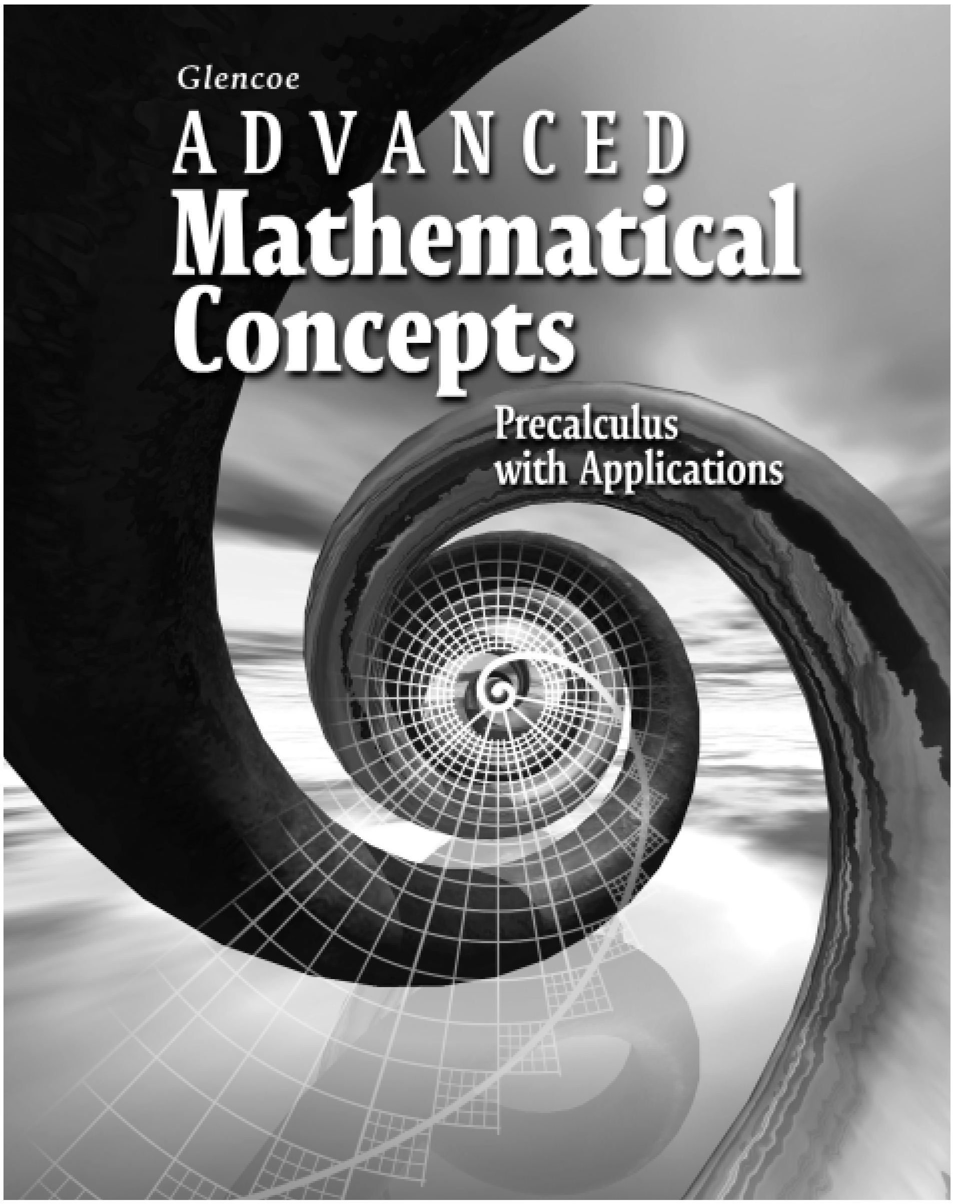
Chapter 10
Resource Masters



Glencoe

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StudentWorks™ This CD-ROM includes the entire Student Edition along with the Study Guide, Practice, and Enrichment masters.

TeacherWorks™ All of the materials found in this booklet are included for viewing and printing in the *Advanced Mathematical Concepts TeacherWorks* CD-ROM.

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Advanced Mathematical Concepts
Chapter 10 Resource Masters

1 2 3 4 5 6 7 8 9 10 XXX 11 10 09 08 07 06 05 04

Contents

Vocabulary Builder . . . . vii-viii

Lesson 10-1
Study Guide **417**
Practice **418**
Enrichment **419**

Lesson 10-2
Study Guide **420**
Practice **421**
Enrichment **422**

Lesson 10-3
Study Guide **423**
Practice **424**
Enrichment **425**

Lesson 10-4
Study Guide **426**
Practice **427**
Enrichment **428**

Lesson 10-5
Study Guide **429**
Practice **430**
Enrichment **431**

Lesson 10-6
Study Guide **432**
Practice **433**
Enrichment **434**

Lesson 10-7
Study Guide **435**
Practice **436**
Enrichment **437**

Lesson 10-8
Study Guide **438**
Practice **439**
Enrichment **440**

Chapter 10 Assessment
Chapter 10 Test, Form 1A **441-442**
Chapter 10 Test, Form 1B **443-444**
Chapter 10 Test, Form 1C **445-446**
Chapter 10 Test, Form 2A **447-448**
Chapter 10 Test, Form 2B **449-450**
Chapter 10 Test, Form 2C **451-452**
Chapter 10 Extended Response
Assessment **453**
Chapter 10 Mid-Chapter Test **454**
Chapter 10 Quizzes A & B **455**
Chapter 10 Quizzes C & D **456**
Chapter 10 SAT and ACT Practice **457-458**
Chapter 10 Cumulative Review **459**

SAT and ACT Practice Answer Sheet,
10 Questions **A1**
SAT and ACT Practice Answer Sheet,
20 Questions **A2**
ANSWERS **A3-A17**

A Teacher's Guide to Using the Chapter 10 Resource Masters

The ***Fast File*** Chapter Resource system allows you to conveniently file the resources you use most often. The *Chapter 10 Resource Masters* include the core materials needed for Chapter 10. These materials include worksheets, extensions, and assessment options. The answers for these pages appear at the back of this booklet.

All of the materials found in this booklet are included for viewing and printing in the *Advanced Mathematical Concepts TeacherWorks* CD-ROM.

Vocabulary Builder Pages vii-viii include a student study tool that presents the key vocabulary terms from the chapter. Students are to record definitions and/or examples for each term. You may suggest that students highlight or star the terms with which they are not familiar.

When to Use Give these pages to students before beginning Lesson 10-1. Remind them to add definitions and examples as they complete each lesson.

Study Guide There is one Study Guide master for each lesson.

When to Use Use these masters as reteaching activities for students who need additional reinforcement. These pages can also be used in conjunction with the Student Edition as an instructional tool for those students who have been absent.

Practice There is one master for each lesson. These problems more closely follow the structure of the Practice section of the Student Edition exercises. These exercises are of average difficulty.

When to Use These provide additional practice options or may be used as homework for second day teaching of the lesson.

Enrichment There is one master for each lesson. These activities may extend the concepts in the lesson, offer a historical or multicultural look at the concepts, or widen students' perspectives on the mathematics they are learning. These are not written exclusively for honors students, but are accessible for use with all levels of students.

When to Use These may be used as extra credit, short-term projects, or as activities for days when class periods are shortened.

Assessment Options

The assessment section of the *Chapter 10 Resources Masters* offers a wide range of assessment tools for intermediate and final assessment. The following lists describe each assessment master and its intended use.

Chapter Assessments

Chapter Tests

- *Forms 1A, 1B, and 1C* Form 1 tests contain multiple-choice questions. Form 1A is intended for use with honors-level students, Form 1B is intended for use with average-level students, and Form 1C is intended for use with basic-level students. These tests are similar in format to offer comparable testing situations.

- *Forms 2A, 2B, and 2C* Form 2 tests are composed of free-response questions. Form 2A is intended for use with honors-level students, Form 2B is intended for use with average-level students, and Form 2C is intended for use with basic-level students. These tests are similar in format to offer comparable testing situations.

  All of the above tests include a challenging Bonus question.

- The **Extended Response Assessment** includes performance assessment tasks that are suitable for all students. A scoring rubric is included for evaluation guidelines. Sample answers are provided for assessment.

Intermediate Assessment

- A **Mid-Chapter Test** provides an option to assess the first half of the chapter. It is composed of free-response questions.

- Four free-response **quizzes** are included to offer assessment at appropriate intervals in the chapter.

Continuing Assessment

- The **SAT and ACT Practice** offers continuing review of concepts in various formats, which may appear on standardized tests that they may encounter. This practice includes multiple-choice, quantitative-comparison, and grid-in questions. Bubble-in and grid-in answer sections are provided on the master.

- The **Cumulative Review** provides students an opportunity to reinforce and retain skills as they proceed through their study of advanced mathematics. It can also be used as a test. The master includes free-response questions.

Answers

- Page A1 is an answer sheet for the SAT and ACT Practice questions that appear in the Student Edition on page 693. Page A2 is an answer sheet for the SAT and ACT Practice master. These improve students' familiarity with the answer formats they may encounter in test taking.

- The answers for the lesson-by-lesson masters are provided as reduced pages with answers appearing in red.

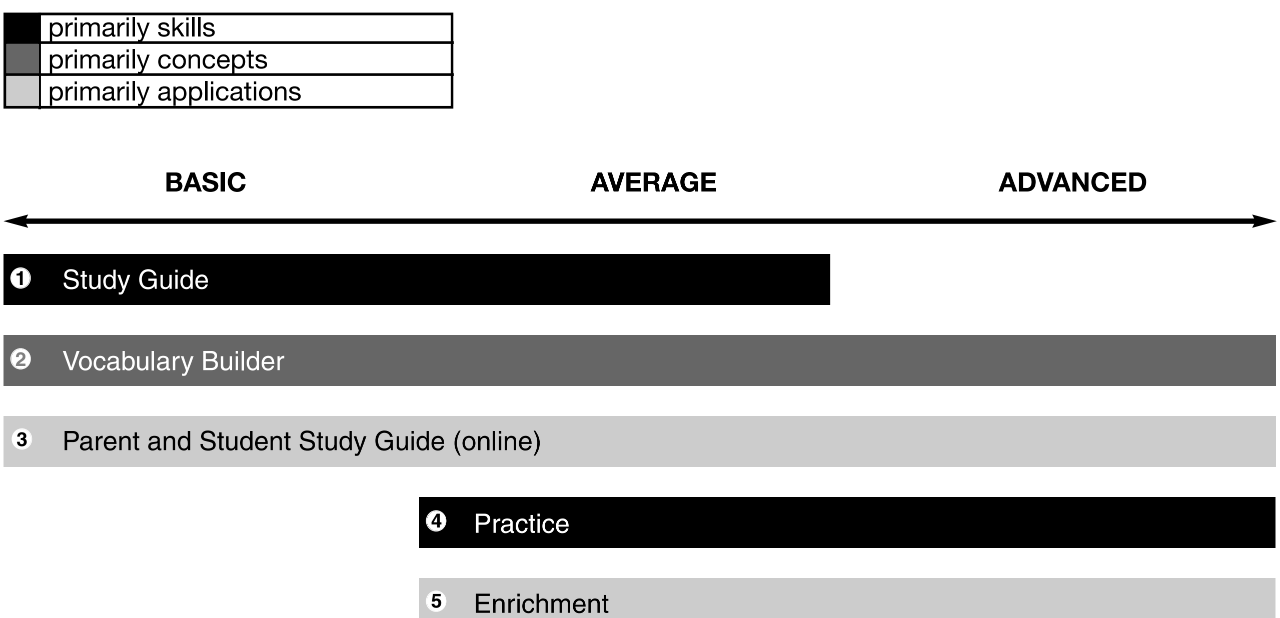
- Full-size answer keys are provided for the assessment options in this booklet.

Chapter 10 Leveled Worksheets

Glencoe's **leveled worksheets** are helpful for meeting the needs of every student in a variety of ways. These worksheets, many of which are found in the **FAST FILE Chapter Resource Masters**, are shown in the chart below.

- **Study Guide** masters provide worked-out examples as well as practice problems.
- Each chapter's **Vocabulary Builder** master provides students the opportunity to write out key concepts and definitions in their own words.
- **Practice** masters provide average-level problems for students who are moving at a regular pace.
- **Enrichment** masters offer students the opportunity to extend their learning.

Five Different Options to Meet the Needs of Every Student in a Variety of Ways



NAME ______________ DATE ______________ PERIOD ______________

Chapter 10

Reading to Learn Mathematics

Vocabulary Builder

This is an alphabetical list of the key vocabulary terms you will learn in Chapter 10. As you study the chapter, complete each term's definition or description. Remember to add the page number where you found the term.

Vocabulary Term	Found on Page	Definition/Description/Example
analytic geometry		
asymptotes		
axis of symmetry		
center		
conic section		
conjugate axis		
degenerate conic		
directrix		
eccentricity		
ellipse		

(continued on the next page)

NAME ______________________________ DATE ______________ PERIOD ________

Reading to Learn Mathematics

Vocabulary Builder *(continued)*

Vocabulary Term	Found on Page	Definition/Description/Example
equilateral hyperbola		
focus		
hyperbola		
locus		
major axis		
minor axis		
rectangular hyperbola		
semi-major axis		
semi-minor axis		
transverse axis		
vertex		

NAME ______________________ DATE ______________ PERIOD ________

Study Guide

Introduction to Analytic Geometry

Example 1 **Find the distance between points at (−2, 2) and (5, −4). Then find the midpoint of the segment that has endpoints at the given coordinates.**

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ *Distance Formula*

$d = \sqrt{[5 - (-2)]^2 + [(-4) - 2]^2}$ *Let $(x_1, y_1) = (-2, 2)$ and $(x_2, y_2) = (5, -4)$.*

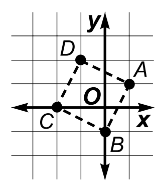
$d = \sqrt{7^2 + (-6)^2}$ or $\sqrt{85}$

midpoint $= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ *Midpoint Formula*

The midpoint is at $\left(\frac{-2 + 5}{2}, \frac{2 + (-4)}{2}\right)$ or $\left(\frac{3}{2}, -1\right)$.

Example 2 **Determine whether quadrilateral *ABCD* with vertices *A*(1, 1), *B*(0, −1), *C*(−2, 0), and *D*(−1, 2) is a parallelogram.**

First, graph the figure.



To determine if $\overline{DA} \parallel \overline{CB}$, find the slopes of $\overline{DA}$ and $\overline{CB}$.

slope of $\overline{DA}$

$m = \frac{y_2 - y_1}{x_2 - x_1}$ *Slope formula*

$= \frac{1 - 2}{1 - (-1)}$ *D(−1, 2) and A(1, 1)*

$= -\frac{1}{2}$

slope of $\overline{CB}$

$m = \frac{y_2 - y_1}{x_2 - x_1}$ *Slope formula*

$= \frac{-1 - 0}{0 - (-2)}$ *C(−2, 0) and B(0, −1)*

$= -\frac{1}{2}$

Their slopes are equal. Therefore, $\overline{DA} \parallel \overline{CB}$.

To determine if $\overline{DA} \cong \overline{CB}$, use the distance formula to find $\overline{DA}$ and $\overline{CB}$.

$DA = \sqrt{[1 - (-1)]^2 + (1 - 2)^2}$
$= \sqrt{5}$

$CB = \sqrt{[0 - (-2)]^2 + (-1 - 0)^2}$
$= \sqrt{5}$

The measures of $\overline{DA}$ and $\overline{CB}$ are equal. Therefore, $\overline{DA} \cong \overline{CB}$.

Since $\overline{DA} \parallel \overline{CB}$ and $\overline{DA} \cong \overline{CB}$, quadrilateral *ABCD* is a parallelogram.

NAME ______________________________ DATE ______________ PERIOD ________

Practice

Introduction to Analytic Geometry

Find the distance between each pair of points with the given coordinates. Then find the midpoint of the segment that has endpoints at the given coordinates.

1. $(-2, 1), (3, 4)$

2. $(1, 1), (9, 7)$

3. $(3, -4),(5, 2)$

4. $(-1, 2), (5, 4)$

5. $(-7, -4), (2, 8)$

6. $(-4, 10), (4, -5)$

Determine whether the quadrilateral having vertices with the given coordinates is a parallelogram.

7. $(4, 4), (2, -2), (-5, -1), (-3, 5)$

8. $(3, 5), (-1, 1), (-6, 2), (-3, 7)$

9. $(4, -1), (2, -5), (-3, -3), (-1, 1)$

10. $(2, 6), (1, 2), (-4, 4), (-3, 9)$

11. ***Hiking*** Jenna and Maria are hiking to a campsite located at (2, 1) on a map grid, where each side of a square represents 2.5 miles. If they start their hike at $(-3, 1)$, how far must they hike to reach the campsite?

NAME ______________________________ DATE _______________ PERIOD ________

10-1 Enrichment

Mathematics and History: Hypatia

Hypatia (A.D. 370–415) is the earliest woman mathematician whose life is well documented. Born in Alexandria, Egypt, she was widely known for her keen intellect and extraordinary mathematical ability. Students from Europe, Asia, and Africa flocked to the university at Alexandria to attend her lectures on mathematics, astronomy, philosophy, and mechanics.

Hypatia wrote several major treatises in mathematics. Perhaps the most significant of these was her commentary on the *Arithmetica* of Diophantus, a mathematician who lived and worked in Alexandria in the third century. In her commentary, Hypatia offered several observations about the *Arithmetica's* Diophantine problems—problems for which one was required to find only the rational solutions. It is believed that many of these observations were subsequently incorporated into the original manuscript of the *Arithmetica*.

In modern mathematics, the solutions of a **Diophantine equation** are restricted to integers. In the exercises, you will explore some questions involving simple Diophantine equations.

For each equation, find three solutions that consist of an ordered pair of integers.

1. $2x - y = 7$

2. $x + 3y = 5$

3. $6x - 5y = -8$

4. $-11x - 4y = 6$

5. Refer to your answers to Exercises 1–4. Suppose that the integer pair (x_1, y_1) is a solution of $Ax - By = C$. Describe how to find other integer pairs that are solutions of the equation.

6. Explain why the equation $3x + 6y = 7$ has no solutions that are integer pairs.

7. *True* or *false*: Any line on the coordinate plane must pass through at least one point whose coordinates are integers. Explain.

NAME ______________________ DATE ______________ PERIOD ________

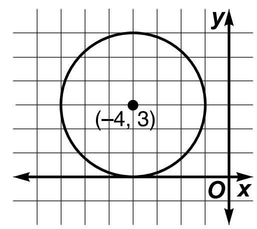
Study Guide

Circles

The standard form of the equation of a **circle** with **radius** r and **center** at (h, k) is $(x - h)^2 + (y - k)^2 = r^2$.

Example 1 **Write the standard form of the equation of the circle that is tangent to the x-axis and has its center at $(-4, 3)$. Then graph the equation.**

Since the circle is tangent to the x-axis, the distance from the center to the x-axis is the radius. The center is 3 units above the x-axis. Therefore, the radius is 3.



$(x - h)^2 + (y - k)^2 = r^2$ *Standard form*

$[x - (-4)]^2 + (y - 3)^2 = 3^2$ *$(h, k) = (-4, 3)$ and $r = 3$*

$(x + 4)^2 + (y - 3)^2 = 9$

Example 2 **Write the standard form of the equation of the circle that passes through the points at $(1, -1)$, $(5, 3)$, and $(-3, 3)$. Then identify the center and radius of the circle.**

Substitute each ordered pair (x, y) in the general form $x^2 + y^2 + Dx + Ey + F = 0$ to create a system of equations.

$(1)^2 + (-1)^2 + D(1) + E(-1) + F = 0$ *$(x, y) = (1, -1)$*

$(5)^2 + (3)^2 + D(5) + E(3) + F = 0$ *$(x, y) = (5, 3)$*

$(-3)^2 + (3)^2 + D(-3) + E(3) + F = 0$ *$(x, y) = (-3, 3)$*

Simplify the system of equations.

$D - E + F + 2 = 0$

$5D + 3E + F + 34 = 0$

$-3D + 3E + F + 18 = 0$

The solution to the system is $D = -2$, $E = -6$, and $F = -6$.

The general form of the equation of the circle is $x^2 + y^2 - 2x - 6y - 6 = 0$.

$x^2 + y^2 - 2x - 6y - 6 = 0$

$(x^2 - 2x + ?) + (y^2 - 6y + ?) = 6$ *Group to form perfect square trinomials.*

$(x^2 - 2x + 1) + (y^2 - 6y + 9) = 6 + 1 + 9$ *Complete the square.*

$(x - 1)^2 + (y - 3)^2 = 16$ *Factor the trinomials.*

After completing the square, the standard form of the circle is $(x - 1)^2 + (y - 3)^2 = 16$. Its center is at $(1, 3)$, and its radius is 4.

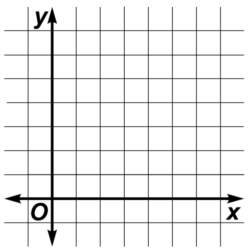
NAME ______________________ DATE ______________ PERIOD ________

Practice

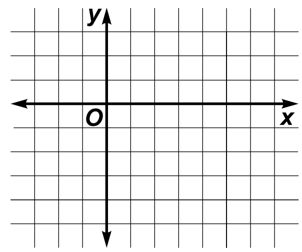
Circles

Write the standard form of the equation of each circle described. Then graph the equation.

1. center at (3, 3) tangent to the x-axis

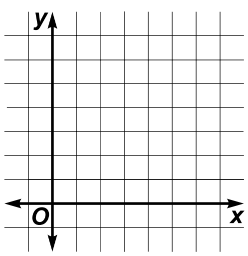


2. center at (2, -1), radius 4

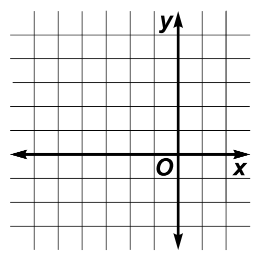


Write the standard form of each equation. Then graph the equation.

3. $x^2 + y^2 - 8x - 6y + 21 = 0$



4. $4x^2 + 4y^2 + 16x - 8y - 5 = 0$



Write the standard form of the equation of the circle that passes through the points with the given coordinates. Then identify the center and radius.

