

# FACTORIZING POLYNOMIALS

- 1) First determine if a common monomial factor (Greatest Common Factor) exists. Factor trees may be used to find the GCF of difficult numbers. Be aware of opposites: Ex. (a-b) and (b-a) These may become the same by factoring -1 from one of them.

$$\begin{aligned}3x - 12 &= 3(x - 4) \\x^2y^2 - 3xy^2 &= xy^2(x - 3) \\6(x - y) + a(x - y) &= (x - y)(6 + a)\end{aligned}$$

- 2) If the problem to be factored is a binomial, see if it fits one of the following situations.

- A. Difference of two squares:

$$\begin{aligned}a^2 - b^2 &= (a + b)(a - b) \\9x^2 - 25y^2 &= (3x + 5y)(3x - 5y) \\(a + b)^2 - 25 &= [(a + b) + 5][(a + b) - 5] = (a + b + 5)(a + b - 5)\end{aligned}$$

- B. Sum of two squares:

$$a^2 + b^2 \text{ does not factor (it is prime).}$$

- C. Sum of two cubes:

$$\begin{aligned}a^3 + b^3 &= (a + b)(a^2 - ab + b^2) \\8x^3 + 27y^3 &= (2x + 3y)(4x^2 - 6xy + 9y^2)\end{aligned}$$

**Note: Resulting trinomial does not factor.**

- D. Difference of two cubes:

$$\begin{aligned}a^3 - b^3 &= (a - b)(a^2 + ab + b^2) \\x^3 - 64 &= (x - 4)(x^2 + 4x + 16)\end{aligned}$$

**Note: Resulting trinomial does not factor.**

- E. If none of these occur, the binomial does not factor.

- 3) If the problem is a trinomial, check for one of the following possibilities.

- A. Square of a binomial:

$$\begin{aligned}a^2 + 2ab + b^2 &= (a + b)(a + b) = (a + b)^2 \\x^2 + 6x + 9 &= (x + 3)(x + 3) = (x + 3)^2 \\4x^2 - 20xy + 25y^2 &= (2x - 5y)^2\end{aligned}$$

- B. If  $a = 1$ , use reverse foil or trial and error method:

$$\begin{aligned}x^2 + 7x + 12 &= (x + 3)(x + 4) \\x^2 - 7x + 12 &= (x - 3)(x - 4) \\x^2 + 3x - 18 &= (x + 6)(x - 3) \\x^2 - 3x - 18 &= (x - 6)(x + 3)\end{aligned}$$

- C. If  $a \neq 1$ , use trial and error method. (Grouping may also be used.)

- 4) If factoring a polynomial with four terms, possible choices are below.

- A. Group first two terms together and last two terms together.

$$\begin{aligned}5a - 5b - xa + xb &= (5a - 5b) + (-xa + xb) = 5(a - b) - x(a - b) = (a - b)(5 - x) \\x^3 - 3x^2 + 2x - 6 &= (x^3 - 3x^2) + (2x - 6) = x^2(x - 3) + 2(x - 3) = (x - 3)(x^2 + 2)\end{aligned}$$

- B. Group first three terms together.

$$x^2 + 6x + 9 - y^2 = (x^2 + 6x + 9) - y^2 = (x + 3)^2 - y^2 = [(x + 3) + y][(x + 3) - y] = (x + 3 + y)(x + 3 - y)$$

- C. Group last three terms together.

$$y^2 - x^2 + 6x - 9 = y^2 - (x^2 - 6x + 9) = y^2 - (x - 3)^2 = [y + (x - 3)][y - (x - 3)] = (y + x - 3)(y - x + 3)$$

**BE SURE YOUR ANSWERS WILL NOT FACTOR FURTHER!**

All answers may be checked by multiplication.

