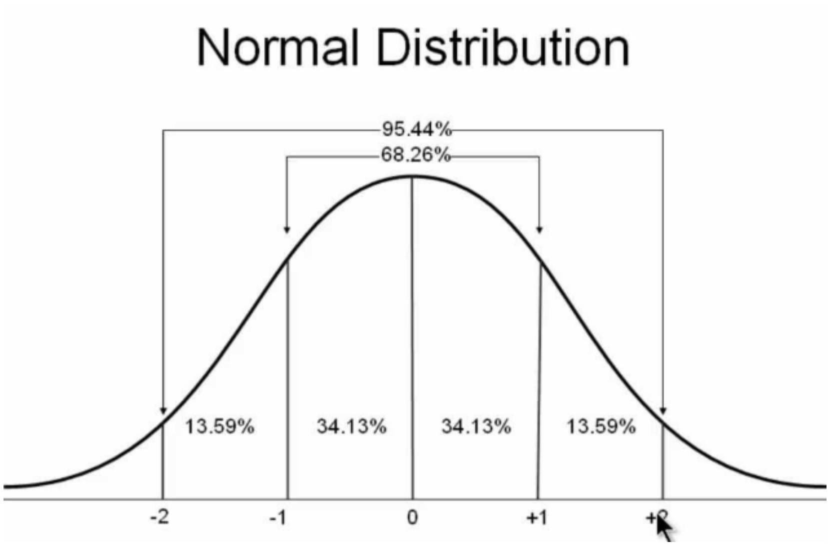
MATH 1610/MATH 1552
6.2 Normal Probability Distribution

Normal Probability Distribution

- The most important probability distribution for describing a continuous variable is the normal probability distribution. It is widely used in statistical inference.
- Application include:
  - Heights of people
  - Rainfall amounts
  - Test Scores Scientific measurements
- The normal distribution provides a description of the likely results obtain through sampling.
- The probability density function defines a bell-shaped curve.
- Abraham de Moivre, a French mathematician, published "The Doctrine of Chances" in 1773. He derived the normal distribution.



Normal Probability Distribution Function

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

where $\mu = mean$
$\sigma = standard\ deviation$
$\pi = 3.14159$
$e = 2.71828$

Understanding the normal probability distribution is the ***<u>most important</u>*** statistics concept of the course. We will use this concept REGULARLY in the next course.

Let's practice understanding the areas under a normal distribution curve by applying the Empirical Rule.

<u>Example 1</u> Draw a normal standard distribution curve for each scenario.

a. Draw the normal distribution with the mean.

_____________ of the area is below the mean.

_____________ of the area is above the mean.

b. Draw the normal distribution with the mean, and **one standard deviation** from the mean.

_____________ of the area is lies within

_____ standard deviation from the mean.

c. Draw the normal distribution with the mean, and **two standard deviations** from the mean.

_____________ of the area is lies within

_____ standard deviations from the mean.

d. Draw the normal distribution with the mean, and **three standard deviations** from the mean.

_____________ of the area is lies within

_____ standard deviations from the mean.

Question: What happens if the data is not exactly 1, 2 or 3 standard deviations from the mean?

Answer: To find the probability that the normal random variable (NRV) falls in a given interval, we need to use $\boldsymbol{f(x)} = \frac{\mathbf{1}}{\boldsymbol{\sigma\sqrt{2\pi}}}\boldsymbol{e}^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^{2}}$ and compute the area under f, but the topic is not covered in this class.

Therefore, we will typically use the following process to compute the probabilities.

1. Compute the z-score to convert everything to the standard normal distribution.

$$z = \frac{x - \mu}{\sigma}$$

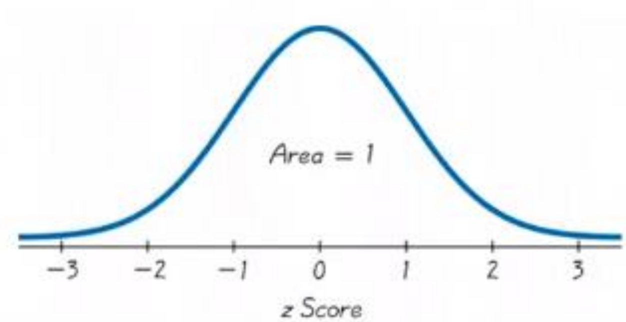
2. Use the standard normal probability distribution table to estimate the area.

