MATH 1080 TRIGONOMETRY NOTES
7.1 Angles

Objectives: Draw angles in standard position
Convert between degrees and radians
Identify angles that are coterminal with a given angle.
Find the length of a circular arc
Use linear and angular speed to describe motion on a circular path

BASIC TERMINOLOGY

Angle – consists of two rays in a plane with a common endpoint

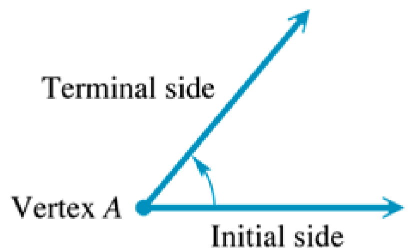
Sides – the two rays that form the angle

Vertex – the common endpoint of the rays

Initial Side – the ray in its initial position

Terminal Side – the ray its location after a rotation

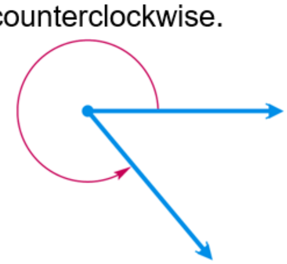
Measure - the degree generated by a rotation about the vertex



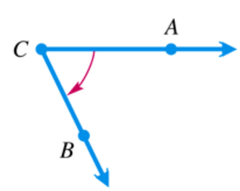
POSITIVE and NEGATIVE ANGLES

Positive angle: The rotation of the terminal side of an angle is counterclockwise.

Negative angle: The rotation of the terminal side is clockwise.

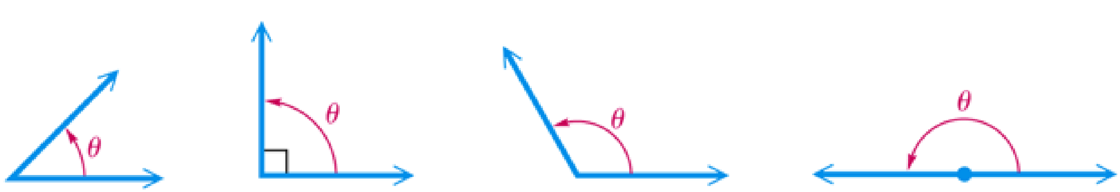


Positive angle



Negative angle

TYPES of ANGLES



Acute angle $0° < \theta < 90°$

Right angle $\theta = 90°$

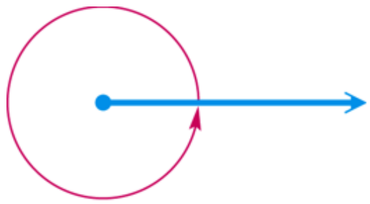
Obtuse angle $90° < \theta < 180°$

Straight angle $\theta = 180°$

MEASURING ANGLES

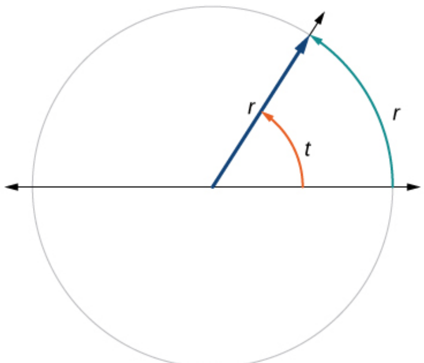
Degree - the most common unit for measuring angles

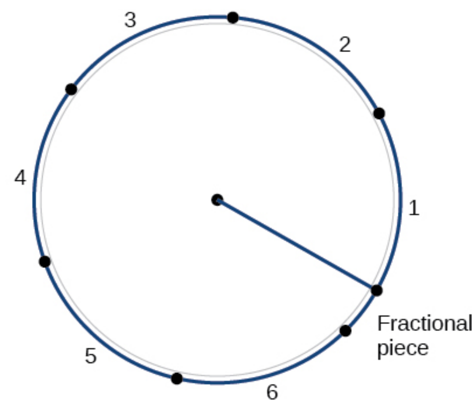
A complete counterclockwise rotation of a ray produces an angle whose measure is 360°.



Radian - one radian is the measure of a central angle of a circle that intercepts an arc equal in length to the radius of that circle. Each circle measures 2π radians.

$$2\pi\ radians = 360°$$





Example 1 Convert each measurement from degrees to radians.

a. $-150°$

b. $315°$

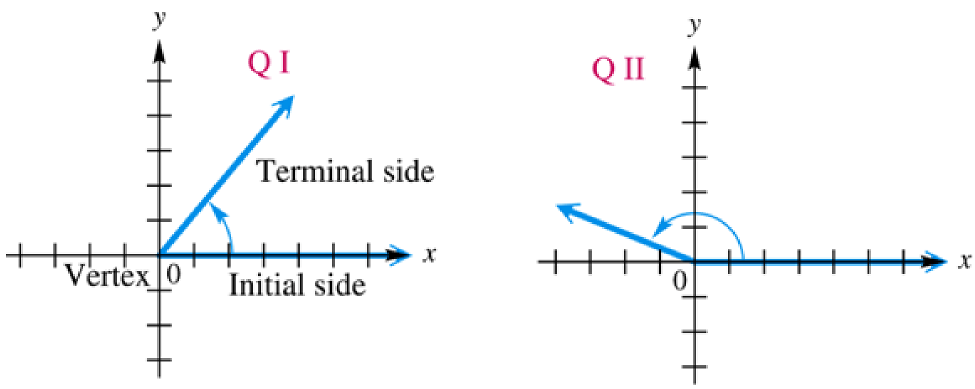
Example 2 Convert each measurement from radians to degrees.

a. $\frac{3\pi}{4}$

b. $-\frac{7\pi}{3}$

STANDARD POSITION

An angle in **standard position** if its vertex is at the origin and its initial is along the positive x-axis.

