quantity varies directly as the product of two or more other quantities and can be described by the equation $y = kx^n z^n$.

Example 1 **Suppose y varies directly as x and $y = 14$ when $x = 8$.**

a. Find the constant of variation and write an equation of the form $y = kx^n$.

b. Use the equation to find the value of y when $x = 4$.

a. The power of x is 1, so the direct variation equation is $y = kx$.

$$y = kx$$
$$14 = k(8) \quad \textit{y = 14, x = 8}$$
$$1.75 = k \quad \textit{Divide each side by 8.}$$

The constant of variation is 1.75. The equation relating x and y is $y = 1.75x$.

b. $y = 1.75x$

$y = 1.75(4)$ *$x = 4$*

$y = 7$

When $x = 4$, the value of y is 7.

Example 2 **If y varies inversely as x and $y = 102$ when $x = 7$, find x when $y = 12$.**

Use a proportion that relates the values.

$$\frac{x_1{}^n}{y_2} = \frac{x_2{}^n}{y_1}$$

$$\frac{7}{12} = \frac{x}{102} \quad \textit{Substitute the known values.}$$

$$12x = 714 \quad \textit{Cross multiply.}$$

$$x = \frac{714}{12} \text{ or } 59.5 \quad \textit{Divide each side by 12.}$$

When $y = 12$, the value of x is 59.5.

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Practice

Direct, Inverse, and Joint Variation

Write a statement of variation relating the variables of each equation. Then name the constant of variation.

1. $-\frac{x^2}{y} = 3$

2. $E = IR$

3. $y = 2x$

4. $d = 6t^2$

Find the constant of variation for each relation and use it to write an equation for each statement. Then solve the equation.

5. Suppose y varies directly as x and $y = 35$ when $x = 5$. Find y when $x = 7$.

6. If y varies directly as the cube of x and $y = 3$ when $x = 2$, find x when $y = 24$.

7. If y varies inversely as x and $y = 3$ when $x = 25$, find x when $y = 10$.

8. Suppose y varies jointly as x and z, and $y = 64$ when $x = 4$ and $z = 8$. Find y when $x = 7$ and $z = 11$.

9. Suppose V varies jointly as h and the square of r, and $V = 45\pi$ when $r = 3$ and $h = 5$. Find r when $V = 175\pi$ and $h = 7$.

10. If y varies directly as x and inversely as the square of z, and $y = -5$ when $x = 10$ and $z = 2$, find y when $x = 5$ and $z = 5$.

11. ***Finances*** Enrique deposited \$200.00 into a savings account. The simple interest I on his account varies jointly as the time t in years and the principal P. After one quarter (three months), the interest on Enrique's account is \$2.75. Write an equation relating interest, principal, and time. Find the constant of variation. Then find the interest after three quarters.

Enrichment

Reading Mathematics: Interpreting Conditional Statements

The conditional statement below is written in “if-then” form. It has the form $p \to q$ where p is the hypothesis and q is the consequent.

If a matrix A has a determinant of 0, then A^{-1} does not exist.

It is important to recognize that a conditional statement need not appear in “if-then” form. For example, the statement

Any point that lies in Quadrant I has a positive x-coordinate.

can be rewritten as

If the point $P(x, y)$ lies in Quadrant I, then x is positive.

Notice that P lying in Quadrant I is a *sufficient* condition for its x-coordinate to be positive. Another way to express this is to say that P lying in Quadrant I *guarantees* that its x-coordinate is positive. On the other hand, we can also say that x being positive is a *necessary* condition for P to lie in Quadrant I. In other words, P does not lie in Quadrant I if x is not positive.

To change an English statement into “if-then” form requires that you understand the meaning and syntax of the English statement. Study each of the following equivalent ways of expressing $p \to q$.

- If p then q
- p only if q
- p is a sufficient condition for q
- q is a necessary condition for p.
- p implies q
- only if q, p
- not p unless q

Rewrite each of the following statements in “if-then” form.

1. A consistent system of equations has at least one solution.

2. When the region formed by the inequalities in a linear programming application is unbounded, an optimal solution for the problem may not exist.

3. Functions whose graphs are symmetric with respect to the y-axis are called even functions.

4. In order for a decimal number d to be odd, it is sufficient that d end in the digit 7.

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Chapter 3 Test, Form 1A

Write the letter for the correct answer in the blank at the right of each problem.

1. The graph of the equation $y = x^3 - x$ is symmetric with respect to which of the following? **1.** ________

A. the x-axis **B.** the y-axis **C.** the origin **D.** none of these

2. If the graph of a function is symmetric about the origin, which of the following must be true? **2.** ________

A. $f(x) = f(-x)$ **B.** $f(-x) = -f(x)$ **C.** $f(x) = |f(x)|$ **D.** $f(x) = \frac{1}{f(x)}$

3. The graph of an odd function is symmetric with respect to which of the following? **3.** ________

A. the x-axis **B.** the y-axis **C.** the line $y = x$ **D.** none of these

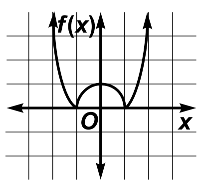
4. Given the parent function $p(x) = [\![x]\!]$, what transformations occur in the graph of $p(x) = 2[\![x - 3]\!] + 4$? **4.** ________

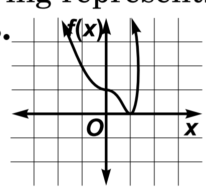
A. vertical expansion by factor of 2, left 3 units, up 4 units
B. vertical compression by factor of 0.5, down 3 units, left 4 units
C. vertical expansion by factor of 2, right 3 units, up 4 units
D. vertical compression by factor of 0.5, down 3 units, left 4 units

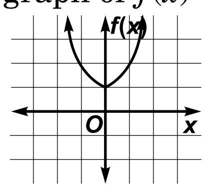
5. Which of the following results in the graph of $f(x) = x^2$ being expanded vertically by a factor of 3 and reflected over the x-axis? **5.** ________

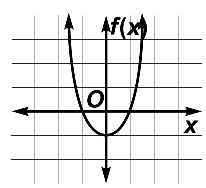
A. $f(x) = \frac{1}{3}x^2$ **B.** $f(x) = -3x^2$ **C.** $f(x) = -\frac{1}{x^2} + 3$ **D.** $f(x) = -\frac{1}{3}x^2$

6. Which of the following represents the graph of $f(x) = |x|^3 - 1$? **6.** ________

A.


B.


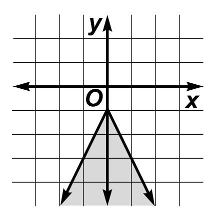
C.


D.


7. Solve $|2x - 5| \geq 7$. **7.** ________

A. $x \leq -1$ or $x \geq 6$
B. $-1 \leq x \leq 6$
C. $x \leq 6$
D. $x \geq -1$

8. Choose the inequality shown by the graph. **8.** ________

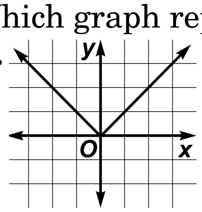


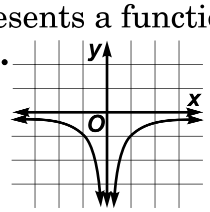
A. $y \leq 2|x| + 1$
B. $y \leq -2|x| + 1$
C. $y \leq 2|x| - 1$
D. $y \leq -2|x| - 1$

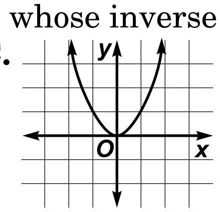
9. Find the inverse of $f(x) = 2\sqrt{x} + 3$. **9.** ________

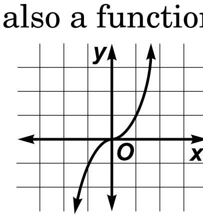
A. $f^{-1}(x) = \left(\frac{x - 3}{2}\right)^2$
B. $f^{-1}(x) = \left(\frac{x + 3}{2}\right)^2$
C. $f^{-1}(x) = \frac{1}{2}\sqrt{x} - 3$
D. $f^{-1}(x) = \frac{1}{2}\sqrt{x} + 3$

10. Which graph represents a function whose inverse is also a function? **10.** ________

A.


B.


C.


D.


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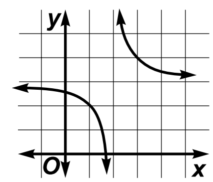
Chapter 3 Test, Form 1A (continued)

11. Describe the end behavior of $f(x) = 2x^4 - 5x + 1$. **11.** ________

A. $x \to \infty, f(x) \to \infty$; $x \to -\infty, f(x) \to -\infty$
B. $x \to \infty, f(x) \to \infty$; $x \to -\infty, f(x) \to \infty$
C. $x \to \infty, f(x) \to -\infty$; $x \to -\infty, f(x) \to \infty$
D. $x \to \infty, f(x) \to -\infty$; $x \to -\infty, f(x) \to -\infty$

12. Which type of discontinuity, if any, is shown in the graph at the right? **12.** ________



A. jump **B.** infinite
C. point **D.** The graph is continuous.

13. Choose the statement that is true for the graph of $f(x) = x^3 - 12x$. **13.** ________

A. $f(x)$ increases for $x > -2$. **B.** $f(x)$ decreases for $x > -2$.
C. $f(x)$ increases for $x > 2$. **D.** $f(x)$ decreases for $x < 2$.

14. Which type of critical point, if any, is present in the graph of $f(x) = (-x + 4)^5 + 1$? **14.** ________

A. maximum **B.** minimum
C. point of inflection **D.** none of these

15. Which is true for the graph of $f(x) = -x^3 + 3x - 2$? **15.** ________

A. relative maximum at $(1, 0)$ **B.** relative minimum at $(1, 0)$
C. relative maximum at $(-1, -4)$ **D.** relative minimum at $(0, -2)$

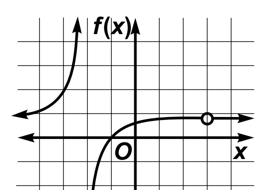
16. Which is true for the graph of $y = \frac{x^2 - 4}{x^2 + 9}$? **16.** ________

A. vertical asymptotes at $x = \pm 3$ **B.** horizontal asymptotes at $y = \pm 2$
C. vertical asymptotes at $x = \pm 2$ **D.** horizontal asymptote at $y = 1$

17. Find the equation of the slant asymptote for the graph of $y = \frac{x^3 - 4x^2 + 2x - 5}{x^2 + 2}$. **17.** ________

A. $y = x - 4$ **B.** $y = x + 1$ **C.** $y = x$ **D.** $y = 1$

18. Which of the following could be the function represented by the graph at the right? **18.** ________



A. $f(x) = \frac{x + 1}{x + 2}$ **B.** $f(x) = \frac{(x + 1)(x - 3)}{(x - 3)(x + 2)}$
C. $f(x) = \frac{x + 1}{x - 2}$ **D.** $f(x) = \frac{(x + 1)(x + 3)}{(x + 3)(x + 2)}$

19. ***Chemistry*** The volume V of a gas varies inversely as pressure P is exerted. If $V = 3.5$ liters when $P = 5$ atmospheres, find V when $P = 8$ atmospheres. **19.** ________

A. 2.188 liters **B.** 5.600 liters **C.** 11.429 liters **D.** 17.5 liters

20. If y varies jointly as x and the cube of z, and $y = 378$ when $x = 4$ and $z = 3$, find y when $x = 9$ and $z = 2$. **20.** ________

A. $y = 283.5$ **B.** $y = 84$ **C.** $y = 567$ **D.** $y = 252$

Bonus An even function f has a vertical asymptote at $x = 3$ and a maximum at $x = 0$. Which of the following could be f? **Bonus:** ________

A. $f(x) = \frac{x}{x^2 - 9}$ **B.** $f(x) = \frac{x}{x - 3}$ **C.** $f(x) = \frac{x^2}{x^2 + 9}$ **D.** $f(x) = \frac{x^2}{x^4 - 81}$

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Chapter 3 Test, Form 1B

Write the letter for the correct answer in the blank at the right of each problem.