5. $(-3, -2), (-2, -3), (-4, -3)$

6. $(0, -1), (2, -3), (4, -1)$

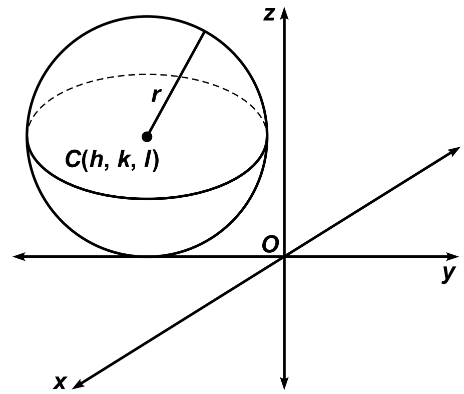
7. ***Geometry*** A square inscribed in a circle and centered at the origin has points at (2, 2), $(-2, 2)$, $(2, -2)$ and $(-2, -2)$. What is the equation of the circle that circumscribes the square?

NAME ______________________ DATE ____________ PERIOD ________

Enrichment

Spheres

The set of all points in three-dimensional space that are a fixed distance r (the **radius**), from a fixed point C (the **center**), is called a **sphere**. The equation below is an algebraic representation of the sphere shown at the right.

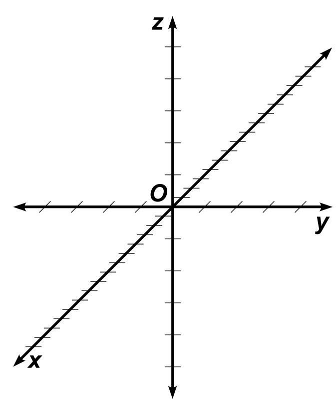


$$(x - h)^2 + (y - k)^2 + (z - l)^2 = r^2$$

A line segment containing the center of a sphere and having its endpoints on the sphere is called a **diameter** of the sphere. The endpoints of a diameter are called **poles** of the sphere. A **great circle** of a sphere is the intersection of the sphere and a plane containing the center of the sphere.

1. If $x^2 + y^2 - 4y + z^2 + 2z - 4 = 0$ is an equation of a sphere and $(1, 4, -3)$ is one pole of the sphere, find the coordinates of the opposite pole.

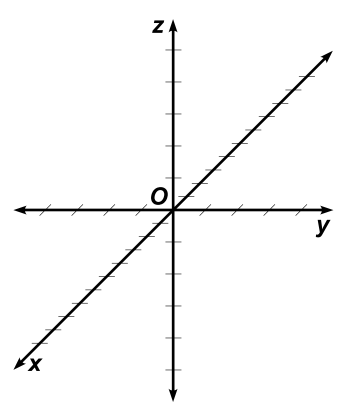
2. a. On the coordinate system at the right, sketch the sphere described by the equation $x^2 + y^2 + z^2 = 9$.



b. Is $P(2, -2, -2)$ inside, outside, or on the sphere?

c. Describe a way to tell if a point with coordinates $P(a, b, c)$ is inside, outside, or on the sphere with equation $x^2 + y^2 + z^2 = r^2$.

3. If $x^2 + y^2 + z^2 - 4x + 6y - 2z - 2 = 0$ is an equation of a sphere, find the circumference of a great circle, and the surface area and volume of the sphere.



4. The equation $x^2 + y^2 = 4$ represents a set of points in three-dimensional space. Describe that set of points in your own words. Illustrate with a sketch on the coordinate system at the right.

NAME ______________________ DATE ______________ PERIOD ________

Study Guide

Ellipses

The standard form of the equation of an **ellipse** is
$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ when the **major axis** is horizontal.
In this case, a^2 is in the denominator of the x term. The standard form is $\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1$ when the major axis is vertical. In this case, a^2 is in the denominator of the y term. In both cases, $c^2 = a^2 - b^2$.

Example **Find the coordinates of the center, the foci, and the vertices of the ellipse with the equation $4x^2 + 9y^2 + 24x - 36y + 36 = 0$. Then graph the equation.**

First write the equation in standard form.

$$4x^2 + 9y^2 + 24x - 36y + 36 = 0$$

$$4(x^2 + 6x + ?) + 9(y^2 - 4y + ?) = -36 + ? + ?$$ *GCF of x terms is 4; GCF of y terms is 9.*

$$4(x^2 + 6x + 9) + 9(y^2 - 4y + 4) = -36 + 4(9) + 9(4)$$ *Complete the square.*

$$4(x+3)^2 + 9(y-2)^2 = 36$$ *Factor.*

$$\frac{(x+3)^2}{9} + \frac{(y-2)^2}{4} = 1$$ *Divide each side by 36.*

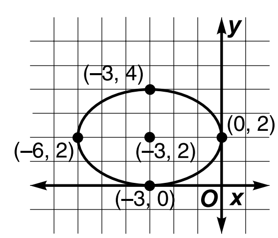
Now determine the values of a, b, c, h, and k. In all ellipses, $a^2 > b^2$. Therefore, $a^2 = 9$ and $b^2 = 4$. Since a^2 is the denominator of the x term, the major axis is parallel to the x-axis.

$a = 3$ $b = 2$ $c = \sqrt{a^2 - b^2}$ or $\sqrt{5}$ $h = -3$ $k = 2$

center: $(-3, 2)$ (h, k)
foci: $(-3 \pm \sqrt{5}, 2)$ $(h \pm c, k)$

major axis vertices:
$(0, 2)$ and $(-6, 2)$ $(h \pm a, k)$

minor axis vertices:
$(-3, 4)$ and $(-3, 0)$ $(h, k \pm b)$



Graph these ordered pairs. Then complete the ellipse.

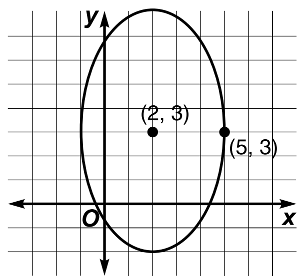
10-3

Practice

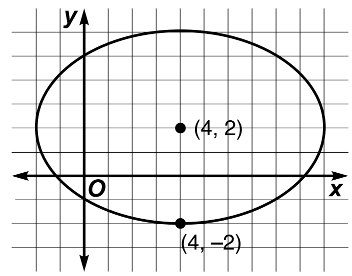
Ellipses

Write the equation of each ellipse in standard form. Then find the coordinates of its foci.

1.

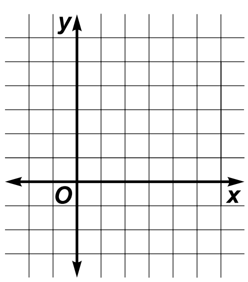


2.

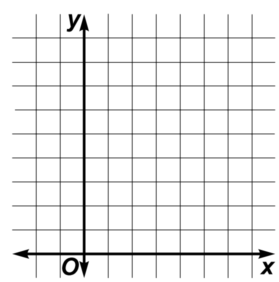


For the equation of each ellipse, find the coordinates of the center, foci, and vertices. Then graph the equation.

3. $4x^2 + 9y^2 - 8x - 36y + 4 = 0$



4. $25x^2 + 9y^2 - 50x - 90y + 25 = 0$



Write the equation of the ellipse that meets each set of conditions.

5. The center is at (1, 3), the major axis is parallel to the y-axis, and one vertex is at (1, 8), and $b = 3$.

6. The foci are at $(-2, 1)$ and $(-2, -7)$, and $a = 5$.

7. ***Construction*** A semi elliptical arch is used to design a headboard for a bed frame. The headboard will have a height of 2 feet at the center and a width of 5 feet at the base. Where should the craftsman place the foci in order to sketch the arch?

NAME ______________________ DATE ____________ PERIOD ________

Enrichment

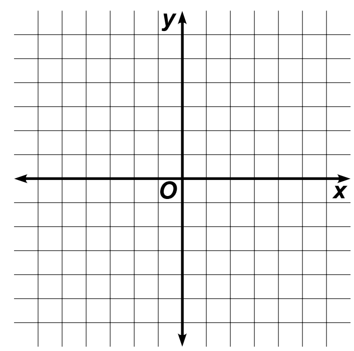
Superellipses

The circle and the ellipse are members of an interesting family of curves that were first studied by the French physicist and mathematician Gabriel Lamé (1795-1870). The general equation for the family is

$\left|\frac{x}{a}\right|^n + \left|\frac{y}{b}\right|^n = 1$, with $a \neq 0$, $b \neq 0$, and $n > 0$.

For even values of n greater than 2, the curves are called **superellipses**.

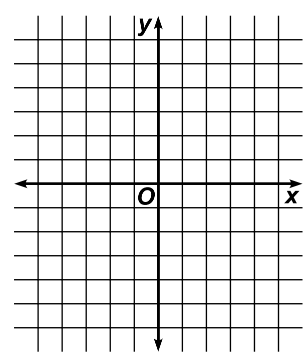
1. Consider two curves that are *not* superellipses. Graph each equation on the grid at the right. State the type of curve produced each time.



a. $\left|\frac{x}{2}\right|^2 + \left|\frac{y}{2}\right|^2 = 1$

b. $\left|\frac{x}{3}\right|^2 + \left|\frac{y}{2}\right|^2 = 1$

2. In each of the following cases you are given values of a, b, and n to use in the general equation. Write the resulting equation. Then graph. Sketch each graph on the grid at the right.



a. $a = 2, b = 3, n = 4$

b. $a = 2, b = 3, n = 6$

c. $a = 2, b = 3, n = 8$

3. What shape will the graph of $\left|\frac{x}{2}\right|^n + \left|\frac{y}{3}\right|^n = 1$ approximate for greater and greater even, whole-number values of n?

NAME ______________________ DATE ____________ PERIOD _______

10-4

Study Guide

Hyperbolas

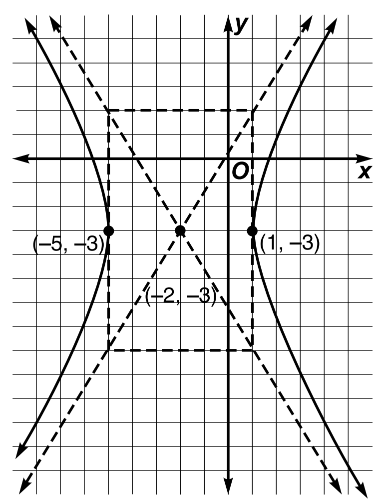
The standard form of the equation of a **hyperbola** is
$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ when the **transverse axis** is horizontal, and
$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$ when the transverse axis is vertical. In both cases, $b^2 = c^2 - a^2$.

Example **Find the coordinates of the center, foci, and vertices, and the equations of the asymptotes of the graph of $25x^2 - 9y^2 + 100x - 54y - 206 = 0$. Then graph the equation.**

Write the equation in standard form.

$$25x^2 - 9y^2 + 100x - 54y - 206 = 0$$
$$25(x^2 + 4x + ?) - 9(y^2 + 6y + ?) = 206 + ? + ?$$ *GCF of x terms is 25; GCF of y terms is 9.*
$$25(x^2 + 4x + 4) - 9(y^2 + 6y + 9) = 206 + 25(4) + (-9)(9)$$ *Complete the square.*
$$25(x + 2)^2 - 9(y + 3)^2 = 225$$ *Factor.*
$$\frac{(x+2)^2}{9} - \frac{(y+3)^2}{25} = 1$$ *Divide each side by 225.*

From the equation, $h = -2$, $k = -3$, $a = 3$, $b = 5$, and $c = \sqrt{34}$. The center is at $(-2, -3)$.



Since the x terms are in the first expression, the hyperbola has a horizontal transverse axis.

The vertices are at $(h \pm a, k)$ or $(1, -3)$ and $(-5, -3)$.

The foci are at $(h \pm c, k)$ or $(-2 \pm \sqrt{34}, -3)$.

The equations of the asymptotes are $y - k = \pm\frac{b}{a}(x - h)$ or $y + 3 = \pm\frac{5}{3}(x + 2)$.

Graph the center, the vertices, and the rectangle guide, which is $2a$ units by $2b$ units. Next graph the asymptotes. Then sketch the hyperbola.

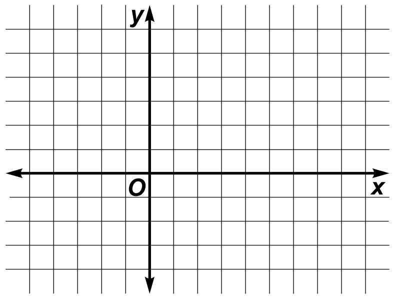
10-4

Practice

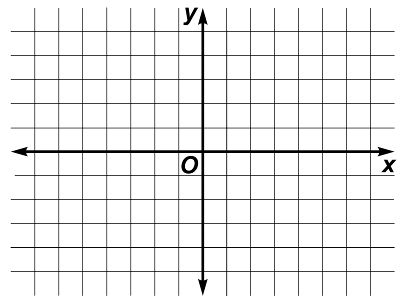
Hyperbolas

For each equation, find the coordinates of the center, foci, and vertices, and the equations of the asymptotes of its graph. Then graph the equation.

1. $x^2 - 4y^2 - 4x + 24y - 36 = 0$

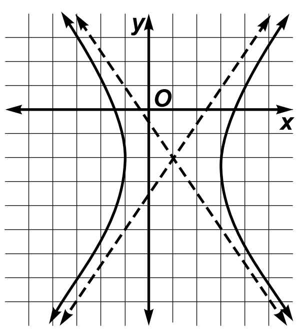


2. $y^2 - 4x^2 + 8x = 20$

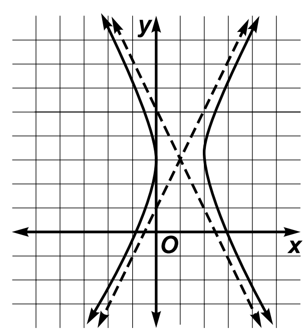


Write the equation of each hyperbola.

3.



4.



5. Write an equation of the hyperbola for which the length of the transverse axis is 8 units, and the foci are at (6, 0) and (−4, 0).

6. ***Environmental Noise*** Two neighbors who live one mile apart hear an explosion while they are talking on the telephone. One neighbor hears the explosion two seconds before the other. If sound travels at 1100 feet per second, determine the equation of the hyperbola on which the explosion was located.

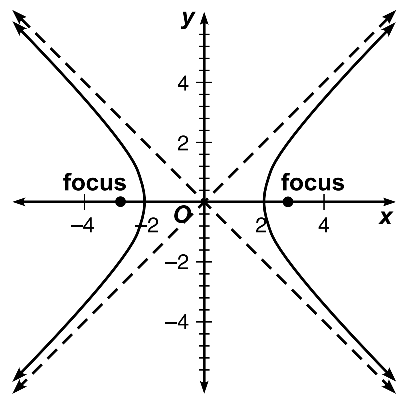
NAME ______________________ DATE ____________ PERIOD ________

10-4

Enrichment

Moving Foci

Recall that the equation of a hyperbola with center at the origin and horizontal transverse axis has the equation $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. The foci are at $(-c, 0)$ and $(c, 0)$, where $c^2 = a^2 + b^2$, the vertices are at $(-a, 0)$ and $(a, 0)$, and the asymptotes have equations $y = \pm\frac{b}{a}x$. Such a hyperbola is shown at the right.



What happens to the shape of the graph as c grows very large or very small?

Refer to the hyperbola described above.

1. Write a convincing argument to show that as c approaches 0, the foci, the vertices, and the center of the hyperbola become the same point.

2. Use a graphing calculator or computer to graph $x^2 - y^2 = 1$, $x^2 - y^2 = 0.1$, and $x^2 - y^2 = 0.01$. (Such hyperbolas correspond to smaller and smaller values of c.) Describe the changes in the graphs. What shape do the graphs approach as c approaches 0?

3. Suppose a is held fixed and c approaches infinity. How does the graph of the hyperbola change?

4. Suppose b is held fixed and c approaches infinity. How does the graph of the hyperbola change?

NAME ______________________ DATE ______________ PERIOD ________

Study Guide

Parabolas

The standard form of the equation of the **parabola** is $(y - k)^2 = 4p(x - h)$ when the parabola opens to the right. When p is negative, the parabola opens to the left. The standard form is $(x - h)^2 = 4p(y - k)$ when the parabola opens upward. When p is negative, the parabola opens downward.

Example 1 **Given the equation $x^2 = 12y + 60$, find the coordinates of the focus and the vertex and the equations of the directrix and the axis of symmetry. Then graph the equation of the parabola.**

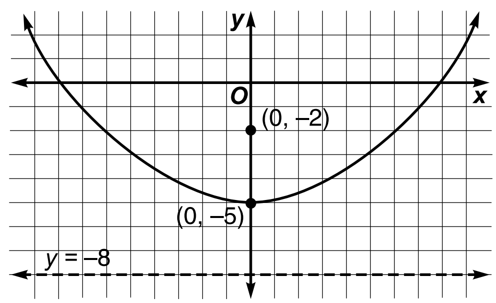
First write the equation in the form $(x - h)^2 = 4p(y - k)$.

$x^2 = 12y + 60$
$x^2 = 12(y + 5)$ — *Factor.*
$(x - 0)^2 = 4(3)(y + 5)$ — *$4p = 12$, so $p = 3$.*

In this form, we can see that $h = 0$, $k = -5$, and $p = 3$.

Vertex: $(0, -5)$	*(h, k)*	Focus: $(0, -2)$	*$(h, k + p)$*
Directrix: $y = -8$	*$y = k - p$*	Axis of Symmetry: $x = 0$	*$x = h$*

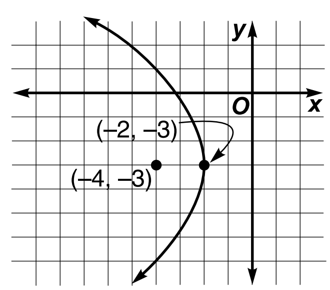
The axis of symmetry is the y-axis. Since p is positive, the parabola opens upward. Graph the directrix, the vertex, and the focus. To determine the shape of the parabola, graph several other ordered pairs that satisfy the equation and connect them with a smooth curve.



Example 2 **Write the equation $y^2 + 6y + 8x + 25 = 0$ in standard form. Find the coordinates of the focus and the vertex, and the equations of the directrix and the axis of symmetry. Then graph the parabola.**

$y^2 + 6y + 8x + 25 = 0$
$y^2 + 6y = -8x - 25$ — *Isolate the x terms and the y terms.*
$y^2 + 6y + ? = -8x - 25 + ?$
$y^2 + 6y + 9 = -8x - 25 + 9$ — *Complete the square.*
$(y + 3)^2 = -8(x + 2)$ — *Simplify and factor.*

From the standard form, we can see that $h = -2$ and $k = -3$. Since $4p = -8$, $p = -2$. Since y is squared, the directrix is parallel to the y-axis. The axis of symmetry is the x-axis. Since p is negative, the parabola opens to the left.



Vertex: $(-2, -3)$	*(h, k)*
Focus: $(-4, -3)$	*$(h + p, k)$*
Directrix: $x = 0$	*$x = h - p$*
Axis of Symmetry: $y = -3$	*$y = k$*

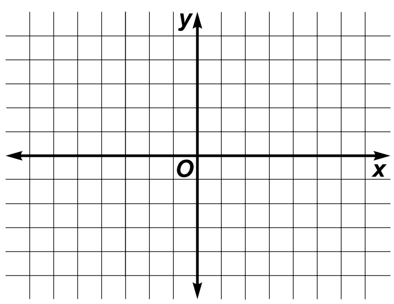
NAME ______________________________ DATE _______________ PERIOD ________

Practice

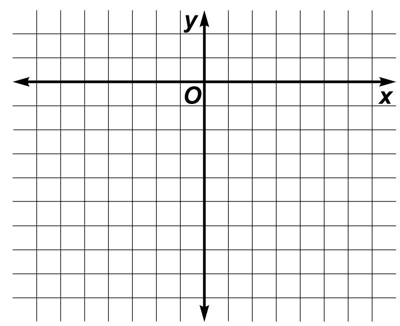
Parabolas

For the equation of each parabola, find the coordinates of the vertex and focus, and the equations of the directrix and axis of symmetry. Then graph the equation.

1. $x^2 - 2x - 8y + 17 = 0$

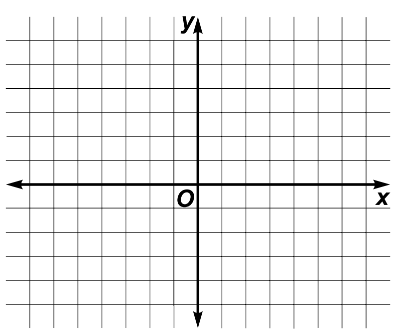


2. $y^2 + 6y + 9 = 12 - 12x$

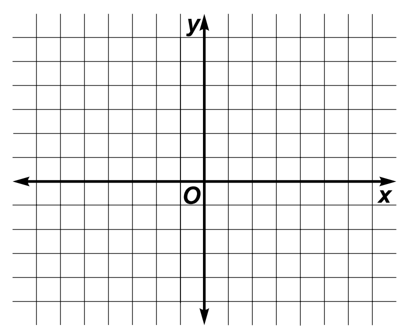


Write the equation of the parabola that meets each set of conditions. Then graph the equation.

3. The vertex is at $(-2, 4)$ and the focus is at $(-2, 3)$.



4. The focus is at $(2, 1)$, and the equation of the directrix is $x = -2$.



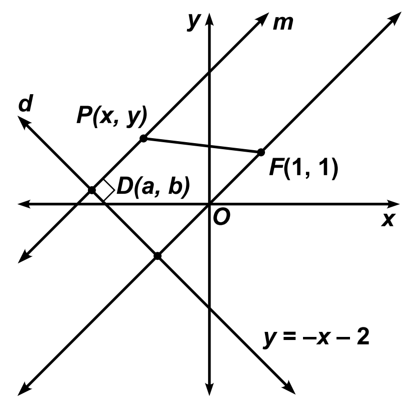
5. ***Satellite Dish*** Suppose the receiver in a parabolic dish antenna is 2 feet from the vertex and is located at the focus. Assume that the vertex is at the origin and that the dish is pointed upward. Find an equation that models a cross section of the dish.

10-5

Enrichment

Tilted Parabolas

The diagram at the right shows a fixed point $F(1, 1)$ and a line d whose equation is $y = -x - 2$. If $P(x, y)$ satisfies the condition that $PD = PF$, then P is on a parabola. Our objective is to find an equation for the tilted parabola; which is the locus of all points that are the same distance from (1,1) and the line $y = -x - 2$.



To do this, first find an equation for the line m through $P(x, y)$ and perpendicular to line d at $D(a, b)$. Using this equation and the equation for line d, find the coordinates (a, b) of point D in terms of x and y. Then use $(PD)^2 = (PF)^2$ to find an equation for the parabola.

