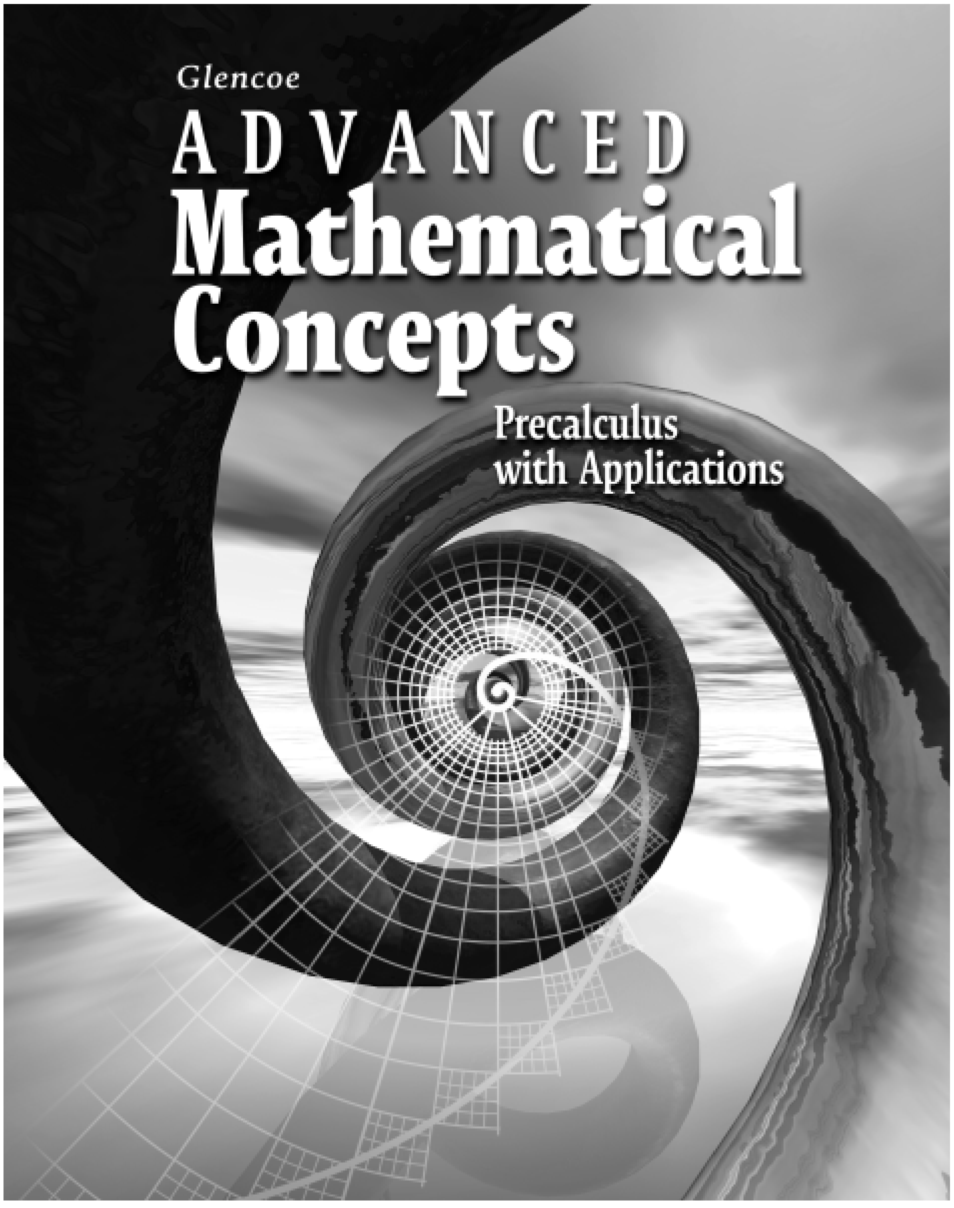
Chapter 2
Resource Masters



Glencoe

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StudentWorks™ This CD-ROM includes the entire Student Edition along with the Study Guide, Practice, and Enrichment masters.

TeacherWorks™ All of the materials found in this booklet are included for viewing and printing in the *Advanced Mathematical Concepts TeacherWorks* CD-ROM.

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Advanced Mathematical Concepts
Chapter 2 Resource Masters

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A Teacher's Guide to Using the Chapter 2 Resource Masters

The ***Fast File*** Chapter Resource system allows you to conveniently file the resources you use most often. The *Chapter 2 Resource Masters* include the core materials needed for Chapter 2. These materials include worksheets, extensions, and assessment options. The answers for these pages appear at the back of this booklet.

All of the materials found in this booklet are included for viewing and printing in the *Advanced Mathematical Concepts TeacherWorks* CD-ROM.

Vocabulary Builder Pages vii-x include a student study tool that presents the key vocabulary terms from the chapter. Students are to record definitions and/or examples for each term. You may suggest that students highlight or star the terms with which they are not familiar.

When to Use Give these pages to students before beginning Lesson 2-1. Remind them to add definitions and examples as they complete each lesson.

Study Guide There is one Study Guide master for each lesson.

When to Use Use these masters as reteaching activities for students who need additional reinforcement. These pages can also be used in conjunction with the Student Edition as an instructional tool for those students who have been absent.

Practice There is one master for each lesson. These problems more closely follow the structure of the Practice section of the Student Edition exercises. These exercises are of average difficulty.

When to Use These provide additional practice options or may be used as homework for second day teaching of the lesson.

Enrichment There is one master for each lesson. These activities may extend the concepts in the lesson, offer a historical or multicultural look at the concepts, or widen students' perspectives on the mathematics they are learning. These are not written exclusively for honors students, but are accessible for use with all levels of students.

When to Use These may be used as extra credit, short-term projects, or as activities for days when class periods are shortened.

Assessment Options

The assessment section of the *Chapter 2 Resources Masters* offers a wide range of assessment tools for intermediate and final assessment. The following lists describe each assessment master and its intended use.

Chapter Assessments

Chapter Tests

- *Forms 1A, 1B, and 1C* Form 1 tests contain multiple-choice questions. Form 1A is intended for use with honors-level students, Form 1B is intended for use with average-level students, and Form 1C is intended for use with basic-level students. These tests are similar in format to offer comparable testing situations.
- *Forms 2A, 2B, and 2C* Form 2 tests are composed of free-response questions. Form 2A is intended for use with honors-level students, Form 2B is intended for use with average-level students, and Form 2C is intended for use with basic-level students. These tests are similar in format to offer comparable testing situations.

  All of the above tests include a challenging Bonus question.
- The **Extended Response Assessment** includes performance assessment tasks that are suitable for all students. A scoring rubric is included for evaluation guidelines. Sample answers are provided for assessment.

Intermediate Assessment

- A **Mid-Chapter Test** provides an option to assess the first half of the chapter. It is composed of free-response questions.
- Four free-response **quizzes** are included to offer assessment at appropriate intervals in the chapter.

Continuing Assessment

- The **SAT and ACT Practice** offers continuing review of concepts in various formats, which may appear on standardized tests that they may encounter. This practice includes multiple-choice, quantitative-comparison, and grid-in questions. Bubble-in and grid-in answer sections are provided on the master.
- The **Cumulative Review** provides students an opportunity to reinforce and retain skills as they proceed through their study of advanced mathematics. It can also be used as a test. The master includes free-response questions.

Answers

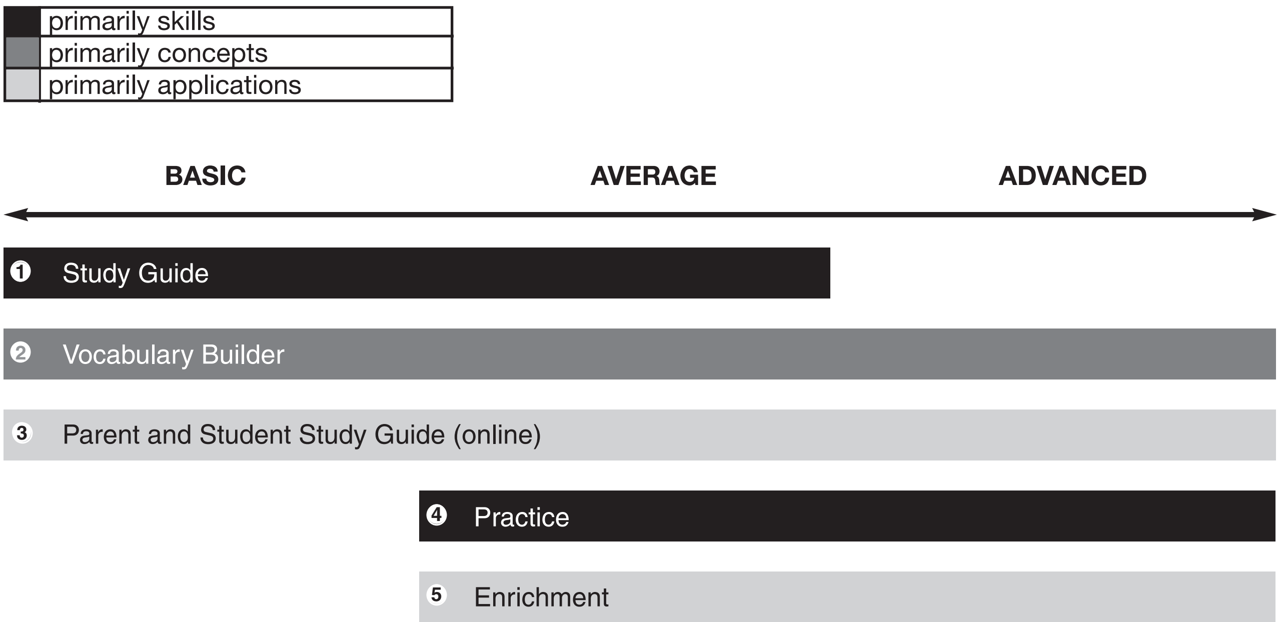
- Page A1 is an answer sheet for the SAT and ACT Practice questions that appear in the Student Edition on page 65. Page A2 is an answer sheet for the SAT and ACT Practice master. These improve students' familiarity with the answer formats they may encounter in test taking.
- The answers for the lesson-by-lesson masters are provided as reduced pages with answers appearing in red.
- Full-size answer keys are provided for the assessment options in this booklet.

Chapter 2 Leveled Worksheets

Glencoe's **leveled worksheets** are helpful for meeting the needs of every student in a variety of ways. These worksheets, many of which are found in the **FAST FILE Chapter Resource Masters**, are shown in the chart below.

- **Study Guide** masters provide worked-out examples as well as practice problems.
- Each chapter's **Vocabulary Builder** master provides students the opportunity to write out key concepts and definitions in their own words.
- **Practice** masters provide average-level problems for students who are moving at a regular pace.
- **Enrichment** masters offer students the opportunity to extend their learning.

Five Different Options to Meet the Needs of Every Student in a Variety of Ways



Chapter 2

NAME ______________________________ DATE ______________ PERIOD ________

Reading to Learn Mathematics

Vocabulary Builder

This is an alphabetical list of the key vocabulary terms you will learn in Chapter 2. As you study the chapter, complete each term's definition or description. Remember to add the page number where you found the term.

Vocabulary Term	Found on Page	Definition/Description/Example
additive identity matrix		
alternate optimal solution		
column matrix		
consistent		
constraints		
dependent		
determinant		
dilation		
dimensions		
element		

(continued on the next page)

NAME ______________________ DATE ____________ PERIOD ________

Reading to Learn Mathematics

Vocabulary Builder *(continued)*

Vocabulary Term	Found on Page	Definition/Description/Example
elimination method		
equal matrices		
identity matrix for multiplication		
image		
inconsistent		
independent		
infeasible		
inverse matrix		
linear programming		
$m \times n$ matrix		
matrix		

(continued on the next page)

NAME ______________________________ DATE ______________ PERIOD ________

Reading to Learn Mathematics

Vocabulary Builder *(continued)*

Vocabulary Term	Found on Page	Definition/Description/Example
minor		
*n*th order		
ordered triple		
polygonal convex set		
pre-image		
reflection		
reflection matrix		
rotation		
rotation matrix		
row matrix		
scalar		

(continued on the next page)

NAME ______________________________ DATE ________________ PERIOD _________

Reading to Learn Mathematics

Vocabulary Builder *(continued)*

Vocabulary Term	Found on Page	Definition/Description/Example
solution		
square matrix		
substitution method		
system of equations		
system of linear inequalities		
transformations		
translation		
translation matrix		
unbounded		
vertex matrix		
Vertex Theorem		
zero matrix		

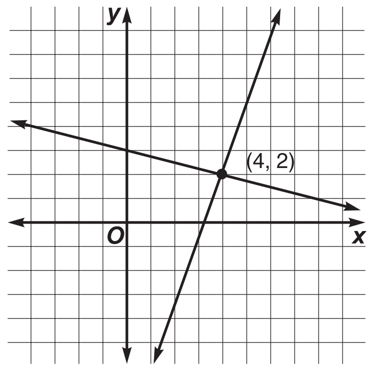
2-1

Study Guide

Solving Systems of Equations in Two Variables

One way to solve a system of equations in two variables is by graphing. The intersection of the graphs is called the solution to the **system of equations.**

Example 1 **Solve the system of equations by graphing.**
$3x - y = 10$
$x + 4y = 12$



First rewrite each equation of the system in slope-intercept form by solving for y.

$3x - y = 10$ → becomes → $y = 3x - 10$

$x + 4y = 12$ → becomes → $y = -\frac{1}{4}x + 3$

The solution to the system is (4, 2).

A **consistent** system has at least one solution.
A system having exactly one solution is **independent.**
If a system has infinitely many solutions, the system is **dependent.**
Systems that have no solution are **inconsistent.**

Systems of linear equations can also be solved algebraically using the **elimination method** or the **substitution method.**

Example 2
Use the elimination method to solve the system of equations.
$2x - 3y = -21$
$5x + 6y = 15$

To solve this system, multiply each side of the first equation by 2, and add the two equations to eliminate y. Then solve the resulting equation.

$2(2x - 3y) = 2(-21) \rightarrow 4x - 6y = -42$

$4x - 6y = -42$
$5x + 6y = 15$
$9x = -27$
$x = -3$

Now substitute -3 for x in either of the *original* equations.

$5x + 6y = 15$
$5(-3) + 6y = 15$ $\quad x = -3$
$6y = 30$
$y = 5$

The solution is $(-3, 5)$.

Example 3
Use the substitution method to solve the system of equations.
$x = 7y + 3$
$2x - y = -7$

The first equation is stated in terms of x, so substitute $7y + 3$ for x in the second equation.

$2x - y = -7$
$2(7y + 3) - y = -7$
$13y = -13$
$y = -1$

Now solve for x by substituting -1 for y in either of the *original* equations.

$x = 7y + 3$
$x = 7(-1) + 3$ $\quad y = -1$
$x = -4$

The solution is $(-4, -1)$.

2-1

Practice

Solving Systems of Equations in Two Variables

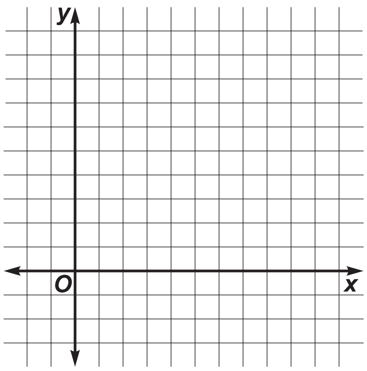
State whether each system is consistent and independent, consistent and dependent, or inconsistent.

1. $-x + y = -4$
$3x - 3y = 12$

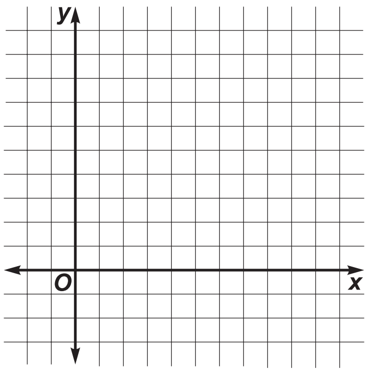
2. $2x - 5y = 8$
$15y - 6x = -24$

Solve each system of equations by graphing.

3. $x + y = 6$
$2x + 3y = 12$



4. $x + y = 6$
$3x - y = 6$



Solve each system of equations algebraically.

5. $x + y = 4$
$3x - 2y = 7$

6. $3x - 4y = 10$
$-3x + 4y = 8$

7. $4x - 3y = 15$
$2x + y = 5$

8. $4x + 5y = 11$
$3x - 2y = -9$

9. $2x + 3y = 19$
$7x - y = 9$

10. $2x - y = 6$
$x + y = 6$

11. ***Real Estate*** AMC Homes, Inc. is planning to build three- and four-bedroom homes in a housing development called Chestnut Hills. Consumer demand indicates a need for three times as many four-bedroom homes as for three-bedroom homes. The net profit from each three-bedroom home is $16,000 and from each four-bedroom home, $17,000. If AMC Homes must net a total profit of $13.4 million from this development, how many homes of each type should they build?

2-1

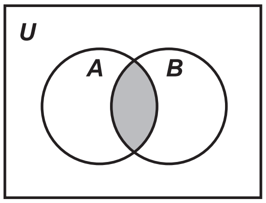
Enrichment

Set Theory and Venn Diagrams

Set theory, which was developed by the nineteenth century German logician and mathematician, Georg Cantor, became a fundamental unifying principle in the study of mathematics in the middle of the twentieth century. The use of sets permits the precise description of mathematical concepts.

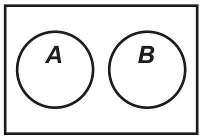
The intersection of two sets determines the elements common to the two sets. Thus, the intersection of two lines in a system of equations refers to the point or points that are common to the sets of points belonging to each of the lines. We can use Venn diagrams, which are named for the British logician, John Venn, to visually represent the intersection of two sets.

Example **Let U = the set of all points in the Cartesian coordinate plane.**
Let A = the set of all points that satisfy the equation $x = 4$.
Let B = the set of all points that satisfy the equation $y = -2$.
Draw a Venn diagram to represent U, A, and B.



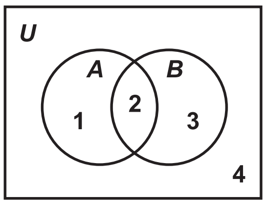
The shaded region represents the intersection of sets A and B, written $A \cap B$. In this example, $A \cap B = \{(4, -2)\}$.

Since the solution sets of two parallel lines have no points in common, the sets are called **disjoint sets**. In a Venn diagram, such sets are drawn as circles that do not overlap.



$A \cap B = \varnothing$.

Use the diagram at the right to answer the following questions.
Let A = the set of all points that satisfy the equation for line p.
Let B = the set of all points that satisfy the equation for line q.



1. In which numbered region do the points that satisfy only the equation for line p lie?

2. In which numbered region do the points that satisfy only the equation for line q lie?

3. In which numbered region do the points that satisfy the equations for neither line p nor line q lie?

4. If the equation of line p is $2x - y = 4$ and the equation of line q is $3x + y = 6$, which point lies in region 2?

Study Guide

Solving Systems of Equations in Three Variables

You can solve systems of three equations in three variables using the same algebraic techniques you used to solve systems of two equations.

Example **Solve the system of equations by elimination.**

$$\mathbf{4x + y + 2z = -8}$$
$$\mathbf{2x - 3y - 4z = 4}$$
$$\mathbf{-7x + 2y + 3z = 2}$$

Choose a pair of equations and then eliminate one of the variables. Because the coefficient of y is 1 in the first equation, eliminate y from the second and third equations.