1. The graph of the equation $y = x^4 - 3x^2$ is symmetric with respect to which of the following? **1.** ________

A. the x-axis **B.** the y-axis **C.** the origin **D.** none of these

2. If the graph of a function is symmetric with respect to the y-axis, which of the following must be true? **2.** ________

A. $f(x) = f(-x)$ **B.** $f(x) = -f(x)$ **C.** $f(x) = |f(x)|$ **D.** $f(x) = \frac{1}{f(x)}$

3. The graph of an even function is symmetric with respect to which of the following? **3.** ________

A. the x-axis **B.** the y-axis **C.** the line $y = x$ **D.** none of these

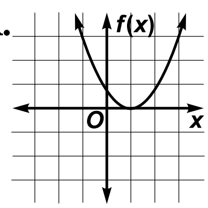
4. Given the parent function $p(x) = x^2$, what translations occur in the graph of $p(x) = (x - 7)^2 + 3$? **4.** ________

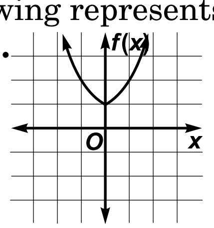
A. right 7 units, up 3 units **B.** down 7 units, left 3 units
C. left 7 units, up 3 units **D.** right 7 units, down 3 units

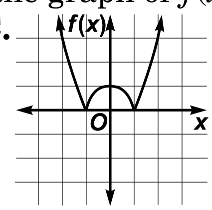
5. Which of the following results in the graph of $f(x) = x^3$ being expanded vertically by a factor of 3? **5.** ________

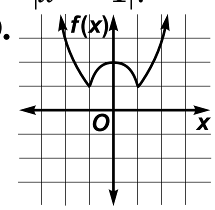
A. $f(x) = x^3 + 3$ **B.** $f(x) = \frac{1}{3}x^3$ **C.** $f(x) = 3x^3$ **D.** $f(x) = -\frac{1}{3}x^3$

6. Which of the following represents the graph of $f(x) = |x^2 - 1|$? **6.** ________

A.


B.


C.


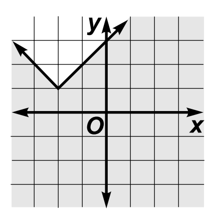
D.


7. Solve $|2x - 4| < 6$. **7.** ________

A. $x < -1$ or $x > 5$ **B.** $-1 < x < 5$
C. $x < 5$ **D.** $x > -1$

8. Choose the inequality shown by the graph. **8.** ________

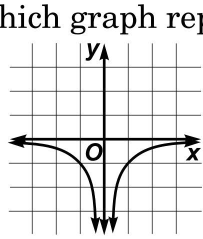
A. $y \le |x + 2| + 1$ **B.** $y \le |x - 2| + 1$
C. $y \ge |x + 2| + 1$ **D.** $y \ge |x - 2| + 1$

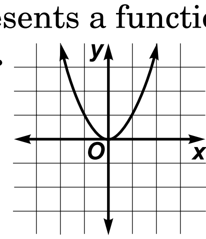


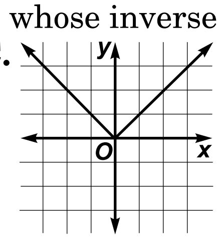
9. Find the inverse of $f(x) = \frac{1}{x - 2}$. **9.** ________

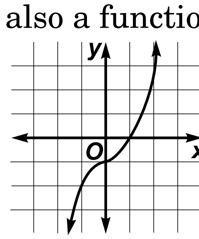
A. $f^{-1}(x) = \frac{1}{x - 2}$ **B.** $f^{-1}(x) = \frac{1}{x} + 2$
C. $f^{-1}(x) = x + 2$ **D.** $f^{-1}(x) = \frac{1}{x} - 2$

10. Which graph represents a function whose inverse is also a function? **10.** ________

A.


B.


C.


D.


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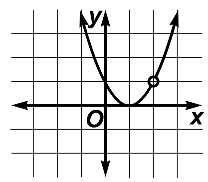
Chapter 3 Test, Form 1B (continued)

11. Which type of discontinuity, if any, is shown in the graph at the right? **11.** ________

A. jump **B.** infinite
C. point **D.** The graph is continuous.



12. Describe the end behavior of $f(x) = 2x^3 - 5x + 1$ **12.** ________

A. $x \to \infty, f(x) \to \infty$; $x \to -\infty, f(x) \to -\infty$
B. $x \to \infty, f(x) \to \infty$; $x \to -\infty, f(x) \to \infty$
C. $x \to \infty, f(x) \to -\infty$; $x \to -\infty, f(x) \to \infty$
D. $x \to \infty, f(x) \to -\infty$; $x \to -\infty, f(x) \to -\infty$

13. Choose the statement that is true for the graph of $f(x) = -(x - 2)^2$. **13.** ________

A. $f(x)$ increases for $x > -2$. **B.** $f(x)$ decreases for $x > -2$.
C. $f(x)$ increases for $x < 2$. **D.** $f(x)$ decreases for $x < 2$.

14. Which type of critical point, if any, is present in the graph of $f(x) = (-x + 4)^3$? **14.** ________

A. maximum **B.** minimum
C. point of inflection **D.** none of these

15. Which is true for the graph of $f(x) = x^3 - 3x + 2$? **15.** ________

A. relative maximum at $(1, 0)$ **B.** relative minimum at $(-1, 4)$
C. relative maximum at $(-1, 4)$ **D.** relative minimum at $(0, 2)$

16. Which is true for the graph of $y = \frac{x^2 - 4}{x^2 - 9}$? **16.** ________

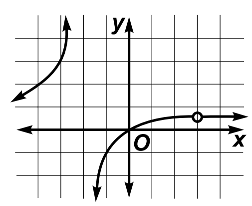
A. vertical asymptotes, $x = \pm 3$ **B.** horizontal asymptotes, $y = \pm 2$
C. vertical asymptotes, $x = \pm 2$ **D.** horizontal asymptote, $y = 0$

17. Find the equation of the slant asymptote for the graph of $y = \frac{3x^2 + 2x - 3}{x - 1}$. **17.** ________

A. $y = x$ **B.** $y = 3x + 1$ **C.** $y = x + 3$ **D.** $y = 3x + 5$

18. Which of the following could be the function represented by the graph at the right? **18.** ________

A. $f(x) = \frac{x}{x + 2}$ **B.** $f(x) = \frac{x(x - 3)}{(x - 3)(x + 2)}$
C. $f(x) = \frac{x}{x - 2}$ **D.** $f(x) = \frac{x(x + 3)}{(x + 3)(x + 2)}$



19. ***Chemistry*** The volume V of a gas varies inversely as pressure P is exerted. If $V = 4$ liters when $P = 3.5$ atmospheres, find V when $P = 2.5$ atmospheres. **19.** ________

A. 5.6 liters **B.** 2.188 liters **C.** 2.857 liters **D.** 10.0 liters

20. If y varies jointly as x and the cube root of z, and $y = 120$ when $x = 3$ and $z = 8$, find y when $x = 4$ and $z = 27$. **20.** ________

A. $y = 540$ **B.** $y = 240$ **C.** $y = 60$ **D.** $y = 26\frac{2}{3}$

Bonus If $f(g(x)) = x$ and $f(x) = 3x - 4$, find $g(x)$. **Bonus:** ________

A. $g(x) = \frac{x - 4}{3}$ **B.** $g(x) = \frac{x + 4}{3}$ **C.** $g(x) = \frac{x}{3} + 4$ **D.** $g(x) = \frac{x}{3} - 4$

Chapter 3

NAME ______________________ DATE ____________ PERIOD ________

Chapter 3 Test, Form 1C

Write the letter for the correct answer in the blank at the right of each problem.

1. The graph of the equation $y = x^2 - 3$ is symmetric with respect to which of the following? **1.** ________

A. the x-axis **B.** the y-axis **C.** the origin **D.** none of these

2. If the graph of a relation is symmetric about the line $y = x$ and the point (a, b) is on the graph, which of the following must also be on the graph? **2.** ________

A. $(-a, b)$ **B.** $(a, -b)$ **C.** (b, a) **D.** $(-a, -b)$

3. The graph of an odd function is symmetric with respect to which of the following? **3.** ________

A. the x-axis **B.** the y-axis **C.** the origin **D.** none of these

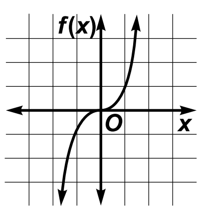
4. Given the parent function $p(x) = \sqrt{x}$, what transformation occurs in the graph of $p(x) = \sqrt{x + 2} - 5$? **4.** ________

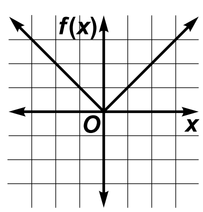
A. right 2 units, up 5 units **B.** up 2 units, right 5 units
C. left 2 units, down 5 units **D.** down 2 units, left 5 units

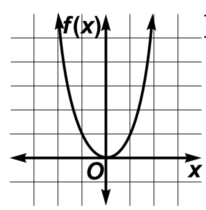
5. Which of the following results in the graph of $f(x) = x^2$ being expanded vertically by a factor of 4? **5.** ________

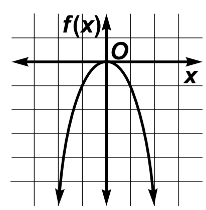
A. $f(x) = x^2 + 4$ **B.** $f(x) = x^2 - 4$ **C.** $f(x) = 4x^2$ **D.** $f(x) = \frac{1}{4}x^2$

6. Which of the following represents the graph of $f(x) = |x^3|$? **6.** ________

A. 

B. 

C. 

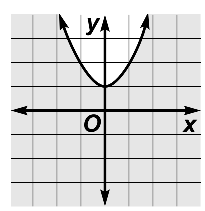
D. 

7. Solve $|x - 4| < 6$. **7.** ________

A. $x > -2$ **B.** $x < 10$
C. $-2 < x < 10$ **D.** $x < -2$ or $x > 10$

8. Choose the inequality shown by the graph. **8.** ________

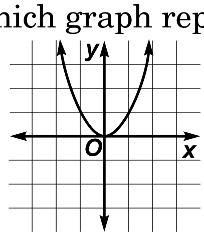
A. $y \leq x^2 + 1$ **B.** $y \leq (x - 1)^2$
C. $y \leq x^2 - 1$ **D.** $y \leq (x + 1)^2$

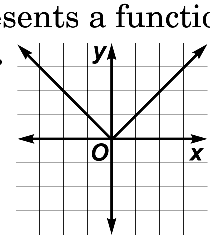


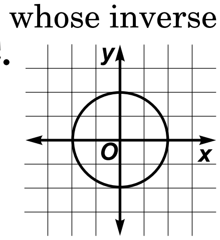
9. Find the inverse of $f(x) = 2x - 4$. **9.** ________

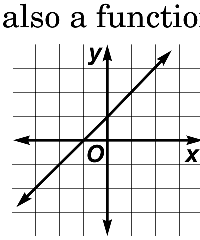
A. $f^{-1}(x) = \frac{1}{2x - 4}$ **B.** $f^{-1}(x) = \frac{x + 4}{2}$
C. $f^{-1}(x) = \frac{x}{2} + 4$ **D.** $f^{-1}(x) = \frac{x}{2} - 4$

10. Which graph represents a function whose inverse is also a function? **10.** ________

A. 

B. 

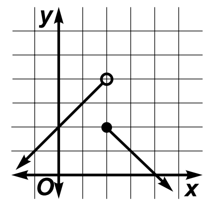
C. 

D. 

NAME ______________________ DATE ____________ PERIOD ________

Chapter 3 Test, Form 1C (continued)

11. Which type of discontinuity, if any, is shown in the graph at the right? **11.** ________



A. jump
B. infinite
C. point
D. The graph is continuous.

12. Describe the end behavior of $f(x) = -x^2$. **12.** ________

A. $x \to \infty, f(x) \to \infty$; $x \to -\infty, f(x) \to -\infty$
B. $x \to \infty, f(x) \to \infty$; $x \to -\infty, f(x) \to \infty$
C. $x \to \infty, f(x) \to -\infty$; $x \to -\infty, f(x) \to \infty$
D. $x \to \infty, f(x) \to -\infty$; $x \to -\infty, f(x) \to -\infty$

13. Choose the statement that is true for the graph of $f(x) = (x + 1)^2$. **13.** ________

A. $f(x)$ increases for $x > -1$. **B.** $f(x)$ decreases for $x > -1$.
C. $f(x)$ increases for $x < 1$. **D.** $f(x)$ decreases for $x < 1$.

14. Which type of critical point, if any, is present in the graph of $f(x) = x^3 - 1$? **14.** ________

A. maximum **B.** minimum
C. point of inflection **D.** none of these

15. Which is true for the graph of $f(x) = x^3 - 3x$? **15.** ________

A. relative maximum at $(1, -2)$ **B.** relative minimum at $(-1, 2)$
C. relative maximum at $(-1, 2)$ **D.** relative minimum at $(0, 0)$

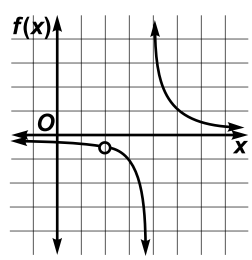
16. Which is true for the graph of $y = \frac{x^2 - 9}{x^2 - 4}$? **16.** ________

A. vertical asymptotes at $x = \pm 3$ **B.** horizontal asymptotes at $y = \pm 2$
C. vertical asymptotes at $x = \pm 2$ **D.** horizontal asymptote at $y = 0$

17. Find the equation of the slant asymptote for the graph of $y = \frac{x^2 - 5}{x + 3}$. **17.** ________

A. $x = 3$ **B.** $y = x - 3$ **C.** $y = x$ **D.** $y = x + 3$

18. Which of the following could be the function represented by the graph? **18.** ________



A. $f(x) = \frac{1}{x - 4}$ **B.** $f(x) = \frac{x - 2}{(x - 2)(x + 4)}$
C. $f(x) = \frac{x + 2}{x - 4}$ **D.** $f(x) = \frac{x - 2}{(x - 2)(x - 4)}$

19. ***Chemistry*** The volume V of a gas varies inversely as pressure P is exerted. If $V = 4$ liters when $P = 3$ atmospheres, find V when $P = 7$ atmospheres. **19.** ________

A. 1.714 liters **B.** 5.25 liters **C.** 9.333 liters **D.** 1.5 liters

20. If y varies inversely as the cube root of x, and $y = 12$ when $x = 8$, find y when $x = 1$. **20.** ________

A. $y = 6144$ **B.** $y = 24$ **C.** $y = 6$ **D.** $y = \frac{3}{128}$

Bonus The graph of $f(x) = \frac{1}{x^2 - c}$ has a vertical asymptote at $x = 3$. Find c. **Bonus:** ________

A. 9 **B.** 3 **C.** -3 **D.** -9

NAME ______________________________ DATE ______________ PERIOD ________

Chapter 3 Test, Form 2A

Determine whether the graph of each equation is symmetric with respect to the origin, the x-axis, the y-axis, the line y = x, the line y = −x, or none of these.