Refer to the discussion above.

1. Find an equation for line m.

2. Use the equations for lines m and d to show that the coordinates of point D are $D(a, b) = D\left(\frac{x - y - 2}{2}, \frac{y - x - 2}{2}\right)$.

3. Use the coordinates of F, P, and D, along with $(PD)^2 = (PF)^2$ to find an equation of the parabola with focus F and directrix d.

4. a. Every parabola has an axis of symmetry. Find an equation for the axis of symmetry of the parabola described above. Justify your answer.

b. Use your answer from part **a** to find the coordinates of the vertex of the parabola. Justify your answer.

NAME ______________________ DATE ______________ PERIOD ________

Study Guide

Rectangular and Parametric Forms of Conic Sections

Use the table to identify a conic section given its equation in general form.

conic	$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$
circle	$A = C$
parabola	Either A or C is zero.
ellipse	A and C have the same sign and $A \neq C$.
hyperbola	A and C have opposite signs.

Example 1 **Identify the conic section represented by the equation $5x^2 + 4y^2 - 10x - 8y + 18 = 0$.**

$A = 5$ and $C = 4$. Since A and C have the same signs and are not equal, the conic is an ellipse.

Example 2 **Find the rectangular equation of the curve whose parametric equations are $x = 2t$ and $y = 4t^2 + 4t - 1$. Then identify the conic section represented by the equation.**

First, solve the equation $x = 2t$ for t.

$x = 2t$

$\frac{x}{2} = t$

Then substitute $\frac{x}{2}$ for t in the equation $y = 4t^2 + 4t - 1$.

$y = 4t^2 + 4t - 1$

$y = 4\left(\frac{x}{2}\right)^2 + 4\left(\frac{x}{2}\right) - 1 \qquad t = \frac{x}{2}$

$y = x^2 + 2x - 1$

Since $C = 0$, the equation $y = x^2 + 2x - 1$ is the equation of a parabola.

Example 3 **Find the rectangular equation of the curve whose parametric equations are $x = 3 \cos t$ and $y = 5 \sin t$, where $0 \leq t \leq 2\pi$. Then graph the equation using arrows to indicate orientation.**

Solve the first equation for $\cos t$ and the second equation for $\sin t$.

$\cos t = \frac{x}{3}$ and $\sin t = \frac{y}{5}$

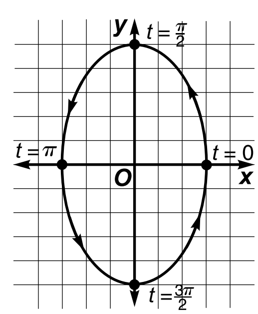
Use the trigonometric identity $\cos^2 t + \sin^2 t = 1$ to eliminate t.

$\cos^2 t + \sin^2 t = 1$

$\left(\frac{x}{3}\right)^2 + \left(\frac{y}{5}\right)^2 = 1 \qquad$ *Substitution*

$\frac{x^2}{9} + \frac{y^2}{25} = 1$

This is the equation of an ellipse with the center at (0, 0). As t increases from 0 to 2π, the curve is traced in a counterclockwise motion.



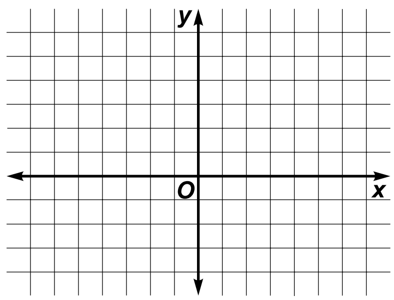
NAME ______________________ DATE ____________ PERIOD ________

Practice

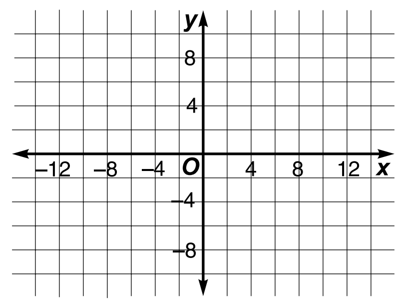
Rectangular and Parametric Forms of Conic Sections

Identify the conic section represented by each equation. Then write the equation in standard form and graph the equation.

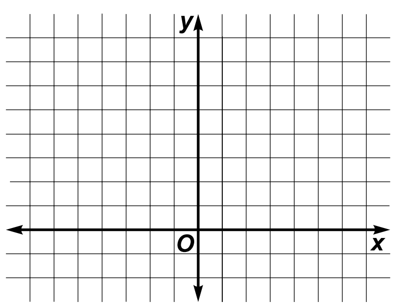
1. $x^2 - 4y + 4 = 0$



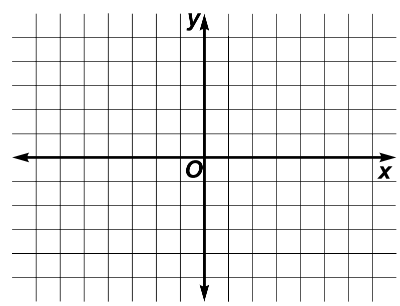
2. $x^2 + y^2 - 6x - 6y - 18 = 0$



3. $4x^2 - y^2 - 8x + 6y = 9$

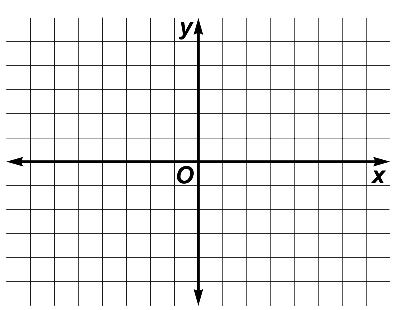


4. $9x^2 + 5y^2 + 18x = 36$

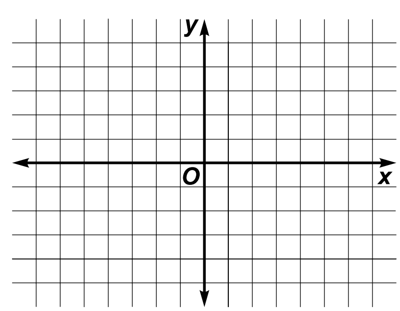


Find the rectangular equation of the curve whose parametric equations are given. Then graph the equation using arrows to indicate orientation.

5. $x = 3 \cos t, y = 3 \sin t, 0 \le t \le 2\pi$



6. $x = -4 \cos t, y = 5 \sin t, 0 \le t \le 2\pi$



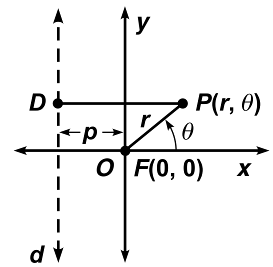
10-6

Enrichment

Polar Graphs of Conics

A conic is the locus of all points such that the ratio e of the distance from a fixed point F and a fixed line d is constant.

$$\frac{FP}{DP} = e$$



To find the polar equation of the conic, use a polar coordinate system with the origin at the focus.

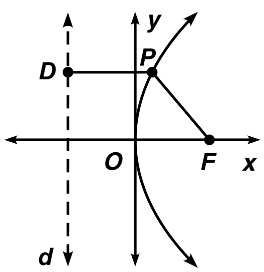
Since $FP = r$ and $DP = p + r\cos\theta$, $\dfrac{r}{p + r\cos\theta} = e$.

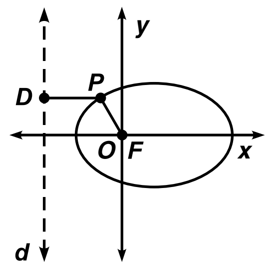
Now solve for r. $\qquad r = \dfrac{ep}{1 - e\cos\theta}$

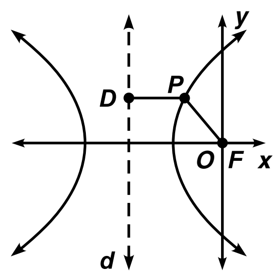
You can classify a conic section by its eccentricity.

$e = 1$: parabola $\qquad 0 < e < 1$: ellipse $\qquad e > 1$: hyperbola

$e = 0$: circle

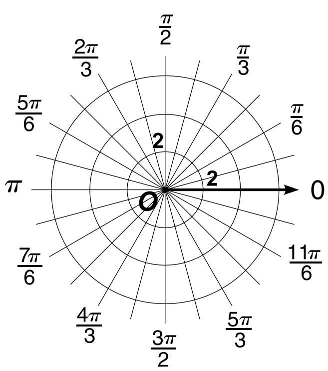




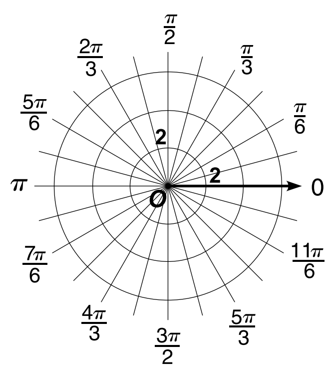


Graph each relation and identify the conic.

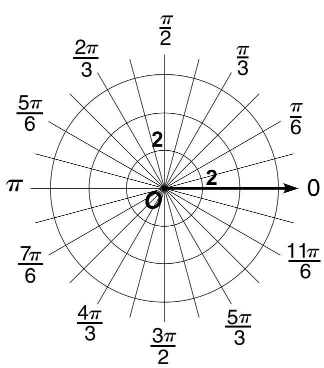
1. $r = \dfrac{4}{1 - \cos\theta}$



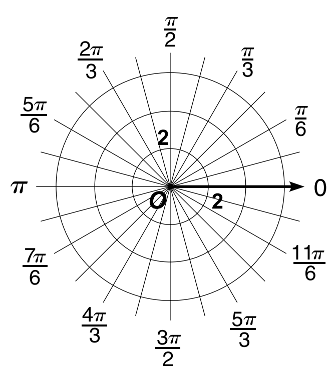
2. $r = \dfrac{4}{2 - \cos\theta}$



3. $r = \dfrac{4}{2 + \sin\theta}$



4. $r = \dfrac{4}{1 + 2\sin\theta}$



NAME ______________________ DATE ____________ PERIOD ________

10-7 Study Guide

Transformation of Conics

Translations are often written in the form $T_{(h,\,k)}$. To find the equation of a rotated conic, replace x with $x'\cos\theta + y'\sin\theta$ and y with $-x'\sin\theta + y'\cos\theta$.

Example **Identify the graph of each equation. Write an equation of the translated or rotated graph in general form.**

a. $\mathbf{4x^2 + y^2 = 12}$ **for** $\mathbf{T_{(-2,\,3)}}$

The graph of this equation is an ellipse. To write the equation of $4x^2 + y^2 = 12$ for $T_{(-2,\,3)}$, let $h = -2$ and $k = 3$. Then replace x with $x - h$ and y with $y - k$.

$x^2 \Rightarrow (x - (-2))^2$ or $(x + 2)^2$
$y^2 \Rightarrow (y - 3)^2$

Thus, the translated equation is $4(x + 2)^2 + (y - 3)^2 = 12$.

Write the equation in general form.

$$4(x + 2)^2 + (y - 3)^2 = 12$$
$$4(x^2 + 4x + 4) + y^2 - 6y + 9 = 12 \qquad \textit{Expand the binomial.}$$
$$4x^2 + y^2 + 16x - 6y + 25 = 12 \qquad \textit{Simplify.}$$
$$4x^2 + y^2 + 16x - 6y + 13 = 0 \qquad \textit{Subtract 12 from both sides.}$$

b. $\mathbf{x^2 - 4y = 0,\ \theta = 45°}$

The graph of this equation is a parabola. Find the expressions to replace x and y.

Replace x with $x'\cos 45° + y'\sin 45°$ or $\frac{\sqrt{2}}{2}x' + \frac{\sqrt{2}}{2}y'$.

Replace y with $-x'\sin 45° + y'\cos 45°$ or $-\frac{\sqrt{2}}{2}x' + \frac{\sqrt{2}}{2}y'$.

$$x^2 - 4y = 0$$
$$\left(\frac{\sqrt{2}}{2}x' + \frac{\sqrt{2}}{2}y'\right)^2 - 4\left(-\frac{\sqrt{2}}{2}x' + \frac{\sqrt{2}}{2}y'\right) = 0 \qquad \textit{Replace x and y.}$$
$$\left[\frac{1}{2}(x')^2 + x'y' + \frac{1}{2}(y')^2\right] - 4\left(-\frac{\sqrt{2}}{2}x' + \frac{\sqrt{2}}{2}y'\right) = 0 \qquad \textit{Expand the binomial.}$$
$$\frac{1}{2}(x')^2 + x'y' + \frac{1}{2}(y')^2 + 2\sqrt{2}x' - 2\sqrt{2}y' = 0 \qquad \textit{Simplify.}$$

The equation of the parabola after the 45° rotation is
$\frac{1}{2}(x')^2 + x'y' + \frac{1}{2}(y')^2 + 2\sqrt{2}x' - 2\sqrt{2}y' = 0$

NAME ______________________________ DATE ______________ PERIOD ________

Practice

Transformations of Conics

Identify the graph of each equation. Write an equation of the translated or rotated graph in general form.

1. $2x^2 + 5y^2 = 9$ for $T_{(-2, 1)}$

2. $2x^2 - 4x + 3 - y = 0$ for $T_{(1, -1)}$

3. $xy = 1, \theta = \frac{\pi}{4}$

4. $x^2 - 4y = 0, \theta = 90°$

Identify the graph of each equation. Then find θ to the nearest degree.

5. $2x^2 + 2y^2 - 2x = 0$

6. $3x^2 + 8xy + 4y^2 - 7 = 0$

7. $16x^2 - 24xy + 9y^2 - 30x - 40y = 0$

8. $13x^2 - 8xy + 7y^2 - 45 = 0$

9. ***Communications*** Suppose the orientation of a satellite dish that monitors radio waves is modeled by the equation $4x^2 + 2xy + 4y^2 + \sqrt{2}x - \sqrt{2}y = 0$. What is the angle of rotation of the satellite dish about the origin?

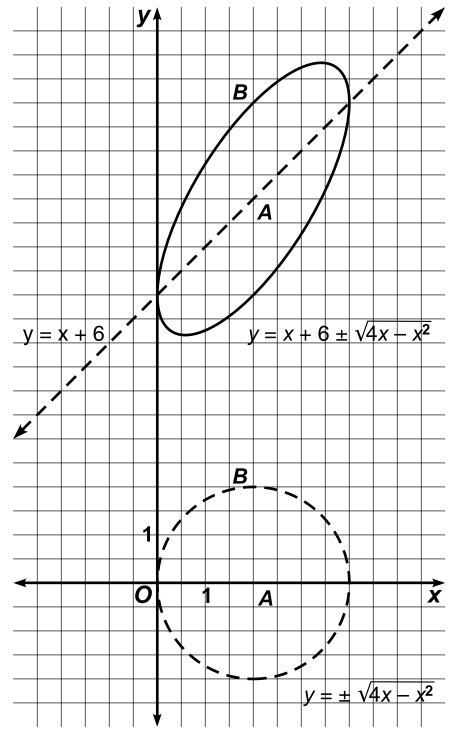
NAME ______________________ DATE ______________ PERIOD ________

Enrichment

Graphing with Addition of y-Coordinates

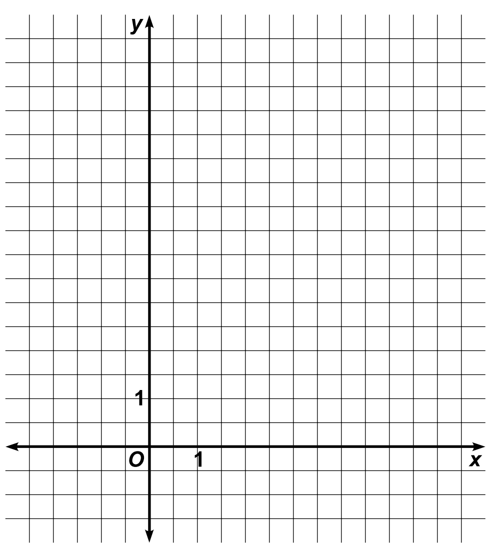
Equations of parabolas, ellipses, and hyperbolas that are “tipped” with respect to the x- and y-axes are more difficult to graph than the equations you have been studying.

Often, however, you can use the graphs of two simpler equations to graph a more complicated equation. For example, the graph of the ellipse in the diagram at the right is obtained by adding the y-coordinate of each point on the circle and the y-coordinate of the corresponding point of the line.

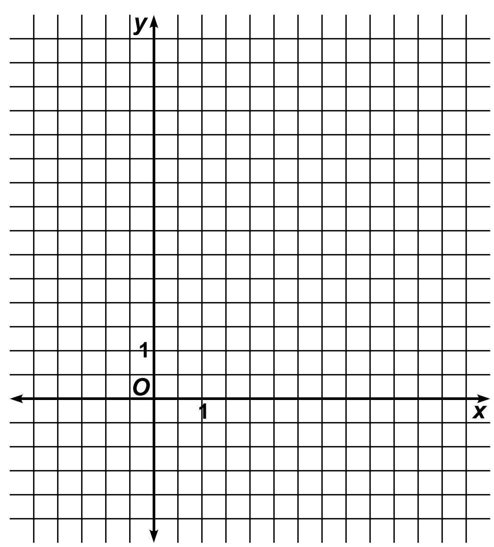


Graph each equation. State the type of curve for each graph.

1. $y = 6 - x \pm \sqrt{4 - x^2}$



2. $y = x \pm \sqrt{x}$



Use a separate sheet of graph paper to graph these equations. State the type of curve for each graph.

3. $y = 2x \pm \sqrt{7 + 6x - x^2}$

4. $y = -2x \pm \sqrt{-2x}$

NAME ______________________________ DATE ______________ PERIOD ________

Study Guide

Systems of Second-Degree Equations and Inequalities

To find the exact solution to a system of second-degree equations, you must use algebra. Graph systems of inequalities involving second-degree equations to find solutions for the inequality.

Example **a. Solve the system of equations algebraically. Round to the nearest tenth.**

$\mathbf{x^2 + 2y^2 = 9}$
$\mathbf{3x^2 - y^2 = 1}$

Since both equations contain a single term involving y, you can solve the system as follows.

First, multiply each side of the second equation by 2.

$2(3x^2 - y^2) = 2(1)$
$6x^2 - 2y^2 = 2$

Then, add the equations.

$$\begin{aligned} 6x^2 - 2y^2 &= 2 \\ x^2 + 2y^2 &= 9 \\ 7x^2 &= 11 \\ x &= \pm\sqrt{\frac{11}{7}} \end{aligned}$$

Now find y by substituting $\pm\sqrt{\frac{11}{7}}$ for x in one of the original equations.

$$x^2 + 2y^2 = 9 \rightarrow \left(\pm\sqrt{\frac{11}{7}}\right)^2 + 2y^2 = 9$$
$$11 + 14y^2 = 63$$
$$y^2 = \frac{26}{7}$$
$$y \approx \pm 1.9$$

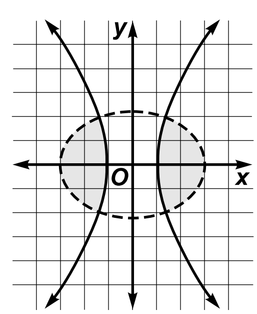
The solutions are $(1.3, \pm 1.9)$, and $(-1.3, \pm 1.9)$.

b. Graph the solutions for the system of inequalities.

$\mathbf{x^2 + 2y^2 < 9}$
$\mathbf{3x^2 - y^2 \geq 1}$

First graph $x^2 + 2y^2 < 9$. The ellipse should be dashed. Test a point either inside or outside the ellipse to see if its coordinates satisfy the inequality. Since (0, 0) satisfies the inequality, shade the interior of the ellipse.

Then graph $3x^2 - y^2 \geq 1$. The hyperbola should be a solid curve. Test a point inside the branches of the hyperbola or outside its branches. Since (0, 0) does not satisfy the inequality, shade the regions inside the branches. The intersection of the two graphs represents the solution of the system.



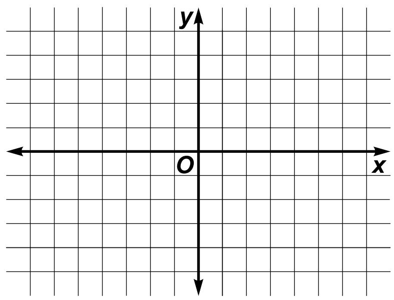
NAME ______________________ DATE ______________ PERIOD ________

Practice

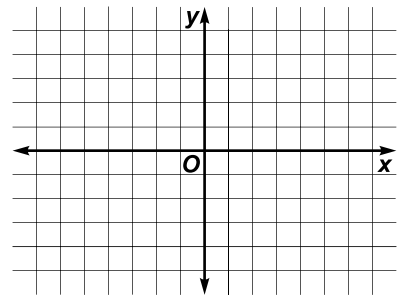
Systems of Second-Degree Equations and Inequalities

Solve each system of equations algebraically. Round to the nearest tenth. Check the solutions by graphing each system.

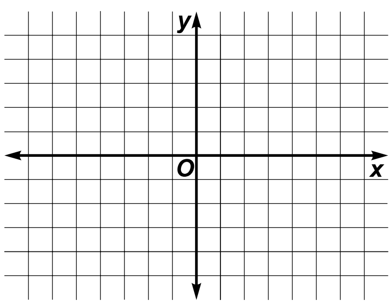
1. $2x - y = 8$
$x^2 + y^2 = 9$



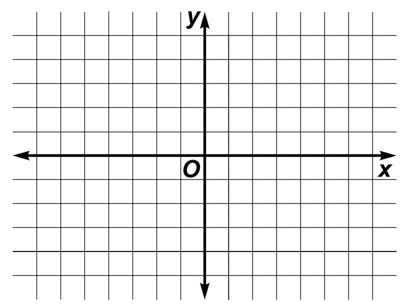
2. $x^2 - y^2 = 4$
$y = 1$



3. $xy = 4$
$x^2 = y^2 + 1$

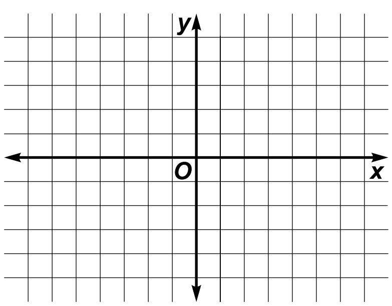


4. $x^2 + y^2 = 4$
$4x^2 + 9y^2 = 36$

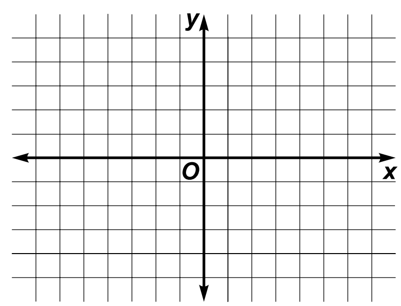


Graph each system of inequalities.

