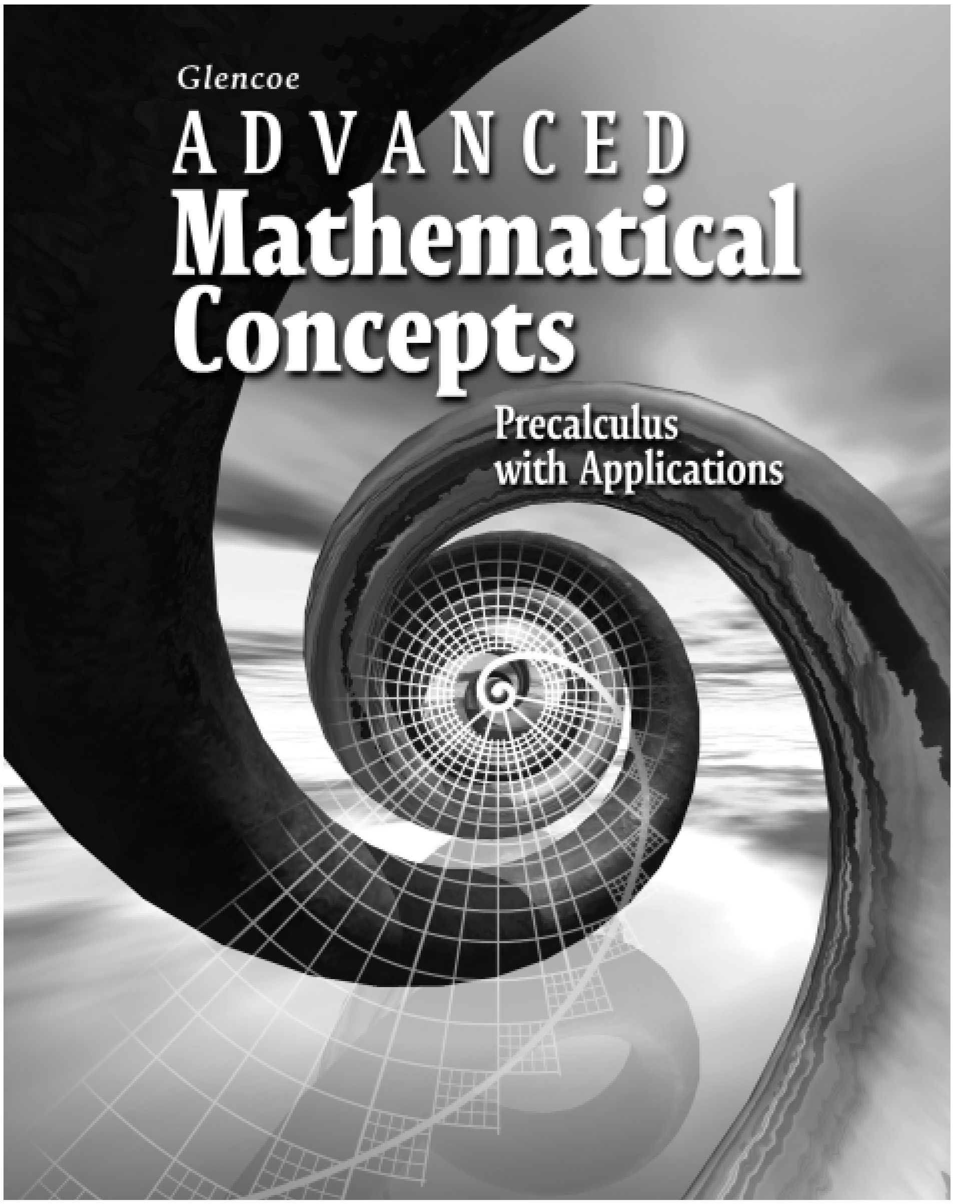
Chapter 13
Resource Masters



Glencoe

New York, New York Columbus, Ohio Woodland Hills, California Peoria, Illinois

StudentWorks™ This CD-ROM includes the entire Student Edition along with the Study Guide, Practice, and Enrichment masters.

TeacherWorks™ All of the materials found in this booklet are included for viewing and printing in the *Advanced Mathematical Concepts TeacherWorks* CD-ROM.

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Advanced Mathematical Concepts
Chapter 13 Resource Masters

1 2 3 4 5 6 7 8 9 10 XXX 11 10 09 08 07 06 05 04

Contents

A Teacher's Guide to Using the Chapter 13 Resource Masters

The ***Fast File*** Chapter Resource system allows you to conveniently file the resources you use most often. The *Chapter 13 Resource Masters* include the core materials needed for Chapter 13. These materials include worksheets, extensions, and assessment options. The answers for these pages appear at the back of this booklet.

All of the materials found in this booklet are included for viewing and printing in the *Advanced Mathematical Concepts TeacherWorks* CD-ROM.

Vocabulary Builder Pages vii-viii include a student study tool that presents the key vocabulary terms from the chapter. Students are to record definitions and/or examples for each term. You may suggest that students highlight or star the terms with which they are not familiar.

When to Use Give these pages to students before beginning Lesson 13-1. Remind them to add definitions and examples as they complete each lesson.

Study Guide There is one Study Guide master for each lesson.

When to Use Use these masters as reteaching activities for students who need additional reinforcement. These pages can also be used in conjunction with the Student Edition as an instructional tool for those students who have been absent.

Practice There is one master for each lesson. These problems more closely follow the structure of the Practice section of the Student Edition exercises. These exercises are of average difficulty.

When to Use These provide additional practice options or may be used as homework for second day teaching of the lesson.

Enrichment There is one master for each lesson. These activities may extend the concepts in the lesson, offer a historical or multicultural look at the concepts, or widen students' perspectives on the mathematics they are learning. These are not written exclusively for honors students, but are accessible for use with all levels of students.

When to Use These may be used as extra credit, short-term projects, or as activities for days when class periods are shortened.

Assessment Options

The assessment section of the *Chapter 13 Resources Masters* offers a wide range of assessment tools for intermediate and final assessment. The following lists describe each assessment master and its intended use.

Chapter Assessments

Chapter Tests

- *Forms 1A, 1B, and 1C* Form 1 tests contain multiple-choice questions. Form 1A is intended for use with honors-level students, Form 1B is intended for use with average-level students, and Form 1C is intended for use with basic-level students. These tests are similar in format to offer comparable testing situations.
- *Forms 2A, 2B, and 2C* Form 2 tests are composed of free-response questions. Form 2A is intended for use with honors-level students, Form 2B is intended for use with average-level students, and Form 2C is intended for use with basic-level students. These tests are similar in format to offer comparable testing situations.

  All of the above tests include a challenging Bonus question.
- The **Extended Response Assessment** includes performance assessment tasks that are suitable for all students. A scoring rubric is included for evaluation guidelines. Sample answers are provided for assessment.

Intermediate Assessment

- A **Mid-Chapter Test** provides an option to assess the first half of the chapter. It is composed of free-response questions.
- Four free-response **quizzes** are included to offer assessment at appropriate intervals in the chapter.

Continuing Assessment

- The **SAT and ACT Practice** offers continuing review of concepts in various formats, which may appear on standardized tests that they may encounter. This practice includes multiple-choice, quantitative-comparison, and grid-in questions. Bubble-in and grid-in answer sections are provided on the master.
- The **Cumulative Review** provides students an opportunity to reinforce and retain skills as they proceed through their study of advanced mathematics. It can also be used as a test. The master includes free-response questions.

Answers

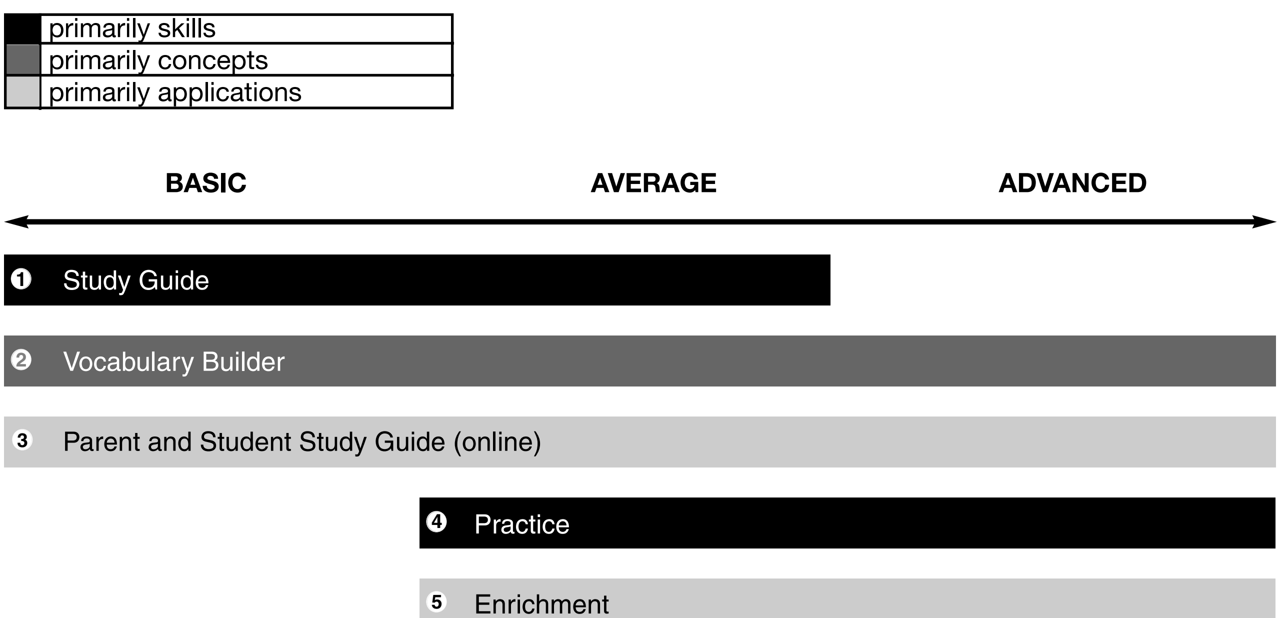
- Page A1 is an answer sheet for the SAT and ACT Practice questions that appear in the Student Edition on page 887. Page A2 is an answer sheet for the SAT and ACT Practice master. These improve students' familiarity with the answer formats they may encounter in test taking.
- The answers for the lesson-by-lesson masters are provided as reduced pages with answers appearing in red.
- Full-size answer keys are provided for the assessment options in this booklet.

Chapter 13 Leveled Worksheets

Glencoe's **leveled worksheets** are helpful for meeting the needs of every student in a variety of ways. These worksheets, many of which are found in the **FAST FILE Chapter Resource Masters**, are shown in the chart below.

- **Study Guide** masters provide worked-out examples as well as practice problems.
- Each chapter's **Vocabulary Builder** master provides students the opportunity to write out key concepts and definitions in their own words.
- **Practice** masters provide average-level problems for students who are moving at a regular pace.
- **Enrichment** masters offer students the opportunity to extend their learning.

Five Different Options to Meet the Needs of Every Student in a Variety of Ways



NAME ______________________________ DATE ________________ PERIOD _________

Reading to Learn Mathematics

Vocabulary Builder

This is an alphabetical list of the key vocabulary terms you will learn in Chapter 13. As you study the chapter, complete each term's definition or description. Remember to add the page number where you found the term.

Vocabulary Term	Found on Page	Definition/Description/Example
Basic Counting Principle		
binomial experiments		
circular permutation		
combination		
combinatorics		
complements		
conditional probability		
dependent event		
experimental probability		
failure		
inclusive event		

(continued on the next page)

NAME ______________________________ DATE ________________ PERIOD _________

Reading to Learn Mathematics

Vocabulary Builder *(continued)*

Vocabulary Term	Found on Page	Definition/Description/Example
independent event		
mutually exclusive		
odds		
permutation		
permutation with repetition		
probability		
reduced sample space		
sample space		
simulation		
success		
theoretical probability		
tree diagram		

NAME ______________________________ DATE ______________ PERIOD ________

Study Guide

Permutations and Combinations

Use the **Basic Counting Principle** to determine different possibilities for the arrangement of objects. The arrangement of objects in a certain order is called a **permutation**. A **combination** is an arrangement in which order is *not* a consideration.

Example 1 **Eight students on a student council are assigned 8 seats around a U-shaped table.**

a. How many different ways can the students be assigned seats at the table?

Since order is important, this situation is a permutation. The eight students are taken all at once, so the situation can be represented as $P(8, 8)$.

$P(8, 8) = 8!$ $\qquad P(n, n) = n!$

$= 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$ or 40,320

There are 40,320 ways the students can be seated.

b. How many ways can a president and a vice-president be elected from the eight students?

This is a permutation of 8 students being chosen 2 at a time.

$P(8, 2) = \frac{8!}{(8-2)!}$ $\qquad P(n, r) = \frac{n!}{(n-r)!}$

$= \frac{8 \cdot 7 \cdot 6!}{6!}$ or 56

There are 56 ways a president and vice-president can be chosen.

Example 2 **The Outdoor Environmental Club consists of 20 members, of which 9 are male and 11 are female. Seven members will be selected to form an event-planning committee. How many committees of 4 females and 3 males can be formed?**

Order is not important. There are three questions to consider.

How many ways can 3 males be chosen from 9?
How many ways can 4 females be chosen from 11?
How many ways can 3 males and 4 females be chosen together?

