PRECALC §7-1 NOTES

PRECALCULUS NOTES

7.1 Basic Trigonometric Identities

Objectives: **Identify and use reciprocal identities, quotient identities, Pythagorean identities, symmetry identities, and opposite angle identities.**

Definitions

Identity – a statement of equality between two expressions that is true for ALL values of the variable(s) for which the expressions are defined.

Trigonometric Identity – an identity involving trigonometric expressions.

Counterexample – an example that shows that an equation is false.

Example 1 **Prove that each equation is not a trigonometric identity by producing a counterexample.**

a. $\sin x \cos x = \tan x$

let $x = \frac{\pi}{4}$

$(\sin \frac{\pi}{4})(\cos \frac{\pi}{4}) \stackrel{?}{=} \tan(\frac{\pi}{4})$

$(\frac{\sqrt{2}}{2})(\frac{\sqrt{2}}{2}) \stackrel{?}{=} 1$

$\frac{2}{4} \stackrel{?}{=} 1$

$\frac{1}{2} \neq 1$

b. $\sin x \tan x = \cos x$

let $x = \frac{\pi}{3}$

$(\sin \frac{\pi}{3})(\tan \frac{\pi}{3}) \stackrel{?}{=} \cos \frac{\pi}{3}$

$(\frac{\sqrt{3}}{2})(\sqrt{3}) \stackrel{?}{=} \frac{1}{2}$

$\frac{3}{2} \neq \frac{1}{2}$

RECIPROCAL IDENTITIES

$\sin\theta = \frac{1}{\csc\theta}$ $\quad$ $\csc\theta = \frac{1}{\sin\theta}$

$\cos\theta = \frac{1}{\sec\theta}$ $\quad$ $\sec\theta = \frac{1}{\cos\theta}$

$\tan\theta = \frac{1}{\cot\theta}$ $\quad$ $\cot\theta = \frac{1}{\tan\theta}$

QUOTIENT IDENTITIES

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

PYTHAGOREAN IDENTITIES

$$\sin^2\theta + cos^2\theta = 1^* \qquad \tan^2\theta + 1 = sec^2\theta \qquad 1 + \cot^2\theta = csc^2\theta$$

*Recall the equation in a unit circle $x^2 + y^2 = 1$, since $x = \sin\theta \; and \; y = \cos\theta$ then $\sin^2\theta + cos^2\theta = 1$

Take one Pythagorean Identity to create a new identity.

$$\sin^2\theta + cos^2\theta = 1$$

$$\frac{\sin^2\theta}{\sin^2\theta} + \frac{cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$$ Divide each side by $\sin^2\theta$.

$$1 + cot^2\theta = csc^2\theta$$ Simplify to obtain another Pythagorean Identity

Try again...

$$\sin^2\theta + cos^2\theta = 1$$

$$\frac{\sin^2\theta}{\cos^2\theta} + \frac{cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$ Divide each side by $\cos^2\theta$.

$$tan^2\theta + 1 = sec^2\theta$$ Simplify to obtain another Pythagorean Identity

OPPOSITE ANGLES IDENTITIES

sin (-θ) = - sin θ	cos (-θ) = cos θ	tan (-θ) = - tan θ
csc (-θ) = - csc θ	sec (-θ) = sec θ	cot (-θ) = - cot θ

Example 2 **Use the given information to determine the exact trigonometric value.**

a. Given $\tan\theta = \frac{\sqrt{3}}{4}, 0 < \theta < \frac{\pi}{2}, find\ cot\theta$

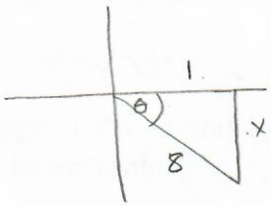
$$\cot\theta = \frac{1}{\tan\theta}$$

$$= \frac{1}{\frac{\sqrt{3}}{4}}$$

$$= \frac{4}{\sqrt{3}}$$

$$= \frac{4\sqrt{3}}{3}$$

b. Given $\cos\theta = \frac{1}{8}, \frac{3\pi}{2} < \theta < 2\pi, find\ tan\theta$



$$x^2 + 1^2 = 8^2$$

$$x^2 = 63$$

$$x = \sqrt{63} = 3\sqrt{7}$$

$$\sin\theta = -\frac{3\sqrt{7}}{8}$$

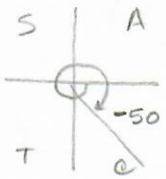
$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$= \frac{-\frac{3\sqrt{7}}{8}}{\frac{1}{8}}$$

$$= -3\sqrt{7}$$

Example 3 **Express each value as a trigonometric function of an angle in Quadrant I.**

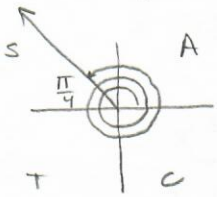
a. $\cos(-410°)$



$$= \cos(-360 - 50)$$

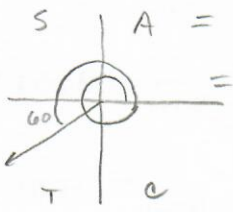
$$= \cos 50$$

b. $\sin\frac{19\pi}{4}$



$$= \sin\frac{\pi}{4}$$

c. $\sin 600°$



$$= \sin(60 + 540)$$

$$= \sin(60 + 3(180))$$

$$= -\sin 60$$

Example 4 **Simplify each expression.**

a. $\tan x \cos x$

$= \left(\frac{\sin x}{\cos x}\right)(\cos x)$

$= \sin x$

b. $\frac{\sec x}{\csc x} = \frac{\frac{1}{\cos x}}{\frac{1}{\sin x}}$

$= \left(\frac{1}{\cos x}\right)\left(\frac{\sin x}{1}\right)$

$= \frac{\sin x}{\cos x}$

$= \tan x$

Example 5 **Simplify each expression.**

a. $\frac{\csc x}{\cot x} = \frac{\frac{1}{\sin x}}{\frac{\cos x}{\sin x}}$

$= \left(\frac{1}{\sin x}\right)\left(\frac{\sin x}{\cos x}\right)$

$= \frac{1}{\cos x}$

$= \sec x$

b. $\cos x \csc x \tan x$

$= \cos x \left(\frac{1}{\sin x}\right)(\tan x)$

$= \left(\frac{\cos x}{\sin x}\right)\left(\frac{\sin x}{\cos x}\right)$

$= 1$

c. $\cos x \cot x + \sin x$

$= \cos x \left(\frac{\cos x}{\sin x}\right) + \sin x$

$= \frac{\cos^2 x}{\sin x} + \frac{\sin^2 x}{\sin x}$

$= \frac{\cos^2 x + \sin^2 x}{\sin x}$

$= \frac{1}{\sin x}$

$= \csc x$

d. $\sin x + \sin x \cot^2 x$

$= \sin x \left(1 + \cot^2 x\right)$

$= \sin x \left(\csc^2 x\right)$

$= \sin x \left(\frac{1}{\sin^2 x}\right)$

$= \frac{1}{\sin x}$

$= \csc x$