PRECALC – §7-3 SOLNS

PRECALCULUS NOTES

7.3 Sum and Difference Identities

Objectives: Use the sum and difference identities for the sine, cosine, and tangent functions.

Warm-Up Verify that $\sin x \tan x = \sec x - \cos x$.

$$\sin x\left(\frac{\sin x}{\cos x}\right) \overset{?}{=} \sec x - \cos x$$

$$\frac{\sin^2 x}{\cos x} \overset{?}{=} \sec x - \cos x$$

$$\frac{1-\cos^2 x}{\cos x} \overset{?}{=} \sec x - \cos x$$

$$\frac{1}{\cos x} - \frac{\cos^2 x}{\cos x} \overset{?}{=} \sec x - \cos x$$

$$\sec x - \cos x = \sec x - \cos x \checkmark$$

Sum and Difference Identities **(note: α is alpha and β is beta)**

Sine Function Identities

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

Cosine Function Identities

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

Tangent Function Identities

$$\tan(\alpha+\beta) = \frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$$

$$\tan(\alpha-\beta) = \frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$$

Example 1 Using the sum and difference identities find the exact value of cos 75°.

Q: * How can you get 75° using the special angles 0, 30, 45, 60, 90, 180, 270, 360?

A: 30 + 45 = 75

Now, $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

So, $\cos(30+45) = \cos 30 \cos 45 - \sin 30 \sin 45$

$$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$$

$$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$$

$$= \frac{\sqrt{6}-\sqrt{2}}{4}$$

Example 2 Using the sum and difference identities find the exact value of $\sin\frac{\pi}{12}$.

Q: How do you get $\frac{\pi}{12}$ using the special radian values $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$?

Use the denom

A: to guide you

$$\frac{4\pi}{12} - \frac{3\pi}{12} = \frac{\pi}{12}$$

$$\frac{\pi}{3} - \frac{\pi}{4} = \frac{\pi}{12}$$

Now, $\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

So, $\sin\left(\frac{\pi}{3}-\frac{\pi}{4}\right) = \sin\frac{\pi}{3}\cos\frac{\pi}{4} - \cos\frac{\pi}{3}\sin\frac{\pi}{4}$

$$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$$

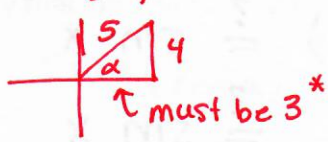
$$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$$

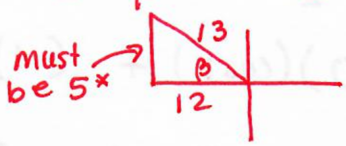
$$= \frac{\sqrt{6}-\sqrt{2}}{4}$$

Example 3 Using the sum and difference identities, find the exact value of $\sin(\alpha+\beta)$ *given* $\sin\alpha = \frac{4}{5}$ *where* $0<\alpha<\frac{\pi}{2}$ *and* $\cos\beta = -\frac{12}{13}$ *where* $\frac{\pi}{2}<\beta<\pi$.

Start by drawing the appropriate triangles.

$\sin\alpha = \frac{4}{5}$; $0<\alpha<\frac{\pi}{2}$ $\qquad$ $\cos\beta = -\frac{12}{13}$; $\frac{\pi}{2}<\beta<\pi$





* use Pythag Thm

So, $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$$= \left(\frac{4}{5}\right)\left(-\frac{12}{13}\right) + \left(\frac{3}{5}\right)\left(\frac{5}{13}\right)$$

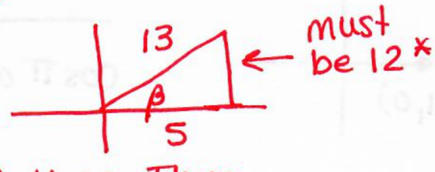
$$= -\frac{48}{65} + \frac{15}{65}$$

$$= -\frac{33}{65}$$

Example 4 Using the sum and difference identities, find the exact value of $\tan(\alpha+\beta)$ *given* $\csc\alpha = \frac{5}{3}$ *where* $0<\alpha<\frac{\pi}{2}$ *and* $\cos\beta = \frac{5}{13}$ *where* $0<\beta<\frac{\pi}{2}$.

Start by drawing the appropriate triangles.

$\csc\alpha = \frac{5}{3}$; $0<\alpha<\frac{\pi}{2}$ $\qquad$ $\cos\beta = \frac{5}{13}$; $0<\beta<\frac{\pi}{2}$



* use Pythag. Thm

So, $\tan(\alpha+\beta) = \dfrac{\tan\alpha + \tan\beta}{1-\tan\alpha\tan\beta}$

$$= \frac{\frac{3}{4}+\frac{12}{5}}{1-\left(\frac{3}{4}\right)\left(\frac{12}{5}\right)}$$

$$= \frac{\frac{15}{20}+\frac{48}{20}}{1-\frac{9}{5}}$$

$$= \frac{\frac{63}{20}}{-\frac{4}{5}}$$

$$= \left(\frac{63}{20}\right)\left(-\frac{5}{4}\right)$$

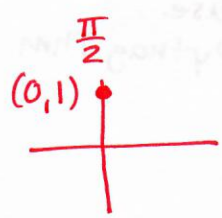
$$= -\frac{63}{16}$$

Example 5 Verify that $\cos\left(\frac{\pi}{2} - x\right) = \sin x$ is an identity.

Consider that $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

So, $\cos\frac{\pi}{2}\cos x + \sin\frac{\pi}{2}\sin x \stackrel{?}{=} \sin x$

$(0)(\cos x) + (1)(\sin x) \stackrel{?}{=} \sin x$



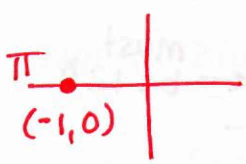
$0 + \sin x \stackrel{?}{=} \sin x$

$\sin x = \sin x$ ✓

Example 6 Verify that $\sec(\pi + A) = -\sec A$ is an identity.

$$\frac{1}{\cos(\pi + A)} \stackrel{?}{=} -\sec A$$

$$\frac{1}{\cos\pi\cos A - \sin\pi\sin A} \stackrel{?}{=} -\sec A$$



$$\frac{1}{(-1)(\cos A) - (0)(\sin A)} \stackrel{?}{=} -\sec A$$

$$\frac{1}{-\cos A} \stackrel{?}{=} -\sec A$$

$$-\sec A = -\sec A \ ✓$$