

MATH 1080 TRIGONOMETRY NOTES

7.4 Other Trigonometric Functions & Identities

Objectives: Use properties of even and odd trigonometric functions.
Recognize and use fundamental identities to rewrite and simplify a given problem.

NOTE: We discussed how to find the exact values of ALL the trigonometric functions in the 7-3 notes. Therefore, the focus of these notes will be the odd and even trigonometric functions and trig identities.

REVIEW PROBLEMS

Example 1 Given the point on the unit circle $\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$, determine the exact value of $\sec \theta$.

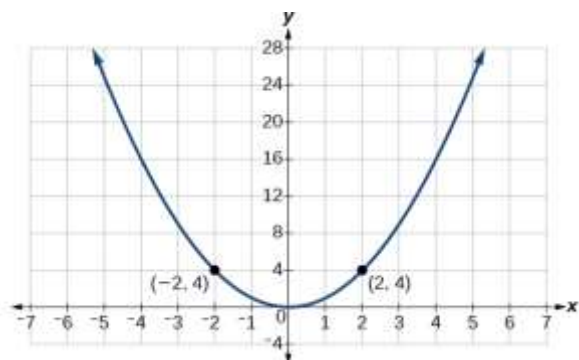
Example 2 Determine the exact value of the $\cot \theta$ if $\theta = -\frac{2\pi}{3}$.

Example 3 If $\cos \theta = \frac{12}{13}$, where $\frac{3\pi}{2} < \theta < 2\pi$, determine the exact value of $\csc \theta$.

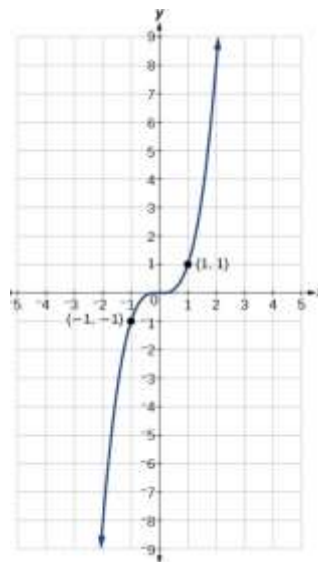
Example 4 If $\tan \theta = \frac{7}{2}$, where $0 < \theta < \frac{\pi}{2}$, determine the exact value of $\sin \theta$.

ALGEBRA REVIEW OF EVEN AND ODD FUNCTIONS

EVEN FUNCTION: $y = x^2$



ODD FUNCTION: $y = x^3$



ODD and EVEN FUNCTIONS TRIGONOMETRIC FUNCTIONS

Even Function $f(-x) = f(x)$

$$\cos(-t) = \cos(t)$$

$$\sec(-t) = \sec(t)$$

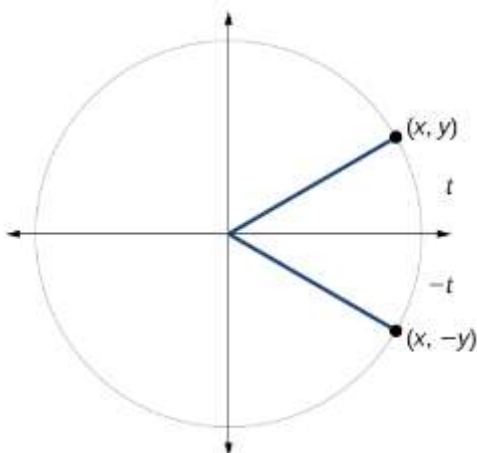
Odd Function $f(-x) = -f(x)$

$$\sin(-t) = -\sin(t)$$

$$\csc(-t) = -\csc(t)$$

$$\tan(-t) = -\tan(t)$$

$$\cot(-t) = -\cot(t)$$



Example 5

- a. If $\cos t = \frac{1}{2}$, determine the value of $\cos(-t)$.
- b. If $\csc t = 0.34$, determine the value of $\csc(-t)$.
- c. If $\cot t = 9.23$, determine the value of $\cot(-t)$.

RECIPROCAL IDENTITIES

$$\sin \theta = \frac{1}{\csc \theta}$$

$$\csc \theta = \frac{1}{\sin \theta}$$

$$\cos \theta = \frac{1}{\sec \theta}$$

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\tan \theta = \frac{1}{\cot \theta}$$

$$\cot \theta = \frac{1}{\tan \theta}$$

QUOTIENT IDENTITIES

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

PYTHAGOREAN IDENTITIES

$$\sin^2 \theta + \cos^2 \theta = 1^* \quad \tan^2 \theta + 1 = \sec^2 \theta \quad 1 + \cot^2 \theta = \csc^2 \theta$$

*Recall the equation in a unit circle $x^2 + y^2 = 1$, since $x = \sin \theta$ and $y = \cos \theta$
then $\sin^2 \theta + \cos^2 \theta = 1$

Example 6 Using identities, evaluate each trigonometric function.

a. Given $\sin(45^\circ) = \frac{\sqrt{2}}{2}$ and $\cos(45^\circ) = \frac{\sqrt{2}}{2}$, determine $\tan(45^\circ)$.

b. Given $\sin\left(\frac{5\pi}{6}\right) = \frac{1}{2}$ and $\cos\left(\frac{5\pi}{6}\right) = -\frac{\sqrt{3}}{2}$, determine $\cot\left(\frac{5\pi}{6}\right)$.

Example 7 Using identities, simplify each trigonometric expression.

a. $\frac{\sec \theta}{\tan \theta}$

b. $\tan x \cos x$

Example 8 Using identities, simplify each trigonometric expression.

a. $\frac{\sec x}{\csc x}$

b. $\cos x \csc x \tan x$

Example 9 Using identities, simplify each trigonometric expression.

a. $\frac{\csc x}{\cot x}$

b. $\cos \theta \cot \theta + \sin \theta$

c. $\frac{\cot A}{\tan A}$

d. $\sin^2 \theta \cos^2 \theta - \cos^2 \theta$

Example 10 Using identities, simplify each trigonometric expression.

a. $\cos x + \sin x \tan x$

b. $\sin x + \sin x \cot^2 x$