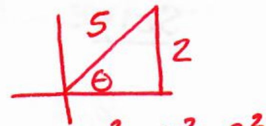
PRE CALC – §7-5 NOTES

PRECALCULUS NOTES
7.5 Solving Trigonometric Equations

Objectives: Solve trigonometric equations and inequalities.

Warm-Up

1. $If\ \sin\theta = \frac{2}{5}\ and\ 0 \le \theta < \frac{\pi}{2}\ , find\ the\ exact\ value\ of:$



$x^2 = 5^2 - 2^2$
$x^2 = 25 - 4$
$x = \sqrt{21}$

a. $\cos 2\theta = 1 - 2\sin^2\theta$
$= 1 - 2\left(\frac{2}{5}\right)^2$
$= 1 - 2\left(\frac{4}{25}\right)$
$= 1 - \frac{8}{25}$
$= \frac{17}{25}$

b. $\sin 2\theta = 2\sin\theta\cos\theta$
$= 2\left(\frac{2}{5}\right)\left(\frac{\sqrt{21}}{5}\right)$
$= \frac{4\sqrt{21}}{25}$

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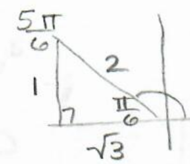
2. Use a half-angle identity to find the exact value of $\cos\frac{5\pi}{12}$. $= \sqrt{\frac{1+\cos\frac{5\pi}{6}}{2}}$



$= \sqrt{\frac{1 - \frac{\sqrt{3}}{2}}{2}}\left(\frac{2}{2}\right)$

$= \frac{\sqrt{2-\sqrt{3}}}{2}$

DEFINITION: A **<u>trigonometric equation</u>** is an equation that contains a trigonometric function.

Examples:

$sin^2x + cos^2x = 1;$ $2sinx - 1;$ $tan^2x - 1 = 0$

Trigonometric equations do not have unique solutions. They have infinitely many solutions, differing by the period of the function (2π for sine and cosine; π for tangent).

Principal values – the restricted domain of a trig equation

sin x; where $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$

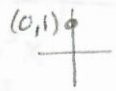
tan x; where $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$

cos x; where $0 \le x \le \pi$

Example 1 Solve $sinx\ cosx - \frac{1}{2}cosx = 0$ for principal values of x. Express solutions in degrees.

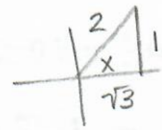
Factor $\cos x\left(\sin x - \frac{1}{2}\right) = 0$

Solve $\cos x = 0$ or $\sin x - \frac{1}{2} = 0$



$x = 90°$

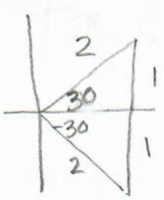
$\sin x = \frac{1}{2}$



$x = 30°$

Example 2 Find *ALL real* solutions to the following :

a. $\sqrt{csc^2 x} = \sqrt{4}$



$\csc x = \pm 2$

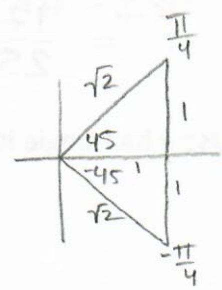
$\left[\begin{array}{l} x = \frac{\pi}{6} + 2n \\ x = -\frac{\pi}{6} + 2n \end{array}\right. ;\ n \in \mathbb{Z}$

b. $\sqrt{2}\,cosx - 1 = 0$

$\sqrt{2}\cos x = 1$

$\cos x = \frac{1}{\sqrt{2}}$

$\cos x = \frac{\sqrt{2}}{2}$



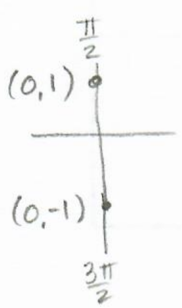
$\left[\begin{array}{l} x = \frac{\pi}{4} + 2\pi n \\ x = -\frac{\pi}{4} + 2\pi n \end{array}\right. ;\ n \in \mathbb{Z}$

Example 3 Solve $cos^2x - cosx + 1 = sin^2x\ \ for\ \ 0 \le x < 2\pi$.