## PRACTICE PROBLEMS:

- $y^3 + 9y^2$
- $5x^2y^3 + 15x^3y^2$
- $12t^5 - 20t^4 + 8t^2 - 16$
- $p^2 - 36$
- $25 - x^2$
- $4a^3 - 49a$
- $(a + b)^2 - 100$
- $9 - (x - y)^2$
- $y^3 + 8$
- $64y^4 + y$
- $x^3 - 27$
- $5x^3 - 40y^3$
- $2y^4 - 128y$
- $t^6 - 64$
- $x^2 - 10x + 25$
- $4a^2 + 16a + 16$
- $16y^2 + 56y + 49$
- $-20xy + 4y^2 + 25x^2$
- $x^2 + 9x + 20$
- $2y^2 - 16y + 32$
- $3x + x^2 - 10$
- $y^2 + 5y - 84$
- $8x^2 - 16 - 28x$
- $12x^3 - 31x^2 + 20x$
- $6a^2 - 7a - 10$
- $8 - 6x - 9x^2$
- $6x^6 + x^3 - 2$
- $2x^8 - 14x^4 + 20$
- $y^3 - y^2 + 2y - 2$
- $x^4 - x^3 - x + x^2$
- $x^3 + 8x^2 - x - 8$
- $p^2q - 25q + 3p^2 - 75$
- $16 - x^2 + 2xy - y^2$
- $2xy - x^2y - 6 + 3x$
- $6x^2 + 23x + 20$
- $9x^2 + 15x + 4$
- $8m^2 - 6m - 9$
- $25 - 10x + x^2$
- $16 - w^4$
- $ay - yx - x^2 + ax$

## ANSWERS:

- $y^2(y + 9)$
- $5x^2y^2(y + 3x)$
- $4(3t^5 - 5t^4 + 2t^2 - 4)$
- $(p + 6)(p - 6)$
- $(5 + x)(5 - x)$
- $a(2a + 7)(2a - 7)$
- $(a + b + 10)(a + b - 10)$
- $(3 + x - y)(3 - x + y)$
- $(y + 2)(y^2 - 2y + 4)$
- $y(4y + 1)(16y^2 - 4y + 1)$
- $(x - 3)(x^2 + 3x + 9)$
- $5(x - 2y)(x^2 + 2xy + 4y^2)$
- $2y(y - 4)(y^2 + 4y + 16)$
- $(t + 2)(t^2 - 2t + 4)(t - 2)(t^2 + 2t + 4)$
- $(x - 5)^2$
- $4(a + 2)^2$
- $(4y + 7)^2$
- $(5x - 2y)^2$
- $(x + 5)(x + 4)$
- $2(y - 4)^2$
- $(x + 5)(x - 2)$
- $(y + 12)(y - 7)$
- $4(2x + 1)(x - 4)$
- $x(4x - 5)(3x - 4)$
- $(a - 2)(6a + 5)$
- $(4 + 3x)(2 - 3x)$
- $(3x^3 + 2)(2x^3 - 1)$
- $2(x^4 - 5)(x^4 - 2)$
- $(y - 1)(y^2 + 2)$
- $x(x^2 + 1)(x - 1)$
- $(x + 8)(x + 1)(x - 1)$
- $(q + 3)(p + 5)(p - 5)$
- $(4 + x - y)(4 - x + y)$
- $(2 - x)(xy - 3)$
- $(3x + 4)(2x + 5)$
- $(3x + 1)(3x + 4)$
- $(4m + 3)(2m - 3)$
- $(5 - x)^2$  or  $(x - 5)^2$
- $(4 + w^2)(2 + w)(2 - w)$
- $(y + x)(a - x)$

## MORE PRACTICE PROBLEMS:

41.  $x^2 - 6x - 16$

42.  $x^2 - 10xy + 24y^2$

43.  $x^2 + 3x + 2$

44.  $x^2 - 3x + 2$

45.  $x^2 - x - 30$

46.  $x^2 + 7x - 8$

47.  $x^2 + x - 2$

48.  $x^2 - 5xy + 6y^2$

49.  $x^2 + 10x + 16$

50.  $x^2 + x - 72$

51.  $x^2 - 8x - 9$

52.  $x^2 + 2x - 48$

53.  $x^2 - 13xy + 42y^2$

54.  $x^2 + 8x + 12$

55.  $4x^3 - 8x^2 - 12x$

56.  $2x^3 - 2x^2 - 4x$

57.  $2x^3 - 4x^2 - 6x$

58.  $3x^3 - 6x^2 - 9x$

59.  $5x^3y - 35x^2y + 50xy$

60.  $3x^3y + 18x^2y - 21xy$

61.  $4x^2 + 1 - 4x$

62.  $15x^2 + 12 + 29x$

63.  $8r^2 - 2r - 3$

64.  $35a^2 + 3a - 20$

65.  $25x^2 + 8 + 30x$

66.  $12x^2 + 3 + 13x$

67.  $9x^2 - 27xy + 20y^2$

68.  $25u^2 - 15u - 18$

69.  $12f^2 - 4f - 5$

70.  $5z^2 + 3z + 4$

71.  $4x^2 + 15 + 16x$

72.  $20x^2 + 6 + 23x$

73.  $6x^2 - 19xy + 10y^2$

74.  $35p^2 + 13p - 4$

75.  $50x^2 + 10x - 12$

76.  $-30x^2 - 25x + 30$

77.  $-18x^2 + 18x + 20$

78.  $3x^3 - 22x^2 + 7x$

79.  $15x^2 - 18x - 24$

80.  $4x^3 - 25x^2 + 6x$

## ANSWERS:

41.  $(x - 8)(x + 2)$  42.  $(x - 6y)(x - 4y)$  43.  $(x + 2)(x + 1)$  44.  $(x - 2)(x - 1)$