Normal Distribution

1. Bell-shaped curve
2. Total Area = 1
3. Symmetrical

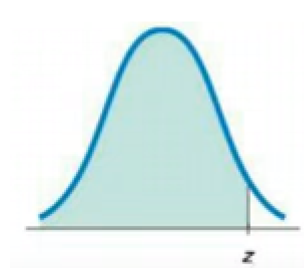
"Standard" Normal Distribution

4. Mean is equal to 0 ($\mu = 0$)
5. Standard deviation is equal to 1 ($\sigma = 1$)



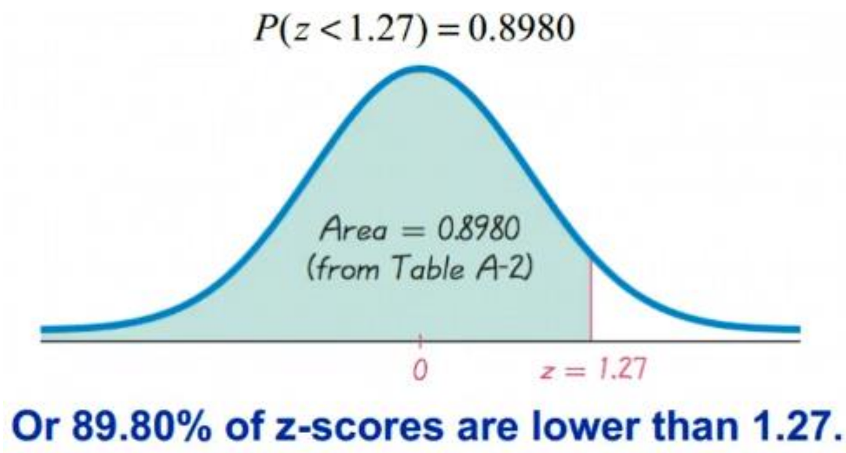
Cumulative probability for the standard normal distribution

Entries in the table provide the cumulative area from the ***LEFT*** up to the vertical line above a specific value of z.



<u>Example 2</u> Given that z is a standard normal random variable:

a. determine the probability of P(z < 1.27).



b. Determine the probability of $P(z \geq 1.27)$.

<u>Example 3</u> Given that z is a standard normal random variable:

a. determine the probability of $P(z < 1.58)$.

b. Determine the probability of $P(z \geq 1.58)$

Example 4 Given that z is a standard normal random variable:

a. determine the probability of $P(-1 \leq z \leq 1)$

b. Determine the probability of $P(-0.50 \leq z \leq 1.25)$

Example 5

a. Determine the z value associated with the probability of 0.35.

b. Determine the z value associated with the probability of 0.83

<u>Example 6</u> Determine the z_0 value that corresponds to the given area.

$a.\ P(z < z_0) = 0.0212$

$b.\ P(z \geq z_0) = 0.0212$

c. $P(z < z_0) = 0.0375$

d. $P(z \geq z_0) = 0.0375$

e. Determine the z score for the lower 40th percentile.

f. Determine the z score for the upper 20th percentile.

Converting to the Standard Normal Random Variable

$$z = \frac{x - \mu}{\sigma}$$

z is a random variable that has the standard normal distribution.
z is distributed normally with a mean of 0 and a standard deviation of 1.

APPLICATIONS

Example 7

Suppose the height of adult American females is approximately normally distributed with a mean of 162.2 cm and a standard deviation of 6.8 cm.

a. What is the probability that a randomly selected adult American female is taller than 170.5 cm?

b. What is the probability that a randomly selected adult American female has a height between 150.5 and 170.5 cm?

APPLICATIONS (con't)

Example 8

The average stock price for companies making up the S & P 500 is \$30, and the standard deviation is \$8.20. Assume the stock prices are normally distributed.

a. What is the probability that a company will have a stock price no higher than \$20?

b. What is the probability that a company will have a stock price of at least \$40?

c. How high would a stock price have to be for the company to be in the top 10%?

MATH 1610/MATH 1552 Name____________________________

6.2 WS #1 Normal Probability Distribution

1. $\bar{x} = 600, \quad s = 204.4$, what percent of all McDonald's food would you expect to have:
    a. More than 500 calories?
    b. 600 to 900 calories?
    c. Less than 250 calories?

2. $\overline{x} = 600, \quad s = 204.4$, how many calories would you expect any food from McDonald's above the 80th percentile to have? **This answer will be k or more calories.**

3. $\overline{x} = 600, \quad s = 204.4$, how many calories would foods from McDonald's in the middle 80% have? Your answer will read: From_____________calories to__________calories.

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6.2 WS #2 Normal Probability Distribution

1. Find the area/probability under the normal curve to the left of $z = 1.24$. ____________________

2. Find the area/probability under the normal curve to the right of $z = 1.35$. ____________________

3. Find the area/probability under the normal curve over the interval from $z = 0.27$ to $z = 1.58$. ____________________

4. Determine the z-value that is associated with a probability of 0.35. ____________________

5. Determine the z-value that is associated with a probability of 0.85. ____________________

6. Determine the z-value that is associated with the 60th percentile of the normal distribution. ____________________

7. Determine the z-values associated with the middle 60% of the normal distribution.________________

8. For a normal distribution, find the z-score that separates:

a. the highest 30% from the rest of the distribution.

b. the lowest 40% from the rest of the distribution.

c. the highest 75% from the rest of the distribution.

9. A fifth grader took a standardized achievement test which has a mean score of 125 and a standard deviation of 15. The student's score was 148. What is the student's percentile rank?

10. Women have a mean height of 63.6 inches and a standard deviation of 2.5 inches. What is the probability that a woman is 70 inches or taller?

11. Scores on the SAT form a normal distribution with a mean of 500 and standard deviation of 100.

a. What is the minimum score necessary to be in the top 15% of the SAT distribution?

b. Determine the range of scores that defines the middle 80% of the distribution of SAT scores.