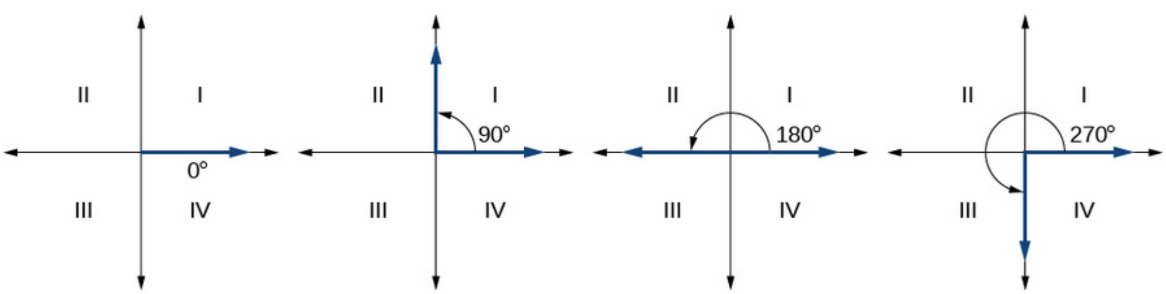


Positive angle – if the rotation is in a counterclockwise rotation

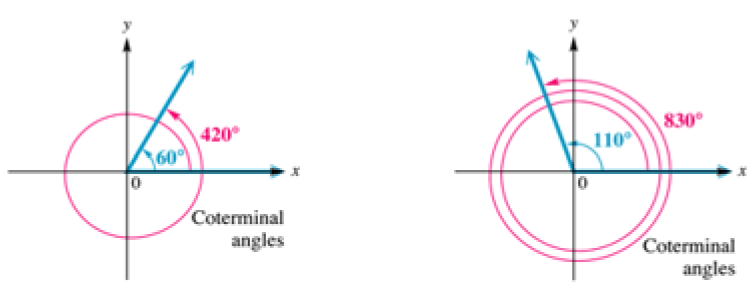
Negative angle – if the rotation is in a clockwise rotation

QUADRANTAL ANGLES

Quadrantal Angles - Angles in standard position having their terminal sides along the x-axis or y-axis, such as angles with measures 90°, 180°, 270°, and so on.



COTERMINAL ANGLES - two angles in standard position that have the same terminal side



The measures of coterminal angles differ by multiples of 360° or 2π radians.

Example 3 Draw each angle in standard position. Then find one positive angle and one negative angle that are coterminal with the given angle.

a. $40°$

b. $-150°$

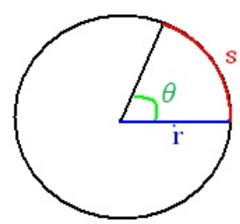
c. $-\frac{3\pi}{4}$

d. $\frac{2\pi}{3}$

ARC LENGTH OF A CIRCLE

The length of an arc s subtended by an angle with measure θ is $s = r\theta$

Note: θ is measured in radians



Example 4

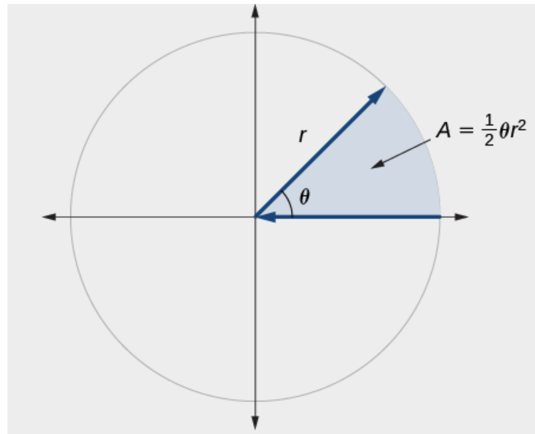
a. Given a central angle of 52.8°, determine the length of the intercepted arc in a circle of radius 8 cm.

b. Determine the length of the arc intercepted by a central angle of $\frac{\pi}{8}$ radians on a circle of radius 6 inches.

AREA OF A SECTOR

The area of a sector of a circle with radius r subtended by an angle θ is
$A = \frac{1}{2}\theta\, r^2$

Note: θ is measured in radians



Example 5

An automatic lawn sprinkler sprays a distance of 20 feet while rotating 30 degrees. Determine the area of the sector of grass that the sprinkler waters to the nearest hundredth.

LINEAR AND ANGULAR SPEED

Linear speed – the speed along a straight path which can be determined by the distance travelled (its displacement), arc length s, in a given time interval.

$$v = \frac{s}{t}$$ where s is displacement and t is time

Angular speed – the speed resulting from a circular motion which can be determined by the angle through which a point rotates in a given time interval

$$\omega = \frac{\theta}{t}$$ where θ is the angular rotation per unit t time t

When angular speed is measured in radians per unit time, linear speed and angular speed are related by the equation (using $s = r\theta$)

$$v = r\omega$$

Example 6

An old vinyl record is played on a turntable rotating clockwise at a rate of 45 rotations per minute. Determine the angular speed in radians per second.

Example 7

A bicycle has wheels 28 inches in diameter. If the wheels are rotating at 180 RPM (revolutions per minute), determine the speed that the bicycle is traveling down the road in miles per hour.