MATH 1080 TRIGONOMETRY NOTES

7.2 Right Triangle Trigonometry

Objectives: Use right triangles to evaluate trigonometric functions.
Use equal cofunctions of complementary angles.
Use the definitions of trigonometric functions of any angle.
Use right-triangle trigonometry to solve applied problems.

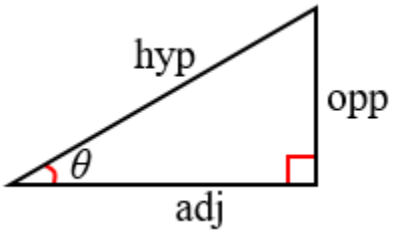
RIGHT TRIANGLE TRIGONOMETRY

Trigonometry is based upon ratios of the sides of right triangles.

TRIGONOMETRIC FUNCTIONS

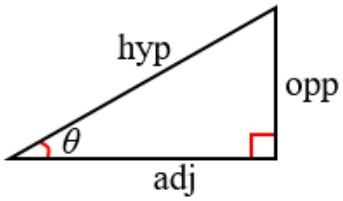
The six **trigonometric functions** of a right triangle, with an acute angle θ, are defined by **ratios** of two sides of the triangle.

The sides are labeled in relation to the location of θ.



DEFINITION OF TRIGONOMETRIC FUNCTIONS

The six trigonometric functions are: sine, cosine, tangent, cotangent, secant, and cosecant.



$$\sin\theta = \frac{opp}{hyp} \qquad \cos\theta = \frac{adj}{hyp} \qquad \tan\theta = \frac{opp}{adj}$$

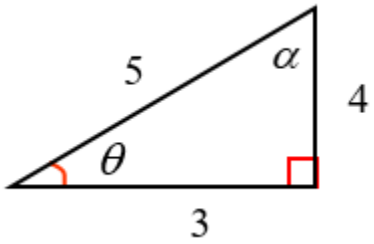
$$\csc\theta = \frac{hyp}{opp} \qquad \sec\theta = \frac{hyp}{adj} \qquad \cot\theta = \frac{adj}{opp}$$

RECIPROCAL FUNCTIONS

$\sin\theta = \frac{1}{\csc\theta}$ $\quad \cos\theta = \frac{1}{\sec\theta}$ $\quad \tan\theta = \frac{1}{\cot\theta}$

$\csc\theta = \frac{1}{\sin\theta}$ $\quad \sec\theta = \frac{1}{\cos\theta}$ $\quad \cot\theta = \frac{1}{\tan\theta}$

Example 1 Determine the value of each trig function as a reduced fraction.



$\sin\theta =$ $\quad \sin\alpha =$

$\cos\theta =$ $\quad \cos\alpha =$

$\tan\theta =$ $\quad \tan\alpha =$

$\cot\theta =$ $\quad \cot\alpha =$

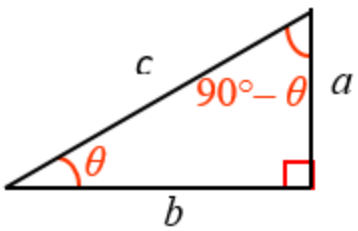
$\sec\theta =$ $\quad \sec\alpha =$

$\csc\theta =$ $\quad \csc\alpha =$

NOTE: $\theta \; and \; (90° - \theta) \; are \; complementary \; angles$

$\sin\theta = \frac{a}{c} \; and \; \cos(90° - \theta) = \frac{a}{c}$

$So, \; \sin\theta = \cos(90° - \theta), for \; 0° \leq \theta \leq 90°$



The functions of the complements are called ***cofunctions***.

COFUNCTIONS

$\sin\theta = \cos(90^\circ - \theta)$	$\cos\theta = \sin(90^\circ - \theta)$
$\sin\theta = \cos(\pi/2 - \theta)$	$\cos\theta = \sin(\pi/2 - \theta)$
$\tan\theta = \cot(90^\circ - \theta)$	$\cot\theta = \tan(90^\circ - \theta)$
$\tan\theta = \cot(\pi/2 - \theta)$	$\cot\theta = \tan(\pi/2 - \theta)$
$\sec\theta = \csc(90^\circ - \theta)$	$\csc\theta = \sec(90^\circ - \theta)$
$\sec\theta = \csc(\pi/2 - \theta)$	$\csc\theta = \sec(\pi/2 - \theta)$

Example 2 Evaluate using cofunction identities.

a. $\cos(34°) = \sin(\quad\quad °)$

b. $\sec\left(\frac{\pi}{6}\right) = \csc(\quad\quad)$

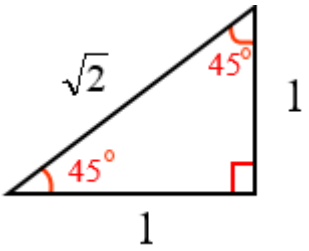
c. $\tan(48°) = \cot(\quad\quad °)$

d. If $\sin\theta = \frac{5}{12}$, find $cos\left(\frac{\pi}{2} - \theta\right)$.

