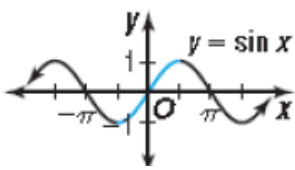
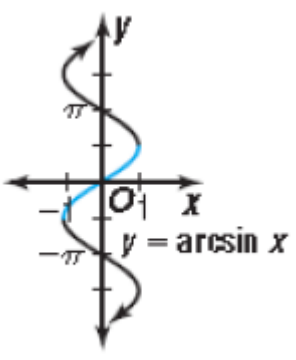
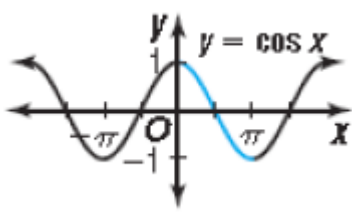
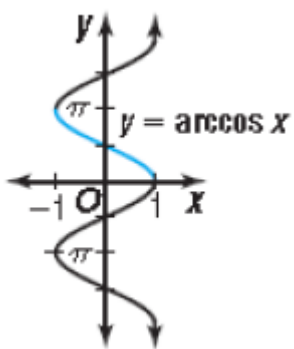


## MATH 1080 NOTES

### 8.3 Inverse Trigonometric Functions

**Objectives:** Understand and use inverse sine, cosine, and tangent functions.  
 Find exact values of expressions using the inverse sine, cosine, and tangent functions.  
 Use a calculator to evaluate inverse trigonometric functions.  
 Find exact values of composite functions using inverse trigonometric functions.

FUNCTION	GRAPH	DOMAIN	RANGE
$y = \sin x$		$(-\infty, \infty)$	$-1 \leq y \leq 1$
$y = \arcsin x$		$-1 \leq x \leq 1$	$(-\infty, \infty)$

FUNCTION	GRAPH	DOMAIN	RANGE
$y = \cos x$		$(-\infty, \infty)$	$-1 \leq y \leq 1$
$y = \arccos x$		$-1 \leq x \leq 1$	$(-\infty, \infty)$

FUNCTION	GRAPH	DOMAIN	RANGE
$y = \tan x$		All real numbers except $\frac{\pi}{2}n$ , where n is an odd integer	$(-\infty, \infty)$
$y = \arctan x$		$(-\infty, \infty)$	All real numbers except $\frac{\pi}{2}n$ , where n is an odd integer

**NOTE:** None of the inverses of the trigonometric functions are functions.

In order for the inverse to represent a function, we must restrict the domain of the given function.

Function	Domain	Range
$y = \sin x$	$-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$	$-1 \leq y \leq 1$
$y = \arcsin x$	$-1 \leq x \leq 1$	$-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$
$y = \cos x$	$0 \leq x \leq \pi$	$-1 \leq y \leq 1$
$y = \arccos x$	$-1 \leq x \leq 1$	$0 \leq y \leq \pi$
$y = \tan x$	$-\frac{\pi}{2} < x < \frac{\pi}{2}$	all real numbers
$y = \arctan$	all real numbers	$-\frac{\pi}{2} < y < \frac{\pi}{2}$

**Example 1** Evaluate each of the following in radians.

a.  $\sin^{-1}\left(\frac{1}{2}\right)$

b.  $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

c.  $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

d.  $\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right)$

**Example 2** Evaluate the radian and degree measurement of each using a calculator.  
Round to the nearest hundredth.

a.  $\sin^{-1}(0.97)$

b.  $\cos^{-1}(-0.4)$

c.  $\tan^{-1}(4)$

**Example 3** Evaluate each of the following with exact values.

a.  $\sin^{-1}\left(\sin\left(\frac{\pi}{3}\right)\right)$

b.  $\sin^{-1}\left(\sin\left(\frac{2\pi}{3}\right)\right)$

c.  $\cos^{-1}\left(\sin\left(\frac{\pi}{2}\right)\right)$

d.  $\tan^{-1}\left(\sin\left(\frac{\pi}{2}\right)\right)$

e.  $\cos^{-1}\left(\sin\left(-\frac{11\pi}{4}\right)\right)$

**Example 4** Evaluate each of the following with exact values.

a.  $\sin\left(\cos^{-1}\left(\frac{4}{5}\right)\right)$

b.  $\cos\left(\tan^{-1}\left(\frac{5}{12}\right)\right)$

c.  $\sin\left(\tan^{-1}\left(\frac{7}{4}\right)\right)$

d.  $\cos\left(\sin^{-1}\left(-\frac{7}{9}\right)\right)$