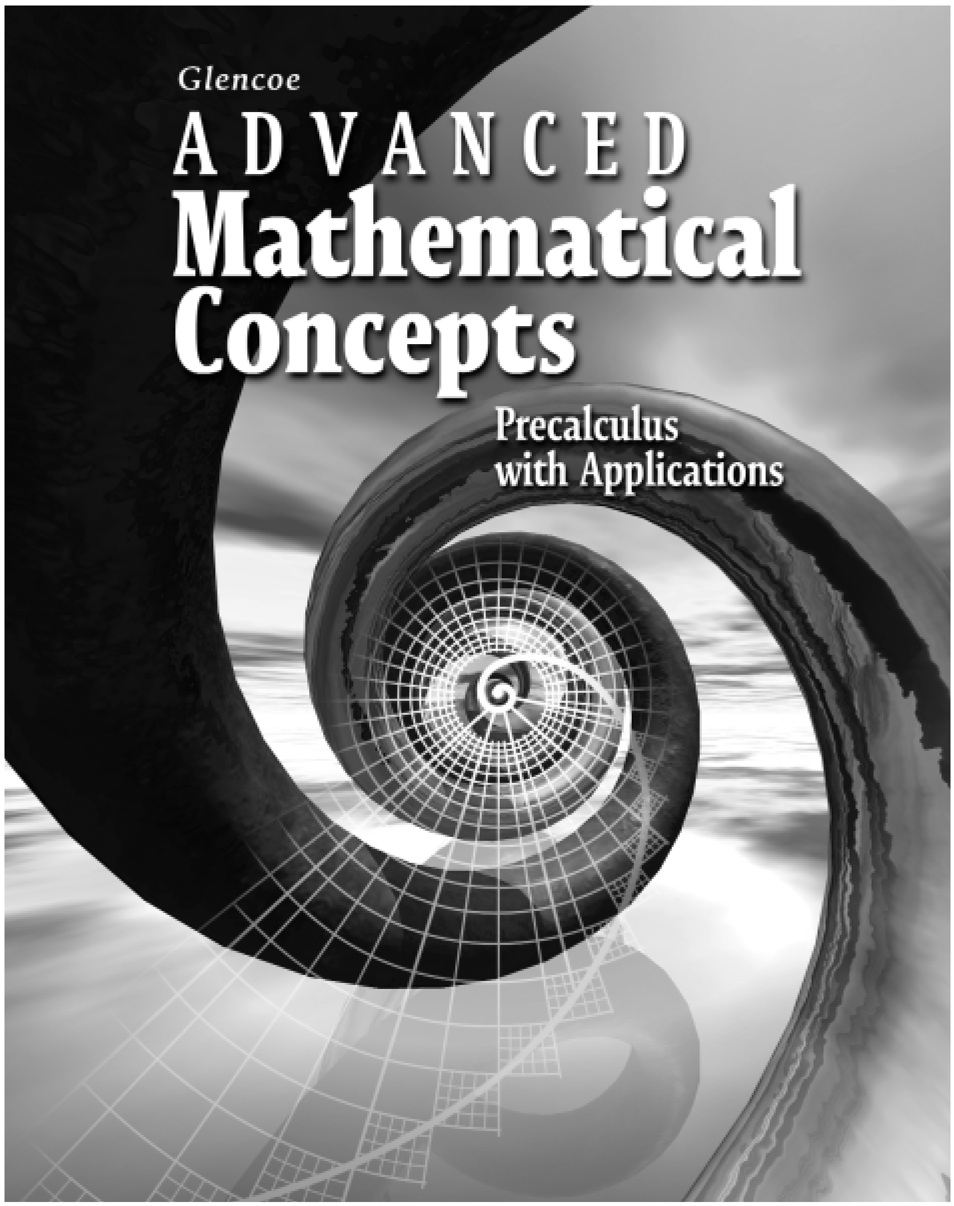
# Chapter 11
# Resource Masters




Glencoe

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**StudentWorks™** This CD-ROM includes the entire Student Edition along with the Study Guide, Practice, and Enrichment masters.

**TeacherWorks™** All of the materials found in this booklet are included for viewing and printing in the *Advanced Mathematical Concepts TeacherWorks* CD-ROM.

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*Advanced Mathematical Concepts*
*Chapter 11 Resource Masters*

1 2 3 4 5 6 7 8 9 10 XXX 11 10 09 08 07 06 05 04

# Contents

# A Teacher's Guide to Using the Chapter 11 Resource Masters

The ***Fast File*** Chapter Resource system allows you to conveniently file the resources you use most often. The *Chapter 11 Resource Masters* include the core materials needed for Chapter 11. These materials include worksheets, extensions, and assessment options. The answers for these pages appear at the back of this booklet.

All of the materials found in this booklet are included for viewing and printing in the *Advanced Mathematical Concepts TeacherWorks* CD-ROM.

**Vocabulary Builder** Pages vii-viii include a student study tool that presents the key vocabulary terms from the chapter. Students are to record definitions and/or examples for each term. You may suggest that students highlight or star the terms with which they are not familiar.

*When to Use* Give these pages to students before beginning Lesson 11-1. Remind them to add definitions and examples as they complete each lesson.

**Study Guide** There is one Study Guide master for each lesson.

*When to Use* Use these masters as reteaching activities for students who need additional reinforcement. These pages can also be used in conjunction with the Student Edition as an instructional tool for those students who have been absent.

**Practice** There is one master for each lesson. These problems more closely follow the structure of the Practice section of the Student Edition exercises. These exercises are of average difficulty.

*When to Use* These provide additional practice options or may be used as homework for second day teaching of the lesson.

**Enrichment** There is one master for each lesson. These activities may extend the concepts in the lesson, offer a historical or multicultural look at the concepts, or widen students' perspectives on the mathematics they are learning. These are not written exclusively for honors students, but are accessible for use with all levels of students.

*When to Use* These may be used as extra credit, short-term projects, or as activities for days when class periods are shortened.

## Assessment Options

The assessment section of the *Chapter 11 Resources Masters* offers a wide range of assessment tools for intermediate and final assessment. The following lists describe each assessment master and its intended use.

## Chapter Assessments

### *Chapter Tests*

- *Forms 1A, 1B, and 1C* Form 1 tests contain multiple-choice questions. Form 1A is intended for use with honors-level students, Form 1B is intended for use with average-level students, and Form 1C is intended for use with basic-level students. These tests are similar in format to offer comparable testing situations.
- *Forms 2A, 2B, and 2C* Form 2 tests are composed of free-response questions. Form 2A is intended for use with honors-level students, Form 2B is intended for use with average-level students, and Form 2C is intended for use with basic-level students. These tests are similar in format to offer comparable testing situations.

  All of the above tests include a challenging Bonus question.
- The **Extended Response Assessment** includes performance assessment tasks that are suitable for all students. A scoring rubric is included for evaluation guidelines. Sample answers are provided for assessment.

## Intermediate Assessment

- A **Mid-Chapter Test** provides an option to assess the first half of the chapter. It is composed of free-response questions.
- Four free-response **quizzes** are included to offer assessment at appropriate intervals in the chapter.

## Continuing Assessment

- The **SAT and ACT Practice** offers continuing review of concepts in various formats, which may appear on standardized tests that they may encounter. This practice includes multiple-choice, quantitative-comparison, and grid-in questions. Bubble-in and grid-in answer sections are provided on the master.
- The **Cumulative Review** provides students an opportunity to reinforce and retain skills as they proceed through their study of advanced mathematics. It can also be used as a test. The master includes free-response questions.

## Answers

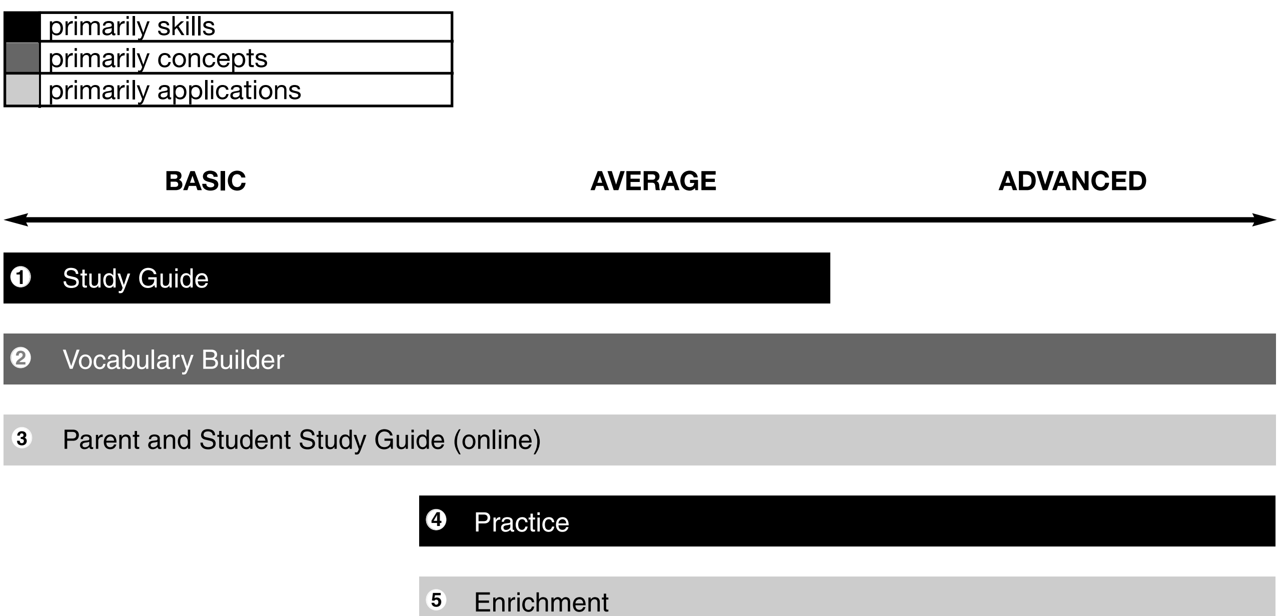
- Page A1 is an answer sheet for the SAT and ACT Practice questions that appear in the Student Edition on page 755. Page A2 is an answer sheet for the SAT and ACT Practice master. These improve students' familiarity with the answer formats they may encounter in test taking.
- The answers for the lesson-by-lesson masters are provided as reduced pages with answers appearing in red.
- Full-size answer keys are provided for the assessment options in this booklet.

# Chapter 11 Leveled Worksheets

Glencoe's **leveled worksheets** are helpful for meeting the needs of every student in a variety of ways. These worksheets, many of which are found in the **FAST FILE Chapter Resource Masters**, are shown in the chart below.

- **Study Guide** masters provide worked-out examples as well as practice problems.
- Each chapter's **Vocabulary Builder** master provides students the opportunity to write out key concepts and definitions in their own words.
- **Practice** masters provide average-level problems for students who are moving at a regular pace.
- **Enrichment** masters offer students the opportunity to extend their learning.

## Five Different Options to Meet the Needs of Every Student in a Variety of Ways



Chapter 11

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# Reading to Learn Mathematics

### *Vocabulary Builder*

**This is an alphabetical list of the key vocabulary terms you will learn in Chapter 11. As you study the chapter, complete each term's definition or description. Remember to add the page number where you found the term.**

| Vocabulary Term | Found on Page | Definition/Description/Example |
|---|---|---|
| antiln $x$ | | |
| antilogarithm | | |
| characteristic | | |
| common logarithm | | |
| doubling time | | |
| exponential decay | | |
| exponential function | | |
| exponential growth | | |
| linearizing data | | |
| ln $x$ | | |

*(continued on the next page)*

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# Reading to Learn Mathematics

***Vocabulary Builder*** *(continued)*

| Vocabulary Term | Found on Page | Definition/Description/Example |
|---|---|---|
| logarithm | | |
| logarithmic function | | |
| mantissa | | |
| natural logarithm | | |
| nonlinear regression | | |
| power function | | |
| scientific notation | | |

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# Study Guide

## Rational Exponents

***Example 1*** **Simplify each expression.**

**a.** $\left(\dfrac{c^5d^3}{c^3d^2}\right)^{\frac{1}{2}}$

$$\left(\frac{c^5d^3}{c^3d^2}\right)^{\frac{1}{2}} = (c^2d)^{\frac{1}{2}} \qquad \frac{a^m}{a^n} = a^{m-n}$$
$$= c^{\frac{2}{2}}d^{\frac{1}{2}} \qquad (a^m)^n = a^{mn}$$
$$= |c|\sqrt{d}$$

**b.** $\left(\dfrac{p^2}{q^3}\right)^{-3}$

$$\left(\frac{p^2}{q^3}\right)^{-3} = \frac{p^{-6}}{q^{-9}} \qquad \left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$$
$$= \frac{q^9}{p^6} \qquad b^{-n} = \frac{1}{b^n}$$

***Example 2*** **Evaluate each expression.**

**a.** $64^{\frac{2}{3}}$

$$64^{\frac{2}{3}} = (4^3)^{\frac{2}{3}} \qquad 64 = 4^3$$
$$= 4^2 \text{ or } 16 \qquad (a^m)^n = a^{mn}$$

**b.** $\dfrac{27^{\frac{2}{3}}}{27^{\frac{1}{3}}}$

$$\frac{27^{\frac{2}{3}}}{27^{\frac{1}{3}}} = 27^{\frac{2}{3}-\frac{1}{3}} \qquad \frac{a^m}{a^n} = a^{m-n}$$
$$= 27^{\frac{1}{3}} \text{ or } 3$$

**c.** $\sqrt{35} \cdot \sqrt{10}$

$$\sqrt{35} \cdot \sqrt{10} = 35^{\frac{1}{2}} \cdot 10^{\frac{1}{2}} \qquad \sqrt[n]{b} = b^{\frac{1}{n}}$$
$$= (7 \cdot 5)^{\frac{1}{2}} \cdot (5 \cdot 2)^{\frac{1}{2}}$$
$$= 7^{\frac{1}{2}} \cdot 5^{\frac{1}{2}} \cdot 5^{\frac{1}{2}} \cdot 2^{\frac{1}{2}} \qquad (ab)^m = a^mb^m$$
$$= 7^{\frac{1}{2}} \cdot 5 \cdot 2^{\frac{1}{2}} \qquad a^ma^n = a^{m+n}$$
$$= 5 \cdot \sqrt{7} \cdot \sqrt{2}$$
$$= 5\sqrt{14} \qquad \sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$$

***Example 3*** **Express $\sqrt[3]{8x^6y^{12}}$ using rational exponents.**

$$\sqrt[3]{8x^6y^{12}} = (8x^6y^{12})^{\frac{1}{3}} \qquad b^{\frac{1}{n}} = \sqrt[n]{b}$$
$$= 8^{\frac{1}{3}}x^{\frac{6}{3}}y^{\frac{12}{3}} \qquad (ab)^m = a^mb^m$$
$$= 2x^2y^4$$

***Example 4*** **Express $16x^{\frac{3}{4}}y^{\frac{1}{2}}$ using radicals.**

$$16x^{\frac{3}{4}}y^{\frac{1}{2}} = 16(x^3y^2)^{\frac{1}{4}} \qquad (ab)^m = a^mb^m$$
$$= 16\sqrt[4]{x^3y^2}$$

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# Practice

## Rational Exponents

***Evaluate each expression.***

**1.** $\dfrac{8^{\frac{2}{3}}}{8^{\frac{1}{3}}}$

**2.** $\left(\frac{4}{5}\right)^{-2}$

**3.** $343^{\frac{2}{3}}$

**4.** $\sqrt[3]{8^3}$

**5.** $\sqrt{5} \cdot \sqrt{10}$

**6.** $9^{-\frac{1}{2}}$

***Simplify each expression.***

**7.** $(5n^3)^2 \cdot n^{-6}$

**8.** $\left(\dfrac{x^2}{4y^{-2}}\right)^{-\frac{1}{2}}$

**9.** $(64x^6)^{\frac{1}{3}}$

**10.** $(5x^6y^4)^{\frac{1}{2}}$

**11.** $\sqrt{x^2y^3} \cdot \sqrt{x^3y^4}$

**12.** $\left(\dfrac{p^{6a}}{p^{-3a}}\right)^{\frac{1}{3}}$

***Express each using rational exponents.***

**13.** $\sqrt{x^5y^6}$

**14.** $\sqrt[5]{27x^{10}y^5}$

**15.** $\sqrt{144x^6y^{10}}$

**16.** $21\sqrt[3]{c^7}$

**17.** $\sqrt{1024a^3}$

**18.** $\sqrt[4]{36a^8b^5}$

***Express each using radicals.***

**19.** $64^{\frac{1}{3}}$

**20.** $2^{\frac{1}{2}}a^{\frac{3}{2}}b^{\frac{5}{2}}$

**21.** $s^{\frac{2}{3}}t^{\frac{1}{3}}v^{\frac{2}{3}}$

**22.** $y^{\frac{3}{2}}$

**23.** $x^{\frac{2}{5}}y^{\frac{3}{5}}$

**24.** $(x^6y^3)^{\frac{1}{2}}\,z^{\frac{3}{2}}$

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# Enrichment

## Look for Cases

Many problems can be partitioned into a few cases. The cases are classifications within the problem that are exclusive one to another but which taken together comprise all the possibilities in the problem. Divisions into cases are made where the characteristics of the problem are most critical.

***Example*** **If $a > b$ and $\frac{1}{a} > \frac{1}{b}$, what must be true of $a$ and $b$?**

One of the critical characteristics of inequalities is that changing the signs of the left and right sides requires changing the sense of the inequality. This suggests classifying $a$ and $b$ according to signs. Since neither $a$ nor $b$ can equal zero in the second inequality and $a > b$, the cases are:

(i) $a$ and $b$ are both positive.
(ii) $a$ and $b$ are both negative.
(iii) $a$ is positive and $b$ is negative.

An important characteristic of reciprocals is that 1 and –1 are their own reciprocals. Therefore, within each of the above cases consider examples of these three subclassifications:

(A) $a$ and $b$ are both greater than 1 (or < –1).
(B) $a$ and $b$ are both fractions whose absolute values are less than one.
(C) $a$ is greater than 1 (or –1) and $b$ is less than 1 (or –1).

For case (i), consider these examples.

(A) $3 > 2 \rightarrow \frac{1}{3} < \frac{1}{2}$ (B) $\frac{1}{4} > \frac{1}{5} \rightarrow 4 < 5$ (C) $\frac{3}{2} > \frac{3}{4} \rightarrow \frac{2}{3} < \frac{4}{3}$

For case (ii), consider these examples.

(A) $-2 > -3 \rightarrow -\frac{1}{2} < -\frac{1}{3}$ (B) $-\frac{1}{5} > -\frac{1}{4} \rightarrow -5 < -4$ (C) $-\frac{3}{4} > -\frac{3}{2} \rightarrow -\frac{4}{3} < -\frac{2}{3}$

For case (iii), consider these examples.

(A) $2 > -3 \rightarrow \frac{1}{2} > -\frac{1}{3}$ (B) $\frac{1}{5} > -\frac{1}{4} \rightarrow 5 > -4$ (C) $\frac{3}{4} > -\frac{3}{2} \rightarrow \frac{4}{3} > -\frac{2}{3}$

Notice that the second given inequality, $\frac{1}{a} > \frac{1}{b}$, only holds true in case (iii). We conclude that $a$ must be positive and $b$ negative to satisfy both given inequalities.

***Complete.***

**1.** Find the positive values of $b$ such that $b^{x_1} > b^{x_2}$ whenever $x_1 < x_2$.

**2.** Solve $\frac{x}{x+1} > 0$.

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# Study Guide

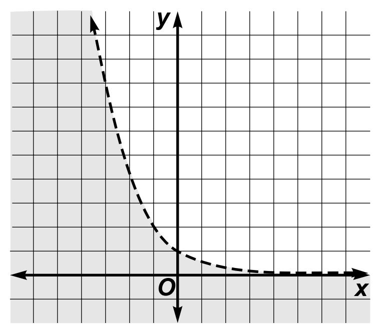
## Exponential Functions

Functions of the form $y = b^x$, in which the base $b$ is a positive real number and the exponent is a variable, are known as **exponential functions.** Many real-world situations can be modeled by exponential functions. The equation $N = N_0(1 + r)^t$, where $N$ is the final amount, $N_0$ is the initial amount, $r$ is the rate of growth or decay, and $t$ is time, is used for modeling exponential growth. The compound interest equation is $A = P\left(1 + \frac{r}{n}\right)^{nt}$, where $P$ is the principal or initial investment, $A$ is the final amount of the investment, $r$ is the annual interest rate, $n$ is the number of times interest is compounded each year, and $t$ is the number of years.

***Example 1*** **Graph $y < 2^{-x}$.**

First, graph $y = 2^{-x}$. This graph is a function, since there is a unique $y$-value for each $x$-value.

| $x$ | −3 | −2 | −1 | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|---|---|
| $2^{-x}$ | 8 | 4 | 2 | 1 | $\frac{1}{2}$ | $\frac{1}{4}$ | $\frac{1}{8}$ | $\frac{1}{16}$ |




Since the points on this curve are not in the solution of the inequality, the graph of $y = 2^{-x}$ is shown as a dashed curve.

Then, use (0, 0) as a test point to determine which area to shade.

$y < 2^{-x}$

$0 < 2^0$

$0 < 1$

Since (0,0) satisfies the inequality, the region that contains (0,0) should be shaded.

***Example 2*** ***Biology*** **Suppose a researcher estimates that the initial population of a colony of cells is 100. If the cells reproduce at a rate of 25% per week, what is the expected population of the colony in six weeks?**

$N = N_0(1 + r)^t$

$N = 100(1 + 0.25)^6$ *$N_0 = 100, r = 0.25, t = 6$*

$N \approx 381.4697266$ *Use a calculator.*

There will be about 381 cells in the colony in 6 weeks.

***Example 3*** ***Finance*** **Determine the amount of money in a money market account that provides an annual rate of 6.3% compounded quarterly if \$1700 is invested and left in the account for eight years.**

$A = P\left(1 + \frac{r}{n}\right)^{nt}$

$A = 1700\left(1 + \frac{0.063}{4}\right)^{4\cdot 8}$ *$P = 1700, r = 0.063, n = 4, t = 8$*

$A \approx 2803.028499$ *Use a calculator.*

After 8 years, the \$1700 investment will have a value of \$2803.03.

