

Math 1553/Math 1610
Worksheet Section 7.5

1. An automotive repair shop has determined that the average service time on an automobile is 2 hours with a standard deviation of 32 minutes. A random sample of 64 services is selected.

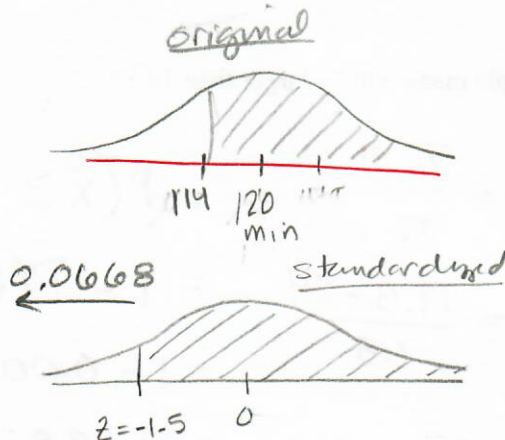
- a. What is the probability that the sample of 64 will have a mean service time greater than 114 minutes?

$$\mu = 2 \text{ hrs} \\ = 120 \text{ min}$$

$$\sigma = 32 \text{ min}$$

$$n = 64 \text{ cars}$$

$$\bar{x} = 114$$



$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} \\ = \frac{32}{\sqrt{64}} \\ = \frac{32}{8} \\ = 4$$

$$z = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}} \\ z = \frac{114 - 120}{4} \\ z = \frac{-6}{4} \\ z = -1.5$$

$$P(x \geq 114) \\ = 1 - P(z \leq -1.5) \\ = 1 - 0.0668 \\ = 0.9332$$

There is a 93.32% that the service time is greater than 114 min

- b. Assume the population consists of 400 services. Determine the standard error of the mean.

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{32}{\sqrt{400}} = \frac{32}{20} = 1.6$$

More samples \Rightarrow smaller error

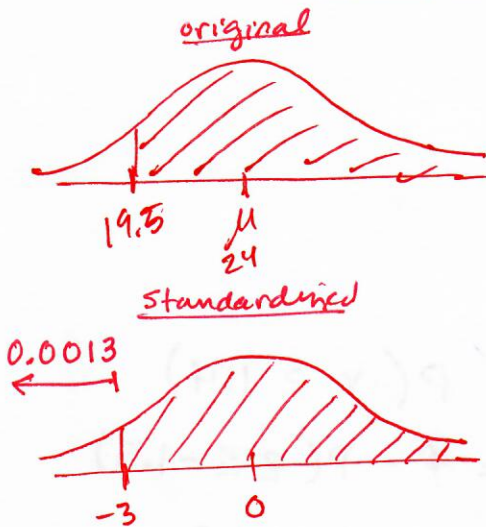
2. There are 8,000 students at the University of Tennessee at Chattanooga. The average age of all the students is 24 years with a standard deviation of 9 years. A random sample of 36 students is selected.

a. Determine the standard error of the mean.

$N = 8000$
 $\mu = 24$
 $\sigma = 9$
 $n = 36$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{9}{\sqrt{36}} = \frac{9}{6} = 1.5$$

b. What is the probability that the sample mean will be larger than 19.5?



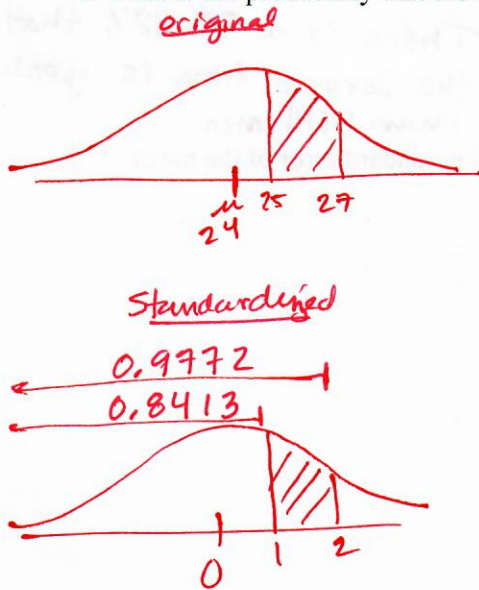
$$z = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}}$$

$$z = \frac{19.5 - 24}{1.5} = -3$$

$$P(\bar{x} \geq 19.5) = 1 - P(z \leq -3) = 1 - 0.0013 = 0.9987$$

There is a 99.87% chance that the sample mean is greater than 19.5 yrs.

c. What is the probability that the sample mean will be between 25.5 and 27 years?



$$z = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}}$$

$\bar{x} = 25.5$	$\bar{x} = 27$	$P(25 \leq \bar{x} \leq 27)$ $= P(1 \leq z \leq 2)$ $= P(z \leq 2) - P(z \leq 1)$ $= 0.9772 - 0.8413$ $= 0.1359$
$z = \frac{25.5 - 24}{1.5}$	$z = \frac{27 - 24}{1.5}$	
$z = \frac{1.5}{1.5}$	$z = \frac{3}{1.5}$	
$z = 1$	$z = 2$	

13.59% of the students are between 25.5 and 27 yrs old.