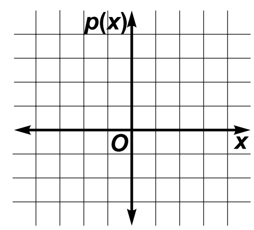
1. $xy = -4$ **1.** ________________

2. $x = 5y^2 - 2$ **2.** ________________

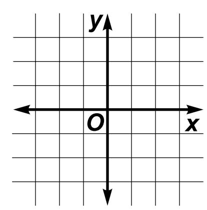
3. Determine whether the function $f(x) = \frac{x}{x^2 - 4}$ is odd, even, or neither. **3.** ________________

4. Describe the transformations relating the graph of $y = -2x^3 + 4$ to its parent function, $y = x^3$. **4.** ________________

5. Use transformations of the parent graph $p(x) = \frac{1}{x}$ to sketch the graph of $p(x) = \frac{1}{|x|} - 1$. **5.**



6. Graph the inequality $y > 2x^2 - 1$. **6.**



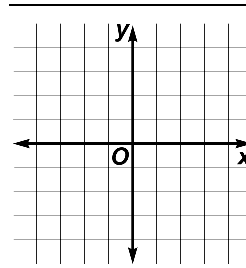
7. Solve $|5 - 2x| \geq 11$. **7.** ________________

Find the inverse of each function and state whether the inverse is a function.

8. $f(x) = \frac{x}{x + 2}$ **8.** ________________

9. $f(x) = x^2 - 4$ **9.** ________________

10. Graph $f(x) = x^3 - 2$ and its inverse. State whether the inverse is a function. **10.** ________________



Chapter 3 Test, Form 2A (continued)

Determine whether each function is continuous at the given x-value. If discontinuous, state the type of discontinuity (point, jump, or infinite).

11. $f(x) = \begin{cases} x^2 + 1 \text{ if } x < 1 \\ -x^3 + 2 \text{ if } x \geq 1 \end{cases}$; $x = 1$ **11.** ______________

12. $f(x) = \frac{x^2 + 9}{x + 3}$; $x = -3$ **12.** ______________

13. Describe the end behavior of $y = -3x^4 - 2x$. **13.** ______________

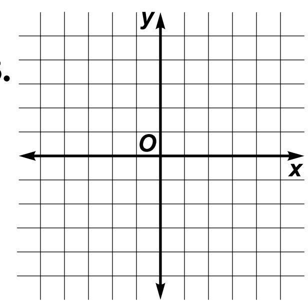
14. Locate and classify the extrema for the graph of $y = x^4 - 3x^2 + 2$. **14.** ______________

15. The function $f(x) = x^3 - 3x^2 + 3x$ has a critical point when $x = 1$. Identify the point as a maximum, a minimum, or a point of inflection, and state its coordinates. **15.** ______________

16. Determine the vertical and horizontal asymptotes for the graph of $y = \frac{x^2 - 4}{x^3 - 5x^2 + 6x}$. **16.** ______________

17. Find the slant asymptote for $y = \frac{3x^2 - 5x + 1}{x - 2}$. **17.** ______________

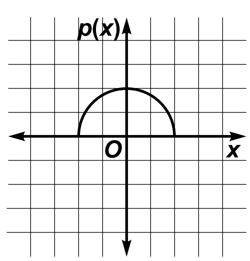
18. Sketch the graph of $y = \frac{x^2 - 1}{x^3 - x^2 - 12x}$. **18.**



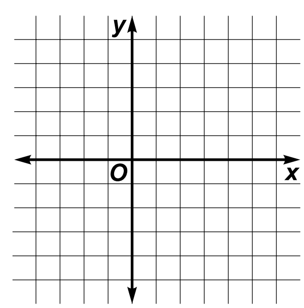
19. If y varies directly as x and inversely as the square root of z, and $y = 8$ when $x = 4$ and $z = 16$, find y when $x = 10$ and $z = 25$. **19.** ______________

20. ***Physics*** The kinetic energy E_k of a moving object, measured in joules, varies jointly as the mass m of the object and the square of the speed v. Find the constant of variation k if E_k is 36 joules, m is 4.5 kilograms, and v is 4 meters per second. **20.** ______________

Bonus Given the graph of $p(x)$, sketch the graph of $y = -2p\left[\frac{1}{2}(x - 2)\right] + 2$.



Bonus:



NAME ______________________ DATE ______________ PERIOD ________

Chapter 3 Test, Form 2B

Determine whether the graph of each equation is symmetric with respect to the origin, the x-axis, the y-axis, the line y = x, the line y = −x, or none of these.

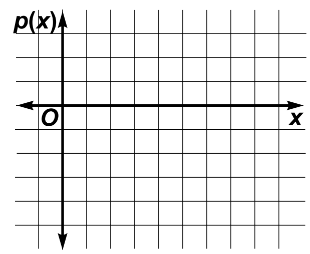
1. $xy = 2$ — **1.** ________________

2. $y = 5x^3 - 2x$ — **2.** ________________

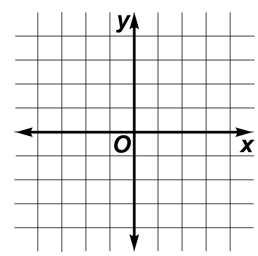
3. Determine whether the function $f(x) = |x|$ is odd, even, or neither. — **3.** ________________

4. Describe the transformations relating the graph of $y = \frac{1}{2}(x - 3)^2$ to its parent function, $y = x^2$. — **4.** ________________

5. Use transformations of the parent graph of $p(x) = \sqrt{x}$ to sketch the graph of $p(x) = -\sqrt{x} - 3$. — **5.**



6. Graph the inequality $y \leq (x - 2)^3$. — **6.**



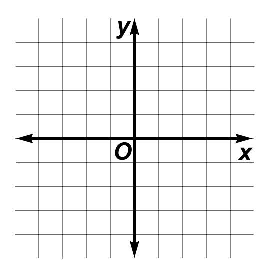
7. Solve $|2x - 4| \leq 10$. — **7.** ________________

Find the inverse of each function and state whether the inverse is a function.

8. $f(x) = x^3 - 4$ — **8.** ________________

9. $f(x) = (x - 1)^2$ — **9.** ________________

10. Graph $f(x) = x^2 + 2$ and its inverse. State whether the inverse is a function. — **10.** ________________



NAME ______________________ DATE ____________ PERIOD ________

Chapter 3 Test, Form 2B (continued)

Determine whether each function is continuous at the given x-value. If discontinuous, state the type of discontinuity (point, jump, or infinite).

11. $f(x) = \begin{cases} x^2 + 1 & \text{if } x < 1 \\ -x + 3 & \text{if } x \geq 1 \end{cases}; x = 1$ **11.** ______________

12. $f(x) = \frac{x^2 - 9}{x + 3}; x = -3$ **12.** ______________

13. Describe the end behavior of $y = 3x^3 - 2x$. **13.** ______________

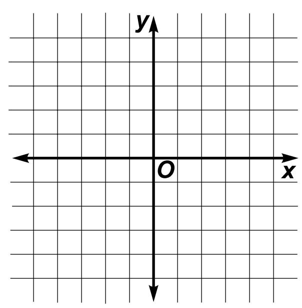
14. Locate and classify the extrema for the graph of $y = x^4 - 3x^2$. **14.** ______________

15. The function $f(x) = -x^3 - 6x^2 - 12x - 7$ has a critical point when $x = -2$. Identify the point as a maximum, a minimum, or a point of inflection, and state its coordinates. **15.** ______________

16. Determine the vertical and horizontal asymptotes for the graph of $y = \frac{x^2 - 4}{x^3 - x^2}$. **16.** ______________

17. Find the slant asymptote for $y = \frac{2x^2 - 5x + 2}{x - 3}$. **17.** ______________

18. Sketch the graph of $y = \frac{x - 1}{x^2 - 3x}$. **18.**



19. If y varies directly as x and inversely as the square of z, and $y = 8$ when $x = 4$ and $z = 3$, find y when $x = 6$ and $z = -2$. **19.** ______________

20. ***Geometry*** The volume V of a sphere varies directly as the cube of the radius r. Find the constant of variation k if V is 288π cubic centimeters and r is 6 centimeters. **20.** ______________

Bonus Determine the value of k such that $f(x) = \begin{cases} 2x^2 \text{ if } x < 2 \\ x + k \text{ if } x \geq 2 \end{cases}$ is continuous when $x = 2$. **Bonus:** ______________

NAME ______________________ DATE ______________ PERIOD ________

Chapter 3 Test, Form 2C

Determine whether the graph of each equation is symmetric with respect to the origin, the x-axis, the y-axis, the line y = x, the line y = −x, or none of these.

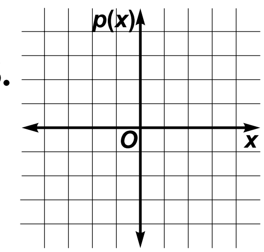
1. $y = 2|x|$ — **1.** ______________

2. $y = x^3 - x$ — **2.** ______________

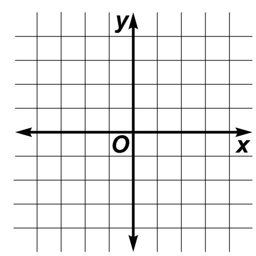
3. Determine whether the function $f(x) = x^2 + 2$ is odd, even, or neither. — **3.** ______________

4. Describe the transformation relating the graph of $y = (x - 1)^2$ to its parent function, $y = x^2$. — **4.** ______________

5. Use transformations of the parent graph $p(x) = x^3$ to sketch the graph of $p(x) = (x + 2)^3 - 1$. — **5.**



6. Graph the inequality $y \geq x^2 - 1$. — **6.**



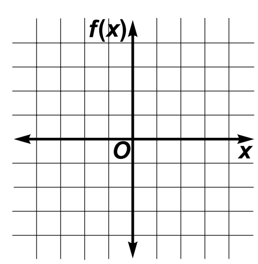
7. Solve $|x - 4| \leq 8$. — **7.** ______________

Find the inverse of each function and state whether the inverse is a function.

8. $f(x) = x^2$ — **8.** ______________

9. $f(x) = x^3 - 1$ — **9.** ______________

10. Graph $f(x) = -2x + 4$ and its inverse. State whether the inverse is a function. — **10.** ______________



NAME ______________________ DATE ______________ PERIOD ________

Chapter 3 Test, Form 2C (continued)

Determine whether each function is continuous at the given x-value. If discontinuous, state the type of discontinuity (point, jump, or infinite).

11. $f(x) = \begin{cases} x^2 + 1 \text{ if } x < 0 \\ -x \text{ if } x \geq 0 \end{cases}; x = 0$ **11.** ______________

12. $f(x) = \frac{x + 3}{x^2 + 9}; x = -3$ **12.** ______________

13. Describe the end behavior of $y = x^4 - x^2$. **13.** ______________

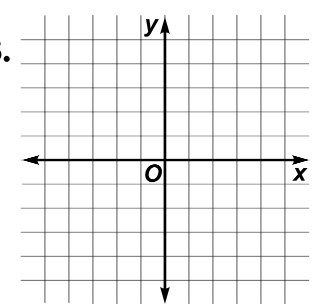
14. Locate and classify the extrema for the graph of $y = -x^4 - 2x^2$. **14.** ______________

15. The function $f(x) = x^3 - 3x$ has a critical point when $x = 0$. Identify the point as a maximum, a minimum, or a point of inflection, and state its coordinates. **15.** ______________

16. Determine the vertical and horizontal asymptotes for the graph of $y = \frac{x - 4}{x^2 - 25}$. **16.** ______________

17. Find the slant asymptote for $y = \frac{x^2 - x + 2}{x - 1}$. **17.** ______________

18. Sketch the graph of $y = \frac{x}{x^2 - 4}$. **18.**



19. If y varies directly as the square of x, and $y = 200$ when $x = 5$, find y when $x = 2$. **19.** ______________

20. If y varies inversely as the cube root of x, and $y = 10$ when $x = 27$, find y when $x = 8$. **20.** ______________

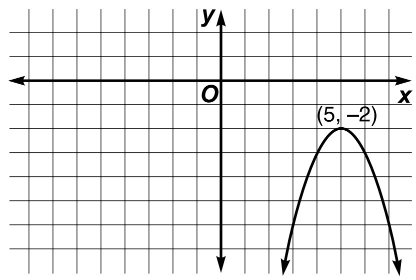
Bonus Determine the value of k such that $f(x) = 3x^2 + kx - 4$ is an even function. **Bonus:** ______________

NAME ______________________ DATE ______________ PERIOD ________

Chapter 3 Open-Ended Assessment

Instructions: Demonstrate your knowledge by giving a clear, concise solution to each problem. Be sure to include all relevant drawings and justify your answers. You may show your solution in more than one way or investigate beyond the requirements of the problem.