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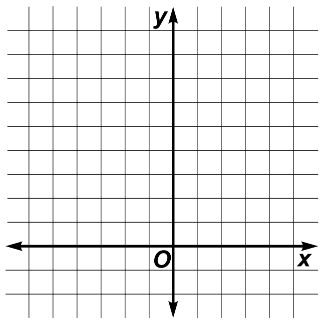
# Practice

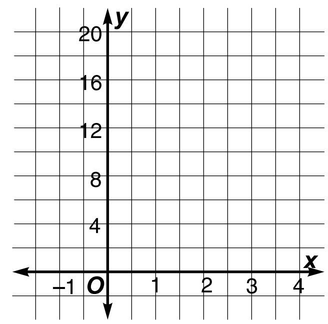
## Exponential Functions

***Graph each exponential function or inequality.***

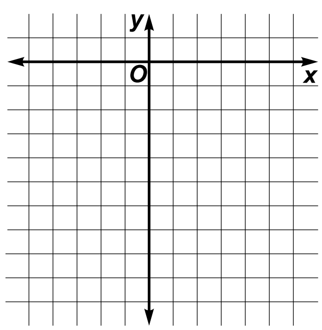
**1.** $y = 2^{x-1}$



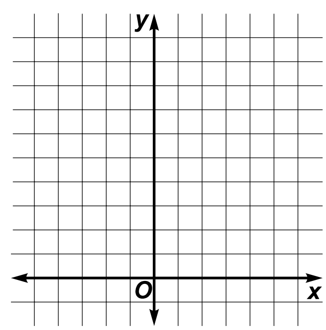

**2.** $y = 4^{-x+2}$




**3.** $y > -3^x + 1$




**4.** $y \geq 0.5^x$




**5.** ***Demographics*** An area in North Carolina known as The Triangle is principally composed of the cities of Durham, Raleigh, and Chapel Hill. The Triangle had a population of 700,000 in 1990. The average yearly rate of growth is 5.9%. Find the projected population for 2010.

**6.** ***Finance*** Determine the amount of money in a savings account that provides an annual rate of 4% compounded monthly if the initial investment is $1000 and the money is left in the account for 5 years.

**7.** ***Investments*** How much money must be invested by Mr. Kaufman if he wants to have $20,000 in his account after 15 years? He can earn 5% compounded quarterly.

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# Enrichment

## Finding Solutions of $x^y = y^x$

Perhaps you have noticed that if $x$ and $y$ are interchanged in equations such as $x = y$ and $xy = 1$, the resulting equation is equivalent to the original equation. The same is true of the equation $x^y = y^x$. However, finding solutions of $x^y = y^x$ and drawing its graph is not a simple process.

***Solve each problem. Assume that x and y are positive real numbers***

**1**. If $a > 0$, will $(a, a)$ be a solution of $x^y = y^x$? Justify your answer.

**2.** If $c > 0$, $d > 0$, and $(c, d)$ is a solution of $x^y = y^x$, will $(d, c)$ also be a solution? Justify your answer.

**3.** Use 2 as a value for $y$ in $x^y = y^x$. The equation becomes $x^2 = 2^x$.

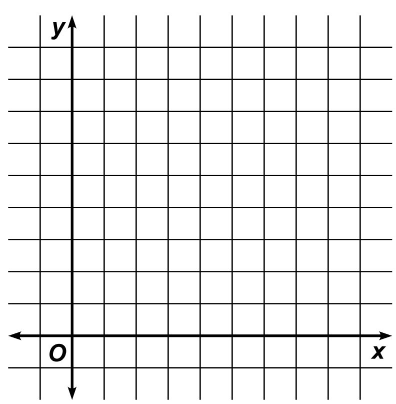
**a.** Find equations for two functions, $f(x)$ and $g(x)$ that you could graph to find the solutions of $x^2 = 2^x$. Then graph the functions on a separate sheet of graph paper.

**b.** Use the graph you drew for part **a** to state two solutions for $x^2 = 2^x$. Then use these solutions to state two solutions for $x^y = y^x$.

**4.** In this exercise, a graphing calculator will be very helpful. Use the technique from Exercise 3 to complete the tables below. Then graph $x^y = y^x$ for positive values of $x$ and $y$. If there are a symptotes, show them in your diagram using dotted lines. Note that in the table, some values of $y$ call for one value of $x$, others call for two.

| x | y |
|---|---|
| | $\frac{1}{2}$ |
| | $\frac{3}{4}$ |
| | 1 |
| | 2 |
| | 2 |
| | 3 |
| | 3 |

| x | y |
|---|---|
| | 4 |
| | 4 |
| | 5 |
| | 5 |
| | 8 |
| | 8 |



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# Study Guide

## The Number *e*

The number $e$ is a special irrational number with an approximate value of 2.718 to three decimal places. The formula for exponential growth or decay is $N = N_0e^{kt}$, where $N$ is the final amount, $N_0$ is the initial amount, $k$ is a constant, and $t$ is time. The equation $A = Pe^{rt}$, where $P$ is the initial amount, $A$ is the final amount, $r$ is the annual interest rate, and $t$ is time in years, is used for calculating interest that is compounded continuously.

**Example 1** ***Demographics*** **The population of Dubuque, Iowa, declined at a rate of 0.4% between 1997 1998. In 1998, the population was 87,806.**

**a. Let $t$ be the number of years since 1998 and write a function to model the population.**

**b. Suppose that the rate of decline remains steady at 0.4%. Find the projected population of Dubuque in 2010.**

**a.** $y = ne^{kt}$

$y = 87{,}806e^{-0.004t}$ *$n = 87{,}806$; $k = -0.004$*

**b.** In 2010, it will have been 2010 − 1998 or 12 years since the initial population figure. Thus, $t = 12$.

$y = 87{,}806e^{-0.004t}$

$y = 87{,}806e^{-0.004(12)}$ *$t = 12$*

$y \approx 83690.86531$ *Use a calculator.*

Given a population of 87,806 in 1998 and a steady rate of decline of 0.4%, the population of Dubuque, Iowa, will be approximately 83,691 in 2010.

**Example 2** ***Finance*** **Compare the balance after 10 years of a \$5000 investment earning 8.5% interest compounded continuously to the same investment compounded quarterly.**

In both cases, $P = 5000$, $r = 0.085$, and $t = 10$. When the interest is compounded quarterly, $n = 4$. Use a calculator to evaluate each expression.

**Continuously**

$A = Pe^{rt}$

$A = 5000e^{(0.085)(10)}$

$A = 11{,}698.23$

**Quarterly**

$A = P\left(1 + \frac{r}{n}\right)^{nt}$

$A = 5000\left(1 + \frac{0.085}{4}\right)^{4\cdot 10}$

$A = 11{,}594.52$

You would earn \$11,698.23 − \$11,594.52 = \$103.71 more by choosing the account that compounds continuously.

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11-3

# Practice

## The Number $e$

1. ***Demographics*** In 1995, the population of Kalamazoo, Michigan, was 79,089. This figure represented a 0.4% annual decline from 1990.
   **a.** Let $t$ be the number of years since 1995 and write a function that models the population in Kalamazoo in 1995.

   **b.** Predict the population in 2010 and 2015. Assume a steady rate of decline.

2. ***Biology*** Suppose a certain type of bacteria reproduces according to the model $P(t) = 100e^{0.271t}$, where $t$ is time in hours.
   **a.** At what rate does this type of bacteria reproduce?

   **b.** What was the initial number of bacteria?

   **c.** Find the number of bacteria at $P(5)$, $P(10)$, $P(24)$, and $P(72)$. Round to the nearest whole number.

3. ***Finance*** Suppose Karyn deposits $1500 in a savings account that earns 6.75% interest compounded continuously. She plans to withdraw the money in 6 years to make a $2500 down payment on a car. Will there be enough funds in Karyn's account in 6 years to meet her goal?

4. ***Banking*** Given the original principal, the annual interest rate, the amount of time for each investment, and the type of compounded interest, find the amount at the end of the investment.
   **a.** $P = \$1250$, $r = 8.5\%$, $t = 3$ years, semiannually

   **b.** $P = \$2575$, $r = 6.25\%$, $t = 5$ years 3 months, continuously

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# Enrichment

## Approximations for $\pi$ and $e$

The following expression can be used to approximate $e$. If greater and greater values of $n$ are used, the value of the expression approximates $e$ more and more closely.

$$\left(1 + \frac{1}{n}\right)^n$$

Another way to approximate $e$ is to use this infinite sum. The greater the value of $n$, the closer the approximation.

$$e = 1 + 1 + \frac{1}{2} + \frac{1}{2 \cdot 3} + \frac{1}{2 \cdot 3 \cdot 4} + \ldots + \frac{1}{2 \cdot 3 \cdot 4 \cdot \ldots \cdot n} + \ldots$$

In a similar manner, $\pi$ can be approximated using an infinite product discovered by the English mathematician John Wallis (1616-1703).

$$\frac{\pi}{2} = \frac{2}{1} \cdot \frac{2}{3} \cdot \frac{4}{3} \cdot \frac{4}{5} \cdot \frac{6}{5} \cdot \frac{6}{7} \ldots \frac{2n}{2n-1} \cdot \frac{2n}{2n+1} \ldots$$

***Solve each problem.***

**1.** Use a calculator with an $e^x$ key to find $e$ to 7 decimal places.

**2.** Use the expression $\left(1 + \frac{1}{n}\right)^n$ to approximate $e$ to 3 decimal places. Use 5, 100, 500, and 7000 as values of $n$.

**3.** Use the infinite sum to approximate $e$ to 3 decimal places. Use the whole numbers from 3 through 6 as values of $n$.

**4.** Which approximation method approaches the value of $e$ more quickly?

**5.** Use a calculator with a $\pi$ key to find $\pi$ to 7 decimal places.

**6.** Use the infinite product to approximate $\pi$ to 3 decimal places. Use the whole numbers from 3 through 6 as values of $n$.

**7.** Does the infinite product give good approximations for $\pi$ quickly?

**8.** Show that $\pi^4 + \pi^5$ is equal to $e^6$ to 4 decimal places.

**9.** Which is larger, $e^{\pi}$ or $\pi^e$?

**10.** The expression $x^{\frac{1}{x}}$ reaches a maximum value at $x = e$. Use this fact to prove the inequality you found in Exercise 9.

 

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11-4

# Study Guide

## Logarithmic Functions

In the function $x = a^y$, $y$ is called the **logarithm** of $x$. It is usually written as $y = \log_a x$ and is read "$y$ equals the log, base $a$, of $x$." Knowing that if $a^u = a^v$ then $u = v$, you can evaluate a logarithmic expression to determine its logarithm.

***Example 1*** **Write $\log_7 49 = 2$ in exponential form.**

The base is 7 and the exponent is 2.
$7^2 = 49$

***Example 2*** **Write $2^5 = 32$ in logarithmic form.**

The base is 2, and the exponent or logarithm is 5.
$\log_2 32 = 5$

***Example 3*** **Evaluate the expression $\log_5 \frac{1}{25}$.**

Let $x = \log_5 \frac{1}{25}$.

| | |
|---|---|
| $x = \log_5 \frac{1}{25}$ | |
| $5^x = \frac{1}{25}$ | *Definition of logarithm.* |
| $5^x = (25)^{-1}$ | $a^{-m} = \frac{1}{a^m}$ |
| $5^x = (5^2)^{-1}$ | $5^2 = 25$ |
| $5^x = 5^{-2}$ | $(a^m)^n = a^{mn}$ |
| $x = -2$ | *If* $a^u = a^v$, *then* $u = v$. |

***Example 4*** **Solve each equation.**

**a. $\log_6 (4x + 6) = \log_6 (8x - 2)$**

| | |
|---|---|
| $\log_6 (4x + 6) = \log_6 (8x - 2)$ | |
| $4x + 6 = 8x - 2$ | *If* $\log_b m = \log_b n$, *then* $m = n$. |
| $-4x = -8$ | |
| $x = 2$ | |

**b. $\log_9 x + \log_9 (x - 2) = \log_9 3$**

| | |
|---|---|
| $\log_9 x + \log_9 (x - 2) = \log_9 3$ | |
| $\log_9 [x(x - 2)] = \log_9 3$ | $\log_b mn = \log_b m + \log_b n$ |
| $x^2 - 2x = 3$ | *If* $\log_b m = \log_b n$, *then* $m = n$. |
| $x^2 - 2x - 3 = 0$ | |
| $(x - 3)(x + 1) = 0$ | *Factor.* |
| $x - 3 = 0$ or $x + 1 = 0$ | *Find the zeros.* |
| $x = 3$ or $x = -1$. | |

The log of a negative value does not exist, so the answer is $x = 3$.

11-4

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# Practice

## Logarithmic Functions

***Write each equation in exponential form.***

**1.** $\log_3 81 = 4$

**2.** $\log_8 2 = \frac{1}{3}$

**3.** $\log_{10} \frac{1}{100} = -2$

***Write each equation in logarithmic form.***

**4.** $3^3 = 27$

**5.** $5^{-3} = \frac{1}{125}$

**6.** $\left(\frac{1}{4}\right)^{-4} = 256$

***Evaluate each expression.***

**7.** $\log_7 7^3$

**8.** $\log_{10} 0.001$

**9.** $\log_8 4096$

**10.** $\log_4 32$

**11.** $\log_3 1$

**12.** $\log_6 \frac{1}{216}$

***Solve each equation.***

**13.** $\log_x 64 = 3$

**14.** $\log_4 0.25 = x$

**15.** $\log_4 (2x - 1) = \log_4 16$

**16.** $\log_{10} \sqrt{10} = x$

**17.** $\log_7 56 - \log_7 x = \log_7 4$

**18.** $\log_5 (x + 4) + \log_5 x = \log_5 12$

**19.** ***Chemistry*** How long would it take 100,000 grams of radioactive iodine, which has a half-life of 60 days, to decay to 25,000 grams? Use the formula $N = N_0\left(\frac{1}{2}\right)^t$, where $N$ is the final amount of the substance, $N_0$ is the initial amount, and $t$ represents the number of half-lives.

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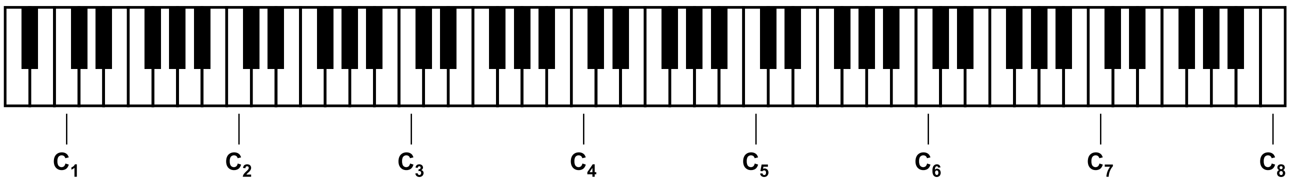
# Enrichment

## Musical Relationships

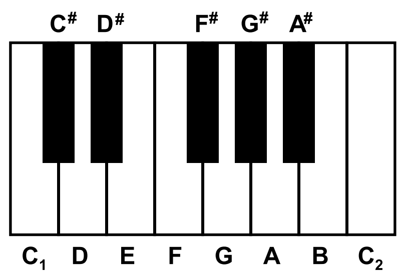
The frequencies of notes in a musical scale that are one octave apart are related by an exponential equation. For the eight C notes on a piano, the equation is $C_n = C_1 2^{n-1}$, where $C_n$ represents the frequency of note $C_n$.




**1.** Find the relationship between $C_1$ and $C_2$.

**2.** Find the relationship between $C_1$ and $C_4$.

The frequencies of consecutive notes are related by a common ratio *r*. The general equation is $f_n = f_1 r^{n-1}$.

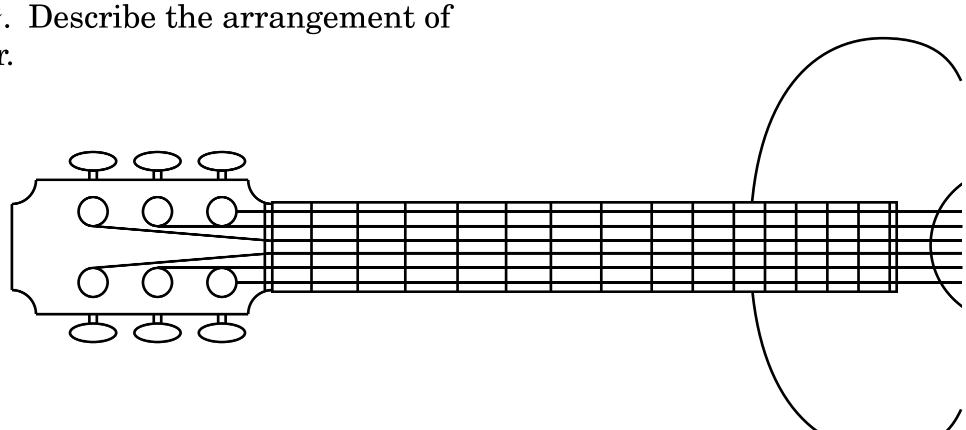



**3.** If the frequency of middle C is 261.6 cycles per second and the frequency of the next higher C is 523.2 cycles per second, find the common ratio *r*. (Hint: The two Cs are 12 notes apart.) Write the answer as a radical expression.

**4.** Substitute decimal values for *r* and $f_1$ to find a specific equation for $f_n$.

**5.** Find the frequency of F# above middle C.

**6.** The frets on a guitar are spaced so that the sound made by pressing a string against one fret has about 1.0595 times the wavelength of the sound made by using the next fret. The general equation is $w_n = w_0 (1.0595)^n$. Describe the arrangement of the frets on a guitar.



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11-5

# Study Guide

## Common Logarithms

Logarithms with base 10 are called **common logarithms.** The change of base formula, $\log_a n = \frac{\log_b n}{\log_b a}$, where $a$, $b$, and $n$ are positive numbers and neither $a$ nor $b$ is 1, allows you to evaluate logarithms in other bases with a calculator. Logarithms can be used to solve **exponential equations.**

***Example 1*** **Evaluate each expression.**

**a.** $\mathbf{\log 8(3)^2}$

$$\log 8(3)^2 = \log 8 + 2\log 3 \quad \textit{log ab = log a + log b, log } b^n = n \textit{ log b}$$
$$\approx 0.9031 + 2(0.4771) \quad \textit{Use a calculator.}$$
$$\approx 0.9031 + 0.9542$$
$$\approx 1.8573$$

**b.** $\mathbf{\log \frac{15^3}{7}}$

$$\log \frac{15^3}{7} = 3\log 15 - \log 7 \quad \log \frac{a}{b} = \log a - \log b,\ \log a^m = m \log a$$
$$\approx 3(1.1761) - 0.8451 \quad \textit{Use a calculator.}$$
$$\approx 3.5283 - 0.8451$$
$$\approx 2.6832$$

***Example 2*** **Find the value of $\log_8 2037$ using the change of base formula.**

$$\log_8 2037 = \frac{\log_{10} 2037}{\log_{10} 8} \quad \log_a n = \frac{\log_b n}{\log_b a}$$
$$\approx \frac{3.3090}{0.9031} \quad \textit{Use a calculator.}$$
$$\approx 3.6641$$

***Example 3*** **Solve $7^{2x} = 93$.**

$$7^{2x} = 93$$
$$\log 7^{2x} = \log 93 \quad \textit{Take the logarithm of each side.}$$
$$2x \log 7 = \log 93 \quad \log_b m^p = p \cdot \log_b m$$
$$2x = \frac{\log 93}{\log 7} \quad \textit{Divide each side by log 7.}$$
$$2x \approx 2.3293 \quad \textit{Use a calculator.}$$
$$x \approx 1.1646$$

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11-5

# Practice

## Common Logarithms

***Given that log 3 = 0.4771, log 5 = 0.6990, and log 9 = 0.9542, evaluate each logarithm.***

**1.** log 300,000 **2.** log 0.0005 **3.** log 9000

**4.** log 27 **5.** log 75 **6.** log 81

***Evaluate each expression.***

**7.** log 66.3 **8.** $\log \frac{17^4}{5}$ **9.** $\log 7(4^3)$

***Find the value of each logarithm using the change of base formula.***

**10.** $\log_6 832$ **11.** $\log_{11} 47$ **12.** $\log_3 9$

***Solve each equation or inequality.***

**13.** $8^x = 10$ **14.** $2.4^x \leq 20$ **15.** $1.8^{x-5} = 19.8$

**16.** $3^{5x} = 85$ **17.** $4^{2x} > 25$ **18.** $3^{2x-2} = 2^x$

**19.** ***Seismology*** The intensity of a shock wave from an earthquake is given by the formula $R = \log_{10} \frac{I}{I_0}$, where $R$ is the magnitude, $I$ is a measure of wave energy, and $I_0 = 1$. Find the intensity per unit of area for the following earthquakes.

**a.** Northridge, California, in 1994, $R = 6.7$

**b.** Hector Mine, California, in 1999, $R = 7.1$

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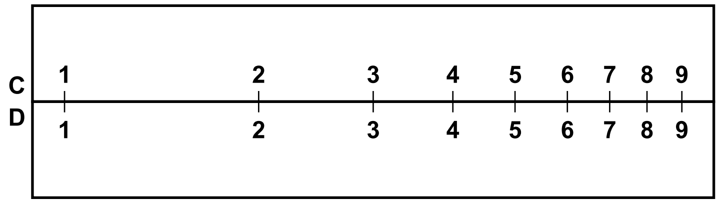
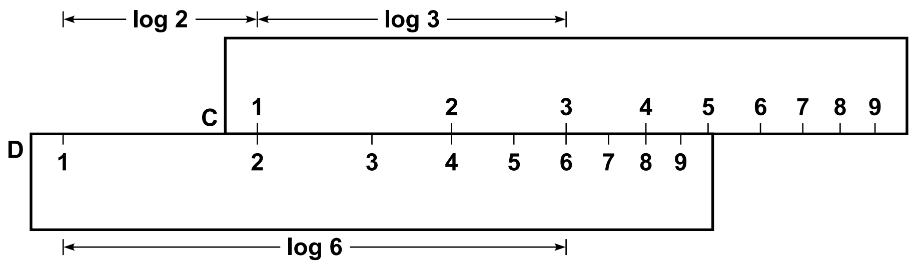
# Enrichment

## The Slide Rule

Before the invention of electronic calculators, computations were often performed on a slide rule. A slide rule is based on the idea of logarithms. It has two movable rods labeled with C and D scales. Each of the scales is logarithmic.

