MATH 1080 TRIGONOMETRY NOTES

8.1 Graphs of Sine and Cosine

Objectives: Graph variations of $y = \sin(x)$ *and* $y = \cos(x)$.

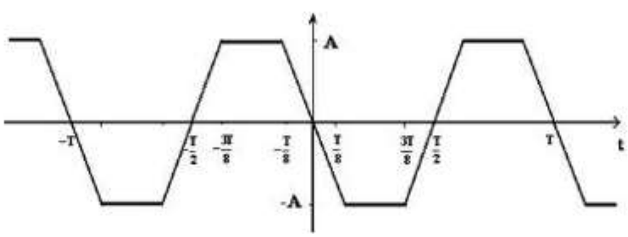
DEFINITIONS: Periodic Function and Period

A function is ***periodic*** if, for some real number *a*, $f(x+a) = f(x)$ for each *x* in the domain of f.

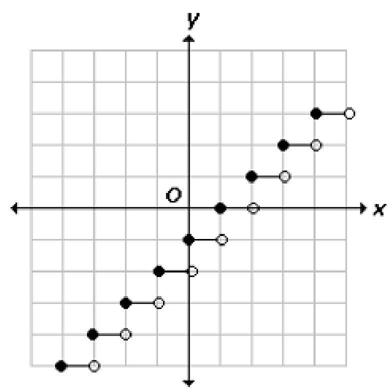
The least positive value of *a* for which $f(x) = f(x+a)$ is the ***period*** of the function.

Example 1 Determine if the function is a periodic function. If so, determine the period of the function.

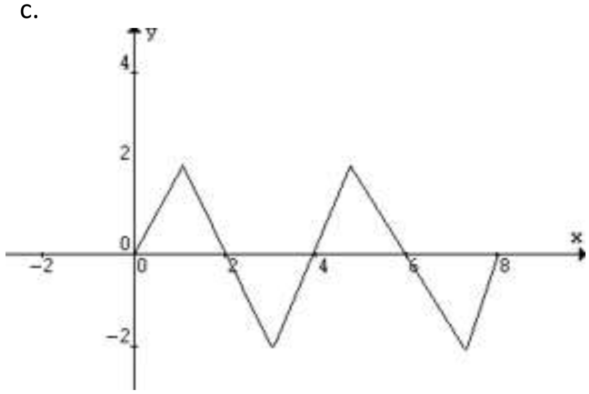
a.



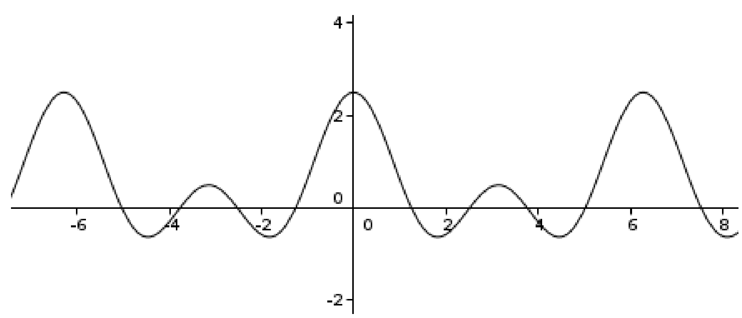
b.