To eliminate y using the first and second equations, multiply both sides of the first equation by 3.

$$3(4x + y + 2z) = 3(-8)$$
$$12x + 3y + 6z = -24$$

Then add the second equation to that result.

$$\begin{aligned} 12x + 3y + 6z &= -24 \\ 2x - 3y - 4z &= 4 \\ \hline 14x \qquad + 2z &= -20 \end{aligned}$$

To eliminate y using the first and third equations, multiply both sides of the first equation by -2.

$$-2(4x + y + 2z) = -2(-8)$$
$$-8x - 2y - 4z = 16$$

Then add the third equation to that result.

$$\begin{aligned} -8x - 2y - 4z &= 16 \\ -7x + 2y + 3z &= 2 \\ \hline -15x \qquad - z &= 18 \end{aligned}$$

Now you have two linear equations in two variables. Solve this system. Eliminate z by multiplying both sides of the second equation by 2. Then add the two equations.

$$2(-15x - z) = 2(18)$$
$$-30x - 2z = 36$$

$$\begin{aligned} 14x + 2z &= -20 \\ -30x - 2z &= 36 \\ \hline -16x \qquad &= 16 \\ x &= -1 \end{aligned}$$

The value of x is -1.

By substituting the value of x into one of the equations in two variables, we can solve for the value of z.

$$14x + 2z = -20$$
$$14(-1) + 2z = -20 \qquad x = -1$$
$$z = -3$$

The value of z is -3.

Finally, use one of the original equations to find the value of y.

$$4x + y + 2z = -8$$
$$4(-1) + y + 2(-3) = -8 \qquad x = -1,\ z = -3$$
$$y = 2$$

The value of y is 2.

The solution is $x = -1$, $y = 2$, $z = -3$. This can be written as the **ordered triple** $(-1, 2, -3)$.

2-2

Practice

Solving Systems of Equations in Three Variables

Solve each system of equations.

1. $x + y - z = -1$
$x + y + z = 3$
$3x - 2y - z = -4$

2. $x + y = 5$
$3x + z = 2$
$4y - z = 8$

3. $3x - 5y + z = 8$
$4y - z = 10$
$7x + y = 4$

4. $2x + 3y + 3z = 2$
$10x - 6y + 3z = 0$
$4x - 3y - 6z = 2$

5. $2x - y + z = -1$
$x - y + z = 1$
$x - 2y + z = 2$

6. $4x + 4y - 2z = 3$
$-6x - 6y + 6z = 5$
$2x - 3y - 4z = 2$

7. $x - z = 5$
$y + 3z = 12$
$2x + y = 7$

8. $2x + 4y - 2z = 9$
$4x - 6y + 2z = -9$
$x - y + 3z = -4$

9. ***Business*** The president of Speedy Airlines has discovered that her competitor, Zip Airlines, has purchased 13 new airplaines from Commuter Aviation for a total of \$15.9 million. She knows that Commuter Aviation produces three types of planes and that type A sells for \$1.1 million, type B sells for \$1.2 million, and type C sells for \$1.7 million. The president of Speedy Airlines also managed to find out that Zip Airlines purchased 5 more type A planes than type C planes. How many planes of each type did Zip Airlines purchase?

Enrichment

Graph Coloring

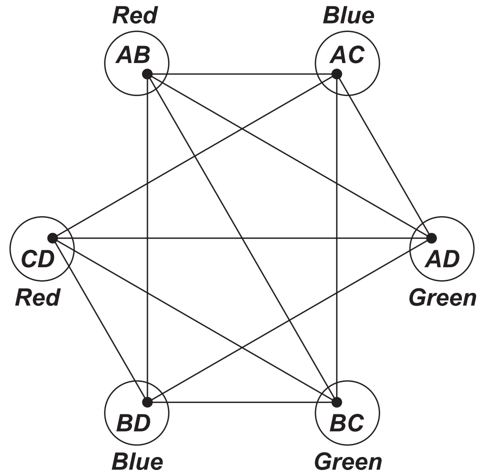
The student council is scheduling a volleyball tournament for teams from all four classes. They want each class team to play every other class team exactly once. How should they schedule the tournament?

If we call the teams *A*, *B*, *C*, and *D*, all of the possible games among the four teams can be represented as *AB*, *AC*, *AD*, *BC*, *BD*, and *CD*.

Draw a graph to represent the problem. Then color the graph. To **color** a graph means to color the vertices of that graph so that no two vertices connected by an edge have the same color.

Let the vertices represent the possible games. Let the edges represent games that cannot be scheduled at the same time. For example, if team *B* is playing team *C*, then team *C* cannot play team *D*.

Choose a vertex at which to begin.



- Color *AB* red. Since *CD* is not connected to *AB*, color it red as well. All of the other vertices are connected to *AB*, so do not color them red.
- Color *AC* blue. Since *BD* is not connected to *AC*, color it blue.
- Color *AD* green. Since *BC* is not connected to *AD*, color it green.

The colored graph shows that pairs of games can be scheduled as follows

AB with *CD*; *AC* with *BD*; *AD* with *BC*

The **chromatic number** of a graph is the least number of colors necessary to color the graph. The chromatic number of the graph above is 3.

1. What does the chromatic number of the graph in the example above represent?

2. Draw and color a graph to represent the same type of tournament, but with 6 teams playing.

3. What is the chromatic number for your graph?

NAME ______________________________ DATE ______________ PERIOD ________

Study Guide

Modeling Real-World Data with Matrices

A **matrix** is a rectangular array of terms called **elements.**
A matrix with m rows and n columns is an $\boldsymbol{m \times n}$ **matrix.**
The dimensions of the matrix are m and n.

Example 1 **Find the values of x and y for which $\begin{bmatrix} 3x + 5 \\ y \end{bmatrix} = \begin{bmatrix} y - 9 \\ -4x \end{bmatrix}$ is true.**

Since the corresponding elements must be equal, we can express the equal matrices as two equations.

$3x + 5 = y - 9$
$y = -4x$

Solve the system of equations by using substitution.

$3x + 5 = y - 9$
$3x + 5 = (-4x) - 9$ *Substitute $-4x$ for y.*
$x = -2$

$y = -4x$
$y = -4(-2)$ *Substitute -2 for x.*
$y = 8$

The matrices are equal if $x = -2$ and $y = 8$.

To add or subtract matrices, the dimensions of the matrices must be the same.

Example 2 **Find $C - D$ if $C = \begin{bmatrix} 3 & 6 \\ -2 & 4 \end{bmatrix}$ and $D = \begin{bmatrix} 1 & 5 \\ -7 & 8 \end{bmatrix}$.**

$$C - D = C + (-D)$$
$$= \begin{bmatrix} 3 & 6 \\ -2 & 4 \end{bmatrix} + \begin{bmatrix} -1 & -5 \\ 7 & -8 \end{bmatrix}$$
$$= \begin{bmatrix} 3 + (-1) & 6 + (-5) \\ -2 + 7 & 4 + (-8) \end{bmatrix}$$
$$= \begin{bmatrix} 2 & 1 \\ 5 & -4 \end{bmatrix}$$

To multiply two matrices, the number of columns in the first matrix must be equal to the number of rows in the second matrix. The product of two matrices is found by multiplying columns and rows.

Example 3 **Find each product if $X = \begin{bmatrix} 4 & 3 \\ 5 & 2 \end{bmatrix}$ and $Y = \begin{bmatrix} 8 & 0 \\ 9 & -6 \end{bmatrix}$.**

a. XY

$$XY = \begin{bmatrix} 4(8) + 3(9) & 4(0) + 3(-6) \\ 5(8) + 2(9) & 5(0) + 2(-6) \end{bmatrix} \text{ or } \begin{bmatrix} 59 & -18 \\ 58 & -12 \end{bmatrix}$$

b. YX

$$YX = \begin{bmatrix} 8(4) + 0(5) & 8(3) + 0(2) \\ 9(4) + (-6)(5) & 9(3) + (-6)(2) \end{bmatrix} \text{ or } \begin{bmatrix} 32 & 24 \\ 6 & 15 \end{bmatrix}$$

NAME ______________________ DATE ____________ PERIOD ______

Practice

Modeling Real-World Data with Matrices

Find the values of x and y for which each matrix equation is true.

1. $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2y - 4 \\ 2x \end{bmatrix}$

2. $\begin{bmatrix} 2x - 3 \\ 4y \end{bmatrix} = \begin{bmatrix} y \\ 3x \end{bmatrix}$

Use matrices A, B, and C to find each sum, difference, or product.

$A = \begin{bmatrix} -1 & 5 & 6 \\ 2 & -7 & -2 \\ 4 & 4 & 2 \end{bmatrix}$ $B = \begin{bmatrix} 2 & 3 & 1 \\ -1 & 1 & 4 \\ 5 & -2 & 3 \end{bmatrix}$ $C = \begin{bmatrix} 8 & 10 & -9 \\ -6 & 12 & 14 \end{bmatrix}$

3. $A + B$

4. $A - B$

5. $B - A$

6. $-2A$

7. CA

8. AB

9. AA

10. CB

11. $(CA)B$

12. $C(AB)$

13. ***Entertainment*** On one weekend, the Goxfield Theater reported the following ticket sales for three first-run movies, as shown in the matrix at the right. If the ticket prices were \$6 for each adult and \$4 for each child, what were the weekend sales for each movie.

	Adults	Children
Movie 1	1021	523
Movie 2	2547	785
Movie 3	3652	2456

2-3

Enrichment

Elementary Matrix Transformations

Elementary row transformations can be made by multiplying a matrix on the left by the appropriate transformation matrix. For example, to interchange rows 2 and 3 in a 3 × 3 matrix, multiply the matrix on the left by the matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$.

Example **Let $C = \begin{bmatrix} -5 & 3 & 1 \\ 1 & 5 & 0 \\ 10 & -6 & -2 \end{bmatrix}$. Multiply on the left by $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$.**

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} -5 & 3 & 1 \\ 1 & 5 & 0 \\ 10 & -6 & -2 \end{bmatrix} = \begin{bmatrix} -5 & 3 & 1 \\ 10 & -6 & -2 \\ 1 & 5 & 0 \end{bmatrix}$$

More complicated row transformations can also be made by a matrix multiplier.

Example **Find the elementary matrix that, when matrix A is multiplied by it on the left, row 1 will be replaced by the sum of 2 times row 1 and row 3.**

Let $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$. Find M such that $M \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} =$

$\begin{bmatrix} 2a_{11} + a_{31} & 2a_{12} + a_{32} & 2a_{13} + a_{33} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$. Try $M = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

Check to see that the product MA meets the required conditions.

1. Find the elementary matrix that will interchange rows 1 and 3 of matrix A.

2. Find the elementary matrix that will multiply row 2 of matrix A by 5.

3. Find the elementary matrix that will multiply the elements of the first row of matrix A by –2 and add the results to the corresponding elements of the third row.

4. Find the elementary matrix that will interchange rows 2 and 3 of matrix A and multiply row 1 by –2.

5. Find the elementary matrix that will multiply the elements of the second row of matrix A by –3 and add the results to the corresponding elements of row 1.

2-4

Study Guide

Modeling Motion with Matrices

You can use matrices to perform many **transformations,** such as **translations** (slides), **reflections** (flips), **rotations** (turns), and **dilations** (enlargements or reductions).

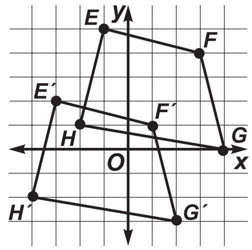
Example 1 **Suppose quadrilateral *EFGH* with vertices *E*(−1, 5), *F*(3, 4), *G*(4, 0), and *H*(−2, 1) is translated 2 units left and 3 units down.**

The **vertex matrix** for the quadrilateral is $\begin{bmatrix} -1 & 3 & 4 & -2 \\ 5 & 4 & 0 & 1 \end{bmatrix}$.

The **translation matrix** is $\begin{bmatrix} -2 & -2 & -2 & -2 \\ -3 & -3 & -3 & -3 \end{bmatrix}$.

Adding the two matrices gives the coordinates of the vertices of the translated quadrilateral.

$$\begin{bmatrix} -1 & 3 & 4 & -2 \\ 5 & 4 & 0 & 1 \end{bmatrix} + \begin{bmatrix} -2 & -2 & -2 & -2 \\ -3 & -3 & -3 & -3 \end{bmatrix} = \begin{bmatrix} -3 & 1 & 2 & -4 \\ 2 & 1 & -3 & -2 \end{bmatrix}$$



Graphing the **pre-image** and the **image** of the translated quadrilateral on the same axes, we can see the effect of the translation.

The chart below summarizes the matrices needed to produce specific reflections or rotations. All rotations are counterclockwise about the origin.

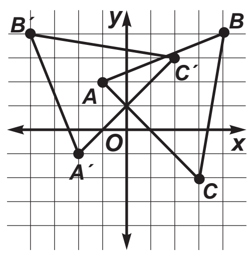
Reflections	$R_{x\text{-axis}} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$	$R_{y\text{-axis}} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$	$R_{y=x} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
Rotations	$Rot_{90} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$	$Rot_{180} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$	$Rot_{270} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

Example 2 **A triangle has vertices *A*(−1, 2), *B*(4, 4), and *C*(3, −2). Find the coordinates of the image of the triangle after a rotation of 90° counterclockwise about the origin.**

The vertex matrix is $\begin{bmatrix} -1 & 4 & 3 \\ 2 & 4 & -2 \end{bmatrix}$.

Multiply it by the 90° **rotation matrix.**

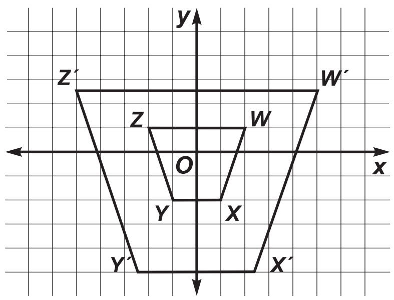
$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} -1 & 4 & 3 \\ 2 & 4 & -2 \end{bmatrix} = \begin{bmatrix} -2 & -4 & 2 \\ -1 & 4 & 3 \end{bmatrix}$$



Example 3 **Trapezoid *WXYZ* has vertices *W*(2, 1), *X*(1, −2), *Y*(−1, −2), and *Z*(−2, 1). Find the coordinates of dilated trapezoid *W′X′Y′Z′* for a scale factor of 2.5.**

Perform scalar multiplication on the vertex matrix for the trapezoid.

$$2.5\begin{bmatrix} 2 & 1 & -1 & -2 \\ 1 & -2 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 2.5 & -2.5 & -5 \\ 2.5 & -5 & -5 & 2.5 \end{bmatrix}$$



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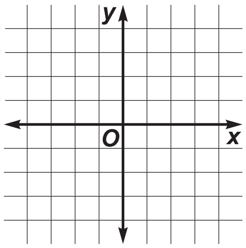
2-4

Practice

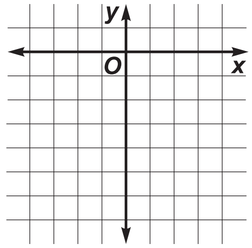
Modeling Motion with Matrices

Use scalar multiplication to determine the coordinates of the vertices of each dilated figure. Then graph the pre-image and the image on the same coordinate grid.

1. triangle with vertices $A(1, 2)$, $B(2, -1)$, and $C(-2, 0)$; scale factor 2

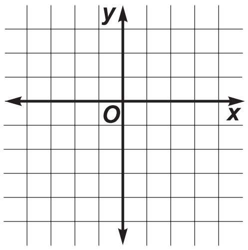


2. quadrilateral with vertices $E(-2, -7)$, $F(4, -3)$, $G(0, 1)$, and $H(-4, -2)$; scale factor 0.5

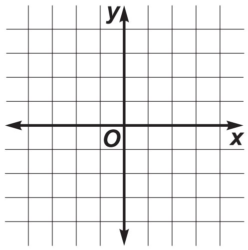


Use matrices to determine the coordinates of the vertices of each translated figure. Then graph the pre-image and the image on the same coordinate grid.

3. square with vertices $W(1, -3)$, $X(-4, -2)$, $Y(-3, 3)$, and $Z(2, 2)$ translated 2 units right and 3 units down

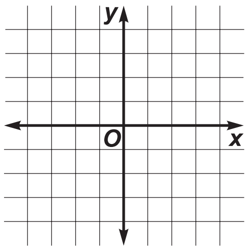


4. triangle with vertices $J(3, 1)$, $K(2, -4)$, and $L(0, -2)$ translated 4 units left and 2 units up

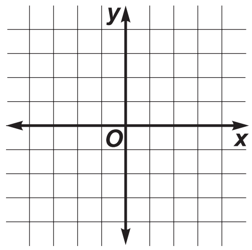


Use matrices to determine the coordinates of the vertices of each reflected figure. Then graph the pre-image and the image on the same coordinate grid.

5. $\triangle MNP$ with vertices $M(-3, 4)$, $N(3, 1)$, and $P(-4, -3)$ reflected over the y-axis

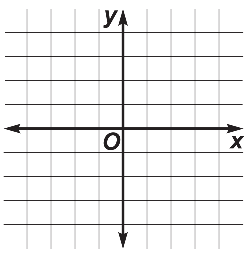


6. a rhombus with vertices $Q(2, 3)$, $R(4, -1)$, $S(-1, -2)$, and $T(-3, 2)$ reflected over the line $y = x$

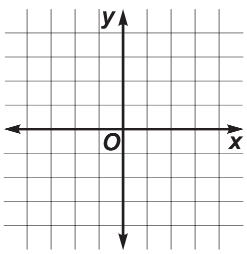


Use matrices to determine the coordinates of the vertices of each rotated figure. Then graph the pre-image and the image on the same coordinate grid.

7. quadrilateral $CDFG$ with vertices $C(-2, 3)$, $D(3, 4)$, $F(3, -1)$, and $G(-3, -4)$ rotated 90°



8. Pentagon $VWXYZ$ with vertices $V(1, 3)$, $W(4, 2)$, $X(3, -2)$, $Y(-1, -4)$, $Z(-2, 1)$ rotated 180°



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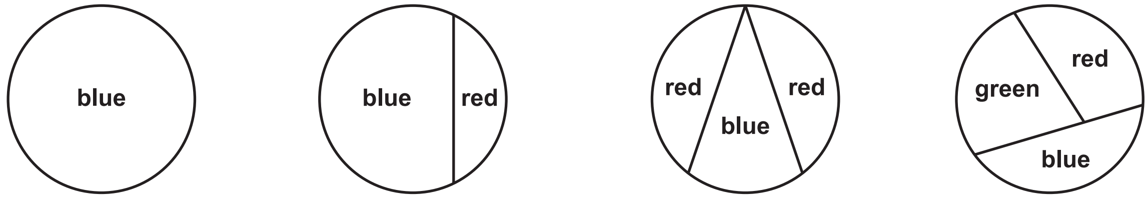
Enrichment

Map Coloring

Each of the graphs below is drawn on a plane with no accidental crossings of its edges. This kind of graph is called a planar graph. A planar graph separates the plane into **regions**. A very famous rule called the *four-color theorem* states that every planar graph can be colored with at most four colors, without any adjacent regions having the same color.



Graph A	**Graph B**	**Graph C**	**Graph D**
one region	two regions	three regions	three regions
one color	two colors	two colors	three colors

If there are four or more regions, two, three, or four colors may be required.

Example **Color the map at the right using the least number of colors.**

Look for the greatest number of isolated states or regions. Virginia, Georgia, and Mississippi are isolated. Color them pink.

Alabama and North Carolina are also isolated. Color them blue.

South Carolina and Tennessee are remaining. Because they do not share a common edge, they may be colored with the same color. Color them both green.

Thus, three colors are needed to color this map.

