PRE CALC – §10-4 NOTES

PRECALCULUS NOTES
10.4 Hyperbolas

<u>Objectives</u>: Use and determine the standard and general forms of the equation of a hyperbola. Graph hyperbolas.

<u>Warm-Up</u> Consider the equation of the ellipse: $3x^2 + 4y^2 - 6x + 16y + 7 = 0$

a. Write the equation in standard form.

$3x^2 - 6x + \_ + 4y^2 + 16y + \_ = -7 + \_ + \_$

$3(x^2 - 2x + \underline{1}) + 4(y^2 + 4y + \underline{4}) = -7 + \underline{3} + \underline{16}$

$3(x-1)^2 + 4(y+2)^2 = 12$

$\frac{(x-1)^2}{4} + \frac{(y+2)^2}{3} = 1$

$c^2 = a^2 - b^2$
$c^2 = 4 - 3$
$c^2 = 1$
$c = \pm 1$

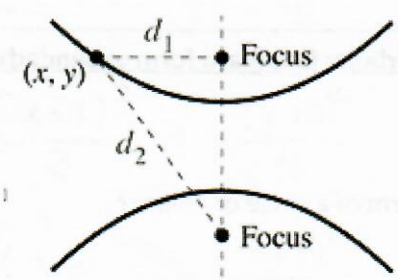
b. Determine the center. $(h, k) = (1, -2)$

c. Determine the foci. $(h \pm c, k) = (1 \pm 1, -2)$ $(2, -2)$, $(0, -2)$

d. Determine the vertices.
<u>major vertices</u> $(h \pm a, k) = (1 \pm 2, -2)$ $(3, -2)$, $(-1, -2)$
<u>minor vertices</u> $(h \pm b, k) = (1 \pm \sqrt{3}, -2)$

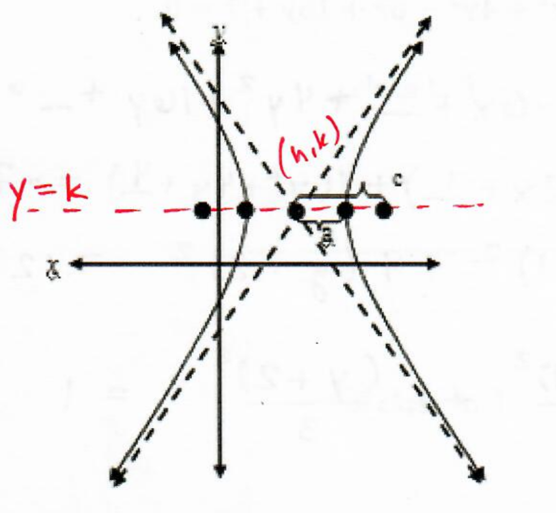
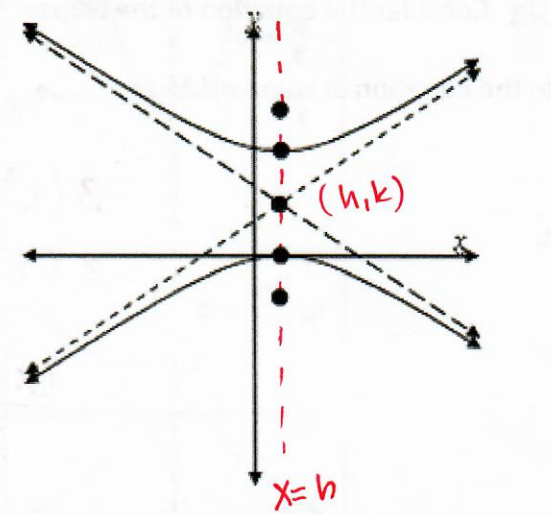
<u>DEFINITION – HYPERBOLA</u>

A hyperbola is the set of all points in a plane, the ***<u>difference</u>*** of whose distances from two distinct fixed points (foci) is a positive constant.



$d_2 - d_1$ is a positive constant.