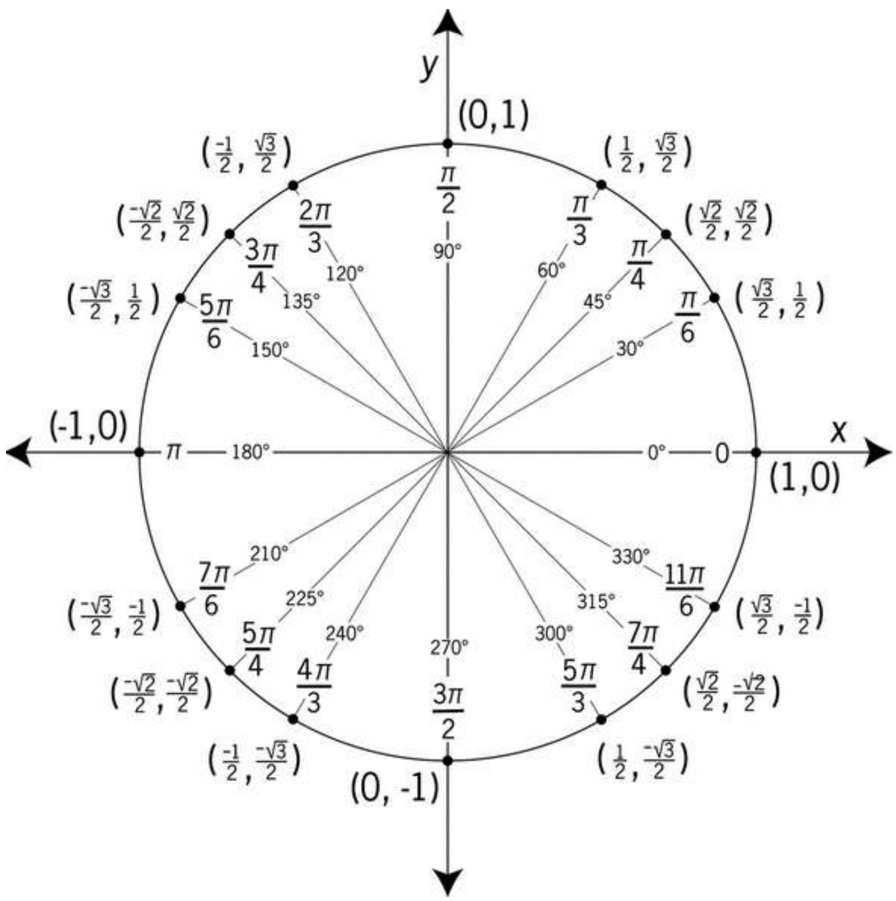
c.



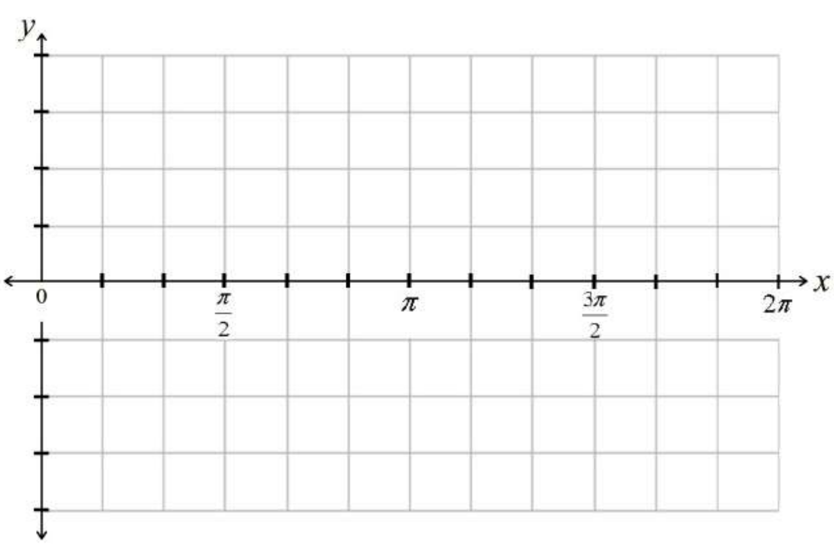
d.