1. a. Draw the parent graph and each of the three transformations that would result in the graph shown below. Write the equation for each graph and describe the transformation.



b. Draw the parent graph of a polynomial function. Then draw a transformation of the graph. Describe the type of transformation you performed. Use equations as needed to clarify your answer.

c. Sketch the graph of $y = \frac{x - 4}{x(x + 3)(x - 4)}$. Describe how you determined the shape of the graph and its vertical asymptotes.

d. Write the equation of a graph that has a vertical asymptote at $x = 2$ and point discontinuity at $x = -1$. Describe your reasoning.

2. The critical points of $f(x) = \frac{1}{5}x^5 - \frac{1}{4}x^4 - \frac{2}{3}x^3 + 2$ are at $x = 0$, 2, and -1. Determine whether each of these critical points is the location of a maximum, a minimum, or a point of inflection. Explain why $x = 3$ is not a critical point of the function.

3. Mrs. Custer has 100 bushels of soybeans to sell. The current price for soybeans is \$6.00 a bushel. She expects the market price of a bushel to rise in the coming weeks at a rate of \$0.10 per week. For each week she waits to sell, she loses 1 bushel due to spoilage.

a. Find a function to express Mrs. Custer's total income from selling the soybeans. Graph the function and determine when Mrs. Custer should sell the soybeans in order to maximize her income. Justify your answer by showing that selling a short time earlier or a short time later would result in less income.

b. Suppose Mrs. Custer loses 5 bushels per week due to spoilage. How would this affect her selling decision?

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Chapter 3 Mid-Chapter Test (Lessons 3-1 through 3-4)

Determine whether the graph of each equation is symmetric with respect to the origin, the x-axis, the y-axis, the line y = x, the line y = −x, or none of these.

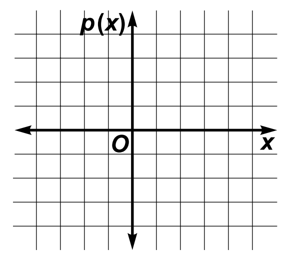
1. $x = y^2 - 1$ **1.** ______________

2. $xy = 8$ **2.** ______________

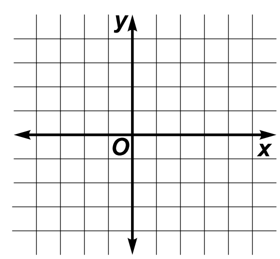
3. Determine whether the function $f(x) = 2(x - 1)^2 + 4x$ is odd, even, or neither. **3.** ______________

4. Describe the transformations that relate the graph of $p(x) = (0.5x)^3 + 1$ to its parent function $p(x) = x^3$. **4.** ______________

5. Use transformations of the parent graph $p(x) = [\![x]\!]$ to sketch the graph of $p(x) = 2[\![x - 3]\!]$. **5.**



6. Graph the inequality $y \leq \sqrt{x - 1}$. **6.**



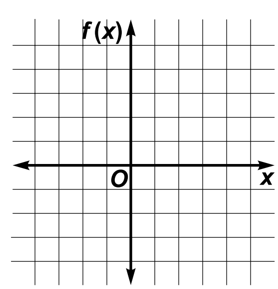
7. Solve $|x - 4| \geq 6$. **7.** ______________

Find the inverse of each function and state whether the inverse is a function.

8. $f(x) = x^3 + 1$ **8.** ______________

9. $f(x) = -\frac{1}{x}$ **9.** ______________

10. Graph $f(x) = (x - 3)^2$ and its inverse. State whether the inverse is a function. **10.** ______________



Chapter 3

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Chapter 3 Quiz A (Lessons 3-1 and 3-2)

Determine whether the graph of each equation is symmetric with respect to the origin, the x-axis, the y-axis, the line y = x, the line y = −x, or none of these.

1. $y = (x - 2)^3$ **2.** $y = \frac{1}{x^4}$

1. ______________

2. ______________

3. Determine whether the function $f(x) = x^3 - 4x$ is odd, even, or neither.

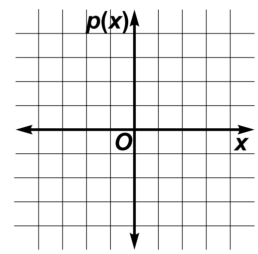
3. ______________

4. Describe how the graphs of $f(x) = |x|$ and $g(x) = -4|x|$ are related.

4. ______________

5. Use transformations of the parent graph of $p(x) = x^2$ to sketch the graph of $p(x) = (x - 1)^2 - 2$.

5.



Chapter 3

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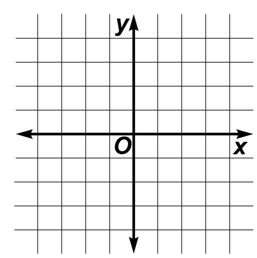
Chapter 3 Quiz B (Lessons 3-3 and 3-4)

1. Graph $y > -x^2 + 1$.

1.



2. Solve $|x + 3| > 4$.

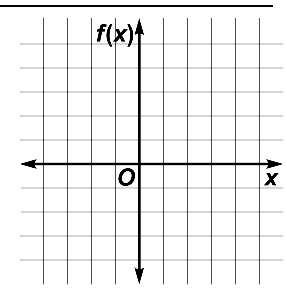
2. ______________

3. Find the inverse of $f(x) = 3x - 6$. Is the inverse a function?

3. ______________

4. Graph $f(x) = (x - 1)^2 + 2$ and its inverse.

4.



5. What is the equation of the line that acts as a line of symmetry between the graphs of $f(x)$ and $f^{-1}(x)$?

5. ______________

Chapter 3

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Chapter 3 Quiz C (Lessons 3-5 and 3-6)

Determine whether the function is continuous at the given x-value. If discontinuous, state the type of discontinuity (jump, infinite, or point).

1. $f(x) = \begin{cases} x^2 + 1 \text{ if } x < 0 \\ x \text{ if } x \geq 0 \end{cases}$; $x = 0$ **1.** ______________

2. $f(x) = \frac{1}{x^2 - 1}$; $x = 1$ **2.** ______________

3. Describe the end behavior of $y = x^4 - 3x + 1$. **3.** ______________

4. Locate and classify the extrema for the graph of $y = 2x^3 - 6x$. **4.** ______________

5. The function $f(x) = \frac{x^5}{5} - \frac{4}{3}x^3$ has a critical point at $x = 2$. Identify the point as a maximum, a minimum, or a point of inflection, and state its coordinates. **5.** ______________

Chapter 3

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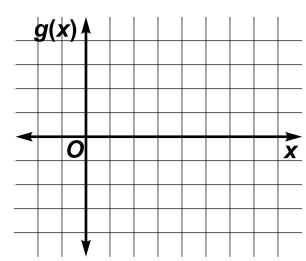
Chapter 3 Quiz D (Lessons 3-7 and 3-8)

1. Determine the equations of the vertical and horizontal asymptotes, if any, for the graph of $y = \frac{x^2}{x^2 - 5x + 6}$. **1.** ______________

2. Find the slant asymptote for $f(x) = \frac{x^2 - 4x + 3}{x - 2}$. **2.** ______________

3. Use the parent graph $f(x) = \frac{1}{x}$ to graph the function $g(x) = \frac{2}{x - 3}$. Describe the transformation(s) that have taken place. Identify the new locations of the asymptotes. **3.** ______________



Find the constant of variation for each relation and use it to write an equation for each statement. Then solve the equation.

4. If y varies directly as x, and $y = 12$ when $x = 8$, find y when $x = 14$. **4.** ______________

5. If y varies jointly as x and the cube of z, and $y = 48$ when $x = 3$ and $z = 2$, find y when $x = -2$ and $z = 5$. **5.** ______________

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Chapter 3 SAT and ACT Practice

After working each problem, record the correct answer on the answer sheet provided or use your own paper.

Multiple Choice

1. If $x = y - 4$, what is the value of $|y - x| + |x - y|$?
A 0
B 4
C 8
D 16
E It cannot be determined from the information given.

2. If $2x = 5 - 4y$ and $12y = 6x - 15$, then which of the following is true?
A $2y = 5$
B $2x = 5$
C $y + 2x < 5$
D $x + 2y > 5$
E $2y - x > 5$

3. If $x = \frac{4}{5}$, then $3(x - 1) + 4x$ is how many fifths?
A 13
B 15
C 17
D 23
E 25

4. If $x^2 + 3y = 10$ and $\frac{x}{y} = -1$, then which of the following could be a value of x?
A -5
B -2
C -1
D 2
E 3

5. 168:252 as 6: __?__
A 4
B 5
C 9
D 18
E 36

6. $(3.1 - 12.4) - (3.1 + 12.4) =$
A -24.8
B -6.2
C 0
D 6.2
E 24.8

7. If $x = 5z$ and $y = \frac{1}{15z + 2}$, then which of the following is equivalent to y?
A $\frac{5}{x + 2}$
B $\frac{1}{3x}$
C $\frac{1}{x + 2}$
D $\frac{1}{3x + 2}$
E None of these

8. If $\frac{x + y}{x} = 3$ and $\frac{z + y}{z} = 5$, then which of the following is the value of $\frac{z}{x}$?
A $\frac{1}{2}$
B $\frac{3}{5}$
C $\frac{5}{3}$
D 2
E 15

9. $\sqrt{1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32}} =$
A $\frac{3\sqrt{6}}{8}$
B $\frac{\sqrt{62}}{8}$
C $\frac{\sqrt{2}}{4}$
D $\frac{1}{4}$
E $\frac{3\sqrt{14}}{8}$

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Chapter 3 SAT and ACT Practice (continued)

10. If 12 and 18 each divide R without a remainder, which of the following could be the value of R?
A 24
B 36
C 48
D 54
E 120

11. If $x + y = 5$ and $x - y = 3$, then $x^2 - y^2 =$
A 2
B 4
C 8
D 15
E 16

12. If $n < 0$, then which of the following statements are true?
I. $\sqrt[3]{-n} < 0$
II. $\sqrt{1 - n} < 1$
III. $\sqrt[5]{n^3} > 0$
A I only
B III only
C I and III only
D II and III only
E None of the statements are true.

13. If x^* is defined to be $x - 2$, which of the following is the value of $(3^* + 4^*)^*$?
A 0
B 1
C 2
D 3
E 4

14. Which of the following is the total cost of $3\frac{1}{2}$ pounds of apples at \$0.56 per pound and $4\frac{1}{2}$ pounds of bananas at \$0.44 per pound?
A \$2.00
B \$3.00
C \$3.94
D \$4.00
E \$4.06

15. If a and b are real numbers where $a < b$ and $a + b < 0$, which of the following MUST be true?
A $a < 0$
B $b < 0$
C $|a| < |b|$
D $ab > 0$
E It cannot be determined from the information given.

16. For all x, $(3x^3 - 13x^2 + 6x - 8)(x - 4) =$
A $3x^4 - 25x^3 - 46x^2 + 16x - 32$
B $3x^4 - 25x^3 - 46x^2 - 32x + 32$
C $3x^4 - x^3 - 46x^2 + 16x - 32$
D $3x^4 - 25x^3 + 58x^2 - 32x + 32$
E $3x^4 - 25x^3 + 58x^2 + 16x + 32$

17–18. Quantitative Comparison
A if the quantity in Column A is greater
B if the quantity in Column B is greater
C if the two quantities are equal
D if the relationship cannot be determined from the information given

	Column A	Column B
17.	The smaller value of x in the equation $x^2 - 7x + 12 = 0$	The larger value of x in the equation $x^2 + 7x + 12 = 0$
18.	$(2x - 3)^2$	$4x(x - 3)$

19. Grid-In For nonzero real numbers $a, b, c,$ and d, $d = 2a$, $a = 2b$, and $b = 2c$. What is the value of $\frac{d}{c}$?