Color each map with the least number of colors.

1.

2.

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Study Guide

Determinants and Multiplicative Inverses of Matrices

Each square matrix has a **determinant.** The determinant of a 2×2 matrix is a number denoted by $\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$ or $\det\begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix}$. Its value is $a_1b_2 - a_2b_1$.

Example 1 **Find the value of** $\begin{vmatrix} 5 & -2 \\ 3 & 1 \end{vmatrix}$.

$$\begin{vmatrix} 5 & -2 \\ 3 & 1 \end{vmatrix} = 5(1) - 3(-2) \text{ or } 11$$

The **minor** of an element can be found by deleting the row and column containing the element.

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

The minor of a_1 *is* $\begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix}$.

Example 2 **Find the value of** $\begin{vmatrix} 8 & 9 & 3 \\ 3 & 5 & 7 \\ -1 & 2 & 4 \end{vmatrix}$.

$$\begin{vmatrix} 8 & 9 & 3 \\ 3 & 5 & 7 \\ -1 & 2 & 4 \end{vmatrix} = 8\begin{vmatrix} 5 & 7 \\ 2 & 4 \end{vmatrix} - 9\begin{vmatrix} 3 & 7 \\ -1 & 4 \end{vmatrix} + 3\begin{vmatrix} 3 & 5 \\ -1 & 2 \end{vmatrix}$$
$$= 8(6) - 9(19) + 3(11) \text{ or } -90$$

The multiplicative inverse of a matrix is defined as follows.

If $A = \begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix}$ and $\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} \neq 0$, then $A^{-1} = \qquad \begin{bmatrix} b_2 & -b_1 \\ -a_2 & a_1 \end{bmatrix}$.

Example 3 **Solve the system of equations by using matrix equations.**
$5x + 4y = -3$
$3x - 5y = -24$

Write the system as a matrix equation.

$$\begin{bmatrix} 5 & 4 \\ 3 & -5 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -3 \\ -24 \end{bmatrix}$$

To solve the matrix equation, first find the inverse of the coefficient matrix.

$$\frac{1}{\begin{vmatrix} 5 & 4 \\ 3 & -5 \end{vmatrix}}\begin{bmatrix} -5 & -4 \\ -3 & 5 \end{bmatrix} = -\frac{1}{37}\begin{bmatrix} -5 & -4 \\ -3 & 5 \end{bmatrix}$$

Now multiply each side of the matrix equation by the inverse and solve.

$$-\frac{1}{37}\begin{bmatrix} -5 & -4 \\ -3 & 5 \end{bmatrix} \cdot \begin{bmatrix} 5 & 4 \\ 3 & -5 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix}$$
$$= -\frac{1}{37}\begin{bmatrix} -5 & -4 \\ -3 & 5 \end{bmatrix} \cdot \begin{bmatrix} -3 \\ -24 \end{bmatrix}$$
$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -3 \\ 3 \end{bmatrix}$$

The solution is $(-3, 3)$.

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Practice

Determinants and Multiplicative Inverses of Matrices

Find the value of each determinant.

1. $\begin{vmatrix} -2 & 3 \\ 8 & -12 \end{vmatrix}$

2. $\begin{vmatrix} 3 & -5 \\ 7 & 9 \end{vmatrix}$

3. $\begin{vmatrix} 1 & -1 & 0 \\ 2 & 1 & 4 \\ 5 & -3 & 5 \end{vmatrix}$

4. $\begin{vmatrix} 2 & 3 & 1 \\ -3 & -1 & 5 \\ 1 & -4 & 2 \end{vmatrix}$

Find the inverse of each matrix, if it exists.

5. $\begin{vmatrix} 3 & 8 \\ -1 & 5 \end{vmatrix}$

6. $\begin{vmatrix} 5 & 2 \\ 10 & 4 \end{vmatrix}$

Solve each system by using matrix equations.

7. $2x - 3y = 17$
$3x + y = 9$

8. $4x - 3y = -16$
$2x + 5y = 18$

Solve each matrix equation.

9. $\begin{bmatrix} 2 & -1 & 3 \\ 1 & 2 & 1 \\ -1 & -3 & -2 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -8 \\ 3 \\ -7 \end{bmatrix}$, if the inverse is $-\frac{1}{6}\begin{bmatrix} -1 & -11 & -7 \\ 1 & -1 & 1 \\ -1 & 7 & 5 \end{bmatrix}$

10. $\begin{bmatrix} 5 & -2 & 4 \\ 3 & -4 & 2 \\ 1 & -3 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ \mathrm{z} \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$, if the inverse is $-\frac{1}{8}\begin{bmatrix} 2 & -10 & 12 \\ -1 & 1 & 2 \\ -5 & 13 & -14 \end{bmatrix}$

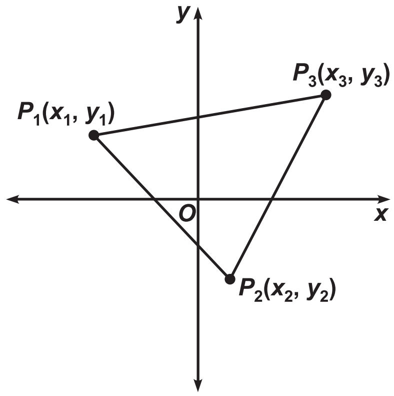
11. ***Landscaping*** Two dump truck have capacities of 10 tons and 12 tons. They make a total of 20 round trips to haul 226 tons of topsoil for a landscaping project. How many round trips does each truck make?

2-5

Enrichment

Area of a Triangle

Determinants can be used to find the area of a triangle on the coordinate plane. For a triangle with vertices $P_1(x_1, y_1)$, $P_2(x_2, y_2)$, and $P_3(x_3, y_3)$, the area is given by the following formula.



$$\text{Area} = \pm\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

The sign is chosen so that the result is positive.

Example **Find the area of the triangle with vertices *A*(3, 8), *B*(−2, 5), and *C*(0, −1).**

$$\text{Area} = \pm\frac{1}{2}\begin{vmatrix} 3 & 8 & 1 \\ -2 & 5 & 1 \\ 0 & -1 & 1 \end{vmatrix} = \pm\frac{1}{2}\left[3\begin{vmatrix} 5 & 1 \\ -1 & 1 \end{vmatrix} - 8\begin{vmatrix} -2 & 1 \\ 0 & 1 \end{vmatrix} + 1\begin{vmatrix} -2 & 5 \\ 0 & -1 \end{vmatrix}\right]$$

$$= \pm\frac{1}{2}[3(5-(-1)) - 8(-2-0) + 1(2-0)]$$

$$= \pm\frac{1}{2}[36] = 18$$

The area is 18 square units.

Find the area of the triangle having vertices with the given coordinates.

1. $A(0, 6)$, $B(0, -4)$, $C(0, 0)$

2. $A(5, 1)$, $B(-3, 7)$, $C(-2, -2)$

3. $A(8, -2)$, $B(0, -4)$, $C(-2, 10)$

4. $A(12, 4)$, $B(-6, 4)$, $C(3, 16)$

5. $A(-1, -3)$, $B(7, -5)$, $C(-3, 9)$

6. $A(1.2, 3.1)$, $B(5.7, 6.2)$, $C(8.5, 4.4)$

7. What is the sign of the determinant in the formula when the points are taken in clockwise order? in counterclockwise order?

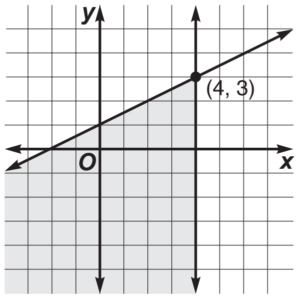
2-6

Study Guide

Solving Systems of Linear Inequalities

To solve a **system of linear inequalities**, you must find the ordered pairs that satisfy all inequalities. One way to do this is to graph the inequalities on the same coordinate plane. The intersection of the graphs contains points with ordered pairs in the solution set. If the graphs of the inequalities do not intersect, then the system has no solution.

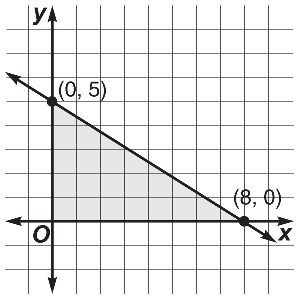
Example 1 **Solve the system of inequalities by graphing.** $-x + 2y \leq 2$, $x \leq 4$



The shaded region represents the solution to the system of inequalities.

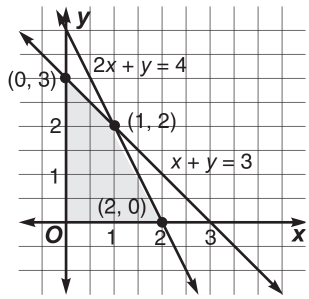
A system of more than two linear inequalities can have a solution that is a bounded set of points called a **polygonal convex set.**

Example 2 **Solve the system of inequalities by graphing and name the coordinates of the vertices of the polygonal convex set.** $x \geq 0$, $y \geq 0$, $5x + 8y \leq 40$



The region shows points that satisfy all three inequalities. The region is a triangle whose vertices are the points at (0, 0), (0, 5), and (8, 0).

Example 3 **Find the maximum and minimum values of $f(x, y) = x + 2y + 1$ for the polygonal convex set determined by the following inequalities.**
$x \geq 0 \quad y \geq 0 \quad 2x + y \leq 4 \quad x + y \leq 3$



First, graph the inequalities and find the coordinates of the vertices of the resulting polygon.

The coordinates of the vertices are (0, 0), (2, 0), (1, 2), and (0, 3).

Then, evaluate the function $f(x, y) = x + 2y + 1$ at each vertex.

$f(0, 0) = 0 + 2(0) + 1 = 1$ $\qquad f(1, 2) = 1 + 2(2) + 1 = 6$

$f(2, 0) = 2 + 2(0) + 1 = 3$ $\qquad f(0, 3) = 0 + 2(3) + 1 = 7$

Thus, the maximum value of the function is 7, and the minimum value is 1.

2-6

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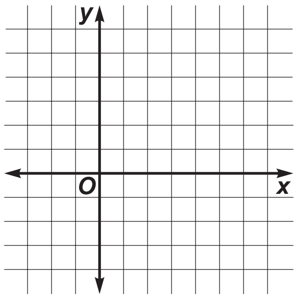
Practice

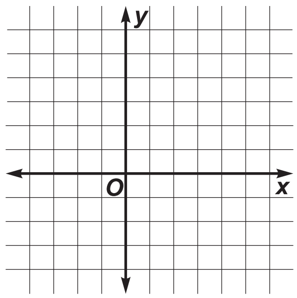
Solving Systems of Linear Inequalities

Solve each system of inequalities by graphing.

1. $-4x + 7y \geq -21;\ 3x + 7y \leq 28$

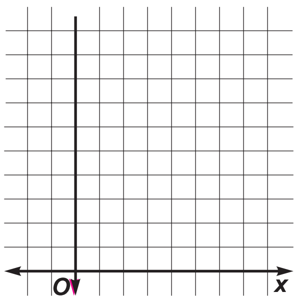


2. $x \leq 3;\ y \leq 5;\ x + y \geq 1$

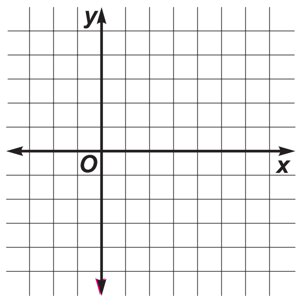


Solve each system of inequalities by graphing. Name the coordinates of the vertices of the polygonal convex set.

3. $x \geq 0;\ y \geq 0;\ y \geq x - 4;\ 7x + 6y \leq 54$

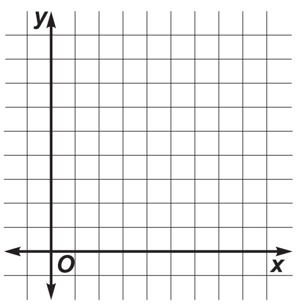


4. $x \geq 0;\ y + 2 \geq 0;\ 5x + 6y \leq 18$

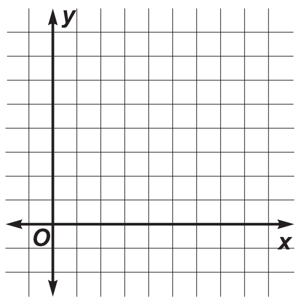


Find the maximum and minimum values of each function for the polygonal convex set determined by the given system of inequalities.

5. $3x - 2y \geq 0 \quad y \geq 0$
$3x + 2y \leq 24 \quad f(x, y) = 7y - 3x$



6. $y \leq -x + 8 \quad 4x - 3y \geq -3$
$x + 8y \geq 8 \quad f(x, y) = 4x - 5y$



7. ***Business*** Henry Jackson, a recent college graduate, plans to start his own business manufacturing bicycle tires. Henry knows that his start-up costs are going to be \$3000 and that each tire will cost him at least \$2 to manufacture. In order to remain competitive, Henry cannot charge more than \$5 per tire. Draw a graph to show when Henry will make a profit.

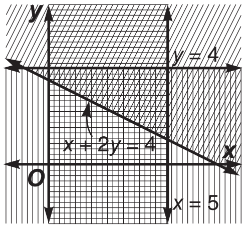
2-6

Enrichment

Reading Mathematics: Reading the Graph of a System of Inequalities

Like all other kinds of graphs, the graph of a system of linear inequalities is useful to us only as long as we are able to "read" the information presented on it.

The line of each equation separates the xy-plane into two **half-planes**. Shading is used to tell which half-plane satisfies the inequality bounded by the line. Thus, the shading below the boundary line $y = 4$ indicates the solution set of $y \leq 4$. By drawing horizontal or vertical shading lines or lines with slopes of 1 or -1, and by using different colors, you will make it easier to "read" your graph.



$$0 \leq x \leq 5$$
$$y \leq 4$$
$$x + 2y \geq 7$$

The shading in between the boundary lines $x \geq 0$ and $x \leq 5$, indicate that the inequality $0 \leq x \leq 5$ is equivalent to the *intersection* $x \geq 0 \cap x \leq 5$.

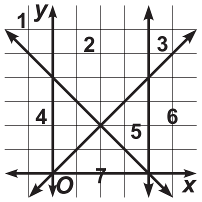
Similarly, the solution set of the system, indicated by the cross-hatching, is the intersection of the solutions of all the inequalities in the system. We can use set notation to write a description of this solution set:

$$\{(x, y) \mid 0 \leq x \leq 5 \cap y \leq 4 \cap x + 2y \geq 7\}.$$

We read this " The set of all ordered pairs, (x, y), such that x is greater than or equal to 0 *and* x is less than or equal to 5 *and* y is less than or equal to 4 *and* $x + 2y$ is greater than or equal to 7."

The graph also shows where any maximum and minimum values of the function f occur. Notice, however, that these values occur only at points belonging to the set containing the *union* of the vertices of the convex polygonal region.

Use the graph at the right to show which numbered region or regions belong to the graph of the solution set of each system.



1. $x \geq 0$
$y \leq x$

2. $0 \leq x \leq 4$
$y \leq x$
$x + y \geq 4$

3. The domain of $f(x, y) = 2x + y$ is the xy-plane. If we graphed this function, where would we represent its range?

Study Guide

Linear Programming

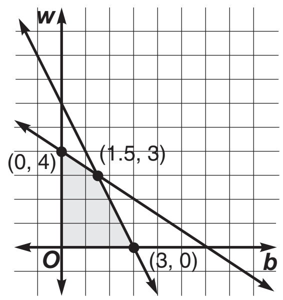
The following example outlines the procedure used to solve **linear programming** problems.

Example **The B & W Leather Company wants to add handmade belts and wallets to its product line. Each belt nets the company $18 in profit, and each wallet nets $12. Both belts and wallets require cutting and sewing. Belts require 2 hours of cutting time and 6 hours of sewing time. Wallets require 3 hours of cutting time and 3 hours of sewing time. If the cutting machine is available 12 hours a week and the sewing machine is available 18 hours per week, what mix of belts and wallets will produce the most profit within the constraints?**

Define variables. Let b = the number of belts.
Let w = the number of wallets.

Write inequalities.
$b \geq 0$
$w \geq 0$
$2b + 3w \leq 12$ cutting
$6b + 3w \leq 18$ sewing

Graph the system.



Write an equation. Since the profit on belts is $18 and the profit on wallets is $12, the profit function is $B(b, w) = 18b + 12w$.

Substitute values.
$B(0, 0) = 18(0) + 12\ (0) = 0$
$B(0, 4) = 18(0) + 12(4) = 48$
$B(1.5, 3) = 18(1.5) + 12(3) = 63$
$B(3, 0) = 18(3) + 12(0) = 54$

Answer the problem. The B & W Company will maximize profit if it makes and sells 1.5 belts for every 3 wallets.

When constraints of a linear programming problem cannot be satisfied simultaneously, then **infeasibility** is said to occur.

The solution of a linear programming problem is **unbounded** if the region defined by the constraints is infinitely large.

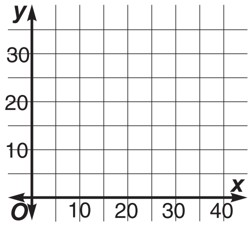
2-7

Practice

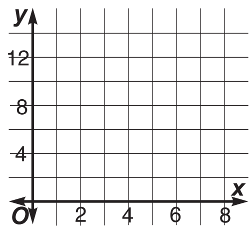
Linear Programming

Graph each system of inequalities. In a problem asking you to find the maximum value of ƒ(x, y), state whether the situation is infeasible, has alternate optimal solutions, or is unbounded. In each system, assume that x ≥ 0 and y ≥ 0 unless stated otherwise.

1. $-2y \geq 2x - 36$
$x + y \geq 30$
$f(x, y) = 3x + 3y$

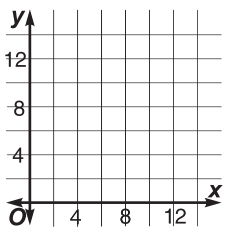


2. $2x + 2y \geq 10$
$2x + y \geq 8$
$f(x, y) = x + y$

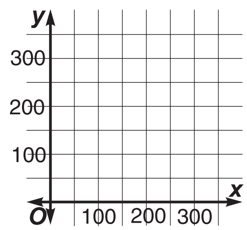


Solve each problem, if possible. If not possible, state whether the problem is infeasible, has alternate optimal solutions, or is unbounded.

3. ***Nutrition*** A diet is to include at least 140 milligrams of Vitamin A and at least 145 milligrams of Vitamin B. These requirements can be obtained from two types of food. Type X contains 10 milligrams of Vitamin A and 20 milligrams of Vitamin B per pound. Type Y contains 30 milligrams of Vitamin A and 15 milligrams of Vitamin B per pound. If type X food costs \$12 per pound and type Y food costs \$8 per pound how many pounds of each type of food should be purchased to satisfy the requirements at the minimum cost?



4. ***Manufacturing*** The Cruiser Bicycle Company makes two styles of bicycles: the Traveler, which sells for \$200, and the Tourester, which sells for \$600. Each bicycle has the same frame and tires, but the assembly and painting time required for the Traveler is only 1 hour, while it is 3 hours for the Tourister. There are 300 frames and 360 hours of labor available for production. How many bicycles of each model should be produced to maximize revenue?

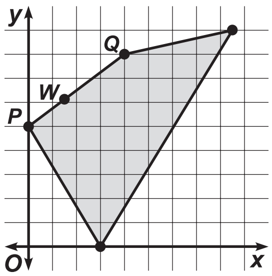


2-7

Enrichment

Convex Polygons

You have already learned that over a closed convex polygonal region, the maximum and minimum values of any linear function occur at the vertices of the polygon. To see why the values of the function at any point on the boundary of the region must be between the values at the vertices, consider the convex polygon with vertices P and Q.