5. $3 \geq (y - 1)^2 + 2x$
$y \geq -3x + 1$



6. $(x - 1)^2 + (y - 2)^2 < 9$
$4(y + 1)^2 + x^2 \leq 16$

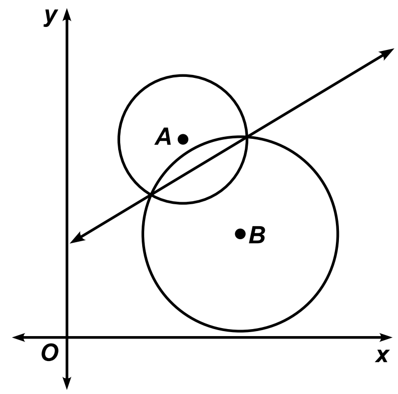


7. ***Sales*** Vincent's Pizzeria reduced prices for large specialty pizzas by $5 for 1 week in March. In the previous week, sales for large specialty pizzas totaled $400. During the sale week, the number of large pizzas sold increased by 20 and total sales amounted to $600. Write a system of second-degree equations to model this situation. Find the regular price and the sale price of large specialty pizzas.

10-8

Enrichment

Intersections of Circles

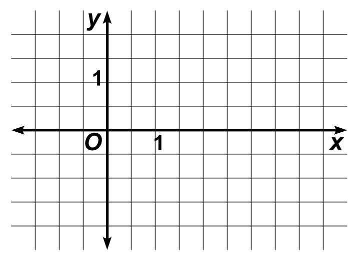


Many interesting problems involving circles can be solved by using a system of equations. Consider the following problem.

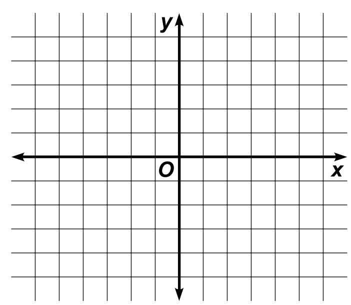
Find an equation for the straight line that contains the two points of intersection of two intersecting circles whose equations are given.

You may be surprised to find that if the given circles intersect in two points, then the difference of their equations is the equation of the line containing the intersection points.

1. Circle A has equation $x^2 + y^2 = 1$ and circle B has equation $(x - 3)^2 + y^2 = 1$. Use a sketch to show that the circles do not intersect. Use an algebraic argument to show that circles A and B do not intersect.



2. Circle A has equation $(x - 2)^2 + (y + 1)^2 = 16$ and circle B has equation $(x + 3)^2 + y^2 = 9$. Use a sketch to show that the circles meet in two points. Then find an equation in standard form for the line containing the points of intersection.



3. Without graphing the equations, decide if the circles with equations $(x - 2)^2 + (y - 2)^2 = 8$ and $(x - 3)^2 + (y - 4)^2 = 4$ are tangent. Justify your answer.

NAME ______________________ DATE ______________ PERIOD ________

Chapter 10

Chapter 10 Test, Form 1A

Write the letter for the correct answer in the blank at the right of each problem.

Exercises 1–3 refer to the ellipse represented by $9x^2 + 16y^2 - 18x + 64y - 71 = 0$.

1. Find the coordinates of the center. **1.** ________

A. $(1, 2)$ **B.** $(1, -2)$ **C.** $(-1, 2)$ **D.** $(-2, 1)$

2. Find the coordinates of the foci. **2.** ________

A. $(1 \pm \sqrt{7}, -2)$ **B.** $(1, -2 \pm \sqrt{7})$

C. $(5, -2), (-3, -2)$ **D.** $(1, 4), (1, -8)$

3. Find the coordinates of the vertices. **3.** ________

A. $(1, 2), (1, -6), (4, -2), (-2, -2)$ **B.** $(4, 2), (-2, 2), (1, 1), (1, -5)$

C. $(5, -2), (-3, -2), (1, 1), (1, -5)$ **D.** $(5, -2), (-3, -2), (1, 2), (1, -6)$

4. Write the standard form of the equation of the circle that passes through the points at $(4, 5)$, $(-2, 3)$, and $(-4, -3)$. **4.** ________

A. $(x - 5)^2 + (y + 4)^2 = 49$ **B.** $(x - 3)^2 + (y + 2)^2 = 50$

C. $(x + 4)^2 + (y - 2)^2 = 36$ **D.** $(x - 2)^2 + (y + 2)^2 = 25$

5. For $4x^2 - 4xy + y^2 = 4$, find θ, the angle of rotation about the origin, to the nearest degree. **5.** ________

A. $27°$ **B.** $63°$ **C.** $333°$ **D.** $307°$

6. Find the rectangular equation of the curve whose parametric equations are $x = 5 \cos 2t$ and $y = -\sin 2t$, $0° \leq t \leq 180°$. **6.** ________

A. $\frac{x^2}{5} + y^2 = 1$ **B.** $\frac{x^2}{5} - y^2 = 1$

C. $\frac{x^2}{25} + y^2 = 1$ **D.** $\frac{(x - 2)^2}{5} + (y - 2)^2 = 1$

7. Find the distance between points at $(m + 4, n)$ and $(m, n - 3)$. **7.** ________

A. 3.5 **B.** 5 **C.** 1 **D.** 7

8. Write the standard form of the equation of the circle that is tangent to the line $x = -3$ and has its center at $(2, -7)$. **8.** ________

A. $(x - 2)^2 + (y + 7)^2 = 25$ **B.** $(x - 2)^2 + (y + 7)^2 = 5$

C. $(x - 2)^2 + (y + 7)^2 = 16$ **D.** $(x + 2)^2 + (y - 7)^2 = 25$

9. Find the coordinates of the point(s) of intersection for the graphs of $x^2 + 2y^2 = 33$ and $x^2 + y^2 = 2x + 19$. **9.** ________

A. $(5, 2), (-1, 4)$ **B.** $(5, 4), (-1, 2)$

C. $(5, \pm 2), (-1, \pm 4)$ **D.** Graphs do not intersect.

10. Identify the conic section represented by $9y^2 + 4x^2 - 108y + 24x = -144$. **10.** ________

A. parabola **B.** hyperbola **C.** ellipse **D.** circle

11. Write the equation of the conic section $y^2 - x^2 = 5$ after a rotation of $45°$ about the origin. **11.** ________

A. $x'y' = -2.5$ **B.** $x'y' = -5$

C. $(y')^2 - (x')^2 = 2.5$ **D.** $(x')^2 = 2.5y'$

12. Find parametric equations for the rectangular equation $(x + 2)^2 = 4(y - 1)$. **12.** ________

A. $x = t, y = t^2 + 2, -\infty < t < \infty$

B. $x = t, y = \frac{1}{4}t^2 + t + 2, -\infty < t < \infty$

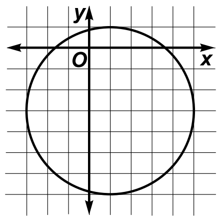
C. $x = t, y = \frac{1}{4}t^2 - t + 2, -\infty < t < \infty$

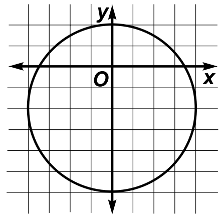
D. $x = t, y = 4t^2 + t + 2, -\infty < t < \infty$

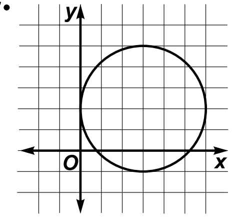
NAME ______________________ DATE ______________ PERIOD __________

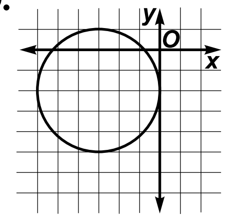
Chapter 10 Test, Form 1A (continued)

13. Which is the graph of $6x^2 - 12x + 6y^2 + 36y = 36$? **13.** ________

A.


B.


C.


D.


Exercises 14 and 15 refer to the hyperbola represented by $-2x^2 + y^2 + 4x + 6y = -3$.

14. Write the equations of the asymptotes. **14.** ________

A. $y + 3 = \pm 2(x - 1)$ **B.** $y + 3 = \pm\frac{1}{2}(x - 1)$

C. $y + 3 = \pm\sqrt{2}(x - 1)$ **D.** $y + 3 = \pm\frac{\sqrt{2}}{2}(x - 1)$

15. Find the coordinates of the foci. **15.** ________

A. $(1 \pm \sqrt{2}, -3)$ **B.** $(1 \pm \sqrt{6}, -3)$ **C.** $(1, -3 \pm \sqrt{2})$ **D.** $(1, -3 \pm \sqrt{6})$

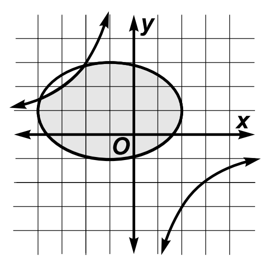
16. Write the standard form of the equation of the hyperbola for which the transverse axis is 4 units long and the coordinates of the foci are $(1, -4 \pm \sqrt{7})$. **16.** ________

A. $\frac{(x-1)^2}{3} - \frac{(y+4)^2}{4} = 1$ **B.** $\frac{(y+4)^2}{4} - \frac{(x-1)^2}{3} = 1$

C. $\frac{(y+4)^2}{3} - \frac{(x-1)^2}{4} = 1$ **D.** $\frac{(x-1)^2}{4} - \frac{(y+4)^2}{3} = 1$

17. The graph at the right shows the solution set for which system of inequalities? **17.** ________

A. $xy \geq -6, 9(x + 1)^2 + 4(y - 1)^2 \leq 36$
B. $xy \leq -6, 4(x + 1)^2 + 9(y - 1)^2 \leq 36$
C. $xy \geq -6, 4(x + 1)^2 + 9(y - 1)^2 \leq 36$
D. $xy \leq -6, 9(x + 1)^2 + 4(y - 1)^2 \leq 36$



18. Find the coordinates of the vertex and the equation of the axis of symmetry for the parabola represented by $x^2 + 4x - 6y + 10 = 0$. **18.** ________

A. $(-2, 1), y = 1$ **B.** $(1, -2), y = -2$

C. $(-2, 1), x = -2$ **D.** $(1, -2), x = 1$

19. Write the standard form of the equation of the parabola whose directrix is $x = -1$ and whose focus is at $(5, -2)$. **19.** ________

A. $(y + 2)^2 = 12(x + 2)$ **B.** $y - 2 = 12(x + 2)^2$

C. $x + 2 = \frac{1}{12}(y + 2)^2$ **D.** $x - 2 = \frac{1}{12}(y + 2)^2$

20. Identify the graph of the equation $16x^2 - 24xy + 9y^2 - 30x - 40y = 0$. Then, find θ to the nearest degree. **20.** ________

A. hyperbola; $-37°$ **B.** parabola; $-37°$

C. parabola; $45°$ **D.** circle; $-74°$

Bonus Find the coordinates of the points of intersection of the graphs of $x^2 - y^2 = 3$, $xy = 2$, and $y = -2x + 5$. **Bonus:** ________

A. $(\pm 2, \pm 1)$ **B.** $(1, -2)$

C. $(2, 1)$ **D.** Graphs do not intersect.

Chapter 10

NAME ______________________ DATE ______________ PERIOD ________

Chapter 10 Test, Form 1B

Write the letter for the correct answer in the blank at the right of each problem.
Exercises 1-3 refer to the ellipse represented by $x^2 + 25y^2 - 6x - 100y + 84 = 0$.

1. Find the coordinates of the center. **1.** ________
A. (2, 3) **B.** (3, 2) **C.** (−3, −2) **D.** (−2, −3)

2. Find the coordinates of the foci. **2.** ________
A. $(3, 2 \pm \sqrt{26})$ **B.** (−2, 2), (8, 2) **C.** $(3 \pm \sqrt{26}, 2)$ **D.** $(2 \pm \sqrt{26}, 3)$

3. Find the coordinates of the vertices. **3.** ________
A. (8, 2), (−2, 2), (3, 3), (3, 1) **B.** (8, 2), (−2, 2), (3, 7), (3 , −3)
C. (4, 2), (2, 2), (3, 3), (3, 1) **D.** (4, 2), (2, 2), (3, 7), (3, −3)

4. Write $6x^2 - 12x + 6y^2 + 36y = 36$ in standard form. **4.** ________
A. $(x - 3)^2 + (y - 1)^2 = 16$ **B.** $(x + 1)^2 + (y - 3)^2 = 16$
C. $(x - 1)^2 + (y + 3)^2 = 16$ **D.** $(x - 3)^2 + (y + 1)^2 = 16$

5. For $2x^2 + 3xy + y^2 = 1$, find θ, the angle of rotation about the origin, to the nearest degree. **5.** ________
A. −9° **B.** 36° **C.** −36° **D.** 324°

6. Find the rectangular equation of the curve whose parametric equations are $x = 3 \cos t$ and $y = \sin t$, $0° \le t \le 360°$. **6.** ________
A. $\frac{x^2}{9} + y^2 = 1$ **B.** $\frac{x^2}{9} - y^2 = 1$ **C.** $y^2 - \frac{x^2}{3} = 1$ **D.** $y^2 + \frac{x^2}{3} = 1$

7. Find the distance between points at (−5, 2) and (7, −3). **7.** ________
A. $\sqrt{5}$ **B.** 13 **C.** $\sqrt{29}$ **D.** $\sqrt{119}$

8. Write the standard form of the equation of the circle that is tangent to the y-axis and has its center at (−3, 5). **8.** ________
A. $(x + 3)^2 + (y - 5)^2 = 9$ **B.** $(x + 3)^2 + (y - 5)^2 = 25$
C. $(x + 3)^2 + (y - 5)^2 = 3$ **D.** $(x - 3)^2 + (y + 5)^2 = 9$

9. Find the coordinates of the point(s) of intersection for the graphs of $x^2 + y^2 = 4$ and $y = 2x - 1$. **9.** ________
A. (1.3, 1.5) **B.** (1.3, 1.5), (−0.5, −1.9)
C. (±1.3, ±1.5), (±0.5, ±1.9) **D.** Graphs do not intersect.

10. Identify the conic section represented by $3y^2 - 3x^2 + 12y + 18x = 42$. **10.** ________
A. parabola **B.** hyperbola **C.** ellipse **D.** circle

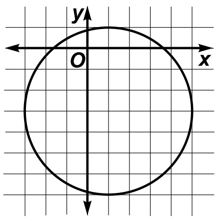
11. Write the equation of the conic section $y^2 - x^2 = 2$ after a rotation of 45° about the origin. **11.** ________
A. $x'y' = -1$ **B.** $x'y' = -2$ **C.** $(y')^2 - (x')^2 = 2$ **D.** $(x')^2 = y'$

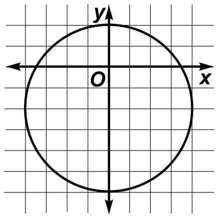
12. Find parametric equations for the rectangular equation $x^2 + y^2 - 25 = 0$. **12.** ________
A. $x = \cos 5t, y = \sin 5t, 0 \le t \le 2\pi$ **B.** $x = \cos t, y = 5 \sin t, 0 \le t \le 2\pi$
C. $x = 5 \cos t, y = \sin t, 0 \le t \le 2\pi$ **D.** $x = 5 \cos t, y = 5 \sin t, 0 \le t \le 2\pi$

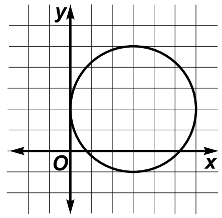
NAME ______________________ DATE ____________ PERIOD ________

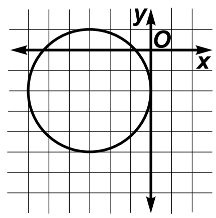
Chapter 10 Test, Form 1B (continued)

13. Which is the graph of $x^2 + (y + 2)^2 = 16$? **13.** ________

A.


B.


C.


D.


Exercises 14 and 15 refer to the hyperbola represented by $36x^2 - y^2 - 4y = -32$.

14. Write the equations of the asymptotes. **14.** ________

A. $y - 1 = \pm 6(x - 2)$ **B.** $y = \pm 6x$
C. $y + 2 = \pm 6(x - 1)$ **D.** $y + 2 = \pm 6x$

15. Find the coordinates of the foci. **15.** ________

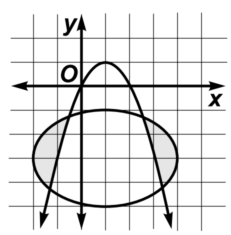
A. $(1 \pm \sqrt{37}, -2)$ **B.** $(\pm \sqrt{37}, -2)$
C. $(6 \pm \sqrt{37}, -2)$ **D.** $(0, -2 \pm \sqrt{37})$

16. Write the standard form of the equation of the hyperbola for which $a = 2$, the transverse axis is vertical, and the equations of the asymptotes are $y = \pm 2x$. **16.** ________

A. $\frac{x^2}{4} - y^2 = 1$ **B.** $y^2 - \frac{x^2}{4} = 1$ **C.** $x^2 - \frac{y^2}{4} = 1$ **D.** $\frac{y^2}{4} - x^2 = 1$

17. The graph at the right shows the solution set for which system of inequalities? **17.** ________

A. $y - 1 \geq -(x - 1)^2, 4(x - 1)^2 + 9(y + 3)^2 \leq 36$
B. $y - 1 \geq -(x - 1)^2, 4(x - 1)^2 + 9(y + 3)^2 \geq 36$
C. $y - 1 \leq -(x - 1)^2, 4(x - 1)^2 + 9(y + 3)^2 \leq 36$
D. $y - 1 \leq -(x - 1)^2, 4(x - 1)^2 + 9(y + 3)^2 \geq 36$



18. Find the coordinates of the vertex and the equation of the axis of symmetry for the parabola represented by $x^2 + 2x + 12y + 37 = 0$. **18.** ________

A. $(-1, -3), x = -1$ **B.** $(-1, -6), x = -1$
C. $(-1, -12), x = -5$ **D.** $(3, 2), y = -9$

19. Write the standard form of the equation of the parabola whose directrix is $y = -4$ and whose focus is at $(2, 2)$. **19.** ________

A. $(y - 2)^2 = 12(x + 2)$ **B.** $y + 1 = 12(x - 2)^2$
C. $(x + 2)^2 = 12(y - 2)$ **D.** $(x - 2)^2 = 12(y + 1)$

20. Identify the graph of the equation $4x^2 - 5xy + 16y^2 - 32 = 0$. **20.** ________

A. circle **B.** ellipse **C.** parabola **D.** hyperbola

Bonus Find the coordinates of the points of intersection of the graphs of $x^2 + y^2 = 5$, $xy = -2$, and $y = -3x + 1$. **Bonus:** ________

A. $(\pm 2, \pm 1)$ **B.** $(1, -2)$ **C.** $(2, 1)$ **D.** $(\pm 1, \pm 2)$

Chapter 10

NAME ______________________________ DATE ________________ PERIOD ________

Chapter 10 Test, Form 1C

Write the letter for the correct answer in the blank at the right of each problem.

Exercises 1–3 refer to the ellipse represented by $4x^2 + 9y^2 - 18y - 27 = 0$.

1. Find the coordinates of the center. **1.** ________

A. $(-1, 0)$ **B.** $(0, -1)$ **C.** $(1, 0)$ **D.** $(0,1)$

2. Find the coordinates of the foci. **2.** ________

A. $(0, 1 \pm \sqrt{5})$ **B.** $(\sqrt{5}, 1), (-\sqrt{5}, 1)$

C. $(\sqrt{5}, 3), (-\sqrt{5}, 3)$ **D.** $(1, 5), (1, -5)$

3. Find the coordinates of the vertices. **3.** ________

A. $(2, 1), (-2, 1)$ $(0, 4), (0, -2)$ **B.** $(3, 1), (-3, 1), (0, 3), (0, -1)$

C. $(3, 1), (-3, 1), (0, 4), (0, -2)$ **D.** $(2, 1), (-2, 1), (0, 3), (0, -1)$

4. Write the standard form of the equation of the circle that is tangent to the x-axis and has its center at $(3, -2)$. **4.** ________

A. $(x - 3)^2 + (y + 2)^2 = 4$ **B.** $(x + 3)^2 + (y - 2)^2 = 4$

C. $(x - 3)^2 + (y + 2)^2 = 2$ **D.** $(x + 3)^2 + (y - 2)^2 = 2$

5. For $2x^2 + xy + 2y^2 = 1$, find θ, the angle of rotation about the origin, to the nearest degree. **5.** ________

A. $215°$ **B.** $150°$ **C.** $45°$ **D.** $-30°$

6. Find the rectangular equation of the curve whose parametric equations are $x = -\cos t$ and $y = \sin t$, $0° \le t \le 360°$. **6.** ________

A. $y^2 - x^2 = 1$ **B.** $x^2 + y^2 = -1$ **C.** $x^2 + y^2 = 1$ **D.** $x^2 - y^2 = 1$

7. Find the distance between points at $(-1, 6)$ and $(5, -2)$. **7.** ________

A. $\sqrt{14}$ **B.** 10 **C.** $\sqrt{34}$ **D.** 8

8. Write $x^2 + 4x + y^2 + 2y = 4$ in standard form. **8.** ________

A. $(x - 2)^2 + (y - 1)^2 = 9$ **B.** $(x + 2)^2 + (y + 1)^2 = 9$

C. $(x + 2)^2 + (y + 1)^2 = 3$ **D.** $(x + 2)^2 - (y - 1)^2 = 4$

9. Find the coordinates of the point(s) of intersection for the graphs of $x^2 + y^2 = 20$ and $y = x - 2$. **9.** ________

A. $(\pm 2, \pm 4), (4, 2)$ **B.** $(\pm 2, \pm 4), (\pm 4, \pm 2)$

C. $(2, 4), (-4, -2)$ **D.** $(-2, -4), (4, 2)$

10. Identify the conic section represented by $x^2 - y^2 + 12y + 18x = 42$. **10.** ________

A. parabola **B.** hyperbola **C.** ellipse **D.** circle

11. Write the equation of the conic section $x^2 + y^2 = 16$ after a rotation of $45°$ about the origin. **11.** ________

A. $(x')^2 + (y')^2 = 16$ **B.** $(x')^2 - (y')^2 = 16$

C. $(x')^2 - 2x'y' + (y')^2 = 16$ **D.** $(x')^2 + 2x'y' + (y')^2 = 16$

12. Find parametric equations for the rectangular equation $x^2 + y^2 = 16$. **12.** ________

A. $x = \cos 4t, y = \sin 4t, 0° \le t \le 360°$

B. $x = \cos 16t, y = \sin 16t, 0° \le t \le 360°$

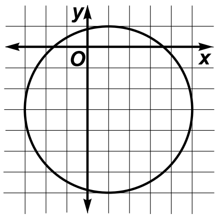
C. $x = 4 \cos t, y = 4 \sin t, 0° \le t \le 360°$

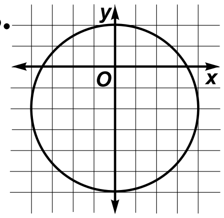
D. $x = 16 \cos t, y = 16 \sin t, 0° \le t \le 360°$

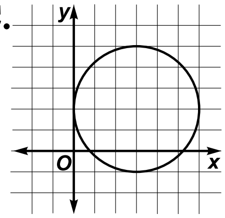
NAME ______________________ DATE ______________ PERIOD ________

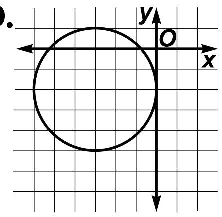
Chapter 10 Test, Form 1C (continued)

13. Which is the graph of $(x - 3)^2 + (y - 2)^2 = 9$? **13.** ________

A. 