20. Grid-In If $\dfrac{3}{4 + \frac{x+1}{x}} = \dfrac{3}{4}$, what is the value of x?

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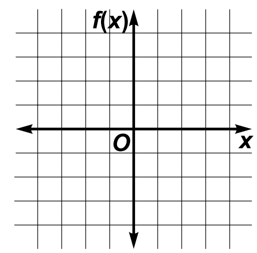
Chapter 3, Cumulative Review (Chapters 1-3)

1. State the domain and range of $\{(-2, 2), (0, 2), (2, 2)\}$. Then state whether the relation is a function. Write *yes* or *no*. **1.** ______________

2. If $f(x) = \frac{3}{x - 1}$ and $g(x) = x + 1$, find $f(g(x))$. **2.** ______________

3. Write the standard form of the equation of the line that is parallel to the line with equation $y = 2x - 3$ and passes through the point at $(-1, 5)$. **3.** ______________

4. Graph $f(x) = [\![x + 1]\!]$. **4.**



Solve each system of equations algebraically.

5. $3x + 5y = 21$
$x + y = 5$ **5.** ______________

6. $x - 2y + z = 7$
$3x + y - z = 2$
$2x + 3y + 2z = 7$ **6.** ______________

7. How many solutions does a consistent and independent system of linear equations have? **7.** ______________

8. Find the value of $\begin{vmatrix} 3 & -7 \\ 5 & -2 \end{vmatrix}$. **8.** ______________

9. Determine whether the graph of $y = x^3 - 2x$ is symmetric with respect to the origin, the x-axis, the y-axis, or none of these. **9.** ______________

10. Describe the transformations relating the graph of $y = 2(x - 1)^2$ to the graph of $y = x^2$. **10.** ______________

11. Solve $|3x - 6| < 9$. **11.** ______________

12. Find the inverse of the function $f(x) = \frac{3}{x - 2}$. **12.** ______________

13. State the type of discontinuity (jump, infinite, or point) that is present in the graph of $f(x) = [\![x]\!]$. **13.** ______________

14. Determine the horizontal asymptote for the graph of $f(x) = \frac{x - 3}{x + 5}$. **14.** ______________

15. If y varies inversely as the square root of x, and $y = 20$ when $x = 9$, find y when $x = 16$. **15.** ______________

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SAT and ACT Practice Answer Sheet

(10 Questions)

1 (A) (B) (C) (D) (E)

2 (A) (B) (C) (D) (E)

3 (A) (B) (C) (D) (E)

4 (A) (B) (C) (D) (E)

5 (A) (B) (C) (D) (E)

6 (A) (B) (C) (D) (E)

7 (A) (B) (C) (D) (E)

8 (A) (B) (C) (D) (E)

9 (A) (B) (C) (D) (E)

10

	/	/	
.	.	.	.
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

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SAT and ACT Practice Answer Sheet

(20 Questions)

1 (A) (B) (C) (D) (E)

2 (A) (B) (C) (D) (E)

3 (A) (B) (C) (D) (E)

4 (A) (B) (C) (D) (E)

5 (A) (B) (C) (D) (E)

6 (A) (B) (C) (D) (E)

7 (A) (B) (C) (D) (E)

8 (A) (B) (C) (D) (E)

9 (A) (B) (C) (D) (E)

10 (A) (B) (C) (D) (E)

11 (A) (B) (C) (D) (E)

12 (A) (B) (C) (D) (E)

13 (A) (B) (C) (D) (E)

14 (A) (B) (C) (D) (E)

15 (A) (B) (C) (D) (E)

16 (A) (B) (C) (D) (E)

17 (A) (B) (C) (D) (E)

18 (A) (B) (C) (D) (E)

19

	(/)	(/)	
(.)	(.)	(.)	(.)
	(0)	(0)	(0)
(1)	(1)	(1)	(1)
(2)	(2)	(2)	(2)
(3)	(3)	(3)	(3)
(4)	(4)	(4)	(4)
(5)	(5)	(5)	(5)
(6)	(6)	(6)	(6)
(7)	(7)	(7)	(7)
(8)	(8)	(8)	(8)
(9)	(9)	(9)	(9)

20

	(/)	(/)	
(.)	(.)	(.)	(.)
	(0)	(0)	(0)
(1)	(1)	(1)	(1)
(2)	(2)	(2)	(2)
(3)	(3)	(3)	(3)
(4)	(4)	(4)	(4)
(5)	(5)	(5)	(5)
(6)	(6)	(6)	(6)
(7)	(7)	(7)	(7)
(8)	(8)	(8)	(8)
(9)	(9)	(9)	(9)

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3-1

Practice

Symmetry and Coordinate Graphs

Determine whether the graph of each function is symmetric with respect to the origin.

1. $f(x) = \frac{-12}{x}$ **yes**

2. $f(x) = x^5 - 2$ **no**

3. $f(x) = x^3 - 4x$ **yes**

4. $f(x) = \frac{x^2}{3 - x}$ **no**

Determine whether the graph of each equation is symmetric with respect to the x-axis, the y-axis, the line y = x, the line y = −x, or none of these.

5. $x + y = 6$ **$y = x$**

6. $x^2 + y = 2$ **y-axis**

7. $xy = 3$ **$y = x$ and $y = -x$**

8. $x^3 + y^2 = 4$ **x-axis**

9. $y = 4x$ **none of these**

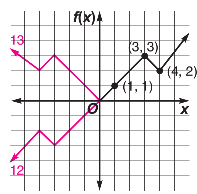
10. $y = x^2 - 1$ **y-axis**

11. Is $f(x) = |x|$ an even function, an odd function, or neither? **even**

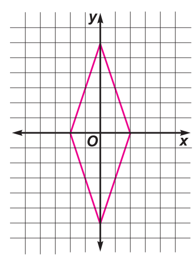
Refer to the graph at the right for Exercises 12 and 13.

12. Complete the graph so that it is the graph of an odd function.

13. Complete the graph so that it is the graph of an even function.



14. ***Geometry*** Cameron told her friend Juanita that the graph of $|y| = 6 - |3x|$ has the shape of a geometric figure. Determine whether the graph of $|y| = 6 - |3x|$ is symmetric with respect to the x-axis, the y-axis, both, or neither. Then make a sketch of the graph. Is Cameron correct? **x-axis and y-axis; yes, the graph is that of a diamond or parallelogram.**



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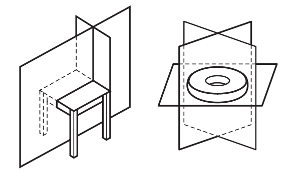
3-1

Enrichment

Symmetry in Three-Dimensional Figures

A solid figure that can be superimposed, point for point, on its mirror image has a *plane of symmetry*. A symmetrical solid object may have a finite or infinite number of planes of symmetry. The chair in the illustration at the right has just one plane of symmetry; the doughnut has infinitely many planes of symmetry, three of which are shown.



Determine the number of planes of symmetry for each object and describe the planes.

1. a brick

3 planes of symmetry; each plane is parallel to a pair of opposite faces.

2. a tennis ball

An infinite number of planes; each plane passes through the center.

3. a soup can

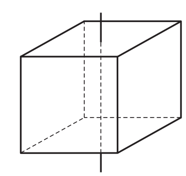
an infinite number of planes passing through the central axis, plus one plane cutting the center of the axis at right angles

4. a square pyramid

4 planes all passing through the top vertex: 2 planes are parallel to a pair of opposite edges of the base and the other 2 cut along the diagonals of the square base.

5. a cube

9 planes: 3 planes are parallel to pairs of opposite faces and the other 6 pass through pairs of opposite edges.

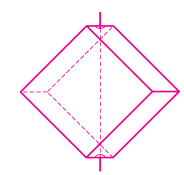


Solid figures can also have *rotational symmetry*. For example, the axis drawn through the cube in the illustration is a fourfold axis of symmetry because the cube can be rotated about this axis into four different positions that are exactly alike.

6. How many four-fold axes of symmetry does a cube have? Use a die to help you locate them.

3; each axis passes through the centers of a pair of opposite faces.

7. A cube has 6 two-fold axes of symmetry. In the space at the right, draw one of these axes.



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3-2

Practice

Families of Graphs

Describe how the graphs of f(x) and g(x) are related.

1. $f(x) = x^2$ and $g(x) = (x + 3)^2 - 1$

g(x) is the graph of f(x) translated left 3 units and down 1 unit.

2. $f(x) = |x|$ and $g(x) = -|2x|$

g(x) is the graph of f(x) reflected over the x-axis and compressed horizontally by a factor of 0.5.

Use the graph of the given parent function to describe the graph of each related function.

3. $f(x) = x^3$

a. $y = 2x^3$

expanded vertically by a factor of 2

b. $y = -0.5(x - 2)^3$

reflected over the x-axis, translated right 2 units, compressed vertically by a factor of 0.5

c. $y = |(x + 1)^3|$

translated left 1 unit, portion below the x-axis reflected so that it is above the x-axis

4. $f(x) = \sqrt{x}$

a. $y = \sqrt{x + 3} + 1$

translated left 3 units and up 1 unit

b. $y = \sqrt{-x} - 2$

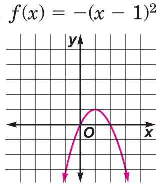
reflected over the y-axis, translated down 2 units

c. $y = \sqrt{0.25x} - 4$

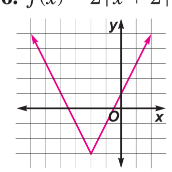
expanded horizontally by a factor of 4, translated down 4 units

Sketch the graph of each function.

5. $f(x) = -(x - 1)^2 + 1$

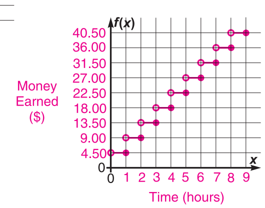


6. $f(x) = 2|x + 2| - 3$



7. ***Consumer Costs*** During her free time, Jill baby-sits the neighborhood children. She charges $4.50 for each whole hour or any fraction of an hour. Write and graph a function that shows the cost of x hours of baby-sitting.

$$f(x) = \begin{cases} 4.5 \text{ if } [\![x]\!] = x \\ 4.5[\![x + 1]\!] \text{ if } [\![x]\!] < x \end{cases}$$



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3-2

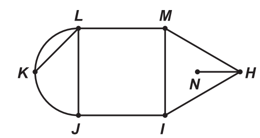
Enrichment

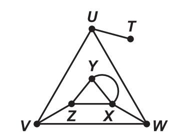
Isomorphic Graphs

A **graph** G is a collection of points in which a pair of points, called **vertices**, are connected by a set of segments or arcs, called **edges**. The **degree** of vertex C, denoted deg (C), is the number of edges connected to that vertex. We say two graphs are **isomorphic** if they have the same structure. The definition below will help you determine whether two graphs are isomorphic.

> A graph G' is isomorphic to a graph G if the following conditions hold.
> 1. G and G' have the same number of vertices and edges.
> 2. The degree of each vertex in G is the same as the degree of each corresponding vertex in G'.
> 3. If two vertices in G are joined by k ($k \geq 0$) edges, then the two corresponding vertices in G' are also joined by k edges.

Example **In the graphs below $HIJKLMN \leftrightarrow TUVWXYZ$. Determine whether the graphs are isomorphic.**



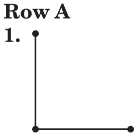


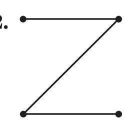
Number of vertices in G: 7		Number of vertices in G': 7	
Number of edges in G: 10		Number of edges in G': 10	
deg (H): 3	deg (I): 3	deg (T): 1	deg (U): 3
deg (J): 3	deg (K): 3	deg (V): 3	deg (W): 3
deg (L): 4	deg (M): 3	deg (X): 4	deg (Y): 3
deg (N): 1		deg (Z): 3	

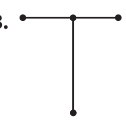
Since there are the same number of vertices and the same number of edges and there are five vertices of degree 3, one vertex of degree 4, and one vertex of degree 1 in both graphs, we can assume they are isomorphic.

Each graph in Row A is isomorphic to one graph in Row B. Match the graphs that are isomorphic.

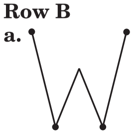
Row A

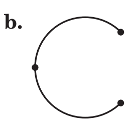
1.  **b**

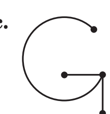
2.  **a**

3.  **c**

Row B

a. 

b. 

c. 

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3-3 Practice

Graphs of Nonlinear Inequalities

Determine whether the ordered pair is a solution for the given inequality. Write yes or no.

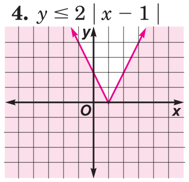
1. $y > (x + 2)^2 + 3$, $(-2, 6)$ **yes**

2. $y < (x - 3)^3 + 2$, $(4, 5)$ **no**

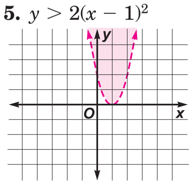
3. $y \leq |2x - 4| - 1$, $(-4, 1)$ **yes**

Graph each inequality.