45.  $(x - 6)(x + 5)$  46.  $(x + 8)(x - 1)$  47.  $(x + 2)(x - 1)$  48.  $(x - 3y)(x - 2y)$

49.  $(x + 8)(x + 2)$  50.  $(x + 9)(x - 8)$  51.  $(x - 9)(x + 1)$  52.  $(x + 8)(x - 6)$

53.  $(x - 7y)(x - 6y)$  54.  $(x + 6)(x + 2)$  55.  $4x(x - 3)(x + 1)$  56.  $2x(x - 2)(x + 1)$

57.  $2x(x - 3)(x + 1)$  58.  $3x(x - 3)(x + 1)$  59.  $5xy(x - 5)(x - 2)$  60.  $3xy(x + 7)(x - 1)$  61.

$(2x - 1)^2$  62.  $(3x + 4)(5x + 3)$  63.  $(2r + 1)(4r - 3)$  64.  $(5a + 4)(7a - 5)$

65.  $(5x + 4)(5x + 2)$  66.  $(3x + 1)(4x + 3)$  67.  $(3x - 5y)(3x - 4y)$  68.  $(5u + 3)(5u - 6)$  69.

$(2f + 1)(6f - 5)$  70. *Prime (Cannot be factored)* 71.  $(2x + 3)(2x + 5)$

72.  $(5x + 2)(4x + 3)$  73.  $(2x - 5y)(3x - 2y)$  74.  $(7p + 4)(5p - 1)$

75.  $2(5x + 3)(5x - 2)$  76.  $-5(2x + 3)(3x - 2)$  77.  $-2(3x - 5)(3x + 2)$

78.  $x(3x - 1)(x - 7)$  79.  $3(5x + 4)(x - 2)$  80.  $x(4x - 1)(x - 6)$

## MORE PRACTICE PROBLEMS:

81.  $125x^3 - 1$   
82.  $w^2 - 64$   
83.  $y^2 - 12y + 36$   
84.  $x^2 - 8x - 48$   
85.  $a^3 - 7a^2 + 12a$   
86.  $25a^2 + 8b^2$   
87.  $(x - 3)(x + 7) + (x - 3)(x - 4)$   
88.  $6x^2 + 12x + 6$   
89.  $y^2 - 11y + 18$   
90.  $40 + 3b - b^2$   
91.  $3x^5 - 12x^2$   
92.  $250x^3 + 2$   
93.  $7xy^4 - 7xz^4$   
94.  $2y^4 + 5y^3 - 12y^2$   
95.  $24x^2 - 7x - 5$   
96.  $y^2 + 14y - 32$   
97.  $0.04w^2 + 0.28w + 0.49$   
98.  $4x^3 + 40x^2 + 64x$   
99.  $64y^3 + 27$   
100.  $\frac{1}{81} - x^2$
101.  $5x^2 - 2x + 3$   
102.  $x^3 - 343$   
103.  $40y^2 + 28y - 48$   
104.  $3ab - 5bc + bd$   
105.  $8c^6 - 125d^6$   
106.  $81 - 18z + z^2$   
107.  $x^4 + 10x^3 + 25x^2$   
108.  $xz - xw - yz + yw$   
109.  $y^2 + 5y - 36$   
110.  $x^2 - 11x - 42$   
111.  $7a^2 - 7b^2$   
112.  $216 - a^3$   
113.  $81 + 18y + y^2$   
114.  $b^2 - 5b - 14$   
115.  $q^4 - 10q^3 + 21q^2$   
116.  $9x^2y^2 - 25y^4$   
117.  $105 + 8x - x^2$   
118.  $x^2 - 3x - 2$   
119.  $6y^3 + 48$   
120.  $a^3 - 14a^2 + 49a$