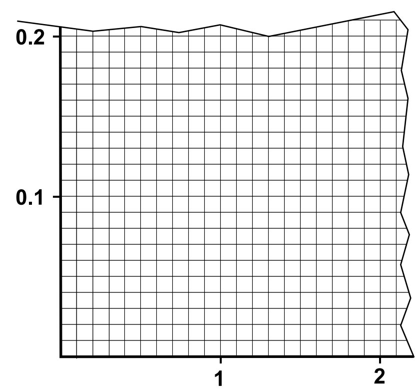



To multiply $2 \times 3$ on a slide rule, move the C rod to the right as shown below. You can find $2 \times 3$ by adding log 2 to log 3, and the slide rule adds the lengths for you. The distance you get is 0.778, or the logarithm of 6.

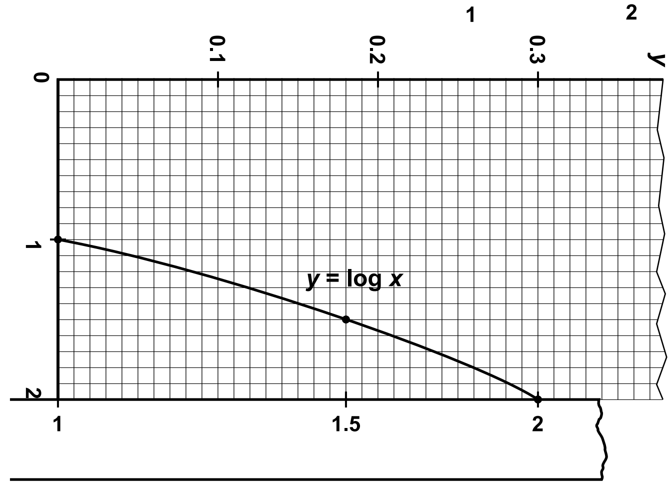



***Follow the steps to make a slide rule.***

1. Use graph paper that has small squares, such as 10 squares to the inch. Using the scales shown at the right, plot the curve $y = \log x$ for $x = 1, 1.5$, and the whole numbers from 2 through 10. Make an obvious heavy dot for each point plotted.




2. You will need two strips of cardboard. A 5-by-7 index card, cut in half the long way, will work fine. Turn the graph you made in Exercise 1 sideways and use it to mark a logarithmic scale on each of the two strips. The figure shows the mark for 2 being drawn.
3. Explain how to use a slide rule to divide 8 by 2.



# Study Guide

## Natural Logarithms

Logarithms with base $e$ are called **natural logarithms** and are usually written **ln $x$.** Logarithms with a base other than $e$ can be converted to natural logarithms using the change of base formula. The antilogarithm of a natural logarithm is written **antiln $x$.** You can use the properties of logarithms and antilogarithms to simplify and solve exponential and logarithmic equations or inequalities with natural logarithms.

***Example 1*** **Convert $\log_4 381$ to a natural logarithm and evaluate.**

$$\log_a n = \frac{\log_b n}{\log_b a}$$

$$\log_4 381 = \frac{\log_e 381}{\log_e 4} \qquad a = 4, b = e, n = 381$$

$$= \frac{\ln 381}{\ln 4} \qquad \log_e x = \ln x$$

$$\approx 4.2868 \qquad \text{Use a calculator.}$$

So, $\log_4 381$ is about 4.2868.

***Example 2*** **Solve $3.75 = -7.5 \ln x$.**

$$3.75 = -7.5 \ln x$$

$$-0.5 = \ln x \qquad \text{Divide each side by } -7.5$$

$$\text{antiln}(-0.5) = x \qquad \text{Take the antilogarithm of each side.}$$

$$0.6065 \approx x \qquad \text{Use a calculator.}$$

The solution is about 0.6065.

***Example 3*** **Solve each equation or inequality by using natural logarithms.**

**a. $4^{3x} = 6^{x+1}$**

$$4^{3x} = 6^{x+1}$$

$$\ln 4^{3x} = \ln 6^{x+1} \qquad \text{Take the natural logarithm of each side.}$$

$$3x \ln 4 = (x + 1) \ln 6 \qquad \ln a^n = n \ln a$$

$$3x(1.3863) = (x + 1)(1.7918) \qquad \text{Use a calculator.}$$

$$4.1589x = 1.7918x + 1.7918$$

$$2.3671x = 1.7918$$

$$x \approx 0.7570$$

**b. $25 > e^{0.2t}$**

$$25 > e^{0.2t}$$

$$\ln 25 > \ln e^{0.2t} \qquad \text{Take the natural logarithm of each side.}$$

$$\ln 25 > 0.2t \ln e \qquad \ln a^n = n \ln a$$

$$3.2189 > 0.2t \qquad \text{Use a calculator.}$$

$$16.0945 > t$$

Thus, $t < 16.0945$

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# 11-6 Practice

## Natural Logarithms

***Evaluate each expression.***

**1.** ln 71 **2.** ln 8.76 **3.** ln 0.532

**4.** antiln −0.256 **5.** antiln 4.62 **6.** antiln −1.62

***Convert each logarithm to a natural logarithm and evaluate.***

**7.** $\log_7 94$ **8.** $\log_5 256$ **9.** $\log_9 0.712$

***Use natural logarithms to solve each equation or inequality.***

**10.** $6^x = 42$ **11.** $7^x = 4^{x+3}$ **12.** $1249 = 175e^{-0.04t}$

**13.** $10^{x+1} > 3^x$ **14.** $12 < e^{0.048y}$ **15.** $8.4 < e^{t-2}$

**16.** ***Banking*** Ms. Cubbatz invested a sum of money in a certificate of deposit that earns 8% interest compounded continuously. The formula for calculating interest that is compounded continuously is $A = Pe^{rt}$. If Ms. Cubbatz made the investment on January 1, 1995, and the account was worth $12,000 on January 1, 1999, what was the original amount in the account?

11-6

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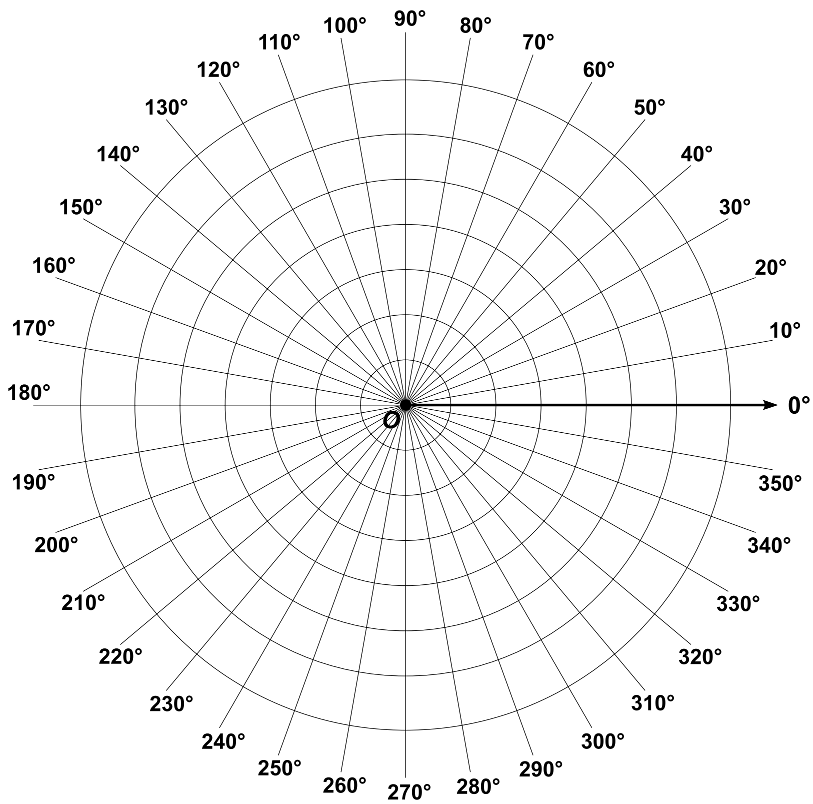
# Enrichment

## Spirals

Consider an angle in standard position with its vertex at a point $O$ and its initial side on a polar axis. Remember that point $P$ on the terminal side of the angle can be named by $(r, \theta)$, where $r$ is the directed distance of the point from $O$ and $\theta$ is the measure of the angle. As you learned, graphs in this system may be drawn on polar coordinate paper such as the kind shown below.




**1.** Use a calculator to complete the table for $\log_2 r = \frac{\theta}{120}$. (To find $\log_2 a$ on a calculator, press LOG $a$ ÷ LOG 2). Round $\theta$ to the nearest degree if necessary.

| *r* | | 1 | 2 | 3 | 4 | 5 | 6 | 7 8 |
|---|---|---|---|---|---|---|---|---|
| *θ* | | | | | | | | |

**2.** Plot the points found in Exercise 1 on the grid above and connect them to form a smooth curve.

This type of spiral is called a *logarithmic spiral* because the angle measures are proportional to the logarithms of the radii.

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# Study Guide

## Modeling Real-World Data with Exponential and Logarithmic Functions

The **doubling time,** or amount of time $t$ required for a quantity modeled by the exponential equation $N = N_0e^{kt}$ to double, is given by $t = \frac{\ln 2}{k}$.

***Example*** ***Finance*** **Tara's parents invested $5000 in an account that earns 11.5% compounded continuously. They would like to double their investment in 5 years to help finance Tara's college education.**

**a. Will the initial investment of $5000 double within 5 years?**

Find the doubling time for the investment. For continuously compounded interest, the constant $k$ is the interest rate written as a decimal.

$t = \frac{\ln 2}{k}$

$= \frac{\ln 2}{0.115}$ *The decimal for 11.5% is 0.115.*

$\approx 6.03$ years *Use a calculator.*

Five years is not enough time for the initial investment to double.

**b. What interest rate is required for an investment with continuously compounded interest to double in 5 years?**

$t = \frac{\ln 2}{k}$

$5 = \frac{\ln 2}{k}$

$\frac{1}{5} = \frac{k}{\ln 2}$ *Take the reciprocal of each side.*

$\frac{\ln 2}{5} = k$ *Multiply each side by ln 2 to solve for k.*

$0.1386 \approx k$

An interest rate of 13.9% is required for an investment with continuously compounded interest to double in 5 years.

11-7

# Practice

## Modeling Real-World Data with Exponential and Logarithmic Functions

***Find the amount of time required for an amount to double at the given rate if the interest is compounded continuously.***

**1.** 4.75%

**2.** 6.25%

**3.** 5.125%

**4.** 7.1%

**5.** ***City Planning*** At a recent town council meeting, proponents of increased spending claimed that spending should be doubled because the population of the city would double over the next three years. Population statistics for the city indicate that population is growing at the rate of 16.5% per year. Is the claim that the population will double in three years correct? Explain.

**6.** ***Conservation*** A wildlife conservation group released 14 black bears into a protected area. Their goal is to double the population of black bears every 4 years for the next 12 years.

**a.** If they are to meet their goal at the end of the first four years, what should be the yearly rate of increase in population?

**b.** Suppose the group meets its goal. What will be the minimum number of black bears in the protected area in 12 years?

**c.** What type of model would best represent such data?

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**11-7**

# Enrichment

## Hyperbolic Functions

The *hyperbolic functions* are a family of functions of great importance in calculus and higher-level mathematics. Because they are defined in terms of the hyperbola, their name is derived from that word. These functions have an interesting relationship to the number $e$ and to the trigonometric functions, uniting those seemingly unrelated subjects with the conic sections.

The hyperbolic functions can be written in terms of $e$.

Hyperbolic sine of $x$: $\sinh x = \dfrac{e^x - e^{-x}}{2}$

Hyperbolic cosine of $x$: $\cosh x = \dfrac{e^x + e^{-x}}{2}$

Hyperbolic tangent of $x$: $\tanh x = \dfrac{\sinh x}{\cosh x} = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$

Identities involving hyperbolic functions exhibit strong resemblances to trigonometric identities.

***Example*** **Show that $\sinh 2x = 2 \sinh x \cosh x$.**

$$\sinh 2x = \frac{e^{2x} - e^{-2x}}{2} \qquad \leftarrow \text{ Replace } x \text{ in the definition above by } 2x.$$

$$= 2\left(\frac{e^{2x} - e^{-2x}}{4}\right)$$

$$= 2\left(\frac{(e^x)^2 - (e^{-x})^2}{4}\right) \qquad \leftarrow \text{ difference of two squares}$$

$$= 2\left(\frac{e^x - e^{-x}}{2}\right)\left(\frac{e^x + e^{-x}}{2}\right)$$

$$= 2 \sinh x \cosh x$$

**1.** Find $\cosh^2 x - \sinh^2 x$.

***Prove each identity.***

**2.** $\sinh(-x) = -\sinh x$

**3.** $\sinh(x + y) = \sinh x \cosh y + \cosh x \sinh y$

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Chapter 11

# Chapter 11 Test, Form 1A

***Write the letter for the correct answer in the blank at the right of each problem.***

**1.** Evaluate $\left(9^{\frac{1}{2}} + 216^{\frac{1}{3}}\right)^{-\frac{1}{2}}$. **1.** ________

**A.** $-\frac{1}{3}$ **B.** $\frac{1}{3}$ **C.** $-3$ **D.** $3$

**2.** Simplify $\left(\frac{32x^4y^4}{4x^{-2}y}\right)^{\frac{2}{3}}$. **2.** ________

**A.** $2x^{\frac{4}{3}}y$ **B.** $4x^{\frac{4}{3}}y^2$ **C.** $4x^4y^2$ **D.** $2x^4y^2$

**3.** Express $\sqrt[3]{27x^4y^6}$ using rational exponents. **3.** ________

**A.** $3x^{\frac{4}{3}}y^2$ **B.** $9x^{\frac{4}{3}}y^2$ **C.** $9x^{\frac{3}{4}}y$ **D.** $9x^{\frac{3}{4}}y^2$

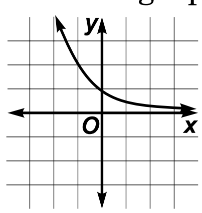
**4.** Express $(2x^2)^{\frac{1}{3}}(2x)^{\frac{1}{2}}$ using radicals. **4.** ________

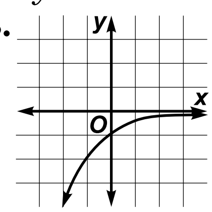
**A.** $\sqrt[6]{32x^5}$ **B.** $\sqrt[6]{4x^7}$ **C.** $x\sqrt[6]{32x}$ **D.** $x\sqrt[6]{4x}$

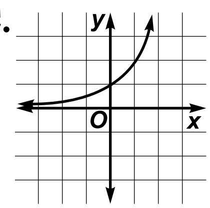
**5.** Evaluate $9^{\frac{\pi}{2}}$ to the nearest thousandth. **5.** ________

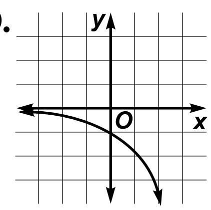
**A.** 14.137 **B.** 31.544 **C.** 497.521 **D.** 799.438

**6.** Choose the graph of $y = 2^{-x}$. **6.** ________

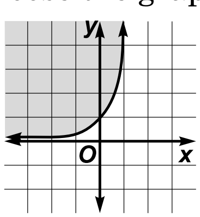
**A.**


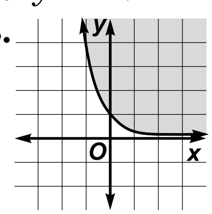

**B.**


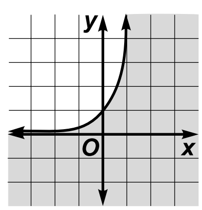

**C.**


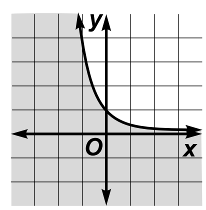

**D.**



**7.** Choose the graph of $y \leq 4^x$. **7.** ________

**A.**



**B.**



**C.**



**D.**



**8.** In 2000, the bird population in a certain area is 10,000. The number of birds increases exponentially at a rate of 9% per year. Predict the population in 2005. **8.** ________

**A.** 15,137 **B.** 15,683 **C.** 15,489 **D.** 15,771

**9.** A scientist has 86 grams of a radioactive substance that decays at an exponential rate. Assuming $k = -0.4$, how many grams of radioactive substance remain after 10 days? **9.** ________

**A.** 21.5 g **B.** 15.8 g **C.** 3.7 g **D.** 1.6 g

**10.** Write $3^{-2} = \frac{1}{9}$ in logarithmic form. **10.** ________

**A.** $\log_3(-2) = \frac{1}{9}$ **B.** $\log_3 \frac{1}{9} = -2$ **C.** $\log_{-2} \frac{1}{9} = 3$ **D.** $\log_{-2} 3 = \frac{1}{9}$

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# Chapter 11 Test, Form 1A (continued)

**11.** Evaluate $\log_9 \frac{1}{27}$. **11.** ________

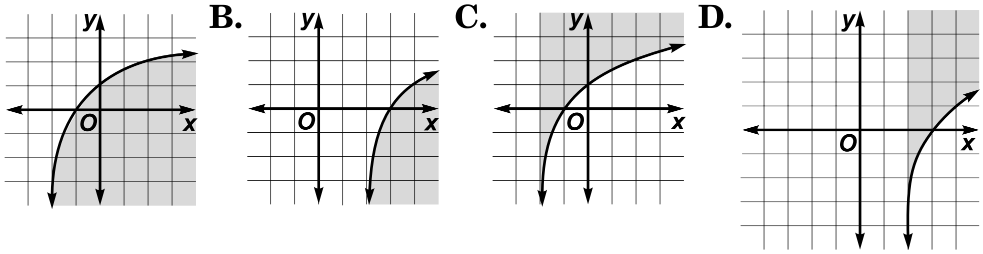
**A.** $\frac{2}{3}$ **B.** $\frac{3}{2}$ **C.** $-\frac{2}{3}$ **D.** $-\frac{3}{2}$

**12.** Solve $\log_4 x + \log_4 (x - 2) = \log_4 15$. **12.** ________

**A.** $-3$ only **B.** 5 only **C.** $-3$ or 5 **D.** $-5$ or 3

**13.** Choose the graph of $y \leq \log_2 (x + 2)$. **13.** ________

**A.** **B.** **C.** **D.**




**14.** Find the value of $\log_6 27.5$ using the change of base formula. **14.** ________

**A.** 0.661 **B.** 1.439 **C.** 1.850 **D.** 2.232

**15.** Solve $5^x = 3^{x+2}$ using common logarithms. **15.** ________

**A.** 2.732 **B.** 3.109 **C.** 4.117 **D.** 4.301

**16.** The pH of a water supply is 7.3. What is the concentration of hydrogen ions in the tested water? **16.** ________

**A.** $5.012 \times 10^{-8}$ **B.** $-0.863$ **C.** 5.012 **D.** $1.995 \times 10^7$

**17.** Convert $\log_5 47$ to a natural logarithm and evaluate. **17.** ________

**A.** 0.770 **B.** 2.241 **C.** 2.392 **D.** 2.516

**18.** Solve $e^{0.2x} < 21.2$ by using natural logarithms. **18.** ________

**A.** $x < -1.898$ **B.** $x < 4.663$ **C.** $x < 8.234$ **D.** $x < 15.270$

**19.** ***Banking*** Find the amount of time required for an investment to double at a rate of 12.3% if the interest is compounded continuously. **19.** ________

**A.** 5.635 years **B.** 6.241 years **C.** 7.770 years **D.** 8.325 years

**20.** ***Biology*** The table below shows the population of a given bacteria colony. **20.** ________

| Time (days) | 0 | 3 | 6 | 9 | 12 |
|---|---|---|---|---|---|
| Population (thousands) | 95 | 120 | 155 | 190 | 250 |

Let $x$ be the number of days and let $y$ be the population in thousands. Linearize the data and find a regression equation for the linearized data.