B. 

C. 

D. 

Exercises 14 and 15 refer to the hyperbola represented by $-16y^2 - 54x + 9x^2 = 63$.

14. Write the equations of the asymptotes. **14.** ________

A. $y - 3 = \pm\frac{4}{3}x$ **B.** $y - 3 = \pm\frac{3}{4}x$

C. $y = \pm\frac{4}{3}(x - 3)$ **D.** $y = \pm\frac{3}{4}(x - 3)$

15. Find the coordinates of the foci. **15.** ________

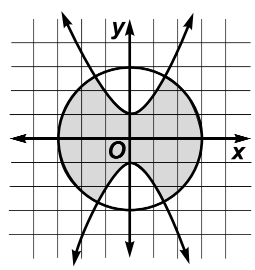
A. $(5, 0), (-5, 0)$ **B.** $(0, 5), (0, -5)$

C. $(3, 5), (3, -5)$ **D.** $(8, 0), (-2, 0)$

16. Write the standard form of the equation of the hyperbola for which $a = 5$, $b = 6$, the transverse axis is vertical, and the center is at the origin. **16.** ________

A. $\frac{y^2}{25} - \frac{x^2}{36} = 1$ **B.** $\frac{x^2}{36} - \frac{y^2}{25} = 1$ **C.** $\frac{x^2}{25} - \frac{y^2}{36} = 1$ **D.** $\frac{y^2}{36} - \frac{x^2}{25} = 1$

17. The graph at the right shows the solution set for which system of inequalities? **17.** ________



A. $y^2 - 4x^2 \leq 1, x^2 + y^2 \leq 9$

B. $y^2 - 4x^2 \geq 1, x^2 + y^2 \leq 9$

C. $y^2 - 4x^2 \leq 1, x^2 + y^2 \leq 3$

D. $y^2 - 4x^2 \geq 1, x^2 + y^2 \leq 3$

18. Find the coordinates of the vertex and the equation of the axis of symmetry for the parabola represented by $y^2 - 8x - 4y + 28 = 0$. **18.** ________

A. $(3, 2), x = 3$ **B.** $(3, 2), y = 2$

C. $(2, 3), y = 3$ **D.** $(2, 3), x = 2$

19. Write the standard form of the equation of the parabola whose directrix is $x = -2$ and whose focus is at $(2, 0)$. **19.** ________

A. $(y - 2)^2 = 8(x - 2)$ **B.** $(y - 2)^2 = 4(x - 2)$

C. $y^2 = 8x$ **D.** $x^2 = 8y$

20. Write the equation for the translation of the graph of $y^2 - 4x - 12 = 0$ for $T_{(-1, 1)}$. **20.** ________

A. $(y - 1)^2 = 4(x + 2)$ **B.** $(y - 1)^2 = 4(x + 4)$

C. $(y + 1)^2 = 4(x - 2)$ **D.** $(y + 1)^2 = 4(x - 4)$

Bonus Find the coordinates of the points of intersection of the graphs of $x^2 - y^2 = 4$, $x^2 + y^2 = 4$, and $x - y = 2$. **Bonus:** ________

A. $(2, 0)$ **B.** $(0, 2)$ **C.** $(0, -2)$ **D.** $(-2, 0)$

NAME ________________ DATE ________ PERIOD ______

Chapter 10

Chapter 10 Test, Form 2A

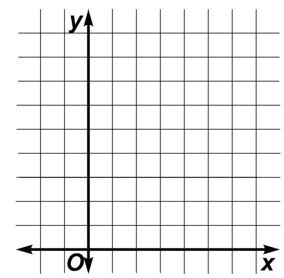
1. Find the distance between points at $(m, n - 5)$ and $(m - 3, n + 2)$. **1.** ________

2. Determine whether the quadrilateral $ABCD$ with vertices $A\left(-1, \frac{1}{2}\right)$, $B\left(\frac{1}{2}, -\frac{1}{2}\right)$, $C\left(1, -\frac{3}{2}\right)$, and $D\left(-\frac{1}{2}, -1\right)$ is a parallelogram. **2.** ________

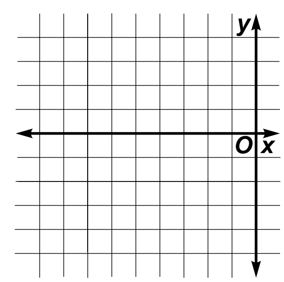
3. Write the standard form of the equation of the circle that passes through the point at $(2, -2)$ and has its center at $(-2, 3)$. **3.** ________

4. Write the standard form of the equation of the circle that passes through the points at $(1, 3)$, $(7, 3)$, and $(8, 2)$. **4.** ________

5. Write $2x^2 - 10x + 2y^2 - 18y - 1 = 0$ in standard form. Then, graph the equation, labeling the center. **5.** ________



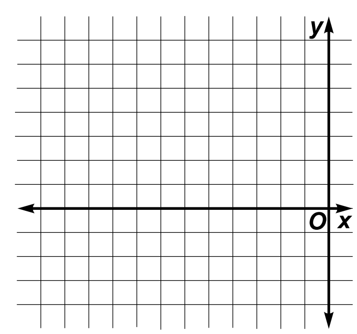
6. Write $3x^2 + 2y^2 + 24x - 4y + 26 = 0$ in standard form. Then, graph the equation, labeling the center, foci, and vertices. **6.** ________



7. Find the equation of the ellipse that has its major axis parallel to the y-axis and its center at $(4, -3)$, and that passes through points at $(1, -3)$ and $(4, 2)$. **7.** ________

8. Find the equation of the equilateral hyperbola that has its foci at $\left(-2, -3 - 2\sqrt{3}\right)$ and $\left(-2, -3 + 2\sqrt{3}\right)$, and whose conjugate axis is 6 units long. **8.** ________

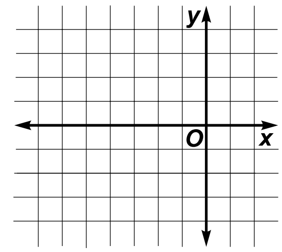
9. Write $-2x^2 + 3y^2 - 24x - 6y - 93 = 0$ in standard form. Find the equations of the asymptotes of the graph. Then, graph the equation, labeling the center, foci, and vertices. **9.** ________



NAME ______________________ DATE ____________ PERIOD ________

Chapter 10 Test, Form 2A (continued)

10. Write $x = y^2 - 2y - 5$ in standard form. Find the equations of the directrix and axis of symmetry. Then, graph the equation, labeling the focus, vertex, and directrix. **10.** ______________



11. Find the equation of the parabola that passes through the point at $\left(0, -\frac{1}{2}\right)$, has a vertical axis, and has a maximum at $(-2, 1)$. **11.** ______________

12. Find the equation of the hyperbola that has eccentricity $\frac{9}{5}$ and foci at $(4, 6)$ and $(4, -12)$. **12.** ______________

13. Identify the conic section represented by $xy + 3y + 4x = 0$. **13.** ______________

14. Find a rectangular equation for the curve whose parametric equations are $x = -\frac{1}{2}\cos 4t + 2$, $y = -2\sin 4t - 3$, $0° \leq t \leq 90°$. **14.** ______________

15. Find parametric equations for the equation $\frac{(x+1)^2}{16} + 2(y-3)^2 = 1$. **15.** ______________

Identify the graph of each equation. Write an equation of the translated or rotated graph in general form.

16. $x^2 + 12y = -31 - 10x$ for $T_{(3,\,-5)}$ **16.** ______________

17. $x^2 - xy + 2y^2 = 2$, $\theta = -30°$ **17.** ______________

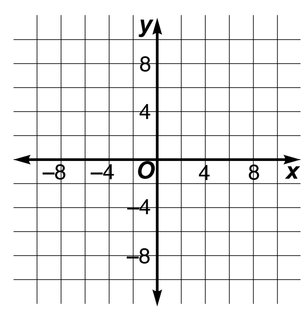
18. Identify the graph of $4x^2 + 7xy - 5y^2 = -3$. Then, find θ, the angle of rotation about the origin, to the nearest degree. **18.** ______________

19. Solve the system $4x^2 + y^2 = 32 - 4y$ and $x^2 = 7 + y$ algebraically. Round to the nearest tenth. **19.** ______________

20. Graph the solutions for the system of inequalities. **20.**

$9x^2 > y^2$

$x^2 \leq 100 - y^2$



Bonus Find the coordinates of the point(s) of intersection of the graphs of $2x + 1 = y$, $x^2 = 10 - y^2$, and $y - 4x^2 = -1$. **Bonus:** ______________

NAME ______________________ DATE ______________ PERIOD ________

Chapter 10 Test, Form 2B

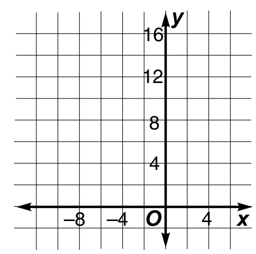
1. Find the distance between points at $(-2, -5)$ and $(6, -1)$. **1.** ______________

2. Determine whether the quadrilateral $ABCD$ with vertices $A(-1, 2)$, $B(2, 0)$, $C(-2, -2)$, and $D(-5, 0)$ is a parallelogram. **2.** ______________

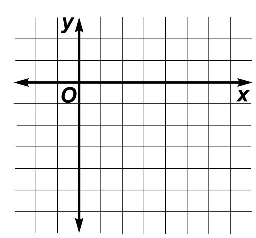
3. Write the standard form of the equation of the circle that is tangent to $x = -3$ and has its center at $(1, -3)$. **3.** ______________

4. Write the standard form of the equation of the circle that passes through the points at $(-6, 3)$, $(-4, -1)$, and $(-2, 5)$. **4.** ______________

5. Write $x^2 + y^2 + 6x - 14y - 42 = 0$ in standard form. Then, graph the equation, labeling the center. **5.** ______________



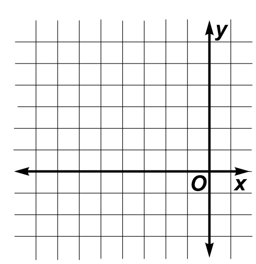
6. Write $4x^2 + 9y^2 - 24x + 18y + 9 = 0$ in standard form. Then, graph the equation, labeling the center, foci, and vertices. **6.** ______________



7. Find the equation of the ellipse that has its foci at $(2, 1)$ and $(2, -7)$ and $b = 2$. **7.** ______________

8. Find the equation of the hyperbola that has its foci at $(0, -4)$ and $(10, -4)$, and whose conjugate axis is 6 units long. **8.** ______________

9. Write $4x^2 - y^2 + 24x + 4y + 28 = 0$ in standard form. Find the equations of the asymptotes of the graph. Then, graph the equation, labeling the center, foci, and vertices. **9.** ______________

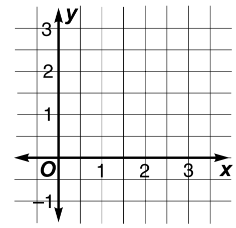


NAME ______________________ DATE ______________ PERIOD ________

Chapter 10 Test, Form 2B (continued)

10. Write $-2x + y^2 - 2y + 5 = 0$ in standard form. Find the equations of the directrix and axis of symmetry. Then, graph the equation, labeling the focus, vertex and directrix.

10. ________________



11. Find the equation of the parabola that passes through the point at $(-8, 15)$, has its vertex at $(2, -5)$, and opens to the left.

11. ________________

12. Find the equation of the ellipse that has its center at the origin, eccentricity $\frac{2\sqrt{2}}{3}$, and a vertical major axis of 6 units.

12. ________________

13. Identify the conic section represented by $x^2 - 3xy + y^2 = 5$.

13. ________________

14. Find a rectangular equation for the curve whose parametric equations are $x = \cos 3t$, $y = -2 \sin 3t$, $0° \le t \le 120°$.

14. ________________

15. Find parametric equations for the equation $\frac{x^2}{16} + \frac{y^2}{36} = 1$.

15. ________________

Identify the graph of each equation. Write an equation of the translated or rotated graph in general form.

16. $x^2 - 2x - 2y - 9 = 0$ for $T_{(-2, 3)}$

16. ________________

17. $2x^2 + 5y^2 - 20 = 0$, $\theta = 30°$

17. ________________

18. Identify the graph of $3x^2 - 8xy - 3y^2 = 3$. Then, find θ, the angle of rotation about the origin, to the nearest degree.

18. ________________

19. Solve the system $5x^2 = 10 - 2y^2$ and $3y^2 = 84 - 2x^2$ algebraically. Round to the nearest tenth.

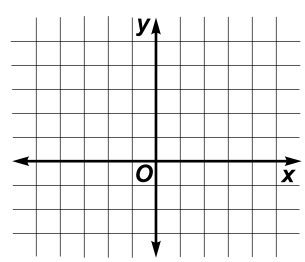
19. ________________

20. Graph the solutions for the system of inequalities.

$(x + 2)^2 + (y - 1)^2 > 9$

$\frac{x^2}{16} - y^2 \le 1$

20.



Bonus Find the coordinates of the point(s) of intersection of the graphs of $x + y + 1 = 0$, $x^2 + y^2 = 5$, $y = -3x^2 + 1$.

Bonus: ________________

NAME ______________________ DATE ______________ PERIOD ________

Chapter 10 Test, Form 2C

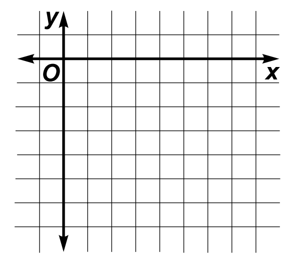
1. Find the distance between points at (2, 1) and (5, −3). **1.** ________________

2. Determine whether the quadrilateral $ABCD$ with vertices $A(5, 3)$, $B(7, 3)$, $C(5, 1)$, and $D(2, 1)$ is a parallelogram. **2.** ________________

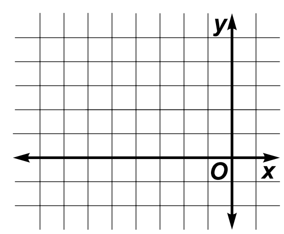
3. Write the standard form of the equation of the circle that has its center at (−4, 3) and a radius of 5. **3.** ________________

4. Write the standard form of the equation of the circle that passes through the points at (−2, 2), (2, 2), and (2, −2). **4.** ________________

5. Write $x^2 - 8x + y^2 + 4y + 16 = 0$ in standard form. Then, graph the equation, labeling the center. **5.** ________________



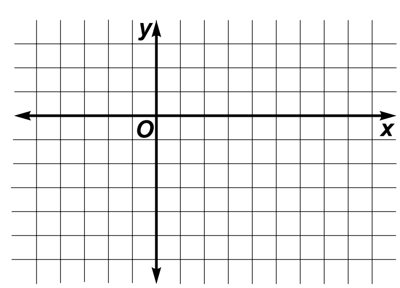
6. Write $9x^2 + 54x + 4y^2 - 16y + 61 = 0$ in standard form. Then, graph the equation, labeling the center, foci, and vertices. **6.** ________________



7. Find the equation of the ellipse that has its center at (−1, 3), a horizontal major axis of 6 units, and a minor axis of 2 units. **7.** ________________

8. Find the equation of the hyperbola that has its center at (−2, 4), $a = 3$, $b = 5$, and a vertical transverse axis. **8.** ________________

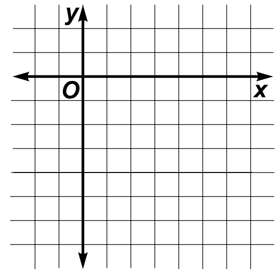
9. Write $x^2 - 4x - 4y^2 - 8y - 16 = 0$ in standard form. Find the equations of the asymptotes of the graph. Then, graph the equation, labeling the center, foci, and vertices. **9.** ________________



NAME ______________________ DATE ______________ PERIOD ________

Chapter 10 Test, Form 2C (continued)

10. Write $y^2 + 4y - 4x + 12 = 0$ in standard form. Find the equations of the directrix and axis of symmetry. Then, graph the equation, labeling the focus, vertex, and directrix. **10.** ______________



11. Find the equation of the parabola that has its vertex at $(5, -1)$, and the focus at $(5, -2)$. **11.** ______________

12. Find the eccentricity of the ellipse $\frac{(x-2)^2}{4} + \frac{(y+3)^2}{8} = 1$. **12.** ______________

13. Identify the conic section represented by $x^2 - 2y^2 + 16y - 42 = 0$. **13.** ______________

14. Find a rectangular equation for the curve whose parametric equations are $x = 2 \cos t$, $y = -3 \sin t$, $0° \le t \le 360°$. **14.** ______________

15. Find parametric equations for the equation $x^2 + y^2 = 8$. **15.** ______________

Identify the graph of each equation. Write an equation of the translated or rotated graph in general form.

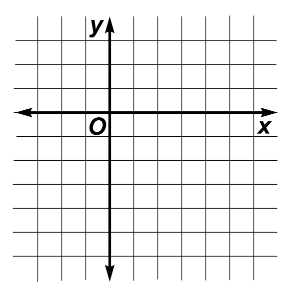
16. $y^2 + 8y + 8x + 32 = 0$ for $T_{(1,\,-4)}$ **16.** ______________

17. $5x^2 + 4y^2 = 20$, $\theta = 45°$ **17.** ______________

18. Identify the graph of $4x^2 - 9xy - 3y^2 = 5$. Then, find θ, the angle of rotation about the origin, to the nearest degree. **18.** ______________

19. Solve the system algebraically. Round to the nearest tenth. **19.** ______________
$x^2 + y^2 = 9$
$2x^2 + 3y^2 = 18$

20. Graph the solutions for the system of inequalities. **20.**
$(x - 1)^2 + 2(y + 3)^2 < 16$
$(x - 3)^2 \le -8(y + 2)$



Bonus Find the coordinates of the point(s) of intersection of the graphs of $-\frac{3}{2}x + y = 0$, $x^2 + y^2 = 13$, and $y + 1 = \frac{1}{4}(x + 2)^2$. **Bonus:** ______________

NAME ______________________ DATE ____________ PERIOD ________

Chapter 10 Open-Ended Assessment

Instructions: Demonstrate your knowledge by giving a clear, concise solution to each problem. Be sure to include all relevant drawings and justify your answers. You may show your solution in more than one way or investigate beyond the requirements of the problem.

1. Consider the equation of a conic section written in the form $Ax^2 + By^2 + Cx + Dy + E = 0$.

a. Explain how you can tell if the equation is that of a circle. Write an equation of a circle whose center is not the origin. Graph the equation.

b. Explain how you can tell if the equation is that of an ellipse. Write an equation of an ellipse whose center is not the origin. Graph the equation.

c. Explain how you can tell if the equation is that of a parabola. Write an equation of a parabola with its vertex at $(-1, 2)$. Graph the equation.

d. Explain how you can tell if the equation is that of a hyperbola. Write an equation of a hyperbola with a vertical transverse axis.

e. Identify the graph of $3x^2 - xy + 2y^2 - 3 = 0$. Then find the angle of rotation θ to the nearest degree.

2. a. Describe the graph of $x^2 - 4y^2 = 0$.

b. Graph the relation to verify your conjecture.

c. What conic section does this graph represent?

3. Give a real-world example of a conic section. Discuss how you know the object is a conic section and analyze the conic section if possible.

NAME ______________________________ DATE ______________ PERIOD ________

Chapter 10 Mid-Chapter Test (Lessons 10-1 through 10-4)

Find the distance between each pair of points with the given coordinates. Then, find the midpoint of the segment that has endpoints at the given coordinates.

1. ________________

1. $(1, -4), (2, -9)$ **2.** $(s, -t), (6 + s, -5 - t)$

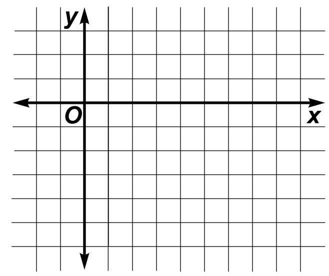
2. ________________

3. Determine whether the quadrilateral $ABCD$ with vertices $A(-2, 2), B(1, 3), C(4, -1)$, and $D(1, -2)$ is a parallelogram.

3. ________________

4. Write the standard form of $x^2 - 6x + y^2 + 4y - 12 = 0$. Then, graph the equation labeling the center.

4. ________________



5. Write the standard form of the equation of the circle that passes through the points $(-2, 16)$, $(-2, 0)$, and $(-32, 0)$. Then, identify the center and radius.

5. ________________

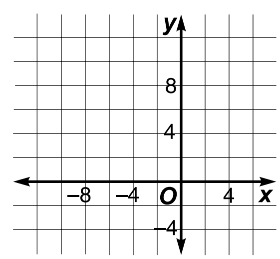
For the equation of each ellipse, find the coordinates of the center, foci, and vertices. Then, graph the equation.

6. Write the standard form of $9x^2 + y^2 + 18x - 6y + 9 = 0$. Then, find the coordinates of the center, the foci, and the vertices of the ellipse.