The answer is the product of the combinations $C(9, 3)$ and $C(11, 4)$.

$C(9, 3) \cdot C(11, 4) = \frac{9!}{(9-3)!3!} \cdot \frac{11!}{(11-4)!4!}$ $\qquad C(n, r) = \frac{n!}{(n-r)!r!}$

$= \frac{9!}{6!3!} \cdot \frac{11!}{7!4!}$

$= 84 \cdot 330$ or 27,720

There are 27,720 possible committees.

NAME ______________________________ DATE ______________ PERIOD ________

Practice

Permutations and Combinations

1. A golf manufacturer makes irons with 7 different shaft lengths, 3 different grips, and 2 different club head materials. How many different combinations are offered?

2. A briefcase lock has 3 rotating cylinders, each containing 10 digits. How many numerical codes are possible?

3. How many 7-digit telephone numbers can be formed if the first digit cannot be 0 or 1?

Find each value.

4. $P(10, 7)$

5. $P(7, 7)$

6. $P(6, 3)$

7. $C(7, 2)$

8. $C(10, 4)$

9. $C(12, 4) \cdot C(8, 3)$

10. How many ways can the 4 call letters of a radio station be arranged if the first letter must be W or K and no letters can be repeated?

11. There are 5 different routes that a commuter can take from her home to her office. How many ways can she make a roundtrip if she uses different routes for coming and going?

12. How many committees of 5 students can be selected from a class of 25?

13. A box contains 12 black and 8 green marbles. How many ways can 3 black and 2 green marbles be chosen?

14. ***Basketball*** How many ways can a coach select a starting team of one center, two forwards, and two guards if the basketball team consists of three centers, five forwards, and three guards?

NAME ______________________ DATE ______________ PERIOD ________

13-1

Enrichment

Permutation and Combination Algebra

Expressions involving $P(n, r)$ and $C(n, r)$, the symbols for permutations and combinations, can sometimes be simplified or used in equations as though they were algebraic expressions. You can solve problems involving such expressions by applying the definitions of $P(n, r)$ and $C(n, r)$.

Example **Simplify $C(n, n-1)$.**

By the definition of $C(n, r)$, $C(n, n-1) = \frac{n!}{(n-[n-1])!(n-1)!}$

$$= \frac{n!}{(n-n+1)!(n-1)!}$$

$$= \frac{n!}{1!(n-1)!}$$

$$= \frac{n!}{(n-1)!}$$

$$= n$$

Simplify.

1. $P(n, n-1)$

2. $C(n, n)$

3. $C(n, 1)$

4. $P(n, n)$

5. $C(n+1, n)$

6. $C(n+1, n-1)$

Solve for n.

7. $P(n, 5) = 7\,P(n, 4)$

8. $C(n, n-2) = 6$

9. $C(n+2, 4) = 6\,C(n, 2)$

10. $P(n, 5) = 9\,P(n-1, 4)$

13-2

Study Guide

Permutations with Repetitions and Circular Permutations

For permutations involving repetitions, the number of permutations of n objects of which p are alike and q are alike is $\frac{n!}{p!q!}$. When n objects are arranged in a circle, there are $\frac{n!}{n}$, or $(n - 1)!$, permutations of the objects around the circle. If n objects are arranged relative to a fixed point, then there are $n!$ permutations.

Example 1 **How many 10-letter patterns can be formed from the letters of the word *basketball*?**

The ten letters can be arranged in $P(10, 10)$, or $10!$, ways. However, some of these 3,628,800 ways have the same appearance because some of the letters appear more than once.

$\frac{10!}{2!2!2!}$ *There are 2 as, 2 bs, and 2 ls in basketball.*

$$\frac{10!}{2!2!2!} = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1 \cdot 2 \cdot 1 \cdot 2 \cdot 1}$$

$$= 453{,}600$$

There are 453,600 ten-letter patterns that can be formed from the letters of the word *basketball*.

Example 2 **Six people are seated at a round table to play a game of cards.**

a. Is the seating arrangement around the table a linear or circular permutation? Explain.

b. How many possible seating arrangements are there?

a. The arrangement of people is a circular permutation since the people form a circle around the table.

b. There are 6 people, so the number of arrangements can be described by $(6 - 1)!$.

$$(6 - 1)! = 5!$$
$$= 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \text{ or } 120$$

There are 120 possible seating arrangements.

13-2

NAME ______________________________ DATE _______________ PERIOD ________

Practice

Permutations with Repetitions and Circular Permutations

How many different ways can the letters of each word be arranged?

1. *members*

2. *annually*

3. *Missouri*

4. *concert*

5. How many different 5-digit street addresses can have the digits 4, 7, 3, 4, and 8?

6. Three hardcover books and 5 paperbacks are placed on a shelf. How many ways can the books be arranged if all the hardcover books must be together and all the paperbacks must be together?

Determine whether each arrangement of objects is a linear or circular permutation. Then determine the number of arrangements for each situtation.

7. 9 keys on a key ring with no chain

8. 5 charms on a bracelet with no clasp

9. 6 people seated at a round table with one person seated next to a door

10. 12 different symbols around the face of a watch

11. ***Entertainment*** Jasper is playing a word game and has the following letters in his tray: QUOUNNTAGGRA. How many 12-letter arrangements could Jasper make to check if a single word could be formed from all the letters?

NAME ______________________ DATE ____________ PERIOD ________

13-2 Enrichment

Approximating Factorials

James Stirling (1692-1770) was a teacher, a friend of Sir Isaac Newton, and a mathematician who made important contributions to calculus. Today he is best remembered as the creator of a formula for approximating factorials.

Stirling's Formula	$n! \approx \sqrt{2n\pi}\left(\frac{n}{e}\right)^n$, where e is the irrational number 2.7182818....

1. Complete the chart. By examining the ratio $\frac{n!}{\sqrt{2n\pi}\left(\frac{n}{e}\right)^n}$, we can see how closely Stirling's formula approximates $n!$.

n	$n\,!$	$\sqrt{2n\pi}\left(\ \right)^n$	$\frac{n!}{\sqrt{2n\pi}\left(\frac{n}{e}\right)^n}$
10			
20			
30			
40			
50			
60			

2. Based on the completed chart, as n increases, will the approximations obtained using Stirling's formula become more accurate or less accurate? Explain.

NAME ______________________ DATE ______________ PERIOD ________

13-3

Study Guide

Probability and Odds

The **probability** of an event is the ratio of the number of ways an event can happen to the total number of ways an event can and cannot happen.

Example **A bag contains 3 black, 5 green, and 4 yellow marbles.**

a. What is the probability that a marble selected at random will be green?

The probability of selecting a green marble is written P(green). There are 5 ways to select a green marble from the bag and 3 + 4, or 7, ways not to select a green marble. So, success (s) = 5 and failure (f) = 7. Use the probability formula.

$P(\text{green}) = \frac{5}{5+7}$ or $\frac{5}{12}$ $\qquad P(s) = \frac{s}{s+f}$

The probability of selecting a green marble is $\frac{5}{12}$.

b. What is the probability that a marble selected at random will *not* be yellow?

There are 8 ways not to select a yellow marble and 4 ways to select a yellow marble.

$P(\text{not yellow}) = \frac{8}{4+8}$ or $\frac{2}{3}$ $\qquad P(f) = \frac{f}{s+f}$

The probability of not selecting a yellow marble is $\frac{2}{3}$.

c. What is the probability that 2 marbles selected at random will both be black?

Use counting methods to determine the probability. There are $C(3, 2)$ ways to select 2 black marbles out of 3, and $C(12, 2)$ ways to select 2 marbles out of 12.

$$P(\text{2 black marbles}) = \frac{C(3, 2)}{C(12, 2)}$$

$$= \frac{\frac{3!}{1!2!}}{\frac{12!}{10!2!}} \text{ or } \frac{1}{22}$$

The probability of selecting 2 black marbles is $\frac{1}{22}$.

NAME ______________________________ DATE _______________ PERIOD ________

13-3

Practice

Probability and Odds

A kitchen drawer contains 7 forks, 4 spoons, and 5 knives. Three are selected at random. Find each probability.

1. P(3 forks)

2. P(2 forks, 1 knife)

3. P(3 spoons)

4. P(1 fork, 1 knife, 1 spoon)

A laundry bag contains 5 red, 9 blue, and 6 white socks. Two socks are selected at random. Find each probability.

5. P(2 red)

6. P(2 blue)

7. P(1 red, 1 blue)

8. P(1 red, 1 white)

Sharon has 8 mystery books and 9 science-fiction books. Four are selected at random. Find each probability.

9. P(4 mystery books)

10. P(4 science-fiction books)

11. P(2 mysteries, 2 science-fiction)

12. P(3 mysteries, 1 science-fiction)

From a standard deck of 52 cards, 5 cards are drawn. What are the odds of each event occurring?

13. 5 aces

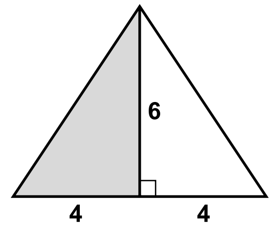
14. 5 face cards

15. ***Meteorology*** A local weather forecast states that the chance of sunny weather on Wednesday is 70%. What are the odds that it will be sunny on Wednesday?

NAME ______________________________ DATE ________________ PERIOD _________

Enrichment

Geometric Probability



If a dart, thrown at random, hits the triangular board shown at the right, what is the chance that is will hit the shaded region? This chance, also called a probability, can be determined by analyzing the area of the board. This ratio indicates what fraction of the tosses should hit in the shaded region.

$$= \frac{(4)(6)}{\frac{1}{2}(8)(6)}$$

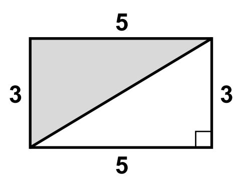
$$= \frac{12}{24} \text{ or } \frac{1}{2}$$

In general, if S is a subregion of some region R, then the probability $P(S)$ that a point, chosen at random, belongs to subregion S is given by the following.

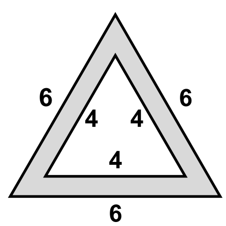
$$P(S) = \frac{\text{area of subregion } S}{\text{area of region } R}$$

Find the probability that a point, chosen at random, belongs to the shaded subregions of the following regions.

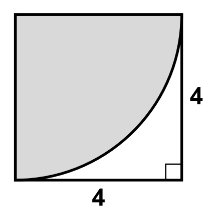
1.



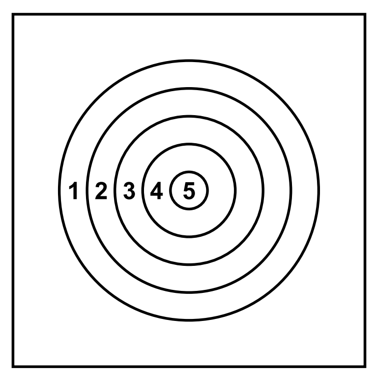
2.



3.



The dart board shown at the right has 5 concentric circles whose centers are also the center of the square board. Each side of the board is 38 cm, and the radii of the circles are 2 cm, 5 cm, 8 cm, 11 cm, and 14 cm. A dart hitting within one of the circular regions scores the number of points indicated on the board, while a hit anywhere else scores 0 points. If a dart, thrown at random, hits the board, find the probability of scoring the indicated number of points. Write your answer in terms of π.



4. 0 points

5. 1 point

6. 2 points

7. 3 points

8. 4 points

9. 5 points

NAME ______________________ DATE ______________ PERIOD ________

Study Guide

Probabilities of Compound Events

Example 1 **Using a standard deck of playing cards, find the probability of drawing a king, replacing it, then drawing a second king.**

Since the first card is returned to the deck, the outcome of the second draw is not affected by the first. The events are independent. The probability is the product of each individual probability.

$P(A \text{ and } B) = P(A) \cdot P(B)$ *Let A represent the first draw and B the second draw.*

$P(A) = P(B) = \frac{4}{52} = \frac{1}{13}$ $\frac{4 \text{ kings}}{52 \text{ cards in a standard deck}}$

$P(A \text{ and } B) = \frac{1}{13} \cdot \frac{1}{13} \cdot = \frac{1}{169}$

The probability of selecting a king, replacing it, and then selecting another king is $\frac{1}{169}$.

Example 2 **What is the probability of selecting a yellow or a blue marble from a box of 5 green, 3 yellow, and 2 blue marbles?**

A yellow marble and a blue marble cannot be selected at the same time. Thus, the events are mutually exclusive. Find the sum of the individual probabilities.

$P(\text{yellow or blue}) = P(\text{yellow}) + P(\text{blue})$

$= \frac{3}{10} + \frac{2}{10}$ $P(yellow) = \frac{3}{10}$; $P(blue) = \frac{2}{10}$

$= \frac{5}{10} \text{ or } \frac{1}{2}$

Example 3 **What is the probability that a card drawn from a standard deck is either a face card or black?**

The card drawn could be both a face card and black, so the events are mutually inclusive.