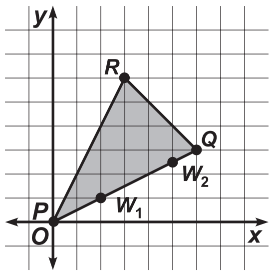
Let W be a point on $\overline{PQ}$.
If W lies between P and Q, let $\frac{PW}{PQ} = w$.

Then $0 < w < 1$ and the coordinates of W are $((1 - w)x_1 + wx_2, (1 - w)y_1 + wy_2)$. Now consider the function $f(x, y) = 3x - 5y$.

$$
\begin{aligned}
f(W) &= 3[(1-w)x_1 + wx_2] - 5[(1-w)y_1 + wy_2] \\
&= (1-w)(3x_1) + 3wx_2 + (1-w)(-5y_1) - 5wy_2 \\
&= (1-w)(3x_1 - 5y_1) + w(3x_2 - 5y_2) \\
&= (1-w)f(P) + wf(Q)
\end{aligned}
$$

This means that $f(W)$ is between $f(P)$ and $f(Q)$, or that the greatest and least values of $f(x, y)$ must occur at P or Q.

Example **If $f(x, y) = 3x + 2y$, find the maximum value of the function over the shaded region at the right.**



The maximum value occurs at the vertex (6, 3). The minimum value occurs at (0, 0). The values of $f(x, y)$ at W_1 and W_2 are between the maximum and minimum values.

$f(Q) = f(6, 3) = 24$
$f(W_1) = f(2, 1) = 8$
$f(W_2) = f(5, 2.5) = 20$
$f(P) = f(0, 0) = 0$

Let P and Q be vertices of a closed convex polygon, and let W lie on $\overline{PQ}$. Let f(x, y) = ax + by.

1. If $f(Q) = f(P)$, what is true of f? of $f(W)$?

2. If $f(Q) = f(P)$, find an equation of the line containing P and Q.

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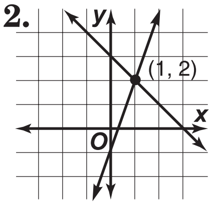
Chapter 2 Test, Form 1A

Write the letter for the correct answer in the blank at the right of each problem.

1. Which system is inconsistent? **1.** ________

A. $y = 0.5x$, $x - 2y = 1$ **B.** $2x - y = 8$, $4x + y = 5$ **C.** $x + y = 0$, $2x = y$ **D.** $y = -1$, $x = 2$

2. Which system of equations is shown by the graph? **2.** ________



A. $-3x + y = 1$, $x + y = 3$ **B.** $3x - y = 1$, $x + y = 3$

C. $x + y = 3$, $-x + y = 1$ **D.** $-6x + 3y = -3$, $3x - y = 1$

3. Solve algebraically. $2x + 7y = 5$, $6x + y = 5$ **3.** ________

A. $\left(\frac{-15}{8}, \frac{5}{4}\right)$ **B.** $(1, -1)$ **C.** $(6, -1)$ **D.** $\left(\frac{3}{4}, \frac{1}{2}\right)$

4. Solve algebraically. $6x - 3y + 8z = 4$, $-x + 9y - 2z = 2$, $4x + 6y - 4z = 3$ **4.** ________

A. $\left(\frac{5}{2}, \frac{5}{3}, -\frac{3}{4}\right)$ **B.** $(1, -2, -1)$ **C.** $\left(\frac{1}{2}, \frac{1}{3}, \frac{1}{4}\right)$ **D.** $\left(5, -2, \frac{5}{4}\right)$

5. Find $2A - B$ if $A = \begin{bmatrix} 6 & -3 \\ 1 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} -4 & -1 \\ 2 & 5 \end{bmatrix}$. **5.** ________

A. $\begin{bmatrix} 10 & -2 \\ -1 & -7 \end{bmatrix}$ **B.** $\begin{bmatrix} 16 & -5 \\ 0 & -9 \end{bmatrix}$ **C.** $\begin{bmatrix} 20 & -4 \\ -2 & -14 \end{bmatrix}$ **D.** $\begin{bmatrix} 8 & -7 \\ 4 & 1 \end{bmatrix}$

6. Find the values of x and y for which $\begin{bmatrix} 2y + 5 \\ y - 2 \end{bmatrix} = \begin{bmatrix} x - 1 \\ 3x \end{bmatrix}$ is true. **6.** ________

A. $(-1, -1)$ **B.** $(-2, -4)$ **C.** $(5, 1)$ **D.** $(-1, 4)$

7. Find DE if $D = \begin{bmatrix} -2 & 4 & 6 \\ 5 & -7 & -1 \end{bmatrix}$ and $E = \begin{bmatrix} 1 & -2 \\ 0 & 4 \\ -3 & 4 \end{bmatrix}$. **7.** ________

A. $\begin{bmatrix} -20 & 44 \\ 8 & -42 \end{bmatrix}$ **B.** $\begin{bmatrix} -2 & -10 \\ 0 & -28 \\ -18 & -4 \end{bmatrix}$ **C.** $\begin{bmatrix} -20 & 8 \\ 44 & -42 \end{bmatrix}$ **D.** $\begin{bmatrix} -2 & 0 & -18 \\ -10 & -28 & -4 \end{bmatrix}$

8. A triangle with vertices $A(-3, 4)$, $B(3, 1)$ and $C(-1, -5)$ is rotated 90° counterclockwise about the origin. Find the coordinates of A', B', and C'. **8.** ________

A. $A'(-3, -4)$, $B'(3, -1)$, $C'(-1, 5)$ **B.** $A'(3, -4)$, $B'(-3, -1)$, $C'(1, 5)$ **C.** $A'(4, 3)$, $B'(1, -3)$, $C'(-5, 1)$ **D.** $A'(-4, -3)$, $B'(-1, 3)$, $C'(5, -1)$

9. Find the value of $\begin{vmatrix} 1 & 2 & -1 \\ 3 & -2 & 1 \\ 0 & 1 & -3 \end{vmatrix}$. **9.** ________

A. 26 **B.** 22 **C.** −16 **D.** 20

Chapter 2 Test, Form 1A (continued)

10. Find the inverse of $\begin{bmatrix} 3 & 1 \\ -4 & 1 \end{bmatrix}$, if it exists. **10.** ________

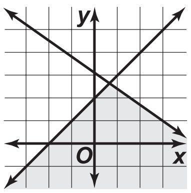
A. does not exist **B.** $-\frac{1}{7}\begin{bmatrix} 1 & -1 \\ 4 & 3 \end{bmatrix}$ **C.** $\frac{1}{7}\begin{bmatrix} 1 & -1 \\ 4 & 3 \end{bmatrix}$ **D.** $\frac{1}{7}\begin{bmatrix} -3 & -4 \\ 1 & -1 \end{bmatrix}$

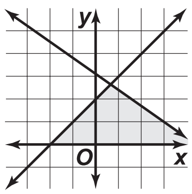
11. Which product represents the solution to the system? $-y + 7x = 14$ **11.** ________
$-x + 4y = 1$

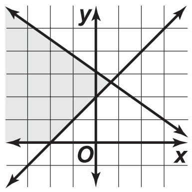
A. $\begin{bmatrix} \frac{1}{27} & \frac{4}{27} \\ \frac{7}{27} & \frac{1}{27} \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 14 \end{bmatrix}$ **B.** $\begin{bmatrix} -\frac{7}{3} & \frac{4}{3} \\ -\frac{1}{3} & \frac{1}{3} \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 14 \end{bmatrix}$

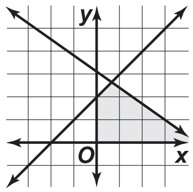
C. $\begin{bmatrix} \frac{7}{3} & -\frac{4}{3} \\ \frac{1}{3} & -\frac{1}{3} \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 14 \end{bmatrix}$ **D.** $\begin{bmatrix} -\frac{1}{27} & -\frac{4}{27} \\ -\frac{4}{27} & -\frac{1}{27} \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 14 \end{bmatrix}$

12. Which is the graph of the system $y \geq 0$, $2x + 3y \leq 9$, and $x + 2 \geq y$? **12.** ________

A.


B.


C.


D.


13. Find the minimum value of $f(x, y) = 2x - y + 2$ for the polygonal convex set determined by this system of inequalities. **13.** ________

$x \geq 1$ $x \leq 3$ $y \geq 0$ $\frac{1}{2}x + y \leq 5$

A. 0 **B.** -3 **C.** 3 **D.** -0.5

14. Describe the linear programming situation for this system of inequalities where you are asked to find the maximum value of $f(x, y) = x + y$. $x \geq 0$ $y \geq 0$ $6x + 3y \leq 18$ $x + 3y \leq 9$ **14.** ________

A. infeasible **B.** unbounded
C. an optimal solution **D.** alternate optimal solutions

15. A farmer can plant a combination of two different crops on 20 acres of land. Seed costs \$120 per acre for crop A and \$200 per acre for crop B. Government restrictions limit acreage of crop A to 15 acres, but do not limit crop B. Crop A will take 15 hours of labor per acre at a cost of \$5.60 per hour, and crop B will require 10 hours of labor per acre at \$5.00 per hour. If the expected income from crop A is \$600 per acre and from crop B is \$520 per acre, how many acres of crop A should be planted in order to maximize profit? **15.** ________

A. 5 acres **B.** 20 acres **C.** 15 acres **D.** infeasible

Bonus The system below forms a polygonal convex set. **Bonus:** ________
$x \leq 0$ $y \geq -x$, if $-6 \leq x \leq 0$
$y \leq 10$ $2x + 3y \geq 6$, if $-12 \leq x \leq -6$
What is the area of the closed figure?

A. 60 units^2 **B.** 52 units^2 **C.** 54 units^2 **D.** 120 units^2

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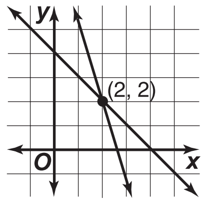
Chapter 2 Test, Form 1B

Write the letter for the correct answer in the blank at the right of each problem.

1. Which term(s) describe(s) this system? $4x + y = 8$, $3x - 2y = -5$ **1.** ________

A. dependent **B.** consistent and dependent
C. consistent and independent **D.** inconsistent

2. Which system of equations is shown by the graph? **2.** ________



A. $3x + y = 8$, $x = 1$ **B.** $y = x$, $3x = -4y$
C. $y = 2x - 1$, $x + y = 4$ **D.** $x + y = 4$, $3x + y = 8$

3. Solve algebraically. $5x - y = 16$, $2x + 3y = 3$ **3.** ________

A. (4, 4) **B.** (−1, 3) **C.** (9, −5) **D.** (3, −1)

4. Solve algebraically. **4.** ________

$$x - 3y - 3z = 0$$
$$2x + 5y - 5z = 1$$
$$-x + 5y - 6z = -9$$

A. (3, 4, −3) **B.** (3, 0, 1) **C.** (−2, 4, 3) **D.** (0, −3, 3)

5. Find $A - B$ if $A = \begin{bmatrix} -5 & 0 \\ 1 & 1 \\ 4 & -7 \end{bmatrix}$ and $B = \begin{bmatrix} 6 & -2 \\ -3 & -3 \\ 0 & 1 \end{bmatrix}$. **5.** ________

A. $\begin{bmatrix} -11 & 2 \\ 4 & 4 \\ 4 & -8 \end{bmatrix}$ **B.** $\begin{bmatrix} -1 & -2 \\ -2 & -2 \\ 4 & 6 \end{bmatrix}$ **C.** $\begin{bmatrix} 1 & -2 \\ -2 & -2 \\ 4 & -6 \end{bmatrix}$ **D.** $\begin{bmatrix} -1 & -2 \\ 4 & 4 \\ 4 & -6 \end{bmatrix}$

6. Find the values of x and y for which $\begin{bmatrix} 3y \\ 2x \\ 6y \end{bmatrix} = \begin{bmatrix} x + 5 \\ 0 \\ 10 \end{bmatrix}$ is true. **6.** ________

A. $\left(0, -\frac{3}{5}\right)$ **B.** $\left(0, \frac{5}{3}\right)$ **C.** $\left(0, \frac{3}{5}\right)$ **D.** $\left(-2, \frac{5}{3}\right)$

7. Find DE if $D = [5 \quad 2]$ and $E = \begin{bmatrix} 9 \\ -6 \end{bmatrix}$. **7.** ________

A. $\begin{bmatrix} 45 \\ -12 \end{bmatrix}$ **B.** [57] **C.** [45 −12] **D.** [33]

8. A triangle with vertices $A(-3, 4)$, $B(3, 1)$, and $C(-1, -5)$ is translated 5 units left and 3 units up. Find the coordinates of A', B', and C'. **8.** ________

A. $A'(-2, 7)$, $B'(8, 4)$, $C'(-7, -2)$ **B.** $A'(-8, 7)$, $B'(2, -4)$, $C'(6, 2)$ **C.** $A'(-8, 7)$, $B'(-2, 4)$, $C'(-6, -2)$ **D.** $A'(-8, 1)$, $B'(-2, -2)$, $C'(-6, -8)$

9. Find the value of $\begin{vmatrix} 5 & -1 \\ 2 & 7 \end{vmatrix}$. **9.** ________

A. 39 **B.** 37 **C.** −37 **D.** 33

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Chapter 2 Test, Form 1B (continued)

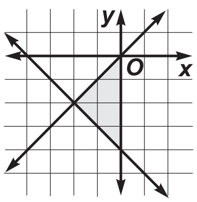
10. Find the inverse of $\begin{bmatrix} 3 & -6 \\ 1 & -2 \end{bmatrix}$, if it exists. **10.** ________

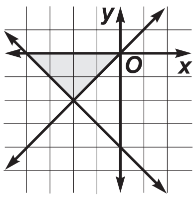
A. does not exist **B.** $-\frac{1}{12}\begin{bmatrix} -2 & 6 \\ -1 & 3 \end{bmatrix}$ **C.** $-\frac{1}{12}\begin{bmatrix} 3 & -6 \\ 1 & -2 \end{bmatrix}$ **D.** $-\frac{1}{12}\begin{bmatrix} 3 & 6 \\ -1 & -2 \end{bmatrix}$

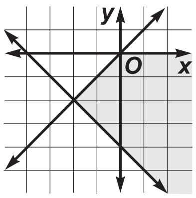
11. Which product represents the solution to the system? $3x + 4y = -2$, $2x + 5y = 6$ **11.** ________

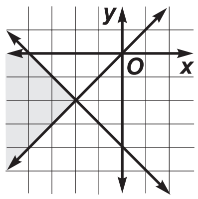
A. $\begin{bmatrix} 3 & 2 \\ 4 & 5 \end{bmatrix} \cdot \begin{bmatrix} -2 \\ 6 \end{bmatrix}$ **B.** $\frac{1}{7}\begin{bmatrix} 3 & 2 \\ 4 & 5 \end{bmatrix} \cdot \begin{bmatrix} -2 \\ 6 \end{bmatrix}$ **C.** $\begin{bmatrix} 5 & -2 \\ -4 & 3 \end{bmatrix} \cdot \begin{bmatrix} -2 \\ 6 \end{bmatrix}$ **D.** $\frac{1}{7}\begin{bmatrix} 5 & -4 \\ -2 & 3 \end{bmatrix} \cdot \begin{bmatrix} -2 \\ 6 \end{bmatrix}$

12. Which is the graph of the system? $y \le 0$, $x \le 0$, $x \ge y$, $x + y \ge -4$ **12.** ________

A.


B.


C.


D.


13. Find the maximum and minimum values of $f(x, y) = 2y + x$ for the polygonal convex set determined by this system of inequalities. **13.** ________
$x \ge 1$ $y \ge 0$ $x \le 4 - y$

A. minimum: 1; maximum: 4 **B.** minimum: 0; maximum: 5
C. minimum: 1; maximum: 7 **D.** minimum: 4; maximum: 8

14. Describe the linear programming situation for this system of inequalities. **14.** ________
$x \le 1$ $y \ge 0$ $3x + y \le 5$

A. infeasible **B.** unbounded
C. an optimal solution **D.** alternate optimal solutions

15. A farm supply store carries 50-lb bags of both grain pellets and grain mash for pig feed. It can store 600 bags of pig feed. At least twice as many of its customers prefer the mash to the pellets. The store buys the pellets for \$3.75 per bag and sells them for \$6.00. It buys the mash for \$2.50 per bag and sells it for \$4.00. If the store orders no more than \$1400 worth of pig feed, how many bags of mash should the store order to make the most profit? **15.** ________

A. 160 bags **B.** 320 bags **C.** 200 bags **D.** 400 bags

Bonus Find the value of c for which the system is consistent and dependent. **Bonus:** ________
$3x + y = 5$
$4.5y = c + 9x$

A. no value **B.** -8 **C.** 9.5 **D.** 7.5

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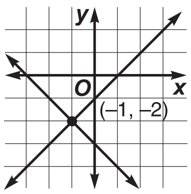
Chapter 2 Test, Form 1C

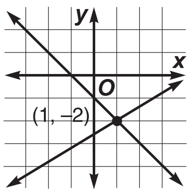
Write the letter for the correct answer in the blank at the right of each problem.

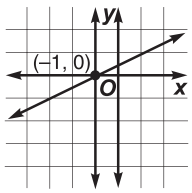
1. Which term(s) describe(s) this system? $3x - 2y = 7$ **1.** ________
$-9x + 6y = -10$

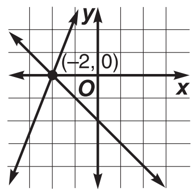
A. dependent **B.** consistent and dependent
C. consistent and independent **D.** inconsistent

2. Solve by graphing. $x + y = -1$ **2.** ________
$x - 2y = 5$

A. 

B. 

C. 

D. 