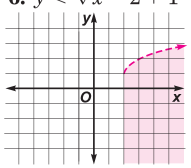
4. $y \leq 2|x - 1|$



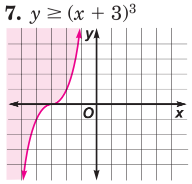
5. $y > 2(x - 1)^2$



6. $y < \sqrt{x - 2} + 1$



7. $y \geq (x + 3)^3$



Solve each inequality.

8. $|4x - 10| \leq 6$ **$\{x \mid 1 \leq x \leq 4\}$**

9. $|x + 5| + 2 > 6$ **$\{x \mid x < -9$ or $x > -1\}$**

10. $|2x - 2| - 1 < 7$ **$\{x \mid -3 < x < 5\}$**

11. ***Measurement*** Instructions for building a birdhouse warn that the platform, which ideally measures 14.75 cm^2, should not vary in size by more than 0.30 cm^2. If it does, the preconstructed roof for the birdhouse will not fit properly.

a. Write an absolute value inequality that represents the range of possible sizes for the platform. Then solve for x to find the range. **$|x - 14.75| \leq 0.30$; $\{x \mid 14.45 \leq x \leq 15.05\}$**

b. Dena cut a board 14.42 cm^2. Does the platform that Dena cut fit within the acceptable range? **no**

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3-3 Enrichment

Some Parametric Graphs

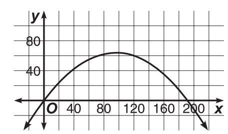
For some curves, the coordinates x and y can be written as functions of a third variable. The conditions determining the curve are given by two equations, rather than by a single equation in x and y. The third variable is called a *parameter*, and the two equations are called *parametric equations* of the curve.

For the curves you will graph on this page, the parameter is t and the parametric equations of each curve are in the form $x = f(t)$ and $y = g(t)$.

Example **Graph the curve associated with the parametric equations $x = 48t$ and $y = 64t - 16t^2$.**

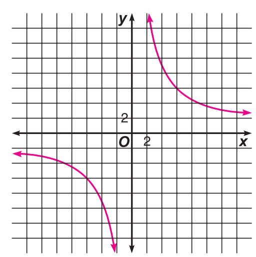
Choose values for t and make a table showing the values of all three variables. Then graph the x- and y-values.

t	x	y
–1	–48	–80
0	0	0
0.5	24	28
1	48	48
2	96	64
3	144	48
4	192	0

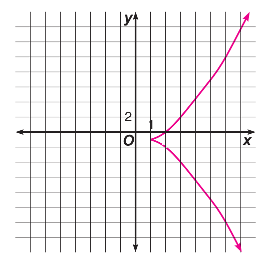


Graph each curve.

1. $x = 3t$, $y = \frac{12}{t}$



2. $x = t^2 + 1$, $y = t^3 - 1$



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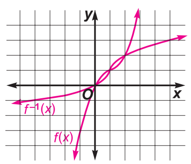
3-4 Practice

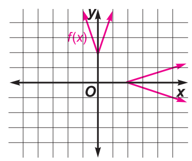
Inverse Functions and Relations

Graph each function and its inverse.

1. $f(x) = (x - 1)^3 + 1$



2. $f(x) = 3|x| + 2$



Find the inverse of f(x). Then state whether the inverse is also a function.

3. $f(x) = -4x^2 + 1$

inverse: $y = \pm\frac{1}{2}\sqrt{1 - x}$
no

4. $f(x) = \sqrt[3]{x - 1}$

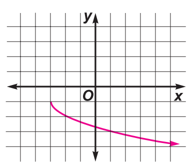
$f^{-1}(x) = x^3 + 1$
yes

5. $f(x) = \frac{4}{(x - 3)^2}$

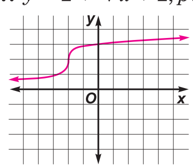
inverse: $y = 3 \pm \frac{2\sqrt{x}}{x}$
no

Graph each equation using the graph of the given parent function.

6. $y = -\sqrt{x + 3} - 1, p(x) = x^2$



7. $y = 2 + \sqrt[5]{x + 2}, p(x) = x^5$



8. ***Fire Fighting*** Airplanes are often used to drop water on forest fires in an effort to stop the spread of the fire. The time t it takes the water to travel from height h to the ground can be derived from the equation $h = \frac{1}{2}gt^2$ where g is the acceleration due to gravity (32 feet/second^2).

a. Write an equation that will give time as a function of height.

$t = \sqrt{\frac{h}{16}}$ or $\frac{\sqrt{h}}{4}$

b. Suppose a plane drops water from a height of 1024 feet. How many seconds will it take for the water to hit the ground? **8 seconds**

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3-4 Enrichment

An Inverse Acrostic

The puzzle on this page is called an acrostic. To solve the puzzle, work back and forth between the clues and the puzzle box. You may need a math dictionary to help with some of the clues.

1. If a relation contains the element (e, v), then the inverse of the relation must contain the element (__, __). — **V E** (17 28)

2. The inverse of the function $2x$ is found by computing __ of x. — **H A L F** (2 29 6 27)

3. The first letter and the last two letters of the meaning of the symbol f^{-1} are __. — **I S E** (31 33 14)

4. This is the product of a number and its multiplicative inverse. — **O N E** (20 11 34)

5. If the second coordinate of the inverse of $(x, f(x))$ is y, then the first coordinate is read "__ of __". — **F Y** (36 7)

6. The inverse ratio of two numbers is the __ of the reciprocals of the numbers. — **R A T I O** (24 16 19 10 4)

7. If $\cdot$ is a binary operation on set S and $x \cdot e = e \cdot x = x$ for all x in S, then an identity element for the operation is __. — **E** (18)

8. To solve a matrix equation, multiply each side of the matrix equation on the __ by the inverse matrix. — **L E F T** (35 3 21 8)

9. Two variables are inversely proportional __ their product is constant. — **W H E N** (13 9 22 5)

10. The graph of the inverse of a linear function is a __ line. — **S T R A I G H T** (26 32 30 23 25 12 15 1)

From President Franklin D. Roosevelt's inaugural address during the Great Depression; delivered March 4, 1933.

	1 T	2 H	3 E		4 O	5 N	6 L	7 Y		8 T	9 H	10 I	11 N	12 G	
13 W	14 E		15 H	16 A	17 V	18 E		19 T	20 O		21 F	22 E	23 A	24 R	
	25 I	26 S		27 F	28 E	29 A	30 R		31 I	32 T	33 S	34 E	35 L	36 F	

3-5

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Practice

Continuity and End Behavior

Determine whether each function is continuous at the given x-value. Justify your answer using the continuity test.

1. $y = \frac{2}{3x^2}$; $x = -1$
Yes; the function is defined at $x = -1$; y approaches $\frac{2}{3}$ as x approaches -1 from both sides; $f(-1) = \frac{2}{3}$.

2. $y = \frac{x^2 + x + 4}{2}$; $x = 1$
Yes; the function is defined at $x = 1$; y approaches 3 as x approaches 1 from both sides; $f(1) = 3$.

3. $y = x^3 - 2x + 2$; $x = 1$
Yes; the function is defined at $x = 1$; y approaches 1 as x approaches 1 from both sides; $f(1) = 1$.

4. $y = \frac{x - 2}{x + 4}$; $x = -4$
No; the function is undefined at $x = -4$.

Describe the end behavior of each function.

5. $y = 2x^5 - 4x$
$y \to \infty$ as $x \to \infty$, $y \to -\infty$ as $x \to -\infty$

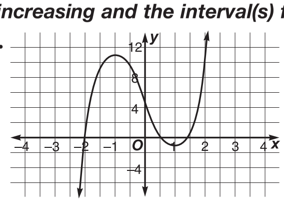
6. $y = -2x^6 + 4x^4 - 2x + 1$
$y \to -\infty$ as $x \to \infty$, $y \to -\infty$ as $x \to -\infty$

7. $y = x^4 - 2x^3 + x$
$y \to \infty$ as $x \to \infty$, $y \to \infty$ as $x \to -\infty$

8. $y = -4x^3 + 5$
$y \to -\infty$ as $x \to \infty$, $y \to \infty$ as $x \to -\infty$

Given the graph of the function, determine the interval(s) for which the function is increasing and the interval(s) for which the function is decreasing.

9.



increasing for $x < -1$ and $x > 1$; decreasing for $-1 < x < 1$

10. *Electronics* Ohm's Law gives the relationship between resistance R, voltage E, and current I in a circuit as $R = \frac{E}{I}$. If the voltage remains constant but the current keeps increasing in the circuit, what happens to the resistance?
Resistance decreases and approaches zero.

3-5

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Enrichment

Reading Mathematics

The following selection gives a definition of a continuous function as it might be defined in a college-level mathematics textbook. Notice that the writer begins by explaining the notation to be used for various types of intervals. It is a common practice for college authors to explain their notation, since, although a great deal of the notation is standard, each author usually chooses the notation he or she wishes to use.

> Throughout this book, the set S, called the domain of definition of a function, will usually be an interval. An interval is a set of numbers satisfying one of the four inequalities $a < x < b$, $a \le x < b$, $a < x \le b$, or $a \le x \le b$. In these inequalities, $a \le b$. The usual notations for the intervals corresponding to the four inequalities are, respectively, (a, b), $[a, b)$, $(a, b]$, and $[a, b]$.
>
> An interval of the form (a, b) is called *open*, an interval of the form $[a, b)$ or $(a, b]$ is called *half-open* or *half-closed*, and an interval of the form $[a, b]$ is called *closed*.
>
> Suppose I is an interval that is either open, closed, or half-open. Suppose $f(x)$ is a function defined on I and x_0 is a point in I. We say that the function $f(x)$ is continuous at the point x_0 if the quantity $|f(x) - f(x_0)|$ becomes small as $x \in I$ approaches x_0.

Use the selection above to answer these questions.

1. What happens to the four inequalities in the first paragraph when $a = b$?
Only the last inequality can be satisfied.

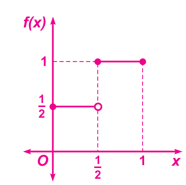
2. What happens to the four intervals in the first paragraph when $a = b$?
The first interval is ø and the others reduce to the point $a = b$.

3. What mathematical term makes sense in this sentence?
If $f(x)$ is not __?__ at x_0, it is said to be discontinuous at x_0.
continuous

4. What notation is used in the selection to express the fact that a number x is contained in the interval I?
$x \in I$

5. In the space at the right, sketch the graph of the function $f(x)$ defined as follows:

$$f(x) = \begin{cases} \frac{1}{2} \text{ if } x \in \left[0, \frac{1}{2}\right) \\ 1, \text{ if } x \in \left[\frac{1}{2}, 1\right] \end{cases}$$



6. Is the function given in Exercise 5 continuous on the interval $[0, 1]$? If not, where is the function discontinuous?
No; it is discontinuous at $x = \frac{1}{2}$.

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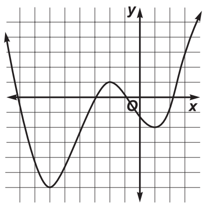
3-6

Practice

Critical Points and Extrema

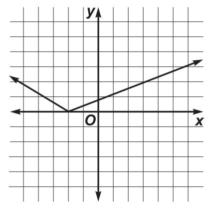
Locate the extrema for the graph of y = f(x). Name and classify the extrema of the function.

1.



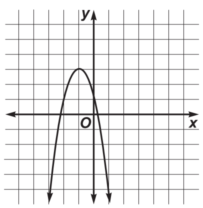
relative maximum: (−2, 1)
relative minimum: (1, −2)
absolute minimum: (−6, −6)

2.



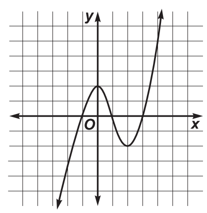
absolute minimum: (−2, 0)

3.



absolute maximum: (−1, 3)

4.



relative maximum: (0, 2)
relative minimum: (2, −2)

Determine whether the given critical point is the location of a maximum, a minimum, or a point of inflection.

5. $y = x^2 - 6x + 1, x = 3$ minimum

6. $y = x^2 - 2x - 6, x = 1$ minimum

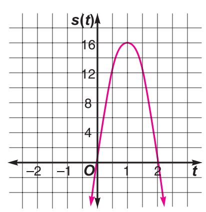
7. $y = x^4 + 3x^2 - 5, x = 0$ minimum

8. $y = x^5 - 2x^3 - 2x^2, x = 0$ maximum

9. $y = x^3 + x^2 - x, x = -1$ maximum

10. $y = 2x^3 + 4, x = 0$ point of inflection

11. ***Physics*** Suppose that during an experiment you launch a toy rocket straight upward from a height of 6 inches with an initial velocity of 32 feet per second. The height at any time t can be modeled by the function $s(t) = -16t^2 + 32t + 0.5$ where $s(t)$ is measured in feet and t is measured in seconds. Graph the function to find the maximum height obtained by the rocket before it begins to fall.
16.5 ft



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3-6

Enrichment

"Unreal" Equations

There are some equations that cannot be graphed on the real-number coordinate system. One example is the equation $x^2 - 2x + 2y^2 + 8y + 14 = 0$. Completing the squares in x and y gives the equation $(x - 1)^2 + 2(y + 2)^2 = -5$.