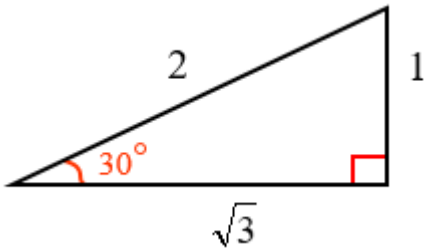
e. If $\csc\left(\frac{\pi}{6}\right) = 2$, find $sec\left(\frac{\pi}{3}\right)$.

SPECIAL RIGHT TRIANGLES

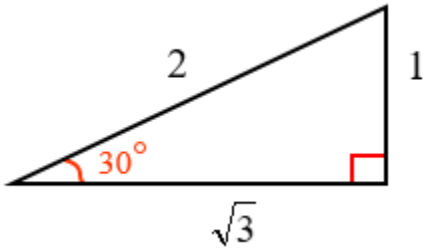
45° - 45° - 90°



30° - 60° - 90°



Example 3 Calculate the trig functions for a 30° - 60° - 90° triangle.



$\sin 30° =$ $\csc 30° =$

$\cos 30° =$ $\sec 30° =$

$\tan 30° =$ $\cot 30° =$

$\sin 60° =$ $\csc 60° =$

$\cos 60° =$ $\sec 60° =$

$\tan 60° =$ $\cot 60° =$

APPLICATIONS OF RIGHT TRIANGLES

Angle of Elevation – angle measurement of objects ***ABOVE*** the horizontal

Angle of Depression - angle measurement of objects ***BELOW*** the horizontal

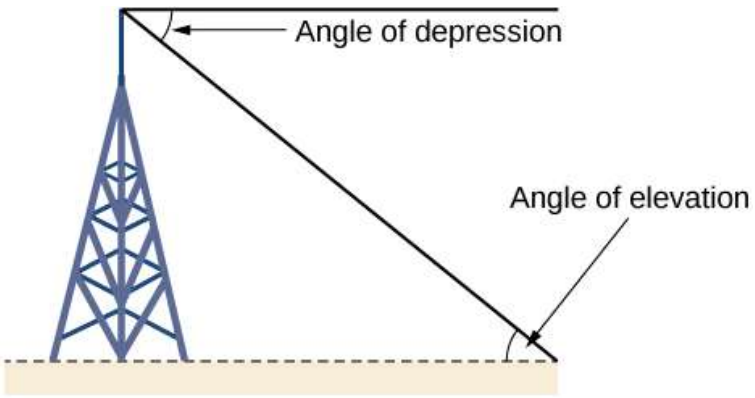


Figure 12

Example 4

A surveyor is standing 115 feet from the base of the Washington Monument.
The surveyor measures the angle of elevation to the top of the monument as 78.3°.
Approximate the height of the Washington Monument to the nearest foot.

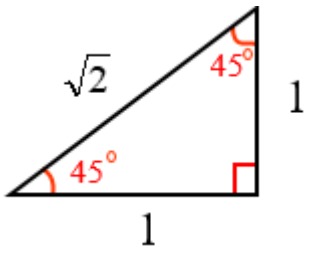
Example 5

An airplane is flying at a height of 2 miles above ground level. The angle of depression from the plane to the foot of the tree is 15°. How far is the plane from the base of the tree?
Approximate the distance from the plane to the tree to the nearest tenth of a mile.

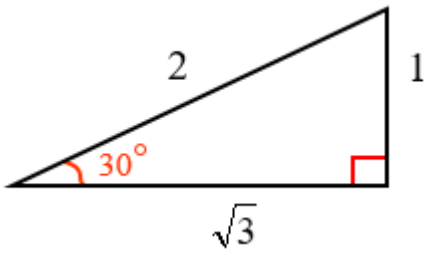
EXTRA PRACTICE

SPECIAL RIGHT TRIANGLES

45° - 45° - 90°



30° - 60° - 90°



Complete the table. (Memorize the $\sin\theta, \cos\theta, and \tan\theta$ values.)

θ	0°	30°	45°	60°	90°	180°	270°
radians							
$\sin\theta$							
$\cos\theta$							
$\tan\theta$							
$\csc\theta$							
$\sec\theta$							
$\cot\theta$							