**A.** $\ln y = 0.0948x + 4.3321$ **B.** $\ln y = 0.0798x + 4.5517$

**C.** $\ln y = 0.0722x + 4.7735$ **D.** $\ln y = 0.0785x + 4.8203$

**Bonus** Express $\sqrt[5]{\sqrt{x^6}}$ in exponential form. Assume $x > 0$. **Bonus:** ________

**A.** $x^{\frac{3}{5}}$ **B.** $x^{\frac{5}{3}}$ **C.** $x^{\frac{1}{60}}$ **D.** $x^{\frac{4}{5}}$

NAME ______________________ DATE ______________ PERIOD ________

# Chapter 11 Test, Form 1B

***Write the letter for the correct answer in the blank at the right of each problem.***

**1.** Evaluate $\left(9^{\frac{1}{2}} + 1^{\frac{1}{3}}\right)^{-\frac{1}{2}}$. **1.** ________

**A.** $-\frac{1}{2}$ **B.** $\frac{1}{2}$ **C.** $-2$ **D.** $2$

**2.** Simplify $\left(\frac{9x^3y^3}{x^{-1}y}\right)^{\frac{3}{2}}$. **2.** ________

**A.** $6x^2|y|^3$ **B.** $3x^{\frac{4}{3}}|y|^3$ **C.** $27x^6|y|^3$ **D.** $27|x|^3|y|^3$

**3.** Express $\sqrt[4]{16xy^4}$ using rational exponents. **3.** ________

**A.** $2x^{\frac{1}{4}}|y|$ **B.** $4x^{\frac{1}{4}}y$ **C.** $2|x|y$ **D.** $4x^4y$

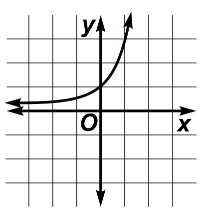
**4.** Express $x^{\frac{2}{3}}y^{\frac{1}{2}}$ using radicals. **4.** ________

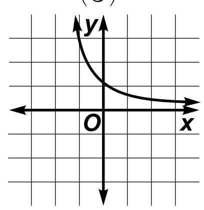
**A.** $\sqrt[3]{x^2y}$ **B.** $\sqrt[6]{x^2y}$ **C.** $\sqrt[6]{x^2y^3}$ **D.** $\sqrt[6]{x^4y^3}$

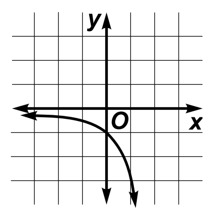
**5.** Evaluate $3^{\pi}$ to the nearest thousandth. **5.** ________

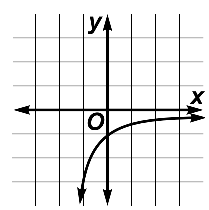
**A.** 9.425 **B.** 27.001 **C.** 31.026 **D.** 31.544

**6.** Choose the graph of $y = \left(\frac{1}{3}\right)^x$. **6.** ________

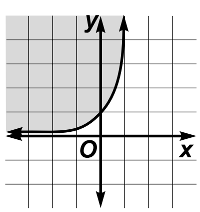
**A.**


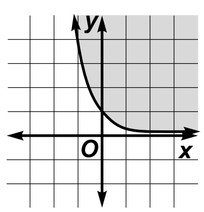

**B.**


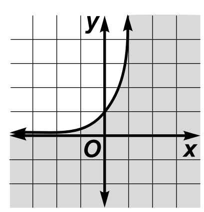

**C.**


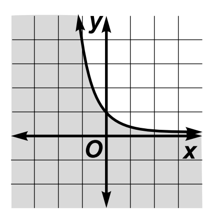

**D.**



**7.** Choose the graph of $y \geq 4^x$. **7.** ________

**A.**



**B.**



**C.**



**D.**



**8.** In 2000, the deer population in a certain area was 800. The number of deer increases exponentially at a rate of 7% per year. Predict the population in 2009. **8.** ________

**A.** 1408 **B.** 1434 **C.** 1502 **D.** 1492

**9.** Find the balance in an account at the end of 8 years if \$6000 is invested at an interest rate of 12% compounded continuously. **9.** ________

**A.** \$15,670.18 **B.** \$15,490.38 **C.** \$14,855.78 **D.** \$14,560.22

**10.** Write $2^{-3} = \frac{1}{8}$ in logarithmic form. **10.** ________

**A.** $\log_{-3} \frac{1}{8} = 2$ **B.** $\log_{-3} 2 = \frac{1}{8}$ **C.** $\log_2 \frac{1}{8} = -3$ **D.** $\log_2 (-3) = \frac{1}{8}$

NAME ______________________ DATE ______________ PERIOD ________

# Chapter 11 Test, Form 1B (continued)

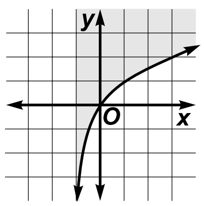
**11.** Evaluate $\log_9 \frac{1}{81}$. **11.** ________

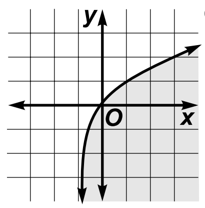
**A.** $-\frac{1}{2}$ **B.** $\frac{1}{2}$ **C.** $-2$ **D.** $2$

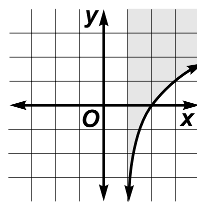
**12.** Solve $\log_4 x^2 - \log_4 5 = \log_4 125$. **12.** ________

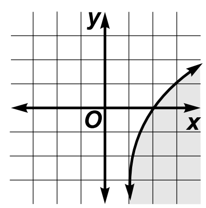
**A.** $-5$ or $5$ **B.** $5$ only **C.** $25$ only **D.** $-25$ or $25$

**13.** Choose the graph of $y \leq \log_2 (x + 1)$. **13.** ________

**A.**



**B.**



**C.**



**D.**



**14.** Find the value of $\log_5 63.2$ using the change of base formula. **14.** ________

**A.** 2.312 **B.** 2.576 **C.** 2.741 **D.** 2.899

**15.** Solve $4^{x-2} = 3$ using common logarithms. **15.** ________

**A.** 2.023 **B.** 2.247 **C.** 2.541 **D.** 2.792

**16.** If the concentration of hydrogen ions in a sample of water is $5.31 \times 10^{-8}$ moles per liter, what is the pH of the water? **16.** ________

**A.** 8.0 **B.** 7.3 **C.** 8.7 **D.** 5.3

**17.** Convert $\log_3 29$ to a natural logarithm and evaluate. **17.** ________

**A.** 2.647 **B.** 2.925 **C.** 3.065 **D.** 3.188

**18.** Solve $e^{3x} > 48$ by using natural logarithms. **18.** ________

**A.** $x > 1.290$ **B.** $x > 1.337$ **C.** $x > 1.452$ **D.** $x > 1.619$

**19.** ***Banking*** How much time would it take for an investment to double at a rate of 10.2% if interest is compounded continuously? **19.** ________

**A.** 6.011 years **B.** 6.241 years **C.** 6.558 years **D.** 6.796 years

**20.** ***Population*** The table below shows the population of a given urban area. **20.** ________

| Year | 1900 | 1920 | 1940 | 1960 | 1980 |
|---|---|---|---|---|---|
| Population (thousands) | 50 | 106 | 250 | 520 | 1170 |

Let $x$ be the number of years since 1900 and let $y$ be the population in thousands. Linearize the data and find a regression equation for the linearized data.

**A.** $\ln y = 0.0395x + 3.9039$ **B.** $\ln y = 0.0327x + 3.8166$
**C.** $\ln y = 0.0412x + 4.0077$ **D.** $\ln y = 0.0365x + 4.2311$

**Bonus** Express $\sqrt[4]{\sqrt{x^6}}$ in exponential form. Assume $x > 0$. **Bonus:** ________

**A.** $x^{\frac{3}{2}}$ **B.** $x^{\frac{3}{4}}$ **C.** $x^{\frac{1}{24}}$ **D.** $x^{\frac{2}{3}}$

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# Chapter 11 Test, Form 1C

***Write the letter for the correct answer in the blank at the right of each problem.***

**1.** Evaluate $\left(16^{\frac{1}{2}}\right)^{-\frac{1}{2}}$. **1.** ________

**A.** $-\frac{1}{2}$ **B.** $\frac{1}{2}$ **C.** $-2$ **D.** $2$

**2.** Simplify $\left(\frac{25x^3y^3}{xy}\right)^{\frac{3}{2}}$. **2.** ________

**A.** $5x^2|y|^3$ **B.** $125x^{\frac{9}{2}}y^{\frac{9}{2}}$ **C.** $25|x|^3|y|^3$ **D.** $125|x|^3|y|^3$

**3.** Express $\sqrt[4]{16x}$ using rational exponents. **3.** ________

**A.** $2x^{\frac{1}{4}}$ **B.** $4x^{\frac{1}{4}}$ **C.** $2x$ **D.** $4x^4$

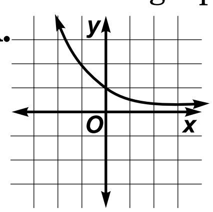
**4.** Express $x^{\frac{2}{3}}$ using radicals. **4.** ________

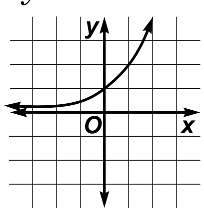
**A.** $\sqrt[3]{x^2}$ **B.** $\sqrt[6]{x}$ **C.** $\sqrt[3]{x}$ **D.** $\sqrt{x^3}$

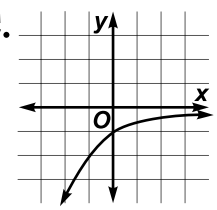
**5.** Evaluate $3^{\sqrt{2}}$ to the nearest thousandth. **5.** ________

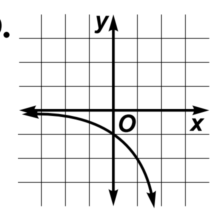
**A.** 4.278 **B.** 4.578 **C.** 4.729 **D.** 4.927

**6.** Choose the graph of $y = 2^x$. **6.** ________

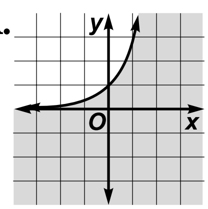
**A.**


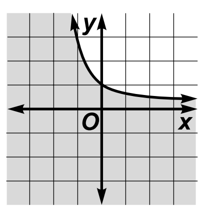

**B.**


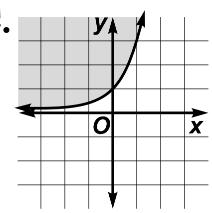

**C.**


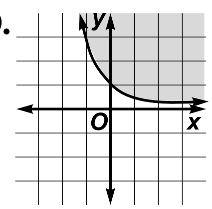

**D.**



**7.** Choose the graph of $y \geq 3^x$. **7.** ________

**A.**



**B.**



**C.**



**D.**



**8.** In 1998, the wolf population in a certain area was 1200. The number of wolves increases exponentially at a rate of 3% per year. Predict the population in 2011. **8.** ________

**A.** 1598 **B.** 1645 **C.** 1722 **D.** 1762

**9.** Find the balance in an account at the end of 14 years if \$5000 is invested at an interest rate of 9% that is compounded continuously. **9.** ________

**A.** \$16,998.14 **B.** \$17,234.72 **C.** \$17,627.11 **D.** \$17,891.23

**10.** Write $4^3 = 64$ in logarithmic form. **10.** ________

**A.** $\log_3 4 = 64$ **B.** $\log_4 64 = 3$ **C.** $\log_3 64 = 4$ **D.** $\log_{64} 3 = 4$

# Chapter 11 Test, Form 1C (continued)

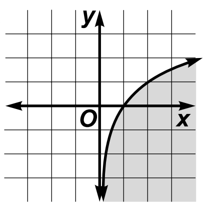
**11.** Evaluate $\log_4 \frac{1}{16}$. **11.** ________

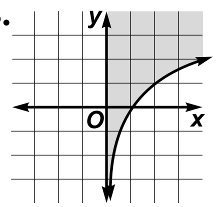
**A.** $-\frac{1}{2}$ **B.** $\frac{1}{2}$ **C.** $-2$ **D.** $2$

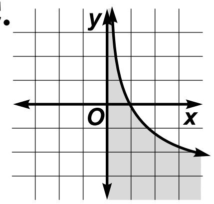
**12.** Solve $\log_4 x - \log_4 5 = \log_4 60$. **12.** ________

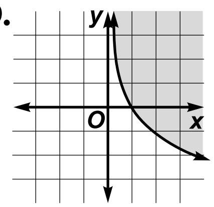
**A.** 3 **B.** 12 **C.** 120 **D.** 300

**13.** Choose the graph of $y \leq \log_2 x$. **13.** ________

**A.**



**B.**



**C.**



**D.**



**14.** Find the value of $\log_3 21.8$ using the change of base formula. **14.** ________

**A.** 2.312 **B.** 2.576 **C.** 2.741 **D.** 2.805

**15.** Solve $5^x = 32$ using common logarithms. **15.** ________

**A.** 2.023 **B.** 2.153 **C.** 2.241 **D.** 2.392

**16.** Evaluate $\log \frac{4^3}{5}$. **16.** ________

**A.** 2.505 **B.** 1.107 **C.** 0.380 **D.** 2.549

**17.** Convert $\log_4 134$ to a natural logarithm and evaluate. **17.** ________

**A.** 3.533 **B.** 3.623 **C.** 3.711 **D.** 3.782

**18.** Solve $e^{2x} > 37$ by using natural logarithms. **18.** ________

**A.** $x > 1.805$ **B.** $x > 1.822$ **C.** $x > 1.931$ **D.** $x > 1.955$

**19.** ***Banking*** What is the amount of time required for an investment to double at a rate of 8.2% if the interest is compounded continuously? **19.** ________

**A.** 8.275 years **B.** 8.453 years **C.** 8.613 years **D.** 8.772 years

**20.** ***Biology*** The table below shows the population for a given ant colony. **20.** ________

| Time (days) | 0 | 5 | 10 | 15 | 20 |
|---|---|---|---|---|---|
| Population (thousands) | 40 | 50 | 73 | 96 | 125 |

Let $x$ be the number of days and let $y$ be the population in thousands. Write a regression equation for the exponential model of the data.

**A.** $y = 39.4033(1.0611)^x$ **B.** $y = 39.2666(1.0723)^x$

**C.** $y = 39.2701(1.0522)^x$ **D.** $y = 39.2741(1.0604)^x$

**Bonus** Express $\sqrt[4]{\sqrt[3]{x}}$ in exponential form. Assume $x > 0$. **Bonus:** ________

**A.** $x^{\frac{3}{2}}$ **B.** $x^{\frac{3}{4}}$ **C.** $x^{\frac{1}{12}}$ **D.** $x^{\frac{4}{3}}$

NAME ____________________ DATE ______________ PERIOD ________

# Chapter 11 Test, Form 2A

**1.** Evaluate $\frac{\sqrt[5]{(-243)^4}}{-3^2}$. **1.** ________

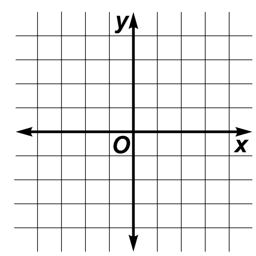
**2.** Simplify $(8x^6 \cdot 32y^5)^{\frac{1}{2}}$. **2.** ________

**3.** Express $5\sqrt[4]{81x^3y^8}$ using rational exponents. **3.** ________

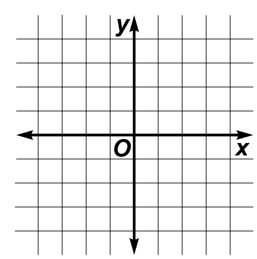
**4.** Express $(3x)^{\frac{1}{5}}\,(3x^2)^{\frac{1}{3}}$ using radicals. **4.** ________

**5.** Evaluate $8^{\frac{7}{\pi}}$ to the nearest thousandth. **5.** ________

**6.** Sketch the graph of $y = 3^{-x}$. **6.**




**7.** Sketch the graph of $y \le \left(\frac{1}{4}\right)^x$. **7.**




**8.** Suppose \$1750 is put into an account that pays an annual rate of 6.25% compounded weekly. How much will be in the account after 36 months? **8.** ________

**9.** A scientist has 37 grams of a radioactive substance that decays exponentially. Assuming $k = -0.3$, how many grams of radioactive substance remain after 9 days? Round your answer to the nearest hundredth. **9.** ________

**10.** Write $\left(\frac{1}{6}\right)^{-4} = 1296$ in logarithmic form. **10.** ________

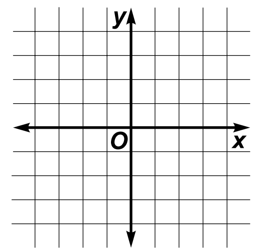
NAME ______________________ DATE ____________ PERIOD ________

# Chapter 11 Test, Form 2A (continued)

**11.** Evaluate $\log_{16} \frac{1}{8}$. **11.** ______________

**12.** Solve $\log_4 x + \log_4 (x + 2) = \log_4 35$. **12.** ______________

**13.** Sketch the graph of $y \geq \log_3 (x + 2)$. **13.**




***For Exercises 14-18, round your answers to the nearest thousandth.***

**14.** Find the value of $\log_5 87.2$ using the change of base formula. **14.** ______________

**15.** Solve $4^{x-3} = 7^x$ using common logarithms. **15.** ______________

**16.** The pH of a sample of seawater is approximately 8.1. What is the concentration of hydrogen ions in the seawater? **16.** ______________

**17.** Convert $\log_7 324$ to a natural logarithm and evaluate. **17.** ______________

**18.** Solve $e^{-0.5x} < 41.6$ by using natural logarithms. **18.** ______________

**19.** ***Banking*** What interest rate is required for an investment with continuously compounded interest to double in 8 years? **19.** ______________

**20.** ***Biology*** The table below shows the population for a given bacteria colony. **20.** ______________

| Time (days) | 0 | 4 | 8 | 12 | 16 |
|---|---|---|---|---|---|
| Population (thousands) | 87 | 112 | 135 | 173 | 224 |

Let $x$ represent the number of days and let $y$ represent a population in thousands. Linearize the data and find a regression equation for the linearized data.

**Bonus** Express $\sqrt[4]{\sqrt{x^{12}}}$ in exponential form. Assume $x > 0$. **Bonus:** ______________

NAME ______________________ DATE ______________ PERIOD ________

# Chapter 11 Test, Form 2B

**1.** Evaluate $(\sqrt[4]{81})^3$. **1.** ______________

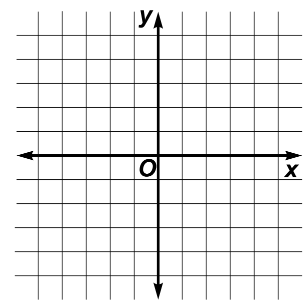
**2.** Simplify $\left(\frac{25}{x^{-4}}\right)^{\frac{3}{2}}$. **2.** ______________

**3.** Express $\sqrt[5]{32x^3y^{10}}$ using rational exponents. **3.** ______________

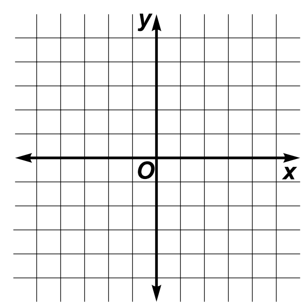
**4.** Express $4x^{\frac{1}{3}}y^{\frac{2}{3}}$ using radicals. **4.**

**5.** Evaluate $5^{\pi}$ to the nearest thousandth. **5.** ______________

**6.** Sketch the graph of $y = 3^x$. **6.**




**7.** Sketch the graph of $y \le \left(\frac{1}{2}\right)^x$. **7.**




**8.** A 1991 report estimated that there were 640 salmon in a certain river. If the population is decreasing exponentially at a rate of 4.3% per year, what is the expected population in 2002? **8.** ______________

**9.** Find the balance in an account at the end of 12 years if $4000 is invested at an interest rate of 9% that is compounded continuously. **9.** ______________

**10.** Write $16^{\frac{3}{4}} = 8$ in logarithmic form. **10.** ______________

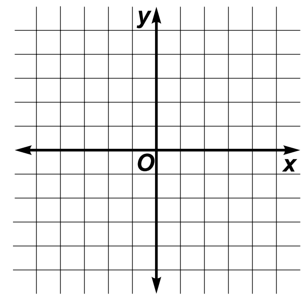
NAME ______________________ DATE ______________ PERIOD ________

# Chapter 11 Test, Form 2B (continued)

**11.** Evaluate $\log_4 \frac{1}{64}$. **11.** ______________

**12.** Solve $\log_2 (x + 6) + \log_2 3 = \log_2 30$. **12.** ______________

**13.** Sketch the graph of $y \geq \log_4 (x + 2)$. **13.**




***For Exercises 14-18, round your answers to the nearest thousandth.***

**14.** Evaluate $\log \frac{9}{2^3}$. **14.** ______________

**15.** Find the value of $\log_3 92.4$ using the change of base formula. **15.** ______________

**16.** Solve $5^{x+2} = 7$ using common logarithms. **16.** ______________

**17.** Convert $\log_5 156$ to a natural logarithm and evaluate. **17.** ______________

**18.** Solve $e^{4x} < 98.6$ by using natural logarithms. **18.** ______________

**19.** ***Banking*** Find the amount of time in years required for an investment to double at a rate of 6.2% if the interest is compounded continuously. **19.** ______________

**20.** ***Biology*** The table below shows the population of mold spores on a given Petri dish. **20.** ______________

| Time (days) | 0 | 2 | 4 | 6 | 8 |
|---|---|---|---|---|---|
| Population (thousands) | 45 | 51 | 63 | 74 | 81 |

Let $x$ represent the number of days and let $y$ represent the populations in thousands. Linearize the data and find a regression equation for the linearized data.

**Bonus** Express $2 \log_b a - \log_b c$ as a single log. Assume $b > 0$. **Bonus:** ______________

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# Chapter 11 Test, Form 2C

**1.** Evaluate $8^{\frac{2}{3}} \cdot 4^{\frac{1}{2}}$. **1.** ______________

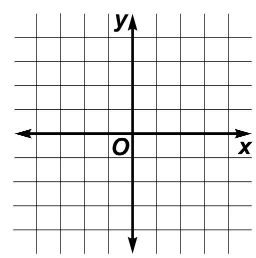
**2.** Simplify $\left(\frac{27y^4}{y}\right)^{\frac{2}{3}}$. **2.** ______________

**3.** Express $\sqrt[3]{125x^5}$ using rational exponents. **3.** ______________

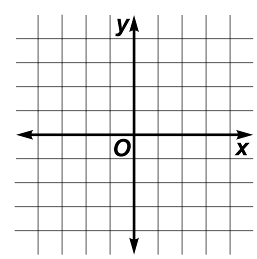
**4.** Express $x^{\frac{2}{5}}$ using radicals. **4.** ______________

**5.** Evaluate $5^{\sqrt{3}}$ to the nearest thousandth. **5.** ______________

**6.** Sketch the graph of $y = 4^x$. **6.**




**7.** Sketch the graph of $y \leq 2^x$. **7.**




**8.** In 1990, the elk population in a certain area was 750. The number of elk increases exponentially at a rate of 6% per year. Predict the elk population in 2004. **8.** ______________

**9.** Find the balance in an account at the end of 8 years if \$7000 is invested at an interest rate of 12% compounded continuously. **9.** ______________

**10.** Write $5^2 = 25$ in logarithmic form. **10.** ______________

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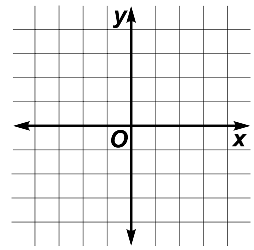
# Chapter 11 Test, Form 2C (continued)

**11.** Evaluate $\log_3 \frac{1}{9}$. **11.** ____________

**12.** Solve $\log_2 x + \log_2 3 = \log_2 12$. **12.** ____________

**13.** Sketch the graph of $y \geq \log_3 x$. **13.**




***For Exercises 14–18, round your answers to the nearest thousandth.***

**14.** Evaluate $\log 3(6)^2$. **14.** ____________

**15.** Find the value of $\log_4 82.4$ using the change of base formula. **15.** ____________

**16.** Solve $3^x = 47$ using common logarithms. **16.** ____________

**17.** Convert $\log_3 59$ to a natural logarithm and evaluate. **17.** ____________

**18.** Solve $e^{3x} < 89$ by using natural logarithms. **18.** ____________

**19.** ***Banking*** Find the amount of time in years required for an investment to double at a rate of 9.5% if the interest is compounded continuously. **19.** ____________

**20.** ***Biology*** The table below shows the population for a given bacteria colony. **20.** ____________

| Time (days) | 0 | 4 | 8 | 12 | 16 |
|---|---|---|---|---|---|
| Population (thousands) | 32 | 40 | 55 | 69 | 85 |

Let $x$ represent the number of days and let $y$ represent the population in thousands. Write a regression equation for the exponential model of the data.