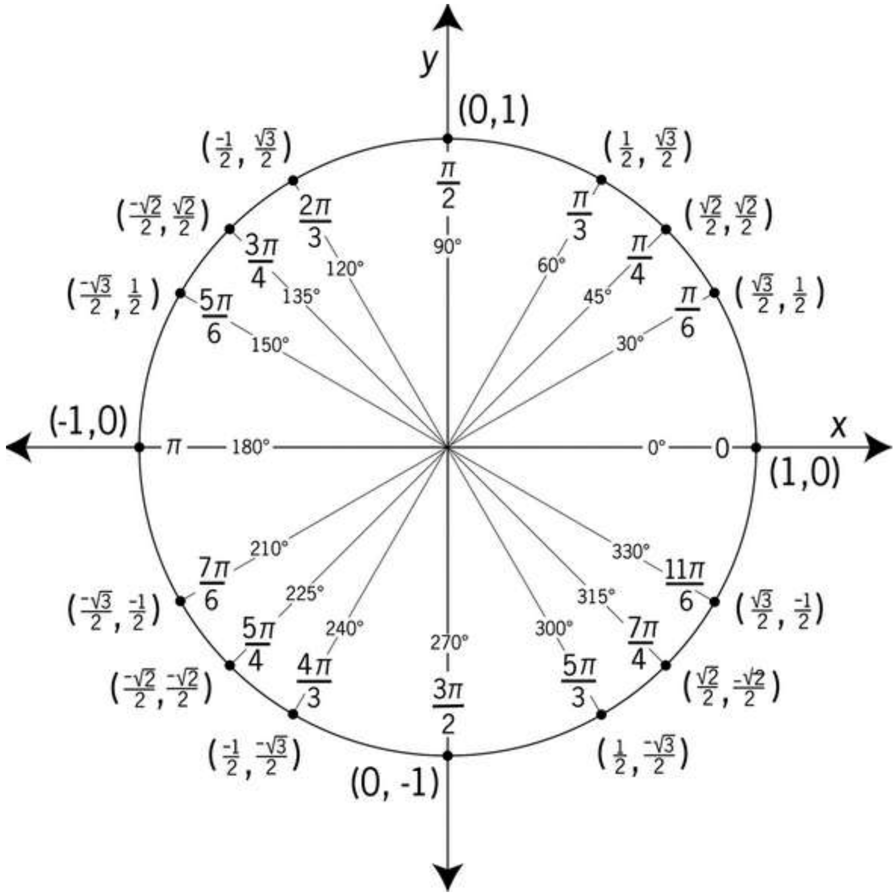
Consider the values on the unit circle where $x = \cos\theta \; and \; y = \sin\theta$



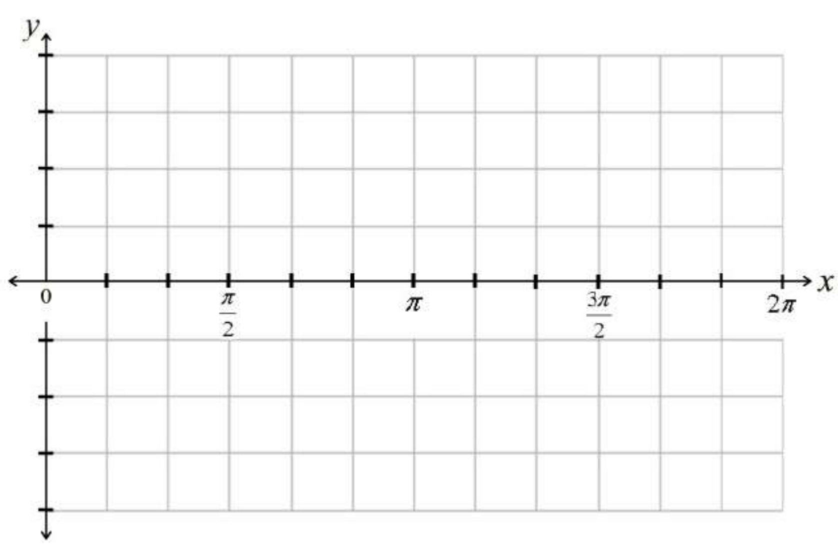
<u>Example 2</u> Using the unit circle, graph $y = \sin\theta$ for $0 \le \theta \le 2\pi$.



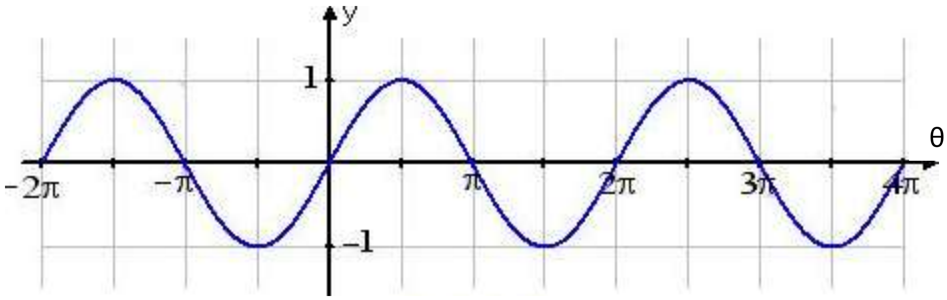
Consider the values on the unit circle where $x = \cos\theta$ *and* $y = \sin\theta$



Example 3 Using the unit circle, graph $y = \cos\theta$ for $0 \leq \theta \leq 2\pi$.