$P(\text{face card}) = \frac{12}{52}$

$P(\text{black}) = \frac{26}{52}$

$P(\text{face card and black}) = \frac{6}{52}$

$P(\text{face card or black}) = \frac{12}{52} + \frac{26}{52} - \frac{6}{52} = \frac{32}{52} \text{ or } \frac{8}{13}$

13-4

Practice

Probabilities of Compound Events

Determine if each event is independent or dependent. Then determine the probability.

1. the probability of drawing a black card from a standard deck of cards, replacing it, then drawing another black card

2. the probability of selecting 1 jazz, 1 country, and 1 rap CD in any order from 3 jazz, 2 country, and 5 rap CDs, replacing the CDs each time

3. the probability that two cards drawn from a deck are both aces

Determine if each event is mutually exclusive or mutually inclusive. Then determine each probability.

4. the probability of rolling a 3 or a 6 on one toss of a number cube

5. the probability of selecting a queen or a red card from a standard deck of cards

6. the probability of selecting at least three white crayons when four crayons are selected from a box containing 7 white crayons and 5 blue crayons

7. ***Team Sports*** Conrad tried out for both the volleyball team and the football team. The probability of his being selected for the volleyball team is $\frac{4}{5}$, while the probability of his being selected for the football team is $\frac{3}{4}$. The probability of his being selected for both teams is $\frac{7}{10}$. What is the probability that Conrad will be selected for either the volleyball team or the football team?

NAME ______________________ DATE ____________ PERIOD _______

13-4 Enrichment

Probability and Tic-Tac-Toe

What would be the chances of winning at tic-tac-toe if it were turned into a game of pure chance? To find out, the nine cells of the tic-tac-toe board are numbered from 1 to 9 and chips (also numbered from 1 to 9) are put into a bag. Player A draws a chip at random and enters an X in the corresponding cell. Player B does the same and enters an O.

To solve the problem, assume that both players draw all their chips without looking and all X and O entries are made at the same time. There are four possible outcomes: a draw, A wins, B wins, and either A or B can win.

There are 16 arrangements that result in a draw. Reflections and rotations must be counted as shown below.

```
o x o     x o x     o o x
x o x     o o x     x x o
x o x     x x o     o x x
  4         4         8
```

There are 36 arrangements in which either player may win because both players have winning triples.

```
x x x     x x x     x o x     x x x     x x x     x x o
o o o     x o x     x x x     x x o     o o o     x x x
x o x     o o o     o o o     o o o     x x o     o o o
  4         4         4         8         8         8
```

In these 36 cases, A's chances of winning are $\frac{13}{40}$.

1. Find the 12 arrangements in which B wins and A cannot.

2. Below are 12 of the arrangements in which A wins and B cannot. Write the numbers to show the reflections and rotations for each arrangement. What is the total number?

```
o x o     x o x     x x x     x x x     x o o     x o x
x x x     o x o     x o o     o x o     x x x     x x o
o x o     x o x     x o o     o x o     o o x     o o x

x x o     x x x     x x x     x x x     x o o     x x o
o x x     o x o     x o o     x o o     x x x     o x o
o o x     o o x     o x o     o o x     o x o     x o x
```

3. There are $\frac{9!}{5!4!}$ different and equally probable distributions. Complete the chart to find the probability for a draw or for A or B to win.

Draw: $\frac{16}{126}$			= ______
A wins:	______	+ $\frac{13}{40}\left(\frac{36}{126}\right)$	= ______
B wins:	______	+ __________	= ______

NAME ______________________________ DATE ______________ PERIOD ________

Study Guide

Conditional Probabilities

The **conditional probability** of event A, given event B, is defined as $P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$, where $P(B) \neq 0$. In some situations, event A is a subset of event B. In these situations, $P(A|B) = \frac{P(A)}{P(B)}$, where $P(B) \neq 0$.

Example **Each of four boxes contains a red marble and a yellow marble. A marble is selected from each box without looking. What is the probability that exactly three red marbles are selected if the third marble is red?**

Sample spaces and reduced sample spaces can be used to help determine the outcomes that satisfy a given condition.

The sample space is $S = \{RRRR, RRRY, RRYR, RRYY, RYRR, RYRY, RYYR, RYYY, YRRR, YRRY, YRYR, YRYY, YYRR, YYRY, YYYR, YYYY\}$ and includes all of the possible outcomes of selecting 1 of the marbles from each of the 4 boxes. All of the outcomes are equally likely.

Event B represents the condition that the third marble is red.
$B = \{RRRR, RRRY, RYRR, RYRY, YRRR, YRRY, YYRR, YYRY\}$

$P(B) = \frac{8}{16}$ or $\frac{1}{2}$

Event A represents the condition that exactly three of the marbles are red.

$A = \{RRRY, RRYR, RYRR, YRRR\}$

(A and B) is the intersection of A and B.
(A and B) = $\{RRRY, RYRR, YRRR\}$.

So, $P(A \text{ and } B) = \frac{3}{16}$.

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$$

$$= \frac{\frac{3}{16}}{\frac{1}{2}} \text{ or } \frac{3}{8}$$

The probability that exactly three marbles are red given that the third marble is red is $\frac{3}{8}$.

NAME ______________________ DATE ____________ PERIOD _______

13-5

Practice

Conditional Probabilities

Find each probability.

1. Two number cubes are tossed. Find the probability that the numbers showing on the cubes match, given that their sum is greater than 7.

2. A four-digit number is formed from the digits 1, 2, 3, and 4. Find the probability that the number ends in the digits 41, given that the number is odd.

3. Three coins are tossed. Find the probability that exactly two coins show tails, given that the third coin shows tails.

A card is chosen from a standard deck of cards. Find each probability, given that the card is red.

4. P(diamond)

5. P(six of hearts)

6. P(queen or 10)

7. P(face card)

A survey taken at Stirers High School shows that 48% of the respondents like soccer, 66% like basketball, and 38% like hockey. Also, 30% like soccer and basketball, 22% like basketball and hockey, and 28% like soccer and hockey. Finally, 12% like all three sports.

8. If Meg likes basketball, what is the probability that she also likes soccer?

9. If Jaime likes soccer, what is the probability that he also likes hockey and basketball?

10. If Ashley likes basketball, what is the probability that she also likes hockey?

11. If Brett likes soccer, what is the probability that he also likes basketball?

NAME ______________________________ DATE _______________ PERIOD ________

13-5

Enrichment

Probability in Genetics

The Austrian monk and botanist Gregor Mendel discovered the basic laws of genetics during the nineteenth century. Through experiments with pea plants, Mendel found that cells in living organisms contain pairs of units that control traits in the offspring of the organism. We now call these units *genes*. If the genes in a cell are identical, the trait is *pure*. If they are different, the trait is *hybrid*. A trait like *tallness* which masks other traits, preventing them from showing up in offspring, is *dominant*. Otherwise, it is *recessive*. A combination of a dominant gene and a recessive gene will always produce a hybrid displaying the dominant trait.

Example **Two hybrid tall pea plants are crossed. What is the probability that the offspring will be short?**

Punnett squares are used to analyze gene combinations. Use capital letters to represent dominant genes and lower-case letters to represent recessive genes.

	T	t
T	TT	Tt
t	Tt	tt

T = tall t = short

The table shows the four equally possible outcomes. One of the outcomes, TT, is a pure tall plant. Two of the outcomes, Tt and Tt, are hybrid tall plants. Only one of the outcomes, tt, is a short plant. Therefore, the probability that an offspring will be short is $\frac{1}{4}$.

Use Punnett squares to solve.

1. A pure dominant yellow pea plant (Y) is crossed with a pure recessive white pea plant (w).

a. What are the possible outcomes?

b. Find the probability that an offspring will be yellow.

2. A hybrid tall pea plant is crossed with a short plant. Find the probability that an offspring will be short.

3. Brown eyes are dominant over blue eyes in humans. What is the probability that a woman with blue eyes and a man with hybrid brown eyes will have a child with blue eyes?

4. What is the probability that the offspring of a hybrid-tall, hybrid-yellow pea plant and a hybrid-tall white plant will be short white?

Study Guide

The Binomial Theorem and Probability

Problems that meet the conditions of a **binomial experiment** can be solved using the binomial expansion. Use the Binomial Theorem to find the probability when the number of trials makes working with the binomial expansion unrealistic.

Example 1 **The probability that Misha will win a word game is $\frac{3}{4}$. If Misha plays the game 5 times, what is the probability that he will win exactly 3 games?**

There are 5 games and each game has only two possible outcomes, win W or lose L. These events are independent and the probability is $\frac{3}{4}$ for each game. So this is a binomial experiment.

When $(W + L)^5$ is expanded, the term W^3L^2 represents 3 wins and 2 losses. The coefficient of W^3L^2 is $C(5, 3)$, or 10.

$$P(\text{exactly 3 wins}) = 10\left(\frac{3}{4}\right)^3\left(\frac{1}{4}\right)^2 \qquad W = \frac{3}{4}, L = \frac{1}{4}$$

$$= 10\left(\frac{27}{64}\right)\left(\frac{1}{16}\right)$$

$$= \frac{270}{1024}$$

$$= \frac{135}{512} \text{ or about } 26.4\%$$

Example 2 **The probability that a computer salesperson will make a sale when approaching a customer is $\frac{1}{2}$. If the salesperson approaches 12 customers, what is the probability that 8 sales will be made?**

Let S be the probability of a sale.
Let N be the probability of not making a sale.

$$(S + N)^{12} = \Sigma \quad \frac{12!}{r!(12 - r)!} P^{12-r} P^{r}$$

Making 8 sales means that 4 sales will not be made. So the probability can be found using the term where $r = 4$.

$$\frac{12!}{4!(12 - 4)!}S^8N^4 = 495S^8N^4$$

$$= 495\left(\frac{1}{2}\right)^8\left(\frac{1}{2}\right)^4 \qquad S = \frac{1}{2}, N = \frac{1}{2}$$

$$= \frac{495}{4096} \text{ or } 0.120849609$$

The probability of making exactly 8 sales is about 12.1%.

NAME ______________________________ DATE ______________ PERIOD ________

Practice

The Binomial Theorem and Probability

Find each probability if six coins are tossed.

1. P(3 heads and 3 tails)

2. P(at least 4 heads)

3. P(2 heads or 3 tails)

4. P(all heads or all tails)

The probability of Chris's making a free throw is $\frac{2}{3}$. Find each probability if she shoots five times.

5. P(all missed)

6. P(all made)

7. P(exactly 4 made)

8. P(at least 3 made)

When Maria and Len play a certain board game, the probability that Maria will win the game is $\frac{3}{4}$. Find each probability if they play five games.

9. P(Len wins only 1 game)

10. P(Maria wins exactly 2 games)

11. P(Len wins at least 2 games)

12. P(Maria wins at least 3 games)

13. ***Gardening*** Assume that 60% of marigold seeds that are sown directly in the ground produce plants. If Tomaso plants 10 seeds, what is the probability that 7 plants will be produced?

NAME ______________________________ DATE ______________ PERIOD ________

Enrichment

Combinations and Pascal's Triangle

Pascal's triangle is a special array of numbers invented by Blaise Pascal (1623-1662). The values in Pascal's triangle can be found using the combinations shown below.

1. Evaluate the expression in each cell of the triangle

$C(1, 0)$	$C(1, 1)$				
$C(2, 0)$	$C(2, 1)$	$C(2, 2)$			
$C(3, 0)$	$C(3, 1)$	$C(3, 2)$	$C(3, 3)$		
$C(4, 0)$	$C(4, 1)$	$C(4, 2)$	$C(4, 3)$	$C(4, 4)$	
$C(5, 0)$	$C(5, 1)$	$C(5, 2)$	$C(5, 3)$	$C(5, 4)$	$C(5, 5)$

2. The pattern shows the relationship between $C(n, r)$ and Pascal's triangle. In general, it is true that $C(n, r) + C(n, r + 1) = C(n + 1, r + 1)$. Complete the proof of this property. In each step, the denominator has been given.

$$C(n, r) + C(n, r + 1) = \frac{\qquad}{r!(n - r)!} + \frac{\qquad}{(r + 1)!(n - r - 1)!}$$

$$= \frac{\qquad}{r!(n - r)!(r + 1)} + \frac{\qquad}{(r + 1)!(n - r - 1)!(n - r)}$$

$$= \frac{\qquad}{(r + 1)!(n - r)!} + \frac{\qquad}{(r + 1)!(n - r)!}$$

$$= \frac{\qquad}{(r + 1)!(n - r)!}$$

$$= \frac{\qquad}{(r + 1)!(n - r)!}$$

$$= \frac{\qquad}{(r + 1)!(n - r)!}$$

$$= \frac{\qquad}{(r + 1)![(n + 1) - (r + 1)]!}$$

$$= C(n + 1, r + 1)$$

Chapter 13

NAME ______________________ DATE ____________ PERIOD ________

Chapter 13 Test, Form 1A

Write the letter for the correct answer in the blank at the right of each problem.

