

MATH 1080 TRIGONOMETRY NOTES

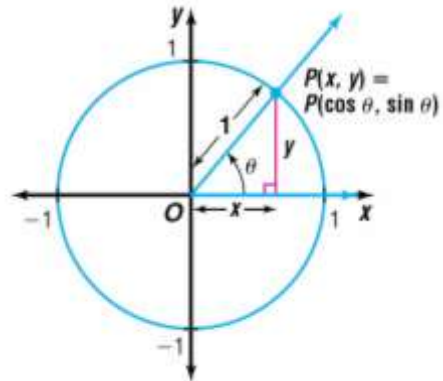
7.3 Unit Circle

Objectives: Determine function values for sine and cosine using special right triangles.
Identify the domain and range of sine and cosine functions.
Determine reference angles and use them to evaluate trigonometric functions.

UNIT CIRCLE

$$x = \cos \theta$$

$$y = \sin \theta$$



NOTE:

The acute angle is always at the origin.

The right angle is always formed with the x-axis.

PYTHAGOREAN IDENTITY $\cos^2 \theta + \sin^2 \theta = 1$

Example 1 Determine the value of each trig function as reduced fractions.

$$\cos 0^\circ =$$

$$\sin 0^\circ =$$

$$\cos 90^\circ =$$

$$\sin 90^\circ =$$

$$\cos 180^\circ =$$

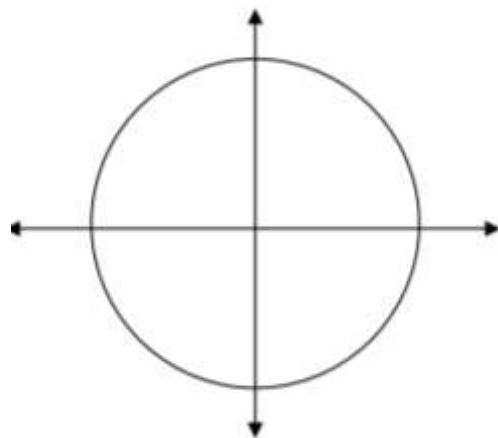
$$\sin 180^\circ =$$

$$\cos 270^\circ =$$

$$\sin 270^\circ =$$

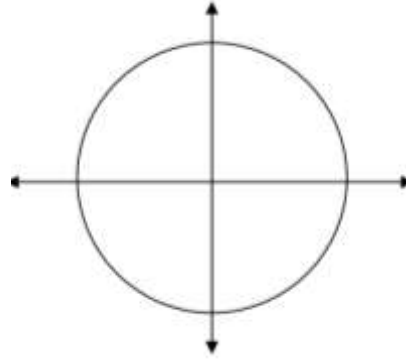
$$\cos 360^\circ =$$

$$\sin 360^\circ =$$



QUADRANT SIGNS

| FUNCTION | QI | QII | QIII | QIV |
|--------------------------------|----|-----|------|-----|
| $\sin \theta$ $\csc \theta$ | | | | |
| $\cos \theta$ $\sec \theta$ | | | | |
| $\tan \theta$ $\cot \theta$ | | | | |



Example 2 Use a unit circle to determine each **exact** value.

a. $\cos(-180^\circ)$

b. $\sin(270^\circ)$

c. $\sec(90^\circ)$

d. $\csc(-270^\circ)$

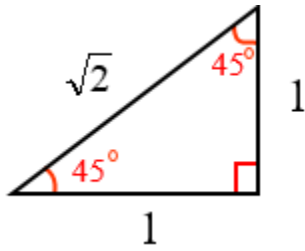
Example 3

a. Determine the exact value of $\cos(t)$, if $\sin(t) = \frac{3}{7}$ in the second quadrant.

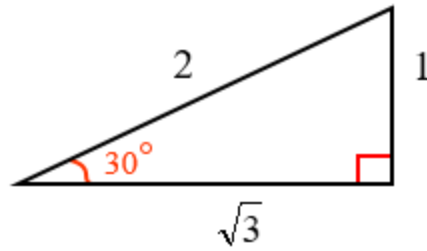
b. Determine the exact value of $\sin(t)$, if $\cos(t) = \frac{24}{25}$ in QIV.

RECALL THE SPECIAL RIGHT TRIANGLES

45° - 45° - 90°



30° - 60° - 90°



Example 4 Complete the table with the exact values.

| θ | radians | $\sin \theta$ | $\cos \theta$ | $\tan \theta$ | $\csc \theta$ | $\sec \theta$ | $\cot \theta$ |
|------------|---------|---------------|---------------|---------------|---------------|---------------|---------------|
| 30° | | | | | | | |
| 45° | | | | | | | |
| 60° | | | | | | | |

Example 5 Using a unit circle, determine the value of the six trigonometric functions for 135° .

Example 6 Using a unit circle, determine the quadrant, the reference angle, and the exact value of each.

a. $\sin 300^\circ$

b. $\cos 225^\circ$

c. $\tan 150^\circ$

d. $\sin \left(\frac{2\pi}{3} \right)$

e. $\cos \left(\frac{11\pi}{6} \right)$

Example 7 Using a calculator **in radian mode**, evaluate each to four decimal places.

a. $\cos\left(\frac{7\pi}{6}\right)$

b. $\sin\left(\frac{9\pi}{4}\right)$

Example 8 Using a unit circle, determine the exact value of each product. **NO CALCULATOR.**

a. $\sin\left(\frac{9\pi}{4}\right)\cos\left(\frac{4\pi}{3}\right)$

b. $\cos\left(-\frac{9\pi}{4}\right)\sin\left(-\frac{2\pi}{3}\right)$

c. $\sin\left(-\frac{3\pi}{2}\right)\cos(\pi)$

Example 9 Suppose θ is an angle in standard position whose terminal side lies in QIV.

If $\sec \theta = \frac{\sqrt{29}}{5}$, find the values of the five remaining functions of θ .

Example 10 Using the unit circle, find the exact measure for each of the following.

a. $\tan 180^\circ$

b. $\sec(-90^\circ)$

c. $\tan 270^\circ$

d. $\cot 270^\circ$

e. $\csc(-3\pi)$

f. $\sec(\pi)$