Example 4 Below is a partial graph of $y = \sin\theta$ for $-2\pi \le \theta \le 4\pi$.



Answer each question with regards to $y = \sin\theta$.

a. Determine the domain. ____

b. Determine the range. ____

c. What is the period of the function? _________________________________

d. Determine the location of the x-intercepts. ___________________________

e. Determine the location of the maximums. ____________________________

f. Determine the location of the minimums. _____________________________

g. On $[0, 2\pi), solve\ f(x) = \frac{1}{2}$. ______________________________________

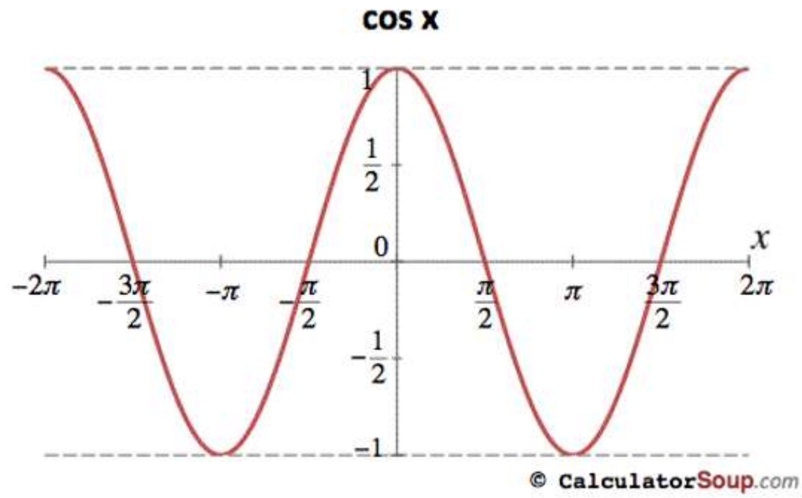
Example 5 Determine each value by referring to the graph of $y = \sin\theta$

a. $\sin\left(\frac{5\pi}{2}\right)$

b. $\sin(3\pi)$

Example 6 Solve $f(x) = \sin x = -1$ on $[0, 2\pi)$.

Example 7 Below is a partial graph of $y = \cos x$ for $-2\pi \leq x \leq 2\pi$.



Answer each question with regards to $y = \cos x$.

a. Determine the domain. ____

b. Determine the range. ____

c. What is the period of the function? ____________________________________

d. Determine the location of the x-intercepts. ______________________________

e. Determine the location of the maximums. ________________________________

f. Determine the location of the minimums. _________________________________

g. On $[0, 2\pi)$, $solve\ f(x) = \frac{\sqrt{3}}{2}$. ____

Example 8 Determine each value by referring to the graph of $y = \cos\theta$.

a. $\cos\left(-\frac{\pi}{2}\right)$

b. $\cos(5\pi)$

Example 9 Solve $f(x) = \cos x = 0$ on $[0, 2\pi)$.

TRANSFORMATIONS OF SINE AND COSINE GRAPHS

General form of each equation: $y = Asin(Bx - C) + D \rightarrow y = A\ sin\left(B(x - \frac{C}{B})\right) + D$

$$y = Acos(Bx - C) + D \rightarrow y = A\ cos\left(B(x - \frac{C}{B})\right) + D$$

Amplitude: $|A|$ Phase Shift (moves left or right): $\frac{C}{B}$

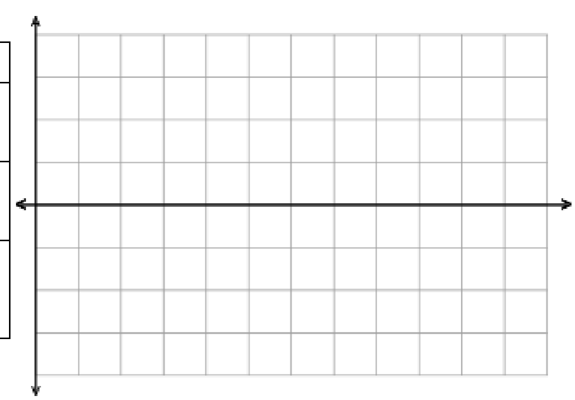
Period: $\frac{2\pi}{|B|}$ Vertical Shift (moves up or down): D

Example 10 Determine the period and amplitude of each function, then graph one cycle of each function.