3. Solve algebraically. $y + 2x = 7$ **3.** ________
$y = 5 - x$

A. $(8, -3)$ **B.** $(2, 3)$ **C.** $(4, 1)$ **D.** $(-2, 3)$

4. Solve algebraically. $x - y + z = 5$ **4.** ________
$-2x - y - z = -6$
$3x + 3y - 2z = -5$

A. $(1, 0, 4)$ **B.** $(0, -1, 6)$ **C.** $(-1, -2, 4)$ **D.** $(-3, 0, 2)$

5. Find $A + B$ if $A = \begin{bmatrix} 2 & 3 & 1 \\ -1 & 0 & -4 \end{bmatrix}$ and $B = \begin{bmatrix} -5 & 2 & -1 \\ 8 & 4 & -3 \end{bmatrix}$. **5.** ________

A. $\begin{bmatrix} 3 & -7 \\ 6 & 4 \\ 0 & 7 \end{bmatrix}$ **B.** $\begin{bmatrix} 3 & 5 & 0 \\ -7 & 4 & 7 \end{bmatrix}$ **C.** $\begin{bmatrix} -3 & 5 & 0 \\ 7 & 4 & -7 \end{bmatrix}$ **D.** $\begin{bmatrix} -3 & 7 \\ 5 & 4 \\ 0 & -7 \end{bmatrix}$

6. Find the values of x and y for which $\begin{bmatrix} y \\ 3x \end{bmatrix} = \begin{bmatrix} 5 - x \\ 2y - 5 \end{bmatrix}$ is true. **6.** ________

A. $(2, 3)$ **B.** $(4, 1)$ **C.** $(-1, 6)$ **D.** $(1, 4)$

7. Find EF if $E = \begin{bmatrix} -1 & -2 \\ 2 & 1 \end{bmatrix}$ and $F = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$. **7.** ________

A. $\begin{bmatrix} -5 & -3 \\ 7 & 0 \end{bmatrix}$ **B.** $\begin{bmatrix} -3 & -2 \\ -2 & 2 \end{bmatrix}$ **C.** $\begin{bmatrix} 2 & -2 \\ -2 & -3 \end{bmatrix}$ **D.** $\begin{bmatrix} -1 & -5 \\ 5 & 4 \end{bmatrix}$

8. A triangle of vertices $A(-3, 4)$, $B(3, 1)$, and $C(-1, -5)$ is dilated by a scale factor of 2. Find the coordinates of A', B', C'. **8.** ________

A. $A'(-3, 4)$, $B'(3, -1)$, $C'(-1, 5)$
B. $A'(-3, 8)$, $B'(3, 2)$, $C'(-1, -10)$
C. $A'(-6, 8)$, $B'(6, 2)$, $C'(-2, -10)$
D. $A'(-1.5, 2)$, $B'(1.5, 0.5)$, $C'(-0.5, -2.5)$

9. Find the value of $\begin{vmatrix} 5 & 1 \\ 4 & 2 \end{vmatrix}$. **9.** ________

A. 6 **B.** -3 **C.** 14 **D.** -6

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Chapter 2 Test, Form 1C (continued)

10. Find the inverse of $\begin{bmatrix} 7 & 1 \\ 3 & 0 \end{bmatrix}$, if it exists. **10.** ________

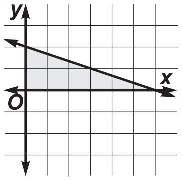
A. does not exist **B.** $-\frac{1}{3}\begin{bmatrix} 0 & -1 \\ -3 & 7 \end{bmatrix}$ **C.** $-\frac{1}{3}\begin{bmatrix} 7 & 1 \\ 3 & 0 \end{bmatrix}$ **D.** $-\frac{1}{4}\begin{bmatrix} 0 & -1 \\ -3 & 7 \end{bmatrix}$

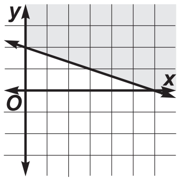
11. Which product represents the solution to the system? $\begin{bmatrix} 1 & 1 \\ 2 & 4 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$ **11.** ________

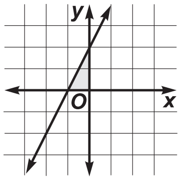
A. $\frac{1}{2}\begin{bmatrix} 1 & -1 \\ -2 & 4 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 2 \end{bmatrix}$ **B.** $\frac{1}{2}\begin{bmatrix} 4 & -1 \\ -2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 2 \end{bmatrix}$

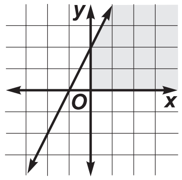
C. $\begin{bmatrix} 1 & 1 \\ 2 & 4 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 2 \end{bmatrix}$ **D.** $\begin{bmatrix} 4 & -1 \\ -2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 2 \end{bmatrix}$

12. Which is the graph of the system $x \geq 0$ $y \geq 0$ $x + 3y \leq 6$? **12.** ________

A.


B.


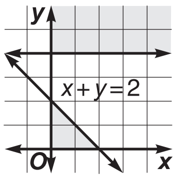
C.


D.


13. Find the maximum value of $f(x, y) = y - x + 1$ for the polygonal convex set determined by this system of inequalities. **13.** ________
$x \geq 0$ $y \geq 0$ $2x + y \leq 4$

A. 1 **B.** 8 **C.** 5 **D.** -2

14. Which term best describes the linear programming situation represented by the graph? **14.** ________

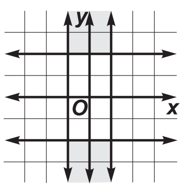


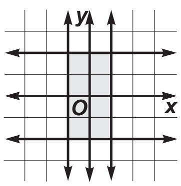
A. infeasible
B. unbounded
C. an optimal solution
D. alternate optimal solutions

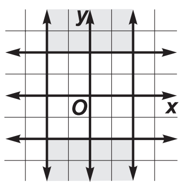
15. Chase Quinn wants to expand his cut-flower business. He has 12 additional acres on which he intends to plant lilies and gladioli. He can plant at most 7 acres of gladiolus bulbs and no more than 11 acres of lilies. In addition, the number of acres planted to gladioli G can be no more than twice the number of acres planted to lilies L. The inequality $L + 2G \geq 10$ represents his labor restrictions. If his profits are represented by the function $f(L, G) = 300L + 200G$, how many acres of lilies should he plant to maximize his profit? **15.** ________

A. 1 acre **B.** 11 acres **C.** 0 acres **D.** 9.5 acres

Bonus Solve the system $|y| \geq 2$ and $|x| \leq 1$ by graphing. **Bonus:** ________

A. no solution **B.**


C.


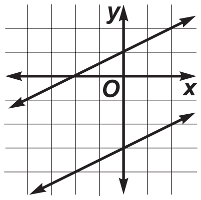
D.


NAME ______________ DATE ______________ PERIOD ______

Chapter 2

Chapter 2 Test, Form 2A

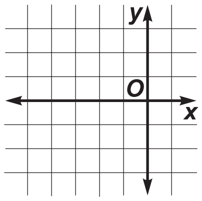
1. Identify the system shown by the graph as consistent and independent, consistent and dependent, or inconsistent.



1. ______________

2. Solve by graphing.
$6 + y = -3x$
$2x + 6y + 4 = 0$

2.



3. Solve algebraically. $4x - y = -3$
$5x + 2y = 1$

3. ______________

4. Solve algebraically. $-3x + y + z = 2$
$5x + 2y - 4z = 21$
$x - 3y - 7z = -10$

4. ______________

Use matrices J, K, and L to evaluate each expression. If the matrix does not exist, write impossible.

$$J = \begin{bmatrix} -5 & 4 \\ 2 & -3 \\ 6 & 1 \end{bmatrix} \quad K = \begin{bmatrix} -8 & 5 & -1 \\ 3 & 2 & 0 \end{bmatrix} \quad L = \begin{bmatrix} -4 & 2 & 3 \\ 7 & -2 & -1 \end{bmatrix}$$

5. ______________

6. ______________

5. $-2K + L$ **6.** $3K - J$ **7.** JL

7. ______________

8. For what values of x, y, and z is $\begin{bmatrix} 2 - 2x \\ -3z + 4x \\ 9z - 6 \end{bmatrix} = \begin{bmatrix} y \\ 0 \\ -4y \end{bmatrix}$ true?

8. ______________

9. A triangle with vertices $A(-3, 4)$, $B(5, -2)$, and $C(7, -4)$ is rotated 90° countrclockwise about the origin. Find the coordinates of A', B', and C'.

9. ______________

10. Find the value of $\begin{vmatrix} 4 & -3 & 1 \\ 7 & 2 & -5 \\ -1 & 1 & 3 \end{vmatrix}$.

10. ______________

11. If it exists, find A^{-1} if $A = \begin{bmatrix} -3 & 7 \\ 5 & 1 \end{bmatrix}$.

11. ______________

NAME ______________________ DATE ______________ PERIOD ________

Chapter 2 Test, Form 2A (continued)

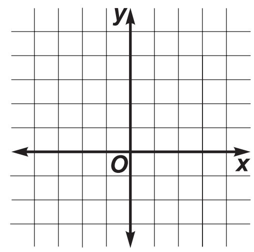
12. Write the matrix product that represents the solution to the system.
$4x + 3y = -1$
$5x - 2y = 3$

12. ________________

Solve each system of inequalities by graphing and name the vertices of each polygonal convex set. Then, find the maximum and minimum values for each set.

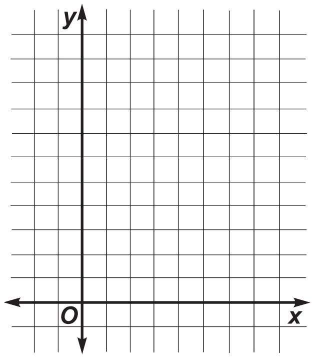
13. $y \leq 5$
$x \leq 3$
$3y + 4x \geq 0$
$f(x, y) = 2x - y + 1$

13.



14. $y \geq 0$
$0 \leq x \leq 4$
$-x + y \leq 6$
$f(x, y) = 3x - 5y$

14.



Solve each problem, if possible. If not possible, state whether the problem is infeasible, has alternate optimal solutions, or is unbounded.

15. The BJ Electrical Company needs to hire master electricians and apprentices for a one-week project. Master electricians receive a salary of \$750 per week and apprentices receive \$350 per week. As part of its contract, the company has agreed to hire at least 30 workers. The local Building Safety Council recommends that each master electrician allow 3 hours for inspection time during the project. This project should require at least 24 hours of inspection time. How many workers of each type should be hired to meet the safety requirements, but minimize the payroll?

15. ________________

16. A company makes two models of light fixtures, A and B, each of which must be assembled and packed. The time required to assemble model A is 12 minutes, and model B takes 18 minutes. It takes 2 minutes to package model A and 1 minute to package model B. Each week, 240 hours are available for assembly time and 20 hours for packing.

a. If model A sells for \$1.50 and model B sells for \$1.70, how many of each model should be made to obtain the maximum weekly profit?

16a. ________________

b. What is the maximum weekly profit?

16b. ________________

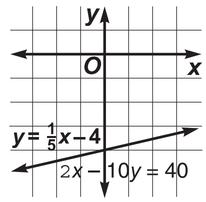
Bonus Use a matrix equation to find the value of x for the system.
$ax + by = c$
$dx + ey = f$

Bonus: ________________

NAME ______________________ DATE ____________ PERIOD ________

Chapter 2 Test, Form 2B

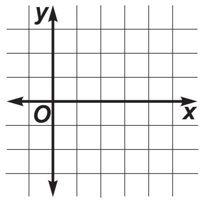
1. Identify the system shown by the graph as consistent and independent, consistent and dependent, or inconsistent.



1. ____________

2. Solve by graphing. $5x + 2y = 8$
$x - y = 3$

2.



3. Solve algebraically. $3x + 2y = 1$
$2x - 3y = 18$

3. ____________

4. Solve. $2x - y + z = -3$
$y + z - 1 = 0$
$x + y - z = 9$

4. ____________

Use matrices D, E, and F to find each sum or product.

$$D = \begin{bmatrix} -2 & 1 \\ 7 & 5 \\ 3 & -4 \end{bmatrix} \quad E = \begin{bmatrix} -3 & -4 \\ 0 & 1 \\ 2 & 6 \end{bmatrix} \quad F = \begin{bmatrix} -2 & 5 & 1 \\ 3 & 4 & -6 \end{bmatrix}$$

5. $E - D$ **6.** $3F$ **7.** DF

5. ____________

6. ____________

7. ____________

8. For what values of x and y is $[5 \quad -3x] = [-4x \quad 5y]$ true?

8. ____________

9. A triangle with vertices $A(3, -3)$, $B(1, 4)$, and $C(-2, -1)$ is reflected over the line with equation $y = x$. Find the coordinates of A', B', and C'.

9. ____________

10. Find the value of $\begin{vmatrix} 3 & -7 \\ 5 & -2 \end{vmatrix}$.

10. ____________

11. Given $A = \begin{bmatrix} 3 & -6 \\ 2 & 4 \end{bmatrix}$, find A^{-1}, if it exists.

11. ____________

NAME ______________________ DATE ______________ PERIOD ________

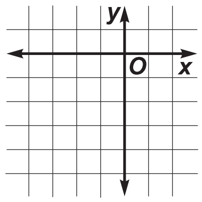
Chapter 2 Test, Form 2B (continued)

12. Write the matrix product that represents the solution to the system.
$3x + 2y = 5$
$5x + 4y = 7$

12. ________________

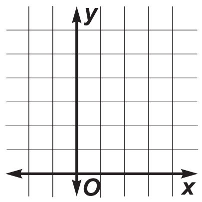
Solve each system of inequalities by graphing and name the vertices of each polygonal convex set. Then, find the maximum and minimum values for each function.

13. $x \leq y$
$y \leq 0$
$x \geq -4$
$f(x, y) = 7x + y$



13. ________________

14. $y \geq 0$
$x \geq -1$
$y \leq x + 3$
$2x + y \leq 6$
$f(x, y) = 5x + 3y$



14. ________________

Solve each problem, if possible. If not possible, state whether the problem is infeasible, has alternate optimal solutions, or is unbounded.

15. A manufacturer of garden furniture makes a Giverny bench and a Kensington bench. The company needs to produce at least 15 Giverny benches a day and at least 20 Kensington benches a day. It must also meet the demand for at least twice as many Kensington benches as Giverny benches. The company can produce no more than 60 benches a day. If each Kensington sells for \$250 and each Giverny sells for \$325, how many of each kind of bench should be produced for maximum daily income?

15. ________________

16. André Gagné caters small dinner parties on weekends. Because of an increase in demand for his work, he needs to hire more chefs and waiters. He will have to pay each chef \$120 per weekend and each waiter \$70 per weekend. He needs at least 2 waiters for each chef he hires. Find the maximum amount André will need to spend to hire extra help for one weekend.

16. ________________

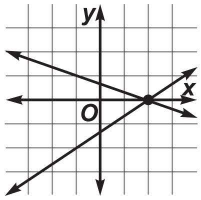
Bonus Find an equation of the line that passes through $P(2, 1)$ and through the intersection of $\frac{3x}{2} + \frac{y}{4} = 7$ and $\frac{x}{5} - \frac{2y}{3} = \frac{7}{3}$.

Bonus: ________________

NAME ______________________ DATE ______________ PERIOD ________

Chapter 2 Test, Form 2C

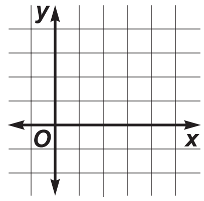
1. Determine whether the system shown by the graph is consistant and independent, consistent and dependent, or inconsistent.



1. ________________

2. Solve by graphing.
$x + y = 3$
$x - y = 1$

2.



3. Solve algebraically. $4x - 3y = 5$
$4x - 3y = 10$

3. ________________

4. Solve algebraically. $x - 2y + z = -5$
$3x - 2y + z = 3$
$2x - y + 2z = -7$

4. ________________

Use matrices A, B, and C to find the sum or product.

$A = \begin{bmatrix} -3 & -2 \\ 0 & 5 \\ 6 & -1 \end{bmatrix}$ $B = \begin{bmatrix} 8 & 6 \\ -5 & 4 \\ 3 & -1 \end{bmatrix}$ $C = \begin{bmatrix} -3 & 2 & 1 \\ 0 & 5 & -6 \end{bmatrix}$

5. ________________

6. ________________

5. $A + B$ **6.** $-2B$ **7.** AC

7. ________________

8. For what values of x and y is $\begin{bmatrix} x \\ y + 3 \end{bmatrix} = \begin{bmatrix} 5y \\ x + 7 \end{bmatrix}$ true?

8. ________________

9. A triangle with vertices $A(1, 2)$, $B(2, -4)$,and $C(-2, 0)$ is translated 2 units left and 3 units up. Find the coordinates A', B', and C'.

9. ________________

10. Find the value of $\begin{vmatrix} -3 & 2 \\ 5 & -7 \end{vmatrix}$.

10. ________________

11. If it exists, find A^{-1} if $A = \begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix}$.

11. ________________

NAME ______________________ DATE ______________ PERIOD _________

Chapter 2 Test, Form 2C (continued)

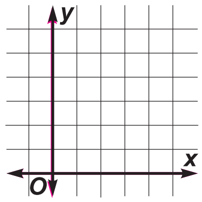
12. Write the matrix product that represents the solution to the system. $3x + 4y = 5$
$x + 2y = 10$

12. ________________

Solve each system of inequalities by graphing, and name the vertices of each polygonal convex set. Then, find the maximum and minimum values for each function.

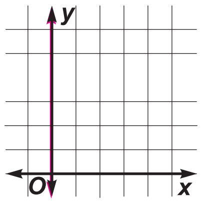
13. $x \geq 0$
$y \geq 0$
$x + y \leq 5$
$f(x, y) = 5x + 3y$

13.



14. $x \geq 0$
$0 \leq y \leq 4$
$2x + y \leq 6$
$f(x, y) = 3x - 2y$

14.



Solve each problem, if possible. If not possible, state whether the problem is infeasible, has alternate optimal solutions, or is unbounded.

15. The members of the junior class at White Mountain High School are selling ice-cream and frozen yogurt cones in the school cafeteria to raise money for their prom. The students have enough ice cream for 50 cones and enough frozen yogurt for 80 cones. They have 100 cones available. If they plan to sell each ice-cream cone for $2 and each frozen yogurt cone for $1, and they sell all 100 cones, what is the maximum amount they can expect to make?

15. ________________

16. Ginny Dettore custom-sews bridal gowns and bridesmaids' dresses on a part-time basis. Each dress sells for $200, and each gown sells for $650. It takes her 2 weeks to produce a bridesmaid's dress and 5 weeks to produce a bridal gown. She accepts orders for at least three times as many bridesmaids' dresses as she does bridal gowns. In the next 22 weeks, what is the maximum amount of money she can expect to earn?

16. ________________

Bonus Find the coordinates of the vertices of the triangle determined by the graphs of $4x + 3y + 1 = 0$, $4x - 3y - 17 = 0$, and $4x - 9y + 13 = 0$.

Bonus: ________________

NAME ______________________ DATE ______________ PERIOD ________

Chapter 2 Open-Ended Assessment

Instructions: Demonstrate your knowledge by giving clear, concise solutions to each problem. Be sure to include all relevant drawings and justify your answers. You may show your solution in more than one way or investigate beyond the requirements of the problem.

1. Stores A and B sell cassettes, CDs, and videotapes. The figures given are in the thousands.

$$A = \begin{array}{r} \text{Cassettes} \\ \text{Videotapes} \\ \text{CDs} \end{array} \overset{\begin{array}{cc} \text{Total Sales} & \text{Units Sold} \end{array}}{\begin{bmatrix} \$1000 & 200 \\ \$1500 & 150 \\ \$2000 & 175 \end{bmatrix}}$$

Store A

$$B = \begin{array}{r} \text{Cassettes} \\ \text{Videotapes} \\ \text{CDs} \end{array} \overset{\begin{array}{cc} \text{Total Sales} & \text{Units Sold} \end{array}}{\begin{bmatrix} \$1750 & 350 \\ \$2500 & 250 \\ \$3000 & 250 \end{bmatrix}}$$

Store B

a. Find $A + B$ and write one or two sentences describing the meaning of the sum.

b. Find $B - A$ and write one or two sentences describing the meaning of the difference.

c. Find $2B$. Interpret its meaning, and describe at least one situation in which a merchant may be interested in this product.

d. If $C = [2\ \ 3\ \ 3]$, find CB and write a word problem to illustrate a possible meaning of the product.

e. Does $CB = BC$? Why or why not?

2. A builder builds two styles of houses: the Executive, on which he makes a profit of $30,000, and the Suburban, on which he makes a $25,000 profit. His construction crew can complete no more than 10 houses each year, and he wishes to build no more than 6 of the Executive per year.

a. How many houses of each style should he build to maximize profit? Explain why your answer makes sense.

b. If he chooses to limit the number of Executives he builds each year to 8, how many houses of each style should he build if he wishes to maximize profit? Explain your answer.

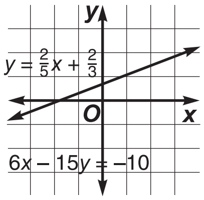
c. If he keeps the original limit on Executives, but raises the limit on the total number of houses he builds, what would be the effect on the number of each style built to maximize profit? Why?