1. A school has different course offerings: 4 in math, 6 in English, 5 in science, and 3 in social studies. How many different 4-course student schedules are possible if a student must have one course from each subject area? **1.** ________

A. 4 **B.** 24 **C.** 120 **D.** 360

2. How many different ways can the letters in the word *social* be arranged if the letter *c* must be directly followed by the letter *i*? **2.** ________

A. 120 **B.** 720 **C.** 24 **D.** 256

3. How many sets of 5 books can be chosen from a set of 8? **3.** ________

A. 32,768 **B.** 120 **C.** 56 **D.** 40,320

4. How many different starting teams consisting of 1 center, 2 forwards, and 2 guards can be chosen from a basketball squad consisting of 3 centers, 6 forwards and 7 guards? **4.** ________

A. 945 **B.** 120 **C.** 126 **D.** 5292

5. Find the possible number of license plates consisting of 2 letters followed by 4 digits if digits can be repeated but letters cannot. **5.** ________

A. 3,276,000 **B.** 3,407,040 **C.** 6,500,000 **D.** 6,760,000

6. How many ways can the letters in the word *bookkeeper* be arranged? **6.** ________

A. 3,628,800 **B.** 151,200 **C.** 302,400 **D.** 362,880

7. How many ways can 10 different chairs be arranged in a circle? **7.** ________

A. 362,880 **B.** 120 **C.** 3,628,800 **D.** 10,000,000,000

8. Find the number of ways that 7 people can be seated at a circular table if 1 seat has a microphone in front of it. **8.** ________

A. 720 **B.** 5040 **C.** 46,656 **D.** 823,543

For Exercises 9 and 10, consider a class with 10 sophomores, 8 juniors, and 6 seniors. Two students are selected at random.

9. What is the probability of selecting 1 junior and 1 senior? **9.** ________

A. $\frac{1}{12}$ **B.** $\frac{2}{23}$ **C.** $\frac{7}{138}$ **D.** $\frac{1}{138}$

10. Find the odds of selecting 2 students who are not seniors. **10.** ________

A. $\frac{51}{41}$ **B.** $\frac{51}{92}$ **C.** $\frac{51}{5}$ **D.** $\frac{3}{4}$

11. The probability of getting 2 heads and 1 tail when three coins are tossed is 3 in 8. Find the odds of *not* getting 2 heads and 1 tail. **11.** ________

A. $\frac{3}{5}$ **B.** $\frac{5}{8}$ **C.** $\frac{5}{3}$ **D.** $\frac{3}{8}$

NAME ______________________ DATE ____________ PERIOD ________

Chapter 13 Test, Form 1A (continued)

12. The odds of rolling a sum of 5 when two number cubes are rolled are $\frac{1}{8}$. What is the probability of rolling a sum of 5 when two number cubes are rolled? **12.** ________

A. $\frac{4}{9}$ **B.** $\frac{1}{9}$ **C.** $\frac{5}{18}$ **D.** $\frac{1}{4}$

13. One red and one green number cube are tossed. What is the probability that the red number cube shows an even number and the green number cube shows a number greater than 2? **13.** ________

A. $\frac{1}{2}$ **B.** $\frac{1}{6}$ **C.** $\frac{1}{3}$ **D.** $\frac{2}{3}$

14. If two cards are drawn at random from a standard deck of cards with no replacement, find the probability that both cards are queens. **14.** ________

A. $\frac{3}{676}$ **B.** $\frac{1}{169}$ **C.** $\frac{1}{52}$ **D.** $\frac{1}{221}$

15. A basket contains 3 red, 4 yellow, and 5 green balls. If one ball is taken at random, what is the probability that it is yellow or green? **15.** ________

A. $\frac{1}{9}$ **B.** $\frac{5}{36}$ **C.** $\frac{3}{22}$ **D.** $\frac{3}{4}$

16. A company survey shows that 50% of employees drive to work, 30% of employees have children, and 20% of employees drive to work and have children. What is the probability that an employee drives to work or has children? **16.** ________

A. 1 **B.** $\frac{3}{5}$ **C.** $\frac{4}{5}$ **D.** $\frac{3}{20}$

17. Three number cubes are tossed. Find the probability of exactly two number cubes showing 6 if the first number cube shows 6. **17.** ________

A. $\frac{5}{18}$ **B.** $\frac{5}{36}$ **C.** $\frac{5}{108}$ **D.** $\frac{5}{216}$

18. In a certain health club, half the members are women, one-third of the members use free weights, and one-fifth of the members are women who use free weights. A female member is elected treasurer. What is the probability that she uses free weights? **18.** ________

A. $\frac{1}{4}$ **B.** $\frac{1}{8}$ **C.** $\frac{19}{30}$ **D.** $\frac{2}{5}$

19. Four coins are tossed. What is the probability that at least 2 of the 4 coins show heads? **19.** ________

A. $\frac{11}{16}$ **B.** $\frac{5}{16}$ **C.** $\frac{1}{256}$ **D.** $\frac{11}{24}$

20. Nine out of every 10 students have a calculator. Expressed as a decimal to the nearest hundredth, what is the probability that exactly 7 out of 8 students in a given class have a calculator? **20.** ________

A. 0.05 **B.** 0.72 **C.** 0.38 **D.** 0.79

Bonus Two number cubes are thrown twice. What is the probability of getting a sum that is a prime number less than 6 on both throws? **Bonus:** ________

A. $\frac{7}{36}$ **B.** $\frac{25}{324}$ **C.** $\frac{49}{1296}$ **D.** $\frac{16}{121}$

NAME ______________________________ DATE ______________ PERIOD ________

Chapter 13 Test, Form 1B

Write the letter for the correct answer in the blank at the right of each problem.

1. Given 3 choices of sandwiches, 4 choices of chips, and 2 choices of cookies, how many different sack lunches can be prepared containing one choice of each item? **1.** ________

A. 12 **B.** 24 **C.** 84 **D.** 288

2. How many ways can the letters in the word *capitol* be arranged if the first letter must be *p*? **2.** ________

A. 120 **B.** 720 **C.** 5040 **D.** 40,320

3. How many 4-letter codes can be formed from the letters in the word *capture* if letters cannot be repeated? **3.** ________

A. 28 **B.** 840 **C.** 2401 **D.** 5040

4. A class consisting of 10 boys and 12 girls must select 2 boys and 2 girls to serve on a committee. How many variations of the committee can there be? **4.** ________

A. 2970 **B.** 120 **C.** 4800 **D.** 7315

5. On a long city block, 4-digit house numbers must begin with the digit 4 and end with either 0 or 1. How many different variations of house numbers are possible? **5.** ________

A. 1000 **B.** 144 **C.** 180 **D.** 200

6. How many ways can 3 identical green candles and 7 identical blue candles be arranged in a row in any variation? **6.** ________

A. 21 **B.** 3,628,800 **C.** 30,240 **D.** 120

7. How many ways can 8 keys be arranged on a key ring with no chain? **7.** ________

A. 2520 **B.** 40,320 **C.** 720 **D.** 64

8. How many ways can 9 numbers be arranged on a small rotating wheel relative to a fixed point? **8.** ________

A. 81 **B.** 5040 **C.** 40,320 **D.** 362,880

For Exercises 9 and 10, consider a basket that contains 15 slips of paper numbered from 1 to 15. Two slips of paper are drawn at random.

9. What is the probability of drawing 2 even numbers? **9.** ________

A. $\frac{49}{225}$ **B.** $\frac{1}{4}$ **C.** $\frac{1}{5}$ **D.** $\frac{7}{15}$

10. What are the odds of drawing an even number less than 7 and an odd number greater than 10? **10.** ________

A. $\frac{2}{35}$ **B.** $\frac{3}{32}$ **C.** $\frac{2}{5}$ **D.** $\frac{33}{35}$

11. The probability of rolling a sum of 4 when two number cubes are tossed is 1 in 12. What are the odds of rolling a sum of 4 when two number cubes are tossed? **11.** ________

A. $\frac{1}{11}$ **B.** $\frac{12}{13}$ **C.** $\frac{1}{12}$ **D.** $\frac{11}{12}$

NAME ______________________ DATE ______________ PERIOD ________

Chapter 13 Test, Form 1B (continued)

12. The odds that it will rain in the city of Houston tomorrow are $\frac{1}{4}$. What is the probability of rain in Houston tomorrow? **12.** ________

A. $\frac{4}{5}$ **B.** $\frac{3}{4}$ **C.** $\frac{1}{5}$ **D.** $\frac{1}{4}$

13. Two number cubes, 1 red and 1 green, are tossed. What is the probability that the red number cube shows a number less than 3 and the green number cube shows a 6? **13.** ________

A. $\frac{1}{2}$ **B.** $\frac{1}{6}$ **C.** $\frac{1}{3}$ **D.** $\frac{1}{18}$

14. If two cards are drawn at random from a standard deck of cards with no replacement, find the probability that both cards are hearts. **14.** ________

A. $\frac{1}{4}$ **B.** $\frac{1}{16}$ **C.** $\frac{1}{17}$ **D.** $\frac{1}{52}$

15. A bucket contains 4 red, 2 yellow, and 3 green balls. If one ball is taken at random, what is the probability that it is red or green? **15.** ________

A. $\frac{2}{3}$ **B.** $\frac{7}{9}$ **C.** $\frac{4}{27}$ **D.** $\frac{7}{81}$

16. A school survey shows that 10% of students are in band, 12% of students are in athletics, and 6% of students are in both band and athletics.What is the probability that a student is in band or athletics? **16.** ________

A. $\frac{1}{60}$ **B.** $\frac{11}{50}$ **C.** $\frac{4}{25}$ **D.** $\frac{7}{25}$

17. Three coins are tossed. Find the probability that exactly 2 coins show heads if the first coin shows heads. **17.** ________

A. $\frac{1}{2}$ **B.** $\frac{3}{8}$ **C.** $\frac{1}{4}$ **D.** $\frac{1}{8}$

18. Given the integers 1 through 33, what is the probability that one of these integers is divisible by 4 if it is a multiple of 6? **18.** ________

A. $\frac{1}{4}$ **B.** $\frac{5}{33}$ **C.** $\frac{5}{8}$ **D.** $\frac{2}{5}$

19. A survey shows that 20% of all cars are white. What is the probability that exactly 3 of the next 4 cars to pass will be white? **19.** ________

A. $\frac{1}{4}$ **B.** $\frac{16}{625}$ **C.** $\frac{4}{625}$ **D.** $\frac{1}{125}$

20. Eight out of every 10 houses have a garage. Express as a decimal to the nearest hundredth the probability that exactly 9 out of 12 houses on a given block have a garage. **20.** ________

A. 0.24 **B.** 0.42 **C.** 0.56 **D.** 0.88

Bonus Two number cubes are thrown twice. What is the probability of getting a sum less than 4 on both throws? **Bonus:** ________

A. $\frac{1}{36}$ **B.** $\frac{1}{144}$ **C.** $\frac{1}{12}$ **D.** $\frac{1}{6}$

NAME ______________________ DATE ______________ PERIOD ________

Chapter 13 Test, Form 1C

Write the letter for the correct answer in the blank at the right of each problem.

1. Susan must wear one of 5 blouses and one of 4 skirts. How many different possible outfits consisting of 1 blouse and 1 skirt does she have? **1.** ________
A. 20 **B.** 24 **C.** 120 **D.** 9

2. How many ways can the letters in the word *country* be arranged? **2.** ________
A. 120 **B.** 720 **C.** 5040 **D.** 40,320

3. How many 3-letter codes can be formed from the letters in the word *picture* if letters cannot be repeated? **3.** ________
A. 21 **B.** 210 **C.** 343 **D.** 5040

4. A class consisting of 24 people must select 3 people among them to serve on a committee. How many different variations are there? **4.** ________
A. 72 **B.** 2024 **C.** 12,144 **D.** 13,824

5. How many 5-digit ZIP codes are possible if the first number cannot be 0? **5.** ________
A. 10,000 **B.** 5040 **C.** 90,000 **D.** 30,240

6. How many ways can the letters in the word *stereo* be arranged? **6.** ________
A. 120 **B.** 46,656 **C.** 720 **D.** 360

7. Given 10 different stones, how many ways can all of the stones be arranged in a circle? **7.** ________
A. 5040 **B.** 40,320 **C.** 362,880 **D.** 3,628,800

8. Find the number of possible arrangements for 6 chairs around a circular table with 1 chair nearest the door. **8.** ________
A. 120 **B.** 5040 **C.** 46,656 **D.** 720

For Exercises 9 and 10, consider a bucket that contains 4 red marbles and 5 blue marbles. Two marbles are drawn at random.