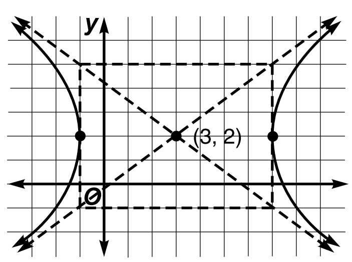
6. ________________

7. Graph the ellipse with equation $\frac{(x + 5)^2}{25} + \frac{(y - 4)^2}{16} = 1$. Label the center and vertices.

7.



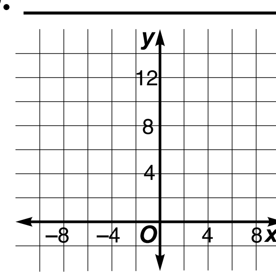
8. Write the equation of the hyperbola graphed at the right.



8. ________________

9. Find the coordinates of the center, the foci, and the vertices for the hyperbola whose equation is $4y^2 - 9x^2 - 48y - 18x + 99 = 0$. Then, find the equations of the asymptotes and graph the equation.

9. ________________



Chapter 10

NAME ______________________ DATE ____________ PERIOD ________

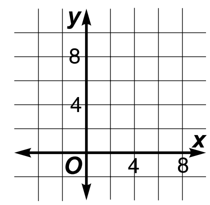
Chapter 10, Quiz A (Lessons 10-1 and 10-2)

Find the distance between each pair of points with the given coordinates. Then, find the midpoint of the segment that has endpoints at the given coordinates.

1. $(-2, 4), (5, -3)$ **2.** $(a, b), (a - 4, b + 3)$

3. Determine whether the quadrilateral $ABCD$ with vertices $A(-1, 1), B(3, 3), C(3, 0), D(-1, -1)$ is a parallelogram.

4. Write the standard form of $x^2 - 6x + y^2 - 10y - 2 = 0$. Then graph the equation, labeling the center.



5. Write the standard form of the equation of the circle that passes through the points $(2, 10)$, $(2, 0)$, and $(-10, 0)$. Then, identify the center and radius.

1. ____________
2. ____________
3. ____________
4. ____________
5. ____________

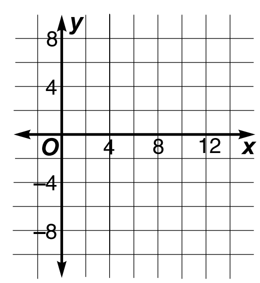
Chapter 10

NAME ______________________ DATE ____________ PERIOD ________

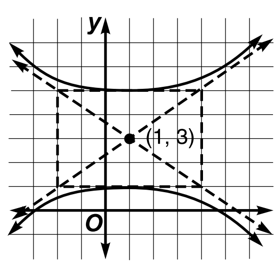
Chapter 10, Quiz B (Lessons 10-3 and 10-4)

1. Write the standard form of $16x^2 + 4y^2 - 96x + 8y + 84 = 0$. Then, find the coordinates of the center, the foci, and the vertices of the ellipse.

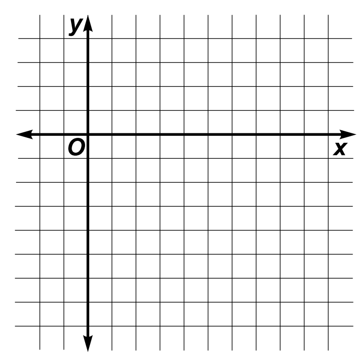
2. Graph the ellipse with equation $\frac{(x-6)^2}{64} + \frac{(y+1)^2}{100} = 1$. Label the center and vertices.



3. Write the equation of the hyperbola shown in the graph at the right.



4. Find the coordinates of the center, the foci, and the vertices for the hyperbola whose equation is $25x^2 - 4y^2 - 150x - 16y = -109$. Then, find the equations of the asymptotes and graph the equation.



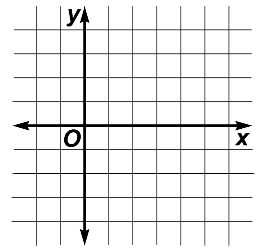
1. ____________
2.
3. ____________
4.

NAME ________________ DATE ________ PERIOD ______

Chapter 10

Chapter 10, Quiz C (Lessons 10-5 and 10-6)

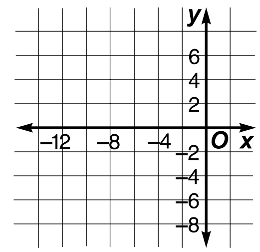
1. Write the standard form of $x^2 - 4x + 8y + 12 = 0$. Identify the coordinates of the focus and vertex, and the equations of the directrix and axis of symmetry. Then, graph the equation. **1.** ________



2. Write the equation of the parabola that has a focus at $(-2, 3)$ and whose directrix is given by the equation $x = 4$. **2.** ________

3. Identify the conic section represented by the equation $4x^2 + 25y^2 + 16x + 50y - 59 = 0$. Then, write the equation in standard form. **3.** ________

4. Find the rectangular equation of the curve whose parametric equations are $x = -3t^2$ and $y = 2t$, $-2 \le t \le 2$. Then graph the equation, using arrows to indicate the orientation. **4.** ________



5. Find parametric equations for the equation $x^2 + y^2 = 100$. **5.** ________

NAME ________________ DATE ________ PERIOD ______

Chapter 10

Chapter 10, Quiz D (Lessons 10-7 and 10-8)

Identify the graph of each equation. Write an equation of the translated or rotated graph in general form. **1.** ________

1. $5x^2 - 8y^2 = 40$ for $T_{(-3, 5)}$ **2.** $4x^2 + 18y^2 = 36$, $\theta = 135°$ **2.** ________

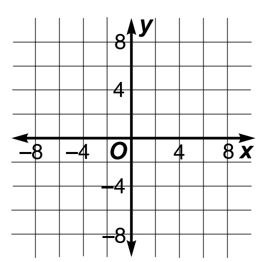
3. Identify the graph of $x^2 - 3xy + 2y^2 + 2x - y + 6 = 0$. **3.** ________

4. Solve the system $x^2 = 25 - y^2$ and $xy = -12$ algebraically. **4.** ________

5. Graph the solutions for the system of inequalities. **5.** ________

$x^2 + 9y^2 \ge 36$

$x^2 - 2y < 4$



NAME ______________________ DATE ______________ PERIOD ________

Chapter 10 SAT and ACT Practice

After working each problem, record the correct answer on the answer sheet provided or use your own paper.

Multiple Choice

1. Three points on a line are X, Y, and Z, in that order. If $XZ - YZ = 6$, what is the ratio $\frac{YZ}{XZ}$?

A 1 to 2
B 1 to 3
C 1 to 4
D 1 to 5
E It cannot be determined from the information given.

2. A train traveling 90 miles per hour for 1 hour covers the same distance as a train traveling 60 miles per hour for how many hours?

A $\frac{1}{2}$ **B** $\frac{1}{3}$
C $\frac{2}{3}$ **D** $\frac{3}{2}$
E 3

3. If x and y are integers and $xy = 48$, then which of the following CANNOT be true?

A $x + y < 14$ **B** $x - y > 14$
C $x + y = 14$ **D** $|x + y| < 14$
E $|x + y| > 14$

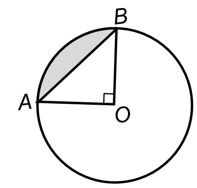
4. If $x \neq 0$, then $\frac{(-4x)^3}{-4x^3} =$

A -16
B -1
C 3
D 1
E 16

5. A framed picture is 5 feet by 8 feet, including the frame. If the frame is 8 inches wide, what is the ratio of the area of the frame to the area of the framed picture, including the frame?

A 7 to 18
B 7 to 11
C 11 to 18
D 143 to 180
E 37 to 180

6. In circle O, $OA = 6$ and $\overline{OA} \perp \overline{OB}$. Find the area of the shaded region.



A 2π units^2
B $(\pi - 2)$ units^2
C $(6\pi - 9\sqrt{3})$ units^2
D $(9\pi - 18)$ units^2
E $(36\pi - 9\sqrt{3})$ units^2

7. If 1 dozen pencils cost 1 dollar, how many dollars will n pencils cost?

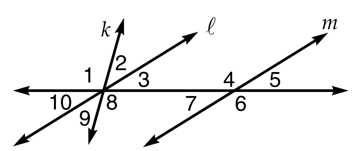
A $12n$
B $\frac{n}{12}$
C $\frac{12}{n}$
D $\frac{1}{12n}$
E It cannot be determined from the information given.

8. On a map drawn to scale, 0.125 inch represents 10 miles. What is the actual distance between two cities that are 2.5 inches apart on the map?

A 12.5 mi
B 25 mi
C 200 mi
D 250 mi
E 1250 mi

9. In the diagram below, if $\ell \parallel m$, then

I. $\angle 3$ and $\angle 4$ are supplementary.
II. $m\angle 2 = m\angle 10$.
III. $m\angle 6 = m\angle 8 + m\angle 9$.



A I only
B II only
C III only
D II and III only
E I and III only

NAME ____________________ DATE ____________ PERIOD ________

Chapter 10 SAT and ACT Practice (continued)

10. $\overline{PC}$ and $\overline{AB}$ intersect at point Q. $m\angle PQB = (2z + 80)°$, $m\angle BQC = (4x + 3y)°$, $m\angle CQA = 5w°$, and $m\angle AQP = 2z°$. Find the value of w.
A 20
B 26
C 75
D 130
E It cannot be determined from the information given.

11. Which point lies the greatest distance from the origin?
A $(0, -9)$
B $(-2, 9)$
C $(-7, -6)$
D $(8, 5)$
E $(-5, 7)$

12. The vertices of rectangle $ABCD$ are the points $A(0, 0)$, $B(8, 0)$, $C(8, k)$, and $D(0, 5)$. What is the value of k?
A 2
B 3
C 4
D 5
E 6

13. The measures of three exterior angles of a quadrilateral are 37°, 58°, and 92°. What is the measure of the exterior angle at the fourth vertex?
A 7°
B 173°
C 83°
D 49°
E 81°

14. The diagonals of parallelogram $ABCD$ intersect at point O. If $AO = 2x + 1$ and $AC = 5x - 5$, then $AO =$
A 5
B 7
C 15
D 30
E It cannot be determined from the information given.

15. If Nancy earns d dollars in h hours, how many dollars will she earn in $h + 25$ hours?
A $\frac{25d}{h}$ **B** $d + \frac{25d}{h}$
C $26d$ **D** $\frac{dh}{h+25}$
E None of these

16. If $6n$ cans fill $\frac{n}{2}$ cartons, how many cans does it take to fill 2 cartons?
A 12 **B** $24n$
C 24 **D** $6n^2$
E $\frac{3}{2}n^2$

17–18. Quantitative Comparison
A if the quantity in Column A is greater
B if the quantity in Column B is greater
C if the two quantities are equal
D if the relationship cannot be determined from the information given

	Column A	Column B
17.	Ratio of $\frac{1}{3}$ to $\frac{1}{5}$	Ratio of $\frac{2}{5}$ to $\frac{1}{3}$
18.	Ratio of boys to girls in a class with half as many boys as girls	Ratio of boys to girls in a class with twice as many girls as boys

19. Grid-In If 4 pounds of fertilizer cover 1500 square feet of lawn, how many pounds of fertilizer are needed to cover 2400 square feet?

20. Grid-In The distance between two cities is 750 miles. How many inches apart will the cities be on a map with a scale of 1 inch = 250 miles?

NAME ______________________ DATE ______________ PERIOD ________

Chapter 10

Chapter 10 Cumulative Review (Chapters 1-10)

1. Write a linear function that has no zero. **1.** ____________

2. Find BC if $B = \begin{bmatrix} 3 & -2 \\ 6 & 1 \end{bmatrix}$ and $C = \begin{bmatrix} 5 & -1 & 0 \\ 2 & 7 & -4 \end{bmatrix}$. **2.** ____________

3. Consider the system of inequalities $3x + 2y \geq 10$, $x + 3y \geq 9$, $x \geq 0$, and $y \geq 0$. In a problem asking you to find the minimum value of $f(x, y) = x + 3y$, state whether the situation is *infeasible,* has *alternate optimal solutions,* or is *unbounded.* **3.** ____________

4. Given $f(x) = \frac{2}{x - 5}$, find $f^{-1}(x)$. Then, state whether $f^{-1}(x)$ is a function. **4.** ____________

5. If y varies directly as the square of x, inversely as w, inversely as the square of z, and $y = 2$ when $x = 1$, $w = 4$, and $z = -2$, find y when $x = 3$, $w = -6$, and $z = -3$. **5.** ____________

6. Solve $\frac{x + 1}{x - 3} = \frac{3}{x} + \frac{12}{x^2 - 3x}$. **6.** ____________

7. Suppose θ is an angle in standard position whose terminal side lies in Quadrant II. If $\cos \theta = -\frac{7}{13}$, find the value of $\csc \theta$. **7.** ____________

8. Solve $\triangle ABC$ if $B = 47°$, $C = 68°$, and $b = 29.2$. **8.** ____________

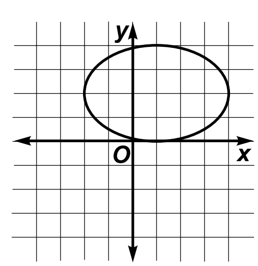
9. Find the linear velocity of the tip of an airplane propeller that is 3 meters long and rotating 500 times per minute. Give the velocity to the nearest meter per second. **9.** ____________

10. Solve $\tan \theta = \cot \theta$ for $0° \leq \theta < 360°$. **10.** ____________

11. Jason is riding his sled down a hill. If the hill is inclined at an angle of 20° with the horizontal, find the force that propels Jason down the hill if he weighs 151 pounds. **11.** ____________

12. Find $(\sqrt{3} + \boldsymbol{i})^5$. Express the result in rectangular form. **12.** ____________

13. Write the equation in standard form of the ellipse graphed at the right. **13.** ____________



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SAT and ACT Practice Answer Sheet

(10 Questions)

1 (A) (B) (C) (D) (E)

2 (A) (B) (C) (D) (E)

3 (A) (B) (C) (D) (E)

4 (A) (B) (C) (D) (E)

5 (A) (B) (C) (D) (E)

6 (A) (B) (C) (D) (E)

7 (A) (B) (C) (D) (E)

8 (A) (B) (C) (D) (E)

9 (A) (B) (C) (D) (E)

10

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3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

NAME ______________________________ DATE ______________ PERIOD ________

SAT and ACT Practice Answer Sheet

(20 Questions)

1 (A) (B) (C) (D) (E)

2 (A) (B) (C) (D) (E)

3 (A) (B) (C) (D) (E)

4 (A) (B) (C) (D) (E)

5 (A) (B) (C) (D) (E)

6 (A) (B) (C) (D) (E)

7 (A) (B) (C) (D) (E)

8 (A) (B) (C) (D) (E)

9 (A) (B) (C) (D) (E)

10 (A) (B) (C) (D) (E)

11 (A) (B) (C) (D) (E)

12 (A) (B) (C) (D) (E)

13 (A) (B) (C) (D) (E)

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16 (A) (B) (C) (D) (E)

17 (A) (B) (C) (D) (E)

18 (A) (B) (C) (D) (E)

19

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20

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8	8	8	8
9	9	9	9

NAME ______________ DATE ________ PERIOD ______

10-1 Practice

Introduction to Analytic Geometry

Find the distance between each pair of points with the given coordinates. Then find the midpoint of the segment that has endpoints at the given coordinates.

1. $(-2, 1), (3, 4)$
$\sqrt{34}$; (0.5, 2.5)

2. $(1, 1), (9, 7)$
10; (5, 4)

3. $(3, -4), (5, 2)$
$2\sqrt{10}$; (4, −1)

4. $(-1, 2), (5, 4)$
$2\sqrt{10}$; (2, 3)

5. $(-7, -4), (2, 8)$
15; (−2.5, 2)

6. $(-4, 10), (4, -5)$
17; (0, 2.5)

Determine whether the quadrilateral having vertices with the given coordinates is a parallelogram.

7. $(4, 4), (2, -2), (-5, -1), (-3, 5)$
yes

8. $(3, 5), (-1, 1), (-6, 2), (-3, 7)$
no

9. $(4, -1), (2, -5), (-3, -3), (-1, 1)$
yes

10. $(2, 6), (1, 2), (-4, 4), (-3, 9)$
no

11. ***Hiking*** Jenna and Maria are hiking to a campsite located at (2, 1) on a map grid, where each side of a square represents 2.5 miles. If they start their hike at (−3, 1), how far must they hike to reach the campsite?
12.5 mi

NAME ______________ DATE ________ PERIOD ______

10-1 Enrichment

Mathematics and History: Hypatia

Hypatia (A.D. 370–415) is the earliest woman mathematician whose life is well documented. Born in Alexandria, Egypt, she was widely known for her keen intellect and extraordinary mathematical ability. Students from Europe, Asia, and Africa flocked to the university at Alexandria to attend her lectures on mathematics, astronomy, philosophy, and mechanics.

Hypatia wrote several major treatises in mathematics. Perhaps the most significant of these was her commentary on the *Arithmetica* of Diophantus, a mathematician who lived and worked in Alexandria in the third century. In her commentary, Hypatia offered several observations about the *Arithmetica's* Diophantine problems—problems for which one was required to find only the rational solutions. It is believed that many of these observations were subsequently incorporated into the original manuscript of the *Arithmetica*.

In modern mathematics, the solutions of a **Diophantine equation** are restricted to integers. In the exercises, you will explore some questions involving simple Diophantine equations.

For each equation, find three solutions that consist of an ordered pair of integers.

1. $2x - y = 7$
(1, −5), (0, −7), (−1, −9)

2. $x + 3y = 5$
(2, 1), (5, 0), (8, −1)

3. $6x - 5y = -8$
(2, 4), (−3, −2), (−8, −8)

4. $-11x - 4y = 6$
(2, −7), (−2, 4), (−6, 15)

5. Refer to your answers to Exercises 1–4. Suppose that the integer pair (x_1, y_1) is a solution of $Ax - By = C$. Describe how to find other integer pairs that are solutions of the equation.
Other integer pairs are of the form $(x_1 + n \cdot B, y_1 - n \cdot A)$, where n is any nonzero integer.

6. Explain why the equation $3x + 6y = 7$ has no solutions that are integer pairs.
Rewrite $3x + 6y = 7$ as $3(x + 2y) = 7$. If x and y are integers 7 would have to be an integral multiple of 3.

7. *True* or *false*: Any line on the coordinate plane must pass through at least one point whose coordinates are integers. Explain.
False; An equation like $3x + 6y = 7$ has no integer-pair solutions, so the graph of such an equation is a line that passes through no point whose coordinates are integers.

10-2

NAME ______________ DATE ______________ PERIOD ________

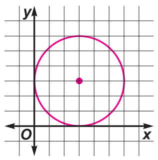
Practice

Circles

Write the standard form of the equation of each circle described. Then graph the equation.

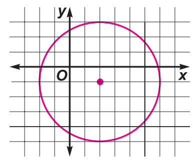
1. center at (3, 3) tangent to the x-axis

$(x - 3)^2 + (y - 3)^2 = 9$



2. center at (2, −1), radius 4

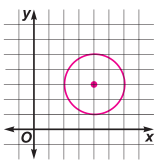
$(x - 2)^2 + (y + 1)^2 = 16$



Write the standard form of each equation. Then graph the equation.

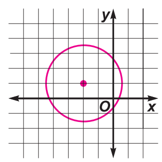
3. $x^2 + y^2 - 8x - 6y + 21 = 0$

$(x - 4)^2 + (y - 3)^2 = 4$



4. $4x^2 + 4y^2 + 16x - 8y - 5 = 0$

$(x + 2)^2 + (y - 1)^2 = \frac{25}{4}$



Write the standard form of the equation of the circle that passes through the points with the given coordinates. Then identify the center and radius.

5. (−3, −2), (−2, −3), (−4, −3)

$(x + 3)^2 + (y + 3)^2 = 1;$

$(-3, -3); 1$

6. (0, −1), (2, −3), (4, −1)

$(x - 2)^2 + (y + 1)^2 = 4;$

$(2, -1); 2$

7. ***Geometry*** A square inscribed in a circle and centered at the origin has points at (2, 2), (−2, 2), (2, −2) and (−2, −2). What is the equation of the circle that circumscribes the square?

$x^2 + y^2 = 8$

10-2

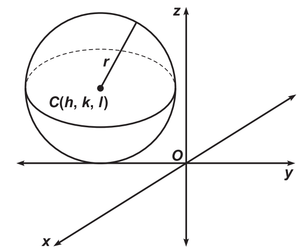
NAME ______________ DATE ______________ PERIOD ________

Enrichment

Spheres

The set of all points in three-dimensional space that are a fixed distance r (the **radius**), from a fixed point C (the **center**), is called a **sphere**. The equation below is an algebraic representation of the sphere shown at the right.

$$(x - h)^2 + (y - k)^2 + (z - l)^2 = r^2$$

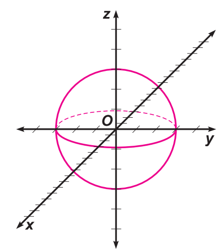


A line segment containing the center of a sphere and having its endpoints on the sphere is called a **diameter** of the sphere. The endpoints of a diameter are called **poles** of the sphere. A **great circle** of a sphere is the intersection of the sphere and a plane containing the center of the sphere.

1. If $x^2 + y^2 - 4y + z^2 + 2z - 4 = 0$ is an equation of a sphere and (1, 4, −3) is one pole of the sphere, find the coordinates of the opposite pole.

(−1, 0, 1)

2. a. On the coordinate system at the right, sketch the sphere described by the equation $x^2 + y^2 + z^2 = 9$.



b. Is $P(2, -2, -2)$ inside, outside, or on the sphere?

outside

c. Describe a way to tell if a point with coordinates $P(a, b, c)$ is inside, outside, or on the sphere with equation $x^2 + y^2 + z^2 = r^2$.

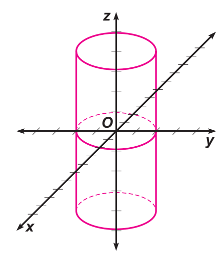
$a^2 + b^2 + c^2 < r^2$: **inside the sphere**
$a^2 + b^2 + c^2 = r^2$: **on the sphere**
$a^2 + b^2 + c^2 > r^2$: **outside the sphere**

3. If $x^2 + y^2 + z^2 - 4x + 6y - 2z - 2 = 0$ is an equation of a sphere, find the circumference of a great circle, and the surface area and volume of the sphere.