Chapter 2

NAME ______________________ DATE ______________ PERIOD __________

Mid-Chapter Test (Lessons 2-1 through 2-4)

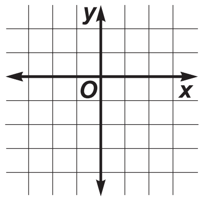
1. Identify the system shown by the graph as consistent and independent, consistent and dependent, or inconsistent



1. ____________

2. Solve by graphing. $x = 2y + 7$
$y = -x - 2$

2.



3. Solve algebraically. $4x + 2y = 6$
$3x - 4y = 10$

3. ____________

4. Solve algebraically. $x + 2y + z = 3$
$2x - 3y + 2z = -1$
$x - 3y + 2z = 1$

4. ____________

Use matrices A, B, and C to evaluate each expression. If the matrix does not exist, write impossible.

$$A = \begin{bmatrix} 5 & 4 \\ -1 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 3 & 2 \\ -5 & 1 \end{bmatrix} \quad C = \begin{bmatrix} 0 & -4 & 2 \\ -3 & 1 & 3 \end{bmatrix}$$

5. ____________

6. ____________

5. $A + (-B)$ **6.** $-4C$ **7.** BC

7. ____________

8. For what values of x and y is $\begin{bmatrix} -3x \\ y \\ 3y + 1 \end{bmatrix} = \begin{bmatrix} y + 2 \\ x - 6 \\ -14 \end{bmatrix}$ true?

8. ____________

9. A quadrilateral with vertices $A(-3, 5)$, $B(1, 2)$, $C(6, 3)$ and $D(-4, -5)$ is translated 2 units left and 7 units down. Find the coordinates of A', B', C', and D'.

9. ____________

10. Find the coordinates of a triangle with vertices $E(3, 4)$, $F(5, -1)$, and $G(-2, 3)$ after a rotation of 180° counterclockwise about the origin.

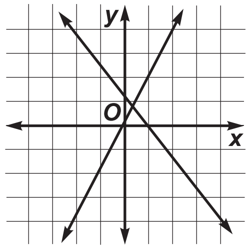
10. ____________

Chapter 2

NAME ______________________ DATE ______________ PERIOD ________

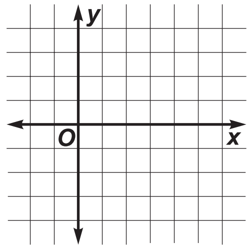
Chapter 2 Quiz A (Lessons 2-1 and 2-2)

1. Identify the system shown by the graph as consistent and independent, consistent and dependent, or inconsistent.



1. ______________________

2. Solve by graphing. $y = \frac{5}{4}x - 3$
$x + 2y = 8$



2.

Solve each system of equations algebraically.

3. $2x + y = -2$
$4x + 2y = -4$

4. $3x + 5y = 21$
$x + y = 5$

5. $2x + y + 3z = 8$
$x + 2y - 2z = 3$
$5x + y + z = 1$

3. ______________________

4. ______________________

5. ______________________

Chapter 2

NAME ______________________ DATE ______________ PERIOD ________

Chapter 2 Quiz B (Lessons 2-3 and 2-4)

Use matrices A, B, and C to evaluate each expression. If the matrix does not exist, write impossible.

$$A = \begin{bmatrix} -6 & 9 \\ 4 & -8 \end{bmatrix} \quad B = \begin{bmatrix} 2 & -3 \\ 4 & 2 \end{bmatrix} \quad C = \begin{bmatrix} 3 & -4 \\ 5 & 2 \\ -8 & 6 \end{bmatrix}$$

1. ______________________

2. ______________________

1. $A + B$ **2.** CB **3.** $2B - A$

3. ______________________

4. For what values of x and y is $\begin{bmatrix} x - 6 \\ 2y + 1 \end{bmatrix} = \begin{bmatrix} 2y + x \\ x \end{bmatrix}$ true?

4. ______________________

5. Each of the following transformations is applied to a triangle with vertices $A(4, 1)$, $B(-5, -3)$, and $C(-2, 6)$. Find the coordinates of A', B', and C' in each case.
a. translation right 3 units and down 5 units
b. reflection over the line $y = x$
c. rotation of 90° counterclockwise about the origin

5a. ______________________

5b. ______________________

5c. ______________________

NAME ______________________ DATE ____________ PERIOD ________

Chapter 2 Quiz C (Lessons 2-5 and 2-6)

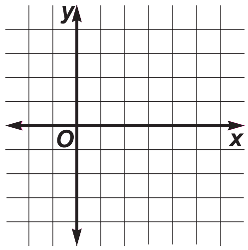
1. Find the value of $\begin{vmatrix} 4 & -1 & 6 \\ -3 & 0 & 1 \\ 5 & -2 & 2 \end{vmatrix}$. **1.** ________________

2. Find the multiplicative inverse of $\begin{bmatrix} -4 & 2 \\ 8 & -4 \end{bmatrix}$, if it exists. **2.** ________________

3. Solve the system $6x + 4y = 0$ and $3x - y = 1$ by using a matrix equation. **3.** ________________

Use the system $x \geq 0$, $y \geq 0$, $y \leq 4$, $2x - y \leq 4$ and the function $f(x, y) = 3x - 2y$ for Exercises 4 and 5.

4. Solve the system by graphing, and name the vertices of the polygonal convex set. **4.**



5. Find the minimum and maximum values of the function. **5.** ________________

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Chapter 2 Quiz D (Lessons 2-7)

1. ***Carpentry*** Emilio has a small carpentry shop where he makes large and small bookcases. His profit on a large bookcase is \$80 and on a small bookcase is \$50. It takes Emilio 6 hours to make a large bookcase and 2 hours to make a small one. He can spend only 24 hours each week on his carpentry work. He must make at least two of each size each week. What is his maximum weekly profit? **1.** ________________

2. ***Painting*** Michael Thomas, the manager of a paint store, is mixing paint for a spring sale. There are 32 units of yellow dye, 54 units of brown dye, and an unlimited supply of base paint available. Mr. Thomas plans to mix as many gallons as possible of Autumn Wheat and Harvest Brown paint. Each gallon of Autumn Wheat requires 4 units of yellow dye and 1 unit of brown dye. Each gallon of Harvest Brown paint requires 1 unit of yellow dye and 6 units of brown dye. Find the maximum number of gallons of paint that Mr. Thomas can mix. **2.** ________________

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SAT and ACT Practice

After working each problem, record the correct answer on the answer sheet provided or use your own paper.

1. If $x - 3 = 2(1 - x) + 1$, then what is the value of x?
A. -8
B. -2
C. 2
D. 5
E. 8

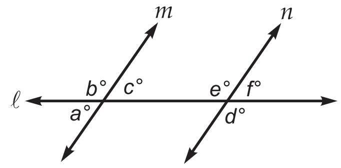
2. If $\frac{a}{b} = 0.625$, then $\frac{b}{a}$ is equal to which of the following?
A. 1.60
B. 2.67
C. 2.70
D. 3.33
E. 4.25

3. Points A, B, C, and D are arranged on a line in that order. If $AC = 13$, $BD = 12$, and $AD = 21$, then $BC = ?$
A. 12
B. 9
C. 8
D. 4
E. 3

4. A group of z people buys x widgets at a price of y dollars each. If the people divide the cost of this purchase evenly, then how much must each person pay in dollars?
A. $\frac{xy}{z}$
B. $\frac{yz}{x}$
C. $\frac{xz}{y}$
D. $xy + z$
E. xyz

5. If $7a + 2b = 11$ and $a - 2b = 5$, then what is the value of a?
A. -2.0
B. -0.5
C. 1.4
D. 2.0
E. It cannot be determined from the information given.

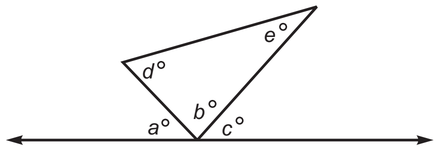
6. In the figure below, if $m \parallel n$ and $b = 125$, then $c + f = ?$



A. 50
B. 55
C. 110
D. 130
E. 180

7. Twenty-five percent of 28 is what percent of 4?
A. 7%
B. 57%
C. 70%
D. 128%
E. 175%

8. Using the figure below, which of the following is equal to $a + c$?



A. $2b$
B. $b + 90$
C. $d + e$
D. $b + d - e$
E. $180 - (d + e)$

9. At what coordinates does the line $3y + 5 = x - 1$ intersect the y-axis?
A. $(0, -2)$
B. $(0, -1)$
C. $\left(0, \frac{1}{3}\right)$
D. $(-2, 0)$
E. $(-6, 0)$

10. $\frac{(n^3)^6 \times (n^4)^5}{n^2} =$
A. n^9
B. n^{16}
C. n^{19}
D. n^{36}
E. n^{40}

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SAT and ACT Practice (continued)

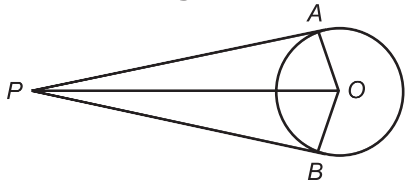
11. If m varies directly as n and $\frac{m}{n} = 5$, then what is the value of m when $n = 2.2$?
A. 0.44
B. 2.27
C. 4.10
D. 8.20
E. 11.00

12. If $\|x\| = x^2 - 3x$, then $\|-3\| =$
A. 0
B. 6.0
C. 18.0
D. 3.3
E. 9.0

13. How many distinct 3-digit numbers contain only nonzero digits?
A. 909
B. 899
C. 789
D. 729
E. 504

14. If $2x^2 - 5x - 9 = 0$, then x could equal which of the following?
A. -1.12
B. 0.76
C. 1.54
D. 2.63
E. 3.71

15. If $\overline{PA}$ and $\overline{PB}$ are tangent to circle O at A and B, respectively, then which of the following must be true?



I. $PB > PO$
II. $\angle APO = \angle BPO$
III. $\angle APB + \angle AOB = \angle PAO + \angle PBO$
A. I only
B. II only
C. I and II only
D. II and III only
E. I, II, and III

16. In the picture below, if the ratio of QR to RS is 2:3 and the ratio of PQ to QS is 1:2, then what is the ratio of PQ to RS?

A. 5:6
B. 7:6
C. 5:4
D. 11:6
E. 11:5

17–18. Quantitive Comparison
A. if the quantity in Column A is greater
B. if the quantity in Column B is greater
C. if the two quantities are equal
D. if the relationship cannot be determined for the information given

n is 12% of m

	Column A	Column B
17.	$6m$	$50n$

$r^5 = t$
$r > 1$

	Column A	Column B
18.	$t^2 - t$	$r^{10} - r^6$

19. Grid-In If the average of m, $m - 3$, $m - 10$, and $2m - 11$ is 4, what is the mode?

20. Grid-In A radio station schedules a one-hour block of time to cover a local music competition. The station runs twelve 30-second advertisements and uses the remaining time to broadcast live music. What is the value of $\frac{\text{live music time}}{\text{advertising time}}$?

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Chapter 2

Chapter 2 Cumulative Review (Chapters 1-2)

1. State the domain and range of $\{(-5, 2), (4, 3), (-2, 0), (-5, 1)\}$. Then state whether the relation is a function. Write *yes* or *no*. **1.** ________

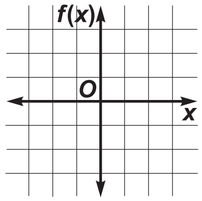
2. If $f(x) = -4x^2$ and $g(x) = \frac{2}{x}$, find $[g \circ f](x)$. **2.** ________

3. Write the slope-intercept form of the equation of the line that passes through the point $(-5, 4)$ and has a slope of -1. **3.** ________

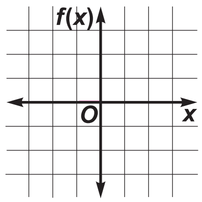
4. Write the standard form of the equation of the line perpendicular to the line $y = 3x - 5$ that passes through $(-3, 7)$. **4.** ________

Graph each function.

5. $f(x) = \begin{cases} -2 \text{ if } x \leq -1 \\ \frac{2}{3}x - \frac{1}{3} \text{ if } x > -1 \end{cases}$ **5.**



6. $f(x) = [\![-x]\!]$ **6.**



7. How many solutions does a consistent and dependent system of linear equations have? **7.** ________

Solve each system of equations algebraically.

8. $4x - 7y = -2$
$x + 2y = 7$ **8.** ________

9. $-3x - 2y + 3z = -1$
$2x + 5y - 3z = -6$
$4x + 3y + 3z = 22$ **9.** ________

10. Find $-2A + B$ if $A = \begin{bmatrix} -5 & 1 & -2 \\ 3 & 4 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} -8 & 3 & 1 \\ 5 & -2 & 4 \end{bmatrix}$. **10.** ________

11. Find the value of $\begin{vmatrix} 3 & 5 \\ 7 & -2 \end{vmatrix}$. **11.** ________

12. A triangle with vertices $A(-2, 5)$, $B(-3, 7)$ and $C(6, -2)$ is reflected over the line with equation $y = x$. Find the coordinates of A', B', and C'. **12.** ________

13. Write the matrix product that represents the solution to the system. $\begin{bmatrix} 1 & 3 \\ -1 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -5 \\ 1 \end{bmatrix}$ **13.** ________

14. Find the maximum and minimum values of $f(x, y) = 3x + y$ for the polygonal convex set determined by $x \geq 1$, $y \geq 0$, and $x + 0.5y \leq 2$. **14.** ________

15. Champion Lumber converts logs into lumber or plywood. In a given week, the total production cannot exceed 800 units, of which 200 units of lumber and 300 units of plywood are required by regular customers. The profit on a unit of lumber is \$20 and on a unit of plywood is \$30. Find the number of units of lumber and plywood that should be produced to maximize profit. **15.** ________

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NAME ______________________________ DATE _______________ PERIOD ________

SAT and ACT Practice Answer Sheet

(10 Questions)

1 (A) (B) (C) (D) (E)

2 (A) (B) (C) (D) (E)

3 (A) (B) (C) (D) (E)

4 (A) (B) (C) (D) (E)

5 (A) (B) (C) (D) (E)

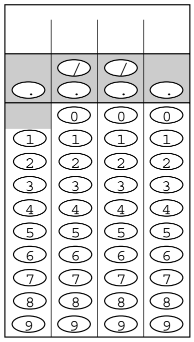
6 (A) (B) (C) (D) (E)

7 (A) (B) (C) (D) (E)

8 (A) (B) (C) (D) (E)

9 (A) (B) (C) (D) (E)

10



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SAT and ACT Practice Answer Sheet

(20 Questions)

1 (A) (B) (C) (D) (E)

2 (A) (B) (C) (D) (E)

3 (A) (B) (C) (D) (E)

4 (A) (B) (C) (D) (E)

5 (A) (B) (C) (D) (E)

6 (A) (B) (C) (D) (E)

7 (A) (B) (C) (D) (E)

8 (A) (B) (C) (D) (E)

9 (A) (B) (C) (D) (E)

10 (A) (B) (C) (D) (E)

11 (A) (B) (C) (D) (E)

12 (A) (B) (C) (D) (E)

13 (A) (B) (C) (D) (E)

14 (A) (B) (C) (D) (E)

15 (A) (B) (C) (D) (E)

16 (A) (B) (C) (D) (E)

17 (A) (B) (C) (D) (E)

18 (A) (B) (C) (D) (E)

19

	/	/	
.	.	.	.
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

20

	/	/	
.	.	.	.
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

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2-1

Practice

Solving Systems of Equations in Two Variables

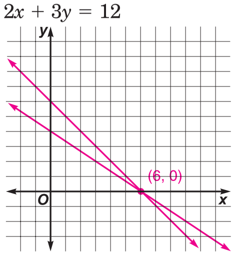
State whether each system is consistent and independent, consistent and dependent, or inconsistent.

1. $-x + y = -4$, $3x - 3y = 12$ **consistent and dependent**

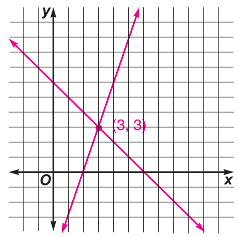
2. $2x - 5y = 8$, $15y - 6x = -24$ **consistent and dependent**

Solve each system of equations by graphing.

3. $x + y = 6$, $2x + 3y = 12$



4. $x + y = 6$, $3x - y = 6$



Solve each system of equations algebraically.

5. $x + y = 4$, $3x - 2y = 7$ **(3, 1)**

6. $3x - 4y = 10$, $-3x + 4y = 8$ **no solution**

7. $4x - 3y = 15$, $2x + y = 5$ **(3, −1)**

8. $4x + 5y = 11$, $3x - 2y = -9$ **(−1, 3)**

9. $2x + 3y = 19$, $7x - y = 9$ **(2, 5)**

10. $2x - y = 6$, $x + y = 6$ **(4, 2)**

11. ***Real Estate*** AMC Homes, Inc. is planning to build three- and four-bedroom homes in a housing development called Chestnut Hills. Consumer demand indicates a need for three times as many four-bedroom homes as for three-bedroom homes. The net profit from each three-bedroom home is \$16,000 and from each four-bedroom home, \$17,000. If AMC Homes must net a total profit of \$13.4 million from this development, how many homes of each type should they build? **200 three-bedroom, 600 four-bedroom**

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2-1

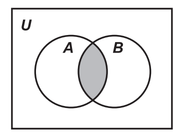
Enrichment

Set Theory and Venn Diagrams

Set theory, which was developed by the nineteenth century German logician and mathematician, Georg Cantor, became a fundamental unifying principle in the study of mathematics in the middle of the twentieth century. The use of sets permits the precise description of mathematical concepts.

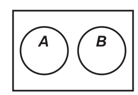
The intersection of two sets determines the elements common to the two sets. Thus, the intersection of two lines in a system of equations refers to the point or points that are common to the sets of points belonging to each of the lines. We can use Venn diagrams, which are named for the British logician, John Venn, to visually represent the intersection of two sets.

Example **Let U = the set of all points in the Cartesian coordinate plane. Let A = the set of all points that satisfy the equation $x = 4$. Let B = the set of all points that satisfy the equation $y = -2$. Draw a Venn diagram to represent U, A, and B.**



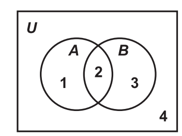
The shaded region represents the intersection of sets A and B, written $A \cap B$. In this example, $A \cap B = \{(4, -2)\}$.

Since the solution sets of two parallel lines have no points in common, the sets are called **disjoint sets**. In a Venn diagram, such sets are drawn as circles that do not overlap.



$A \cap B = \varnothing$.

Use the diagram at the right to answer the following questions.
Let A = the set of all points that satisfy the equation for line p.
Let B = the set of all points that satisfy the equation for line q.



1. In which numbered region do the points that satisfy only the equation for line p lie? **1**

2. In which numbered region do the points that satisfy only the equation for line q lie? **3**

3. In which numbered region do the points that satisfy the equations for neither line p nor line q lie? **4**

4. If the equation of line p is $2x - y = 4$ and the equation of line q is $3x + y = 6$, which point lies in region 2? **(2,0)**

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2-2 Practice

Solving Systems of Equations in Three Variables

Solve each system of equations.