9. What is the probability of drawing 2 red marbles? **9.** ________
A. $\frac{1}{6}$ **B.** $\frac{16}{81}$ **C.** $\frac{16}{25}$ **D.** $\frac{4}{5}$

10. What are the odds of drawing 1 red and 1 blue marble? **10.** ________
A. $\frac{5}{18}$ **B.** $\frac{5}{9}$ **C.** $\frac{4}{5}$ **D.** $\frac{5}{4}$

11. The probability of getting a jack when a card is drawn from a standard deck of cards is 1 in 13. What are the odds of getting a jack when a card is drawn? **11.** ________
A. $\frac{1}{11}$ **B.** $\frac{13}{14}$ **C.** $\frac{1}{12}$ **D.** $\frac{12}{13}$

NAME ______________________ DATE ______________ PERIOD ________

Chapter 13 Test, Form 1C (continued)

12. The odds that Lee will attend a movie this weekend are $\frac{3}{5}$. What is the probability that Lee will attend a movie this weekend? **12.** ________

A. $\frac{3}{8}$ **B.** $\frac{2}{5}$ **C.** $\frac{5}{8}$ **D.** $\frac{3}{2}$

13. Using a standard deck of playing cards, find the probability of selecting a king and then selecting a heart once the king has been returned to the deck. **13.** ________

A. $\frac{4}{221}$ **B.** $\frac{1}{52}$ **C.** $\frac{17}{52}$ **D.** $\frac{1}{16}$

14. Two ribbons are selected at random from a container holding 5 purple and 6 white ribbons. Find the probability that both ribbons are white. **14.** ________

A. $\frac{3}{11}$ **B.** $\frac{36}{121}$ **C.** $\frac{6}{11}$ **D.** $\frac{2}{11}$

15. A number cube is tossed. What is the probability that the number cube shows a 1 or a number greater than 4? **15.** ________

A. $\frac{2}{3}$ **B.** $\frac{1}{3}$ **C.** $\frac{1}{18}$ **D.** $\frac{1}{2}$

16. A school survey shows that 40% of students like rock music, 20% of students like rap, and 10% of students like both rock and rap. What is the probability that a student likes either rock or rap music? **16.** ________

A. $\frac{1}{2}$ **B.** $\frac{3}{5}$ **C.** $\frac{7}{10}$ **D.** $\frac{2}{5}$

17. Two coins are tossed. Find the probability that both coins turn up heads if the first coin turns up heads. **17.** ________

A. $\frac{1}{3}$ **B.** $\frac{3}{4}$ **C.** $\frac{1}{4}$ **D.** $\frac{1}{2}$

18. Given the integers 1 through 14, what is the probability that one of these integers is divisible by 3 if it is less than 10? **18.** ________

A. $\frac{1}{4}$ **B.** $\frac{1}{3}$ **C.** $\frac{3}{14}$ **D.** $\frac{9}{14}$

19. Four coins are tossed. What is the probability of getting 3 heads and 1 tail? **19.** ________

A. $\frac{5}{16}$ **B.** $\frac{1}{4}$ **C.** $\frac{3}{4}$ **D.** $\frac{3}{8}$

20. Two out of every 10 houses in a neighborhood have a front porch. Expressed as a decimal to the nearest hundredth, what is the probability that exactly 2 out of 6 houses on a given block have a front porch? **20.** ________

A. 0.02 **B.** 0.33 **C.** 0.25 **D.** 0.20

Bonus How many ways can 5 cups and 5 glasses be arranged on a shelf if all of the glasses must be kept together? **Bonus:** ________

A. 3,628,800 **B.** 120 **C.** 86,400 **D.** 15,625

NAME ______________________________ DATE _______________ PERIOD ________

Chapter 13 Test, Form 2A

1. A bakery's dessert list consists of 3 kinds of cakes, 9 kinds of pies, and 10 kinds of brownies. How many combinations of three desserts will Jana have if she buys one of each kind? **1.** ________________

2. How many ways can the letters in the word *laughter* be arranged if the *g* must be followed by the letter *h*? **2.** ________________

3. How many 4-digit codes can be formed from the digits 1, 2, 3, 4, 5, 6, and 7 if digits cannot be repeated? **3.** ________________

4. How many different committees of 5 members can be chosen from a club with 25 members? **4.** ________________

5. A test has 4 multiple-choice questions, and each question has 4 answer choices. The multiple-choice questions are followed by 3 true-false questions. How many ways can a student answer the questions if no answers can be left blank? **5.** ________________

6. How many ways can the letters in the word *entertain* be arranged? **6.** ________________

7. How many ways can 9 people arrange themselves in a circle around a campfire? **7.** ________________

8. Find the number of ways that 7 people can sit around a circular table with one seat near a window. **8.** ________________

For Exercises 9 and 10, consider a bag that contains 5 red, 3 blue, and 4 yellow marbles.

9. If five marbles are drawn at random, what is the probability that there will be 2 red, 1 blue, and 2 yellow marbles? **9.** ________________

10. If three marbles are drawn at random, what are the odds of selecting 3 red marbles? **10.** ________________

11. The probability of getting a red queen when a card is drawn from a standard deck of cards is 1 in 26. What are the odds of *not* getting a red queen when a card is drawn? **11.** ________________

12. The odds that Sandra will attend a sporting event each week are $\frac{2}{5}$. What is the probability that Sandra will attend sporting events 2 weeks in a row? **12.** ________________

NAME ______________________ DATE ______________ PERIOD ________

Chapter 13 Test, Form 2A (continued)

13. A coin collection consists of 4 quarters, 5 dimes, and 7 nickels. One coin is selected and replaced. A second coin is selected. What is the probability that 1 dime and 1 nickel are selected? **13.** ______________

14. If two cards are drawn from a standard deck of playing cards without replacement, find the probability of selecting an ace and a face card. **14.** ______________

15. From a collection of 5 blue and 4 red ink pens, three are selected at random. What is the probability that at least two are red? **15.** ______________

16. In a lakeside community, 50% of the residents own a boat, 80% of the residents own fishing equipment, and 40% of the residents own both fishing equipment and a boat. Find the probability that a resident owns a boat or fishing equipment. **16.** ______________

17. Two number cubes are tossed. Find the probability that the sum of the number cubes is an even number, given that the first number cube shows a 3. **17.** ______________

18. At a certain gym, half the members are men, one-fourth of the members swim, and one-sixth of the members are men who swim. What is the probability that a male member who enters the gym is also a swimmer? **18.** ______________

19. A survey shows that 60% of all students at one school complete their homework. Find the probability that at least 3 of 4 students who enter a class have completed their homework. **19.** ______________

20. Six coins are tossed. What is the probability that at least 4 coins turn up tails? **20.** ______________

Bonus Different configurations of flags in a row represent different signals. How many different signals can be sent using 2 red, 4 blue, and 3 green signal flags? **Bonus:** ______________

NAME ______________________________ DATE ________________ PERIOD ________

Chapter 13 Test, Form 2B

1. Stephanie has 3 sweaters, 7 blouses, and 6 pairs of slacks in her closet. If she chooses one of each, how many different outfits could she have? **1.** ________________

2. How many ways can a family of 7 be arranged for a photo if the mother is seated in the middle? **2.** ________________

3. A club has 12 members. How many ways can a president, a secretary and a treasurer be chosen from among the members? **3.** ________________

4. How many color schemes for a backdrop consisting of four colors are possible if there are 10 colors from which to choose? **4.** ________________

5. Find the number of possible 7-digit local phone numbers if the first digit cannot be 0 or 1. **5.** ________________

6. How many ways can the letters in the word *photograph* be arranged? **6.** ________________

7. How many ways can 11 people arrange themselves in a circle around a flagpole? **7.** ________________

8. Find the number of ways that 8 people can sit around a circular table with 7 blue chairs and 1 green chair. **8.** ________________

For Exercises 9 and 10, consider a box containing 6 red, 4 blue, and 3 yellow blocks.

9. If three blocks are drawn at random, what is the probability that 2 blocks are red and 1 block is blue? **9.** ________________

10. If two blocks are drawn at random, what are the odds of drawing 2 blue blocks? **10.** ________________

11. The probability of getting all heads when four coins are tossed is1 in 16. What are the odds of getting all tails when four coins are tossed? **11.** ________________

12. The odds that Jonathan will attend a concert each month are $\frac{2}{9}$. What is the probability that Jonathan will attend concerts 2 months in a row? **12.** ________________

NAME ______________________________ DATE ______________ PERIOD ________

Chapter 13 Test, Form 2B (continued)

13. A book rack contains 5 novels and 7 dictionaries. One book is selected and replaced. A second book is selected. What is the probability that 2 novels are selected? **13.** ________________

14. If two cards are drawn from a standard deck of playing cards without replacement, find the probability of selecting a heart and a diamond. **14.** ________________

15. One pen is randomly selected from a collection of 5 blue, 3 black, and 4 red ink pens. What is the probability that the pen is red or black? **15.** ________________

16. In a certain community, 70% of residents own a VCR, 60% of residents own a stereo, and 50% of residents own both a VCR anda stereo. Find the probability that a resident owns a VCR or a stereo. **16.** ________________

17. Two number cubes are tossed. Find the probability that the sum of the number cubes is less than 6, given that the first number cube shows a 3. **17.** ________________

18. Given the integers 1 through 100, what is the probability that one of these integers is divisible by 4 if it ends in a 0? **18.** ________________

19. A survey shows that 80% of all students wear jeans on Friday. If 4 students enter a class, what is the probability that exactly 3 of them are wearing jeans? **19.** ________________

20. Four out of every 10 college students own a bike. What is the probability that exactly 4 out of 5 students in a project group own bikes? **20.** ________________

Bonus Two number cubes are rolled. What is the probability that at least one number cube shows an 8? **Bonus:** ________________

NAME ______________________ DATE ______________ PERIOD _________

Chapter 13 Test, Form 2C

1. A student has 12 pencils and 10 pens. How many ways can the student choose one pencil and one pen? **1.** ________________

2. How many ways can 8 different videos be arranged in a row for a display? **2.** ________________

3. How many ways can first place, second place, and third place be chosen in a contest in which there are 11 entries? **3.** ________________

4. In an algebra class of 20 students, how many different ways can a subgroup of 6 students be chosen for a group project? **4.** ________________

5. Find the number of possible variations of 4-digit street addresses if the first digit cannot be a 0. **5.** ________________

6. How many ways can the letters in the word *fishing* be arranged? **6.** ________________

7. How many ways can 12 people arrange themselves around a circular trampoline? **7.** ________________

8. Find the number of ways that 6 people can arrange themselves around the circular base of a flagpole at the end of a sidewalk. **8.** ________________

For Exercises 9 and 10, consider a bag containing 5 red, 3 blue, and 6 yellow marbles.

9. If two marbles are drawn at random, what is the probability of getting 1 red marble and 1 blue marble? **9.** ________________

10. If two marbles are drawn at random, what are the odds of selecting 2 red marbles? **10.** ________________

11. The probability of getting all tails when three coins are tossed is 1 in 8. What are the odds of getting all tails when three coins are tossed? **11.** ________________

12. The odds that Marilyn will go to see a movie each week are $\frac{1}{5}$. What is the probability that Marilyn will go to see a movie next week? **12.** ________________

NAME ______________________ DATE ______________ PERIOD ________

Chapter 13 Test, Form 2C (continued)

13. A book rack contains 8 cookbooks and 3 novels. One book is selected at random and replaced. A second book is selected at random. What is the probability that 1 cookbook and 1 novel are selected? **13.** ______________

14. Two cards are drawn from a standard deck of playing cards without replacement. Find the probability of selecting a jack and a king. **14.** ______________

15. From a collection of 2 blue, 6 yellow, and 4 red crayons, one is selected at random. What is the probability that the crayon is red or blue? **15.** ______________

16. In a certain community, 60% of residents own a microwave oven, 40% of residents own a computer, and 30% of residents own both a microwave oven and a computer. Find the probability that a resident owns a microwave oven or a computer. **16.** ______________

17. Two number cubes are tossed. Find the probability that the sum of the number cubes is greater than 5 given that the first number cube shows a 3. **17.** ______________

18. Given the integers 1 through 50, what is the probability that one of these integers is a multiple of 3 if it ends in a 5? **18.** ______________

19. A survey shows that 80% of all students on one campus carry backpacks. If 4 students enter a class, find the probability that exactly 2 of them are carrying backpacks. **19.** ______________

20. Eight out of every 10 working adults own a car. Expressed as a decimal to the nearest hundredth, what is the probability that exactly 6 out of 8 working adults in an office own cars? **20.** ______________

Bonus Eight points lie on a circle. How many different inscribed pentagons can be drawn using the points as vertices? **Bonus:** ______________

NAME ______________________ DATE ______________ PERIOD ________

Chapter 13 Open-Ended Assessment

Instructions: Demonstrate your knowledge by giving a clear, concise solution to each problem. Be sure to include all relevant drawings and justify your answers. You may show your solution in more than one way or investigate beyond the requirements of the problem.

1. Men's socks are to be displayed along an aisle of a department store.

a. If there are 3 styles, 5 colors, and 3 sizes of socks, how many different arrangements are possible?

b. Not all the possible arrangements make sense; that is, some may confuse a customer who is trying to locate a particular pair of socks. Describe a poor arrangement.

c. Sketch an example of a good arrangement of the socks. Explain why this is a better arrangement than the one described in part **b**.

2. Seven different dress styles are to be arranged on a circular rack.

a. How many different arrangements are possible?

b. How does the number of arrangements on a circular rack differ from the number of arrangements on a straight rack? Explain the reason for the differences.

3. A store has 7 different fashion scarves for sale.

a. If the manager wants to display a combination of four of these scarves by the checkout, what is the number of possible combinations?

b. Explain the difference between a combination and a permutation.

4. For the season, Chad's free-throw percentage is 70%.

a. If shooting consecutive free throws are independent events, what is the probability that Chad will make two consecutive shots?

b. Describe a situation in which two free throws would not be independent events. What factors might affect the shots?

5. Eight distinct points are randomly located on a circle.

a. How many different triangles can be formed by using the points as vertices? Justify your answer.

b. How many different quadrilaterals can be formed? Justify your answer.