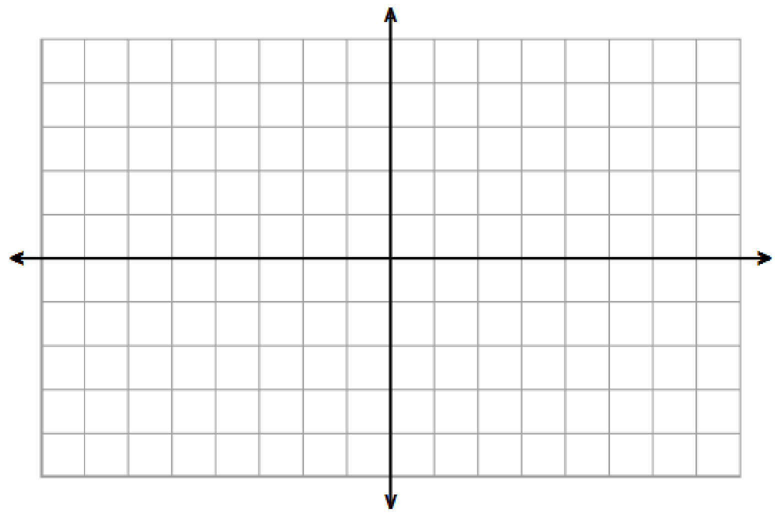
	AMPLITUDE	PERIOD
a. $y = \sin\theta$		
b. $y = -3\sin(2\theta)$		
c. $y = 4\sin\left(\frac{\theta}{2}\right)$		

Example 11 Determine the period and amplitude of each function, then graph one cycle of each function.

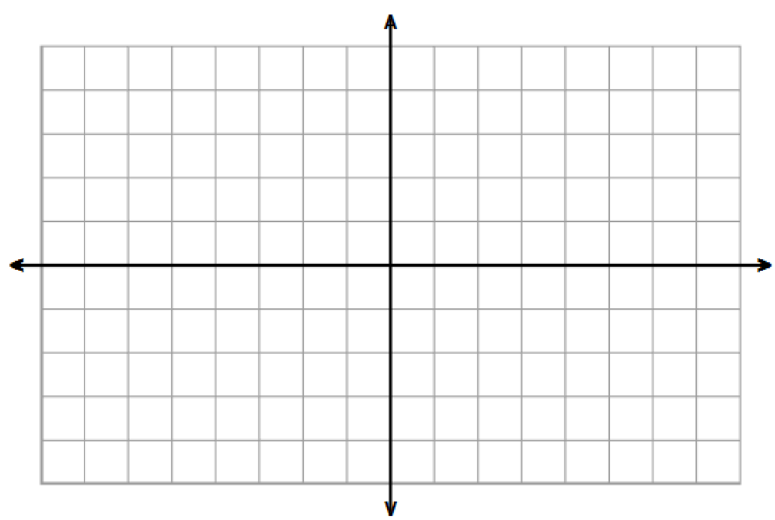
	AMPLITUDE	PERIOD
a. $y = \cos\theta$		
b. $y = 2\cos(4\theta)$		
c. $y = -4\cos\left(\frac{\theta}{3}\right)$		



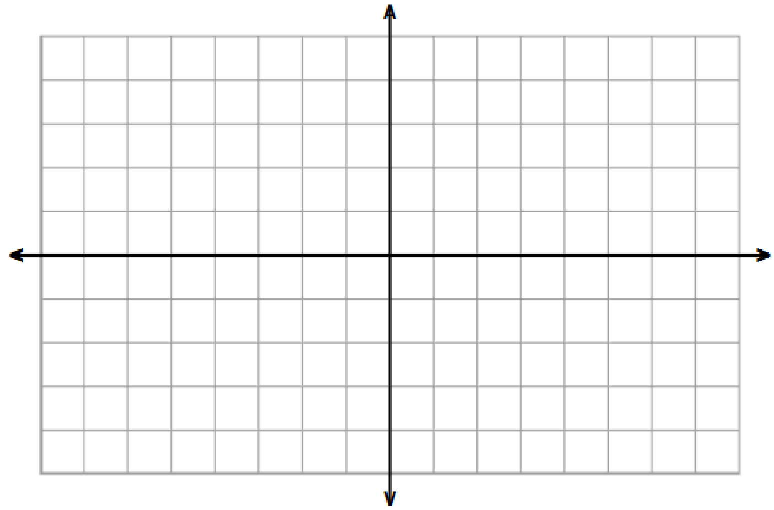
Example 12 Graph $y = 2\sin(x) + 1$. State the amplitude, period, midline, phase & vertical shifts.