1. $x + y - z = -1$
   $x + y + z = 3$
   $3x - 2y - z = -4$
   (0, 1, 2)

2. $x + y = 5$
   $3x + z = 2$
   $4y - z = 8$
   (10, −5, −28)

3. $3x - 5y + z = 8$
   $4y - z = 10$
   $7x + y = 4$
   (2.2, −11.4, −55.6)

4. $2x + 3y + 3z = 2$
   $10x - 6y + 3z = 0$
   $4x - 3y - 6z = 2$
   $\left(\frac{1}{2}, \frac{2}{3}, -\frac{1}{3}\right)$

5. $2x - y + z = -1$
   $x - y + z = 1$
   $x - 2y + z = 2$
   (−2, −1, 2)

6. $4x + 4y - 2z = 3$
   $-6x - 6y + 6z = 5$
   $2x - 3y - 4z = 2$
   $\left(\frac{13}{3}, -2, \frac{19}{6}\right)$

7. $x - z = 5$
   $y + 3z = 12$
   $2x + y = 7$
   (20, −33, 15)

8. $2x + 4y - 2z = 9$
   $4x - 6y + 2z = -9$
   $x - y + 3z = -4$
   $\left(\frac{1}{2}, \frac{3}{2}, -1\right)$

9. ***Business*** The president of Speedy Airlines has discovered that her competitor, Zip Airlines, has purchased 13 new airplaines from Commuter Aviation for a total of \$15.9 million. She knows that Commuter Aviation produces three types of planes and that type A sells for \$1.1 million, type B sells for \$1.2 million, and type C sells for \$1.7 million. The president of Speedy Airlines also managed to find out that Zip Airlines purchased 5 more type A planes than type C planes. How many planes of each type did Zip Airlines purchase?
   7 type A planes, 4 type B planes, and 2 type C planes

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2-2 Enrichment

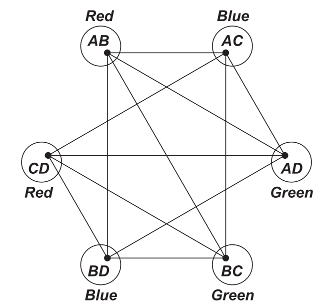
Graph Coloring

The student council is scheduling a volleyball tournament for teams from all four classes. They want each class team to play every other class team exactly once. How should they schedule the tournament?

If we call the teams A, B, C, and D, all of the possible games among the four teams can be represented as AB, AC, AD, BC, BD, and CD.

Draw a graph to represent the problem. Then color the graph. To **color** a graph means to color the vertices of that graph so that no two vertices connected by an edge have the same color.

Let the vertices represent the possible games. Let the edges represent games that cannot be scheduled at the same time. For example, if team B is playing team C, then team C cannot play team D.



Choose a vertex at which to begin.

- Color AB red. Since CD is not connected to AB, color it red as well. All of the other vertices are connected to AB, so do not color them red.
- Color AC blue. Since BD is not connected to AC, color it blue.
- Color AD green. Since BC is not connected to AD, color it green.

The colored graph shows that pairs of games can be scheduled as follows

AB with CD; AC with BD; AD with BC

The **chromatic number** of a graph is the least number of colors necessary to color the graph. The chromatic number of the graph above is 3.

1. What does the chromatic number of the graph in the example above represent?
   the number of game times scheduled

2. Draw and color a graph to represent the same type of tournament, but with 6 teams playing.
   See students' graphs.

3. What is the chromatic number for your graph? **5**

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2-3

Practice

Modeling Real-World Data with Matrices

Find the values of x and y for which each matrix equation is true.

1. $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2y - 4 \\ 2x \end{bmatrix}$ $\left(\frac{4}{3}, \frac{8}{3}\right)$

2. $\begin{bmatrix} 2x - 3 \\ 4y \end{bmatrix} = \begin{bmatrix} y \\ 3x \end{bmatrix}$ $\left(\frac{12}{5}, \frac{9}{5}\right)$

Use matrices A, B, and C to find each sum, difference, or product.

$A = \begin{bmatrix} -1 & 5 & 6 \\ 2 & -7 & -2 \\ 4 & 4 & 2 \end{bmatrix}$ $B = \begin{bmatrix} 2 & 3 & 1 \\ -1 & 1 & 4 \\ 5 & -2 & 3 \end{bmatrix}$ $C = \begin{bmatrix} 8 & 10 & -9 \\ -6 & 12 & 14 \end{bmatrix}$

3. $A + B$ $\begin{bmatrix} 1 & 8 & 7 \\ 1 & -6 & 2 \\ 9 & 2 & 5 \end{bmatrix}$

4. $A - B$ $\begin{bmatrix} -3 & 2 & 5 \\ 3 & -8 & -6 \\ -1 & 6 & -1 \end{bmatrix}$

5. $B - A$ $\begin{bmatrix} 3 & -2 & -5 \\ -3 & 8 & 6 \\ 1 & -6 & 1 \end{bmatrix}$

6. $-2A$ $\begin{bmatrix} 2 & -10 & -12 \\ -4 & 14 & 4 \\ -8 & -8 & -4 \end{bmatrix}$

7. CA $\begin{bmatrix} -24 & -66 & 10 \\ 86 & -58 & -32 \end{bmatrix}$

8. AB $\begin{bmatrix} 23 & -10 & 37 \\ 1 & 3 & -32 \\ 14 & 12 & 26 \end{bmatrix}$

9. AA $\begin{bmatrix} 35 & -16 & -4 \\ -24 & 51 & 22 \\ 12 & 0 & 20 \end{bmatrix}$

10. CB $\begin{bmatrix} -39 & 52 & 21 \\ 46 & -34 & 84 \end{bmatrix}$

11. $(CA)B$ $\begin{bmatrix} 68 & -158 & -258 \\ 70 & 264 & -242 \end{bmatrix}$

12. $C(AB)$ $\begin{bmatrix} 68 & -158 & -258 \\ 70 & 264 & -242 \end{bmatrix}$

13. ***Entertainment*** On one weekend, the Goxfield Theater reported the following ticket sales for three first-run movies, as shown in the matrix at the right. If the ticket prices were $6 for each adult and $4 for each child, what were the weekend sales for each movie.

	Adults	Children
Movie 1	1021	523
Movie 2	2547	785
Movie 3	3652	2456

M1: $8218; M2: $18,422; M3: $31,736

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2-3

Enrichment

Elementary Matrix Transformations

Elementary row transformations can be made by multiplying a matrix on the left by the appropriate transformation matrix. For example, to interchange rows 2 and 3 in a 3 × 3 matrix, multiply the matrix on the left by the matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$.

Example **Let** $C = \begin{bmatrix} -5 & 3 & 1 \\ 1 & 5 & 0 \\ 10 & -6 & -2 \end{bmatrix}$. **Multiply on the left by** $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} -5 & 3 & 1 \\ 1 & 5 & 0 \\ 10 & -6 & -2 \end{bmatrix} = \begin{bmatrix} -5 & 3 & 1 \\ 10 & -6 & -2 \\ 1 & 5 & 0 \end{bmatrix}$$

More complicated row transformations can also be made by a matrix multiplier.

Example **Find the elementary matrix that, when matrix *A* is multiplied by it on the left, row 1 will be replaced by the sum of 2 times row 1 and row 3.**

Let $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$. Find M such that $M \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} =$

$\begin{bmatrix} 2a_{11} + a_{31} & 2a_{12} + a_{32} & 2a_{13} + a_{33} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$. Try $M = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

Check to see that the product MA meets the required conditions.

1. Find the elementary matrix that will interchange rows 1 and 3 of matrix A.

$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

2. Find the elementary matrix that will multiply row 2 of matrix A by 5.

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

3. Find the elementary matrix that will multiply the elements of the first row of matrix A by −2 and add the results to the corresponding elements of the third row.

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$

4. Find the elementary matrix that will interchange rows 2 and 3 of matrix A and multiply row 1 by −2.

$\begin{bmatrix} -2 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

5. Find the elementary matrix that will multiply the elements of the second row of matrix A by −3 and add the results to the corresponding elements of row 1.

5. $\begin{bmatrix} 1 & -3 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

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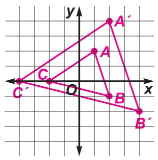
2-4 Practice

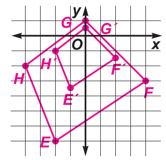
Modeling Motion with Matrices

Use scalar multiplication to determine the coordinates of the vertices of each dilated figure. Then graph the pre-image and the image on the same coordinate grid.

1. triangle with vertices $A(1, 2)$, $B(2, -1)$, and $C(-2, 0)$; scale factor 2
$A'(2, 4)$, $B'(4, -2)$, $C'(-4, 0)$

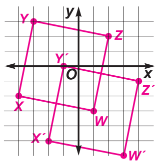


2. quadrilateral with vertices $E(-2, -7)$, $F(4, -3)$, $G(0, 1)$, and $H(-4, -2)$; scale factor 0.5
$E'(-1, -3.5)$ $F'(2, -1.5)$, $G'(0, 0.5)$, $H'(-2, -1)$

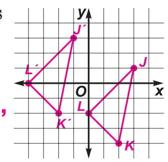


Use matrices to determine the coordinates of the vertices of each translated figure. Then graph the pre-image and the image on the same coordinate grid.

3. square with vertices $W(1, -3)$, $X(-4, -2)$, $Y(-3, 3)$, and $Z(2, 2)$ translated 2 units right and 3 units down **$W'(3, -6)$, $X'(-2, -5)$, $Y'(-1, 0)$, $Z'(4, -1)$**

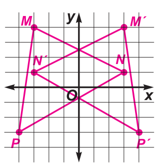


4. triangle with vertices $J(3, 1)$, $K(2, -4)$, and $L(0, -2)$ translated 4 units left and 2 units up **$J'(-1, 3)$, $K'(-2, -2)$, $L'(-4, 0)$**

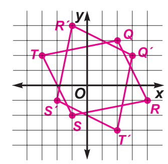


Use matrices to determine the coordinates of the vertices of each reflected figure. Then graph the pre-image and the image on the same coordinate grid.

5. $\triangle MNP$ with vertices $M(-3, 4)$, $N(3, 1)$, and $P(-4, -3)$ reflected over the y-axis
$M'(3, 4)$, $N'(-3, 1)$, $P'(4, -3)$

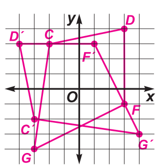


6. a rhombus with vertices $Q(2, 3)$, $R(4, -1)$, $S(-1, -2)$, and $T(-3, 2)$ reflected over the line $y = x$ **$Q'(3, 2)$, $R'(-1, 4)$, $S'(-2, -1)$, $T'(2, -3)$**

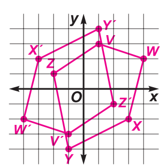


Use matrices to determine the coordinates of the vertices of each rotated figure. Then graph the pre-image and the image on the same coordinate grid.

7. quadrilateral $CDFG$ with vertices $C(-2, 3)$, $D(3, 4)$, $F(3, -1)$, and $G(-3, -4)$ rotated 90°
$C'(-3, -2)$, $D'(-4, 3)$, $F'(1, 3)$, $G'(4, -3)$



8. Pentagon $VWXYZ$ with vertices $V(1, 3)$, $W(4, 2)$, $X(3, -2)$, $Y(-1, -4)$, $Z(-2, 1)$ rotated 180°
$V'(-1, -3)$, $W'(-4, -2)$, $X'(-3, 2)$, $Y'(1, 4)$, $Z'(2, -1)$



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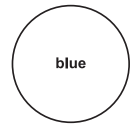
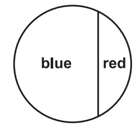
2-4 Enrichment

Map Coloring

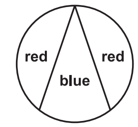
Each of the graphs below is drawn on a plane with no accidental crossings of its edges. This kind of graph is called a planar graph. A planar graph separates the plane into **regions**. A very famous rule called the *four-color theorem* states that every planar graph can be colored with at most four colors, without any adjacent regions having the same color.



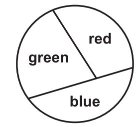
Graph A
one region
one color



Graph B
two regions
two colors



Graph C
three regions
two colors



Graph D
three regions
three colors

If there are four or more regions, two, three, or four colors may be required.

Example **Color the map at the right using the least number of colors.**

Look for the greatest number of isolated states or regions. Virginia, Georgia, and Mississippi are isolated. Color them pink.

Alabama and North Carolina are also isolated. Color them blue.

South Carolina and Tennessee are remaining. Because they do not share a common edge, they may be colored with the same color. Color them both green.

Thus, three colors are needed to color this map.

Color each map with the least number of colors. **See students' maps.**

1.

2.

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2-5

Practice

Determinants and Multiplicative Inverses of Matrices

Find the value of each determinant.

1. $\begin{vmatrix} -2 & 3 \\ 8 & -12 \end{vmatrix}$ **0**

2. $\begin{vmatrix} 3 & -5 \\ 7 & 9 \end{vmatrix}$ **62**

3. $\begin{vmatrix} 1 & -1 & 0 \\ 2 & 1 & 4 \\ 5 & -3 & 5 \end{vmatrix}$ **7**

4. $\begin{vmatrix} 2 & 3 & 1 \\ -3 & -1 & 5 \\ 1 & -4 & 2 \end{vmatrix}$ **82**

Find the inverse of each matrix, if it exists.

5. $\begin{vmatrix} 3 & 8 \\ -1 & 5 \end{vmatrix}$ $\frac{1}{23}\begin{vmatrix} 5 & -8 \\ 1 & 3 \end{vmatrix}$

6. $\begin{vmatrix} 5 & 2 \\ 10 & 4 \end{vmatrix}$ **does not exist**

Solve each system by using matrix equations.

7. $2x - 3y = 17$
$3x + y = 9$
(4, −3)

8. $4x - 3y = -16$
$2x + 5y = 18$
(−1, 4)

Solve each matrix equation.

9. $\begin{bmatrix} 2 & -1 & 3 \\ 1 & 2 & 1 \\ -1 & -3 & -2 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -8 \\ 3 \\ -7 \end{bmatrix}$, if the inverse is $-\frac{1}{6}\begin{bmatrix} -1 & -11 & -7 \\ 1 & -1 & 1 \\ -1 & 7 & 5 \end{bmatrix}$ **(−4, 3, 1)**

10. $\begin{bmatrix} 5 & -2 & 4 \\ 3 & -4 & 2 \\ 1 & -3 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$, if the inverse is $-\frac{1}{8}\begin{bmatrix} 2 & -10 & 12 \\ -1 & 1 & 2 \\ -5 & 13 & -14 \end{bmatrix}$ $\left(-1, -\frac{1}{2}, \frac{1}{2}\right)$

11. ***Landscaping*** Two dump truck have capacities of 10 tons and 12 tons. They make a total of 20 round trips to haul 226 tons of topsoil for a landscaping project. How many round trips does each truck make?
7 trips by the 10-ton truck, 13 trips by the 12-ton truck

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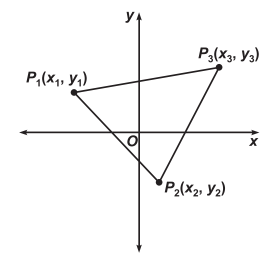
2-5

Enrichment

Area of a Triangle

Determinants can be used to find the area of a triangle on the coordinate plane. For a triangle with vertices $P_1(x_1, y_1)$, $P_2(x_2, y_2)$, and $P_3(x_3, y_3)$, the area is given by the following formula.



$$\text{Area} = \pm\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

The sign is chosen so that the result is positive.

Example **Find the area of the triangle with vertices $A(3, 8)$, $B(-2, 5)$, and $C(0, -1)$.**

$$\text{Area} = \pm\frac{1}{2}\begin{vmatrix} 3 & 8 & 1 \\ -2 & 5 & 1 \\ 0 & -1 & 1 \end{vmatrix} = \pm\frac{1}{2}\left[3\begin{vmatrix} 5 & 1 \\ -1 & 1 \end{vmatrix} - 8\begin{vmatrix} -2 & 1 \\ 0 & 1 \end{vmatrix} + 1\begin{vmatrix} -2 & 5 \\ 0 & -1 \end{vmatrix}\right]$$

$$= \pm\frac{1}{2}[3(5 - (-1)) - 8(-2 - 0) + 1(2 - 0)]$$

$$= \pm\frac{1}{2}[36] = 18$$

The area is 18 square units.

Find the area of the triangle having vertices with the given coordinates.

1. $A(0, 6)$, $B(0, -4)$, $C(0, 0)$
Points are collinear; there is no triangle.

2. $A(5, 1)$, $B(-3, 7)$, $C(-2, -2)$
33 units^2

3. $A(8, -2)$, $B(0, -4)$, $C(-2, 10)$
58 units^2

4. $A(12, 4)$, $B(-6, 4)$, $C(3, 16)$
108 units^2

5. $A(-1, -3)$, $B(7, -5)$, $C(-3, 9)$
46 units^2

6. $A(1.2, 3.1)$, $B(5.7, 6.2)$, $C(8.5, 4.4)$
8.39 units^2

7. What is the sign of the determinant in the formula when the points are taken in clockwise order? in counterclockwise order?
negative; positive

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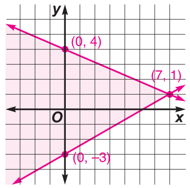
Practice

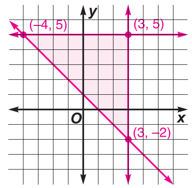
Solving Systems of Linear Inequalities

Solve each system of inequalities by graphing.

1. $-4x + 7y \geq -21$; $3x + 7y \leq 28$

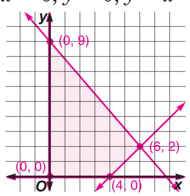


2. $x \leq 3$; $y \leq 5$; $x + y \geq 1$



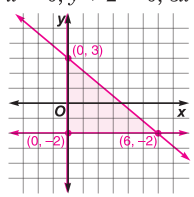
Solve each system of inequalities by graphing. Name the coordinates of the vertices of the polygonal convex set.

3. $x \geq 0$; $y \geq 0$; $y \geq x - 4$; $7x + 6y \leq 54$



vertices: (0, 0), (0, 9), (6, 2), (4, 0)

4. $x \geq 0$; $y + 2 \geq 0$; $5x + 6y \leq 18$

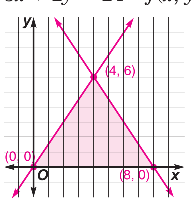


vertices: (0, 3), (0, −2), (6, −2)

Find the maximum and minimum values of each function for the polygonal convex set determined by the given system of inequalities.

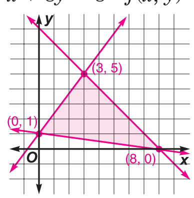
5. $3x - 2y \geq 0$ $\quad y \geq 0$
$3x + 2y \leq 24$ $\quad f(x, y) = 7y - 3x$

max = 30
min = −24



6. $y \leq -x + 8$ $\quad 4x - 3y \geq -3$
$x + 8y \geq 8$ $\quad f(x, y) = 4x - 5y$

max = 32
min = −13



7. ***Business*** Henry Jackson, a recent college graduate, plans to start his own business manufacturing bicycle tires. Henry knows that his start-up costs are going to be \$3000 and that each tire will cost him at least \$2 to manufacture. In order to remain competitive, Henry cannot charge more than \$5 per tire. Draw a graph to show when Henry will make a profit.

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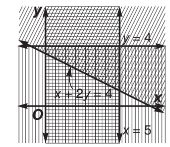
Enrichment

Reading Mathematics: Reading the Graph of a System of Inequalities

Like all other kinds of graphs, the graph of a system of linear inequalities is useful to us only as long as we are able to "read" the information presented on it.

The line of each equation separates the xy-plane into two **half-planes**. Shading is used to tell which half-plane satisfies the inequality bounded by the line. Thus, the shading below the boundary line $y = 4$ indicates the solution set of $y \leq 4$. By drawing horizontal or vertical shading lines or lines with slopes of 1 or −1, and by using different colors, you will make it easier to "read" your graph.