## ANSWERS:

81.  $(5x - 1)(25x^2 + 5x + 1)$  82.  $(w + 8)(w - 8)$  83.  $(y - 6)^2$  84.  $(x - 12)(x + 4)$   
85.  $a(a - 4)(a - 3)$  86. *Prime (Cannot be factored)* 87.  $(x - 3)(2x + 3)$   
88.  $6(x + 1)^2$  89.  $(y - 9)(y - 2)$  90.  $(8 - b)(5 + b)$  91.  $3x^2(x^3 - 4)$   
92.  $2(5x + 1)(25x^2 - 5x + 1)$  93.  $7x(y^2 + z^2)(y + z)(y - z)$  94.  $y^2(2y - 3)(y + 4)$   
95.  $(8x - 5)(3x + 1)$  96.  $(y - 2)(y + 16)$  97.  $(0.2w + 0.7)^2$  98.  $4x(x + 2)(x + 8)$   
99.  $(4y + 3)(16y^2 - 12y + 9)$  100.  $\left(\frac{1}{9} + x\right)\left(\frac{1}{9} - x\right)$  101. *Prime (Cannot be factored)* 102.  
 $(x - 7)(x^2 + 7x + 49)$  103.  $4(2y + 3)(5y - 4)$  104.  $b(3a - 5c + d)$   
105.  $(2c^2 - 5d^2)(4c^4 + 10c^2d^2 + 25d^4)$  106.  $(9 - z)^2$  107.  $x^2(x + 5)^2$   
108.  $(x - y)(z - w)$  109.  $(y - 4)(y + 9)$  110.  $(x - 14)(x + 3)$  111.  $7(a + b)(a - b)$   
112.  $(6 - a)(36 + 6a + a^2)$  113.  $(9 + y)^2$  114.  $(b - 7)(b + 2)$  115.  $q^2(q - 3)(q - 7)$  116.  
 $y^2(3x + 5y)(3x - 5y)$  117.  $(7 + x)(15 - x)$  118. *Prime (Cannot be factored)*  
119.  $6(y + 2)(y^2 - 2y + 4)$  120.  $a(a - 7)^2$

- |      |                      |      |                             |
|------|----------------------|------|-----------------------------|
| 121. | $3y^2 - 34y - 24$    | 131. | $x^2 - 0.6x + 0.09$         |
| 122. | $a^2 + 8a + 16$      | 132. | $4x^2 - 13x - 35$           |
| 123. | $y^2 - 121$          | 133. | $125x^6 - 81$               |
| 124. | $42 + a - a^2$       | 134. | $49x^3 - 14x^2 + x$         |
| 125. | $9x^3 - 24x^2 + 16x$ | 135. | $40y^2 + 7y - 3$            |
| 126. | $x^3 - \frac{1}{8}$  | 136. | $15w^2 - 15w - 90$          |
| 127. | $10w^2 + 29w - 21$   | 137. | $0.04a^2 - 0.49b^2$         |
| 128. | $16x^2 + 54x - 7$    | 138. | $x^3y^2 + 7x^2y^2 - 18xy^2$ |
| 129. | $27x^2 - 30x - 8$    | 139. | $2x^6 - 54y^6$              |
| 130. | $x^6 - 1$            | 140. | $\frac{1}{4}x^2 - 5x + 25$  |

**ANSWERS:**

121.  $(y - 12)(3y + 2)$  122.  $(a + 4)^2$   
 123.  $(y + 11)(y - 11)$  124.  $(7 - a)(6 + a)$  125.  $x(3x - 4)^2$  126.  $\left(x - \frac{1}{2}\right)\left(x^2 + \frac{1}{2}x + \frac{1}{4}\right)$   
 127.  $(5w - 3)(2w + 7)$  128.  $(2x + 7)(8x - 1)$  129.  $(9x + 2)(3x - 4)$   
 130.  $(x + 1)(x - 1)(x^2 - x + 1)(x^2 + x + 1)$  131.  $(x - 0.3)^2$  132.  $(x - 5)(4x + 7)$   
 133. *Prime (Cannot be factored)* 134.  $x(7x - 1)^2$  135.  $(8y + 3)(5y - 1)$   
 136.  $15(w + 2)(w - 3)$  137.  $(0.2a + 0.7b)(0.2a - 0.7b)$  138.  $xy^2(x - 2)(x + 9)$   
 139.  $2(x^2 - 3y^2)(x^4 + 3x^2y^2 + 9y^4)$  140.  $\left(\frac{1}{2}x - 5\right)^2$