NAME ______________________ DATE ______________ PERIOD ________

Chapter 13 Mid-Chapter Test (Lessons 13-1 through 13-3)

1. A lunch line offers 3 choices of salad, 2 choices of meat, 4 choices of vegetable, and 2 choices of dessert. How many menu combinations are possible that include one of each course? **1.** ______________

2. How many ways can the letters in the word *decimal* be arranged? **2.** ______________

3. Find the number of possible arrangements of 9 different videos in a display window using exactly 4 at a time. **3.** ______________

4. If 5 blocks are drawn at random from a box containing 7 blue and 5 green blocks, how many ways can 3 blue and 2 green blocks be chosen? **4.** ______________

5. How many ways can the letters in the word *attitude* be arranged? **5.** ______________

6. Find the number of ways 6 keys can be arranged on a key ring with no chain. **6.** ______________

7. How many ways can 10 people be seated around a circular conference table if there is a laptop computer on the table in front of one of the seats? **7.** ______________

8. Two number cubes are tossed. Find the probability that the sum of the number cubes is 6. **8.** ______________

9. If two marbles are selected at random from a bag containing 6 red and 4 blue marbles, find the odds that both marbles are red. **9.** ______________

10. The odds of all three coins showing heads when three coins are tossed are 1 to 7. What is the probability of tossing 3 heads when three coins are tossed? **10.** ______________

NAME ______________________ DATE ______________ PERIOD _________

Chapter 13

Chapter 13, Quiz A (Lessons 13-1 and 13-2)

1. A toolbox contains 12 wrenches, 8 screwdrivers, and 5 pairs of pliers. How many ways can a mechanic choose 3 tools, if he needs one of each? **1.** ______________

2. How many ways can a mother, father, and six children be arranged in a row for a photograph? **2.** ______________

3. How many ways can 3 blue, 4 red, and 2 yellow notebooks be arranged in a row in any variation? **3.** ______________

4. For dinner you have chicken, mashed potatoes, and corn. Are eating chicken first and eating mashed potatoes second dependent or independent events? **4.** ______________

5. How many ways can 8 pins be arranged on a circular hatband? **5.** ______________

NAME ______________________ DATE ______________ PERIOD _________

Chapter 13

Chapter 13, Quiz B (Lesson 13-3)

1. What is the probability that a given month of the year begins with the letter *J*? **1.** ______________

2. Two number cubes are tossed. What are the odds that they show a sum greater than 9? **2.** ______________

3. From a box containing 12 slips of paper numbered 1 to 12, 2 slips are drawn. Find the probability that the numbers on both slips are divisible by 3. **3.** ______________

4. The probability of getting a sum of 7 when two number cubes are tossed is 1 in 6. What are the odds of getting a sum of 7 when two number cubes are tossed? **4.** ______________

5. The odds of a student selected at random being a band member are 3 to 10. What is the probability that a student selected at random is in the band? **5.** ______________

NAME ______________________ DATE ______________ PERIOD ________

Chapter 13

Chapter 13, Quiz C (Lessons 13-4 and 13-5)

1. Two number cubes, one red and one blue, are tossed. What is the probability that the red number cube shows a 5 and the blue number cube shows an even number? **1.** ______________

2. Are selecting a king and selecting a black card from a standard deck of cards mutually exclusive or mutually inclusive events? What is the probability of selecting a king or a black card? **2.** ______________

3. A basket contains 8 red, 3 blue, and 5 green balls. If one ball is taken at random, what is the probability that it is blue or green? **3.** ______________

4. A class survey shows 60% of the students like rock music, 40% of the students are juniors, and 30% of the students are both rock music fans and juniors. Find the probability that a student likes rock music or is a junior. **4.** ______________

5. If two number cubes are tossed, what is the probability of getting a sum that is less than 6, given that one number cube shows a 3? **5.** ______________

NAME ______________________ DATE ______________ PERIOD ________

Chapter 13

Chapter 13, Quiz D (Lesson 13-6)

For Exercises 1 and 2, consider that 5 coins are tossed.

1. What is the probability of getting exactly 4 heads? **1.** ______________

2. Find the probability of getting at least 2 tails. **2.** ______________

For Exercises 3 and 4, consider survey results that show 25% of all cars in a community have tinted windows.

3. Find the probability that exactly 3 of the next 5 cars to pass will have tinted windows. **3.** ______________

4. Find the probability that none of the next four cars will have tinted windows. **4.** ______________

5. Three out of every 5 classrooms have a computer. Find the probability, expressed as a decimal to the nearest hundredth, that exactly 8 out of 10 classrooms in a given school have a computer. **5.** ______________

NAME ______________________ DATE ____________ PERIOD ________

Chapter 13 SAT and ACT Practice

After working each problem, record the correct answer on the answer sheet provided or use your own paper.

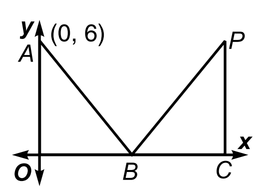
Multiple Choice

1. Determine the number of ways that 5 students can be chosen for a team from a class of 30.
A 1293
B 142,506
C 3,542,292
D 17,100,720
E None of these

2. If a number cube is rolled, what is the probability that the cube will stop with an even number facing up?
A $\frac{1}{2}$
B $\frac{1}{3}$
C $\frac{2}{3}$
D $\frac{3}{2}$
E $\frac{1}{6}$

3. What is the length of a line segment joining two points whose coordinates are $(-2, -7)$ and $(6, 8)$?
A 4
B 5
C $7\frac{1}{2}$
D $8\frac{1}{2}$
E 17

4. In the figure below, $\triangle AOB$ and $\triangle PCB$ are isosceles right triangles with equal areas. What are the coordinates of point P?
A (6, 0)
B (6, 12)
C (12, 0)
D (0, 12)
E (12, 6)



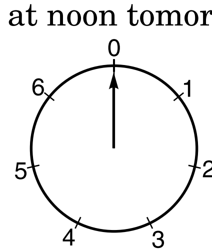
5. C and D are distinct points on $\overline{AB}$ and C is the midpoint of $\overline{AB}$. What is the probability that D is the midpoint of $\overline{AB}$?
A 0
B $\frac{1}{2}$
C $\frac{2}{3}$
D 1
E It cannot be determined from the information given.

6. P is a point on the bisector of $\angle ABC$. What is the probability that P is equidistant from the sides of the angle?
A 0
B $\frac{1}{2}$
C $\frac{1}{4}$
D 1
E It cannot be determined from the information given.

7. If $P = \frac{h(a + b)}{2}$, what is the average of a and b when $P = 30$ and $h = 5$?
A 2
B 6
C 15
D $\frac{35}{2}$
E 150

8. If $a = 7b$, then what is the average of a and b?
A $2b$ **B** $3b$
C $3\frac{1}{2}b$ **D** $4b$
E $8b$

9. A 7-hour clock is shown below. If at noon today the pointer is at 0, where will the pointer be at noon tomorrow?
A 2
B 3
C 4
D 5
E 6



NAME ______________________ DATE ____________ PERIOD ________

Chapter 13 SAT and ACT Practice (continued)

10. $P = 2 - \frac{9}{10}, Q = 2 - 0.099, R = 2 \div 9$
Which list below shows P, Q, and R in order from greatest to least?
A P, Q, R **B** Q, P, R
C R, P, Q **D** P, R, Q
E R, Q, P

11. One number cube is rolled. What is the probability that when the cube stops rolling the number on top is an even number or a number less than 4?
A $\frac{2}{3}$ **B** $\frac{1}{2}$
C $\frac{5}{6}$ **D** $\frac{1}{3}$
E 1

12. Among a group of 6 people, how many committees of 3 people can be formed if 2 of the 6 people cannot be on the same committee?
A 12 **B** 9
C 10 **D** 60
E 16

13. An acute angle can have a measure of:
I. 89.999° **II.** 0.0001° **III.** 90.0001°
A I only
B II only
C III only
D I and II only
E I and III only

14. In circle O, $\overline{AB}$ is a chord, $\overline{OA}$ and $\overline{OB}$ are radii, $m\angle AOB = 120°$, and $AB = 12$. Find the distance from the chord to the center of the circle.
A $2\sqrt{3}$ **B** $4\sqrt{3}$
C 3 **D** 6
E It cannot be determined from the information given.

15. In a detective game, there are 6 suspects, 6 weapons, and 9 rooms. What is the probability that the crime was committed by the housekeeper in the library with a candlestick holder?
A $\frac{1}{108}$ **B** $\frac{1}{216}$
C $\frac{1}{324}$ **D** $\frac{1}{54}$
E None of these

16. Two disks are selected at random from a box containing 10 disks numbered from 1 to 10. What is the probability that one disk has an even number and the other has an odd number if the first disk is not replaced before the second disk is selected?
A $\frac{1}{2}$ **B** $\frac{5}{18}$
C $\frac{5}{9}$ **D** $\frac{3}{5}$
E $\frac{2}{5}$

17–18. Quantitative Comparison
A if the quantity in Column A is greater
B if the quantity in Column B is greater
C if the two quantities are equal
D if the relationship cannot be determined from the information given

	Column A	Column B
17.	The number of ways to select 2 males or 2 females from a group of 6 males and 4 females	The number of ways to select 1 male and 1 female from a group of 6 males and 4 females
18.	$C(50, 0)$	$C(30, 30)$

19. Grid-In José has 6 pennies, 5 nickels, and 4 dimes in his pocket. What is the probability that a coin he draws at random is a penny?

20. Grid-In A 3-person committee is to be chosen from a group of 6 males and 4 females. What is the probability that the committee will consist of 2 males and 1 female?

NAME ______________________ DATE ______________ PERIOD ________

Chapter 13 Cumulative Review (Chapters 1–13)

1. Write the standard form of the equation of the line with an x-intercept of 2 and a y-intercept of 3. **1.** ______________

2. Solve $\sqrt{3t + 7} - 7 > 0$. **2.** ______________

3. Suppose θ is an angle in standard position whose terminal side lies in Quadrant IV. If $\cos \theta = \frac{4}{5}$, what is the value of $\tan \theta$? **3.** ______________

4. Write the equation $y = -x + 4$ in parametric form. **4.** ______________

5. Find the rectangular coordinates of the point with polar coordinates $\left(4, \frac{5\pi}{4}\right)$. **5.** ______________

6. Write the equation of the parabola whose focus is at $(-2, 6)$ and whose directrix has the equation $y = 2$. **6.** ______________

7. Evaluate $\log_9 27$. **7.** ______________

8. Find the sum of the series $18 + 12 + 8 + \cdots$, or state that the sum does not exist. **8.** ______________

9. A sample of 3 fuses from a box of 100 fuses is to be inspected. How many ways can the sample be chosen? **9.** ______________

10. Two number cubes are tossed. What is the probability that they show a sum of either 2 or 11? **10.** ______________

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NAME ______________________________ DATE ______________ PERIOD ________

SAT and ACT Practice Answer Sheet
(10 Questions)

1 (A) (B) (C) (D) (E)

2 (A) (B) (C) (D) (E)

3 (A) (B) (C) (D) (E)

4 (A) (B) (C) (D) (E)

5 (A) (B) (C) (D) (E)

6 (A) (B) (C) (D) (E)

7 (A) (B) (C) (D) (E)

8 (A) (B) (C) (D) (E)

9 (A) (B) (C) (D) (E)

10

	(/)	(/)	
(.)	(.)	(.)	(.)
	(0)	(0)	(0)
(1)	(1)	(1)	(1)
(2)	(2)	(2)	(2)
(3)	(3)	(3)	(3)
(4)	(4)	(4)	(4)
(5)	(5)	(5)	(5)
(6)	(6)	(6)	(6)
(7)	(7)	(7)	(7)
(8)	(8)	(8)	(8)
(9)	(9)	(9)	(9)

NAME ______________________ DATE ____________ PERIOD _______

SAT and ACT Practice Answer Sheet

(20 Questions)

1 (A) (B) (C) (D) (E)

2 (A) (B) (C) (D) (E)

3 (A) (B) (C) (D) (E)

4 (A) (B) (C) (D) (E)

5 (A) (B) (C) (D) (E)

6 (A) (B) (C) (D) (E)

7 (A) (B) (C) (D) (E)

8 (A) (B) (C) (D) (E)

9 (A) (B) (C) (D) (E)

10 (A) (B) (C) (D) (E)

11 (A) (B) (C) (D) (E)

12 (A) (B) (C) (D) (E)

13 (A) (B) (C) (D) (E)

14 (A) (B) (C) (D) (E)

15 (A) (B) (C) (D) (E)

16 (A) (B) (C) (D) (E)

17 (A) (B) (C) (D) (E)

18 (A) (B) (C) (D) (E)

19

	/	/	
.	.	.	.
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

20

	/	/	
.	.	.	.
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

13-1

NAME ______________ DATE ________ PERIOD ____

Practice

Permutations and Combinations

1. A golf manufacturer makes irons with 7 different shaft lengths, 3 different grips, and 2 different club head materials. How many different combinations are offered? **42**

2. A briefcase lock has 3 rotating cylinders, each containing 10 digits. How many numerical codes are possible? **1000**

3. How many 7-digit telephone numbers can be formed if the first digit cannot be 0 or 1? **8,000,000**

Find each value.