8π units; 64π square units; $\frac{256\pi}{3}$ cubic units

4. The equation $x^2 + y^2 = 4$ represents a set of points in three-dimensional space. Describe that set of points in your own words. Illustrate with a sketch on the coordinate system at the right.

a cylinder



10-3

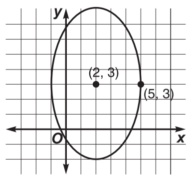
NAME ______________ DATE ______________ PERIOD ________

Practice

Ellipses

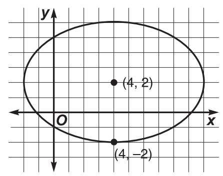
Write the equation of each ellipse in standard form. Then find the coordinates of its foci.

1.



$\frac{(y-3)^2}{25} + \frac{(x-2)^2}{9} = 1$;
(2, −1), (2, 7)

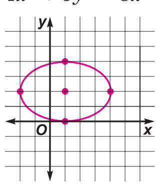
2.



$\frac{(x-4)^2}{36} + \frac{(y-2)^2}{16} = 1$;
$(4 - 2\sqrt{5}, 2)$, $(4 + 2\sqrt{5}, 2)$

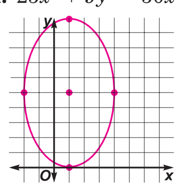
For the equation of each ellipse, find the coordinates of the center, foci, and vertices. Then graph the equation.

3. $4x^2 + 9y^2 - 8x - 36y + 4 = 0$



center: (1, 2);
foci: $(1 \pm \sqrt{5}, 2)$
vertices:
(−2, 2), (1, 4),
(4, 2), (1, 0)

4. $25x^2 + 9y^2 - 50x - 90y + 25 = 0$



center: (1, 5);
foci: (1, 9),
(1, 1)
vertices:
(1, 10), (1, 0),
(4, 5), (−2, 5)

Write the equation of the ellipse that meets each set of conditions.

5. The center is at (1, 3), the major axis is parallel to the y-axis, and one vertex is at (1, 8), and $b = 3$.
$\frac{(y-3)^2}{25} + \frac{(x-1)^2}{9} = 1$

6. The foci are at (−2, 1) and (−2, −7), and $a = 5$.
$\frac{(y+3)^2}{25} + \frac{(x+2)^2}{9} = 1$

7. ***Construction*** A semi elliptical arch is used to design a headboard for a bed frame. The headboard will have a height of 2 feet at the center and a width of 5 feet at the base. Where should the craftsman place the foci in order to sketch the arch?
1.5 ft from the center

10-3

NAME ______________ DATE ______________ PERIOD ________

Enrichment

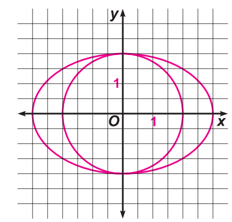
Superellipses

The circle and the ellipse are members of an interesting family of curves that were first studied by the French physicist and mathematician Gabriel Lamé (1795-1870). The general equation for the family is

$\left|\frac{x}{a}\right|^n + \left|\frac{y}{b}\right|^n = 1$, with $a \neq 0$, $b \neq 0$, and $n > 0$.

For even values of n greater than 2, the curves are called **superellipses**.

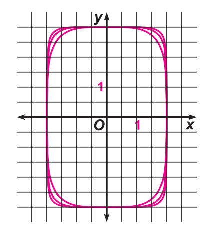
1. Consider two curves that are *not* superellipses. Graph each equation on the grid at the right. State the type of curve produced each time.



a. $\left|\frac{x}{2}\right|^2 + \left|\frac{y}{2}\right|^2 = 1$ **circle**

b. $\left|\frac{x}{3}\right|^2 + \left|\frac{y}{2}\right|^2 = 1$ **ellipse**

2. In each of the following cases you are given values of a, b, and n to use in the general equation. Write the resulting equation. Then graph. Sketch each graph on the grid at the right.



a. $a = 2, b = 3, n = 4$ $\left|\frac{x}{2}\right|^4 + \left|\frac{y}{3}\right|^4 = 1$

b. $a = 2, b = 3, n = 6$ $\left|\frac{x}{2}\right|^6 + \left|\frac{y}{3}\right|^6 = 1$

c. $a = 2, b = 3, n = 8$ $\left|\frac{x}{2}\right|^8 + \left|\frac{y}{3}\right|^8 = 1$

3. What shape will the graph of $\left|\frac{x}{2}\right|^n + \left|\frac{y}{3}\right|^n = 1$ approximate for greater and greater even, whole-number values of n?
a rectangle that is 6 units long and 4 units wide, centered at the origin

NAME ______________ DATE ________ PERIOD ______

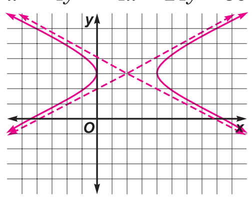
10-4

Practice

Hyperbolas

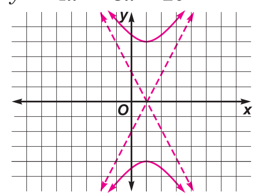
For each equation, find the coordinates of the center, foci, and vertices, and the equations of the asymptotes of its graph. Then graph the equation.

1. $x^2 - 4y^2 - 4x + 24y - 36 = 0$



center: (2, 3); foci $(2 \pm \sqrt{5}, 3)$;
vertices: (0, 3), (4, 3);
asymptotes: $y - 3 = \pm\frac{1}{2}(x - 2)$

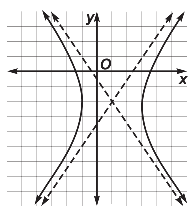
2. $y^2 - 4x^2 + 8x = 20$



center: (1, 0); foci: $(1, \pm 2\sqrt{5})$;
vertices: (1, ±4)
asymptotes: $y = \pm 2(x - 1)$

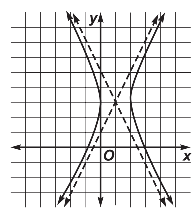
Write the equation of each hyperbola.

3.



$\frac{(x-1)^2}{4} - \frac{(y+2)^2}{9} = 1$

4.



$\frac{(x-1)^2}{1} - \frac{(y-3)^2}{4} = 1$

5. Write an equation of the hyperbola for which the length of the transverse axis is 8 units, and the foci are at (6, 0) and (−4, 0).

$\frac{(x-1)^2}{16} - \frac{y^2}{9} = 1$

6. ***Environmental Noise*** Two neighbors who live one mile apart hear an explosion while they are talking on the telephone. One neighbor hears the explosion two seconds before the other. If sound travels at 1100 feet per second, determine the equation of the hyperbola on which the explosion was located.

$\frac{x^2}{1{,}210{,}000} - \frac{y^2}{5{,}759{,}600} = 1$

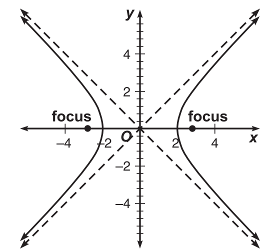
NAME ______________ DATE ________ PERIOD ______

10-4

Enrichment

Moving Foci

Recall that the equation of a hyperbola with center at the origin and horizontal transverse axis has the equation $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. The foci are at $(-c, 0)$ and $(c, 0)$, where $c^2 = a^2 + b^2$, the vertices are at $(-a, 0)$ and $(a, 0)$, and the asymptotes have equations $y = \pm\frac{b}{a}x$. Such a hyperbola is shown at the right.



What happens to the shape of the graph as c grows very large or very small?

Refer to the hyperbola described above.

1. Write a convincing argument to show that as c approaches 0, the foci, the vertices, and the center of the hyperbola become the same point.

Since $0 < a < c$, as c approaches 0, a approaches 0. So the x-coordinates of the foci and vertices approach 0, which is the x-coordinate of the center. Since the y-coordinates are equal, the points become the same.

2. Use a graphing calculator or computer to graph $x^2 - y^2 = 1$, $x^2 - y^2 = 0.1$, and $x^2 - y^2 = 0.01$. (Such hyperbolas correspond to smaller and smaller values of c.) Describe the changes in the graphs. What shape do the graphs approach as c approaches 0?

The asymptotes remain the same, but the branches become sharper near the vertices. The graphs approach the lines $y = x$ and $y = -x$.

3. Suppose a is held fixed and c approaches infinity. How does the graph of the hyperbola change?

The vertices remain at $(\pm a, 0)$, but the branches become more vertical. The graphs approach the vertical lines $x = -a$ and $x = a$.

4. Suppose b is held fixed and c approaches infinity. How does the graph of the hyperbola change?

The vertices recede to infinity and the branches become flatter and farther from the center. As c approaches infinity, the graphs tend to disappear.

Answers (Lesson 10-5)

NAME ______________________ DATE ____________ PERIOD ________

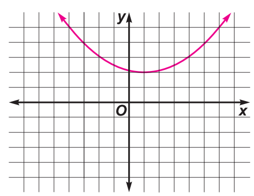
Practice

Parabolas

For the equation of each parabola, find the coordinates of the vertex and focus, and the equations of the directrix and axis of symmetry. Then graph the equation.

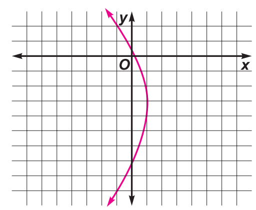
1. $x^2 - 2x - 8y + 17 = 0$

vertex: (1, 2); focus: (1, 4);
directrix: $y = 0$;
axis of symmetry: $x = 1$



2. $y^2 + 6y + 9 = 12 - 12x$

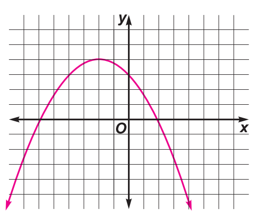
vertex: (1, −3); focus: (−2, −3);
directrix: $x = 4$;
axis of symmetry: $y = -3$



Write the equation of the parabola that meets each set of conditions. Then graph the equation.

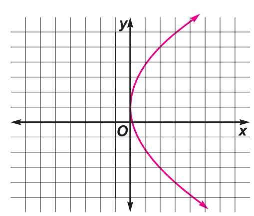
3. The vertex is at $(-2, 4)$ and the focus is at $(-2, 3)$.

$(x + 2)^2 = -4(y - 4)$



4. The focus is at $(2, 1)$, and the equation of the directrix is $x = -2$.

$(y - 1)^2 = 8x$



5. ***Satellite Dish*** Suppose the receiver in a parabolic dish antenna is 2 feet from the vertex and is located at the focus. Assume that the vertex is at the origin and that the dish is pointed upward. Find an equation that models a cross section of the dish.

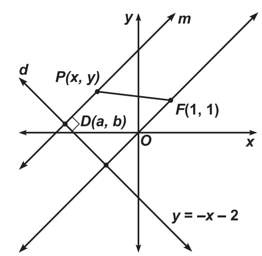
$x^2 = 8y$

NAME ______________________ DATE ____________ PERIOD ________

Enrichment

Tilted Parabolas

The diagram at the right shows a fixed point $F(1, 1)$ and a line d whose equation is $y = -x - 2$. If $P(x, y)$ satisfies the condition that $PD = PF$, then P is on a parabola. Our objective is to find an equation for the tilted parabola; which is the locus of all points that are the same distance from (1,1) and the line $y = -x - 2$.



To do this, first find an equation for the line m through $P(x, y)$ and perpendicular to line d at $D(a, b)$. Using this equation and the equation for line d, find the coordinates (a, b) of point D in terms of x and y. Then use $(PD)^2 = (PF)^2$ to find an equation for the parabola.

Refer to the discussion above.

1. Find an equation for line m.

$x - y + (b - a) = 0$

2. Use the equations for lines m and d to show that the coordinates of point D are $D(a, b) = D\left(\frac{x - y - 2}{2}, \frac{y - x - 2}{2}\right)$.

From the equation for line m, $-a + b = -x + y$. From the equation for d, $a + b = -2$. Subtract to get $a = \frac{x - y - 2}{2}$. Add to get $b = \frac{y - x - 2}{2}$.

3. Use the coordinates of F, P, and D, along with $(PD)^2 = (PF)^2$ to find an equation of the parabola with focus F and directrix d.

$x^2 - 2xy + y^2 - 8x - 8y = 0$

4. a. Every parabola has an axis of symmetry. Find an equation for the axis of symmetry of the parabola described above. Justify your answer.

$y = x$, since $y = x$ contains $F(1,1)$ and is perpendicular to d.

b. Use your answer from part **a** to find the coordinates of the vertex of the parabola. Justify your answer.

(0,0), since (0,0) is midway between point F and line d.

NAME ______________ DATE ________ PERIOD ______

10-6

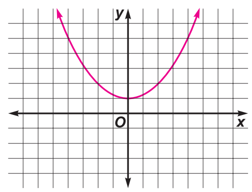
Practice

Rectangular and Parametric Forms of Conic Sections

Identify the conic section represented by each equation. Then write the equation in standard form and graph the equation.

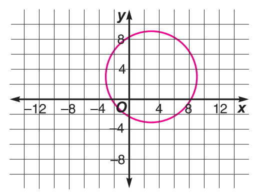
1. $x^2 - 4y + 4 = 0$

parabola; $x^2 = 4(y - 1)$



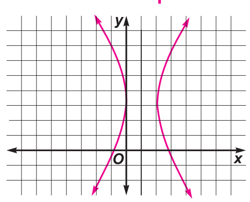
2. $x^2 + y^2 - 6x - 6y - 18 = 0$

circle;
$(x - 3)^2 + (y - 3)^2 = 36$



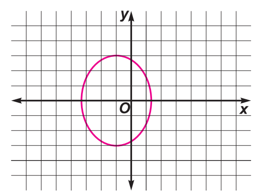
3. $4x^2 - y^2 - 8x + 6y = 9$

hyperbola; $\frac{(x-1)^2}{1} - \frac{(y-3)^2}{4} = 1$



4. $9x^2 + 5y^2 + 18x = 36$

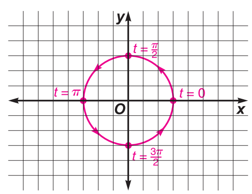
ellipse; $\frac{(x+1)^2}{5} + \frac{y^2}{9} = 1$



Find the rectangular equation of the curve whose parametric equations are given. Then graph the equation using arrows to indicate orientation.

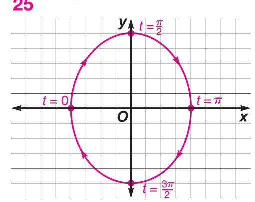
5. $x = 3 \cos t$, $y = 3 \sin t$, $0 \le t \le 2\pi$

$x^2 + y^2 = 9$



6. $x = -4 \cos t$, $y = 5 \sin t$, $0 \le t \le 2\pi$

$\frac{x^2}{16} + \frac{y^2}{25} = 1$



NAME ______________ DATE ________ PERIOD ______

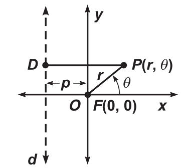
10-6

Enrichment

Polar Graphs of Conics

A conic is the locus of all points such that the ratio e of the distance from a fixed point F and a fixed line d is constant.

$$\frac{FP}{DP} = e$$



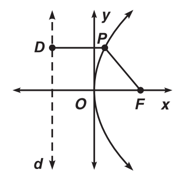
To find the polar equation of the conic, use a polar coordinate system with the origin at the focus.

Since $FP = r$ and $DP = p + r \cos\theta$, $\frac{r}{p + r\cos\theta} = e$.

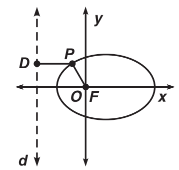
Now solve for r. $\quad r = \frac{ep}{1 - e\cos\theta}$

You can classify a conic section by its eccentricity.

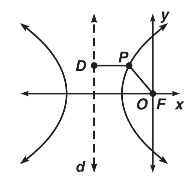
$e = 1$: parabola



$0 < e < 1$: ellipse
$e = 0$: circle

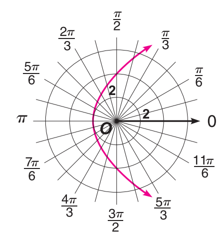


$e > 1$: hyperbola

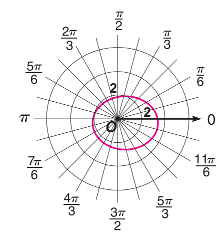


Graph each relation and identify the conic.

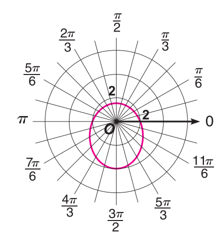
1. $r = \frac{4}{1 - \cos\theta}$ **parabola**



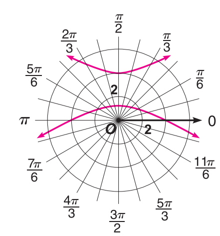
2. $r = \frac{4}{2 - \cos\theta}$ **ellipse**



3. $r = \frac{4}{2 + \sin\theta}$ **ellipse**



4. $r = \frac{4}{1 + 2\sin\theta}$ **hyperbola**



NAME ______________ DATE ________ PERIOD ______

10-7 Practice

Transformations of Conics

Identify the graph of each equation. Write an equation of the translated or rotated graph in general form.

1. $2x^2 + 5y^2 = 9$ for $T_{(-2, 1)}$
ellipse
$\mathbf{2x^2 + 5y^2 + 8x - 10y + 4 = 0}$

2. $2x^2 - 4x + 3 - y = 0$ for $T_{(1, -1)}$
parabola
$\mathbf{2x^2 - 8x - y + 8 = 0}$

3. $xy = 1$, $\theta = \frac{\pi}{4}$
hyperbola
$\mathbf{-\frac{1}{2}x^2 + \frac{1}{2}y^2 - 1 = 0}$

4. $x^2 - 4y = 0$, $\theta = 90°$
parabola
$\mathbf{y^2 + 4x = 0}$

Identify the graph of each equation. Then find θ to the nearest degree.

5. $2x^2 + 2y^2 - 2x = 0$
circle
45°

6. $3x^2 + 8xy + 4y^2 - 7 = 0$
hyperbola
−41°

7. $16x^2 - 24xy + 9y^2 - 30x - 40y = 0$
parabola
−37°

8. $13x^2 - 8xy + 7y^2 - 45 = 0$
ellipse
−27°

9. ***Communications*** Suppose the orientation of a satellite dish that monitors radio waves is modeled by the equation $4x^2 + 2xy + 4y^2 + \sqrt{2}x - \sqrt{2}y = 0$. What is the angle of rotation of the satellite dish about the origin?
45°

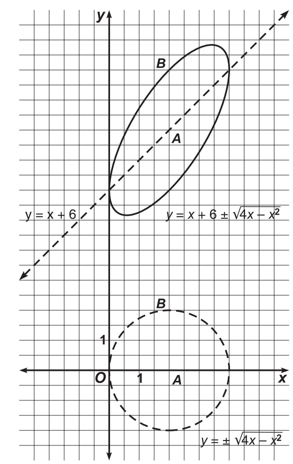
NAME ______________ DATE ________ PERIOD ______

10-7 Enrichment

Graphing with Addition of y-Coordinates

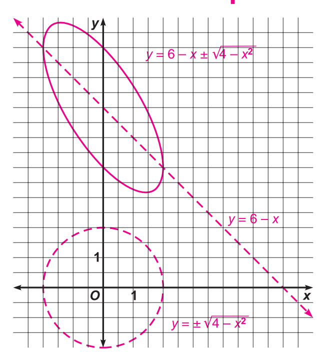
Equations of parabolas, ellipses, and hyperbolas that are "tipped" with respect to the x- and y-axes are more difficult to graph than the equations you have been studying.

Often, however, you can use the graphs of two simpler equations to graph a more complicated equation. For example, the graph of the ellipse in the diagram at the right is obtained by adding the y-coordinate of each point on the circle and the y-coordinate of the corresponding point of the line.

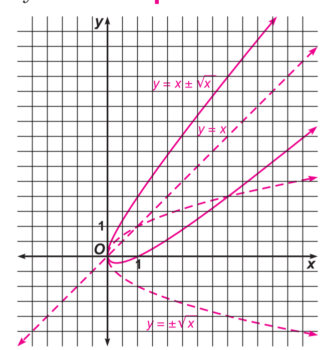


Graph each equation. State the type of curve for each graph.

1. $y = 6 - x \pm \sqrt{4 - x^2}$ **ellipse**



2. $y = x \pm \sqrt{x}$ **parabola**



Use a separate sheet of graph paper to graph these equations. State the type of curve for each graph.

3. $y = 2x \pm \sqrt{7 + 6x - x^2}$ **ellipse; See students' graphs.**

4. $y = -2x \pm \sqrt{-2x}$ **parabola; See students' graphs.**

NAME ______________ DATE ________ PERIOD ______

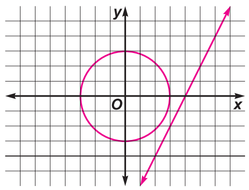
10-8

Practice

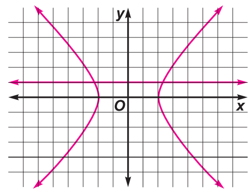
Systems of Second-Degree Equations and Inequalities

Solve each system of equations algebraically. Round to the nearest tenth. Check the solutions by graphing each system.

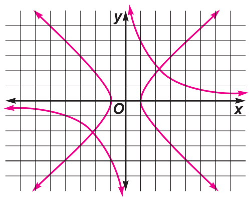
1. $2x - y = 8$
$x^2 + y^2 = 9$
no real solutions



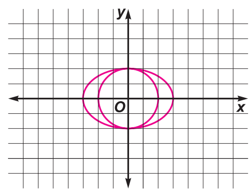
2. $x^2 - y^2 = 4$
$y = 1$
(−2.2, 1) and (2.2, 1)



3. $xy = 4$
$x^2 = y^2 + 1$
(2.1, 1.9) and (−2.1, −1.9)

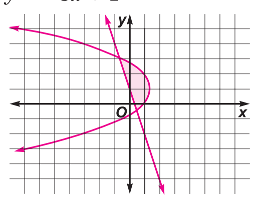


4. $x^2 + y^2 = 4$
$4x^2 + 9y^2 = 36$
(0, −2) and (0, 2)

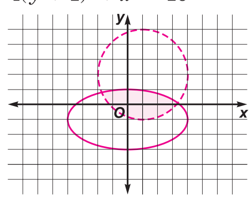


Graph each system of inequalities.