For any real numbers, x and y, the values of $(x - 1)^2$ and $2(y + 2)^2$ are nonnegative. So, their sum cannot be −5. Thus, no real values of x and y satisfy the equation; only imaginary values can be solutions.

Determine whether each equation can be graphed on the real-number plane. Write yes or no.

1. $(x + 3)^2 + (y - 2)^2 = -4$
no

2. $x^2 - 3x + y^2 + 4y = -7$
no

3. $(x + 2)^2 + y^2 - 6y + 8 = 0$
yes

4. $x^2 + 16 = 0$
no

5. $x^4 + 4y^2 + 4 = 0$
no

6. $x^2 + 4y^2 + 4xy + 16 = 0$
no

In Exercises 7 and 8, for what values of k :

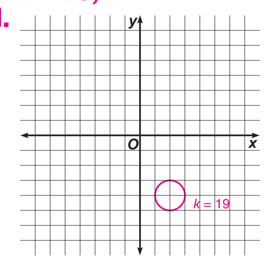
a. will the solutions of the equation be imaginary?

b. will the graph be a point?

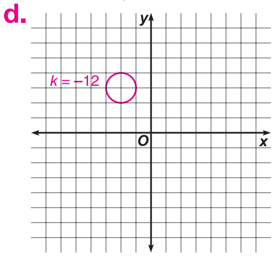
c. will the graph be a curve?

d. Choose a value of k for which the graph is a curve and sketch the curve on the axes provided.

7. $x^2 - 4x + y^2 + 8y + k = 0$
a. $k > 20$; b. $k = 20$;
c. $k < 20$;
d.



8. $x^2 + 4x + y^2 - 6y - k = 0$
a. $k < -13$; b. $k = -13$;
c. $k > -13$;
d.



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3-7 Practice

Graphs of Rational Functions

Determine the equations of the vertical and horizontal asymptotes, if any, of each function.

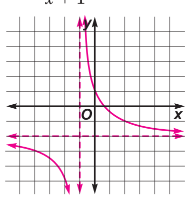
1. $f(x) = \frac{4}{x^2 + 1}$ **$y = 0$**

2. $f(x) = \frac{2x + 1}{x + 1}$ **$x = -1, y = 2$**

3. $g(x) = \frac{x + 3}{(x + 1)(x - 2)}$ **$x = -1, x = 2, y = 0$**

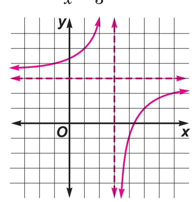
Use the parent graph $f(x) = \frac{1}{x}$ to graph each equation. Describe the transformation(s) that have taken place. Identify the new locations of the asymptotes.

4. $y = \frac{3}{x + 1} - 2$



expanded vertically by a factor of 3; translated left 1 unit and down 2 units; $x = -1, y = -2$

5. $y = -\frac{4}{x - 3} + 3$



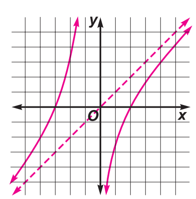
reflected over *x*-axis; expanded vertically by a factor of 4; translated right 3 units and up 3 units; $x = 3, y = 3$

Determine the slant asymptotes of each equation.

6. $y = \frac{5x^2 - 10x + 1}{x - 2}$ **$y = 5x$**

7. $y = \frac{x^2 - x}{x + 1}$ **$y = x - 2$**

8. Graph the function $y = \frac{x^2 + x - 6}{x + 1}$.



9. ***Physics*** The illumination I from a light source is given by the formula $I = \frac{k}{d^2}$, where k is a constant and d is distance. As the distance from the light source doubles, how does the illumination change? **It decreases by one fourth.**

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3-7 Enrichment

Slant Asymptotes

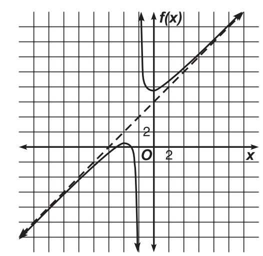
The graph of $y = ax + b$, where $a \neq 0$, is called a slant asymptote of $y = f(x)$ if the graph of $f(x)$ comes closer and closer to the line as $x \to \infty$ or $x \to -\infty$.

For $f(x) = 3x + 4 + \frac{2}{x}$, $y = 3x + 4$ is a slant asymptote because

$f(x) - (3x + 4) = \frac{2}{x}$, and $\frac{2}{x} \to 0$ as $x \to \infty$ or $x \to -\infty$.

Example **Find the slant asymptote of $f(x) = \frac{x^2 + 8x + 15}{x + 2}$.**

$$\begin{array}{r|rrr} -2 & 1 & 8 & 15 \\ & & -2 & -12 \\ \hline & 1 & 6 & 3 \end{array}$$ *Use synthetic division.*



$y = \frac{x^2 + 8x + 15}{x + 2} = x + 6 + \frac{3}{x + 2}$

Since $\frac{3}{x + 2} \to 0$ as $x \to \infty$ or $x \to -\infty$,

$y = x + 6$ is a slant asymptote.

Use synthetic division to find the slant asymptote for each of the following.

1. $y = \frac{8x^2 - 4x + 11}{x + 5}$

$y = 8x - 44$

2. $y = \frac{x^2 + 3x - 15}{x - 2}$

$y = x + 5$

3. $y = \frac{x^2 - 2x - 18}{x - 3}$

$y = x + 1$

4. $y = \frac{ax^2 + bx + c}{x - d}$

$y = ax + b + ad$

5. $y = \frac{ax^2 + bx + c}{x + d}$

$y = ax + b - ad$

NAME ______________ DATE ________ PERIOD _____

3-8

Practice

Direct, Inverse, and Joint Variation

Write a statement of variation relating the variables of each equation. Then name the constant of variation.

1. $-\frac{x^2}{y} = 3$
y varies directly as the square of x; $-\frac{1}{3}$

2. $E = IR$
E varies jointly as I and R; 1

3. $y = 2x$
y varies directly as x; 2

4. $d = 6t^2$
d varies directly as the square of t; 6

Find the constant of variation for each relation and use it to write an equation for each statement. Then solve the equation.

5. Suppose y varies directly as x and $y = 35$ when $x = 5$. Find y when $x = 7$.
7; $y = 7x$; 49

6. If y varies directly as the cube of x and $y = 3$ when $x = 2$, find x when $y = 24$.
$\frac{3}{8}$; $y = \frac{3}{8}x^3$; 4

7. If y varies inversely as x and $y = 3$ when $x = 25$, find x when $y = 10$.
75; $y = \frac{75}{x}$; 7.5

8. Suppose y varies jointly as x and z, and $y = 64$ when $x = 4$ and $z = 8$. Find y when $x = 7$ and $z = 11$.
2; $y = 2xz$; 154

9. Suppose V varies jointly as h and the square of r, and $V = 45\pi$ when $r = 3$ and $h = 5$. Find r when $V = 175\pi$ and $h = 7$.
π; $V = \pi r^2 h$; 5

10. If y varies directly as x and inversely as the square of z, and $y = -5$ when $x = 10$ and $z = 2$, find y when $x = 5$ and $z = 5$.
-2; $y = -2\frac{x}{z^2}$; -0.4

11. ***Finances*** Enrique deposited \$200.00 into a savings account. The simple interest I on his account varies jointly as the time t in years and the principal P. After one quarter (three months), the interest on Enrique's account is \$2.75. Write an equation relating interest, principal, and time. Find the constant of variation. Then find the interest after three quarters.
$I = kPt$; 0.055; \$8.25

NAME ______________ DATE ________ PERIOD _____

3-8

Enrichment

Reading Mathematics: Interpreting Conditional Statements

The conditional statement below is written in "if-then" form. It has the form $p \rightarrow q$ where p is the hypothesis and q is the consequent.

If a matrix A has a determinant of 0, then A^{-1} does not exist.

It is important to recognize that a conditional statement need not appear in "if-then" form. For example, the statement

Any point that lies in Quadrant I has a positive x-coordinate.

can be rewritten as

If the point $P(x, y)$ lies in Quadrant I, then x is positive.

Notice that P lying in Quadrant I is a *sufficient* condition for its x-coordinate to be positive. Another way to express this is to say that P lying in Quadrant I *guarantees* that its x-coordinate is positive. On the other hand, we can also say that x being positive is a *necessary* condition for P to lie in Quadrant I. In other words, P does not lie in Quadrant I if x is not positive.

To change an English statement into "if-then" form requires that you understand the meaning and syntax of the English statement. Study each of the following equivalent ways of expressing $p \rightarrow q$.

- If p then q
- p only if q
- p is a sufficient condition for q
- q is a necessary condition for p.
- p implies q
- only if q, p
- not p unless q

Rewrite each of the following statements in "if-then" form.

1. A consistent system of equations has at least one solution.
If a system of equations is consistent, then the system has at least one solution.

2. When the region formed by the inequalities in a linear programming application is unbounded, an optimal solution for the problem may not exist.
If the region formed by the inequalities in a linear programming application is unbounded, then an optimal solution for the problem may not exist.

3. Functions whose graphs are symmetric with respect to the y-axis are called even functions.
If the graph of a function is symmetric with respect to the y-axis, then the function is even.

4. In order for a decimal number d to be odd, it is sufficient that d end in the digit 7.
If a decimal number d ends in the digit 7, then d is odd.

Chapter 3 Answer Key

Form 1A

Page 111

1. C
2. B
3. D
4. C
5. B
6. D
7. A
8. D
9. A
10. D

Page 112

11. B
12. B
13. C
14. C
15. A
16. D
17. A
18. B
19. A
20. D

Bonus: D

Form 1B

Page 113

1. B
2. A
3. B
4. A
5. C
6. C
7. B
8. A
9. B
10. D

Page 114

11. C
12. A
13. C
14. C
15. C
16. A
17. D
18. B
19. A
20. B

Bonus: B

Chapter 3 Answer Key

Form 1C

Page 115

1. B
2. C
3. C
4. C
5. C
6. C
7. C
8. A
9. B
10. D

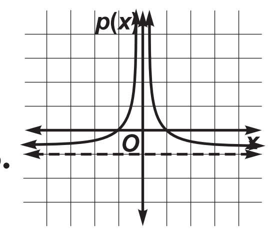
Page 116

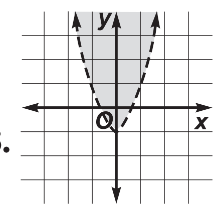
11. A
12. D
13. A
14. C
15. C
16. C
17. B
18. D
19. A
20. B

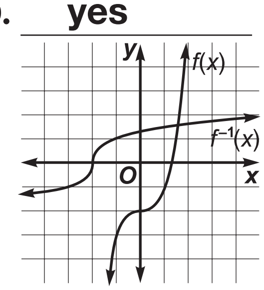
Bonus: A

Form 2A

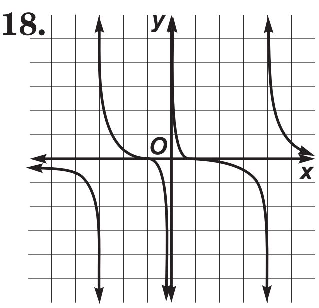
Page 117

1. $y = x$, $y = -x$, *the origin*
2. x-axis
3. odd
4. reflected over the x-axis, expanded vertically by factor of 2, translated up 4 units
5. 

6. 

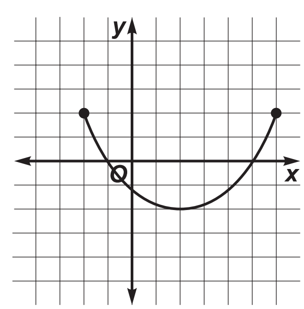
7. $x \leq -3$ or $x \geq 8$
8. $f^{-1}(x) = \frac{2x}{1 - x}$; yes
9. $f^{-1}(x) = \pm\sqrt{x + 4}$; no
10. yes


Page 118

11. discontinuous jump
12. discontinuous infinite
13. $x \to \infty$, $y \to -\infty$, $x \to -\infty$, $y \to -\infty$
14. rel. max. at (0, 2); abs. min. at ($\pm$1.22, $-$0.25)
15. point of inflection at (1, 1)
16. vertical at $x = 0, 3$; horizontal at $y = 0$
17. $y = 3x + 1$
18. 

19. $y = 16$
20. $k = \frac{1}{2}$

Bonus:

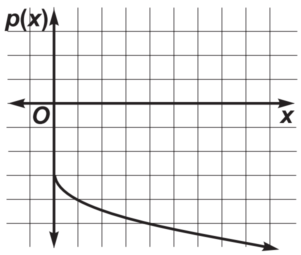


Chapter 3 Answer Key

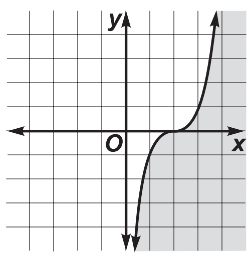
Form 2B

Page 119

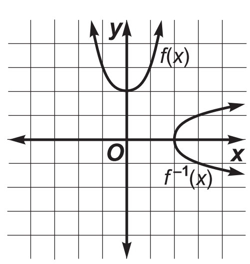
1. $y = x$, $y = -x$, the origin
2. origin
3. even
4. translated right 3 units, compressed vertically by a factor of 0.5
5.