4. $P(10, 7)$ **604,800**
5. $P(7, 7)$ **5040**
6. $P(6, 3)$ **120**
7. $C(7, 2)$ **21**
8. $C(10, 4)$ **210**
9. $C(12, 4) \cdot C(8, 3)$ **27,720**

10. How many ways can the 4 call letters of a radio station be arranged if the first letter must be W or K and no letters can be repeated? **27,600**

11. There are 5 different routes that a commuter can take from her home to her office. How many ways can she make a roundtrip if she uses different routes for coming and going? **20**

12. How many committees of 5 students can be selected from a class of 25? **53,130**

13. A box contains 12 black and 8 green marbles. How many ways can 3 black and 2 green marbles be chosen? **6160**

14. ***Basketball*** How many ways can a coach select a starting team of one center, two forwards, and two guards if the basketball team consists of three centers, five forwards, and three guards? **90**

13-1

NAME ______________ DATE ________ PERIOD ____

Enrichment

Permutation and Combination Algebra

Expressions involving $P(n, r)$ and $C(n, r)$, the symbols for permutations and combinations, can sometimes be simplified or used in equations as though they were algebraic expressions. You can solve problems involving such expressions by applying the definitions of $P(n, r)$ and $C(n, r)$.

Example **Simplify $C(n, n - 1)$.**

By the definition of $C(n, r)$,

$$C(n, n-1) = \frac{n!}{(n-[n-1])!(n-1)!} = \frac{n!}{(n-n+1)!(n-1)!} = \frac{n!}{1!(n-1)!} = \frac{n!}{(n-1)!} = n$$

Simplify.

1. $P(n, n - 1)$ **$n!$**
2. $C(n, n)$ **1**
3. $C(n, 1)$ **n**
4. $P(n, n)$ **$n!$**
5. $C(n + 1, n)$ **$n + 1$**
6. $C(n + 1, n - 1)$ **$\frac{(n+1)n}{2}$**

Solve for n.

7. $P(n, 5) = 7\,P(n, 4)$ **11**
8. $C(n, n - 2) = 6$ **4**
9. $C(n + 2, 4) = 6\,C(n, 2)$ **7**
10. $P(n, 5) = 9\,P(n - 1, 4)$ **9**

NAME ____________ DATE ________ PERIOD ____

13-2

Practice

Permutations with Repetitions and Circular Permutations

How many different ways can the letters of each word be arranged?

1. *members*
1260

2. *annually*
5040

3. *Missouri*
10,080

4. *concert*
2520

5. How many different 5-digit street addresses can have the digits 4, 7, 3, 4, and 8?
60

6. Three hardcover books and 5 paperbacks are placed on a shelf. How many ways can the books be arranged if all the hardcover books must be together and all the paperbacks must be together?
56

Determine whether each arrangement of objects is a linear or circular permutation. Then determine the number of arrangements for each situtation.

7. 9 keys on a key ring with no chain **circular; 40,320**

8. 5 charms on a bracelet with no clasp **circular; 24**

9. 6 people seated at a round table with one person seated next to a door **linear; 720**

10. 12 different symbols around the face of a watch **circular; 39,916,800**

11. ***Entertainment*** Jasper is playing a word game and has the following letters in his tray: QUOUNNTAGGRA. How many 12-letter arrangements could Jasper make to check if a single word could be formed from all the letters?
29,937,600

NAME ____________ DATE ________ PERIOD ____

13-2

Enrichment

Approximating Factorials

James Stirling (1692-1770) was a teacher, a friend of Sir Isaac Newton, and a mathematician who made important contributions to calculus. Today he is best remembered as the creator of a formula for approximating factorials.

Stirling's Formula	$n! \approx \sqrt{2n\pi}\left(\frac{n}{e}\right)^n$, where e is the irrational number 2.7182818....

1. Complete the chart. By examining the ratio $\frac{n!}{\sqrt{2n\pi}\left(\frac{n}{e}\right)^n}$, we can see how closely Stirling's formula approximates $n!$.

n	$n!$	$\sqrt{2n\pi}\left(\frac{n}{e}\right)^n$	$\frac{n!}{\sqrt{2n\pi}\left(\frac{n}{e}\right)^n}$
10	3.6288×10^{6}	3.5987×10^{6}	1.008
20	2.4329×10^{18}	2.4228×10^{18}	1.004
30	2.6525×10^{32}	2.6452×10^{32}	1.003
40	8.1592×10^{47}	8.1422×10^{47}	1.002
50	3.0414×10^{64}	3.0363×10^{64}	1.002
60	8.3210×10^{81}	8.3094×10^{81}	1.001

2. Based on the completed chart, as n increases, will the approximations obtained using Stirling's formula become more accurate or less accurate? Explain.

**More accurate; as n increases, the ratio $\frac{n!}{\sqrt{2n\pi}\left(\frac{n}{e}\right)^n}$ approaches 1.
So $n!$ is approaching $\sqrt{2n\pi}\left(\frac{n}{e}\right)^n$.**

NAME ______________ DATE ________ PERIOD ______

13-3 Practice

Probability and Odds

A kitchen drawer contains 7 forks, 4 spoons, and 5 knives. Three are selected at random. Find each probability.

1. P(3 forks) $\frac{1}{16}$

2. P(2 forks, 1 knife) $\frac{3}{16}$

3. P(3 spoons) $\frac{1}{140}$

4. P(1 fork, 1 knife, 1 spoon) $\frac{1}{4}$

A laundry bag contains 5 red, 9 blue, and 6 white socks. Two socks are selected at random. Find each probability.

5. P(2 red) $\frac{1}{19}$

6. P(2 blue) $\frac{18}{95}$

7. P(1 red, 1 blue) $\frac{9}{38}$

8. P(1 red, 1 white) $\frac{3}{19}$

Sharon has 8 mystery books and 9 science-fiction books. Four are selected at random. Find each probability.

9. P(4 mystery books) $\frac{1}{34}$

10. P(4 science-fiction books) $\frac{9}{170}$

11. P(2 mysteries, 2 science-fiction) $\frac{36}{85}$

12. P(3 mysteries, 1 science-fiction) $\frac{18}{85}$

From a standard deck of 52 cards, 5 cards are drawn. What are the odds of each event occurring?

13. 5 aces **0**

14. 5 face cards $\frac{33}{108{,}257}$

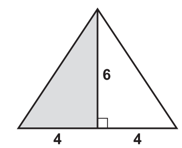
15. ***Meteorology*** A local weather forecast states that the chance of sunny weather on Wednesday is 70%. What are the odds that it will be sunny on Wednesday? $\frac{7}{3}$ **or 7 to 3**

NAME ______________ DATE ________ PERIOD ______

13-3 Enrichment

Geometric Probability

If a dart, thrown at random, hits the triangular board shown at the right, what is the chance that is will hit the shaded region? This chance, also called a probability, can be determined by analyzing the area of the board. This ratio indicates what fraction of the tosses should hit in the shaded region.



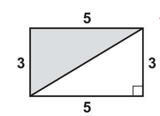
$$\frac{\text{area of shaded region}}{\text{area of triangular board}} = \frac{\frac{1}{2}(4)(6)}{\frac{1}{2}(8)(6)}$$

$$= \frac{12}{24} \text{ or } \frac{1}{2}$$

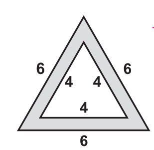
In general, if S is a subregion of some region R, then the probability $P(S)$ that a point, chosen at random, belongs to subregion S is given by the following.

$$P(S) = \frac{\text{area of subregion } S}{\text{area of region } R}$$

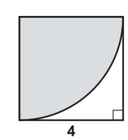
Find the probability that a point, chosen at random, belongs to the shaded subregions of the following regions.

1. 

$\frac{1}{2}$

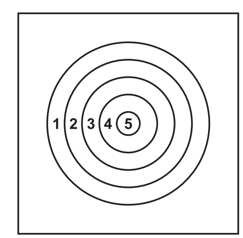
2. 

$\frac{5}{9}$

3. 

$\frac{\pi}{4}$

The dart board shown at the right has 5 concentric circles whose centers are also the center of the square board. Each side of the board is 38 cm, and the radii of the circles are 2 cm, 5 cm, 8 cm, 11 cm, and 14 cm. A dart hitting within one of the circular regions scores the number of points indicated on the board, while a hit anywhere else scores 0 points. If a dart, thrown at random, hits the board, find the probability of scoring the indicated number of points. Write your answer in terms of π.



4. 0 points $\frac{361 - 49\pi}{361}$

5. 1 point $\frac{75\pi}{1444}$

6. 2 points $\frac{3\pi}{76}$

7. 3 points $\frac{39\pi}{1444}$

8. 4 points $\frac{21\pi}{1444}$

9. 5 points $\frac{\pi}{361}$

NAME ______________ DATE ________ PERIOD ______

13-4

Practice

Probabilities of Compound Events

Determine if each event is independent or dependent. Then determine the probability.

1. the probability of drawing a black card from a standard deck of cards, replacing it, then drawing another black card
independent; $\frac{1}{4}$

2. the probability of selecting 1 jazz, 1 country, and 1 rap CD in any order from 3 jazz, 2 country, and 5 rap CDs, replacing the CDs each time
independent; $\frac{9}{50}$

3. the probability that two cards drawn from a deck are both aces
dependent; $\frac{1}{221}$

Determine if each event is mutually exclusive or mutually inclusive. Then determine each probability.

4. the probability of rolling a 3 or a 6 on one toss of a number cube
mutually exclusive; $\frac{1}{3}$

5. the probability of selecting a queen or a red card from a standard deck of cards
mutually inclusive; $\frac{7}{13}$

6. the probability of selecting at least three white crayons when four crayons are selected from a box containing 7 white crayons and 5 blue crayons
mutually exclusive; $\frac{14}{33}$

7. ***Team Sports*** Conrad tried out for both the volleyball team and the football team. The probability of his being selected for the volleyball team is $\frac{4}{5}$, while the probability of his being selected for the football team is $\frac{3}{4}$. The probability of his being selected for both teams is $\frac{7}{10}$. What is the probability that Conrad will be selected for either the volleyball team or the football team? **$\frac{17}{20}$**

NAME ______________ DATE ________ PERIOD ______

13-4

Enrichment

Probability and Tic-Tac-Toe

What would be the chances of winning at tic-tac-toe if it were turned into a game of pure chance? To find out, the nine cells of the tic-tac-toe board are numbered from 1 to 9 and chips (also numbered from 1 to 9) are put into a bag. Player A draws a chip at random and enters an X in the corresponding cell. Player B does the same and enters an O.

To solve the problem, assume that both players draw all their chips without looking and all X and O entries are made at the same time. There are four possible outcomes: a draw, A wins, B wins, and either A or B can win.

There are 16 arrangements that result in a draw. Reflections and rotations must be counted as shown below.

```
o x o    x o x    o o x
x o x    o o x    x x o
x o x    x x o    o x x
  4        4        8
```

There are 36 arrangements in which either player may win because both players have winning triples.

```
x x x    x x x    x o x    x x x    x x x    x x o
o o o    x o x    x x x    x x o    o o o    x x x
x o x    o o o    o o o    o o o    x x o    o o o
  4        4        4        8        8        8
```

In these 36 cases, A's chances of winning are $\frac{13}{40}$.

1. Find the 12 arrangements in which B wins and A cannot.

```
o o x      o x o
x o x 8    x o x 4
x x o      x x o
```

2. Below are 12 of the arrangements in which A wins and B cannot. Write the numbers to show the reflections and rotations for each arrangement. What is the total number? **62**

```
o x o      x o x      x x x      x x x      x o o      x o x
x x x 1    o x o 1    x o o 4    o x o 4    x x x 4    x x o 4
o x o      x o x      x o o      o x o      o o x      o o x

x x o      x x x      x x x      x x x      x o o      x x o
o x x 4    o x o 8    x o o 8    x o o 8    x x x 8    o x o 8
o o x      o o x      o x o      o o x      o x o      x o x
```

3. There are $\frac{9!}{5!4!}$ different and equally probable distributions. Complete the chart to find the probability for a draw or for A or B to win.

Draw: $\frac{16}{126}$			=	**$\frac{8}{63}$**
A wins:	**$\frac{62}{126}$**	+ $\frac{13}{40}\left(\frac{36}{126}\right)$	=	**$\frac{737}{1260}$**
B wins:	**$\frac{12}{126}$**	+ **$\frac{27}{40}\left(\frac{36}{126}\right)$**	=	**$\frac{121}{420}$**

NAME ______________ DATE ________ PERIOD ______

13-5 Practice

Conditional Probabilities

Find each probability.

1. Two number cubes are tossed. Find the probability that the numbers showing on the cubes match, given that their sum is greater than 7. $\frac{1}{5}$
2. A four-digit number is formed from the digits 1, 2, 3, and 4. Find the probability that the number ends in the digits 41, given that the number is odd. $\frac{1}{6}$
3. Three coins are tossed. Find the probability that exactly two coins show tails, given that the third coin shows tails. $\frac{1}{2}$

A card is chosen from a standard deck of cards. Find each probability, given that the card is red.

4. P(diamond) $\frac{1}{2}$
5. P(six of hearts) $\frac{1}{26}$
6. P(queen or 10) $\frac{2}{13}$
7. P(face card) $\frac{3}{13}$

A survey taken at Stirers High School shows that 48% of the respondents like soccer, 66% like basketball, and 38% like hockey. Also, 30% like soccer and basketball, 22% like basketball and hockey, and 28% like soccer and hockey. Finally, 12% like all three sports.