$\cos^2 x - \cos x + 1 = 1 - \cos^2 x$

$2\cos^2 x - \cos x = 0$

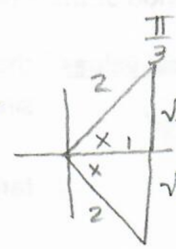
$\cos x\,(2\cos x - 1) = 0$



$\cos x = 0$ or $2\cos x - 1 = 0$

$\cos x = \frac{1}{2}$

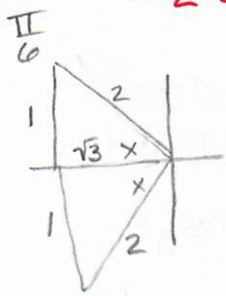
$x = \frac{\pi}{2}$ or $\frac{3\pi}{2}$



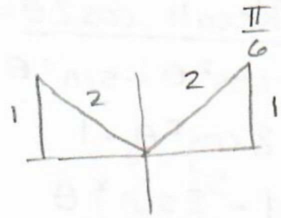
$x = \frac{\pi}{3}$ or $\frac{5\pi}{3}$

$2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$

Example 4 Solve $(2\cos x + \sqrt{3})(2\sin x - 1) = 0$ for $0 \le x < 2\pi$.



$2\cos x + \sqrt{3} = 0$ or $2\sin x - 1 = 0$



$\cos x = -\frac{\sqrt{3}}{2}$ $\qquad$ $\sin x = \frac{1}{2}$

$x = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$ $\qquad$ $x = \frac{\pi}{6}$

$x = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

$\pi + \frac{\pi}{6} = \frac{7\pi}{6}$

$x = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$

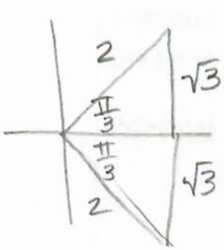
Example 5 Solve $2\sec^2 x - \tan^4 x = -1$ for all real values of x.

$2(1+\tan^2 x) - \tan^4 x = -1$

$2 + 2\tan^2 x - \tan^4 x = -1$

$0 = \tan^4 x - 2\tan^2 x - 3$

$0 = (\tan^2 x - 3)(\tan^2 x + 1)$



$\tan^2 x - 3 = 0$ or $\tan^2 x + 1 = 0$

$\tan^2 x = 3$ $\qquad$ $\tan^2 x = -1$

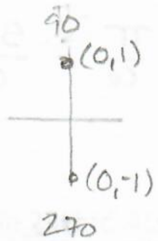
$\tan x = \pm\sqrt{3}$ $\qquad$ $\boxed{\text{No soln.}}$

$$\left[\begin{array}{l} x = \frac{\pi}{3} + \pi n \\ x = -\frac{\pi}{3} + \pi n \end{array}\right. ;\ n \in \mathbb{Z}$$

Example 6 Solve $\cos 2x + cos x + 1 = 0\ for\ 0° \le x < 360°$.

Recall $\cos 2\theta =$
$\cos^2\theta - \sin^2\theta$
$2\cos^2\theta - 1$
$1 - 2\sin^2\theta$

$$(2\cos^2 x - 1) + \cos x + 1 = 0$$
$$2\cos^2 x + \cos x = 0$$
$$\cos x\,(2\cos x + 1) = 0$$



$\cos x = 0$ or $2\cos x + 1 = 0$

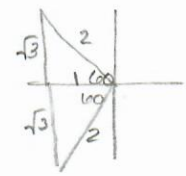
$x = 90$

$x = 270$

$\cos x = -\frac{1}{2}$

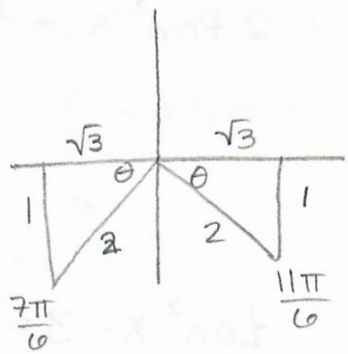
$x = 180 - 60 = 120$

$x = 180 + 60 = 240$



Example 7 Solve $2\sin\theta + 1 > 0\ for\ 0 \le \theta < 2\pi$.

$$\sin\theta > -\frac{1}{2}$$



$0 \le \theta < \frac{7\pi}{6}$

$\frac{11\pi}{6} < \theta < 2\pi$