$0 \leq x \leq 5$
$y \leq 4$
$x + 2y \geq 7$

The shading in between the boundary lines $x \geq 0$ and $x \leq 5$, indicate that the inequality $0 \leq x \leq 5$ is equivalent to the *intersection* $x \geq 0 \cap x \leq 5$.

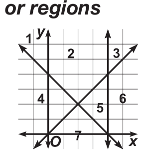
Similarly, the solution set of the system, indicated by the cross-hatching, is the intersection of the solutions of all the inequalities in the system. We can use set notation to write a description of this solution set:

$$\{(x, y) \mid 0 \leq x \leq 5 \cap y \leq 4 \cap x + 2y \geq 7\}.$$

We read this "The set of all ordered pairs, (x, y), such that x is greater than or equal to 0 *and* x is less than or equal to 5 *and* y is less than or equal to 4 *and* $x + 2y$ is greater than or equal to 7."

The graph also shows where any maximum and minimum values of the function f occur. Notice, however, that these values occur only at points belonging to the set containing the *union* of the vertices of the convex polygonal region.

Use the graph at the right to show which numbered region or regions belong to the graph of the solution set of each system.



1. $x \geq 0$
$y \leq x$
5, 6, 7

2. $0 \leq x \leq 4$
$y \leq x$
$x + y \geq 4$
5

3. The domain of $f(x, y) = 2x + y$ is the xy-plane. If we graphed this function, where would we represent its range? **On a 3rd axis, perpendicular to the *xy*-plane.**

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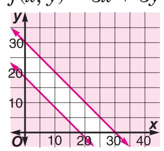
Practice

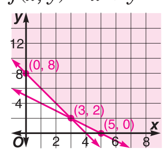
Linear Programming

Graph each system of inequalities. In a problem asking you to find the maximum value of f(x, y), state whether the situation is infeasible, has alternate optimal solutions, or is unbounded. In each system, assume that x ≥ 0 and y ≥ 0 unless stated otherwise.

1. $-2y \ge 2x - 36$ **infeasible**
$x + y \ge 30$
$f(x, y) = 3x + 3y$

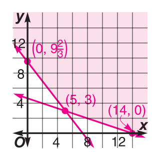


2. $2x + 2y \ge 10$ **unbounded**
$2x + y \ge 8$
$f(x, y) = x + y$

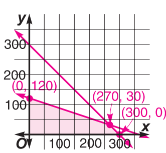


Solve each problem, if possible. If not possible, state whether the problem is infeasible, has alternate optimal solutions, or is unbounded.

3. ***Nutrition*** A diet is to include at least 140 milligrams of Vitamin A and at least 145 milligrams of Vitamin B. These requirements can be obtained from two types of food. Type X contains 10 milligrams of Vitamin A and 20 milligrams of Vitamin B per pound. Type Y contains 30 milligrams of Vitamin A and 15 milligrams of Vitamin B per pound. If type X food costs \$12 per pound and type Y food costs \$8 per pound how many pounds of each type of food should be purchased to satisfy the requirements at the minimum cost?
$9\frac{2}{3}$ pounds of Y and 0 pounds of X



4. ***Manufacturing*** The Cruiser Bicycle Company makes two styles of bicycles: the Traveler, which sells for \$200, and the Tourester, which sells for \$600. Each bicycle has the same frame and tires, but the assembly and painting time required for the Traveler is only 1 hour, while it is 3 hours for the Tourister. There are 300 frames and 360 hours of labor available for production. How many bicycles of each model should be produced to maximize revenue?
alternate optimal



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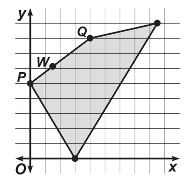
Enrichment

Convex Polygons

You have already learned that over a closed convex polygonal region, the maximum and minimum values of any linear function occur at the vertices of the polygon. To see why the values of the function at any point on the boundary of the region must be between the values at the vertices, consider the convex polygon with vertices P and Q.



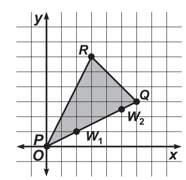
Let W be a point on $\overline{PQ}$.
If W lies between P and Q, let $\frac{PW}{PQ} = w$.

Then $0 < w < 1$ and the coordinates of W are $((1 - w)x_1 + wx_2, (1 - w)y_1 + wy_2)$. Now consider the function $f(x, y) = 3x - 5y$.

$$\begin{aligned} f(W) &= 3[(1-w)x_1 + wx_2] - 5[(1-w)y_1 + wy_2] \\ &= (1-w)(3x_1) + 3wx_2 + (1-w)(-5y_1) - 5wy_2 \\ &= (1-w)(3x_1 - 5y_1) + w(3x_2 - 5y_2) \\ &= (1-w)f(P) + wf(Q) \end{aligned}$$

This means that $f(W)$ is between $f(P)$ and $f(Q)$, or that the greatest and least values of $f(x, y)$ must occur at P or Q.

Example **If $f(x, y) = 3x + 2y$, find the maximum value of the function over the shaded region at the right.**



The maximum value occurs at the vertex (6, 3). The minimum value occurs at (0, 0). The values of $f(x, y)$ at W_1 and W_2 are between the maximum and minimum values.

$f(Q) = f(6, 3) = 24$
$f(W_1) = f(2, 1) = 8$
$f(W_2) = f(5, 2.5) = 20$
$f(P) = f(0, 0) = 0$

Let P and Q be vertices of a closed convex polygon, and let W lie on $\overline{PQ}$. Let f(x, y) = ax + by.

1. If $f(Q) = f(P)$, what is true of f? of $f(W)$? **It produces alternate optimal solutions all along $\overline{PQ}$; $f(W) = f(P)$.**

2. If $f(Q) = f(P)$, find an equation of the line containing P and Q.
Sample answer: $ax + by = f(P)$

Chapter 2 Answer Key

Form 1A

Page 67

1. A
2. B
3. D
4. C
5. B
6. B
7. A
8. D
9. D

Page 68

10. C
11. A
12. B
13. D
14. C
15. C

Bonus: C

Form 1B

Page 69

1. C
2. D
3. D
4. B
5. A
6. B
7. D
8. C
9. B

Page 70

10. A
11. D
12. A
13. C
14. B
15. B

Bonus: A

Chapter 2 Answer Key

Form 1C

Page 71

1. D
2. B
3. B
4. A
5. C
6. D
7. D
8. C
9. A

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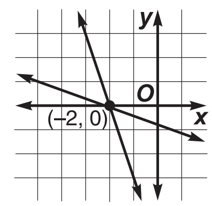
10. B
11. B
12. A
13. C
14. A
15. B

Bonus: B

Form 2A

Page 73

1. inconsistent

2. 

3. $\left(-\frac{5}{13}, \frac{19}{13}\right)$

4. (1, 6, −1)

5. $\begin{bmatrix} 12 & -8 & 5 \\ 1 & -6 & -1 \end{bmatrix}$

6. impossible

7. $\begin{bmatrix} 48 & -18 & -19 \\ -29 & 10 & 9 \\ -17 & 10 & 17 \end{bmatrix}$

8. $\left(-\frac{1}{2}, 3, -\frac{2}{3}\right)$

9. $A'(-4, -3)$, $B'(2, 5)$, $C'(4, 7)$

10. 101

11. $\begin{bmatrix} -\frac{1}{38} & \frac{7}{38} \\ \frac{5}{38} & \frac{3}{38} \end{bmatrix}$

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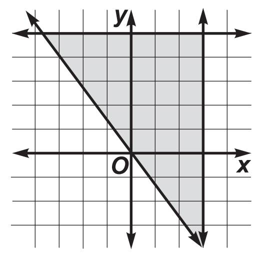
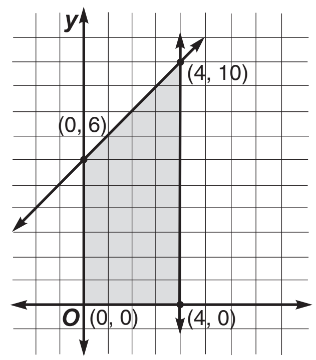
12. $-\frac{1}{23}\begin{bmatrix} -2 & -3 \\ -5 & 4 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 3 \end{bmatrix}$

13. 

Vertices: (−3.75, 5), (3, 5), (3, −4); min.: −11.5; max.: 11

14. 

Vertices: (0, 0), (0, 6), (4, 10), (4, 0); min.: −38; max.: 12

15. 8 master and 22 apprentice

16a. 300 A, 600 B

16b. $1470

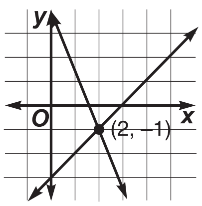
Bonus: $x = \frac{ce - bf}{ae - bd}$

Chapter 2 Answer Key

Form 2B

Page 75

1. consistent and dependent

2. 

3. (3, −4)

4. (2, 4, −3)

5. $\begin{bmatrix} -1 & -5 \\ -7 & -4 \\ -1 & 10 \end{bmatrix}$

6. $\begin{bmatrix} -6 & 15 & 3 \\ 9 & 12 & -18 \end{bmatrix}$

7. $\begin{bmatrix} 7 & -6 & -8 \\ 1 & 55 & -23 \\ -18 & -1 & 27 \end{bmatrix}$

8. $\left(-\frac{5}{4}, \frac{3}{4}\right)$

9. $A'(-3, 3)$, $B'(4, 1)$, $C'(-1, -2)$

10. 29

11. $\begin{bmatrix} \frac{1}{6} & \frac{1}{4} \\ -\frac{1}{12} & \frac{1}{8} \end{bmatrix}$

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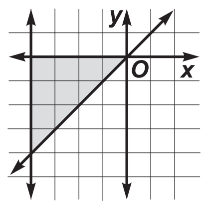
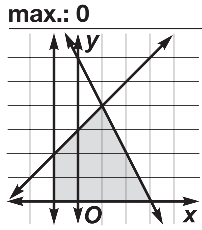
12. $\frac{1}{2}\begin{bmatrix} 4 & -2 \\ -5 & 3 \end{bmatrix} \cdot \begin{bmatrix} 5 \\ 7 \end{bmatrix}$



13. vertices: (0, 0), (−4, 0), (−4, −4); min.: −32; max.: 0



14. vertices: (−1, 0), (−1, 2), (1, 4), (3, 0); min.: −5; max.: 17

15. 20 Giverny benches and 40 Kensington benches

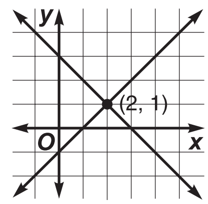
16. unbounded

Bonus: $x + y = 3$

Form 2C

Page 77

1. consistent and independent

2. 

3. no solution

4. (4, 1, −7)

5. $\begin{bmatrix} 5 & 4 \\ -5 & 9 \\ 9 & -2 \end{bmatrix}$

6. $\begin{bmatrix} -16 & -12 \\ 10 & -8 \\ -6 & 2 \end{bmatrix}$

7. $\begin{bmatrix} 9 & -16 & 9 \\ 0 & 25 & -30 \\ -18 & 7 & 12 \end{bmatrix}$

8. (−5, −1)

9. $A'(-1, 5)$, $B'(0, -1)$, $C'(-4, 3)$

10. 11

11. $\begin{bmatrix} \frac{1}{2} & 0 \\ -\frac{1}{6} & \frac{1}{3} \end{bmatrix}$

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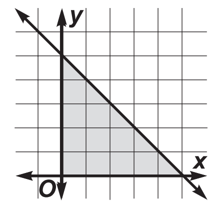
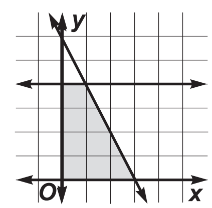
12. $\frac{1}{2}\begin{bmatrix} 2 & -4 \\ -1 & 3 \end{bmatrix} \cdot \begin{bmatrix} 5 \\ 10 \end{bmatrix}$



13. Vertices: (0, 0), (0, 5), (5, 0); min.: 0; max.: 25

14. Vertices: (0, 0), (0, 4), (1, 4), (3, 0); min.: −8; max.: 9



15. $150

16. $2500

Bonus: (2, −3), (−1, 1), (8, 5)

Chapter 2 Answer Key

CHAPTER 2 SCORING RUBRIC

Level	Specific Criteria
3 Superior	• Shows thorough understanding of the concepts *addition, subtraction,* and *multiplication of matrices, sclar multiplication of a matrix,* and *finding the maximum value of a function for a polygonal convex set.* • Uses appropriate strategies to solve problem. • Computations are correct. • Written explanations are exemplary. • Word problem concerning linear inequalities is appropriate and makes sense. • Goes beyond requirements of some of all problems.
2 Satisfactory, with Minor Flaws	• Shows thorough understanding of the concepts *addition, subtraction,* and *multiplication of matrices, scalar multiplication of a matrix,* and *finding the maximum value of a function for a polygonal convex set.* • Uses appropriate strategies to solve problem. • Computations are mostly correct. • Written explanations are effective. • Word problem concerning the product of matrices is appropriate and mades sense. • Satisfies most requirements of problems.
1 Nearly Satisfactory, with Serious Flaws	• Shows understanding of the concepts *addition, subtraction,* and *multiplication of matrices, scalar multiplication of a matrix,* and *finding the maximum multiplication value of a function for a polygonal convex set.* • May not use appropriate strategies to solve problem. • Computations are mostly correct. • Written explanations are mostly correct. • Word problem concerning the product of matrices is mostly appropriate and sensible. • Satisfies most requirements of problems.
0 Unsatisfactory	• Shows little or no understanding of the concepts *addition, subtraction*, and *multiplication of matrices, scalar multiplication of a matrix,* and *finding the maximum value of a function for a polygonal convex set.* • May not use appropriate strategies to solve problem. • Computations are incorrect. • Written explanations are not satisfactory. • Word problem concerning the product of matrices is not appropriate or sensible. • Does not satisfy requirements of problems.

Chapter 2 Answer Key

Open-Ended Assessment

Page 79

1a. $A + B$:

	Total Sales	Units Sold
Cassettes	$2750	550
Videotape	$4000	400
CDs	$5000	425

$A + B$ gives the total sales and number of units sold for both stores. For example, the two stores sold 550,000 cassettes. The combined cassette sales were $2,750,000

1b. $B - A$:

	Total Sales	Units Sold
Cassettes	$750	150
Videotape	$1000	100
CDs	$1000	47

$B - A$ give the difference between total sales and number of units sold for the two years. For example, 100 more albums were sold in 1994 than in 1993.

1c. $2B$:

	Total Sales	Units Sold
Cassettes	$3500	700
Videotape	$5000	500
CDs	$6000	500

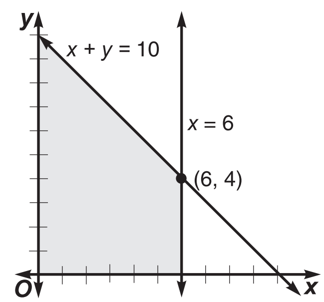
1d.

	Total Sales	Units Sold
CB:	[$20,000	2200]

Mrs. Carl wishes to sell twice as many cassettes, three times as many videotapes, and three times as many CDs in 1995 as in 1994.What is the total value and the total number of units of her 1995 goal?

1e. ***CB* ≠ *BC because BC* is undefined.**

2a. Let x = numbers of Executives
Let y = numbers of Suburbans.
$x + y \leq 10$ total number of houses
$x \leq 6$ number of Executives



Profit = $30,000$x$ + $25,000$y$. Check (0, 10), (0, 0), (6, 0,) and (6, 4).
$30,000(0) + $25,000(10) = $250,000
$30,000(6) + $25,000(0) = $180,000
$30,000(6) + $25,000(4) = $280,000
He should build 6 Executives and 4 Suburbans. This answer makes sense because he would be building the maximum number of Executives, which are the most profitable.

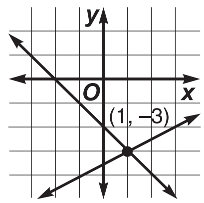
2b. He should build 8 Executives and 2 Suburbans because he makes a greater profit on the Executives.

2c. He should increase the number of Suburbans because he is already at the limit on Executives.

Chapter 2 Answer Key

Mid-Chapter Test
Page 80

1. consistent and dependent

2. 

3. (2, –1)

4. (–2, 1, 3)

5. $\begin{bmatrix} 2 & 2 \\ 4 & 1 \end{bmatrix}$

6. $\begin{bmatrix} 0 & 16 & -8 \\ 12 & -4 & -12 \end{bmatrix}$

7. $\begin{bmatrix} -6 & -10 & 12 \\ -3 & 21 & -7 \end{bmatrix}$

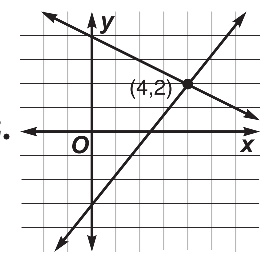
8. (1, –5)

9. *A′*(–5, –2), *B′*(–1, –5), *C′*(4, –4), *D′*(–6, –12)

10. *E′*(–3, –4), *F′*(–5, 1), *G′*(2, –3)

Quiz A
Page 81

1. consistent and independent

2. 

3. infinitely many solutions

4. (2, 3)

5. (–1, 4, 2)

Quiz B
Page 81

1. $\begin{bmatrix} -4 & 6 \\ 8 & -6 \end{bmatrix}$

2. $\begin{bmatrix} -10 & -17 \\ 18 & -11 \\ 8 & 36 \end{bmatrix}$

3. $\begin{bmatrix} 10 & -15 \\ 4 & 12 \end{bmatrix}$

4. (–5, –3)

5a. *A′*(7, –4), *B′*(–2, –8), *C′*(1, 1)

5b. *A′*(1, 4), *B′*(–3, –5), *C′*(6, –2)

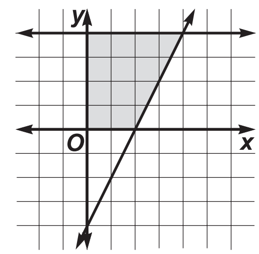
5c. *A′*(–1, 4), *B′*(3, –5), *C′*(–6, –2)

Quiz C
Page 82

1. 33

2. does not exist

3. $\left(\frac{2}{9}, -\frac{1}{3}\right)$

4. 

Vertices: (0, 0), (0, 2), (0, 4), (4, 4)

5. min.: –8; max.: 6

Quiz D
Page 82

1. $460

2. 14 gallons

Chapter 2 Answer Key

SAT/ACT Practice

Page 83

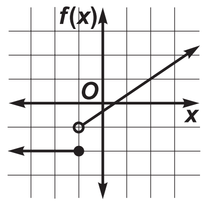
1. C
2. A
3. D
4. A
5 D
6. C
7. E
8. C
9. A
10. D

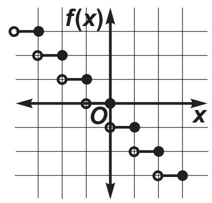
Page 84

11. E
12. C
13. D
14. E
15. D
16. A
17. C
18. A
19. 5
20. 54/6, 27/3, 9/1, or 9

Cumulative Review

Page 85

1. D: {−5, −2, 4}; R: {0, 1, 2, 3}; no
2. $-\frac{1}{2x^2}$
3. $y = -x - 1$
4. $x + 3y - 18 = 0$
5. 

6. 

7. infinitely many
8. (3, 2)
9. (4, −1, 3)
10. $\begin{bmatrix} 2 & 1 & 5 \\ -1 & -10 & 6 \end{bmatrix}$
11. −41
12. $A'(5, -2)$, $B'(7, -3)$, $C'(-2, 6)$
13. $\frac{1}{4}\begin{bmatrix} 1 & -3 \\ 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} -5 \\ 1 \end{bmatrix}$
14. max.: 6, min.:3
15. 200 lumber, 600 plywood