Standard Equations of Hyperbolas

Opens left/right	Opens up/down
$\dfrac{(x-h)^2}{a^2} - \dfrac{(y-k)^2}{b^2} = 1$	$\dfrac{(y-k)^2}{a^2} - \dfrac{(x-h)^2}{b^2} = 1$
	
center: (h, k) foci: (h ± c, k) vertices: (h ± a, k) equation of transverse axis: y = k	center: (h, k) foci: (h, k ± c) vertices: (h, k ± a) equation of transverse axis: x = h
Additional equation: $a^2 + b^2 = c^2$ Asymptotes: $y = k \pm \frac{b}{a}(x-h)$	Additional equation: $a^2 + b^2 = c^2$ Asymptotes: $y = k \pm \frac{a}{b}(x-h)$

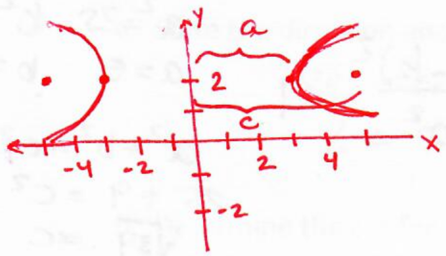
<u>General Form of the Equation of a Hyperbola: (standard form expanded):</u>

$$Ax^2 + Cy^2 + Dx + Ey + F = 0$$

How is this different from the general form of a circle or ellipse?

Example 1

Write an equation of a hyperbola with vertices (-3, 2) and (3, 2) and foci (-5, 2) and (5, 2).



center : (0, 2)

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

$a = 3$

$c = 5$

$$\frac{(x-0)^2}{9} - \frac{(y-2)^2}{16} = 1$$

$b^2 = c^2 - a^2$

$b^2 = 25 - 9$

$b^2 = 16$

$b = 4$

Example 2

Given the general form of a hyperbola, $4x^2 - 3y^2 + 8x + 16 = 0$, write the equation in standard form.

$$4x^2 + 8x + \_\_ - 3y^2 = -16$$

$$4(x^2 + 2x + \_\_) - 3(y^2) = -16$$

$$4(x^2 + 2x + \underline{1}) - 3(y+0)^2 = -16 + \underline{4}$$

$$\frac{4(x+1)^2}{-12} - \frac{3(y+0)^2}{-12} = \frac{-12}{-12}$$

$$-\frac{(x+1)^2}{3} + \frac{(y+0)^2}{4} = 1$$

$$\frac{(y+0)^2}{4} - \frac{(x+1)^2}{3} = 1$$

center $(-1, 0)$

$a^2 = 4$

$b^2 = 3$

Example 3 Given the equation of the hyperbola $\frac{(x-5)^2}{25} - \frac{(y+1)^2}{9} = 1$

$a^2 = 25$ $b^2 = 9$
$a = 5$ $b = 3$

$a^2 + b^2 = c^2$
$25 + 9 = c^2$
$\sqrt{34} = c$

a. State the direction and the equation of the transverse axis.

left/right $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

b. Determine the center. $(\overset{h}{5}, \overset{k}{-1})$

c. Determine the foci. $(5 \pm \sqrt{34}, -1)$

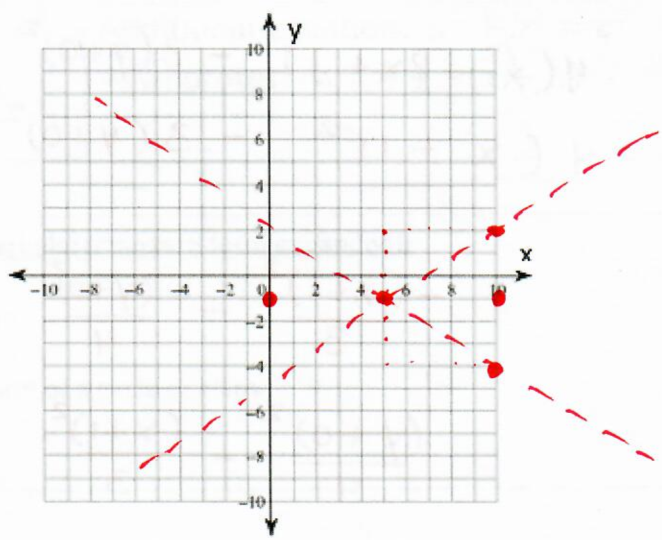
d. Determine the vertices. $(5 \pm 5, -1)$

$(0, -1)$ and $(10, -1)$

e. Determine the equation of the asymptotes.