**Bonus** Express $2 \log_b a + 3 \log_b c$ as a single log. Assume $b > 0$. **Bonus:** ____________

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Chapter 11

# Chapter 11 Open-Ended Assessment

**Instructions:** ***Demonstrate your knowledge by giving a clear, concise solution to each problem. Be sure to include all relevant drawings and justify your answers. You may show your solution in more than one way or investigate beyond the requirements of the problem.***

**1. a**. Graph $y = 3^x$.

**b.** Compare the graphs of $y = 5^x$ and $y = 3^x$. Do the graphs intersect? If so, where? Graph $y = 5^x$.

**c.** Compare the graphs of $y = \left(\frac{1}{3}\right)^x$ and $y = 3^x$. Do the graphs intersect? If so, where? Graph $y = \left(\frac{1}{3}\right)^x$.

**d.** Compare the graphs of $y = 3^x$ and $y = \log_3 x$. Do the graphs intersect? If so, where? Graph $y = \log_3 x$.

**e.** Compare the graphs of $y = \log_5 x$ and $y = \log_3 x$. Do the graphs intersect? If so, where? Graph $y = \log_5 x$.

**f.** Tell how the graphs of $y = \log_2 x$ and $y = \log_8 x$ are related. Justify your answer.

**2.** Write a word problem for the equation below. Then solve for $x$ and explain what the answer means.

$$178 = 9 \cdot 2^x$$

**3.** Solve the equation $\log_2 (x + 3) = 3 - \log_2 (x - 2)$. Explain each step.

**4.** Solve the equation $e^{2x} - 3e^x + 2 = 0$. Explain each step.

**5.** Before calculators and computers were easily accessible, scientists and engineers used slide rules. Using the properties of logarithms, they performed mathematical operations, including finding a product. To see the principle involved, pick two positive numbers that are less than 10 and that each have two places after the decimal point. Calculate their product using only the properties of the logarithm and the exponential function, without calculating the product directly. When you have found the product, check your answer by calculating the product directly.

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# Chapter 11 Mid-Chapter Test (Lessons 11-1 and 11-4)

***For Exercises 1-3, evaluate each expression.***

**1.** $\left(16^{\frac{1}{2}} + 64^{\frac{1}{3}}\right)^{\frac{1}{3}}$ **1.** ______________

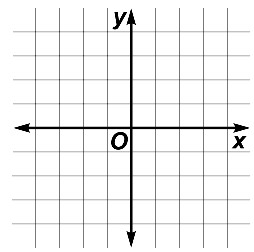
**2.** $\frac{-8^{\frac{1}{3}}}{8}$ **2.** ______________

**3.** $\sqrt{15} \cdot \sqrt{60}$ **3.** ______________

**4.** Express $\sqrt[3]{8x^2y^6}$ using rational exponents. **4.** ______________

**5.** Evaluate $7^{\pi}$ to the nearest thousandth. **5.** ______________

**6.** Sketch the graph of $y = 4^{-x}$. **6.**




**7.** The number of seniors at Freedmont High School was 241 in 1993. If the number of seniors increases exponentially at a rate of 1.7% per year, how many seniors will be in the class of 2005? **7.** ______________

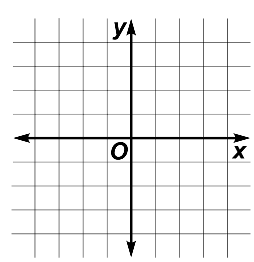
**8.** Jasmine invests \$1500 in an account that earns an interest rate of 11% compounded continuously. Will she have enough money in 4 years to put a \$2500 down payment on a new car? Explain. **8.** ______________

**9.** A city's population can be modeled by the equation $y = 29{,}760e^{-0.021t}$, where $t$ is the number of years since 1986. Find the projected population in 2012. **9.** ______________

**10.** Evaluate $\log_4 \frac{1}{64}$. **10.** ______________

**11.** Solve $\log_3 x + \log_3 (x - 6) = \log_3 16$. **11.** ______________

**12.** Sketch the graph of $y \leq \log_2 (x - 1)$. **12.**



Chapter 11

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# Chapter 11, Quiz A (Lessons 11-1 and 11-2)

***Evaluate each expression.***

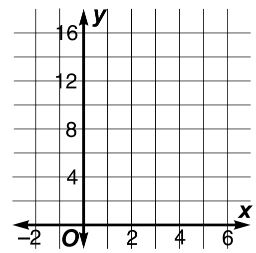
**1.** $\left(81^{\frac{1}{2}} + 4^2\right)^{-\frac{1}{2}}$ **1.** ____________

**2.** $64^{\frac{1}{3}} - 64^{-\frac{1}{3}}$ **2.** ____________

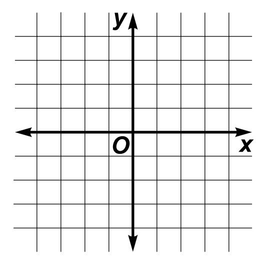
**3.** Express $16^{\frac{1}{7}}$ using radicals. **3.** ____________

***Graph each exponential function or inequality.***

**4.** $y = 2^{x+1}$ **4.**




**5.** $y \leq 3^x$. **5.**

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Chapter 11

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# Chapter 11, Quiz B (Lessons 11-3 and 11-4)

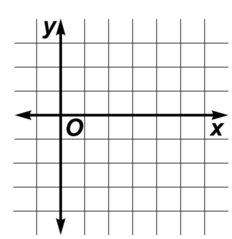
**1.** ***Finance*** Find the balance in an account at the end of 12 years if \$6500 is invested at an interest rate of 8% compounded continuously. **1.** ____________

**2.** Write $3^{-4} = \frac{1}{81}$ in logarithmic form. **2.** ____________

**3.** Evaluate $\log_{\sqrt{5}} 125$. **3.** ____________

**4.** Solve $\log_5 72 - \log_5 x = 3 \log_5 2$. **4.** ____________

**5.** Sketch the graph of $y \geq \log_2 (x + 1)$. **5.**



Chapter 11

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# Chapter 11, Quiz C (Lessons 11-5 and 11-6)

***For Exercises 1-5, round your answers to the nearest thousandth.***

**1.** Find the value of $\log_4 23.9$ using the change of base formula. **1.** ______________

**2.** Solve $5^{x+2} = 87$ using common logarithms. **2.** ______________

**3.** Given that $\log 4 = 0.6021$, evaluate $\log 40{,}000$. **3.** ______________

**4.** Convert $\log_7 235$ to a natural logarithm and evaluate. **4.** ______________

**5.** Evaluate $\ln \frac{1}{0.45}$. **5.** ______________

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Chapter 11

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# Chapter 11, Quiz D (Lesson 11-7)

***Find the amount of time required for an investment to double at the given rate if interest is compounded continuously.***

**1.** 9.5% **1.** ______________

**2.** 5.0% **2.** ______________

**3.** ***Population*** The table shows the population for a given urban area. **3.** ______________

| Year | 1900 | 1910 | 1920 | 1930 | 1940 |
|---|---|---|---|---|---|
| Population (thousands) | 30 | 58 | 120 | 220 | 455 |

Let $x$ be the number of years since 1900 and let $y$ be the population in thousands. Linearize the data and find a regression equation for the linearized data.

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# Chapter 11 SAT and ACT Practice

***After working each problem, record the correct answer on the answer sheet provided or use your own paper.***

**Multiple Choice**

**1.** Bobbi scored 75, 80, and 85 on three tests. What must she score on her fourth test to keep her average test score the same?
**A** 75 **B** 80
**C** 85 **D** 90
**E** None of these

**2.** The average of 3, 4, $x$, and 12 is 7. What is the value of $x$?
**A** 5 **B** 6
**C** 7 **D** 8
**E** 9

**3.** What is $\frac{\sqrt{(x+y)^3}}{\sqrt{(x+y)}}$ in terms of $(x+y)$?
**A** $(x+y)^2$
**B** $(x+y)^{\frac{1}{2}}$
**C** $(x+y)^{\frac{1}{3}}$
**D** $(x+y)$
**E** None of these

**4.** For some positive $x$, if $x^2 + xy = 3(x + y)$, then what is the value of $x$?
**A** 3
**B** $-3$
**C** Both A and B
**D** Neither A and B
**E** It cannot be determined from the information given.

**5.** The average of a set of four numbers is 22. If one of the numbers is removed, the average of the remaining numbers is 21. What is the value of the number that was removed?
**A** 1 **B** 2
**C** 22 **D** 25
**E** It cannot be determined from the information given.

**6.** What is the average age of a group of 15 students if 9 students are 15, 3 are 16, and 3 are 17?
**A** 15.4 years old
**B** 15.5 years old
**C** 15.6 years old
**D** 15.7 years old
**E** 15.8 years old

**7.** $\cot \frac{4\pi}{3} =$
**A** $\frac{\sqrt{3}}{3}$
**B** $\sqrt{3}$
**C** $-\sqrt{3}$
**D** $-\frac{\sqrt{3}}{3}$
**E** None of these

**8.** $\frac{2}{\tan\theta + \cot\theta} =$
**A** $\sin\theta$
**B** $\cos 2\theta$
**C** $\cos\theta$
**D** $2\sin\theta$
**E** $2\sin\theta\cos\theta$

**9.** A basket contains 12 marbles, some green and some blue. Which of the following is *not* a possible ratio of green marbles to blue marbles?
**A** 1:1
**B** 1:2
**C** 1:3
**D** 1:4
**E** 1:5

**10.** The ratio of two integers is 5:4, and their sum is equal to 54. How much larger than the smaller number is the bigger number?
**A** 45
**B** 30
**C** 24
**D** 12
**E** 6

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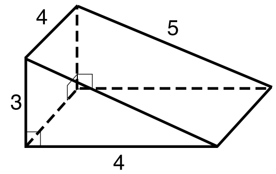
# Chapter 11 SAT and ACT Practice (continued)

**11.** A rectangular solid is cut diagonally as shown below. What is the surface area of the wedge?

**A** 60 $\text{units}^2$
**B** 56 $\text{units}^2$
**C** 54 $\text{units}^2$
**D** 44 $\text{units}^2$
**E** 36 $\text{units}^2$




**12.** Square $ABCD$ is divided into 4 equal squares. If the perimeter of each smaller square is four, what is the area of the larger square?

**A** 2 $\text{units}^2$
**B** 4 $\text{units}^2$
**C** 8 $\text{units}^2$
**D** 16 $\text{units}^2$
**E** None of these

**13.** What is the average price of a dozen rolls if $\frac{1}{3}$ of the customers buy the larger rolls for \$3.00 per dozen and $\frac{2}{3}$ of the customers buy the smaller rolls for \$2.25 per dozen?

**A** \$2.60
**B** \$2.50
**C** \$2.45
**D** \$2.40
**E** \$2.70

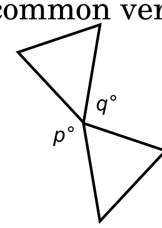
**14.** What is the average speed of Jane's car if Jane drives for 30 minutes at 50 miles per hour, then at 65 miles per hour for 2 hours, then at 45 miles per hour for 20 minutes, and at 30 miles per hour for 10 minutes?

**A** 68.5 mph
**B** 58.3 mph
**C** 56.4 mph
**D** 52.6 mph
**E** None of these

**15.** If $\triangle ABC$ has two sides that are each one unit long, which of the following *cannot* be the length of the third side?

**A** $\frac{\sqrt{2}}{2}$
**B** 1
**C** $\sqrt{2}$
**D** $\sqrt{3}$
**E** $2\sqrt{2}$

**16.** In the figure slown, two equilateral triangles have a common vertex. Find $p + q$.




**A** 240
**B** 180
**C** 120
**D** 90
**E** It cannot be determined from the information given.

**17–18. Quantitative Comparison**

**A** if the quantity in Column A is greater
**B** if the quantity in Column B is greater
**C** if the two quantities are equal
**D** if the relationship cannot be determined from the information given

| | Column A | Column B |
|---|---|---|
| **17.** | Average speed of a train that travels 150 miles in 3 hours | Average speed of a train that travels 50 miles in $\frac{1}{3}$ hour |

**18.** The values of $x$ and $y$ are positive.

| Column A | Column B |
|---|---|
| Average of $x$ and $y$ | Average of $x^2$ and $y^2$ |

**19. Grid-In** For what positive value of $y$ does $\frac{y}{9} = \frac{25}{y}$?

**20. Grid-In** If $xy = 270$ and $x + y = 20 + (x - y)$, what is $x$?

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# Chapter 11 Cumulative Review (Chapters 1–11)

**1.** If $f(x) = 2x^2 - 1$, find $f(4)$. **1.** ____________

**2.** Solve the system algebraically. **2.** ____________

$5x - y = 21$

$2x + 3y = 5$

**3.** Describe the end behavior for $y = x^4 - 3x$. **3.** ____________

**4.** Solve the equation $\sqrt{x - 15} + 7 = 12$. **4.** ____________

**5.** Find $\sin(-180°)$. **5.** ____________

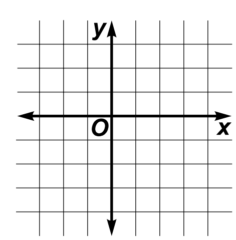
**6.** State the amplitude, period, and phase shift for the graph of $y = 4 \sin(2x - 6\pi)$. **6.** ____________

**7.** Find the polar coordinates of the point with rectangular coordinates $(1, \sqrt{3})$. **7.** ____________

**8.** Write the equation of the circle with center (0, 2) and radius 3 units. **8.** ____________

**9.** Write $5^{-3} = \frac{1}{125}$ in logarithmic form. **9.** ____________

**10.** Sketch the graph of $y \leq \log_3 x$. **10.**



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# UNIT 3

# Unit 3 Review, Chapters 9–11

***Graph the point that has the given polar coordinates. Then, name three other pairs of polar coordinates for each point.***

**1.** $A(2, 60°)$ **2.** $B(-4, 45°)$

**3.** $C\left(1.5, \frac{\pi}{6}\right)$ **4.** $D\left(-2, -\frac{2\pi}{3}\right)$

***Graph each polar equation. Identify the type of curve each represents.***

**5.** $r = \sqrt{5}$

**6.** $\theta = 60°$

**7.** $r = 3 \cos \theta$

**8.** $r = 2 + 2 \sin \theta$

***Find the polar coordinates of each point with the given rectangular coordinates. Use $0 \leq \theta < 2\pi$ and $r \geq 0$.***

**9.** $(-2, -2)$ **10.** $(2, 2)$

**11.** $(2, -3)$ **12.** $(-3, 1)$

***Write each equation in rectangular form.***

**13.** $2 = r \cos\left(\theta - \frac{\pi}{2}\right)$

**14.** $4 = r \cos\left(\theta + \frac{\pi}{3}\right)$

***Simplify.***

**15.** $\boldsymbol{i}^{45}$

**16.** $(3 + 2\boldsymbol{i}) + (3 - 2\boldsymbol{i})$

**17.** $\boldsymbol{i}^4(3 + 3\boldsymbol{i})$

**18.** $(-\boldsymbol{i} - 5)(\boldsymbol{i} - 5)$

**19.** $\frac{2 + \boldsymbol{i}}{2 - 3\boldsymbol{i}}$

***Express each complex number in polar form.***

**20.** $-3\boldsymbol{i}$ **21.** $3 + 3\boldsymbol{i}$

**22.** $-1 + 3\boldsymbol{i}$ **23.** $4 - 5\boldsymbol{i}$

***Find each product. Express the result in rectangular form.***

**24.** $2\left(\cos \frac{\pi}{2} + \boldsymbol{i} \sin \frac{\pi}{2}\right) \cdot 4\left(\cos \frac{\pi}{2} + \boldsymbol{i} \sin \frac{\pi}{2}\right)$

**25.** $1.5(\cos 3.1 + \boldsymbol{i} \sin 3.1) \cdot 2(\cos 0.5 + \boldsymbol{i} \sin 0.5)$

***Solve.***

**26.** Find $(1 + \boldsymbol{i})^7$ using De Moivre's Theorem. Express the result in rectangular form.

**27.** Solve the equation $x^5 - 1 = 0$ for all roots.

***Find the distance between each pair of points with the given coordinates. Then, find the coordinates of the midpoint of the segment that has endpoints at the given coordinates.***

**28.** $(-4, 6), (11, 2)$ **29.** $(5, 0), (3, -2)$

**30.** $(1, 9), (-4, -3)$

***For the equation of each circle, identify the center and radius. Then graph the equation.***

**31.** $4x^2 + 4y^2 = 49$

**32.** $x^2 + 10x + y^2 + 8y = 20$

***For the equation of each ellipse, find the coordinates of the center, foci, and vertices. Then, graph the equation.***

**33.** $(x - 1)^2 + 2(y - 3)^2 = 25$

**34.** $4(x + 2)^2 + 25(y - 2)^2 = 100$

***For the equation of each hyperbola, find the coordinates of the center, foci, and vertices, and the equations of the asymptotes. Then, graph the equation.***

**35.** $4x^2 - y^2 = 27$

**36.** $\frac{(y + 3)^2}{4} - \frac{(x + 1)^2}{9} = 1$

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# Unit 3 Review, Chapters 9–11 (continued)

***For the equation of each parabola, find the coordinates of the focus and vertex, and the equations of the directrix and axis of symmetry. Then graph the equation.***

**37.** $(x - 2)^2 = 2(y - 4)$
**38.** $y^2 + 2y - 5x + 18 = 0$

***Graph each equation and identify the conic section it represents.***

**39.** $12y - 3x + 2x^2 + 1 = 0$
**40.** $4x^2 - 25y^2 - 8x - 150y - 321 = 0$
**41.** $x^2 + 4x + y^2 - 12y + 4 = 0$

***Find the rectangular equation of the curve whose parametric equations are given. Then graph the equation using arrows to indicate orientation.***

**42.** $x = 2t, y = 3t^2, -2 \leq t \leq 2$
**43.** $x = \sin t, y = 3 \cos t, 0 \leq t \leq 2\pi$

***Write an equation in general form of each translated or rotated graph.***

**44.** $x = 3(y + 2)^2 + 1$ for $T_{(1,\ -5)}$
**45.** $x^2 - \frac{y^2}{16} = 1, \theta = \frac{\pi}{3}$

***Graph each system of equations or inequalities. Then solve the system of equations.***

**46.** $x^2 - 2x - 2y - 2 = 0$
$(x - 4)^2 = -8y$

**47.** $x^2 - (y - 1)^2 \geq 1$
$4x^2 + 9(y - 3)^2 < 36$

***Simplify each expression.***

**48.** $\sqrt{16x^2y^7}$
**49.** $\sqrt[3]{54a^4b^3c^8}$
**50.** $\left(3^2c^3d^5\right)^{\frac{1}{5}}$
**51.** $(3x)^2(3x^2)^{-2}$

***Graph each exponential function.***

**52.** $y = 2^{-x}$
**53.** $y = 2^{x+2}$

***A city's population can be modeled by the equation $y = 17{,}492e^{-0.027t}$, where t is the number of years since 1996.***

**54.** What was the city's population in 1996?
**55.** What is the projected population in 2007?

***Solve each equation.***

**56.** $\log_x 36 = 2$
**57.** $\log_2 (2x) = \log_2 27$
**58.** $\log_5 x = \frac{1}{3} \log_5 64 + 2 \log_5 3$

***Find the value of each logarithm using the change of base formula.***

**59.** $\log_6 431$
**60.** $\log_{0.5} 78$
**61.** $\log_7 0.325$

***Use natural logarithms to solve each equation.***

**62.** $2.3^x = 23.4$
**63.** $x = \log_4 16$
**64.** $5^{x-2} = 2^x$

***Solve each equation by graphing. Round solutions to the nearest hundredth.***

**65.** $46 = e^x$
**66.** $18 = e^{4k}$
**67.** $519 = 3e^{0.035t}$

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# Unit 3 Test, Chapters 9–11

**1.** Evaluate $\log_6 \sqrt{6}$. **1.** ______________

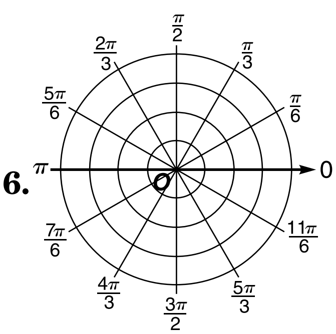
**2.** Identify the conic section represented by $3x^2 - 4xy + 2y^2 - 3y = 0$. **2.** ______________

**3.** Find the coordinates of the vertex and the equation of the axis of symmetry for the parabola with equation $2x^2 + 2x - y = -3$. **3.** ______________

**4.** Write the rectangular equation $x^2 + y^2 = 6$ in polar form. **4.** ______________

**5.** Simplify $\frac{1 - \boldsymbol{i}}{3 + 2\boldsymbol{i}}$. **5.** ______________

**6.** Graph the point with polar coordinates $\left(-2, \frac{3\pi}{2}\right)$. **6.**




**7.** Solve $6 = e^{0.2t}$. Round your answer to the nearest hundredth. **7.** ______________

**8.** Evaluate $(\sqrt{289})^{-3}$. **8.** ______________

**9.** Find the rectangular equation of the curve whose parametric equations are $x = -2 \sin t$ and $y = \cos t$, where $0 \leq t \leq 2\pi$. **9.** ______________

**10.** Find the balance after 15 years of a \$2500 investment earning 5.5% interest compounded continuously. **10.** ______________

**11.** Find the product $2(\cos 10° + \boldsymbol{i} \sin 10°) \cdot 4(\cos 20° + \boldsymbol{i} \sin 20°)$. Then, express the result in rectangular form. **11.** ______________

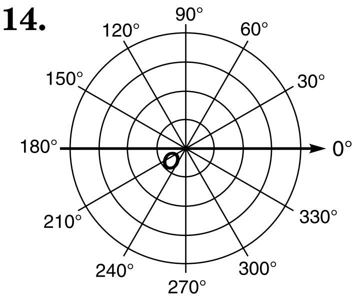
**12.** Identify the classical curve that the graph of $r = 1 + \sin \theta$ represents. **12.** ______________

**13.** Write the standard form of the equation of the circle that passes through (0, 4) and has its center at (−3, −1). **13.** ______________

NAME ______________________ DATE ______________ PERIOD ________

# Unit 3 Test, Chapters 9–11 (continued)

**14.** Graph the polar equation
$2 = r \cos(\theta + 180°)$.

**14.**



**15.** Solve $6 = 15^{1-x}$ by using logarithms. Round your answer to the nearest thousandth.

**15.** ______________

**16.** Find the principal root $(-64)^{\frac{1}{6}}$. Express the result in the form $a + b\boldsymbol{i}$.

**16.** ______________

**17.** Find the distance between points at $(6, -3)$ and $(-1, 4)$.

**17.** ______________

**18.** Write the equation of the ellipse with foci at $\left(0, -\sqrt{3}\right)$ and $\left(0, \sqrt{3}\right)$ and for which $2a = 4$.