8. If Meg likes basketball, what is the probability that she also likes soccer? $\frac{5}{11}$
9. If Jaime likes soccer, what is the probability that he also likes hockey and basketball? $\frac{1}{4}$
10. If Ashley likes basketball, what is the probability that she also likes hockey? $\frac{1}{3}$
11. If Brett likes soccer, what is the probability that he also likes basketball? $\frac{5}{8}$

NAME ______________ DATE ________ PERIOD ______

13-5 Enrichment

Probability in Genetics

The Austrian monk and botanist Gregor Mendel discovered the basic laws of genetics during the nineteenth century. Through experiments with pea plants, Mendel found that cells in living organisms contain pairs of units that control traits in the offspring of the organism. We now call these units *genes*. If the genes in a cell are identical, the trait is *pure*. If they are different, the trait is *hybrid*. A trait like *tallness* which masks other traits, preventing them from showing up in offspring, is *dominant*. Otherwise, it is *recessive*. A combination of a dominant gene and a recessive gene will always produce a hybrid displaying the dominant trait.

Example **Two hybrid tall pea plants are crossed. What is the probability that the offspring will be short?**

Punnett squares are used to analyze gene combinations. Use capital letters to represent dominant genes and lower-case letters to represent recessive genes.

	T	t
T	TT	Tt
t	Tt	tt

T = tall t = short

The table shows the four equally possible outcomes. One of the outcomes, TT, is a pure tall plant. Two of the outcomes, Tt and Tt, are hybrid tall plants. Only one of the outcomes, tt, is a short plant. Therefore, the probability that an offspring will be short is $\frac{1}{4}$.

Use Punnett squares to solve.

1. A pure dominant yellow pea plant (Y) is crossed with a pure recessive white pea plant (w).
   a. What are the possible outcomes? **Yw, Yw, Yw, Yw**
   b. Find the probability that an offspring will be yellow. **1**
2. A hybrid tall pea plant is crossed with a short plant. Find the probability that an offspring will be short. $\frac{1}{2}$
3. Brown eyes are dominant over blue eyes in humans. What is the probability that a woman with blue eyes and a man with hybrid brown eyes will have a child with blue eyes? $\frac{1}{2}$
4. What is the probability that the offspring of a hybrid-tall, hybrid-yellow pea plant and a hybrid-tall white plant will be short white? $\frac{1}{8}$

NAME ______________________ DATE ____________ PERIOD ________

13-6

Practice

The Binomial Theorem and Probability

Find each probability if six coins are tossed.

1. P(3 heads and 3 tails) $\frac{5}{16}$

2. P(at least 4 heads) $\frac{11}{32}$

3. P(2 heads or 3 tails) $\frac{35}{64}$

4. P(all heads or all tails) $\frac{1}{32}$

The probability of Chris's making a free throw is $\frac{2}{3}$. Find each probability if she shoots five times.

5. P(all missed) $\frac{1}{243}$

6. P(all made) $\frac{32}{243}$

7. P(exactly 4 made) $\frac{80}{243}$

8. P(at least 3 made) $\frac{64}{81}$

When Maria and Len play a certain board game, the probability that Maria will win the game is $\frac{3}{4}$. Find each probability if they play five games.

9. P(Len wins only 1 game) $\frac{405}{1024}$

10. P(Maria wins exactly 2 games) $\frac{45}{512}$

11. P(Len wins at least 2 games) $\frac{47}{128}$

12. P(Maria wins at least 3 games) $\frac{459}{512}$

13. ***Gardening*** Assume that 60% of marigold seeds that are sown directly in the ground produce plants. If Tomaso plants 10 seeds, what is the probability that 7 plants will be produced?
about 21.5%

NAME ______________________ DATE ____________ PERIOD ________

13-6

Enrichment

Combinations and Pascal's Triangle

Pascal's triangle is a special array of numbers invented by Blaise Pascal (1623-1662). The values in Pascal's triangle can be found using the combinations shown below.

1. Evaluate the expression in each cell of the triangle

C(1, 0) **1** | C(1, 1) **1**

C(2, 0) **1** | C(2, 1) **2** | C(2, 2) **1**

C(3, 0) **1** | C(3, 1) **3** | C(3, 2) **3** | C(3, 3) **1**

C(4, 0) **1** | C(4, 1) **4** | C(4, 2) **6** | C(4, 3) **4** | C(4, 4) **1**

C(5, 0) **1** | C(5, 1) **5** | C(5, 2) **10** | C(5, 3) **10** | C(5, 4) **5** | C(5, 5) **1**

2. The pattern shows the relationship between $C(n, r)$ and Pascal's triangle. In general, it is true that $C(n, r) + C(n, r + 1) = C(n + 1, r + 1)$. Complete the proof of this property. In each step, the denominator has been given.

$$C(n, r) + C(n, r + 1) = \frac{n!}{r!(n-r)!} + \frac{n!}{(r+1)!(n-r-1)!}$$

$$= \frac{n!(r+1)}{r!(n-r)!(r+1)} + \frac{n!(n-r)}{(r+1)!(n-r-1)!(n-r)}$$

$$= \frac{n!(r+1)}{(r+1)!(n-r)!} + \frac{n!(n-r)}{(r+1)!(n-r)!}$$

$$= \frac{n!(r+1+n-r)}{(r+1)!(n-r)!}$$

$$= \frac{n!(n+1)}{(r+1)!(n-r)!}$$

$$= \frac{(n+1)!}{(r+1)!(n-r)!}$$

$$= \frac{(n+1)!}{(r+1)![(n+1)-(r+1)]!}$$

$$= C(n+1, r+1)$$

Chapter 13 Answer Key

Form 1A

Page 589

1. D
2. A
3. C
4. A
5. C
6. B
7. A
8. B
9. B
10. A
11. C

Page 590

12. B
13. C
14. D
15. D
16. B
17. A
18. C
19. A
20. C

Bonus: C

Form 1B

Page 591

1. B
2. B
3. B
4. A
5. D
6. D
7. A
8. D
9. C
10. B
11. A

Page 592

12. C
13. D
14. C
15. B
16. C
17. C
18. D
19. B
20. A

Bonus: B

Chapter 13 Answer Key

Form 1C

Page 593

1. A
2. C
3. B
4. B
5. C
6. D
7. C
8. D
9. A
10. D
11. C

Page 594

12. A
13. B
14. A
15. D
16. A
17. D
18. B
19. B
20. C

Bonus: C

Form 2A

Page 595

1. 270
2. 5040
3. 840
4. 53,130
5. 2048
6. 45,360
7. 40,320
8. 5040
9. $\frac{5}{22}$
10. $\frac{1}{21}$
11. $\frac{25}{1}$
12. $\frac{4}{49}$

Page 596

13. $\frac{35}{256}$
14. $\frac{4}{221}$
15. $\frac{17}{42}$
16. $\frac{9}{10}$
17. $\frac{1}{2}$
18. $\frac{1}{3}$
19. $\frac{297}{625}$
20. $\frac{11}{32}$

Bonus: 1260

Chapter 13 Answer Key

Form 2B

Page 597

1. 126
2. 720
3. 1320
4. 210
5. 8,000,000
6. 453,600
7. 3,628,800
8. 40,320
9. $\frac{30}{143}$
10. $\frac{1}{12}$
11. $\frac{1}{15}$
12. $\frac{4}{121}$

Page 598

13. $\frac{25}{144}$
14. $\frac{13}{204}$
15. $\frac{7}{12}$
16. $\frac{4}{5}$
17. $\frac{1}{3}$
18. $\frac{1}{2}$
19. $\frac{256}{625}$
20. $\frac{48}{625}$

Bonus: 0

Form 2C

Page 599

1. 120
2. 40,320
3. 990
4. 38,760
5. 9000
6. 2520
7. 39,916,800
8. 720
9. $\frac{15}{91}$
10. $\frac{10}{81}$
11. $\frac{1}{7}$
12. $\frac{1}{6}$

Page 600

13. $\frac{24}{121}$
14. $\frac{4}{663}$
15. $\frac{1}{2}$
16. $\frac{7}{10}$
17. $\frac{2}{3}$
18. $\frac{2}{5}$
19. $\frac{96}{625}$
20. 0.29

Bonus: 56

Chapter 13 Answer Key

CHAPTER 13 SCORING RUBRIC

Level	Specific Criteria
3 Superior	• Shows thorough understanding of the concepts *permutation, combination, probability,* and *independent events.* • Uses appropriate strategies to solve problems. • Computations are correct. • Written explanations are exemplary. • Sketch is detailed and sensible. • Goes beyond requirements of some or all problems.
2 Satisfactory, with Minor Flaws	• Shows understanding of the concepts *permutation, combination, probability,* and *independent events.* • Uses appropriate strategies to solve problems. • Computations are mostly correct. • Written explanations are effective. • Sketch is detailed and sensible. • Satisfies all requirements of problems.
1 Nearly Satisfactory, with Serious Flaws	• Shows understanding of most of the concepts *permutation, combination, probability,* and *independent events.* • May not use appropriate strategies to solve problems. • Computations are mostly correct. • Written explanations are satisfactory. • Sketch is detailed and sensible. • Satisfies all requirements of problems.
0 Unsatisfactory	• Shows little or no understanding of the concepts *polar permutation, combination, probability,* and *independent events.* • May not use appropriate strategies to solve problems. • Computations are incorrect. • Written explanations are not satisfactory. • Sketch is not detailed does not make sense. • Does not satisfy requirements of problems.

Chapter 13 Answer Key

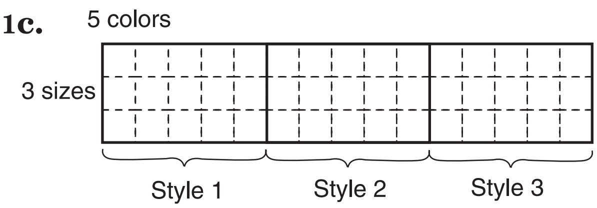
Open-Ended Assessment

Page 601

1a. There are 3 × 5 × 3, or 45, possible arrangements.

1b. A poor arrangement is any that is random, such as an arrangement that is not logically ordered by size, color, or style.

1c.



This arrangement has a pattern that allows the customer to locate a particular pair of socks easily.

2a. $\frac{7!}{7} = 720$ arrangements

2b. On a circular rack, each arrangement has six others just like it, the result of rotating the arrangement. Thus, there are only one-seventh as many arrangements on a circular rack as on a straight rack.

3a. $\frac{7!}{4!\,3!} = 35$ combinations

3b. Order is not considered in a combination.

4a. The probability of Chad making both free throws if the shots are independent events is $0.7 \times 0.7 = 0.49$, or 49%.

4b. If missing the first free throw makes Chad lose confidence in his ability to make the second, then the events are not independent. Likewise, making the first shot may boost his confidence and increase his chances of making the second shot. Fatigue and crowd noise are two other factors that might affect his shots.

5a. Three of the eight points are chosen as vertices for each triangle. Order is not considered in choosing the vertices, so we will use the formula for the number of combinations of 8 objects taken 3 at a time.

$$C(8, 3) = \frac{8!}{5!\,3!}$$
$$= \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 3 \cdot 2 \cdot 1}$$
$$= 8 \cdot 7$$
$$= 56$$

56 different triangles can be formed.

5b. Four of the points are chosen as vertices for each quadrilateral.

$$C(8, 4) = \frac{8!}{4!\,4!}$$
$$= \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{4 \cdot 3 \cdot 2 \cdot 1 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$$
$$= 7 \cdot 5 \cdot 2$$
$$= 70$$

Seventy different quadrilaterals can be formed.

Chapter 13 Answer Key

Mid-Chapter Test
Page 602

1. 48
2. 5040
3. 3024
4. 350
5. 6720
6. 60
7. 3,628,800
8. $\frac{5}{36}$
9. $\frac{1}{2}$
10. $\frac{1}{8}$

Quiz A
Page 603

1. 480
2. 40,320
3. 1260
4. dependent
5. 5040

Quiz B
Page 603

1. $\frac{1}{4}$
2. $\frac{1}{5}$
3. $\frac{1}{11}$
4. $\frac{1}{5}$
5. $\frac{3}{13}$

Quiz C
Page 604

1. $\frac{1}{12}$
2. mutually inclusive: $\frac{7}{13}$
3. $\frac{1}{2}$
4. $\frac{7}{10}$
5. $\frac{2}{3}$

Quiz D
Page 604

1. $\frac{5}{32}$
2. $\frac{13}{16}$
3. $\frac{45}{512}$
4. $\frac{81}{256}$
5. 0.12

Chapter 13 Answer Key

SAT/ACT Practice

Page 605

1. B
2. A
3. E
4. E
5. A
6. D
7. B
8. D
9. B

Page 606

10. B
11. C
12. E
13. D
14. A
15. C
16. C
17. B
18. C
19. $\frac{2}{5}$ or 0.4
20. $\frac{1}{2}$ or 0.5

Cumulative Review

Page 607

1. $3x + 2y - 6 = 0$
2. $t > 14$
3. $-\frac{3}{4}$
4. $x = t$; $y = -t + 4$
5. $(-2\sqrt{2}, -2\sqrt{2})$
6. $(x + 2)^2 = 8(y - 4)$
7. $\frac{3}{2}$
8. 54
9. 161,700
10. $\frac{1}{12}$

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