$y = \pm \frac{b}{a}$ $y = \pm \frac{3}{5}$ $y = -1 \pm \frac{3}{5}(x-5)$

f. Sketch a graph of the hyperbola.



Example 4 Given the equation of the hyperbola $\frac{(y+4)^2}{25} - \frac{(x-2)^2}{9} = 1$

$a^2 = 25$ $\quad$ $b^2 = 9$ $\quad$ $c^2 = a^2 + b^2$

$a = 5$ $\quad$ $b = 3$ $\quad$ $c^2 = 25 + 9$

$c^2 = 34$

$c = \pm\sqrt{34}$

a. State the direction and the equation of the transverse axis.

vertical

$x = 2$

b. Determine the center. $(h, k) = (2, -4)$

c. Determine the foci. $(h, k \pm c) = (2, -4 \pm \sqrt{34})$

d. Determine the vertices. $(h, k \pm a) = (2, -4 \pm 5)$ $(2, 1)$, $(2, -9)$

e. Determine the equation of the asymptotes.

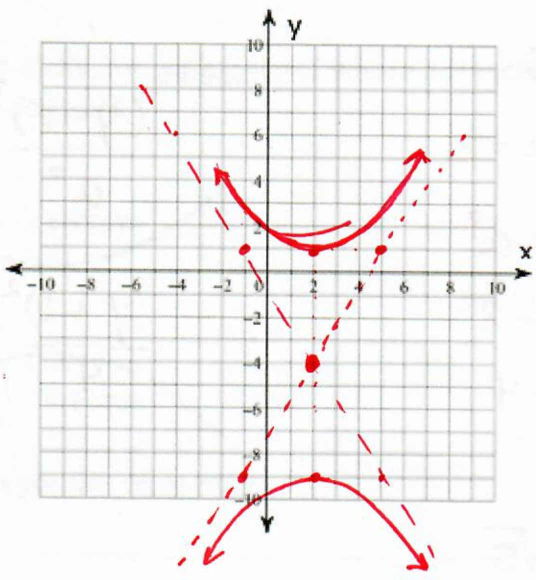
$y - k = \pm \frac{a}{b}(x - h)$

$y - (-4) = \pm \frac{5}{3}(x - 2)$

$y + 4 = \pm \frac{5}{3}(x - 2)$

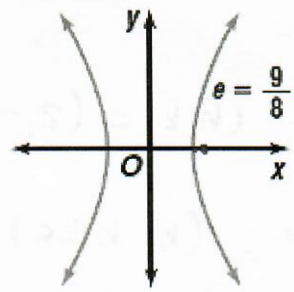
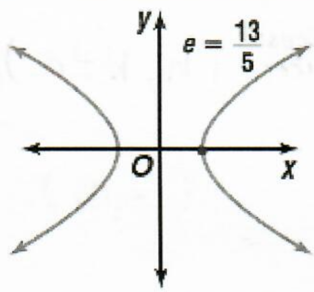
f. Sketch a graph of the hyperbola.

center $(2, -4)$



DEFINITION – ECCENTRICITY

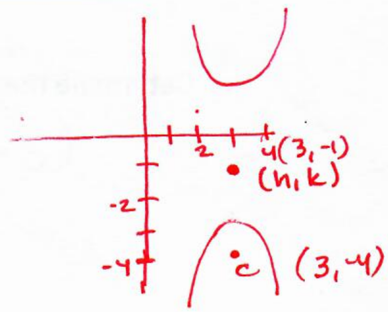
The eccentricity formula, $e = \frac{c}{a}$, is the same as that for an ellipse.

Value of e	Graph
close to 1	
not close to 1	

Example 5

h k

Write an equation of the hyperbola with center at (3, -1), focus at (3, -4), and eccentricity $\frac{3}{2}$.



$$e = \frac{c}{a}$$

$$e = \frac{3}{2}$$

$$c = 3; \quad a = 2$$

$$b^2 = c^2 - a^2$$

$$b^2 = 3^2 - 2^2$$

$$b^2 = 9 - 4$$

$$b^2 = 5$$

$$b = \pm\sqrt{5}$$

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

$$\frac{(y+1)^2}{4} - \frac{(x-3)^2}{5} = 1$$