6.

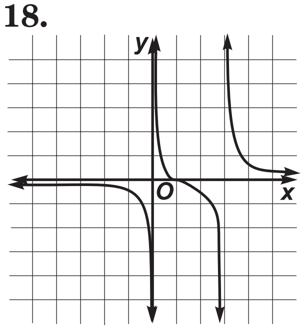


7. $-3 \le x \le 7$
8. $f^{-1}(x) = \sqrt[3]{x + 4}$; yes
9. $f^{-1}(x) = 1 \pm \sqrt{x}$; no
10. no



Page 120

11. continuous
12. discontinuous; point
13. $x \to \infty, y \to -\infty$, $x \to \infty, y \to \infty$
14. rel. max. at (0, 0); abs. min. at (±1.22, −2.25)
15. point of inflection at (−2, 1)
16. vertical at $x = 0$, 1; horizontal at $y = 0$
17. $y = 2x + 1$
18.



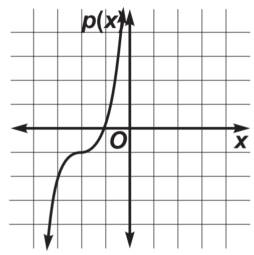
19. $y = 27$
20. $k = \frac{4}{3}\pi$

Bonus: $k = 6$

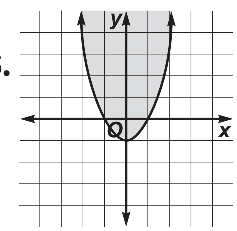
Form 2C

Page 121

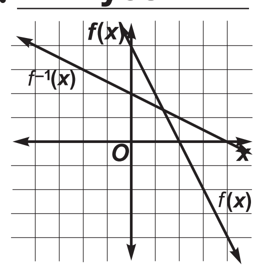
1. y-axis
2. origin
3. even
4. translated right 1 unit
5.



6.

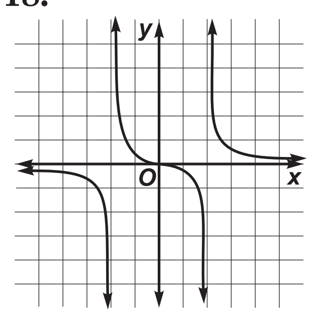


7. $-4 \le x \le 12$
8. $f^{-1}(x) = \pm\sqrt{x}$; no
9. $f^{-1}(x) = \sqrt[3]{x + 1}$; yes
10. yes



Page 122

11. discontinuous; jump
12. continuous
13. $x \to \infty, y \to \infty$, $x \to -\infty, y \to \infty$
14. abs. max. at (0, 0)
15. point of inflection at (0, 0)
16. vertical at $x = \pm 5$; horizontal at $y = 0$
17. $y = x$
18.



19. $y = 32$
20. $y = 15$

Bonus: $k = 0$

Chapter 3 Answer Key

CHAPTER 3 SCORING RUBRIC

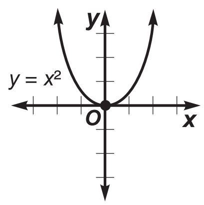
Level	Specific Criteria
3 Superior	• Shows thorough understanding of the concepts *parent graph, transformation asymptote, hole in graph, critical point, maximum, minimum,* and *point of inflection.* • Uses appropriate strategies to solve problems and transform graphs. • Computations are correct. • Written explanations are exemplary. • Graphs are accurate and appropriate. • Goes beyond requirements of some or all problems.
2 Satisfactory, with Minor Flaws	• Shows understanding of the concepts *parent graph, transformation, asymptote, hole in graph, critical point, maximum, minimum,* and *point of inflection.* • Uses appropriate strategies to solve problems and transform graphs. • Computations are mostly correct. • Written explanations are effective. • Graphs are mostly accurate and appropriate. • Satisfies all requirements of problems.
1 Nearly Satisfactory, with Serious Flaws	• Shows understanding of most of the concepts *parent graph, transformation, asymptote, hole in graph, critical point, maximum, minimum,* and *point of inflection.* • May not use appropriate strategies to solve problems and transform graphs. • Computations are mostly correct. • Written explanations are satisfactory. • Graphs are mostly accurate and appropriate. • Satisfies most requirements of problems.
0 Unsatisfactory	• Shows little or no understanding of the concepts *parent graph, transformation, asymptote, hole in graph, critical point, maximum, minimum,* and *point of inflection.* • May not use appropriate strategies to solve problems and transform graphs. • Computations are incorrect. • Written explanations are not satisfactory. • Graphs are not accurate or appropriate. • Does not satisfy requirements of problems.

Open-Ended Assessment

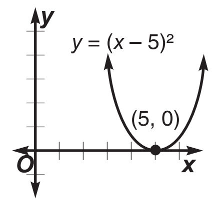
Page 123

1a.

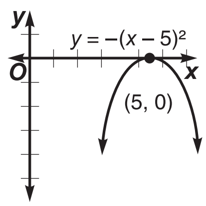
1. parent graph



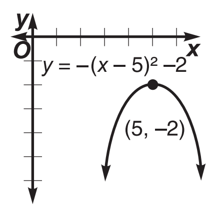
2. transformation



3. transformation



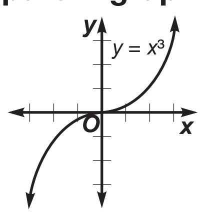
4. transformation



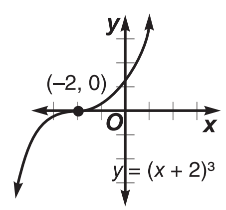
The parent graph $y = x^2$ is reflected over the x-axis and translated 5 units right and 2 units down.

1b.

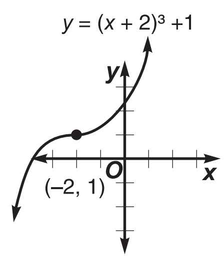
1. parent graph



2. transformation

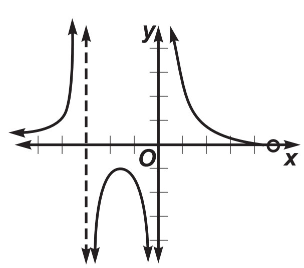


3. transformation



The parent graph $y = x^3$ is translated 2 units left and 1 unit up.

1c.



Since $x - 4$ is a common factor of the numerator and the denominator, the graph has point discontinuity at $x = 4$. Because y increases or decreases without bound close to $x = -3$ and $x = 0$, there are vertical asymptotes at $x = -3$ and $x = 0$.

1d. $f(x) = \dfrac{x + 1}{(x - 2)(x + 1)}$

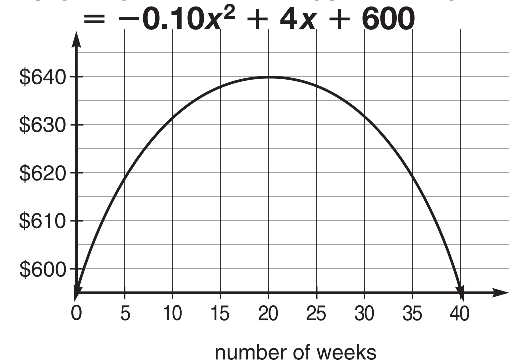
$f(x) = \dfrac{1}{x - 2}$, except at $x = -1$, where it is undefined.

$f(x) = \dfrac{1}{x - 2}$ is undefined at $x = 2$. It approaches $-\infty$ to the left of $x = 2$ and $+\infty$ to the right.

2. $\left(-1, \frac{133}{60}\right)$ is a maximum. $(0, 2)$ is a point of inflection. $\left(2, -\frac{14}{15}\right)$ is a minimum. The value $x = 3$ is not a critical point because a line drawn tangent to the graph at this point is neither horizontal nor vertical. The graph is increasing at $x = 2.99$ and at $x = 3.01$.

3a. $f(x)$ = total income, x = number of weeks

$f(x) = (6.00 + 0.10x)(100 - x)$
$= -0.10x^2 + 4x + 600$

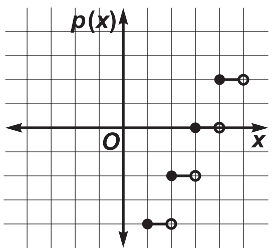


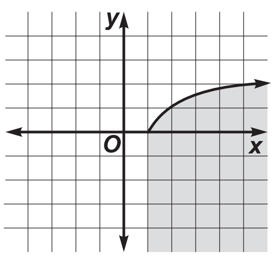
Examining the graph of the function reveals a maximum at $x = 20$. $f(20) = 640$, $f(19.9) = 639.999$, and $f(20.1) = 639.999$, so $f(19.9) < f(20)$ and $f(20) < f(20.1)$, which shows that the function has a maximum of 640 when $x = 20$.

3b. She should sell immediately because the income function would always be decreasing.

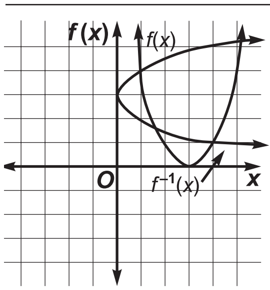
Chapter 3 Answer Key

Mid-Chapter Test
Page 124

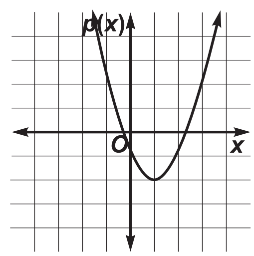
1. x-axis
2. $y = x$, $y = -x$, origin
3. neither
4. translated up 1 unit, expanded horizontally by a factor of 2
5. 

6. 

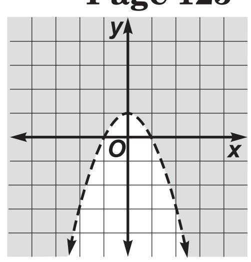
7. $x \leq -2$ or $x \geq 10$
8. $f^{-1}(x) = \sqrt[3]{x - 1}$; yes
9. $f^{-1}(x) = -\frac{1}{x}$; yes
10. no

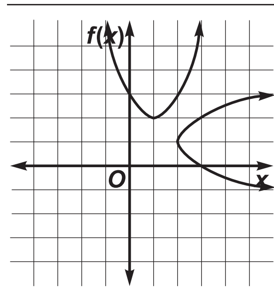


Quiz A
Page 125

1. none of these
2. y-axis
3. odd
4. $g(x)$ is the reflection of $f(x)$ over the x-axis and is expanded vertically by a factor of 4
5. 

Quiz B
Page 125

1. 

2. $\{x \mid x < -7 \text{ or } x > 1\}$
3. $f^{-1}(x) = \frac{x + 6}{3}$; yes
4. 

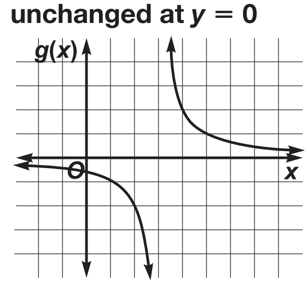
5. $y = x$

Quiz C
Page 126

1. discontinuous; jump
2. discontinuous; infinite
3. $x \to \infty$, $y \to \infty$, $x \to -\infty$, $y \to \infty$
4. rel. max. $(-1, 4)$; rel. min. $(1, -4)$
5. min. at $(2, -4.27)$

Quiz D
Page 126

1. vertical at $x = 2$ and 3, horizontal at $y = 1$
2. $y = x - 2$
3. translated right 3 units, expanded vertically by a factor of 2; vertical asymptote at $x = 3$, horizontal asymptote unchanged at $y = 0$



4. 1.5; $y = 1.5x$; 21
5. 2; $y = 2xz^3$; -500

Chapter 3 Answer Key

SAT/ACT Practice

Page 127

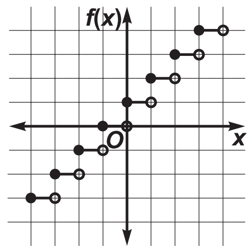
1. C
2. B
3. A
4. B
5. C
6. A
7. D
8. A
9. E

Page 128

10. B
11. D
12. E
13. B
14. C
15. A
16. D
17. A
18. A
19. 8
20. -1

Cumulative Review

Page 129

1. $D = \{-2, 0, 2\}$; $R = \{2\}$; yes
2. $\frac{3}{x}$
3. $2x - y + 7 = 0$
4. 

5. (2, 3)
6. (2, -1, 3)
7. one
8. 29
9. origin
10. expanded vertically by a factor of 2, translated right 1 unit
11. $-1 < x < 5$
12. $f^{-1}(x) = \frac{3}{x} + 2$
13. jump
14. $y = 1$
15. 15