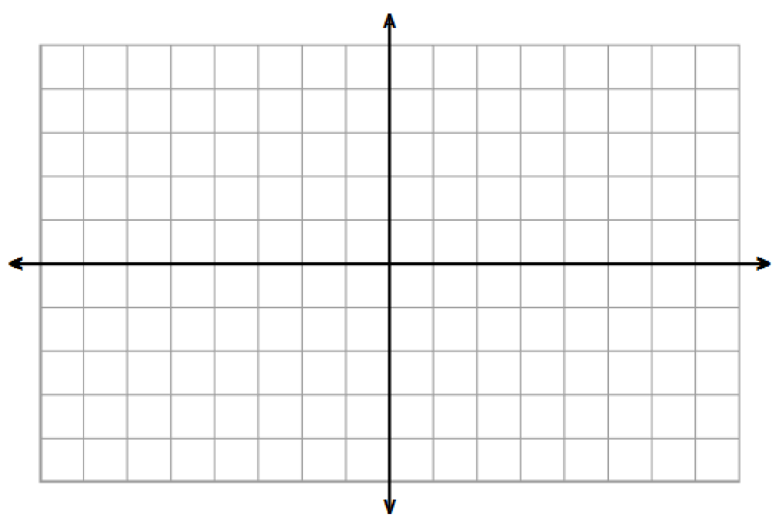
Example 13 Graph $y = 3\cos\left(x - \frac{\pi}{2}\right)$. State the amplitude, period, midline, phase & vertical shifts.



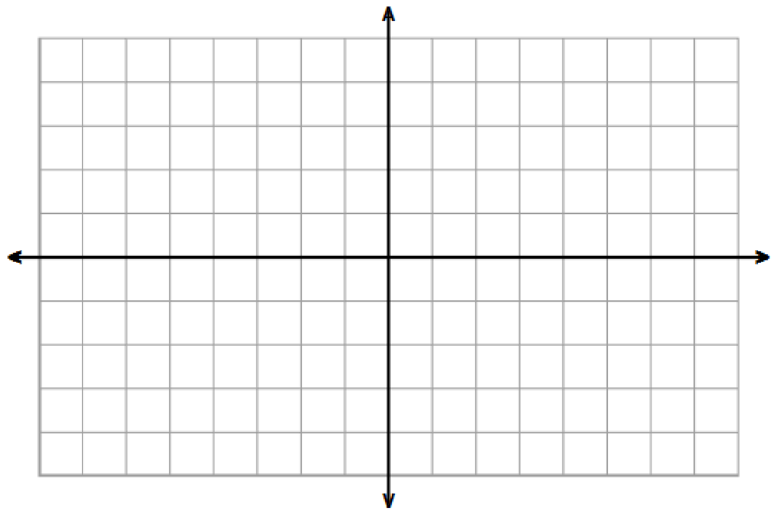
Example 14 Graph $y = -\sin\left(\frac{x}{3}\right) + 2$. State the amplitude, period, midline, phase & vertical shifts