**18.** ______________

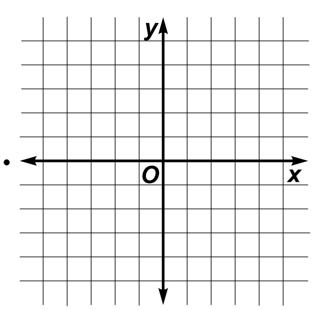
**19.** Find the future value to the nearest dollar of \$2700 invested at 8% for 5 years in an account that compounds interest quarterly.

**19.** ______________

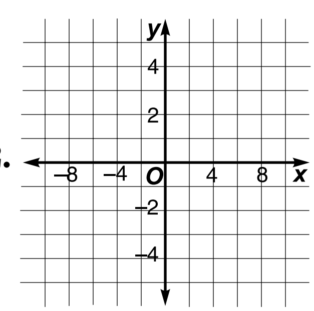
**20.** Use a calculator to find ln 36.9 to the nearest ten thousandth.

**20.** ______________

**21.** Graph the exponential function $y = \left(\frac{1}{4}\right)^x$.

**21.**



**22.** For the ellipse with equation
$5x^2 + 64y^2 + 30x + 128y - 211 = 0$,
find the coordinates of the center, foci, and vertices. Then, graph the equation.

**22.**


# Unit 3 Test, Chapters 9–11 (continued)

**23.** Find the rectangular coordinates of the point whose polar coordinates are (20, 140°). Round to the nearest hundredth. **23.** ________________

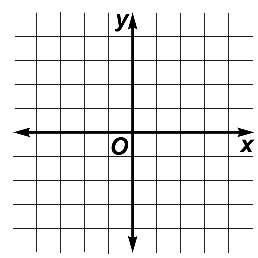
**24.** Write the standard form of the equation of the circle that is tangent to $x = -2$ and has its center at $(2, -4)$. **24.** ________________

**25.** Evaluate $(1 + \boldsymbol{i})^{12}$ by using De Moivre's Theorem. Express the result in rectangular form. **25.** ________________

**26.** Express $\sqrt[3]{8a^3y^5}$ using rational exponents. **26.** ________________

**27.** Simplify $(-1 - 2\boldsymbol{i}) + (4 - 6\boldsymbol{i})$. **27.** ________________

**28.** Graph the system of equations. Then solve.
$x^2 + y^2 = 10$
$xy = 3$
**28.**




**29.** Express $8\boldsymbol{i}$ in polar form. **29.** ________________

**30.** Convert $\log_7 0.59$ to a natural logarithm and evaluate to the nearest ten thousandth. **30.** ________________

**31.** Identify the graph of the equation $4x^2 - 25y^2 = 100$. Then write the equation of the translated graph for $T_{(5,\,-2)}$ in general form. **31.** ________________

**32.** Write the polar equation $3 = r\cos(\theta - 315°)$ in rectangular form. **32.** ________________

**33.** *True* or *false*: The graph of the polar equation $r^2 = 3\sin 2\theta$ is a limaçon. **33.** ________________

**34.** Write $3x - 6y = -14$ in polar form. Round $\phi$ to the nearest degree. **34.** ________________

**35.** Express $x^{\frac{2}{3}}(y^5z)^{\frac{1}{3}}$ using radicals. **35.** ________________

**36.** Write the equation $\log_{343} 7 = \frac{1}{3}$ in exponential form. **36.** ________________

# Unit 3 Test, Chapters 9–11 (continued)

**37.** Write the standard form of the equation of the circle that passes through the points at (0, 8), (8, 0), and (16, 8). Then identify the center and radius of the circle. **37.** ______________

**38.** Use a calculator to find antiln $(-0.049)$ to the nearest ten thousandth. **38.** ______________

**39.** Find the equation of the hyperbola whose vertices are at $(-1, -5)$ and $(-1, 1)$ with a focus at $(-1, -7)$. **39.** ______________

**40.** Find the quotient $3\left(\cos \frac{5\pi}{12} + \boldsymbol{i} \sin \frac{5\pi}{12}\right) \div 6\left(\cos \frac{\pi}{12} + \boldsymbol{i} \sin \frac{\pi}{12}\right)$. The express the quotient in rectangular form. **40.** ______________

**41.** Find the coordinates of the focus and the equation of the directrix of the parabola with equation $y^2 - 8y - 8x + 24 = 0$. **41.** ______________

**42.** Solve $9^{2x-3} > 4$. Round your answer to the nearest hundredth. **42.** ______________

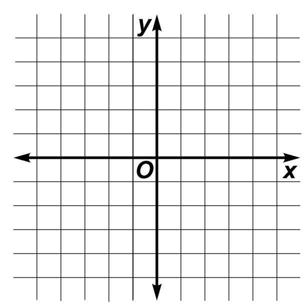
**43.** Express $2(\cos 300° + \boldsymbol{i} \sin 300°)$ in rectangular form. **43.** ______________

**44.** Find the equation of the ellipse whose semi-major axis has length 6 and whose foci are at $\left(3, -2 \pm \sqrt{11}\right)$. **44.** ______________

**45.** Graph the system of inequalities. **45.**

$$\frac{(x-3)^2}{9} + \frac{(y+2)^2}{4} \leq 1$$

$$(x-3)^2 + (y+2)^2 \geq 4$$




**46.** Write the polar equation $\theta = 45°$ in rectangular form. **46.** ______________

**47.** What interest rate is required for an investment with continuously compounded interest to double in 6 years? **47.** ______________

**48.** Write the equation $2^6 = 64$ in logarithmic form. **48.** ______________

**49.** Simplify $(3 + 2\boldsymbol{i})(2 - 5\boldsymbol{i})$. **49.** ______________

**50.** Find the coordinates of the center, the foci, and the vertices, and the equations of the asymptotes of the graph of the equation $\frac{(x+1)^2}{2} - \frac{y^2}{8} = 1$. **50.** ______________

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# SAT and ACT Practice Answer Sheet

## (10 Questions)

1 (A) (B) (C) (D) (E)

2 (A) (B) (C) (D) (E)

3 (A) (B) (C) (D) (E)

4 (A) (B) (C) (D) (E)

5 (A) (B) (C) (D) (E)

6 (A) (B) (C) (D) (E)

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9 (A) (B) (C) (D) (E)

10

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NAME ______________________ DATE ______________ PERIOD ________

# SAT and ACT Practice Answer Sheet

## (20 Questions)

1 (A) (B) (C) (D) (E)
2 (A) (B) (C) (D) (E)
3 (A) (B) (C) (D) (E)
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11-1

# Practice

## Rational Exponents

***Evaluate each expression.***

**1.** $\frac{8^{\frac{2}{3}}}{8^{\frac{1}{3}}}$ **2**

**2.** $\left(\frac{4}{5}\right)^{-2}$ **$\frac{25}{16}$**

**3.** $343^{\frac{2}{3}}$ **49**

**4.** $\sqrt[3]{8^3}$ **8**

**5.** $\sqrt{5} \cdot \sqrt{10}$ **$5\sqrt{2}$**

**6.** $9^{-\frac{1}{2}}$ **$\frac{1}{3}$**

***Simplify each expression.***

**7.** $(5n^3)^2 \cdot n^{-6}$ **25**

**8.** $\left(\frac{x^2}{4y^{-2}}\right)^{-\frac{1}{2}}$ **$2|x|^{-1}|y|^{-1}$ or $\frac{2}{|x||y|}$**

**9.** $(64x^6)^{\frac{1}{3}}$ **$4x^2$**

**10.** $(5x^6y^4)^{\frac{1}{2}}$ **$|x|^3y^2\sqrt{5}$**

**11.** $\sqrt{x^2y^3} \cdot \sqrt{x^3y^4}$ **$|x|^{\frac{5}{2}}|y|^{\frac{7}{2}}$ or $x^2|y|^3\sqrt{xy}$**

**12.** $\left(\frac{p^{6a}}{p^{-3a}}\right)^{\frac{1}{3}}$ **$p^{3a}$**

***Express each using rational exponents.***

**13.** $\sqrt{x^5y^6}$ **$|x|^{\frac{5}{2}}|y|^3$**

**14.** $\sqrt[5]{27x^{10}y^5}$ **$27^{\frac{1}{5}}x^2y$**

**15.** $\sqrt{144x^6y^{10}}$ **$12|x|^3|y|^5$**

**16.** $21\sqrt[3]{c^7}$ **$21c^{\frac{7}{3}}$**

**17.** $\sqrt{1024a^3}$ **$32a^{\frac{3}{2}}$**

**18.** $\sqrt[4]{36a^8b^5}$ **$6^{\frac{1}{2}}a^2b^{\frac{5}{4}}$**

***Express each using radicals.***

**19.** $64^{\frac{1}{3}}$ **$\sqrt[3]{64}$ or 4**

**20.** $2^{\frac{1}{2}}a^{\frac{3}{2}}b^{\frac{5}{2}}$ **$\sqrt{2a^3b^5}$ or $ab^2\sqrt{2ab}$**

**21.** $s^{\frac{2}{3}}t^{\frac{1}{3}}v^{\frac{2}{3}}$ **$\sqrt[3]{s^2tv^2}$**

**22.** $y^{\frac{3}{2}}$ **$\sqrt{y^3}$ or $y\sqrt{y}$**

**23.** $x^{\frac{2}{5}}y^{\frac{3}{5}}$ **$\sqrt[5]{x^2y^3}$**

**24.** $(x^6y^3)^{\frac{1}{2}}z^{\frac{3}{2}}$ **$\sqrt{x^6y^3z^3}$ or $|x|^3yz\sqrt{yz}$**



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11-1

# Enrichment

## Look for Cases

Many problems can be partitioned into a few cases. The cases are classifications within the problem that are exclusive one to another but which taken together comprise all the possibilities in the problem. Divisions into cases are made where the characteristics of the problem are most critical.

***Example*** **If $a > b$ and $\frac{1}{a} > \frac{1}{b}$, what must be true of $a$ and $b$?**

One of the critical characteristics of inequalities is that changing the signs of the left and right sides requires changing the sense of the inequality. This suggests classifying $a$ and $b$ according to signs. Since neither $a$ nor $b$ can equal zero in the second inequality and $a > b$, the cases are:

(i) $a$ and $b$ are both positive.
(ii) $a$ and $b$ are both negative.
(iii) $a$ is positive and $b$ is negative.

An important characteristic of reciprocals is that 1 and –1 are their own reciprocals. Therefore, within each of the above cases consider examples of these three subclassifications:

(A) $a$ and $b$ are both greater than 1 (or < –1).
(B) $a$ and $b$ are both fractions whose absolute values are less than one.
(C) $a$ is greater than 1 (or –1) and $b$ is less than 1 (or –1).

For case (i), consider these examples.

(A) $3 > 2 \rightarrow \frac{1}{3} < \frac{1}{2}$ (B) $\frac{1}{4} > \frac{1}{5} \rightarrow 4 < 5$ (C) $\frac{3}{2} > \frac{3}{4} \rightarrow \frac{2}{3} < \frac{4}{3}$

For case (ii), consider these examples.

(A) $-2 > -3 \rightarrow -\frac{1}{2} < -\frac{1}{3}$ (B) $-\frac{1}{5} > -\frac{1}{4} \rightarrow -5 < -4$

(C) $-\frac{3}{4} > -\frac{3}{2} \rightarrow -\frac{4}{3} < -\frac{2}{3}$

For case (iii), consider these examples.

(A) $2 > -3 \rightarrow \frac{1}{2} > -\frac{1}{3}$ (B) $\frac{1}{5} > -\frac{1}{4} \rightarrow 5 > -4$ (C) $\frac{3}{4} > -\frac{3}{2} \rightarrow \frac{4}{3} > -\frac{2}{3}$

Notice that the second given inequality, $\frac{1}{a} > \frac{1}{b}$, only holds true in case (iii). We conclude that $a$ must be positive and $b$ negative to satisfy both given inequalities.

***Complete.***

**1.** Find the positive values of $b$ such that $b^{x_1} > b^{x_2}$ whenever $x_1 < x_2$.
**$0 < b < 1$**

**2.** Solve $\frac{x}{x+1} > 0$. **$x > 0$ or $x < -1$**

**3.** Find the domain of $f(x) = \sqrt{3 - |x - 2|}$. **$\{x \mid -1 \le x \le 5\}$**

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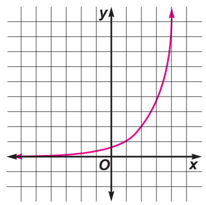
## 11-2 Practice

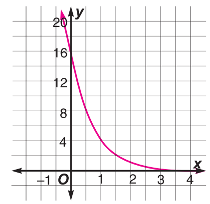
### Exponential Functions

***Graph each exponential function or inequality.***

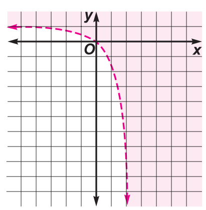
**1.** $y = 2^{x-1}$



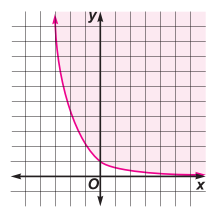

**2.** $y = 4^{-x+2}$




**3.** $y > -3^x + 1$




**4.** $y \geq 0.5^x$




**5.** ***Demographics*** An area in North Carolina known as The Triangle is principally composed of the cities of Durham, Raleigh, and Chapel Hill. The Triangle had a population of 700,000 in 1990. The average yearly rate of growth is 5.9%. Find the projected population for 2010. **about 2,203,014**

**6.** ***Finance*** Determine the amount of money in a savings account that provides an annual rate of 4% compounded monthly if the initial investment is \$1000 and the money is left in the account for 5 years. **about \$1221.00**

**7.** ***Investments*** How much money must be invested by Mr. Kaufman if he wants to have \$20,000 in his account after 15 years? He can earn 5% compounded quarterly.

**about \$9491.35**



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## 11-2 Enrichment

### Finding Solutions of $x^y = y^x$

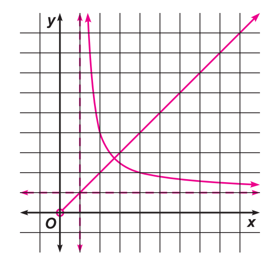
Perhaps you have noticed that if $x$ and $y$ are interchanged in equations such as $x = y$ and $xy = 1$, the resulting equation is equivalent to the original equation. The same is true of the equation $x^y = y^x$. However, finding solutions of $x^y = y^x$ and drawing its graph is not a simple process.

***Solve each problem. Assume that x and y are positive real numbers***

1. If $a > 0$, will $(a, a)$ be a solution of $x^y = y^x$? Justify your answer.
   **Yes, since $a^a = a^a$ must be true by the reflexive prop. of equality.**
2. If $c > 0$, $d > 0$, and $(c, d)$ is a solution of $x^y = y^x$, will $(d, c)$ also be a solution? Justify your answer.
   **Yes; replacing $x$ with $d$, $y$ with $c$ gives $d^c = c^d$; but if $(c, d)$ is a solution, $c^d = d^c$. So, by the symmetric property of equality, $d^c = c^d$ is true.**
3. Use 2 as a value for $y$ in $x^y = y^x$. The equation becomes $x^2 = 2^x$.
   **a.** Find equations for two functions, $f(x)$ and $g(x)$ that you could graph to find the solutions of $x^2 = 2^x$. Then graph the functions on a separate sheet of graph paper.
   **$f(x) = x^2$, $g(x) = 2^x$ See students' graphs.**
   **b.** Use the graph you drew for part **a** to state two solutions for $x^2 = 2^x$. Then use these solutions to state two solutions for $x^y = y^x$.
   **Sample answer: 2 or 4; (2, 2), (4, 2)**
4. In this exercise, a graphing calculator will be very helpful. Use the technique from Exercise 3 to complete the tables below. Then graph $x^y = y^x$ for positive values of $x$ and $y$. If there are asymptotes, show them in your diagram using dotted lines. Note that in the table, some values of $y$ call for one value of $x$, others call for two.

| $x$ | $y$ |
|---|---|
| **$\frac{1}{2}$** | $\frac{1}{2}$ |
| **$\frac{3}{4}$** | $\frac{3}{4}$ |
| **1** | 1 |
| **2** | 2 |
| **4** | 2 |
| **3** | 3 |
| **2.5** | 3 |

| $x$ | $y$ |
|---|---|
| **4** | 4 |
| **2** | 4 |
| **5** | 5 |
| **1.8** | 5 |
| **8** | 8 |
| **1.5** | 8 |



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11-3

# Practice

## The Number $e$

**1. *Demographics*** In 1995, the population of Kalamazoo, Michigan, was 79,089. This figure represented a 0.4% annual decline from 1990.

**a.** Let $t$ be the number of years since 1995 and write a function that models the population in Kalamazoo in 1995.
**$y = 79{,}089e^{-0.004t}$**

**b.** Predict the population in 2010 and 2015. Assume a steady rate of decline. **2010: 74,483; 2015: 73,008**

**2. *Biology*** Suppose a certain type of bacteria reproduces according to the model $P(t) = 100e^{0.271t}$, where $t$ is time in hours.

**a.** At what rate does this type of bacteria reproduce?
**27.1%**

**b.** What was the initial number of bacteria?
**100**

**c.** Find the number of bacteria at $P(5)$, $P(10)$, $P(24)$, and $P(72)$. Round to the nearest whole number.
**$P(5)$: 388**
**$P(10)$: 1503**
**$P(24)$: 66,781**
**$P(72)$: 29,782,004,910**

**3. *Finance*** Suppose Karyn deposits $1500 in a savings account that earns 6.75% interest compounded continuously. She plans to withdraw the money in 6 years to make a $2500 down payment on a car. Will there be enough funds in Karyn's account in 6 years to meet her goal?
**No. Karyn will have $2249 in her account in 6 years. She will be $251 short.**

**4. *Banking*** Given the original principal, the annual interest rate, the amount of time for each investment, and the type of compounded interest, find the amount at the end of the investment.

**a.** $P = \$1250$, $r = 8.5\%$, $t = 3$ years, semiannually
**$1604.60**

**b.** $P = \$2575$, $r = 6.25\%$, $t = 5$ years 3 months, continuously
**$3575.03**



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11-3

# Enrichment

## Approximations for $\pi$ and $e$

The following expression can be used to approximate $e$. If greater and greater values of $n$ are used, the value of the expression approximates $e$ more and more closely.

$$\left(1 + \frac{1}{n}\right)^n$$

Another way to approximate $e$ is to use this infinite sum. The greater the value of $n$, the closer the approximation.

$$e = 1 + 1 + \frac{1}{2} + \frac{1}{2 \cdot 3} + \frac{1}{2 \cdot 3 \cdot 4} + \ldots + \frac{1}{2 \cdot 3 \cdot 4 \cdot \ldots \cdot n} + \ldots$$

In a similar manner, $\pi$ can be approximated using an infinite product discovered by the English mathematician John Wallis (1616-1703).

$$\frac{\pi}{2} = \frac{2}{1} \cdot \frac{2}{3} \cdot \frac{4}{3} \cdot \frac{4}{5} \cdot \frac{6}{5} \cdot \frac{6}{7} \cdots \frac{2n}{2n-1} \cdot \frac{2n}{2n+1} \cdots$$

***Solve each problem.***

**1.** Use a calculator with an $e^x$ key to find $e$ to 7 decimal places.
**2.7182818**

**2.** Use the expression $\left(1 + \frac{1}{n}\right)^n$ to approximate $e$ to 3 decimal places. Use 5, 100, 500, and 7000 as values of $n$.
**2.488, 2.705, 2.716, 2.718**

**3.** Use the infinite sum to approximate $e$ to 3 decimal places. Use the whole numbers from 3 through 6 as values of $n$.
**2.667, 2.708, 2.717, 2.718**

**4.** Which approximation method approaches the value of $e$ more quickly?
**the infinite sum**

**5.** Use a calculator with a $\pi$ key to find $\pi$ to 7 decimal places.
**3.1415927**

**6.** Use the infinite product to approximate $\pi$ to 3 decimal places. Use the whole numbers from 3 through 6 as values of $n$.
**2.926, 2.972, 3.002, 3.023**

**7.** Does the infinite product give good approximations for $\pi$ quickly?
**no**

**8.** Show that $\pi^4 + \pi^5$ is equal to $e^6$ to 4 decimal places.
**To 4 decimal places, they both equal 403.4288.**

**9.** Which is larger, $e^\pi$ or $\pi^e$?
**$e^\pi > \pi^e$**

**10.** The expression $x^{\frac{1}{x}}$ reaches a maximum value at $x = e$. Use this fact to prove the inequality you found in Exercise 9.
**$e^{\frac{1}{e}} > \pi^{\frac{1}{\pi}}$; $\left(e^{\frac{1}{e}}\right)^{\pi e} > \left(\pi^{\frac{1}{\pi}}\right)^{\pi e}$; $e^\pi > \pi^e$**

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11-4

# Practice

## Logarithmic Functions

***Write each equation in exponential form.***

**1.** $\log_3 81 = 4$
**$3^4 = 81$**

**2.** $\log_8 2 = \frac{1}{3}$
**$8^{\frac{1}{3}} = 2$**

**3.** $\log_{10} \frac{1}{100} = -2$
**$10^{-2} = \frac{1}{100}$**

***Write each equation in logarithmic form.***

**4.** $3^3 = 27$
**$\log_3 27 = 3$**

**5.** $5^{-3} = \frac{1}{125}$
**$\log_5 \frac{1}{125} = -3$**

**6.** $\left(\frac{1}{4}\right)^{-4} = 256$
**$\log_{\frac{1}{4}} 256 = -4$**

***Evaluate each expression.***

**7.** $\log_7 7^3$
**3**

**8.** $\log_{10} 0.001$
**−3**

**9.** $\log_8 4096$
**4**

**10.** $\log_4 32$
**$\frac{5}{2}$**

**11.** $\log_3 1$
**0**

**12.** $\log_6 \frac{1}{216}$
**−3**

***Solve each equation.***

**13.** $\log_x 64 = 3$
**4**

**14.** $\log_4 0.25 = x$
**−1**

**15.** $\log_4 (2x - 1) = \log_4 16$
**$\frac{17}{2}$**

**16.** $\log_{10} \sqrt{10} = x$
**$\frac{1}{2}$**

**17.** $\log_7 56 - \log_7 x = \log_7 4$
**14**

**18.** $\log_5 (x + 4) + \log_5 x = \log_5 12$
**2**

**19.** ***Chemistry*** How long would it take 100,000 grams of radioactive iodine, which has a half-life of 60 days, to decay to 25,000 grams? Use the formula $N = N_0\left(\frac{1}{2}\right)^t$, where $N$ is the final amount of the substance, $N_0$ is the initial amount, and $t$ represents the number of half-lives. **120 days**



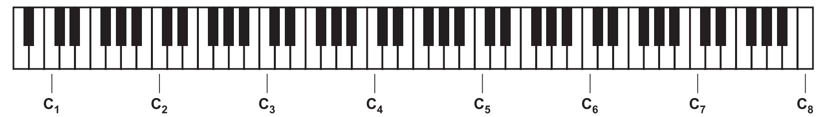
NAME ______________ DATE ________ PERIOD ______

11-4

# Enrichment

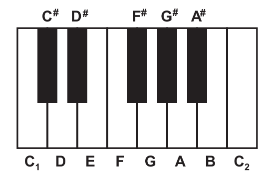
## Musical Relationships

The frequencies of notes in a musical scale that are one octave apart are related by an exponential equation. For the eight C notes on a piano, the equation is $C_n = C_1 2^{n-1}$, where $C_n$ represents the frequency of note $C_n$.