5. $3 \geq (y - 1)^2 + 2x$
$y \geq -3x + 1$



6. $(x - 1)^2 + (y - 2)^2 < 9$
$4(y + 1)^2 + x^2 \leq 16$



7. ***Sales*** Vincent's Pizzeria reduced prices for large specialty pizzas by $5 for 1 week in March. In the previous week, sales for large specialty pizzas totaled $400. During the sale week, the number of large pizzas sold increased by 20 and total sales amounted to $600. Write a system of second-degree equations to model this situation. Find the regular price and the sale price of large specialty pizzas. **$xy = 400$, $(x + 20)(y - 5) = 600$; regular price: $20.00, sale price: $15.00**

NAME ______________ DATE ________ PERIOD ______

10-8

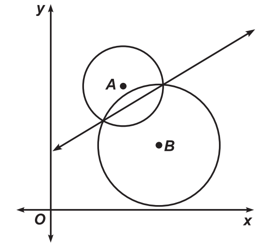
Enrichment

Intersections of Circles

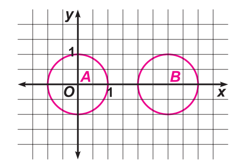
Many interesting problems involving circles can be solved by using a system of equations. Consider the following problem.

Find an equation for the straight line that contains the two points of intersection of two intersecting circles whose equations are given.

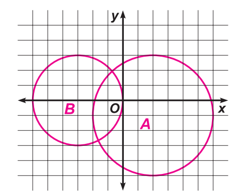
You may be surprised to find that if the given circles intersect in two points, then the difference of their equations is the equation of the line containing the intersection points.



1. Circle A has equation $x^2 + y^2 = 1$ and circle B has equation $(x - 3)^2 + y^2 = 1$. Use a sketch to show that the circles do not intersect. Use an algebraic argument to show that circles A and B do not intersect.
The distance between the centers is 3. Since the sum of the radii is 2, there is 1 unit of space between the circles. Thus, the circles do not intersect.



2. Circle A has equation $(x - 2)^2 + (y + 1)^2 = 16$ and circle B has equation $(x + 3)^2 + y^2 = 9$. Use a sketch to show that the circles meet in two points. Then find an equation in standard form for the line containing the points of intersection.
$10x - 2y + 11 = 0$



3. Without graphing the equations, decide if the circles with equations $(x - 2)^2 + (y - 2)^2 = 8$ and $(x - 3)^2 + (y - 4)^2 = 4$ are tangent. Justify your answer.
The distance between the centers is $\sqrt{5} \approx 2.24$. Since the sum of the radii is $2 + 2\sqrt{2} \approx 4.83$, the circles overlap and meet in two points; they are not tangent.

Chapter 10 Answer Key

Form 1A

Page 441		Page 442	
1.	B	13.	A
2.	A	14.	C
3.	C	15.	D
4.	B	16.	B
5.	C	17.	C
6.	C	18.	C
7.	B	19.	D
8.	A	20.	B
9.	C	Bonus:	C
10.	C		
11.	A		
12.	B		

Form 1B

Page 443		Page 444	
1.	B	13.	B
2.	C	14.	D
3.	A	15.	B
4.	C	16.	D
5.	B	17.	A
6.	A	18.	A
7.	B	19.	D
8.	A	20.	B
9.	B	Bonus:	B
10.	B		
11.	A		
12.	D		

Chapter 10 Answer Key

Form 1C

Page 445

1. D
2. B
3. B
4. A
5. C
6. C
7. B
8. B
9. D
10. B
11. A
12. C

Page 446

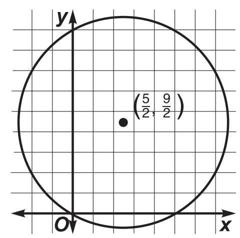
13. C
14. D
15. D
16. A
17. A
18. B
19. C
20. B

Bonus: A

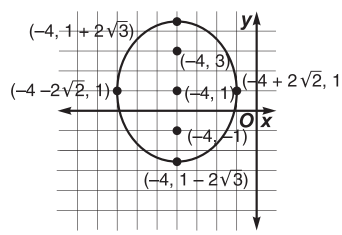
Form 2A

Page 447

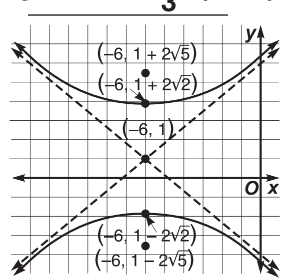
1. $\sqrt{58}$
2. no
3. $(x + 2)^2 + (y - 3)^2 = 41$
4. $(x - 4)^2 + (y + 1)^2 = 25$
5. $\left(x - \frac{5}{2}\right)^2 + \left(y - \frac{9}{2}\right)^2 = 27$



6. $\frac{(x + 4)^2}{8} + \frac{(y - 1)^2}{12} = 1$

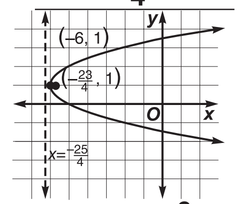


7. $\frac{(x - 4)^2}{9} + \frac{(y + 3)^2}{25} = 1$
8. $\frac{(y + 3)^2}{3} - \frac{(x + 2)^2}{9} = 1$
9. $\frac{(y - 1)^2}{8} - \frac{(x + 6)^2}{12} = 1$; asymptotes: $y - 1 = \pm\frac{\sqrt{6}}{3}(x + 6)$

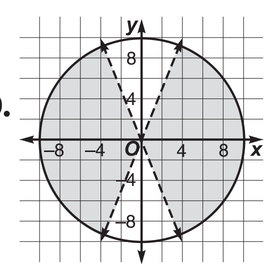


Page 448

10. $(y - 1)^2 = x + 6$; $x = -\frac{25}{4}$; $y = 1$



11. $(x + 2)^2 = -\frac{8}{3}(y - 1)$
12. $\frac{(y + 3)^2}{25} - \frac{(x - 4)^2}{56} = 1$
13. hyperbola
14. $\frac{(x - 2)^2}{\frac{1}{4}} + \frac{(y + 3)^2}{4} = 1$
15. Sample answer: $x = 4\cos t - 1$, $y = \frac{\sqrt{2}}{2}\sin t + 3$, $0° \leq t \leq 360°$
16. parabola; $x^2 + 4x + 12y + 70 = 0$
17. ellipse; $(5 - \sqrt{3})x^2 - (2 - 2\sqrt{3})xy + (7 + \sqrt{3})y^2 - 8 = 0$
18. hyperbola, 19°
19. $(\pm 2.7, 0.5)$
20.



Bonus: (1, 3)

Chapter 10 Answer Key

Form 2B

Page 449

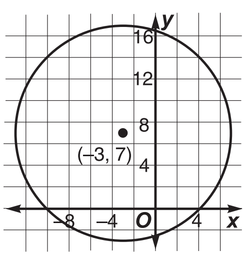
1. $4\sqrt{5}$

2. yes

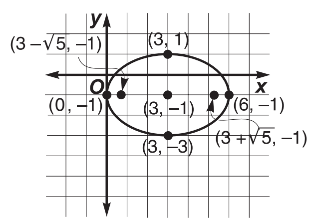
3. $(x-1)^2 + (y+3)^2 = 16$

4. $(x+3)^2 + (y-2)^2 = 10$

5. $(x+3)^2 + (y-7)^2 = 100$



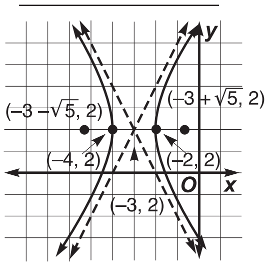
6. $\frac{(x-3)^2}{9} + \frac{(y+1)^2}{4} = 1$



7. $\frac{(x-2)^2}{4} + \frac{(y+3)^2}{20} = 1$

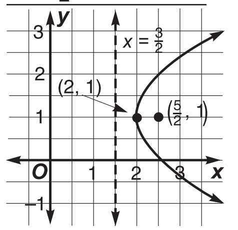
8. $\frac{(x-5)^2}{16} - \frac{(y+4)^2}{9} = 1$

9. $\frac{(x+3)^2}{1} - \frac{(y-2)^2}{4} = 1$
asymptotes:
$y - 2 = \pm 2(x+3)$



Page 450

10. $(y-1)^2 = 2(x-2)$; $x = \frac{3}{2}$; $y = 1$



11. $(y+5)^2 = -40(x-2)$

12. $\frac{x^2}{1} + \frac{y^2}{9} = 1$

13. hyperbola

14. $\frac{x^2}{1} + \frac{y^2}{4} = 1$

15. Sample answer: $x = 4 \cos t$, $y = 6 \sin t$, $0° \le t \le 360°$

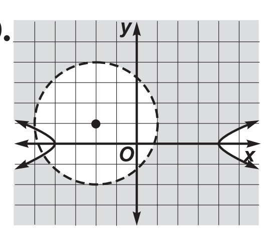
16. parabola; $x^2 + 2x - 2y - 3 = 0$

17. ellipse; $11(x')^2 - 6\sqrt{3}x'y' + 17(y')^2 - 80 = 0$

18. hyperbola, $-27°$

19. no solution

20.



Bonus: $(1, -2)$

Form 2C

Page 451

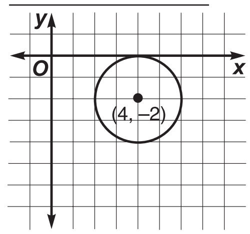
1. 5

2. no

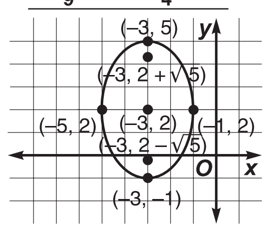
3. $(x+4)^2 + (y-3)^2 = 25$

4. $x^2 + y^2 = 8$

5. $(x-4)^2 + (y+2)^2 = 4$



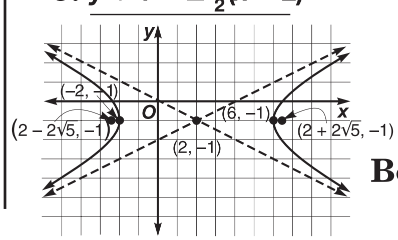
6. $\frac{(y-2)^2}{9} + \frac{(x+3)^2}{4} = 1$



7. $\frac{(x+1)^2}{9} + \frac{(y-3)^2}{1} = 1$

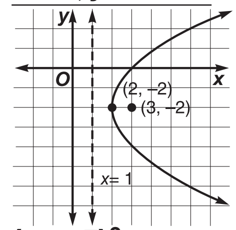
8. $\frac{(y-4)^2}{9} - \frac{(x+2)^2}{25} = 1$

9. $\frac{(x-2)^2}{16} - \frac{(y+1)^2}{4} = 1$
asymptotes:
$y + 1 = \pm \frac{1}{2}(x-2)$



Page 452

10. $(y+2)^2 = 4(x-2)$; $x = 1$; $y = -2$



11. $(x-5)^2 = -4(y+1)$

12. $\frac{\sqrt{2}}{2}$

13. hyperbola

14. $\frac{x^2}{4} + \frac{y^2}{9} = 1$

15. Sample answer: $x = 2\sqrt{2} \cos t$, $y = 2\sqrt{2} \sin t$, $0° \le t \le 360°$

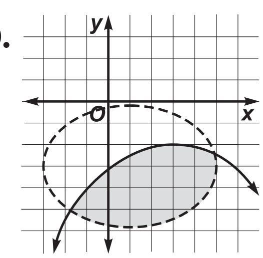
16. parabola; $y^2 + 8x + 16y + 72 = 0$

17. ellipse; $9(x')^2 + 2x'y' + 9(y')^2 - 40 = 0$

18. hyperbola, $-26°$

19. $(3, 0)$, $(-3, 0)$

20.



Bonus: $(2, 3)$

Chapter 10 Answer Key

CHAPTER 10 SCORING RUBRIC

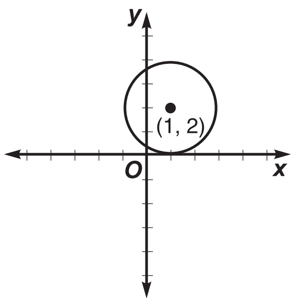
Level	Specific Criteria
3 Superior	• Shows thorough understanding of the concepts *circle*, *ellipse*, *parabola*, *hyperbola*, *center*, *vertex*, and *angle of rotation.* • Uses appropriate strategies to identify equations of conic sections. • Computations are correct. • Written explanations are exemplary. • Real-world example of conic section is appropriate and makes sense. • Graphs are accurate and appropriate. • Goes beyond requirements of some or all problems.
2 Satisfactory, with Minor Flaws	• Shows understanding of the concepts *circle*, *ellipse*, *parabola*, *hyperbola*, *center*, *vertex*, and *angle of rotation.* • Uses appropriate strategies to identify equations of conic sections. • Computations are mostly correct. • Written explanations are effective. • Real-world example of conic section is appropriate and makes sense. • Graphs are mostly accurate and appropriate. • Satisfies all requirements of problems.
1 Nearly Satisfactory, with Serious Flaws	• Shows understanding of most of the concepts *circle*, *ellipse*, *parabola*, *hyperbola*, *center*, *vertex*, and *angle of rotation.* • May not use appropriate strategies to identify equations of conic sections. • Computations are mostly correct. • Written explanations are satisfactory. • Real-world example of conic section is mostly appropriate and sensible. • Graphs are mostly accurate and appropriate. • Satisfies most requirements of problems.
0 Unsatisfactory	• Shows little or no understanding of the concepts *circle*, *ellipse*, *parabola*, *hyperbola*, *center*, *vertex*, and *angle of rotation.* • May not use appropriate strategies to identify equations of conic sections. • Computations are incorrect. • Written explanations are not satisfactory. • Real-world example of conic section is not appropriate or sensible. • Graphs are not accurate or appropriate. • Does not satisfy requirements of problems.

Chapter 10 Answer Key

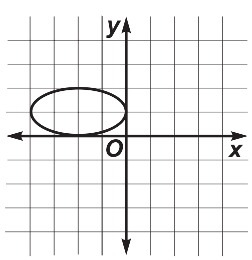
Open-Ended Assessment

Page 453

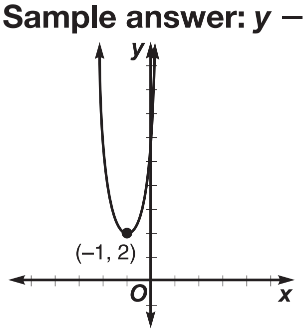
1a. The equation is a circle if $A = B$.
Sample answer:
$(x - 1)^2 + (y - 2)^2 = 4$



1b. The equation is an ellipse if $A \neq B$ and A and B have the same sign.
Sample answer:
$\frac{(x + 2)^2}{4} + \frac{(y - 1)^2}{1} = 1$



1c. The equation is a parabola when A or B is zero, but not both.
Sample answer: $y - 2 = 4(x + 1)^2$



1d. The equation is a hyperbola if A and B have opposite signs. Sample answer: $\frac{y^2}{4} - \frac{x^2}{1} = 1$

1e. The graph is an ellipse since $(-1)^2 - 4(3)(2) < 0$.
$\tan 2\theta = \frac{-1}{3 - 2} = -1$, $2\theta = -45°$,
$\theta = -\frac{\pi}{8}$, or $-22.5°$.

2a. The graph of $x^2 - 4y^2 = 0$ is two lines of slope $\frac{1}{2}$ and slope $-\frac{1}{2}$ that intersect at the origin.

$x^2 - 4y^2 = 0$

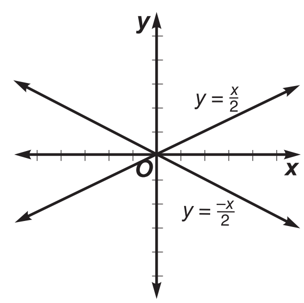
$x^2 = 4y^2$

$|x| = 2|y|$

$\frac{|x|}{2} = |y|$

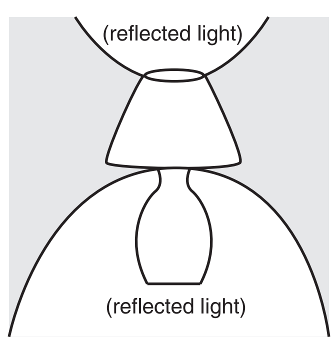
$\frac{x}{2} = y$ or $\frac{-x}{2} = y$

2b.



2c. degenerate hyperbola

3. Sample answer: Most lamps with circular shades shine a cone of light. When this light cone strikes a nearby wall, the resulting shape is a hyperbola. The hyperbola is formed by the cone of light intersecting the plane of the wall.



Chapter 10 Answer Key

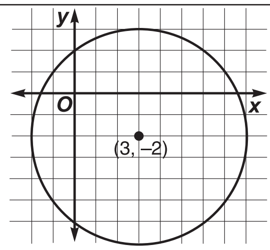
Mid-Chapter Test
Page 454

1. $\sqrt{26}, \left(\frac{3}{2}, -\frac{13}{2}\right)$

2. $\sqrt{61}, \left(3 + s, \frac{-5 - 2t}{2}\right)$

3. yes

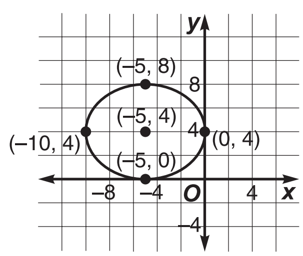
4. $(x - 3)^2 + (y + 2)^2 = 25$



5. $(x + 17)^2 + (y - 8)^2 = 289$; $(-17, 8)$; 17

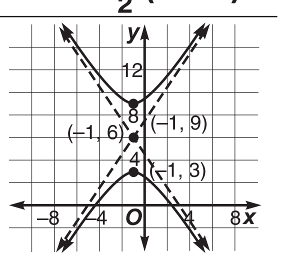
6. $\frac{(y-3)^2}{9} + \frac{(x+1)^2}{1} = 1$; $(-1, 3)$; $(-1, 3 \pm 2\sqrt{2})$; $(-1, 6)$, $(-1, 0)$, $(0, 3)$, $(-2, 3)$

7.



8. $\frac{(x - 3)^2}{16} - \frac{(y - 2)^2}{9} = 1$

9. $(-1, 6)$; $(-1, 6 \pm \sqrt{13})$; $(-1, 9)$, $(-1, 3)$; $y - 6 = \pm\frac{3}{2}(x + 1)$



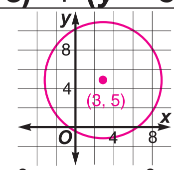
Quiz A
Page 455

1. $7\sqrt{2}, \left(\frac{3}{2}, \frac{1}{2}\right)$

2. $5; \left(a - 2, \frac{2b + 3}{2}\right)$

3. no

4. $(x - 3)^2 + (y - 5)^2 = 36$

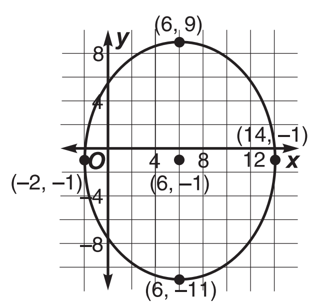


5. $(x + 4)^2 + (y - 5)^2 = 61$; $(-4, 5)$; $\sqrt{61} \approx 7.8$

Quiz B
Page 455

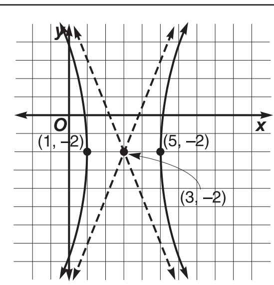
1. $\frac{(y + 1)^2}{16} + \frac{(x - 3)^2}{4} = 1$; $(3, -1)$; $(3, -1 \pm 2\sqrt{3})$; $(3, 3)$, $(3, -5)$, $(5, -1)$, $(1, -1)$

2.



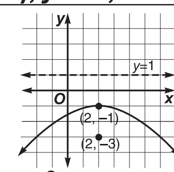
3. $\frac{(y - 3)^2}{4} - \frac{(x - 1)^2}{9} = 1$

4. $(3, -2)$; $(3 \pm \sqrt{29}, -2)$; $(5, -2)$, $(1, -2)$; $y + 2 = \pm\frac{5}{2}(x - 3)$



Quiz C
Page 456

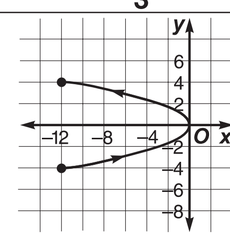
1. $(x - 2)^2 = -8(y + 1)$, $(2, -3)$; $(2, -1)$; $y = 1$; $x = 2$



2. $(y - 3)^2 = -12(x - 1)$

3. ellipse; $\frac{(x + 2)^2}{25} + \frac{(y + 1)^2}{4} = 1$

4. $y^2 = -\frac{4}{3}x$



5. Sample answer: $x = 10 \cos t$, $y = 10 \sin t$, $0 \leq t \leq 2\pi$

Quiz D
Page 456

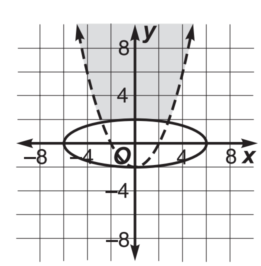
1. hyperbola; $5x^2 - 8y^2 + 30x + 80y - 195 = 0$

2. ellipse; $11(x')^2 + 14x'y' + 11(y')^2 - 36 = 0$

3. hyperbola

4. $(3, -4)$, $(-3, 4)$

5.



Chapter 10 Answer Key

SAT/ACT Practice

Page 457

1. E
2. D
3. D
4. E
5. A
6. D
7. B
8. C
9. E

Page 458

10. B
11. D
12. D
13. B
14. C
15. B
16. C
17. A
18. C
19. 6.4
20. 3

Cumulative Review

Page 459

1. Sample answer: $f(x) = 3$
2. $\begin{bmatrix} 11 & -17 & 8 \\ 32 & 1 & -4 \end{bmatrix}$
3. Alternate optimal solutions
4. $f^{-1}(x) = \frac{2}{x} + 5$; yes
5. $-\frac{16}{3}$
6. -1
7. $\frac{13\sqrt{30}}{60}$
8. $A = 65°$, $a = 36.2$, $c = 37.0$
9. 79 m/s
10. 45°, 135°
11. 51.6 lb
12. $-16\sqrt{3} + 16i$
13. $\frac{(x-1)^2}{9} + \frac{(y-2)^2}{4} = 1$

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