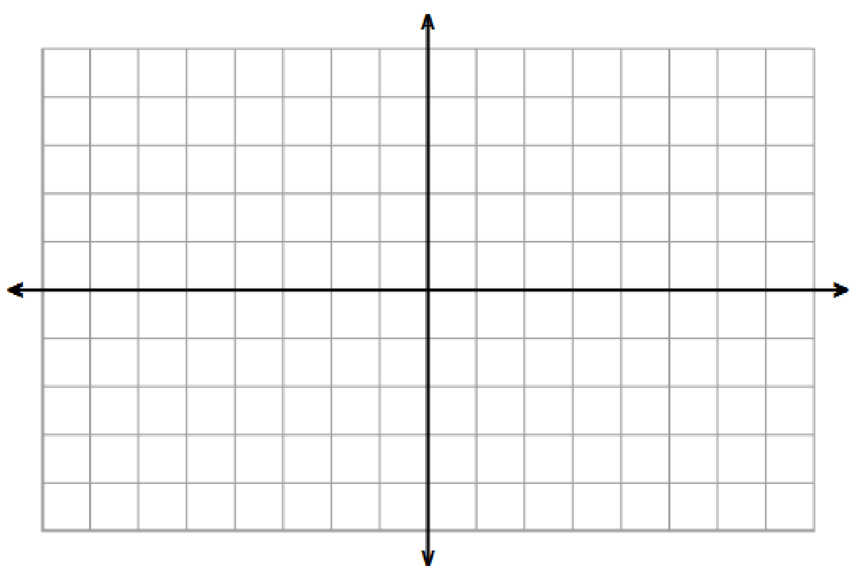
Example 15 Graph $y = \frac{3}{2}\cos(\pi x + \pi)$. State the amplitude, period, midline, phase & vertical shifts.



Example 16 Graph $y = 2\cos\left(\frac{\theta}{4} + \frac{\pi}{2}\right) - 1$. State the amplitude, period, midline, phase & vertical shifts.

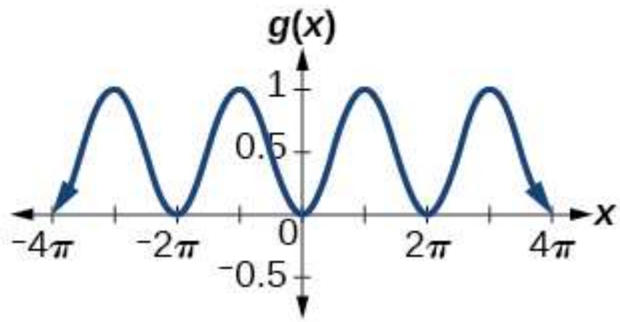


Example 17 Graph $y = -\frac{1}{2}\sin(2\theta - \pi) + 3$. State the amplitude, period, midline, phase & vertical shifts.

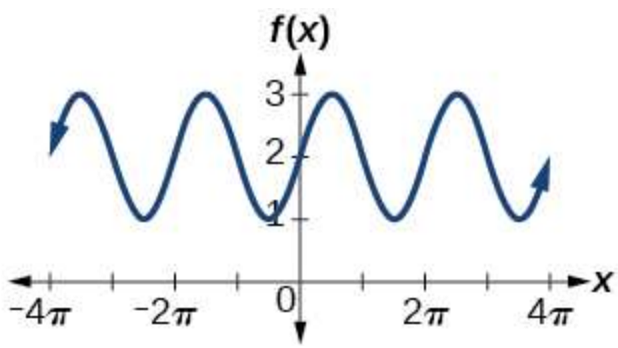


Example 18

a. Determine the equation for the cosine function.

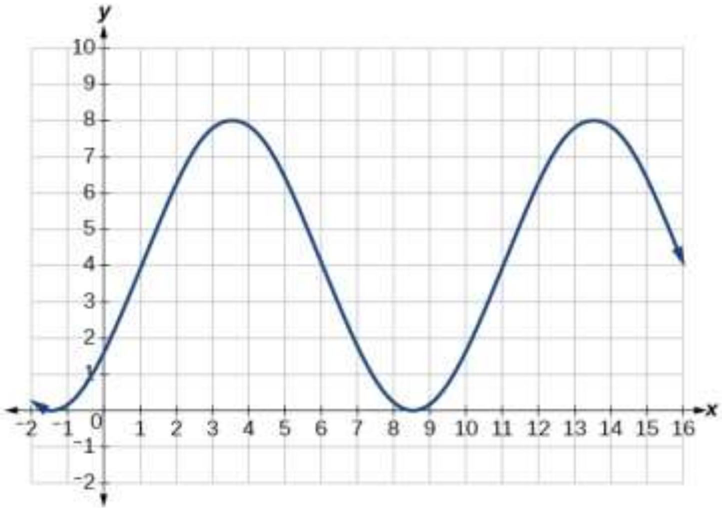


b. Determine the equation for the sine function.



Example 19 Determine the amplitude, midline, period, and an equation for each sinusoidal function.

a.



b.