**1.** Find the relationship between $C_1$ and $C_2$. **$C_2 = 2C_1$**
**2.** Find the relationship between $C_1$ and $C_4$. **$C_4 = 8C_1$**

The frequencies of consecutive notes are related by a common ratio $r$. The general equation is $f_n = f_1 r^{n-1}$.

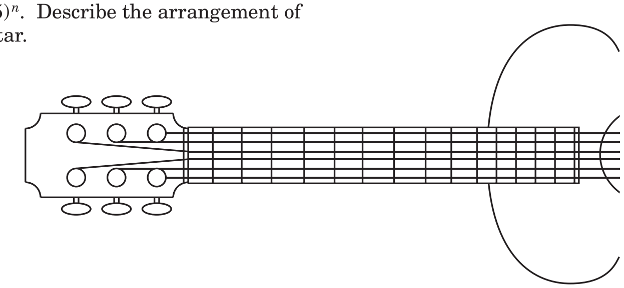



**3.** If the frequency of middle C is 261.6 cycles per second and the frequency of the next higher C is 523.2 cycles per second, find the common ratio $r$. (Hint: The two Cs are 12 notes apart.) Write the answer as a radical expression.
**$r = \sqrt[12]{2}$**

**4.** Substitute decimal values for $r$ and $f_1$ to find a specific equation for $f_n$.
**$f_n = 261.6\,(1.05946)^{n-1}$**

**5.** Find the frequency of F# above middle C.
**$f_7 = 261.6\,(1.05946)^6 \approx 369.95$**

**6.** The frets on a guitar are spaced so that the sound made by pressing a string against one fret has about 1.0595 times the wavelength of the sound made by using the next fret. The general equation is $w_n = w_0\,(1.0595)^n$. Describe the arrangement of the frets on a guitar.
**The frets are spaced in a logarithmic scale.**



NAME ______________ DATE ________ PERIOD ______

# 11-5 Practice

## Common Logarithms

***Given that log 3 = 0.4771, log 5 = 0.6990, and log 9 = 0.9542, evaluate each logarithm.***

**1.** $\log 300{,}000$ **5.4771**

**2.** $\log 0.0005$ **−3.3010**

**3.** $\log 9000$ **3.9542**

**4.** $\log 27$ **1.4313**

**5.** $\log 75$ **1.8751**

**6.** $\log 81$ **1.9084**

***Evaluate each expression.***

**7.** $\log 66.3$ **1.8215**

**8.** $\log \frac{17^4}{5}$ **4.2228**

**9.** $\log 7(4^3)$ **2.6513**

***Find the value of each logarithm using the change of base formula.***

**10.** $\log_6 832$ **3.7526**

**11.** $\log_{11} 47$ **1.6056**

**12.** $\log_3 9$ **2**

***Solve each equation or inequality.***

**13.** $8^x = 10$ **1.1073**

**14.** $2.4^x \leq 20$ **$x \leq 3.4219$**

**15.** $1.8^{x-5} = 19.8$ **10.0795**

**16.** $3^{5x} = 85$ **0.8088**

**17.** $4^{2x} > 25$ **$x > 1.1610$**

**18.** $3^{2x-2} = 2^x$ **1.4608**

**19.** ***Seismology*** The intensity of a shock wave from an earthquake is given by the formula $R = \log_{10} \frac{I}{I_0}$, where $R$ is the magnitude, $I$ is a measure of wave energy, and $I_0 = 1$. Find the intensity per unit of area for the following earthquakes.

**a.** Northridge, California, in 1994, $R = 6.7$ **about 5,011,872**

**b.** Hector Mine, California, in 1999, $R = 7.1$ **about 12,589,254**



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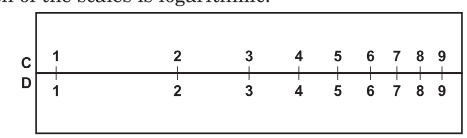
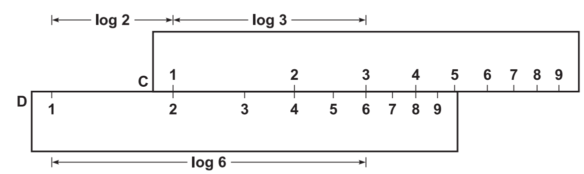
# 11-5 Enrichment

## The Slide Rule

Before the invention of electronic calculators, computations were often performed on a slide rule. A slide rule is based on the idea of logarithms. It has two movable rods labeled with C and D scales. Each of the scales is logarithmic.




To multiply $2 \times 3$ on a slide rule, move the C rod to the right as shown below. You can find $2 \times 3$ by adding log 2 to log 3, and the slide rule adds the lengths for you. The distance you get is 0.778, or the logarithm of 6.

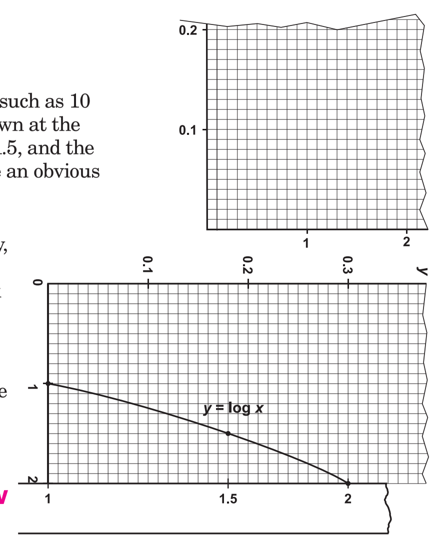



**1-2 See students' work.**

***Follow the steps to make a slide rule.***

**1.** Use graph paper that has small squares, such as 10 squares to the inch. Using the scales shown at the right, plot the curve $y = \log x$ for $x = 1$, 1.5, and the whole numbers from 2 through 10. Make an obvious heavy dot for each point plotted.

**2.** You will need two strips of cardboard. A 5-by-7 index card, cut in half the long way, will work fine. Turn the graph you made in Exercise 1 sideways and use it to mark a logarithmic scale on each of the two strips. The figure shows the mark for 2 being drawn.




**3.** Explain how to use a slide rule to divide 8 by 2. **Line up the 2 on the C scale with the 8 on the D scale. The quotient is the number on the D scale below the 1 on the C scale.**

NAME ______________ DATE ________ PERIOD ______

**11-6**

# Practice

## Natural Logarithms

***Evaluate each expression.***

**1.** ln 71
**4.2627**

**2.** ln 8.76
**2.1702**

**3.** ln 0.532
**−0.6311**

**4.** antiln −0.256
**0.7741**

**5.** antiln 4.62
**101.4940**

**6.** antiln −1.62
**0.1979**

***Convert each logarithm to a natural logarithm and evaluate.***

**7.** $\log_7 94$
**2.3348**

**8.** $\log_5 256$
**3.4454**

**9.** $\log_9 0.712$
**−0.1546**

***Use natural logarithms to solve each equation or inequality.***

**10.** $6^x = 42$
**2.0860**

**11.** $7^x = 4^{x+3}$
**7.4317**

**12.** $1249 = 175e^{-0.04t}$
**−49.1328**

**13.** $10^{x+1} > 3^x$
**$x > -1.9125$**

**14.** $12 < e^{0.048y}$
**$y > 51.7689$**

**15.** $8.4 < e^{t-2}$
**$t > 4.1282$**

**16.** ***Banking*** Ms. Cubbatz invested a sum of money in a certificate of deposit that earns 8% interest compounded continuously. The formula for calculating interest that is compounded continuously is $A = Pe^{rt}$. If Ms. Cubbatz made the investment on January 1, 1995, and the account was worth $12,000 on January 1, 1999, what was the original amount in the account? **$8713.79**



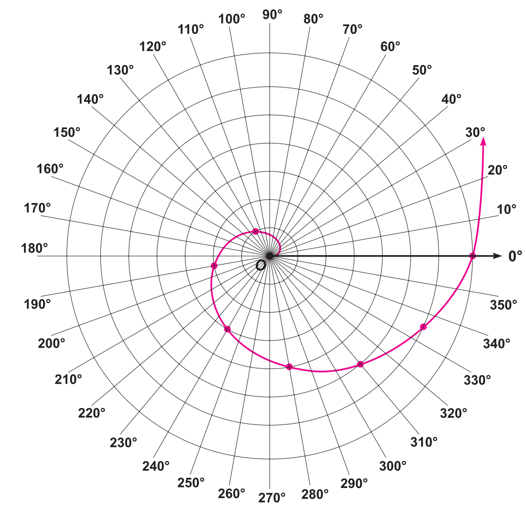
NAME ______________ DATE ________ PERIOD ______

**11-6**

# Enrichment

## Spirals

Consider an angle in standard position with its vertex at a point $O$ and its initial side on a polar axis. Remember that point $P$ on the terminal side of the angle can be named by $(r, \theta)$, where $r$ is the directed distance of the point from $O$ and $\theta$ is the measure of the angle. As you learned, graphs in this system may be drawn on polar coordinate paper such as the kind shown below.




**1.** Use a calculator to complete the table for $\log_2 r = \frac{\theta}{120}$. (To find $\log_2 a$ on a calculator, press LOG $a$ ÷ LOG 2). Round $\theta$ to the nearest degree if necessary.

| *r* | | 1 | 2 | 3 | 4 | 5 | 6 | 7 8 |
|---|---|---|---|---|---|---|---|---|
| *θ* | **0°** | **120°** | **190°** | **240°** | **279°** | **310°** | **337°** | **360°** |

**2.** Plot the points found in Exercise 1 on the grid above and connect them to form a smooth curve. **See graph above.**

This type of spiral is called a *logarithmic spiral* because the angle measures are proportional to the logarithms of the radii.

11-7

NAME ______________ DATE ________ PERIOD ____

## Practice

### Modeling Real-World Data with Exponential and Logarithmic Functions

***Find the amount of time required for an amount to double at the given rate if the interest is compounded continuously.***

**1.** 4.75%
**14.59 years**

**2.** 6.25%
**11.09 years**

**3.** 5.125%
**13.52 years**

**4.** 7.1%
**9.76 years**

**5.** ***City Planning*** At a recent town council meeting, proponents of increased spending claimed that spending should be doubled because the population of the city would double over the next three years. Population statistics for the city indicate that population is growing at the rate of 16.5% per year. Is the claim that the population will double in three years correct? Explain.
**No. To double in size, the population of the city would have to be increasing at the rate of 23.1% per year. The population of the city will double in 4.2 years.**

**6.** ***Conservation*** A wildlife conservation group released 14 black bears into a protected area. Their goal is to double the population of black bears every 4 years for the next 12 years.

**a.** If they are to meet their goal at the end of the first four years, what should be the yearly rate of increase in population?
**17.3%**

**b.** Suppose the group meets its goal. What will be the minimum number of black bears in the protected area in 12 years?
**There will be at least 112 black bears in the protected area.**

**c.** What type of model would best represent such data?
**An exponential model would best represent these data.**



11-7

NAME ______________ DATE ________ PERIOD ____

## Enrichment

### Hyperbolic Functions

The *hyperbolic functions* are a family of functions of great importance in calculus and higher-level mathematics. Because they are defined in terms of the hyperbola, their name is derived from that word. These functions have an interesting relationship to the number $e$ and to the trigonometric functions, uniting those seemingly unrelated subjects with the conic sections.

The hyperbolic functions can be written in terms of $e$.

Hyperbolic sine of $x$: $\sinh x = \frac{e^x - e^{-x}}{2}$

Hyperbolic cosine of $x$: $\cosh x = \frac{e^x + e^{-x}}{2}$

Hyperbolic tangent of $x$: $\tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$

Identities involving hyperbolic functions exhibit strong resemblances to trigonometric identities.

***Example*** **Show that $\sinh 2x = 2 \sinh x \cosh x$.**

$\sinh 2x = \frac{e^{2x} - e^{-2x}}{2}$ ← Replace $x$ in the definition above by $2x$.

$= 2\left(\frac{e^{2x} - e^{-2x}}{4}\right)$

$= 2\left(\frac{(e^x)^2 - (e^{-x})^2}{4}\right)$ ← difference of two squares

$= 2\left(\frac{e^x - e^{-x}}{2}\right)\left(\frac{e^x + e^{-x}}{2}\right)$

$= 2 \sinh x \cosh x$

**1.** Find $\cosh^2 x - \sinh^2 x$. **1**

***Prove each identity.***

**2.** $\sinh(-x) = -\sinh x$

$\sinh(-x) = \frac{e^{-x} - e^{x}}{2} = \frac{-(e^x - e^{-x})}{2} = -\left(\frac{e^x - e^{-x}}{2}\right) = -\sinh x$

**3.** $\sinh(x + y) = \sinh x \cosh y + \cosh x \sinh y$

$\sinh(x + y) = \frac{e^{x+y} - e^{-(x+y)}}{2}$

$= \frac{e^{x+y} - e^{-(x+y)}}{4} + \frac{e^{x+y} - e^{-(x+y)}}{4}$

$= \frac{e^x - e^{-x}}{2} \cdot \frac{e^y + e^{-y}}{2} + \frac{e^x + e^{-x}}{2} \cdot \frac{e^y - e^{-y}}{2}$

$= \sinh x \cosh y + \cosh x \sinh y$

# Chapter 11 Answer Key

## Form 1A

**Page 483**

1. B
2. C
3. A
4. C
5. B
6. A
7. C
8. B
9. D
10. B

**Page 484**

11. D
12. B
13. A
14. C
15. D
16. A
17. C
18. D
19. A
20. B

**Bonus:** A

## Form 1B

**Page 485**

1. B
2. C
3. A
4. D
5. D
6. B
7. A
8. C
9. A
10. C

**Page 486**

11. C
12. D
13. B
14. B
15. D
16. B
17. C
18. A
19. D
20. A

**Bonus:** B

# Chapter 11 Answer Key

## Form 1C

### Page 487

1. B
2. D
3. A
4. A
5. C
6. B
7. C
8. D
9. C
10. B

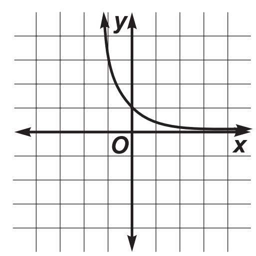
### Page 488

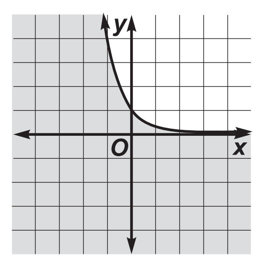
11. C
12. D
13. A
14. D
15. B
16. B
17. A
18. A
19. B
20. D

Bonus: C

## Form 2A

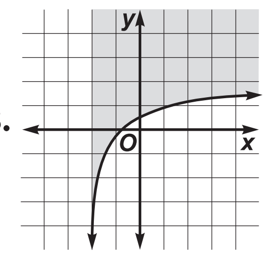
### Page 489

1. $-9$
2. $16|x|^3y^{\frac{5}{2}}$
3. $15x^{\frac{3}{4}}y^2$
4. $\sqrt[15]{6561x^{13}}$
5. 102.858
6. 

7. 

8. \$2110.67
9. 2.49 g
10. $\log_{\frac{1}{6}} 1296 = -4$

### Page 490

11. $-\frac{3}{4}$
12. 5
13. 

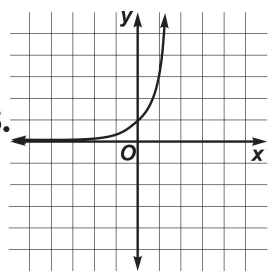
14. 2.776
15. $-7.432$
16. $7.943 \times 10^{-9}$
17. 2.971
18. $x > -7.456$
19. 8.66%
20. $\ln y = 0.0582x + 4.4657$

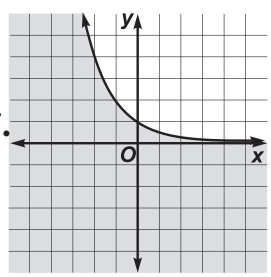
Bonus: $x^{\frac{3}{2}}$

# Chapter 11 Answer Key

## Form 2B

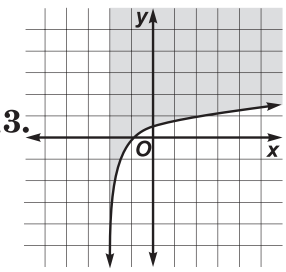
**Page 491**

1. $27$
2. $125x^6$
3. $2x^{\frac{3}{5}}y^2$
4. $4\sqrt[3]{xy^2}$
5. $156.993$
6. 

7. 

8. $399$
9. $11,778.72
10. $\log_{16} 8 = \frac{3}{4}$

**Page 492**

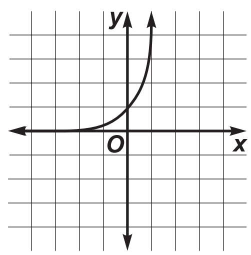
11. $-3$
12. $4$
13. 

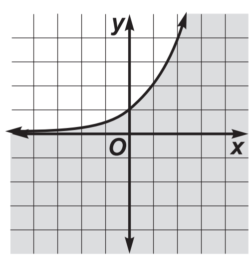
14. $0.051$
15. $4.120$
16. $-0.791$
17. $3.138$
18. $x < 1.148$
19. 11.180 years
20. $\ln y = 0.0774x + 3.8065$

Bonus: $\log_b \frac{a^2}{c}$

## Form 2C

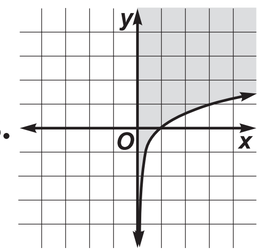
**Page 493**

1. $8$
2. $9y^2$
3. $5x^{\frac{5}{3}}$
4. $\sqrt[5]{x^2}$
5. $16.242$
6. 

7. 

8. $1737$
9. $18,281.88
10. $\log_5 25 = 2$

**Page 494**

11. $-2$
12. $4$
13. 

14. $2.033$
15. $3.182$
16. $3.505$
17. $3.712$
18. $x < 1.496$
19. 7.296 years
20. $y = 32.0703(1.0645)^x$

Bonus: $\log_b a^2c^3$

# Chapter 11 Answer Key

## CHAPTER 11 SCORING RUBRIC

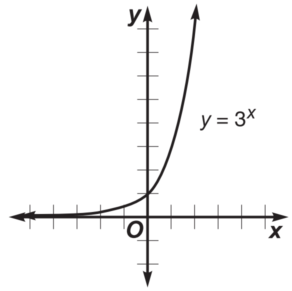
| Level | Specific Criteria |
|---|---|
| 3 Superior | • Shows thorough understanding of the concepts *exponential* and *logarithmic functions* and their *graphs.*<br>• Uses appropriate strategies to solve problems.<br>• Computations are correct.<br>• Written explanations are exemplary.<br>• Word problem concerng exponential equation is appropriate and makes sense.<br>• Graphs are accurate and appropriate.<br>• Goes beyond requirements of some or all problems. |
| 2 Satisfactory, with Minor Flaws | • Shows understanding of the concepts *exponential* and *logarithmic functions* and their graphs.<br>• Uses appropriate strategies to solve problems.<br>• Computations are mostly correct.<br>• Written explanations are effective.<br>• Word problem concerng exponential equation is appropriate and makes sense.<br>• Graphs are accurate and appropriate.<br>• Satisfies all requirements of problems. |
| 1 Nearly Satisfactory, with Serious Flaws | • Shows understanding of most of the concepts *exponential* and *logarithmic functions* and their *graphs.*<br>• May not use appropriate strategies to solve problems.<br>• Computations are mostly correct.<br>• Written explanations are satisfactory.<br>• Word problem concerning exponential equation is mostly appropriate and sensible.<br>• Graphs are mostly accurate and appropriate.<br>• Satisfies most requirements of problems. |
| 0 Unsatisfactory | • Shows little or no understanding of the concepts *exponential* and *logarithmic functions* and their *graphs.*<br>• May not use appropriate strategies to solve problems.<br>• Computations are incorrect.<br>• Written explanations are not satisfactory.<br>• Word problem concerning exponential equation is not appropriate or sensible.<br>• Graphs are not accurate or appropriate.<br>• Does not satisfy requirements of problems. |

# Chapter 11 Answer Key

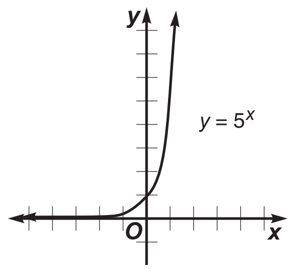
## Open-Ended Assessment

**Page 495**

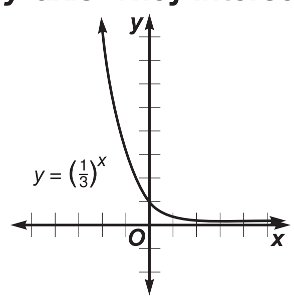
**1a.**



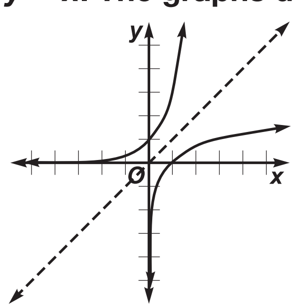

**1b.** For $x > 0$, $5^x > 3^x$. For $x < 0$, $5^x < 3^x$. For $x = 0$, $5^x = 3^x$. They intersect at $x = 0$.



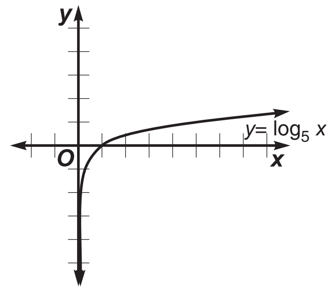

**1c.** The graph of $y = \left(\frac{1}{3}\right)^x$ is the reflection of $y = 3^x$ about the $y$-axis. They intersect at $x = 0$.




**1d.** The graph of $y = \log_3 x$ is the reflection of $y = 3^x$ over the line $y = x$. The graphs do not intersect.




**1e.** For $x < 1$, $\log_5 x > \log_3 x$.
For $x > 1$, $\log_5 x < \log_3 x$.
For $x = 1$, $\log_5 x = \log_3 x$.
They intersect at $x = 1$.




**1f.** The graph of $y = \log_8 x$ is the graph of $y = \log_2 x$ compressed vertically by a factor of $\frac{1}{3}$. By the Change of Base Formula, $\log_8 x = \frac{\log_2 x}{\log_2 8}$, or $\frac{1}{3}\log_2 x$.

**2.** Sample answer: A colony of nine bacteria doubles every minute. When will the population of the colony be 178?

$$178 = 9 \cdot 2^x$$
$$\log 178 = \log 9 + x \log 2$$
$$x = \frac{\log 178 - \log 9}{\log 2}, \text{ or about } 4.3$$

The colony will number 178 in about 4.3 minutes.

**3.** $\log_2 (x + 3) + \log_2 (x - 2) = 3$

$\log_2 [(x + 3)(x - 2)] = 3$ *Product Property*

$(x + 3)(x - 2) = 2^3$ *Definition of logarithm*

$x^2 + x - 6 = 8$ *Multiply.*

$x = \frac{-1 \pm \sqrt{1 + 56}}{2}$

$= \frac{-1 + \sqrt{57}}{2}$

The solution $x = \frac{-1 - \sqrt{57}}{2}$ is extraneous because it makes both logarithms undefined.

**4.** $e^{2x} - 3e^x + 2 = 0$

$(e^x - 1)(e^x - 2) = 0$ *Factor.* $e^{2x} = e^x \cdot e^x$

$e^x - 1 = 0$ or $e^x - 2 = 0$

$e^x = 1$ $\quad$ $e^x = 2$

$x = \ln 1$, or 0 $\quad$ $x = \ln 2$ *Definition of logarithm*

**5.** Sample answer: Let the two positive numbers be 5.36 and 8.45.

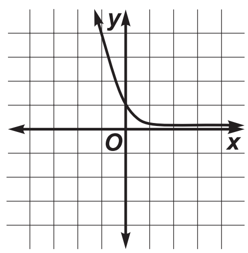
$\ln (5.36 \times 8.45) = \ln 5.36 + \ln 8.45$
$\approx 1.6790 + 2.1342$
$\approx 3.8132$

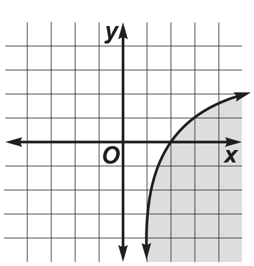
The product is
$5.36 \times 8.45 \approx e^{3.8132}$
$\approx 45.295$

The actual product is 45.292. The difference is due to rounding.

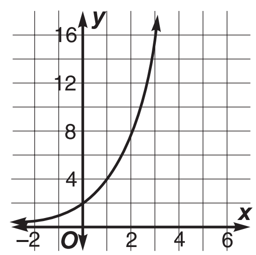
# Chapter 11 Answer Key

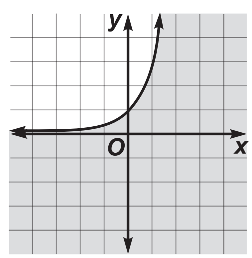
## Mid-Chapter Test
**Page 496**

1. 2
2. $-\frac{1}{4}$
3. 30
4. $2x^{\frac{2}{3}}y^2$
5. 451.808
6. 

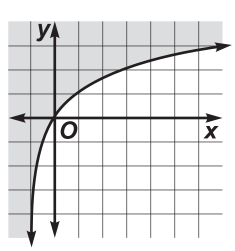
7. 295
8. No; she will have only $2329.06.
9. 17,239
10. $-3$
11. 8
12. 


## Quiz A
**Page 497**

1. $\frac{1}{5}$
2. $\frac{15}{4}$
3. $\sqrt[7]{16}$
4. 

5. 


## Quiz B
**Page 497**

1. $16,976.03
2. $\log_3 \frac{1}{81} = -4$
3. 6
4. 9
5. 


## Quiz C
**Page 498**

1. 2.289
2. 0.775
3. 4.6021
4. 2.806
5. 0.799

## Quiz D
**Page 498**

1. 7.296 years
2. 13.863 years
3. $\ln y = 0.0677x + 3.3983$

# Chapter 11 Answer Key

## SAT/ACT Practice

**Page 499**

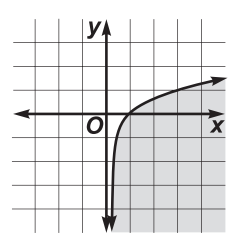
1. B
2. E
3. D
4. E
5. D
6. C
7. A
8. E
9. D
10. E

**Page 500**

11. A
12. B
13. B
14. B
15. E
16. A
17. B
18. D
19. 15
20. 27

## Cumulative Review

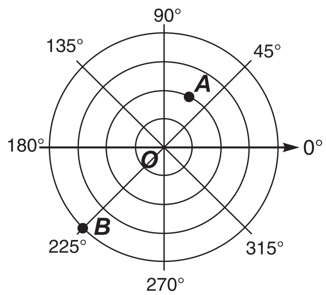
**Page 501**

1. 31
2. (4, −1)
3. As $x \to \infty$, $y \to \infty$
   As $x \to -\infty$, $y \to \infty$
4. 40
5. 0
6. 4; $\pi$; $3\pi$
7. $\left(2, \frac{\pi}{3}\right)$
8. $x^2 + (y - 2)^2 = 9$
9. $\log_5 \frac{1}{125} = -3$
10. 

# Unit 3 Answer Key

## Unit 3 Review

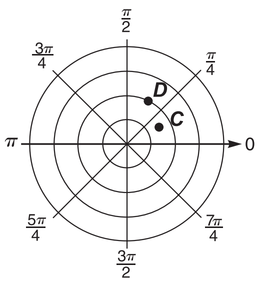
**Graph for Exercises 1 and 2**




1. Sample answer: (2, −300°), (−2, −120°), (−2, 240°)

2. Sample answer: (−4, −315°), (4, −135°), (4, 225°)

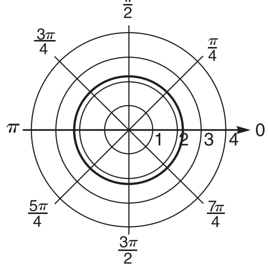
**Graph for Exercises 3 and 4**



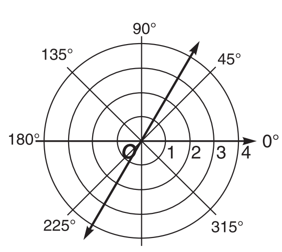

3. Sample answer: $\left(1.5, -\frac{11\pi}{6}\right)$, $\left(-1.5, -\frac{5\pi}{6}\right)$, $\left(-1.5, \frac{7\pi}{6}\right)$

4. Sample answer: $\left(-2, \frac{4\pi}{3}\right)$, $\left(2, \frac{\pi}{3}\right)$, $\left(2, -\frac{5\pi}{3}\right)$

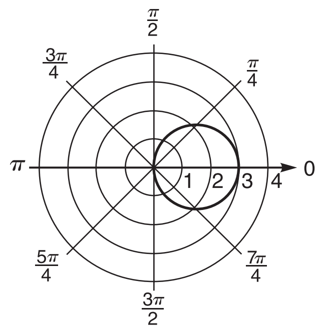
5. circle



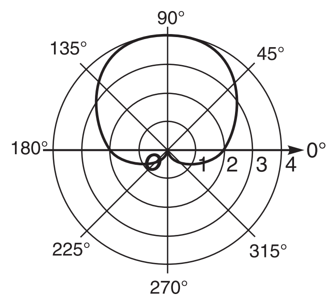
6. line



7. circle



8. cardioid



9. $\left(2\sqrt{2}, \frac{5\pi}{4}\right)$

10. $\left(2\sqrt{2}, \frac{\pi}{4}\right)$

11. (3.61, 5.30)

12. (3.16, 2.82)

13. $y = 2$

14. $x - \sqrt{3}y - 8 = 0$

15. $i$

16. 6

17. $3 + 3i$

18. 26

19. $\frac{1}{13} + \frac{8}{13}i$

20. $3\left(\cos \frac{3\pi}{2} + i \sin \frac{3\pi}{2}\right)$

21. $3\sqrt{2}\left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4}\right)$

22. $\sqrt{10}(\cos 1.89 + i \sin 1.89)$

23. $\sqrt{41}[\cos 5.39 + i \sin 5.39]$

24. −8

25. $-2.69 - 1.33i$

26. $8 - 8i$

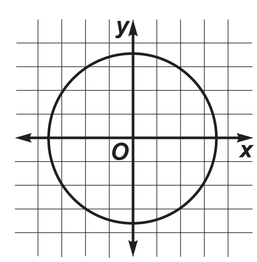
27. $1$, $0.31 + 0.95i$, $-0.81 + 0.59i$, $-0.81 - 0.59i$, $0.31 - 0.95i$

28. $\sqrt{241}$; (3.5, 4)

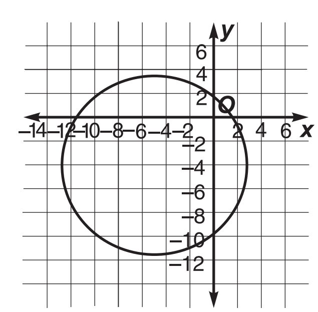
29. $2\sqrt{2}$; (4, −1)

30. 13; (−1.5, 3)

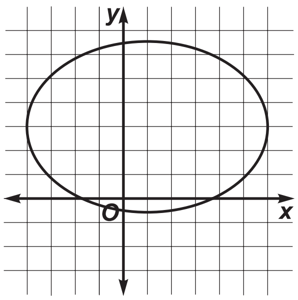
31. (0, 0); $\frac{7}{2}$



32. (−5, −4); $\sqrt{61}$

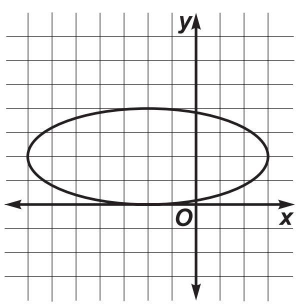


**33.**



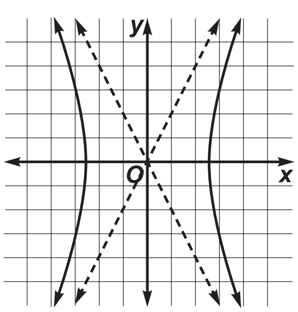

center: (1, 3); foci: $\left(1 \pm \frac{5\sqrt{2}}{2}, 3\right)$; vertices: (6, 3), (−4, 3), $\left(1, 3 \pm \frac{5\sqrt{2}}{2}\right)$

**34.**



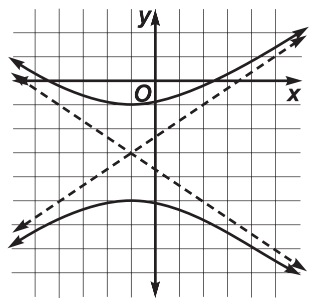

center: (−2, 2); foci: $(-2 \pm \sqrt{21}, 2)$; vertices: (−7, 2), (3, 2), (−2, 4), (−2, 0)

**35.**



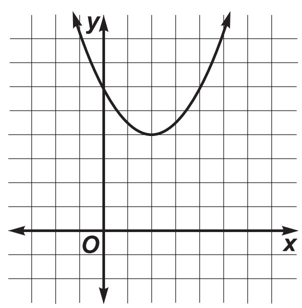

center: (0, 0); foci: $\left(\pm\frac{3\sqrt{15}}{2}, 0\right)$; vertices: $\left(\pm\frac{3\sqrt{3}}{2}, 0\right)$; asymptotes: $y = \pm 2x$

**36.**



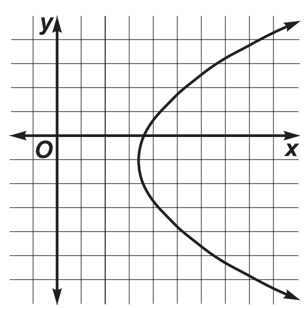

center: (−1, −3); foci: $(-1, -3 \pm \sqrt{13})$; vertices: (−1, −1), (−1, −5); asymptotes: $y + 3 = \pm\frac{2}{3}(x + 1)$

**37.**



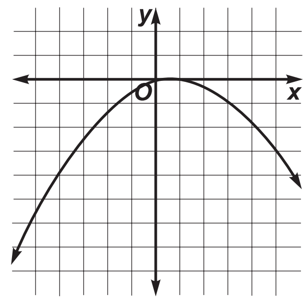

vertex: (2, 4); focus: (2, 4.5); directrix: $y = 3.5$, axis of symmetry: $x = 2$

**38.**

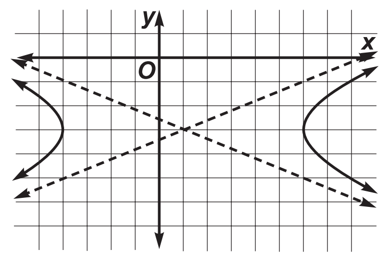



vertex: (3.4, −1); focus: (4.65, −1); directrix: $x = 2.15$; axis of symmetry: $y = -1$

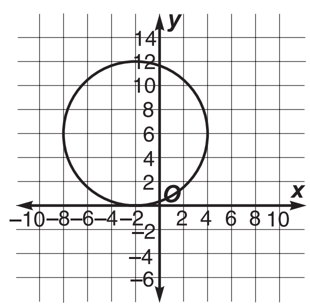
**39.** parabola



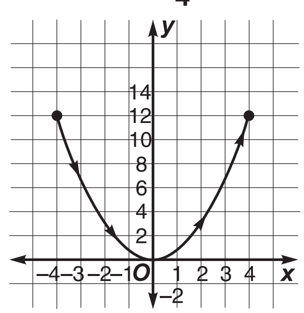

**40.** hyperbola




**41.** circle

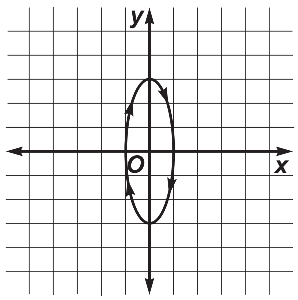



**42.** $y = \frac{3x^2}{4}$



# Unit 3 Answer Key (continued)

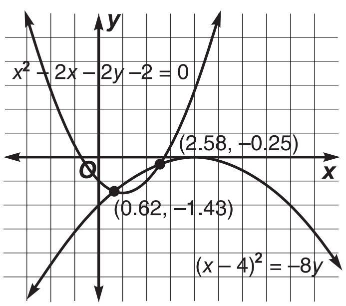
43. $x^2 + \frac{y^2}{9} = 1$



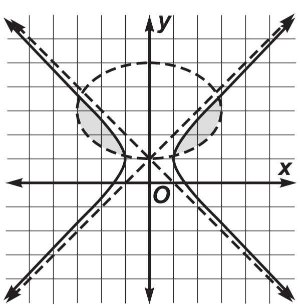

44. $3y^2 - x + 42y + 149 = 0$

45. $13(x')^2 + 34\sqrt{3}x'y' + 47(y')^2 - 64 = 0$

46. (2.58, −0.25), (0.62, −1.43)




47.

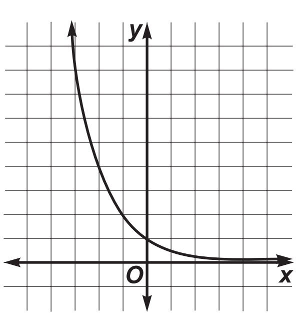



48. $4|x|y^3\sqrt{y}$

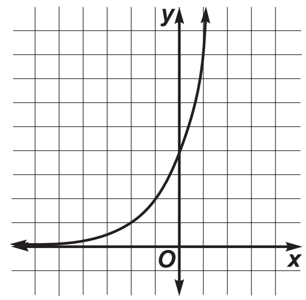
49. $3abc^2\sqrt[3]{2ac^2}$

50. $d\sqrt[5]{9c^3}$ 51. $\frac{1}{x^2}$

52.




53.




54. 17,492 55. 12,997

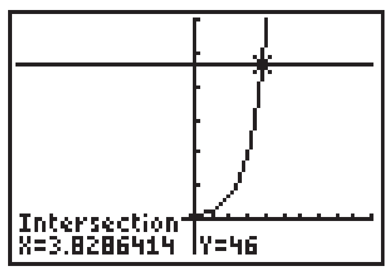
56. 6 57. $\frac{27}{2}$ 58. 36

59. 3.3856 60. −6.2854

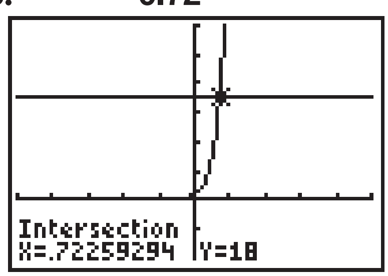
61. −0.5776 62. 3.79

63. 2 64. 3.51

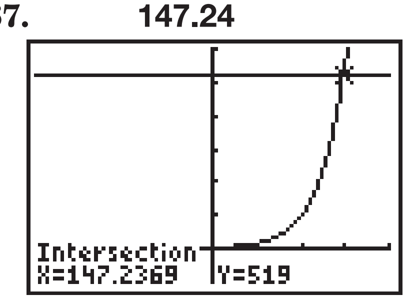
65. 3.83




66. 0.72




67. 147.24




## Unit 3 Test

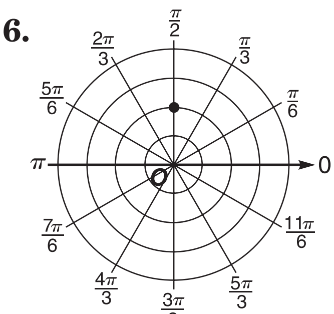
1. $\frac{1}{2}$

2. ellipse

3. $\left(-\frac{1}{2}, \frac{5}{2}\right)$; $x = -\frac{1}{2}$

4. $r = \pm\sqrt{6}$ 5. $\frac{1}{13} - \frac{5}{13}i$

6.




7. 8.96 8. $\frac{1}{4913}$

9. $x^2 + 4y^2 = 4$

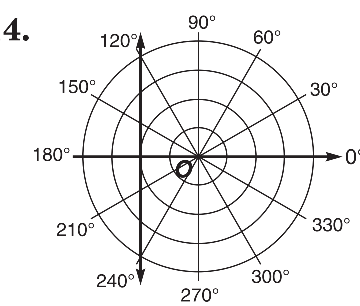
10. \$5704.70

11. $8(\cos 30° + i \sin 30°)$; $4\sqrt{3} + 4i$

12. cardioid

13. $(x + 3)^2 + (y + 1)^2 = 34$

14.



# Unit 3 Answer Key (continued)

15. 0.338

16. $2(\cos 30° + i \sin 30°)$; $\sqrt{3} + i$

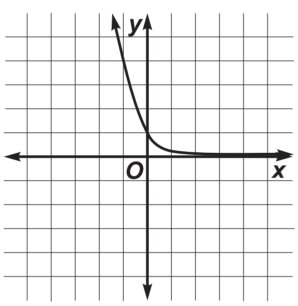
17. $7\sqrt{2}$

18. $\frac{x^2}{1} + \frac{y^2}{4} = 1$

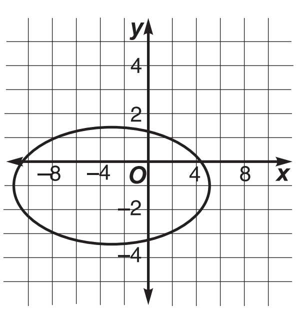
19. $4012

20. 3.6082

21.




22. center: $(-3, -1)$; foci: $(-3 \pm \sqrt{59}, -1)$; vertices: $(5, -1)$, $(-11, -1)$, $(-3, -1 \pm \sqrt{5})$




23. $(-15.32, 12.86)$

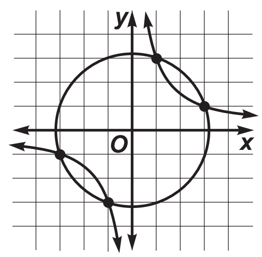
24. $(x - 2)^2 + (y + 4)^2 = 16$

25. $-64$

26. $2ay^{\frac{5}{3}}$

27. $3 - 8i$

28. $(3, 1), (-3, -1), (1, 3), (-1, -3)$




29. $8\left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}\right)$

30. $-0.2711$

31. hyperbola; $4x^2 - 25y^2 - 40x - 100y - 100 = 0$

32. $\sqrt{2}x - \sqrt{2}y - 6 = 0$

33. false

34. $\frac{14\sqrt{5}}{15} = r \cos(\theta - 117°)$

35. $y\sqrt[3]{x^2y^2z}$

36. $343^{\frac{1}{3}} = 7$

37. $(x - 8)^2 + (y - 8)^2 = 64$; $(8, 8)$; 8

38. 0.9522

39. $\frac{(y + 2)^2}{9} - \frac{(x + 1)^2}{16} = 1$

40. $\frac{1}{4} + \frac{\sqrt{3}}{4}i$

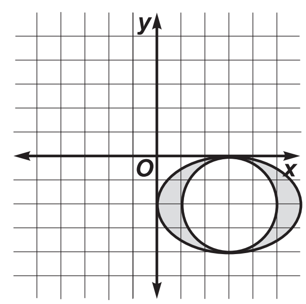
41. $(3, 4)$; $x = -1$

42. $x > 1.82$

43. $1 - \sqrt{3}\,i$

44. $\frac{(y + 2)^2}{36} + \frac{(x - 3)^2}{25} = 1$

45.




46. $y = x$

47. 11.55%

48. $\log_2 64 = 6$

49. $16 - 11i$

50. center: $(-1, 0)$; foci: $(-1 \pm \sqrt{10}, 0)$; vertices: $(-1 \pm \sqrt{2}, 0)$; asymptotes: $y = \pm 2(x + 1)$

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